

# Forces Exerted by Waves on a Pipeline At or Near the Ocean Bottom

by George L. Bowie

# TECHNICAL PAPER NO. 77-11 OCTOBER 1977



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Prepared for

U.S. ARMY, CORPS OF ENGINEERS
COASTAL ENGINEERING
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SECURITY CLASSIFICATION OF THIS PAGE (When Date Bot READ INSTRUCTIONS BEFORE COMPLETING FORM REPORT DOCUMENTATION PAGE 2. GOVT ACCESSION RECIPIENT'S CATALOS NUMBER TP-77-11 Technical Paper FORCES EXERTED BY WAVES ON A PIPELINE AT OR NEAR THE OCEAN POTTOM. Technical Report HEL-9-24 DACW72-74-C-0804 George L. PERFORMING ORGANIZATION NAME AND ADDRESS University of California Hydraulic Engineering Laboratory Berkeley, California 94720 11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Army Coastal Engineering Research Center (CEREN-DE) 177 Kingman Building, Fort Belvoir, Virginia SECURITY CLASS, (of this report) UNCLASSIFIED 15. DECLASSIFICATION/DOWNGRADING Approved for public release; distribution unlimited. 17. DISTRIBUTION STATEMENT (of the obstract entered in Block 20, if different free Report) 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary and identify by block n Submarine pipeline Two- and three-dimensional experiments Wave force analysis Wave-induced lift forces STRACT (Conficue on reverse side if recessary and identify by block number) The wave-induced forces on a submarine pipeline near the ocean floor consist of several components-inertial forces, drag forces, lift forces, and under some conditions, eddy-induced forces. For a pipeline touching the bottom, or at a small clearance above the bottom, the lift force is the predominant force in the vertical direction. This force is generally expressed as  $F_L \approx 1/2$   $C_L \rho$  A  $u^2$  and is added as a lift term to the Morison equation. FSUBL CSABL RHO (Continued)

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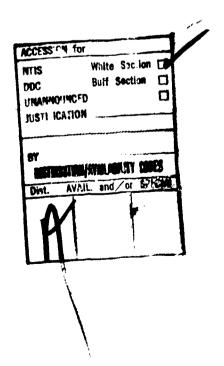
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The experimental results of this investigation, however, show that this steady-flow lift model is inadequate for wave-induced oscillatory flows. For pipelines at small clearances above the bottom, viscous effects near the bottom clearance constriction may result in lift forces acting in both the upward and downward directions during different part of the wave cycle. In addition, the maximum positive and negative lift forces may not correspond to the positions of maximum horizontal velocities in the wave cycle.

A modified lift force model of the form,  $F_L = 1/2 C_L \rho$  A  $u_{max}^2$  [cos² (0-\$\phi) k], is proposed where the parameters,  $C_L$  \$\phi\$, and k, may vary accordingly to allow adequate description of all characteristics of the lift force phenomenon. Quantitative relationships between these unknown lift force parameters and various dimensionless parameters defining the wave and pipe conditions were found. These relationships exhibited good correlation for all wave conditions, bottom clearances, pipe diameters, and orientation angles.

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#### PREFACE

This report is published to provide coastal engineers with an analysis of wave-induced forces on a submarine pipeline near the ccean floor. The work was carried out under the structural design program of the U.S. Army Coastal Engineering Research Center (CERC).

The report was prepared by George L. Bowie, Research Assistant, University of California, Berkeley, under CERC Contract No. DACW72-74-C-0004.

The author acknowledges the help and advice of Professor R.L. Wiegel, Professor J.W. Johnson, and Dr. J.D. Cumming, throughout different stages of the project. Many thanks also go to Dr. M.F. Al-Kazily, J. Allison, and L. Magel for their help in designing and setting up the instrumentation for the experiments, and to W. Krogmoe, E. Parscale, and W. Matthew for their skillful assistance in building the models.

Dr. J.R. Weggel was the CERC contract monitor, under the general supervision of G.M. Watts, Chief, Engineering Development Division.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.

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Colonel, Corps of Engineers

Commander and Director

	Page
	CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) 9
	SYMBOLS AND DEFINITIONS
I	WAVE FORCE ANALYSIS
	1. Wave Force Components on Pipelines
	Near the Bottom
	2. Wave-Induced Lift Forces
	3. Model for Wave-Induced Lift Forces
	4. Extension of Model to Higher Order Theories 32
11	EXPERIMENTAL INVESTIGATION
	I. Experimental Equipment
	2. Procedure for Two-Dimensional Experiments 45
	3. Procedure for Three-Dimensional Experiments 51
	4. Data Reduction
III	RESULTS AND DISCUSSION
	1. Resultant Force Through Wave Cycle
	2. Orientation Angle Considerations
	3. Interrelationships Between $C_1$ , $\phi$ , and $k$
	4. Relationships Between φ and k and Parameters
	Defining the Wave and Pipeline Conditions 89
	<ol><li>Relationships Between ¢ (clear/Dia) and k (clear/Dia)</li></ol>
	and Parameters Defining the Wave and Pipeline
	Conditions
	6. Relationships Between the wefficients of Lift and
	Parameters Defining the Wave and Pipeline Conditions 109
	7. Relationships Between the Lift Forces and Parameters
	Defining the Wave and Pipeline Conditions 109
	8. Relationships Involving the Vertical Coefficients of Mass and Drag and the Vertical Inertial and
	Drag Forces
	9. Relationships Between the Horizontal Coefficient of
	Mass and Parameters Describing the Wave and
	Pipeline Conditions
	10. Relationships Involving the Horizontal Coefficient
	of Drag
	11. Example Problems
IV	CONCLUSIONS
v	RECOMMENDATIONS FOR FURTHER RESEARCH
•	
	ITTED ATTIDE CITED

APPI	ENDIX	Page
	A LEAST SQUARES ANALYSIS OF EXPERIMENTAL DATA	125
	B COMPUTER PROGRAM FOR VERTICAL LEAST SQUARES ANALYSIS (TWO-DIMENSIONAL DATA)	140
•	C COMPUTER PROGRAM FOR VERTICAL LEAST SQUARES ANALYSIS (THREE-DIMENSIONAL DATA)	144
	D COMPUTER PROGRAM FOR HORIZONTAL LEAST SQUARES ANALYSIS (TWO-DIMENSIONAL DATA)	149
	E TABULATED VERTICAL FORCE DATA FROM TWO-DIMENSIONAL EXPERIMENTS	153
	F TABULATED VERTICAL FORCE DATA FROM THREE-DIMENSIONAL EXPERIMENTS	158
	G TABULATED HORIZONTAL FORCE DATA FROM TWO-DIMENSIONAL EXPERIMENTS	176
	TABLE	
	Estimated accuracy of experimental measurements	59
	FIGURES	
1	Change in lift with increasing velocity	21
2	Change in lift force with passing wave crest	23
3	Lift force phenomenon	25
4	Change in lift force record for increasing bottom clearance	27
5	Definition sketch	29
6	Definition of lift force parameters	31
7	Comparison of linear and Stokes' third-order theories: Simultaneous shift of lift force record as $\phi$ increases from 0° to 90° and k increases from 0 to 1 with increasing bottom clearance	
8	Comparison of lift force extreme cases for linear and Stokes' third-order theories	
9	Force meter and support	38

	FIGURESContinued	Page
10	Four-inch cylinder mounted in the flume	. 39
11	Test section (force meter)	. 40
12	Test section mounted in position	41
13	Test section and force transducer	. 12
14	Schematic of pipeline model	. 43
15	Pipeline model	. 44
16	Brush recording instruments	. 46
17	Digitizer and recording instruments	. 47
18	Calibration method for two-dimensional experiments	. 48
19	Experimental arrangement for two-dimensional tests	. 49
20	Calibration method for three-dimensional experiments	. 52
21	Experimental arrangement for three-dimensional tests	54
22	Definition sketch for three-dimensional experiments	. 55
23	Example of data record	57
24	Example of computer output for vertical least squares analysis	. 60
25	Example of computer output for herizontal least squares analysis	. 61
26	Resultant force through wave cycle for 0.001-foot clearance, 1.85-second period, and 0.24-foot height	. 62
27	Resultant force through wave cycle for 1/16-inch clearance, 1.86-second period, and 0.24 foot height	. 63
28	Resultant force through wave cycle for 1/8-inch clearance, 1.85-second period, and 0.25-foot height	. 64
29	Resultant force through wave cycle for 3/16-inch clearance, 1.85-second period, and 0.25-foot height	. 65

	FIGURES Continued	Page
30	Resultant force through wave cycle for 1/4-inch clearance, 1.86-second period, and 0.25-foot height	66
31	Resultant force through wave cycle for 1-inch clearance, 1.86-second period, and 0.24-foot height	67
<b>3</b> 2	Resultant force through wave cycle for 2-inch clearance, 1.86-second period, and 0.25 foot height	68
33	Change in resultant force with increasing clearance	69
34	Resultant force through wave cycle for 1-inch clearance, 1.23-second period, and 0.3-foot height	71
35	Resultant force through wave cycle for 0.001-foot clearance, 0.95-second period, and 0.24-foot height	72
36	Resultant force through wave cycle for 1/16-inch clearance, 0.95-second period, and 0.24-foot height	73
37	Resultant force through wave cycle for 2-inch clearance, 0.96-second period, and 0.25-foot height	74
38	Alternative approaches for handling pipeline orientation angles	76
39	φ versus k	78
<b>4</b> C	CL versus k	79
41	C <sub>L</sub> versus φ	80
42	Effective positive coefficient of lift versus $k$	82
43	Effective positive coefficient of lift versus $\phi$	83
44	Effective negative coefficient of lift versus k	84
45	Effective negative coefficient of lift versus $\phi$	85
46	$C_L(1-k)$ or $C_L(k)$ versus $k$	83
47	φ versus (clear/u <sub>max</sub> T) for 4-inch diameter	91
48	φ versus (clear/u <sub>max</sub> T) for 3-inch diameter	. 92
19	φ versus (clear/u <sub>max</sub> T) for 2-inch diameter	93

	FIGURESContinued Pa	Įе
30	k versus (clear/u <sub>max</sub> T) for 4-inch diameter	94
51	k versus (clear/u <sub>max</sub> T) for 3-inch diameter	95
52	k versus (clear/umaxT) for 2-inch diameter	96
53	k versus (u <sub>max</sub> T/Dia)	98
54	φ versus (u <sub>max</sub> T/Dia)	99
<b>5</b> 5	k versus (umaxclear/v) for 4-inch diameter	)0
56	φ versus (u <sub>max</sub> clear/v) for 4-inch diameter	)1
57	k versus (clear/u <sub>max</sub> T) (Dia/u <sub>max</sub> T)	)2
58	φ versus (clear/u <sub>max</sub> T) (Dia/u <sub>max</sub> T)	)3
59	k(clear/Dia) versus (clear/u <sub>max</sub> T;(clear/Dia)	)5
60	φ(clear/Dia) versus (clear/u <sub>max</sub> T)(clear/Dia)	)6
61	k(clear/Dia) versus √Dia/u <sub>max</sub> T (clear/u <sub>max</sub> T)(clear/Dia) 10	17
62	φ(clear/Dia) versus √Dia/u <sub>max</sub> T (clear/u <sub>max</sub> T)(clear/Dia) 10	8(
63	Maximum lift force (positive or negative) versus the Reynolds number	0
64	Comparison of the horizontal $C_M$ with potential flow theory for a flow with constant acceleration	2
65	Horizontal $C_M$ versus (clear/ $u_{max}T$ )	4

# CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.8532	kilometers per hour
acres	0.4047	hectares
foct-pounds	1.3558	newton meters
millibars	$1.0197 \times 10^{-3}$	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
Po allow	0.4536	ki lograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.1745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvinsl

To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: C = (5/9) (F -32).

To obtain Kelvin (K) readings, use formula: K = (5/9) (F - 32) + 273.15.

#### SYMBOLS AND DEFINITIONS

projected area of pipe section **ANG** orientation angle with respect to wave crests coefficient of drag  $c_n$  $C_{L}$ coefficient of lift C¦ coefficient of transverse force due to eddy shedding clear bottom clearance coefficient of mass  $C_{\mathbf{M}}$ d stillwater depth Dia pipe diameter total wave-induced force FD drag force  $(F_n)_h$ horizontal component of drag force  $(F_{D})_{v}$ vertical component of drag force Fh horizontal component of total wave force  $F_{h}(\theta_{i})$ calculated horizontal force at position  $\boldsymbol{\theta}_i$  in wave cycle inertial force F  $(F_I)_h$ horizontal component of inertial force  $(F_I)_{V}$ vertical component of inertial force F<sub>L</sub> lift force F' transverse "lift" force due to eddy shedding  $F_{oh}(\theta_i)$ observed horizontal force at position  $\theta_i$  in wave cycle  $F_{ov}(\theta_i)$ observed vertical force at position  $\theta_i$  in wave cycle Fv vertical component of total wave force  $F_{\mathbf{v}}(\theta_{i})$ calculated vertical force at position  $\theta_i$  in wave cycle

# SYMBOLS AND DEFIN ITONS--Continued

	Other and and the second designation
Н	wave height
k	negative fraction of lift force cycle
L	wavelength
3.	wave period
t	time since last wave crest passed over center of pipe section
u	horizontal component of water particle velocity if pipeline was absent
umax	maximum horizontal water particle velocity if pipeline was absent
ν	volume of fluid displaced by pipe section
ν	vertical component of water particle velocity if pipeline was absent
v <sub>max</sub>	maximum vertical water particle velocity if pipeline was absent
z	vertical distance of center of pipe section above Dottom
∂u/bt	horizontal component of water particle acceleration if pipeline was absent
∂v/∂t	vertical component of water particle acceleration if pipeline was absent
θ	$2\pi t/T$ = position of wave cycle over center of pipe section with respect to time
ν	kinematic viscosity of fluid
ρ	mass density of fluid
ф	phase shift of maximum lift forces with respect to wave cycle
	Computer Programs
Input Para	meters:

#### Input Parameters:

ANGLE orientation angle

C calibration factor for manual digitizer

#### SYMBOLS AND DEFINITIONS--Continued

CFD downward force calibration factor

CFU upward force calibration factor

CL bottom clearance

DF downward force calibration factor

DIA pipe diameter

DN negative wave (trough) calibration factor

FI(I) wave force readings

FO zero point of wave force record

HI(I) wave surface readings

N number of wave force readings

T wave period

UF upward force calibration factor

UP positive wave (crest) calibration factor

WO zero point of wave record

XC length of pipe test section

XF amplification factor for force record

XW amplification factor for wave record

YI(I) wave surface readings

Program Variables:

ANG orientation angle (in radians)

ANGLE orientation angle (in degrees)

CDH horizontal coefficient of drag

CDV vertical coefficient of drag

CL bottom clearance

#### SYMBOLS AND DEFINITIONS -- Continued

CLV coefficient of lift (calculated using horizontal velocity in direction of wave advance and projected area in plane parallel to the pipeline axis)

CLVA coefficient of lift (calculated using horizontal velocity in direction of wave advance and projected area in plane normal to the direction of wave advance)

CLVU coefficient of lift (calculated using the component of the horizontal velocity in the direction perpendicular to the pipeline axis and the projected area in the plane parallel to the pipeline axis)

CMH horizontal coefficient of mass

CMV vertical coefficient of mass

P stillwater depth

DiA pipe diameter

FDH  $1/2 \rho A u_{max}^2$ 

FDV  $1/2 PA v_{max}^2$ 

FH(1) calculated horizontal wave force

FI(I) measured wave force readings (in grams for two-dimensional data; in 10-grams for three-dimensional data)

FLV  $1/2 \rho A v_{max}^2$ 

FMAX maximum positive wave force (measured)

FMH  $\rho V (\partial u/\partial t)_{max}$ 

FMIN maximum negative wave force (measured)

FMV  $\rho V (\partial v/\partial t)_{max}$ 

FP(1) measured wave force readings (in pounds)

FV(I) calculated vertical wave force

H wave height

HI(I) wave surface profile readings

#### SYMBOLS AND DEFINITIONS--Continued

PHI phase-shift parameter  $\phi$  of modified lift force equation PΙ R mass density of water difference between measured wave force and calculated wave RES(I) force SF wave force averaged through wave cycle ·T wave period U maximum horizontal water particle velocity XC length of pipe section XK parameter K of modified lift force equation XLwavelength ZV vertical distance from bottom to center of pipe section Tabulated Experimental Data **ANG** orientation angle of pipeline with respect to wave crests CDH horizontal coefficient of drag CDV vertical coefficient of drag CLER bottom clearance CLV coefficient of lift (calculated using horizontal velocity in direction of wave advance and projected area in plane parallel to the pipeline axis) CLVA coefficient of lift (calculated using horizontal velocity in direction of wave advance and projected area in the plane normal to the direction of wave advance) CLVU coefficient of lift (calculated using the component of the horizontal velocity in the direction perpendicular to the pipeline axis and the projected area in the plane parallel to the pipeline axis)

horizontal coefficient of mass

CMH

CMV vertical coefficient of mass

DIA pipe diameter

FAVG average horizontal force (averaged over complete wave cycle)

H wave height

K. parameter k of modified lift force equation

L wavelength

PHT phase shift parameter  $\phi$  of modified lift force equation

T wave period

UMAX Maximum horizontal component of water particle velocity at

center of pipe section if absent

# FORCES EXERTED BY WAVES ON A PIPELINE AT OR NEAR THE OCEAN BOTTOM

by George L. Bowie

#### WAVE FORCE ANALYSIS

#### 1. Wave Force Components on Pipelines Near the Bottom.

The most common method of analyzing wave forces on pipelines is the application of the Morison equation (Morison, et al., 1950). Using this approach, the total wave-induced force on a pipeline can be broken into several components, depending on whether the components are due to the water particle velocities or accelerations. These force components can, in turn, be separated into horizontal and vertical components by using the horizontal and vertical components of the water particle velocities and accelerations in their respective force equations. Where there is no lift effect and no eddy-induced forces, the vertical component,  $F_{\rm V}$ , of the total wave force is

$$F_{V} = (F_{I})_{V} + (F_{D})_{V} = C_{M} \rho V \frac{\partial v}{\partial t} + 1/2 C_{D} \rho A v |v|$$
 (1)

and the horizontal component, Fh, is

$$F_h = (F_I)_h + (F_D)_h = C_M \rho V \frac{\partial u}{\partial t} + 1/2 C_D \rho A u |u|,$$
 (2)

where

 $(F_{1})_{v}$  = vertical component of inertial force

 $(F_I)_h$  = horizontal component of inertial force

 $(F_D)_v$  = vertical component of drag force

 $(F_D)_h$  = horizontal component of drag force

v = vertical component of water particle velocity if pipeline was absent

u = horizontal component of the water particle velocity if pipeline was absent

A = projected area of pipe section

V = volume of fluid displaced by pipe section

 $\rho$  = mass density of fluid

 $C_{M}$  = coefficient of mass

 $C_D$  = coefficient of drag

For a pipeline located near the ocean bottom, the water particle orbits are flattened parallel to the boundary. Assuming a horizontal bottom, the vertical motions of the water particles are small in comparison to the horizontal motions, especially in shallow-water depths relative to the wavelength. As a result, the vertical components of the water particle velocities and accelerations are much smaller than the horizontal components, and correspondingly the vertical components of the drag and inertial forces will be smaller than the analogous horizontal forces.

Since the water particles at the bottom are effectively oscillating in a horizontal plane, the vertical excursions of the water particles will generally be less than the diameter of a submarine pipeline lying on or near the bottom. Therefore, the vertical drag forces are generally insignificant, and could probably be neglected from the vertical wave force equation.

Pipelines near the bottom are subject to vertical lift forces. These forces are the result of the asymmetric distortion of the flow field due to the proximity of the bottom boundary, which induces differences in the horizontal flow velocities and corresponding pressure distribution over the top and bottom of the pipeline. Since the water particle velocities near the bottom are at a maximum in the horizontal plane, the lift forces induced by these horizontal motions will generally be the predominant force acting in the vertical direction.

Transverse "lift" forces due to eddy shedding may also be an important component of the vertical wave force, since these forces are also due to the horizontal water particle velocities and excursions which are maximum in the horizontal direction. Certain values of the Keulegan-Carpenter parameter and Reynolds number must be attained for the eddy release phenomenon to occur. The proximity of the bottom boundary will probably have some effect on the formation and release of the eddies, both because it is a solid boundary, and because it affects the orbital motions of the water particles induced by the wave action.

Although the eddy-induced component of the vertical wave force may be significant when compared to the relatively small vertical drag and inertial forces, the experimental results of this investigation show that the eddy-induced lift forces are much smaller than the "Bernoullitype" lift forces for pipelines located near the bottom. At large clearances above the bottom where the Bernoullitype lift effect becomes negligible, the transverse lift forces due to eddy shedding may become a significant component of the total vertical force. At the same time, as the pipeline is raised farther from the bottom boundary, the vertical inertial and drag forces also become more significant.

The vertical component of the total wave-induced force acting on a pipeline near the ocean bottom thus consists of four components—the lift force, the inertial force, the drag force, and the transverse lift force due to eddy shedding. Using the Morison approach, the total vertical wave force is expressed as the sum of these components:

$$F_{v} = F_{L} + (F_{I})_{v} + (F_{D})_{v} + F_{L}'$$
 (3)

where  $F_L$  is the lift force and  $F_L$ ' is the transverse lift force due to eddy shedding.

#### 2. Wave-Induced Lift Forces.

Consider a pipeline in contact with a horizontal rigid, impervious bottom. Water cannot flow between the pipe and the bottom boundary, so the flow must be diverted over the top of the pipe. The asymmetrical distortion of the flow field results in maximum velocities over the top of the pipe section and minimum velocities over the bottom, with zero velocities at the stagnation point on the upstream side of the pipe bottom at the point of contact with the sea floor. Correspondingly, the associated pressure distribution will induce an upward lift force for any velocity field acting on the pipeline. The stagnation pressure at the bottom of the pipe section will increase with increasing velocity, while simultaneously the pressure distribution over the top of the pipeline will decrease with the increased velocities of the flow diverted over the top of the pipe section. The wave-induced lift forces will thus act in the upward direction throughout the wave cycle, increasing with the horizontal water particle velocities to maximum magnitudes under the crests and troughs of the passing waves, and diminishing to zero at the points of horizontal flow reversal.

In contrast, a pipeline located at a small clearance above the bottom boundary is subject to a more complex type of lift phenomenon. At the phase in the wave cycle where the horizontal component of the water particle velocity reverses direction, the horizontal velocity over the pipeline is approximately zero. As the wave crest or trough begins to approach the pipeline, the wave-induced horizontal velocities are initially low, inducing unrestricted flow at low velocities over both the top and bottom of the pipeline. However, the water flows

faster through the bottom clearance constriction than over the top of the pipeline, so the corresponding differences in the pressure distribution exert a downward (negative lift) force toward the bottom boundary (Fig. 1, a).

At first, the negative lift force will increase with the increasing horizontal water particle velocities of the approaching wave, since the flow velocities increase at a faster rate through the bottom clearance constriction than over the top of the pipeline, thus producing larger differences in the corresponding pressure distributions over the top and bottom of the pipe section (Fig. 1, b).

This continues until viscous effects begin to restrict the flow through the narrow bottom clearance. For a given small clearance and a given amount of energy in the horizontal water particle velocities approaching the pipeline, the velocities and flow rates of a viscous fluid through the bottom clearance constriction can attain only certain maximum values. Thus, a "choking" effect is exerted on the restricted flow through the small bottom clearance, and the remainder of the wave-induced flow is forced to flow over the top of the pipe section. Correspondingly, the stagnation point will shift downward, increasing the pressure on the lower upstream side of the pipeline. The larger the proportion of the flow diverted over the top of the pipe, the lower the stagnation point.

At the same time, the increasing velocities associated with the approaching wave crest cause the restricted flow through the bottom clearance to form a turbulent jet with the generation of eddies behind the jet. The generation of increased turbulence and eddies results in an energy loss in the water flowing through the bottom constriction, decreasing the velocities under the pipe section behind the jet.

The above effects associated with the choking phenomenon limit the maximum flow velocities and minimum pressures under the bottom side of the pipe section. In contrast, the unrestricted flow velocities over the top of the pipeline increase freely with the increasing horizontal velocities of the advancing wave. The increased part of the approaching flow that is diverted over the top of the pipe section due to the shift in stagnation point produces a further increase in the flow velocities over the top. Correspondingly, the pressure distribution over the top side of the pipeline decreases at a faster rate than the associated pressures along the bottom side, so the negative lift force gradually decreases and eventually becomes positive (Fig. 1, c, d, and e).

At this stage, the upward lift force becomes larger as the horizontal velocities acting on the pipeline increase further with the advancing wave crest or trough (Fig. 1, f).

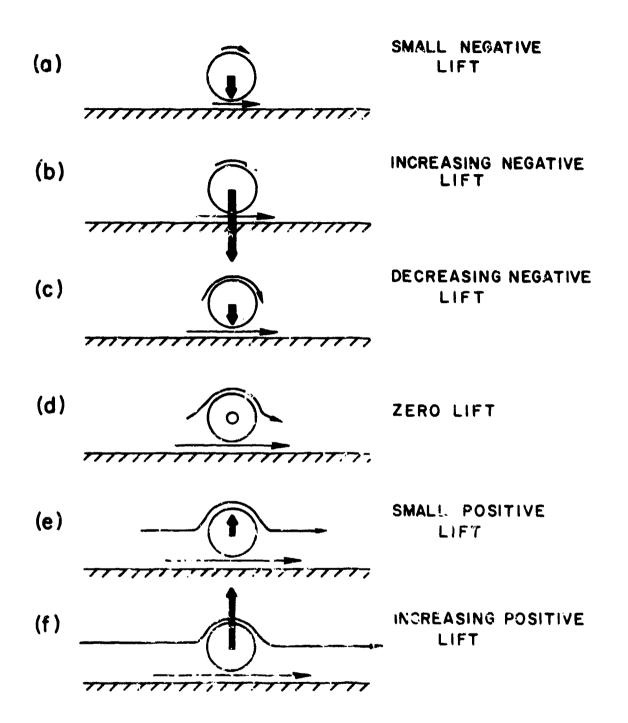


Figure 1. Change in lift with increasing velocity.

As the wave crest or trough passes, this series of steps in the lift force phenomenon is reversed. The horizontal velocities approaching the pipe section begin to decrease, resulting in a decrease in the positive lift force exerted on the pipeling. As the velocities decrease further with the passing wave, the flow under the pipe section begins to become less restricted. The choking effect thus decreases, and the turbulence and eddies near the bottom clearance gradually diminish. As the flow under the pipe section ceases to be restricted, less of the horizontal flow approaching the pipeline is forced to flow over the top of the pipe, so the stagnation point will accordingly shift upward, closer to the center of the pipe section.

The flow velocities decrease simultaneously over the top and bottom of the pipeline as the wave passes, but the rate of decrease is faster over the top of the pipe than in the vicinity of the bottom constriction. The positive lift force decreases until eventually, the flow velocities, location of the stagnation point, and associated pressure distribution are such that the pressure integrated over the pipe section again results in a negative lift force. The downward lift force then increases as the flow through the bottom clearance becomes less restricted with the decreasing velocities of the passing wave.

This lift phenomenon, as shown in Figure 2 for a passing wave crest, is repeated twice during each wave cycle as the direction of the wave-induced horizontal velocities reverses under the crests and troughs of the passing waves.

In reality, the horizontal flow reversal occurs almost instantaneously, so the negative lift force does not return to zero at the point of zero velocity when the flow reverses through the bottom clearance constriction. The instant of zero velocity occurs only at the center of the pipe cross section (the reference point). Since the pipeline has a finite diameter, the wave-induced flow acting on the pipe section at any instant includes the sum of the flow conditions induced by the part of the wave covering the entire diameter of the pipeline. So instead of going to zero with the passing wave crest, and then increasing initially with the approaching trough, the lift force remains negative during the period of minimal velocities as the flow reverses under the pipe section.

In a similar manner, the lift force does not become positive as soon as the choking effect occurs in the bottom clearance constriction. The development of the choking phenomenon involves the formation of a turbulent jet through the constriction, and a downward shift in the stagnation point as more water is diverted over the top of the pipe with increasing restriction of the flow through the clearance. The corresponding changes in the velocities, flow pattern, and associated pressure distribution over the top and bottom of the pipe section produce the transition from negative to positive lift. This process requires some small but finite amount of time. Conversely, the reversal

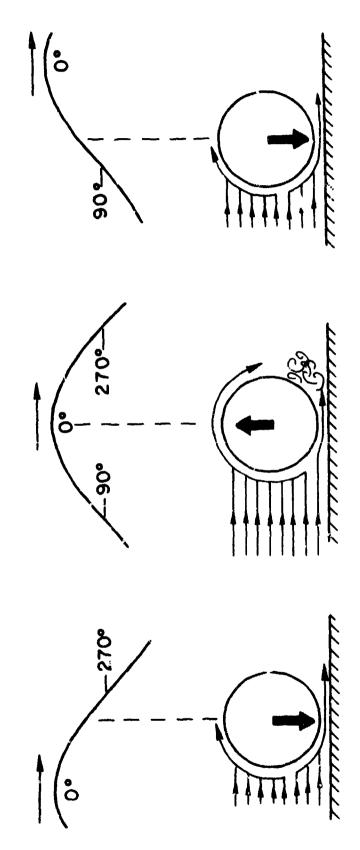


Figure 2. Change in lift force with passing wave crest.

of these processes with the decreasing velocities of the passing wave crest also involves a small but finite amount of time. Thus, there will be a slight timelag in the point of maximum positive lift with reference to the instant of maximum velocity as the wave crest (or trough) passes over the reference point. The smaller the amount of positive lift relative to the amount of negative lift, and the later the positive lift occurs in the wave cycle, the greater the timelag.

An example of the lift force phenomenon over a complete wave cycle for a small bottom clearance is shown in Figure 3.

For a given pipe diameter and wave condition, as the bottom clearance is increased, higher velocities are necessary to produce the choking effect in which the flow becomes restricted through the bottom clearance constriction. Thus, as the bottom clearance is increased, the flow under the pipeline begins to become restricted closer to the approaching wave crest or trough, where the horizontal velocities are at a maximum; this choking effect also diminishes soon after the wave crest or trough has passed. Therefore, as the bottom clearance is increased, the downward lift force occurs during a larger part of the wave cycle.

At the same time, larger clearances permit greater maximum velocities and corresponding lower pressures under the pipe section. Since higher flow rates are possible under the pipe section, less of the wave-induced flow must be diverted over the top of the pipeline. As a result of these changes, the negative lift forces reach a greater magnitude before the choking effect begins, and these maximums are attained later in the wave cycle.

Correspondingly, the upward lift forces occur during a smaller part of the wave cycle, and the maximum magnitude these forces attain decreases with increasing bottom clearance. These maximum values are also reached later in the wave cycle.

If the clearance is increased further, a point is eventually reached at which the clearance is large enough so that the choking effect does not occur. At this stage, the velocities are higher through the bottom clearance constriction than over the top of the pipeline during the entire wave cycle. So the associated pressure distribution results in a negative lift force throughout the wave cycle, with maximum downward forces occurring under the crests and troughs of the passing waves. The negative lift diminishes to zero at the points of horizontal flow reversal.

As the bottom clearance is increased further, the downward lift effect is gradually reduced. The phase of the force cycle relative to the wave cycle remains the same, but the magnitude decreases. Eventually, a point is reached where the bottom clearance no longer acts as a constriction to the wave-induced flow. The flow pattern becomes approximately symmetrical, and the increased velocities of the horizontal flow diverted over the top and bottom of the pipeline, along with the corresponding

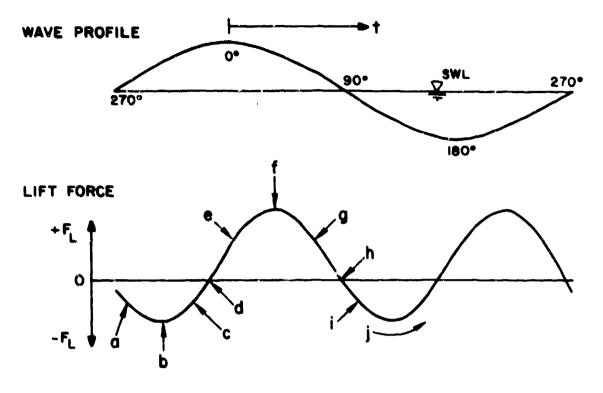


Figure 3. Lift force phenomenon.

- (a) Unrestricted flow through the bottom clearance at low velocities results in downward lift force.
- (b) Unrestricted flow through the bottom clearance at higher velocities increases the negative lift.
- (c) Choking effect begins, so downward lift force decreases.
- (d) Velocities increase and pressures decrease at a faster rate over the top of the pipe section than in the restricted flow through the bottom clearance, so the lift force becomes positive.
- (e) Upward lift force increases with increasing velocities.
- (f) Positive lift reaches a maximum.
- (g) Positive lift force decreases and choking effect diminishes with decreasing velocities of the passing wave crest.
- (h) Lift force again becomes negative as the flow through the bottom clearance becomes less restricted.
- (i) Unrestricted flow through bottom clearance at low velocities results in downward lift force.
- (j) Lift force cycle is repeated as the flow reverses with the approaching wave trough.

pressure distribution, become approximately equal over both sides of the pipe section. At this point, the lift effect is no longer present, and the lift force term may be neglected in calculating the wave-induced forces acting on the pipeline.

The transition in the lift force cycle with increasing bottom clearance is shown in Figure 4.

#### 3. Model for Wave-Induced Lift Forces.

The traditional lift force equation, derived for unidirectional steady-flow situations, is expressed as  $F_L = 1/2$   $C_L$   $\rho$  A  $u^2$ , where  $C_L$  is the coefficient of lift. This equation has been applied to wave-induced lift forces, using the horizontal component of the oscillating water particle velocity, u, in the relationship. The lift force expressed in this way assumes that the force acts in one direction only (either upward or downward) throughout the entire wave cycle.

A pipeline located on the ocean floor with no clearance will experience an upward lift force throughout the entire wave cycle, increasing with the horizontal velocities to reach maximum values under the crests and troughs of the passing waves, and diminishing to zero as the horizontal velocities go to zero at the point of flow reversal. This phenomena is described adequately by the above lift force equation with a positive coefficient of lift C<sub>L</sub>.

A pipeline located at a large enough clearance above the bottom so that the choking effect does not occur will experience a downward lift force throughout the wave cycle, since the flow is always faster through the bottom constriction than over the top of the pipeline. Again, this negative lift force increases with the horizontal water particle velocities, reaching maximum magnitudes under the crests and troughs of the passing waves, and decreasing to zero as the flow reverses. This phenomenon is also suitably expressed by the traditional lift force equation, but using a negative coefficient of lift.

These two situati represent the extreme cases bounding the lift force phenomena. However, the choking phenomenon will occur at any clearance between these two limits, and the traditional lift force equation cannot be used to accurately describe the forces exerted on a pipeline. This equation must be replaced by a model developed specifically for wave-induced lift forces. The experimental results of this investigation demonstrate that the largest wave-induced lift forces occur at these intermediate clearances, where the choking phenomenon does develop.

Since the lift force phenomenon is repeated twice per wave cycle with the reversal of the horizontal flow pattern, the lift force can be described mathematically by a sinusoidal function of twice the frequency of the waves. In addition, the mathematical expression must allow for description of the following lift force properties:

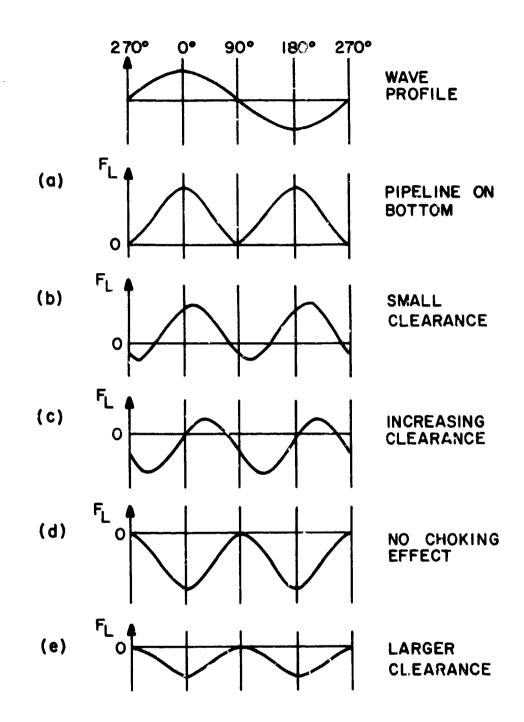


Figure 4. Change in lift force record for increasing bottom clearance.

- (a) The lift force may be positive during part of the wave cycle and negative for the rest of the cycle. The proportion of positive lift to negative lift may range from all positive lift to all negative lift.
- (b) The positions of the maximum values of both the upward and downward lift forces will shift with respect to the position of the wave cycle as the bottom clearance is increased (for a given pipeline and wave condition).
- (c) As the clearance is increased, the maximum value of the upward lift force will decrease, while correspondingly the maximum value of the downward lift force will increase.
- (d) When the bottom clearance is increased to a point at which the lift effect is downward throughout the entire wave cycle, further increases in clearance will result in decreases in the maximum magnitude of the downward lift force, but without a shift in the position of the maximum lift force with respect to the position of the wave cycle over the pipeline.

A lift force equation of the form,

$$F_L = 1/2 C_L \rho A u_{max}^2 [\cos^2 (\theta - \phi) - k],$$
 (4)

allows an adequate mathematical description of all the above properties of the wave-induced lift force phenomena. This equation fits the experimental data reasonably well over the wide range of conditions tested.

The parameters involved in this modified form of the traditional lift force equation are:

C<sub>L</sub> = coefficient of lift

 $\rho$  = mass density of fluid

A = projected area of pipe section

 $\theta = \frac{2\pi t}{T}$  = position of wave cycle over center of pipe section with respect to time, where T is the wave period and t is the time since the last crest passed over the center of the pipe section (see definition sketch in Fig. 5). The wave crest corresponds to  $\theta = 0^{\circ}$  (0 radians) or  $2\pi t/T = 0$  radians. The wave trough corresponds to  $180^{\circ}$  ( $\pi$  radians) or  $2\pi t/T = \pi$  radians

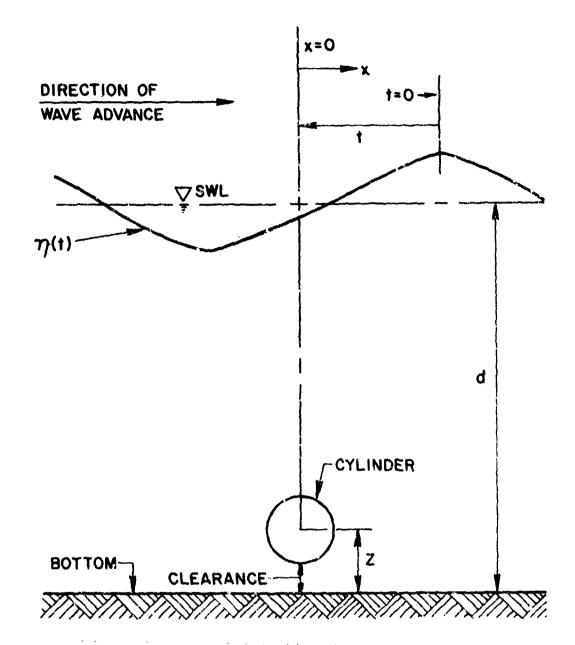


Figure 5. Definition sketch.

- = phase shift of maximum lift forces with respect to wave cycle
- k = negative fraction of lift force cycle

The parameter, k, represents the increase in the magnitude and duration of the negative lift forces acting on a pipeline with increasing bottom clearance, and the corresponding decrease in the magnitude and duration of the positive lift forces. The value of k varies from a minimum of 0 to a maximum value of 1. k = 0 corresponds to the case of a pipeline lying on the bottom with no clearance, in which the lift forces are positive throughout the wave cycle. k increases with increasing bottom clearance to a maximum value of 1, which corresponds to the case of a pipelin. located at a sufficient clearance from the bottom so that the choking phenomenon does not occur, and in which the lift forces are therefore negative throughout the wave cycle.

The phase shift parameter,  $\phi$ , represents the shift in the position of the maximum values of both the positive and negative lift forces with respect to the wave cycle as the bottom clearance increases. The value of  $\phi$  may range from 0° to a maximum value of 90°.  $\phi$  = 0° corresponds to the case of a pipeline located on the ocean floor with no bottom clearance, in which the lift forces are positive throughout the wave cycle with maximum forces occurring under the crests and troughs of the passing waves.  $\phi$  increases with increasing bottom clearance to a maximum value of 90°, corresponding to a pipeline located above the bottom at a sufficient clearance so that the choking effect does not occur; the lift forces are negative throughout the wave cycle with maximums occurring under the crests and troughs of the waves. As defined,  $\phi$  = 0° when k = 0, and  $\phi$  = 90° when k = 1, or vice versa.

The coefficient of lift,  $C_L$ , in this form of the lift force equation will always have a positive value, since negative values of the lift force are accounted for by the value of the parameter, k. The lift force equation is shown graphically in Figure 6.

To apply the lift force equation to a practical design situation, values of  $C_L$ , k, and  $\phi$  must be determined for a given set of pipeline and wave conditions corresponding to the particular case under consideration. Selection of the appropriate values requires quantitative knowledge of the functional relationships between these parameters and the wave conditions, bottom clearance, and pipeline size and configuration. The development of these relationships was the purpose of the experimental part of this investigation.

In a real situation, a pipeline on the ocean floor is often laid over an irregular bottom, supported by the high points in the bottom topography but probably spanning the depressed areas. In this case, the pipeline must be broken into component sections of the same approximate bottom clearance for a separate analysis of each section. The results of the

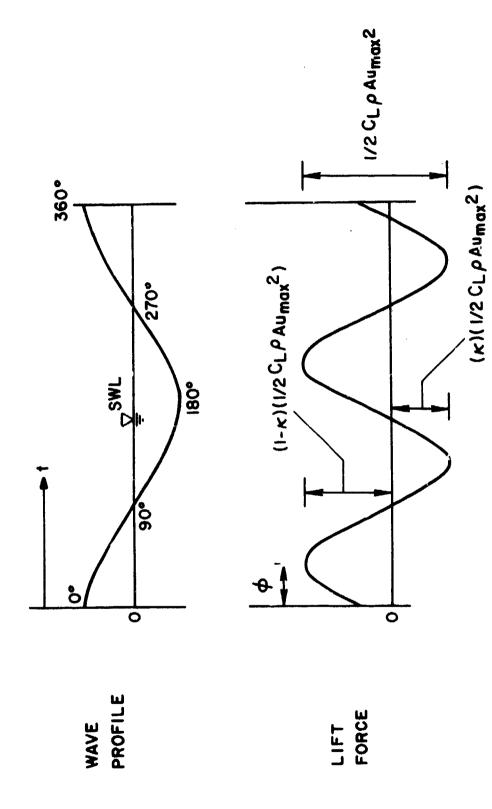


Figure 6. Definition of lift force parameters.

analysis will yield the lift force record (both magnitudes and time history) of each separate component pipe section, which may then be integrated in the appropriate manner to determine the maximum wave-induced stresses exerted on the pipeline at any critical section.

This is important because the maximum lift forces may act upward on a bottom-supported section of a pipeline, while acting downward on the adjacent sections of the pipeline spanning the bottom at a small clearance. Maximum values of both the positive and negative lift forces acting in opposite directions could easily occur at the same point in the wave cycle (under the crests and troughs), thus exerting stresses on the pipeline twice as high as would be calculated considering any pipe section alone, or in using some average clearance for a long section of the pipeline.

#### 4. Extension of Model to Higher Order Theories.

The lift force model (eq. 4) is based on linear theory, assuming the lift force phenomenon is identical as either the wave crest or trough passes over the pipeline. Such a symmetrical expression is not flexible enough to consider slightly different kinematics under the wave crests and troughs, which are expressed in higher order theories. These different kinematics would, in reality, produce slightly different lift forces under the crests and troughs of nonlinear waves.

The lift force model described above was derived as a modification of the traditional lift force equation using linear wave theory to express the horizontal water particle velocities. Using linear wave theory, the traditional lift force equation can be expressed as:

$$F_L = 1/2 C_L \rho A u_{max}^2 cos^2 (\theta).$$
 (5)

This equation was modified to make it a suitable expression for wave-induced lift forces by adding the phase shift parameter,  $\phi$ , to account for maximum lift forces occurring in places other than the crest and trough in the wave cycle, and by adding the parameter, k, to account for positive lift forces during part of the wave cycle and negative forces during the rest of the cycle. This modified equation fits the experimental data very well for all conditions tested in this investigation.

The model was developed after thorough inspection of the experimental data. For a given pipe diameter and wave condition, the force record followed a sinusoidal relationship of twice the frequency of the waves. As the clearance increased, the maximum positive forces gradually diminished while continuously shifting to a maximum of 90° from the wave crest as the forces went to zero (Fig. 4). At the same time, the maximum negative forces slowly grew from a minimum value of zero at a position

90° from the wave crest and increased while continuously shifting positions to reach a maximum negative value at a position 180° from the wave crest (Fig. 7, a).

Since a sinusoidal function of twice the frequency of the wave (sin 20 or cos 20) can be expressed as  $\cos^2\theta$ , using the appropriate trigonometric relationships, and since the lift force is a function of the horizontal velocity squared ( $u_{max}$  cos  $\theta$ )<sup>2</sup>, using linear wave theory, the lift force equation was expressed as  $F_L = 1/2$   $C_L$   $\rho$  A  $u_{max}$  [cos<sup>2</sup> ( $\theta$  -  $\phi$ ) - k].

However, it is the symmetrical properties of this equation and linear wave theory that allow this expression to work so well. When higher order wave theories are applied to this relationship, problems due to nonsymmetry are encountered. This is easily seen by graphically comparing the transition from positive to negative lift forces with increasing bottom clearance with this lift model, using both linear and higher order theories.

The horizontal component of the water particle velocity for both Stokes' third-order waves and linear waves is shown in Figure 8, along with the corresponding lift forces on a pipeline for the two extreme cases of: (a) a pipeline on the bottom with no clearance, and (b) a pipeline with a large enough bottom clearance so that the choking phenomenon does not occur. By gradually shifting the linear theory lift force are for case (a) (no bottom clearance) to the right 90° from the waterest, while simultaneously lowering it so that the forces become negative, the lift force curve for case (b) is obtained (compare Figs. 7 and 8). This same transformation of the wave force record water served with increasing bottom clearance in the experimental data.

ever, if this procedure is repeated with the Stokes third-order lift—ree record, the correct force record for case (b) is not obtained (compare Figs. 7 and 8). In reality, rather than a mere shift of the force record downward and to the right with increasing bottom clearance, a simult neous transformation of the shape of the lift force record would also occur for highly nonlinear waves. This gradual transformation ( he shape occurring simultaneously with the shift would provide a continuous change in the lift force record with increasing clearance between the two limiting cases (a) and (b) (Fig. 8).

However, the lift force phenomenon is not a direct function of the instantaneous water particle velocity acting at the center of the pipe section if the pipeline was absent. Rather, it is a complicated function of the asymmetrical distorted flow pattern and accelerating velocity field acting on the pipeline, which in turn causes the choking phenomenon to occur, with the resulting change in the relative differences in the flow velocities and corresponding pressure distribution over the top and bottom of the pipeline. Boundary layer flow through the bottom constriction, the formation of a turbulent jet and associated eddies, and a cyclic change in the location of the stagnation point with the accelerating velocity field further complicate matters. In addition, the eddies and

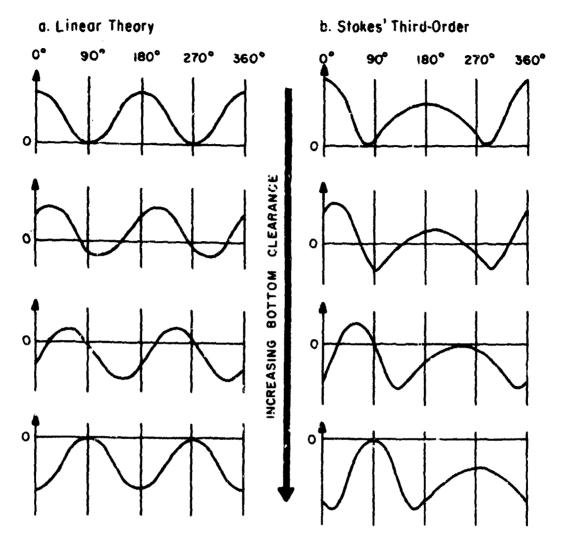
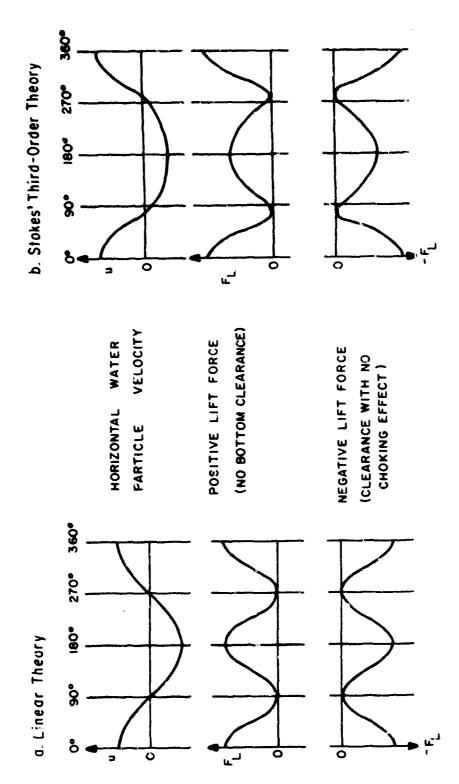


Figure 7. Comparison of linear and Stokes' third-order theories. Simultaneous shift of lift force record as  $\phi$  increases from 0° to 90° and  $\kappa$  increases from 0 to 1 with increasing bottom clearance.



Comparison of lift force extreme cases for linear and Stokes' third-order theories. Figure 8.

increased turbulence generated by the jet may be swept back through the bottom constriction as the flow pattern reverses with the passing waves.

Because of this, development of an accurate mathematical description of the lift force phenomena for nonlinear waves that would cover the complete transformation of the lift force record with increasing bottom clearance, and yet be flexible enough to allow application of any higher order theory, would be a formidable, if not impossible, task. Since the lift force model developed for linear theory seems to fit the experimental data reasonably well, even for waves that were obviously nonlinear, it should provide a useful tool for engineering calculations, even though it may not be flexible enough and theoretically correct to allow the use of higher order wave theories. The value of the maximum horizontal velocity, umax, can be calculated under the wave crest using any higher order wave theory; this value can then be used in the linear lift force model, possibly giving a tetter approximation of the lift forces induced by highly nonlinear waves.

#### II. EXPERIMENTAL INVESTIGATION

### 1. Experimental Equipment.

Model experiments were performed in three different wave tanks. The two-dimensional tests were done in a 1-foot-wide wave channel in the Hydraulic Engineering Laboratory (HEL) at the University of California, Berkeley. The three-dimensional tests were started in the 8-foot-wide Naval Architecture (NA) tow tank, and then continued in the 8-foot-wide HEL wave tank where the majority of the experiments were conducted, both located at the Richmond Field Station of the University of California. The 1-foot wave channel is 100 feet (30.48 meters) long; the 8-foot HEL wave tank and NA tow tank are 180 and 200 feet (54.86 and 60.96 meters) long, respectively. All tests were conducted at approximately the middle of the tanks. A stillwater depth of 2 feet (60.96 centimeters) was used in the two dimensional tests, and a 3-foot (91.44 centimeters) water depth was used in the three-dimensional experiments.

A flapper-type generator is located at one end of each of the HEL wave tanks; the NA tow tank has a piston-type wave generator. The wave period is controlled by varying the speed of the electric motors which drive the wave generators. A cam mechanism with a variable stroke length is connected between the drive motor and the flapper, and the wave height is varied by changing the stroke length. A wave filter, consisting of a series of vertical screens, was placed in front of the wave generator in the 1-foot-wide wave channel to smooth out any irregularities in the generated waves due to reflections from the flapper. A permeable beach was installed at the opposite end of each of the tanks to absorb the wave energy and minimize the wave reflections from that end of the wave tank.

The wave-induced forces on the model pipe section were measured by a wave force meter designed and built by Al-Kazily (1972). A few modifications were made to make the instrument more suitable for this investigation. The same transducer unit was used in all of the experiments, but fittings of different sizes were made to accommodate test cylinders of various diameters.

The force transducer consists of a strain bar mounted between two supports. The model pipe section is mounted to the strain bar in such a way that forces on the pipe induce bending stresses on the strain bar. These forces are measured by four strain gages mounted to the strain bar at sections of maximum strain, with two gages in compression and the other two in tension. The strain gages are wired in a Wheatstone bridge, which is connected to a carrier amplifier which amplifies the output from the strain gages. The signal is then recorded on a strip-chart recorder.

The original strain gages were Bean-type BAB-13-125DD-120S, and were mounted to the steel strain bar with EPY-150 two-part epox, and then coated with Dow Corning Silastic RTV silicon rubber for aterproofing. Shortly after the beginning of the three-dimensional test; problems were encountered in the operation of the transducer. These problems were caused by the deterioration of the original strain gage adhesive and coating, so new strain gages were installed on the transducer unit. The new gages were Micromeasurement-type EA-06-125AD-120, bonded to the strain bar with Micromeasurement M-Bond 610 two-part strain gage adhesive, and then coated with Micromeasurement M-Coat D and M-Coat G for waterproofing protection. About halfway through the three-dimensional tests, further problems were encountered in the operation of the transducer unit, probably due to water leakage into the waterproof coating. There was also evidence of corrosion on the steel strain bar, so it was decided to build a new force transducer using a stainless-steel strain bar to minimize corrosion, and encapsulated strain gages to minimize problems with water leakage. The new strain gages were Micromeasurement-type CEA-06-125UW-120. The same strain gage adhesive and waterproof coatings were used, with Micromeasurement M-Coat B along the lead wires to minimize the change of water "wicking" along the lead wires to the inside of the coating materials.

The transducer mounting arrangement was different for the two-dimensional and three-dimensional experiments. The test cylinder and transducer unit for the two-dimensional tests were mounted between two support brackets on each side of the 1-foot wave channel. For the three-dimensional experiments, the test cylinder and transducer unit were mounted between two long dummy pipe sections, which were in turn mounted to a steel base. The force meter and mounting arrangement is shown in Figures 9 and 10 for the two-dimensional tests, and in Figures 11 to 15 for the three-dimensional tests.

A paralle'-wire resistance-type wave gage was used to record the waves passing over the model pipe section. The gage was mounted directly over

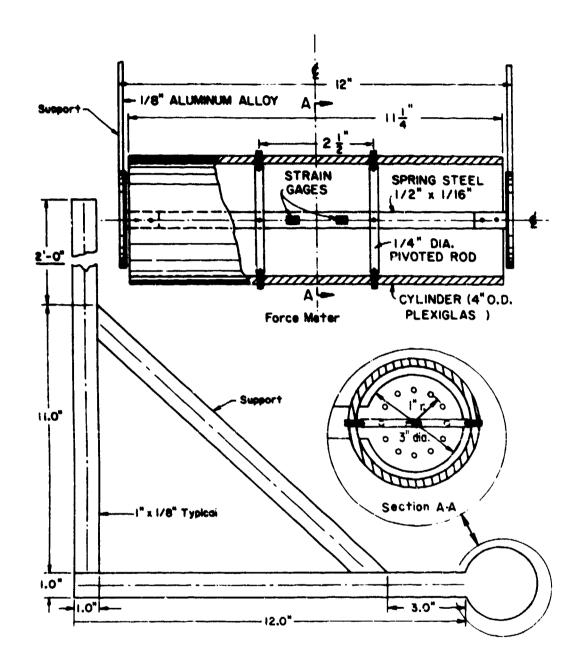


Figure 9. Force meter and support.

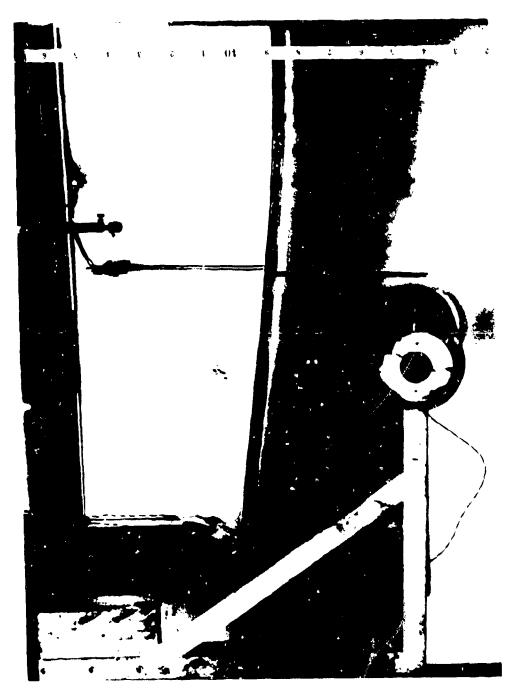


Figure 13. Four-inch cylind., mounted in the flume.

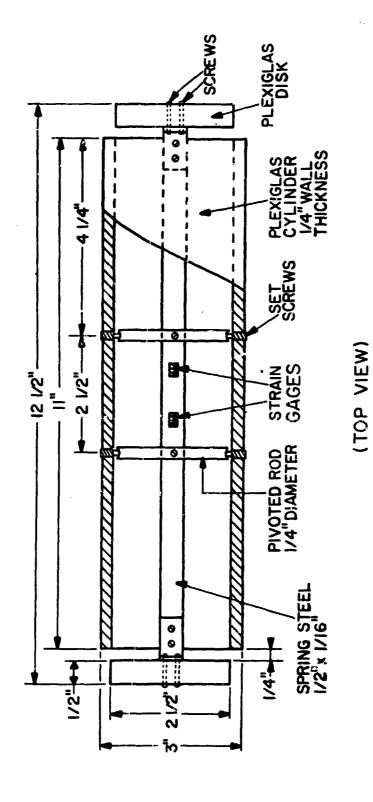


Figure 11. Test section (force meter).

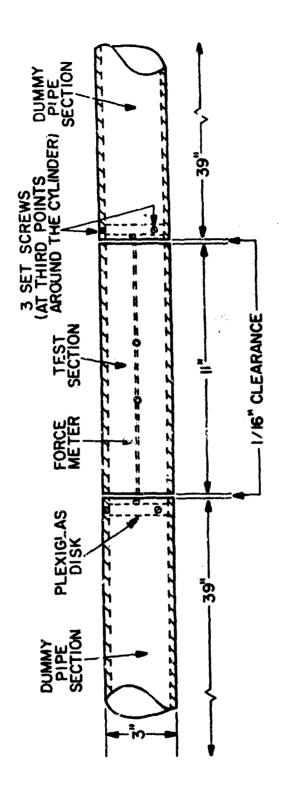


Figure 12. Test section mounted in position.

(SIDE VIEW)



Figure 13. Test section and force transducer.

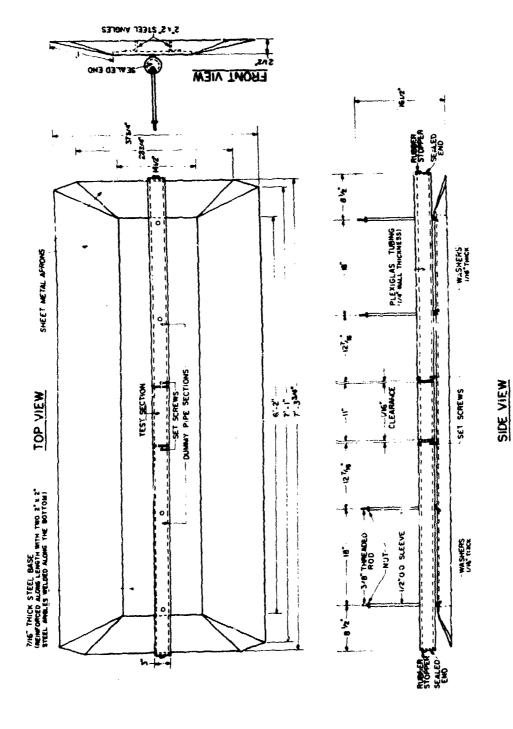


Figure 14. Schematic of pipeline model.



Figure 15. Pipeline model.

the center of the test section, so that the wave records could be correlated directly with the resulting wave-induced force record.

A Brush dual-strain grace amplifier was used in the experiments, with one channel connected to the wave gage, and the other channel connected to the force meter. The amplifier was connected to a Brush two-channel rectilinear writing recorder which continuously recorded the waves and corresponding wave-induced forces on the pipe section (Fig. 16).

An electronic digital data acquisition system (Paulling and Sibul, 1968) was used in the three-dimensional experiments. The digitizer was connected in parallel with the strain gage amplifier to record simultaneously the wave and corresponding force data on magnetic tape, while at the same time the data were being recorded continuously on the stripchart recorder (Fig. 17). The digitizer sampled alternatingly from both the wave record and force record at a rate of 100 samples per second, resulting in 50 samples per second from each of the two channels.

# 2. Procedure for Two-Dimensional Experiments.

- a. Calibration. Both the wave gage and the force transducer were calibrated before each set of experimental runs. The wave gage was calibrated statically by raising and lowering the gage in increments of U.US root (1.52 cent meters) and recording the output. The force meter was also calibrated statically by hanging weights in increasing equal increments from a system of pulleys connected to the force meter and recording the output on the strip chart. The force transducer was calibrated in both the upward and downward directions by rearranging the pulley system and repeating the above procedure. The calibration method is shown in Figure 18.
- b. Procedure. After calibrating the force meter, the model pipe section was lowered and fixed in a horizontal position at the desired clearance above the bottom of the wave channel, with the long axis of the test cylinder parallel to the approaching wave crests. A sliding point gage was mounted to the wave channel above the pipe section and was used to accurately set the model pipe to the desired bottom clearance and aline the pipe section parallel to the wave crests. Once the model was in the correct position, the mounting brackets and support struts were clamped to the sides of the wave channel. The force transducer was mounted in such a way that it was sensitive only to forces acting in the vertical direction.

After the model pipe section was mounted in position, the wave gage was lined up directly over the center of the pipe section with a plumb bob and then clamped in position. The wave gage was then calibrated as described above. The experimental arrangement is shown in Figure 19.

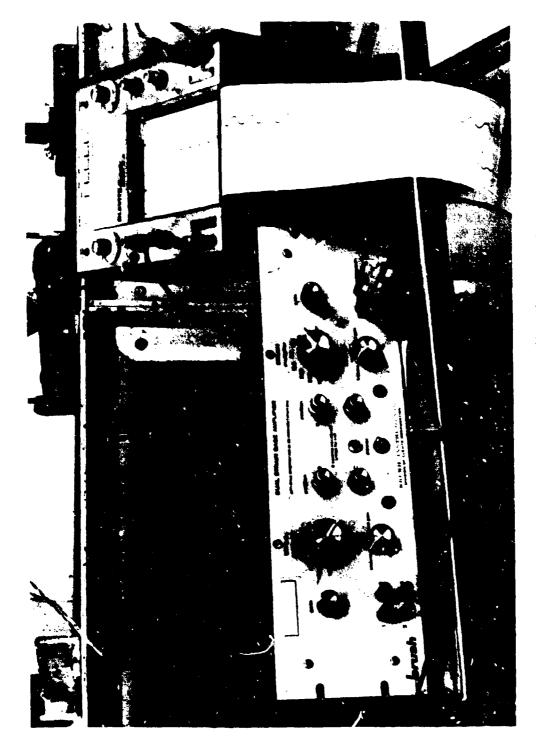


Figure 16. Brush recording instruments.

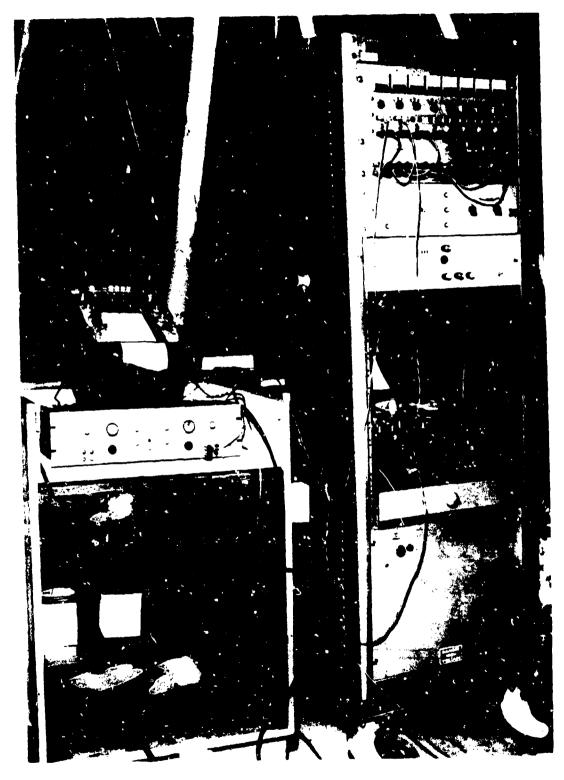


Figure 17. Digitizer and recording instruments.

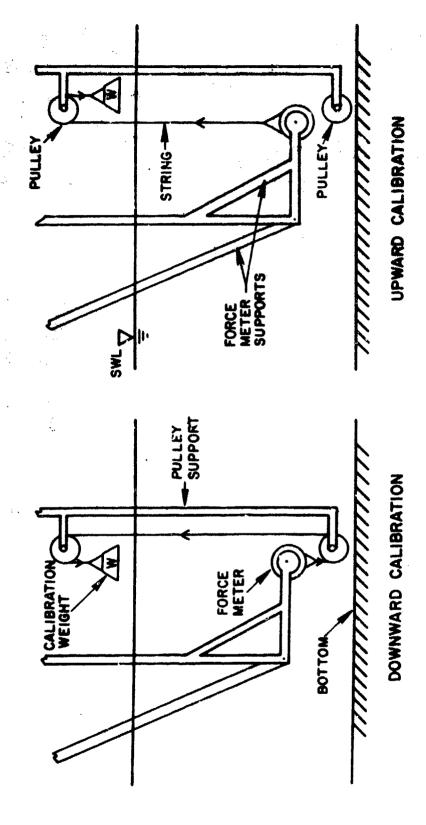


Figure 18. Calibration method for two-dimensional experiments.

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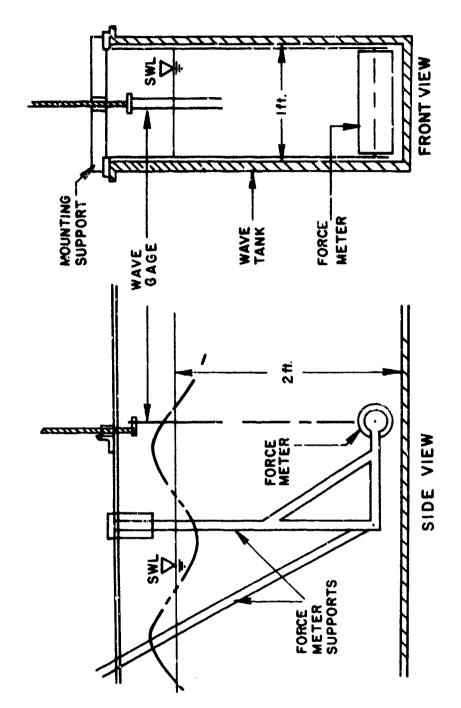


Figure 19. Experimental arrangement for two-dimensional tests.

The pipe model and wave gage were mounted in a glass-walled part of the tank near the middle of the wave channel to facilitate the visual observation of the phenomenon being studied. For each bottom clearance tested, a series of runs was made with waves generated at 19 different wave periods, covering a range of 0.95 to 2.5 seconds. Seven wave heights were generated for each wave period, ranging up to 0.34 foot (10.4 centimeters).

After these runs were completed, the pipeline was set at another bottom clearance, and the precedure was repeated. Seven bottom clearances were tested for each wave condition, ranging from 0.001 foot, 1/16, 1/8, 3/16, 1/4, 1, and 2 inches (0.305, 1.59, 3.18, 4.76, and 6.35 millimeters, 2.54 and 5.08 centimeters), respectively. The minimum clearance tested (0.001 foot) was that which placed the pipe section as close to the bottom as possible without touching the bottom when the waves passed over it. This was necessary to measure any downward forces exerted on the pipe section due to the wave action. The 2-inch bottom clearance placed the pipe section far enough from the bottom so that the vertical lift forces were insignificant.

These experiments were carried out with a 4-inch-diameter (10.16 centimeters) test cylinder. The experiments were repeated with pipe sections of 2-,  $2^{1}$ <sub>2</sub>-, and 3-inch (5.08, 6.35, and 7.62 centimeters) diameters, but only three bottom clearances were tested--0.001 foot, 1/8 inch, and 1/4 inch. The wave conditions covered the same range of wave heights and periods, but were not quite as extensive in number

In addition to the vertical force measurements, a series of experiments was performed to measure the horizontal forces acting on the pipe section, so that the resultant wave-induced force could be determined throughout the entire wave cycle for several of the experimental conditions tested. Only the 4-inch-diameter test cylinder was used in these experiments, since the corresponding vertical experiments were the most extensive for the 4-inch cylinder. The horizontal forces were measured by rotating the force transducer 90° so that it was sensitive only to forces acting in the horizontal direction. The calibration procedure was the same as described above for the vertical force measurements except that the system of pulleys was rearranged so that the calibration weights exerted forces in the horizontal direction only.

All seven of the bottom clearances used in the vertical experiments were also used in the horizontal tests. The wave periods covered the same range as the vertical experiments, but only 6 of the 19 wave periods were used--0.95, 1.25, 1.5, 1.85, 2.25, and 2.55 seconds. Two of the seven wave heights corresponding to each wave period in the vertical experiments were used in the horizontal tests.

The stillwater depth was held constant at a depth of 2 feet throughout the two-dimensional tests.

# 3. Procedure for Three-Dimensional Experiments.

a. Calibration. The wave gage and force meter were calibrated before each set of experimental runs. The wave gage was calibrated in the same manner as the two-dimensional tests, but 0.1-foot (3.05 centimeters) increments were used rather than 0.05-foot increments, since larger waves were used in these experiments.

The force transducer was calibrated in the upward direction in the same manner as the two-dimensional tests, by hanging weights over a pulley to a string attached to the pipe test section. However, because the three-dimensional model was mounted to a base with a small bottom clearance, it was impossible to calibrate the transducer in the downward direction by using a system of pulleys, since there was no room for a pulley between the pipe section and the base to which it was mounted. Rather, the force meter was calibrated in the downward direction by placing the weights directly on top of the center of the submerged test section and using the submerged weight or the weights in calculating the calibration curve. Weight increments of 50 grams were used in calibrating the transducer. The calibration method is shown in Figure 20.

b. Procedure. An overhead crane was used to lower the pipeline model and base into the wave tank. The assembly was first submerged to a depth of about 1½ feet (45.7 centimeters). The model was tilted at both ends to remove all air bubbles from the system, and the ends of the dummy pipe sections were stoppered to prevent waterflow through the pipeline model. The bottom clearance between the base and the pipe model was adjusted by placing spacers on the support rods between the base and the dummy pipe sections, and then tightening the nuts on the support rods above the dummy pipe sections. The test section was then centered and adjusted carefully to the exact bottom clearance desired with the aid of 10 adjusting screws. The calibration string was attached to the test section, and the assembly was lowered to the bottom of the tank.

The calibration string and pulley system was aligned directly over the center of the test section with a plumb bob, and the pulley supports were then clamped to the sides of the wave tank. The transducer was first calibrated in the upward direction, after which the calibration string was removed, and the transducer was calibrated in the downward direction, as described above.

The pipeline model was positioned at the desired angle of orientation on the tank bottom by lining up one of the long edges of the model base parallel to the correct line marked on the bottom of the wave tank. Lines were marked on the tank bottom in 15° increments from 0° to 75°, where 0° corresponds to a pipeline parallel to the approaching wave crests. After the model was calibrated and placed in position, the wave gage was lined up directly over the center of the test section with

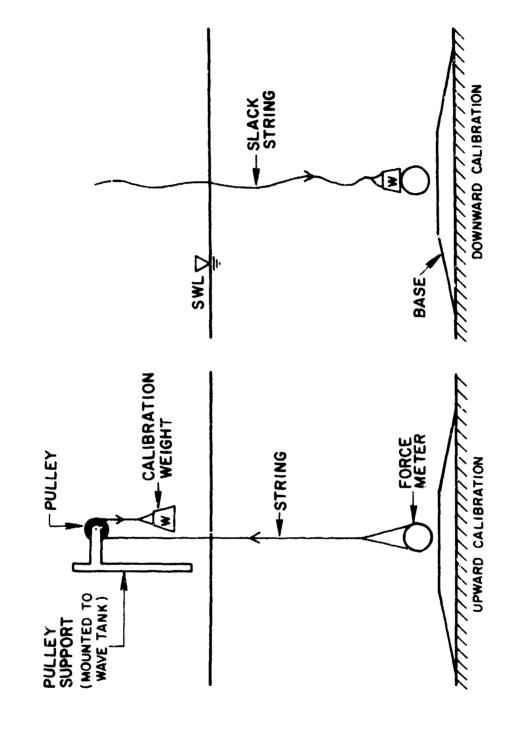


Figure 20. Calibration method for three-dimensional experiments.

a plumb bob, clamped in position, and then calibrated as described above. The experimental arrangement is shown in Figure 21.

For each bottom clearance, six angles of orientation (0°, 15°, 30°, 45°, 60°, and 75°) were tested. Fifteen runs with different wave conditions were made for each bottom clearance and orientation angle. These runs covered four wave periods ranging from 1.4 to 2.6 seconds, with waves generated at four heights for each period, ranging to a maximum of about 0.7 foot (21.3 centimeters). Eight bottom clearances were tested, ranging from 0.001 foot, 1/16 inch, 1/8 inch, 3/16 inch, 1/4 inch, 1/2 inch, 1 inch, and 2 inches.

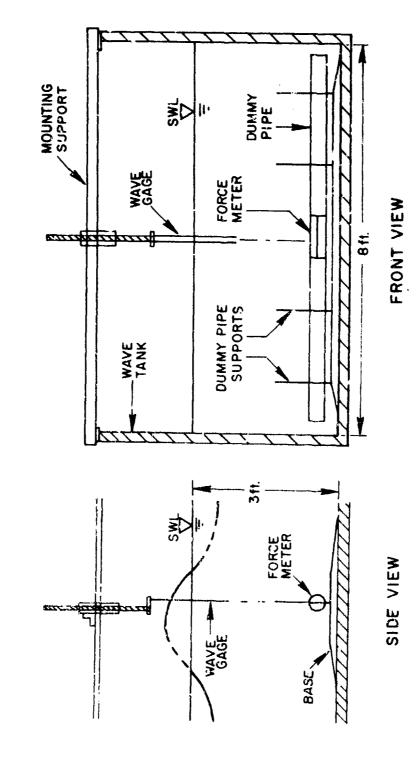
The above experiments were done using a 3-inch-diameter pipeline model. The tests were then repeated using a 2- and 4-inch-diameter pipeline. The 1- and 2-inch clearances were not tested because the lift forces at these clearances prover insignificant in the previous tests. Also, the tests at an orientation angle of 75° were eliminated, since the previous experiments demonstrated that the vertical forces measured at this angle were insignificant, and too small to be measured with any accuracy. Aside from these changes, the 4-inch-diameter pipeline was tested at the same bottom clearances, orientation angles, and wave conditions as the 3-inch-diameter model. The 2-inch-diameter model was tested at the same bottom clearances and wave conditions, but only three of the five orientation angles (0°, 30°, and 60°) were tested.

The stillwater depth in the wave tank was held constant at 3 feet throughout the three-dimensional experiments, but since the base of the pipeline model was located 2-7/16 inches (6.19 centimeters) above the tank bottom, the effective stillwater depth over the pipeline base was 2.797 feet (85.25 centimeters). The definition sketch for the three-dimensional experiments is shown in Figure 22.

#### 4. Data Reduction.

The wave force data were taken on a two-channel strip-chart recorder with the paper advancing at a speed of 25 centimeters per second. One channel recorded the forces while the other channel simultaneously recorded the wave surface profile directly over the center of the pipeline test section, thus allowing direct correlation of the two records.

The two-dimensional experimental data were digitized manually using a Gerber digital data reduction system connected with a card punch to automatically punch the digitized values on computer cards. Using a variable linear scale, each force record was first wided into 20 equally spaced intervals per wave, each interval representing a time interval of 1/20, where T is the wave period. Each force record was digitized at these points over an interval of two consecutive waves (beginning at the wave crest), thus giving 10 values for the analysis and averaging the wave forces over two wave cycles.



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Figure 21. Experimental arrangement for three-dimensional tests.

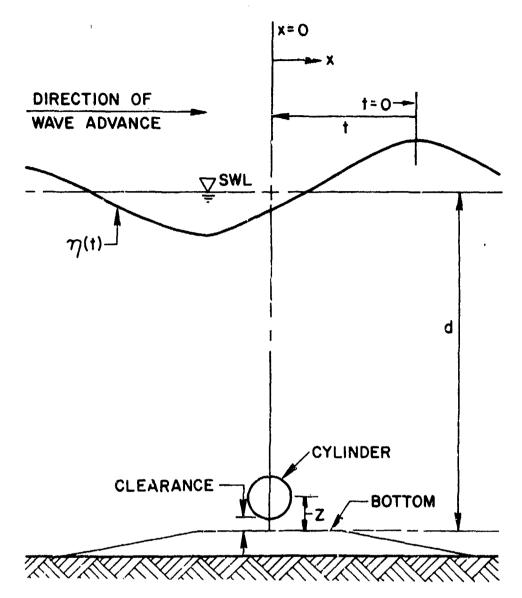


Figure 22. Definition sketch for three-dimensional experiments.

The points in the force records corresponding to the wave crests were chosen as the origin (and end) of the digitized records. These points were determined by averaging the midpoints of three or four horizontal lines drawn through the crests of the wave record at several elevations above the stillwater level (SWL). These midpoints were approximately identical except for some of the larger, longer waves in which the peak of the wave crest did not exactly coincide with the midpoint of the zero crossings of the wave crest. A sample data record is given in Figure 23.

The three-dimensional experimental data were handled differently; the data were recorded on magnetic tape with an electronic digital data acquisition system. This instrument sampled alternatingly from the two channels (wave and force) at a rate of 100 samples per second, resulting in 50 samples per second from each channel.

The origin at the wave crest and the wave period were determined from the digitized wave records, rather than directly from the stripchart records. Since positive readings of the wave profile corresponded to the crest and negative readings corresponded to the trough, the point of origin of the wave crest was determined by taking the midpoint of the positive readings between zero crossings on the wave profile. The crest was thus defined as the data point closest to the midpoint of the zero crossings. The wave period was determined from the number of readings between two successive crests, since there was a time interval of 1/50 second between each reading. Thus, the wave period was determined to the nearest 0.02 second.

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The origin of the force record was taken as the force reading corresponding to the defined origin at the center of the wave crest surface profile. In reality, there was a small timelag of 1/100 second between the wave profile readings and the corresponding force readings. This small timelag was ignored in the analysis, since it was felt that the accuracy of the defined origin at the wave crest was only good to the nearest 1/50 second, the time interval between successive readings of the wave record.

Only one wave cycle was used for analysis of the electronically digitized data. Since the data were on magnetic tape, it was impossible to determine that two successive waves had exactly the same period and height until after the calculations were completed on the computer. Thus, if the waves had slightly different periods, the time phase correlation of the corresponding force readings would be slightly in error when taken over two wave cycles. In addition, since the accuracy, resolution, and rapid sampling rate of the electronic digitizer allowed more readings per wave cycle than the manual digitizing method, a sufficiently large number of force readings could be obtained in one wave cycle.

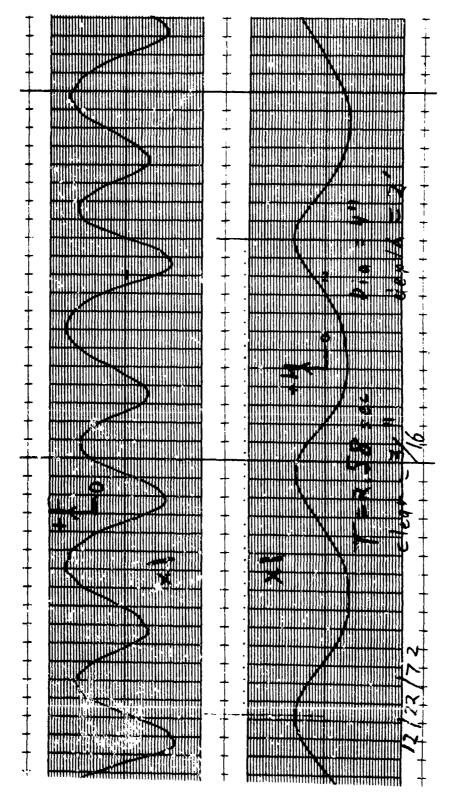


Figure 23. Example of data record.

An estimation of the accuracy of the experimental measurements, along with the sources of error, is tabulated in the Table.

A least squares analysis was performed on the digitized force data to calculate the parameters,  $C_L$ ,  $\phi$ , k,  $C_M$ , and  $C_D$ , of the vertical wave force equation, and the coefficients,  $C_M$  and  $C_D$ , corresponding to the horizontal wave force equation. Using this approach, values of the wave force parameters that best fit the force data throughout the entire wave cycle can be determined. These values were then substituted back into the wave force equation to calculate the force over a complete wave cycle, thus allowing comparison of the results with the original data. The least squares analysis is given in Appendix A. The computer programs used for the analysis are given in Appendixes B, C, and D; the tabulated results of the analysis are in Appendixes E, F, and G. Examples of the computer output showing comparison of the original data with the forces calculated using the results of the least squares analysis are given in Figures 24 and 25.

#### III. RESULTS AND DISCUSSION

### 1. Resultant Force Through Wave Cycle.

Both horizontal and vertical force measurements were made for some test conditions in the two-dimensional experiments using the 4-inch-diameter cylinder. The resultant force throughout the wave cycle could thus be determined for these conditions. Figures 26 to 32 show the resultant force plotted for each bottom clearance under the same wave condition, a period of 1.85 to 1.86 seconds and a wave height of 0.24 to 0.25 foot (7.32 to 7.62 centimeters). Values from the corresponding horizontal and vertical force records were plotted at 20 evenly spaced intervals (18°) through each wave cycle. The forces were plotted for two consecutive wave cycles to indicate the degree of scatter in the data. A rectangle was drawn at each plotted point to illustrate the horizontal and vertical range of the force data over the two wave cycles, and an envelope curve was drawn over these points.

Examination of these plots as a group (Fig. 33) shows the transition of the resultant wave-induced force with increasing clearance for the given wave condition (T = 1.85 to 1.86 seconds, H = 0.24 to 0.25 foot). The vertical component of the wave force is dominated by the lift force, while the horizontal component of the resultant force is due to the inertial and drag forces, with the inertial forces predominating for the experimental conditions tested.

For the smallest clearance (0.001 foot), in which the pipeline is almost in contact with the bottom, the resultant force attains a maximum upward value under the crests and troughs of the passing waves. The total wave force acts in the upward (positive) direction throughout the complete wave cycle, except for small downward forces in the vicinity of 90° and 270°, where the horizontal flow reverses.

Table. Estimated accuracy of experimental measurements.

Variable	Maximum error	Major source of error
Wave height	3 to 5 percent	Stability of amplifier with respect to calibration
Wave period	0.02 seconds (0.8 to 1.4 percent, depending on period)	Accuracy of strip-chart records for two-dimensional experiments; time interval between successive digitizer readings of wave record for three-dimensional experiments
Water depth	<pre>1/8 inch (0.5 percent for two-dimensional tests, 0.35 percent for three-dimensional tests)</pre>	Direct measurement .
Pipe diameter	0.002 foot (0.610 millimeter) (0.6 to 1.2 percent, depending on diameter)	Variations in nominal diameters of tubing from which the models were constructed
Bottom clearance	0.001 foot for two-dimensional tests; 0.0005 foot (0.152 millimeter) for three- dimensional tests	Least count of point gage used to set clearance in two-dimensional tests; accuracy of metal gages used to set clearance in three-dimensional tests
Urientation angle	1.5°	Accuracy of lines marking the angles on the tank bottom, and alinement of pipeline model with the edge of the base to which it was mounted
Wave force	5 percent, except for data taken at largest orientation angles (50° and 75°) in the three-dimensional tests, which are accurate to within 10 percent	Stability of amplifier with respect to force calibration

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<sup>\* =</sup> measured force

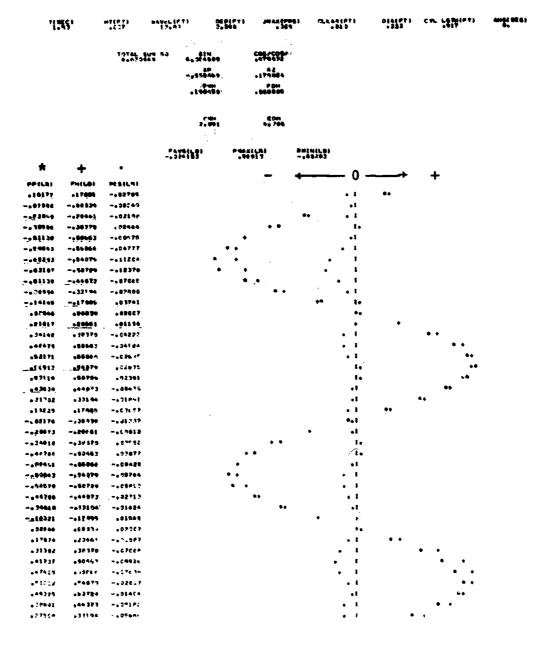
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Figure 24. Example of computer output for vertical least squares analysis.

<sup>+ =</sup> calculated force

<sup>· =</sup> measured force - calculated force

<sup>\$ =</sup> wave surface

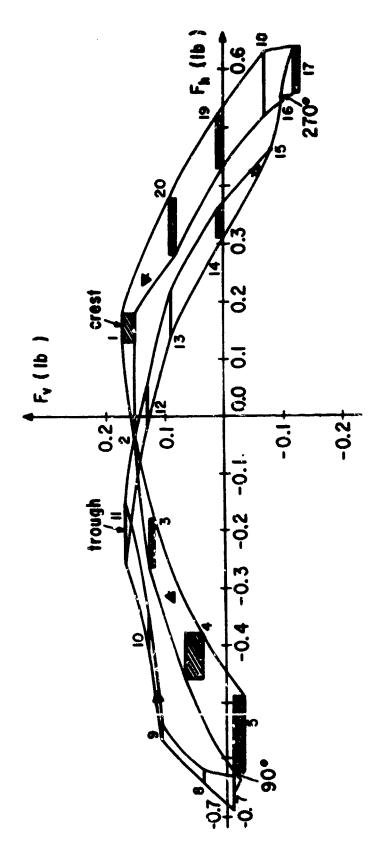


<sup>\* =</sup> measured force

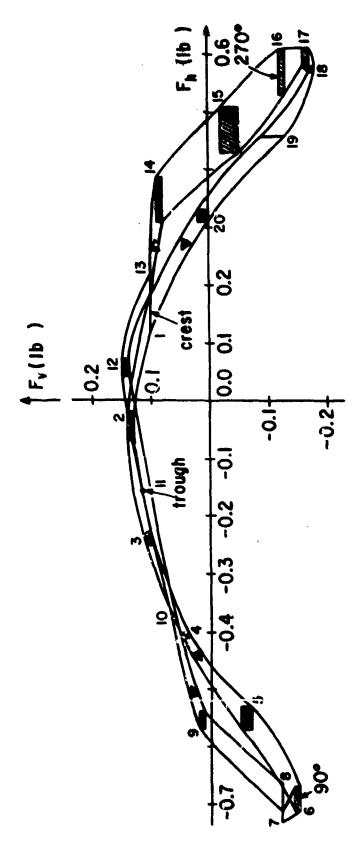
Figure 25. Example of computer output for horizontal least squares analysis.

<sup>+ =</sup> calculated force

<sup>· =</sup> measured force - calculated force

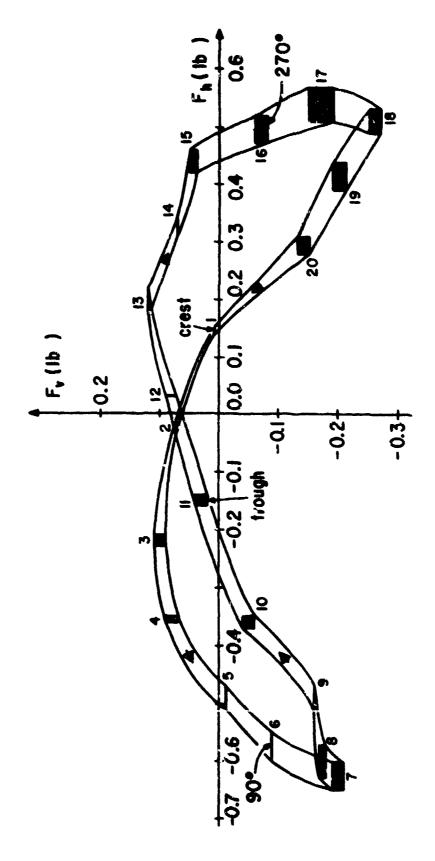


Resultant force through wave cycle for 0.001-foot clearance, 1.85-second period, and 0.24-foot height. Figure 26.

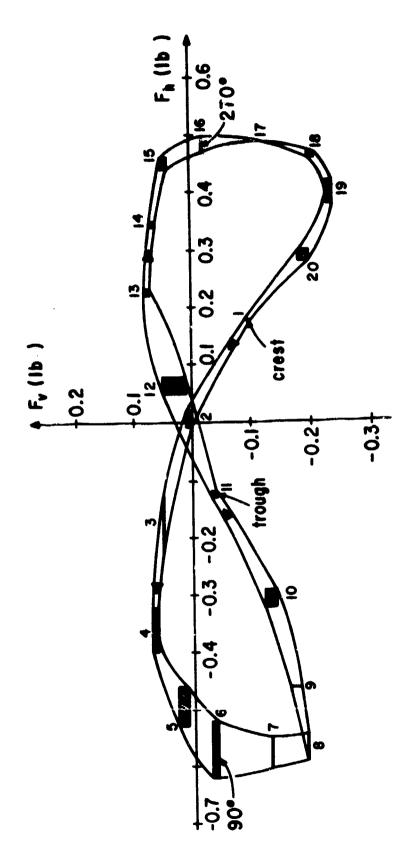


Resultant force through wave cycle for 1/16-inch clearance, 1.86-second period, and 0.24-foot height. Figure 27.

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Resultant force through wave cycle for 1/8-inch clearance, 1.85-second period, and 0.25-foot height. Figure 28.



Resultant force through wave cycle for 3/16-inch clearance, 1.85-second period, and 0.25-foot height. Figure 29.

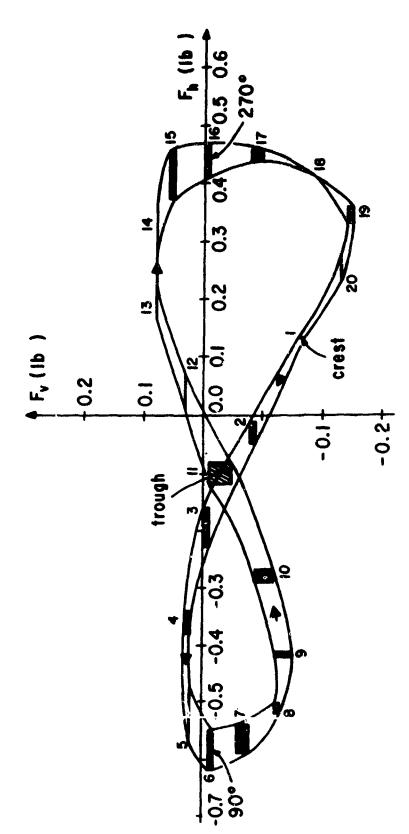
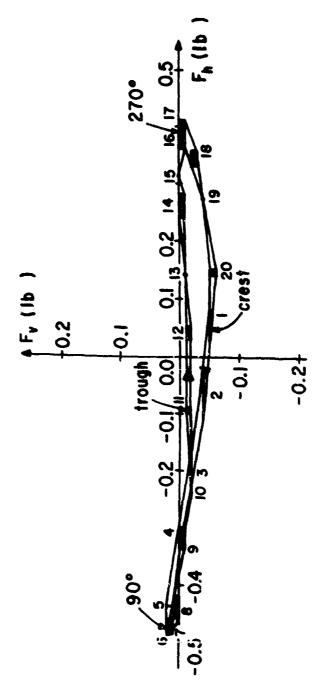


Figure 30. Resultant force through wave cycle for 1/4-inch clearance, 1.86-second period, and 0.25-foot height.



Resultant force through wave cycle for 1-inch clearance, 1.86-second period, and 0.24-foot height. Figure 31.

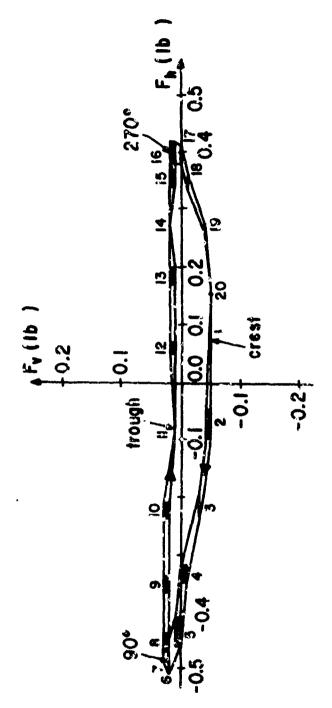


Figure 32. Resultant force through wave cycle for 2-inch clearance, 1.86-second period, and 0.25-foot height.

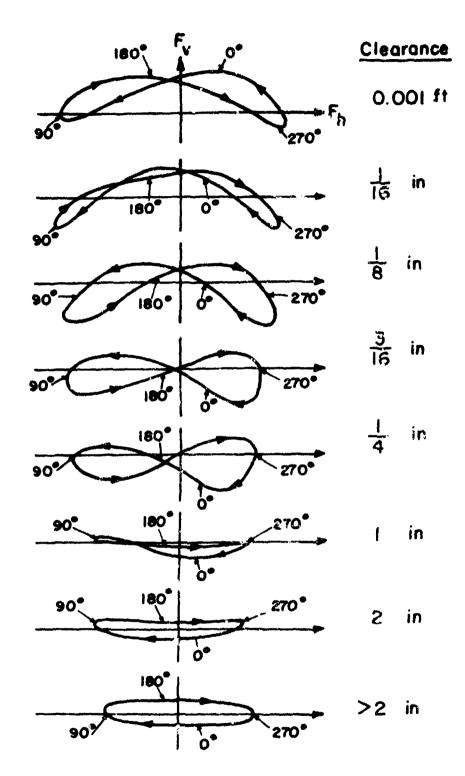


Figure 33. Change in resultant force with increasing clearance.

As the clearance is increased to 1/16 inch, the maximum upward forces decrease in magnitude, and also occur slightly later in the wave cycle. At the same time, the downward forces increase, reaching their maximum values at approximately 90° and 270°.

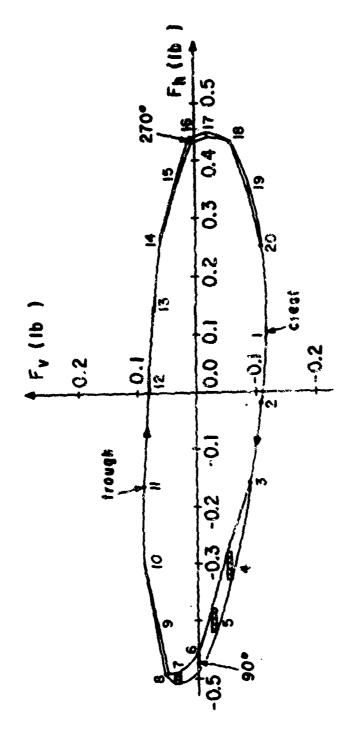
Further increases in the bottom clearance produce a continuous shift of the positions of both the maximum upward and maximum downward forces later in the wave cycle. Simultaneously, the forces become downward rather than upward for a larger part of the cycle. At the same time, the versical components of the wave force under the crasts and troughs become negative and increase in the downward direction, while the negative forces at 90° and 270° gradually decrease to zero.

At a 1-inch clearance, the resultant force acts downward throughout almost the complete wave cycle, with maximum downward forces occurring under the crests and troughs of the passing waves. The vertical forces are zero at 90° and 270°, the positions of the maximum horizontal inertial forces. However, the lift effect is not very large for the 1-inch clearance. The resultant force plot for the 2-inch clearance shows that the lift effect is still present, but is relatively small, even in comparison to the small vertical inertial forces.

At a slightly larger clearance, the lift effect will disappear, and the vertical forces will be due almost entirely to the inertial forces, since the vertical drag forces are negligible near the bottom. At this clearance, the inertial force will act upward under the trough and downward under the crest, so the resultant force plot will take the form of an approximately symmetrical ellipse. This condition is shown in Figure 34 for a smaller wave period (1.23 seconds), with a 1-inch bottom clearance. The ellipse is distorted slightly, due to the small drag forces acting in the horizontal direction, 90° out of phase with the larger horizontal inertial forces.

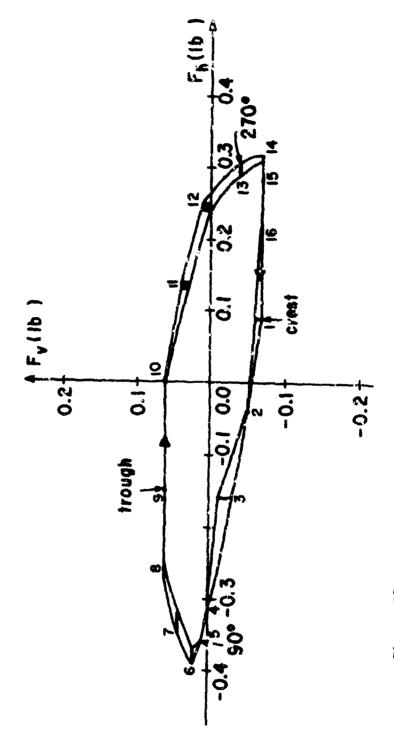
The horizontal components of the resultant wave force are also affected by the proximity of the bottom boundary. Although the horizontal water particle velocities and accelerations increase with distance above the bottom, the corresponding horizontal drag and inertial forces are larger when the pipe is close to the bottom than when it is located above at larger clearances.

Figures 35, 36, and 37 show the resultant force plots at both large and small bottom clearances, for a wave with a period of 0.95 to 0.96 second and a height of 0.24 to 0.25 foot. Because the wave period is small, the horizontal excursions of the water particles at the bottom and the duration of the horizontal flow are too small for the lift effect to develop. So the forces acting in both the horizontal and vertical directions are mostly inertial, with a small drag component in the horizontal direction. The resultant force plots therefore take the form of an ellipse.

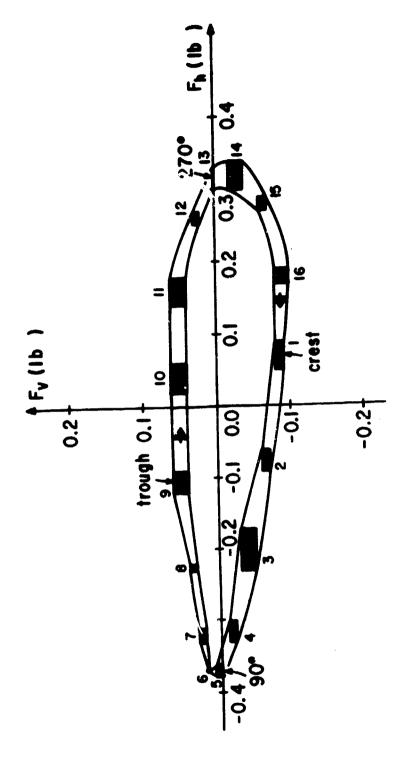


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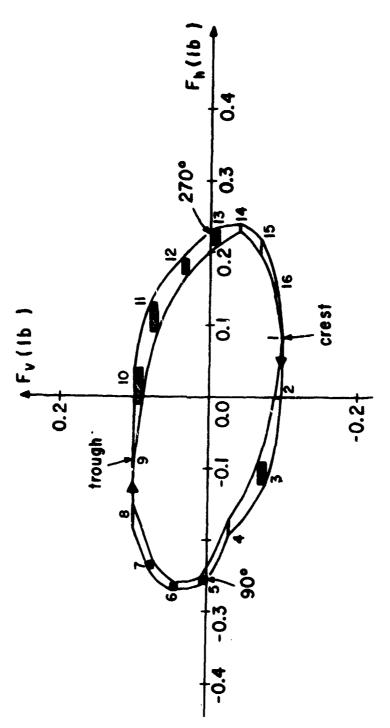
Figure 34. Resultant force through wave cycle for 1-inch clearance, 1.23-second period, and 0.3-foot height.



Resultant force through wave cycle for 0.001-foot clearance, 0.95-second period, and 0.24-foot height. Figure 35.



Resultant force through wave cycle for 1/16-inch clearance, 0.95-second period, and 0.24-foot height. Figure 36.



Resultant force through wave cycle for 2-inch clearance, 0.96-second period, and 0.25-foot height. Figure 37.

However, the horizontal components of the resultant forces are larger at the smallest bottom clearances, even though the lift phenomenon is absent. The presence of the bottom boundary produces an asymmetric flow field around the pipeline. The resulting velocities and accelerations of the water particles over the pipe section are thus modified by the presence of the boundary, and the associated horizontal forces are larger than they would be if subject to the same kinematics in the absence of the boundary. The increased horizontal forces on pipelines located close to the bottom are reflected in increased values of the coefficients of mass and drag,  $C_M$  and  $C_D$ .

## Orientation Angle Considerations.

The coefficient of lift calculated in the least squares analysis of the experimental data was computed using two alternative approaches (Fig. 38): (a) the total horizontal water particle velocity in the direction of wave advance, with the projected area of the pipeline in the plane perpendicular to the direction of wave advance; and (b) only the component of the horizontal water particle velocity perpendicular to the pipeline axis, with the projected area in the plane parallel to the pipeline axis.

After tabulating the data from the three-dimensional experiments, it became apparent that the second method gave consistent values of the coefficient of lift for all angles of orientation. In contrast, the values of  $C_L$  obtained using the first method gave values that were low, and which decreased with increasing angles of orientation (where  $0^{\circ}$  corresponds to a pipeline parallel to the wave crests).

Relationships between the coefficient of lift,  $C_L$ , and the parameters,  $\phi$  and k, of the lift force equation were the same for all angles of orientation when  $C_L$  was calculated considering only the component of the horizontal velocities perpendicular to the pipeline axis.

In addition, relationships involving any of the parameters of the lift force equation ( $C_L$ ,  $\phi$ , or k) and various dimensionless parameters defining the wave and pipeline conditions were consistent for all angles of orientation when the horizontal water particle velocity acting on the pipe section was treated by considering only the component perpendicular to the pipeline, and completely ignoring the parallel component.

Thus, the results of this investigation show that the modified lift force equations presented in this report can be applied to pipelines located at any angle of orientation with respect to the wave crests. However, only the component of the horizontal water particle velocity perpendicular to the pipeline axis should be considered as contributing to the wave-induced lift force acting on the pipeline. Using this approach, the parameters,  $C_L$ ,  $\varphi$ , and k, lefining the lift forces exhibit the same quantitative relationships between the various dimensionless parameters defining the wave and pipe conditions, regardless of the angle of orientation.

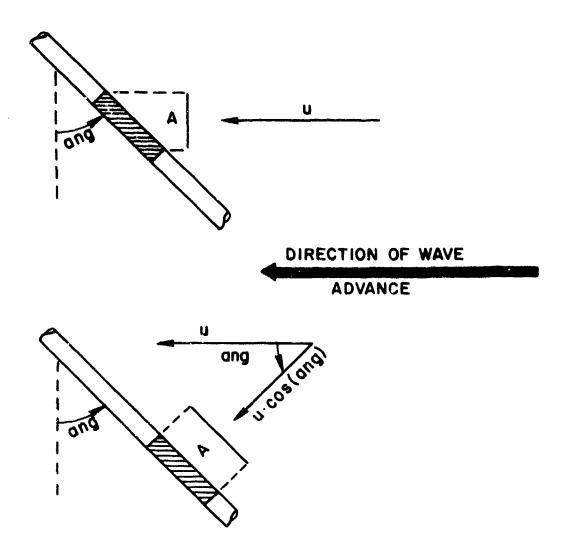


Figure 38. Alternative approaches for handling pipeline orientation angles.

## 3. Interrelationships Between $C_L$ , $\phi$ , and k.

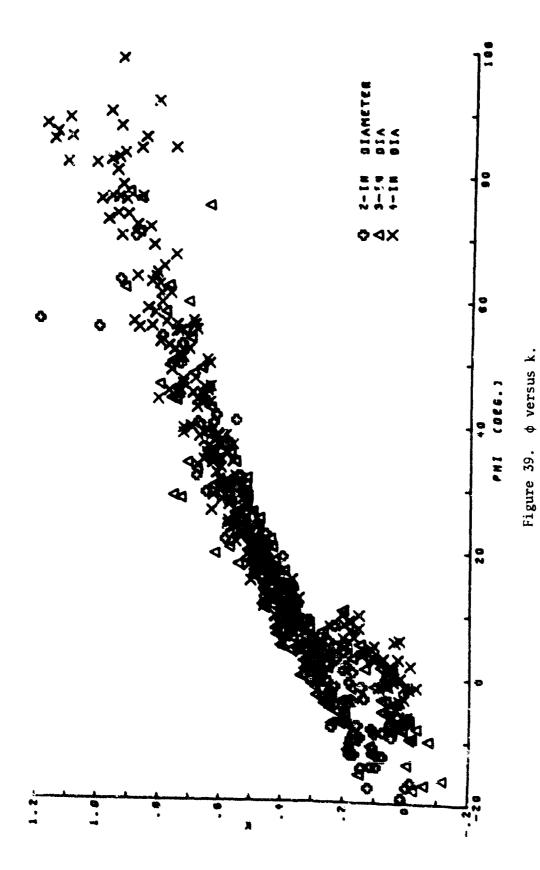
 $\varphi$  and k were defined as varying from 0° to 90° and 0 to 1, respectively, with increasing clearance.  $\varphi=0^\circ$  and k=0 correspond to the case of a pipeline in contact with the bottom (no clearance), while the maximum values of  $\varphi=90^\circ$  and k=1 correspond to the case of a large enough clearance so that the choking phenomenon does not occur at any time throughout the wave cycle. Since a simultaneous increase of both parameters was noted in the data for increasing clearance between the two limiting cases, it was suspected that a direct relationship may exist between  $\varphi$  and k. Such a relationship was found, as shown in Figure 39. The same relationship held for all three pipe diameters tested, regardless of the orientation angle, indicating that the relationship was independent of these two factors, and was thus valid for any pipeline configuration in which the lift effect was present.

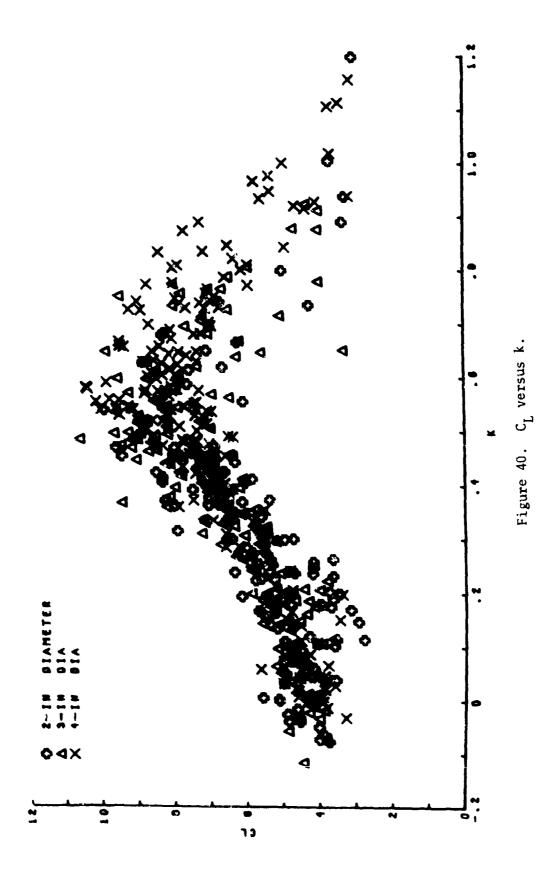
In this plot and the ones that follow, the data for orientation angles from 0° to 30° were plotted for each pipe diameter, without differentiating the data corresponding to each angle. The relationships shown were found to be valid regardless of the angle of orientation, provided the data were handled as discussed above (using the component of the horizontal velocity perpendicular to the pipeline axis). The data corresponding to each pipe diameter are distinguished by using different plot symbols. The same relationships hold for orientation angles of 45°, but these data were not plotted in order to minimize scatter so that differences between the pipe diameters could be detected more easily. In general, the same relationships held for orientation angles up to 60°. But in some cases, the lift effect was negligible at high orientation angles, so the values of the associated parameters  $(C_L, \phi, and k)$  were less accurate. Thus, plotting all of the data corresponding to the larger orientation angles would introduce additional scatter, obscuring the valid relationships which were consistent when the lift forces were significant.

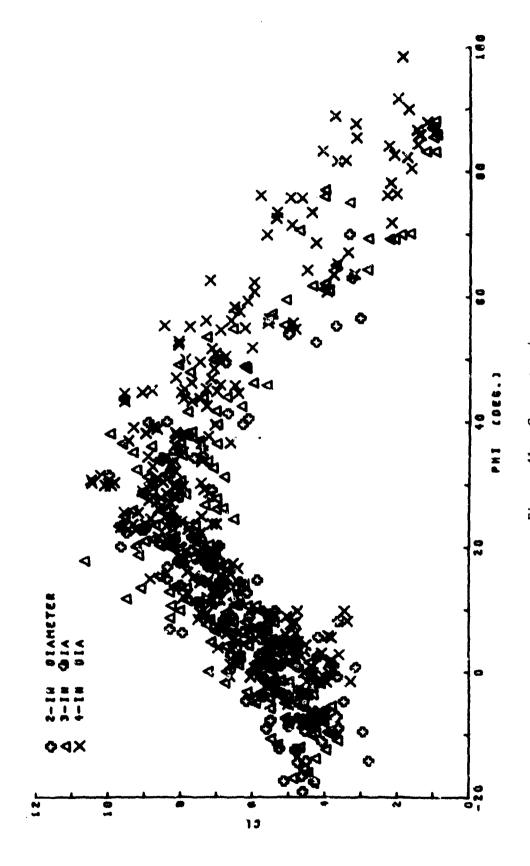
A relationship was found between the coefficient of lift,  $C_L$ , and the parameters,  $\phi$  and k (Figs. 40 and 41).  $C_L$  appears to be better correlated with k than with  $\phi$ . Note that for minimum values of k and  $\phi$ , corresponding to the case of a pipeline in contact with the bottom, the value of  $C_L$  is approximately 4.5. This value is of interest, since it agrees with the potential flow solution ( $C_L = 4.495$ ) for the value of the coefficient of lift for a circular cylinder in contact with a plane wall, subject to an inviscid steady flow (Yamamoto, Nath, and Slotta, 1973).

Maximum values of  $C_L$  occur at approximately k=1/2, corresponding to maximum lift forces that are equal in both the upward and downward directions. The average value of the coefficient of lift at this point is about 9.0, with values extending up to about 10.5. These maximum values of  $C_L$  are attained at approximately  $\phi = 25^\circ$  to  $30^\circ$  in the  $\phi$  versus  $C_L$  plot.

Since the coefficient of lift,  $C_L$ , defines the combined magnitude of both the positive and negative lifts, it can be separated into two parts: (a) the part defining the magnitude of the positive lift,  $C_L(1-k)$ , and







(b) the part defining the magnitude of the negative lift,  $C_L(k)$ . The quantities,  $C_L(1-k)$  and  $C_L(k)$ , can be referred to as the effective positive coefficient of lift and the effective negative coefficient of lift, respectively. Since  $C_L=9.0$  for k=1/2, both  $C_L(1-k)$  and  $C_L(k)$  are equal to 4.5 at this point. This means that the lift forces can reach the same maximum magnitude in both the upward and downward directions as are attained in the upward direction only for the same pipe in contact with the bottom (where  $C_L(1-k)=4.5$ , but  $C_L(k)=0$ ).

The effective positive and negative coefficients of lift are plotted versus both  $\phi$  and k in Figures 42 to 45. Again, the correlations are much better with k than with  $\phi$ . The average value of  $C_L(1-k)$  drops only slightly between k=0 and k=1/2, but for values of k greater than 1/2, the effective positive coefficient of lift drops rapidly to a value of 0 when k=1.

The average value of  $C_L(k)$  increases with k until it reaches a maximum value of about 6.0 when k=0.75, and then decreases to about 4.5 when k=1. Individual maximum values of  $C_L(k)$  attain values slightly greater than 7.0 in the vicinity of k=0.75. But even the average maximum value of 6.0 for the effective negative coefficient of lift indicates that the downward lift forces may attain maximum values 33 percent greater than the maximum possible lift forces acting in the upward direction. Maximum values of  $C_L(k)$  corresponds to a value of  $\phi$  of about 45°, which is half way through the phase shift cycle.

The potential flow theory gives a value of  $C_L = 4.495$  for zero bottom clearance, with a discontinuous jump to very high negative values of  $C_L$  for a very small clearance (Yamamoto, Nath, and Slotta, 1973). In the potential flow solution, the value of  $C_L$  depends only on the relative clearance; i.e., the ratio clearance-diameter. The coefficient of lift is negative whenever the pipe is not in contact with the bottom, and its magnitude decreases as the relative clearance is increased.

Although the potential flow solution appears to work reasonably well when a pipeline is touching the bottom, this approach does not work when there is a small clearance. This is because viscous effects are very important for the flow through the narrow bottom clearance constriction. The choking phenomenon limits the maximum flow velocities and corresponding pressure drops on the bottom side of the pipeline, thereby limiting the maximum possible downward lift forces.

The results of this investigation indicate that the effective negative coefficient of lift,  $C_L(k)$ , can attain a maximum value of only 7.0. This is much less than the values of  $C_L$  suggested for small relative clearances by potential flow theory. The coefficient of lift is obviously not a simple function of relative clearance, since for a given clearance and diameter, both the lift effect and the coefficient of lift will vary with the wave-induced flow conditions. For the smallest relative clearances, the positive lift forces were larger than the negative lift forces, especially where the horizontal water particle velocities and excursions were high.

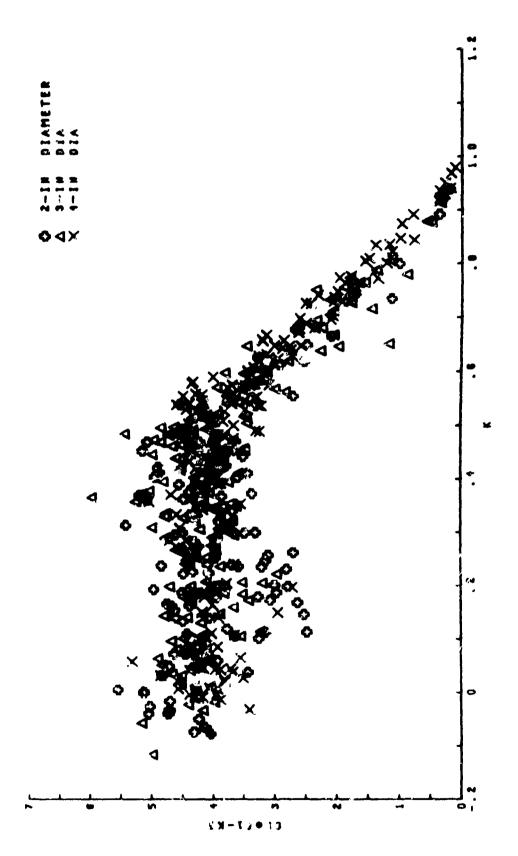


Figure 42. Effective positive coefficient of lift versus k.

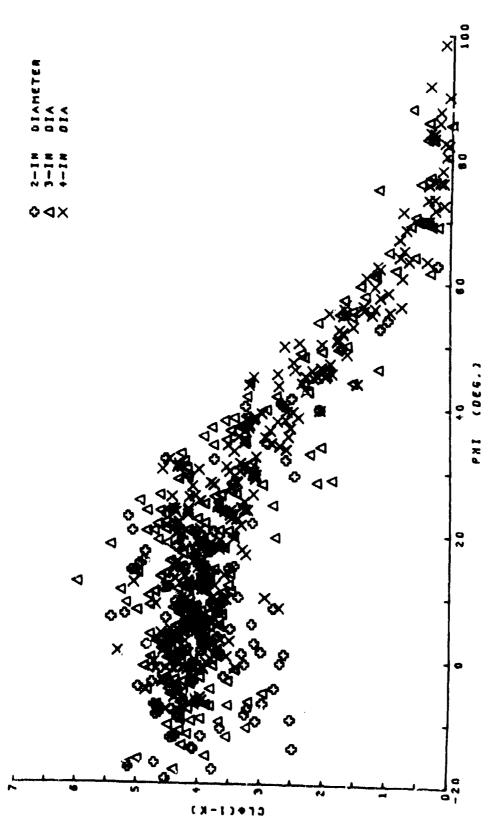


Figure 43. Effective positive coefficient of lift versus  $\phi$ .

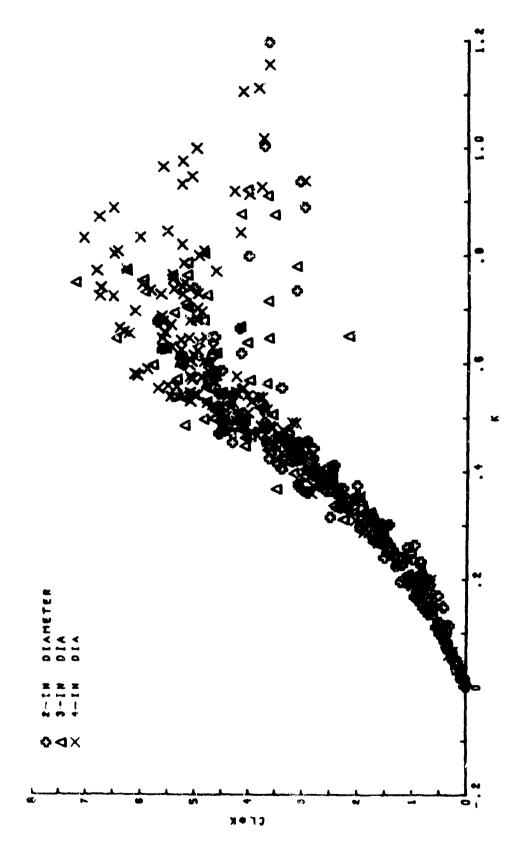


Figure 44. Effective negative coefficient of lift versus k.

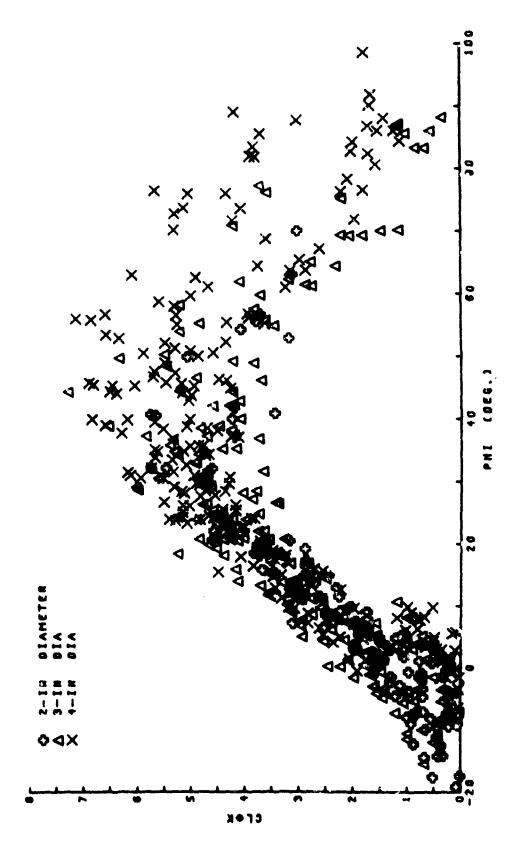


Figure 45. Effective negative coefficient of lift versus  $\phi$ .

The largest negative lift forces do not occur at clearances where the choking effect is absent (corresponding to k = 1 and  $\phi = 90^{\circ}$ ). Rather, the largest values of the effective negative coefficient of lift correspond to values of  $\phi = 45^{\circ}$  and k = 0.75. Interestingly, when k = 1 and  $\phi = 90^{\circ}$  where the positive lift forces have decreased to zero and the choking effect does not develop, the maximum effective negative coefficient of lift is approximately 4.5, the same magnitude as the potential flow solution for the positive coefficient of lift for zero bottom clearance. However, as the bottom clearance is increased further, k and  $\phi$  remain at 1 and 90°, respectively, while the effective negative coefficient of lift decreases to zero (with the diminishing lift forces).

The significance of these results is easily seen by following these relationships for a given pipe and wave as the pipeline is raised from the bottom, and k goes from 0 to 1. In the interval from k=0 to 1/2, the magnitude of the maximum upward lift forces remains the same, which is approximately equal to the potential flow solution for a cylinder in contact with the bottom ( $C_L=4.5$ ). However, at the same time, the negative lift forces increase continuously, reaching a magnitude equal to the positive lift forces at k=1/2 ( $C_L(1-k)=C_L(k)=4.5$ ). Simultaneously, there is a shift in the positions of both the maximum positive and negative lift forces, since  $\phi$  increases from 0° to 30°.

In the interval k=1/2 to !, the maximum positive lift rorces continuously decrease to zero. At the same time, the maximum negative lift forces increase to reach a maximum value at k=0.75 (where  $C_L(k)=6.$  or 7.), and then decrease back to a maximum corresponding to  $C_L(k)=4.5$  at k=1. The point of maximum negative lift corresponds to  $\phi=45^\circ$ , the midpoint of the phase shift cycle.

The phase shift of the maximum lift forces is only half as much in the interval k=0 to 1/2 (where  $\phi$  goes from  $0^{\circ}$  to  $30^{\circ}$ ) as in the interval k=1/2 to 1 (where  $\phi$  goes from  $30^{\circ}$  to  $90^{\circ}$ ).

At k = 1, only negative lift forces exist, and these go to zero as the bottom clearance is increased further.

All of the above interrelationships between  $\phi$ , k,  $C_L$ ,  $C_L(1-k)$ , and  $C_L(k)$  were the same for all pipe diameters tested, regardless of the angle of orientation (provided that  $C_L$  was calculated considering only the component of the horizontal velocity perpendicular to the pipeline axis). Thus, for the range of conditions tested, these interrelationships were independent of the scale and configuration of the pipeline. Also, there is no mention of the wave conditions, which indicates the interrelationships are independent of the wave conditions as well.

The relationships between the parameters,  $C_L$ ,  $\phi$ , and k, defining the lift force equation are useful, since if either  $\phi$  or k is known, the other two parameters can be determined. All that is needed is a relationship between  $\phi$  or k and the wave and pipeline conditions.

There appears to be a better correlation between k and the parameters involving  $C_L(C_L,\,C_L(1-k),\,$  and  $C_L(k))$  than between the analogous relationships using  $\phi$ , so the former relationships should be used. Also, in comparing the plots of  $C_L(1-k)$  versus k (Fig. 42) and  $C_L(k)$  versus k (Fig. 44), the scatter appears minimal in the plot with  $C_L(k)$  for the interval of k between 0 and 1/2. For the interval of k between 1/2 and 1, the scatter is much less on the plot between  $C_L(1-k)$  and k. Therefore, it is suggested that when determining a value of  $C_L$  for a given value of k, the plot of  $C_L(k)$  versus k be used for values of k less than 1/2 (except for k close to 0), and the plot of  $C_L(1-k)$  versus k be used for values of k greater than 1/2 (except for k close to 1) (see Fig. 46). For k close to 0, it can be assumed that  $C_L = 4.5$ . However, for k  $\approx$  1, the value of  $C_L$  can vary from about 4.5 to zero, since as the clearance is increased from the point where  $\phi = 90^\circ$  and k = 1, both  $\phi$  and k remain at their maximum values of  $90^\circ$  and 1, respectively, while the lift effect diminishes to zero.

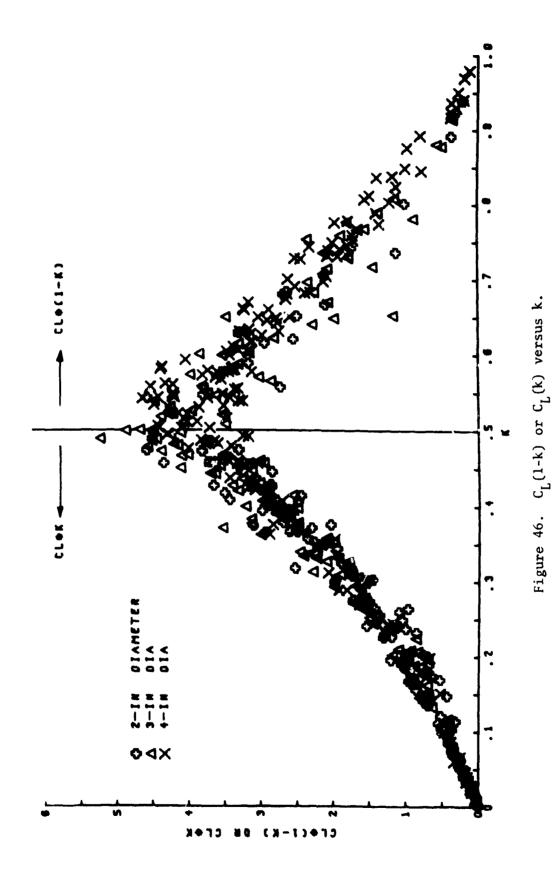
When the above relationships between  $\phi$ , k,  $C_L$ ,  $C_L(1-k)$ , and  $C_L(k)$  are plotted for only the 4-inch-diameter pipe model, the scatter is reduced. Although the data for all three diameters completely overlap (showing the same relationships hold for all diameters), the amount of scatter increases with the smaller diameter models. This is because the data extend to higher relative clearances (clearance-diameter) for the smaller diameter models than the corresponding data for the 4-inch-diameter model, since all models were tested at the same actual clearances.

Since the lift effect diminishes at high values of the relative clearance, the lift forces on the smaller diameter models at the largest bottom clearances were very small in many cases. This is especially true for the smaller waves and higher orientation angles, where the horizontal velocities perpendicular to the pipeline were very low. In such cases, the lift forces were often insignificant, so the values of  $C_L$ ,  $\phi$ , and k calculated from the least squares analysis were not as accurate.

In addition, as the lift forces decrease with high relative clearances, eddy-induced forces may approach the magnitude of the lift forces, thus introducing further error in the calculated values of  $C_L$ ,  $\phi$ , and k.

The lift forces were generally significant for all clearances tested using the 4-inch-diameter pipe section, and since the measured forces were larger, the experimental error involved in measuring them was less than for the smaller diameter models.

Because of this, the data taken for very large bottom clearances were not included in the plotted relationships. For higher clearances, values of k and  $\phi$  equal to 1 and 90°, respectively, would be expected, since the choking phenomenon would not occur throughout the wave cycle. However, as the clearance is increased, the lift effect diminishes, resulting in decreasing values of the coefficient of lift.



If such data were included in the plots of  $C_L$  versus k and  $\phi$ , values of  $C_L$  ranging from 0 to the maximum values shown in Figures 40 and 41 would be present in the vicinity of k=1 and  $\phi=90^\circ$  in the respective plots. The same applies to the plots of  $C_L$  (k) versus k and  $\phi$ .

These trends were observed in the data taken for the largest bottom clearances (1 and 2 inches). However, since these lift forces were so small, a significant amount of error could be introduced into the calculated values of  $C_L$ ,  $\phi$ , and k because of the presence of eddy-induced forces, as discussed above. Therefore, these data were omitted from the plotted relationships, since errors in  $\phi$  or k corresponding to low values of  $C_L$  would produce considerable scatter, obscuring the valid relationships shown.

4. Relationships Between  $\phi$  and k and Parameters Defining the Wave and Pipeline Conditions.

To use the above relationships between  $\mathcal{C}_L$ ,  $\phi$ , and k to determine the wave-induced lift forces acting on a pipeline, either  $\phi$  or k must be known. Thus, a value of one of these parameters must be determined from relationships of  $\phi$  or k with the wave conditions and pipeline configuration.

The lift force phenomenon is a function of the following variables:

- (a) Pipeline configuration
  - (1) Diameter
  - (2) Clearance
  - (3) Orientation angle
- (b) Fluid properties
  - (1) Density
  - (2) Viscosity
- (c) Wave-induced flow conditions
  - (1) Maximum horizontal water particle velocity perpendicular to the pipeline axis
  - (2) Wave period, which represents the duration of the flow in one direction
  - (3) Length of the horizontal excursions of the water particles perpendicular to the pipeline axis (this quantity is directly proportional to the product of the above two parameters)

Assuming that only water with a limited range of temperature is being dealt with, the fluid properties will be ignored for the present. The orientation angle of the pipeline can be handled as discussed above, considering only the components of the horizontal fluid motions

perpendicular to the pipeline axis. Since the length of the horizontal water particle excursions is directly proportional to the product of the wave period and the maximum horizontal water particle velocity, only four independent variables are left: diameter, clearance, horizontal water particle velocity, and wave period. Thus, any single parameter used to relate  $C_L$ ,  $\phi$ , or k to the wave and pipeline conditions must include these four variables. This constraint is necessary if the relationship is expected to be valid for general application under any set of wave and pipeline conditions.

The four variables can be arranged into several dimensionless parameters. The important parameters should include the following:

(1) relative clearance, clear/Dia

where clear = bottom clearance Dia = pipe diameter

(2) Keulegan-Carpenter parameter,  $u_{max}$  T/Dia

where T = wave period

u = component of maximum horizontal water particle
velocity perpendicular to the pipeline axis

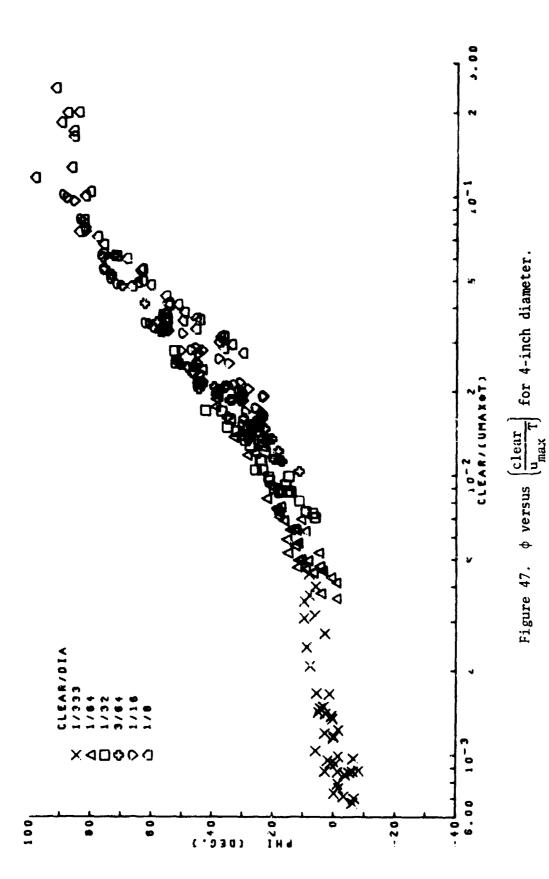
(3) clear/u<sub>max</sub> T

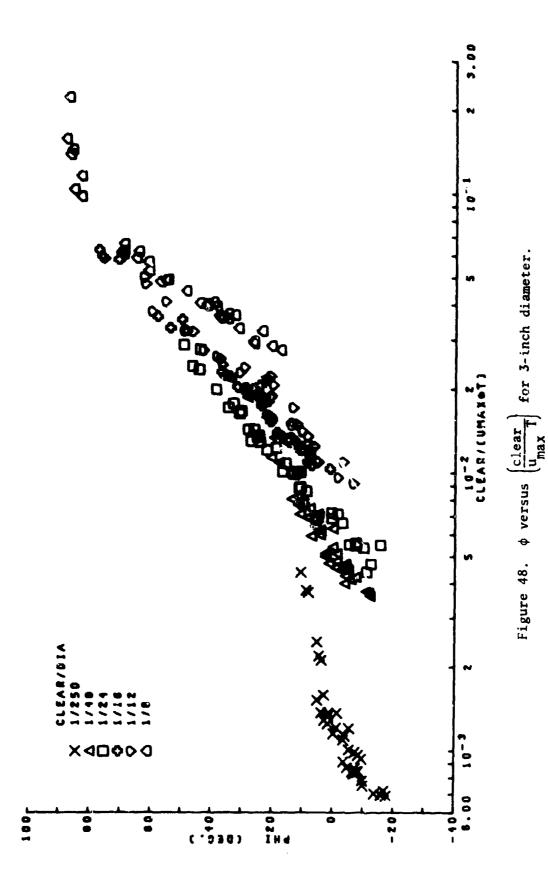
NOTE. -- Not all of these parameters are necessary to describe the system since some are redundant, but some may be more useful than others.

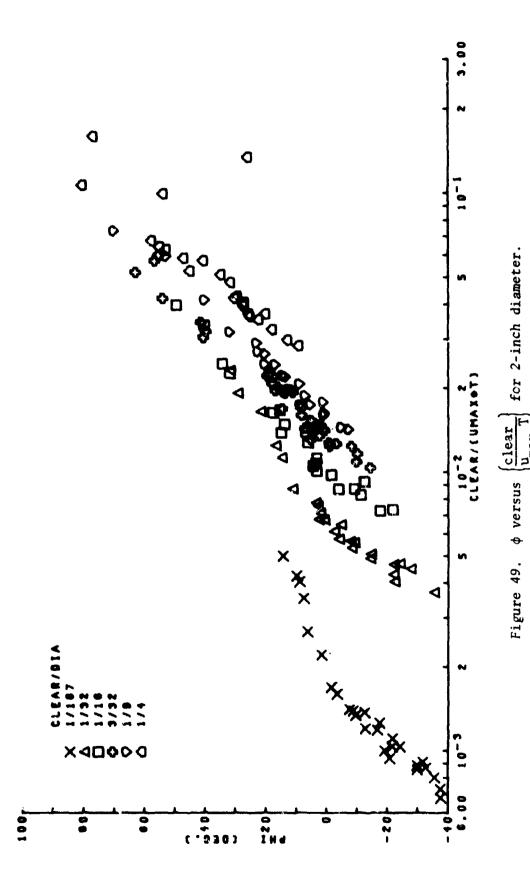
Since viscosity is an important variable involved in the choking phenomenon, the Reynolds number,  $u_{max}$  Dia/v, and a Reynolds number for the clearance,  $u_{max}$  clear/v, are also important parameters (where v = kinematic viscosity).

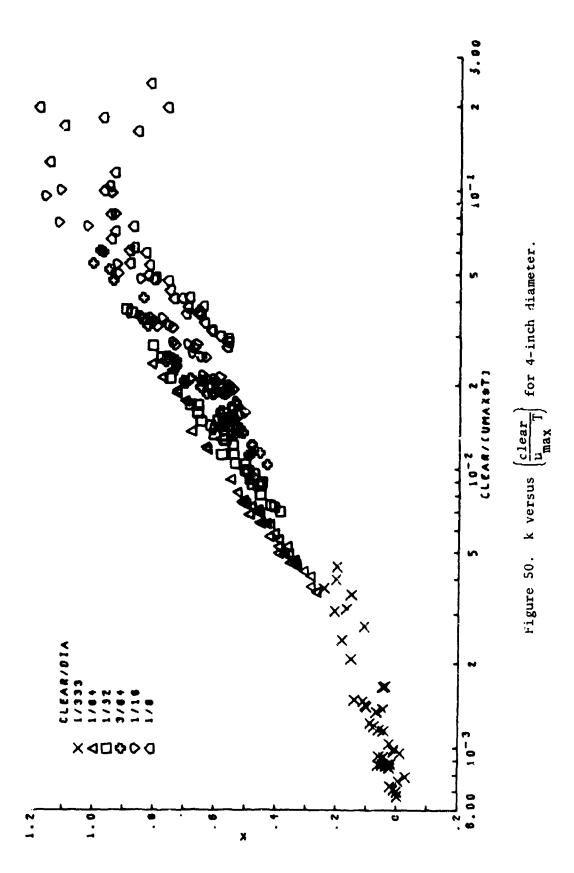
The dimensionless parameters, clear/ $u_{max}$  T,  $u_{max}$  T/Dia,  $u_{max}$  clear/v, and  $u_{max}$  Dia/v, were plotted versus the lift force parameters,  $C_L$ ,  $\phi$ , k,  $C_L(1-k)$ , and  $C_L(k)$ , for constant values of the relative clearance, clear/Dia. The correlation was not good with the parameters involving the coefficient of lift ( $C_L$ ,  $C_L(1-k)$ , and  $C_L(k)$ ). However, good correlation was found between several of the dimensionless parameters and the quantities  $\phi$  and k.

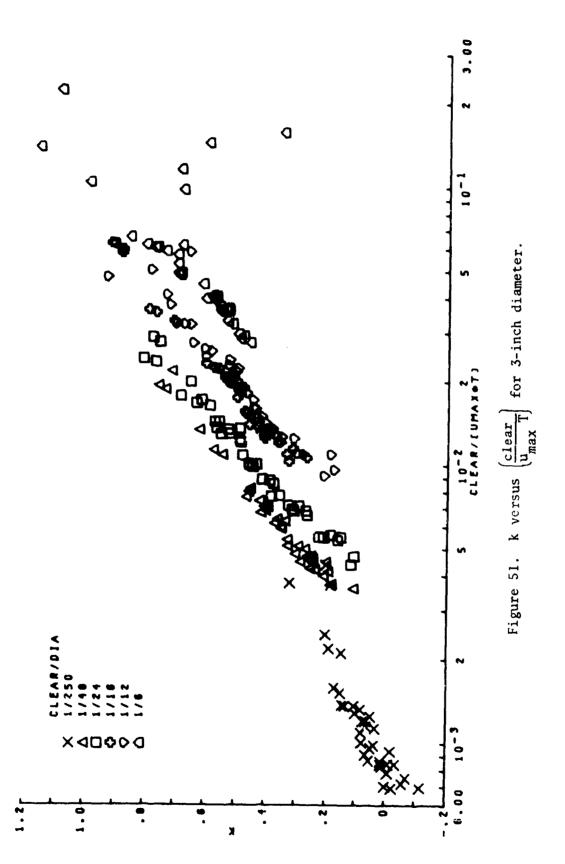
The parameter, clear/ $u_{max}$  T, exhibited the best correlation with both  $\phi$  and k for each relative clearance, although there was some variation in these relationships for the data corresponding to the different pipe diameters (see Figs. 47 to 52). Although the differences are not large, the data do indicate the presence of a scale effect in these relationships.

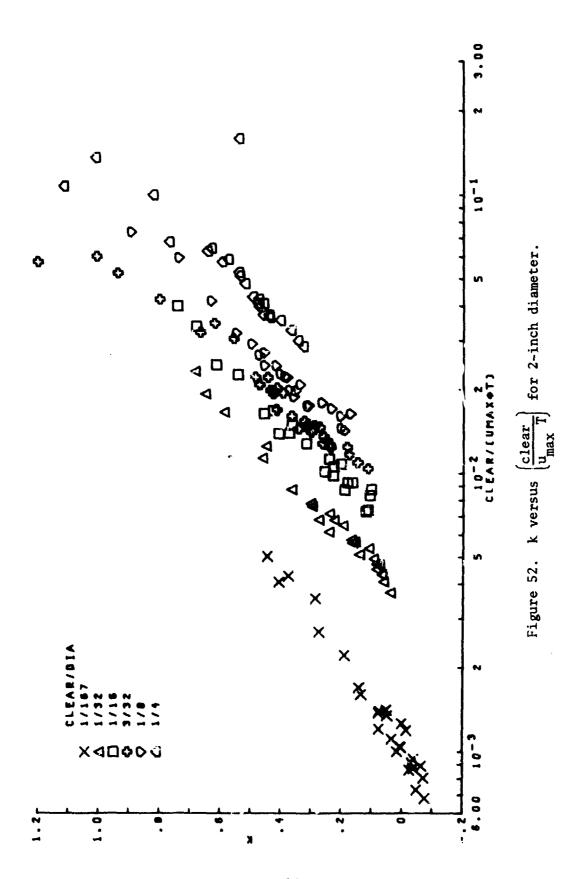












φ and k were also correlated with the Keulegan-Carpenter parameter,  $u_{max}$  T/Dia. However, these relationships were not the same when the data corresponding to a given relative clearance were compared for different pipe diameters. The relationships were the same for a given absolute clearance, rather than a relative clearance (clear/Dia). These relationships are shown in Figures 53 and 54 for the combined data from all three pipe diameters.

The parameter,  $u_{max}$  clear/v, demonstrated correlation with both  $\phi$  and k, but these relationships also exhibited a scale effect, such that the relationships for a given relative clearance were not the same when comparing the data for different pipe diameters. Figures 55 and 56 are examples of these relationships for the 4-inch-diameter pipeline.

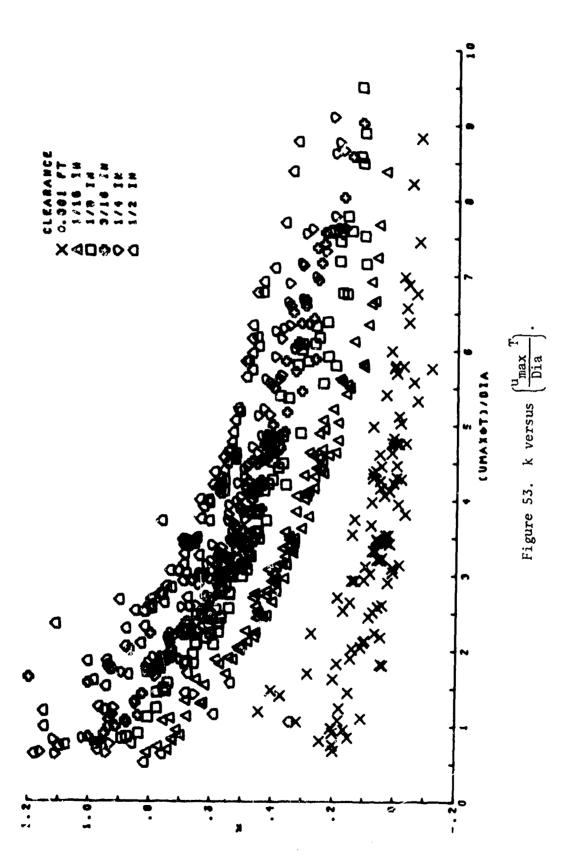
Correlation between the Reynolds number,  $u_{max}$  Dia/v, and the parameters,  $\phi$  and k, was not good, especially when comparing the data for the different pipe diameters.

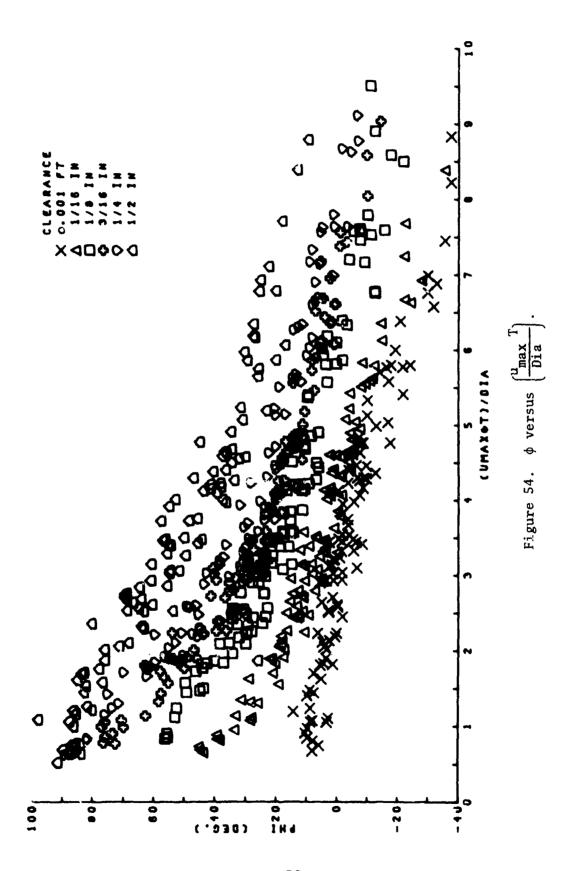
Since none of the above dimensionless parameters alone could be used to determine a value of  $\phi$  or k for any given pipe diameter, clearance, and wave condition due to the presence of scale effects, several of the parameters were combined in various ways to form different dimensionless parameters containing all four of the important variables (clear, Dia,  $u_{max}$ , and T). An attempt was made to find a single parameter containing all of the important variables that was well correlated with  $\phi$  or k for all wave conditions, pipeline sizes, and configurations.

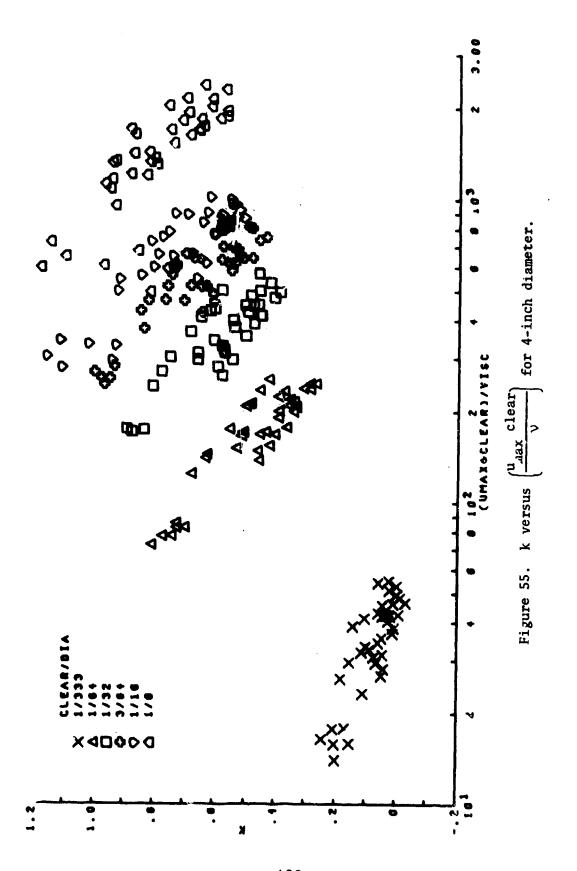
Several relationships were found that exhibited good correlation for all the wave and pipeline conditions tested. However, since this is a model study and, therefore, limited to lower values of the Keulegan-Carpenter parameter and Reynolds number than prototype design situations in the ocean, caution should be used in extrapolating these results.

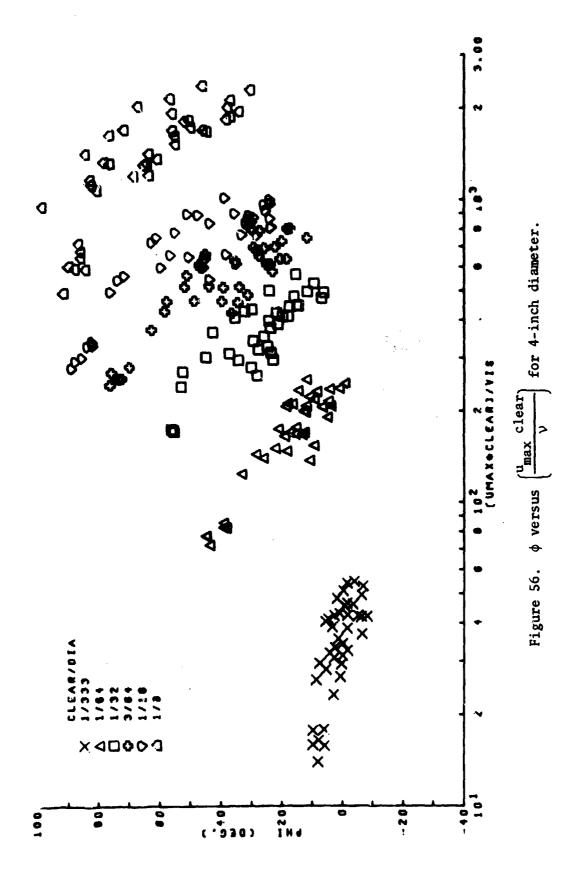
The dimensionless combination, (clear/ $u_{max}$  T) (Dia/ $u_{max}$ T), demonstrated the best correlation with both  $\phi$  and k for all conditions tested. These relationships are given in Figures 57 and 58. Since both k and  $\phi$  define the point at which choking occurs in the wave cycle, it appears that the choking phenomenon is directly dependent on the water particle excursions relative to both the pipe diameter, (Dia/ $u_{max}$  T), and the bottom clearance, (clear/ $u_{max}$  T).

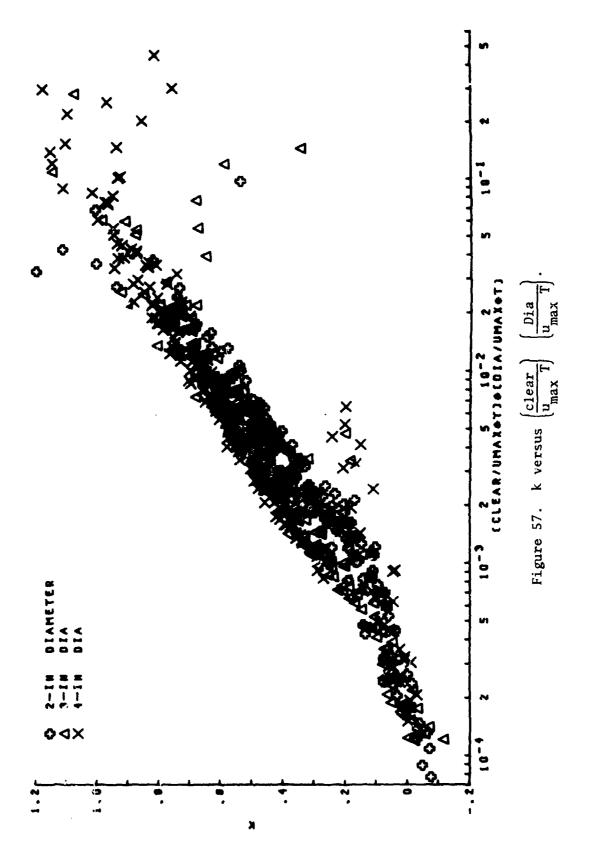
Although the parameter, (clear/ $u_{max}$  T), is equivalent to the ratio of the bottom clearance to the horizontal excursion of the water particles (differing only by the constant  $1/\pi$ ), the quantity ( $u_{max}$  T) should not be thought of only as defining the length of the water particle excursions. Both variables,  $u_{max}$  and T, are independently important in defining the choking phenomenon. The larger  $u_{max}$ , the somer the choking conditions will develop in the wave cycle for a given clearance and pipe diameter. Similarly, since the wave period, T, defines the duration of the horizontal flow in one direction, the larger the wave period, the somer choking will develop relative to the temporal length of the wave cycle.

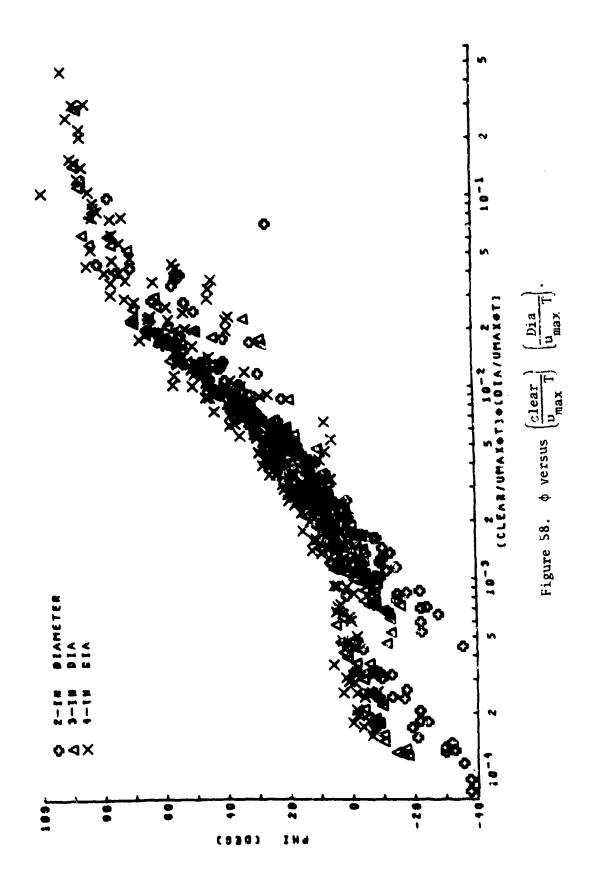












The slight amount of scatter in these plots in the vicinity of k=1 and  $\phi=90^\circ$  is due to the error in calculated values of  $\phi$  and k for the largest bottom clearances where the lift effect was small (as discussed above).

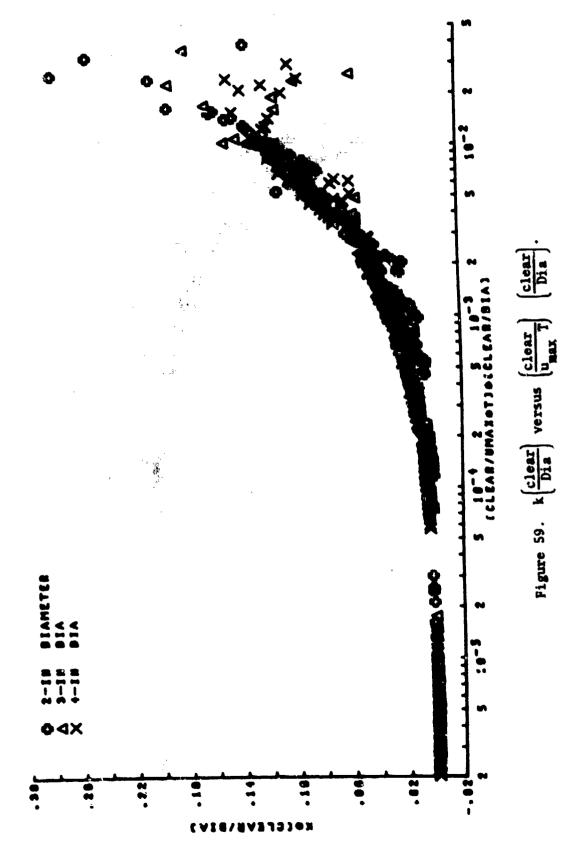
Larger values of the dimensionless combination, (clear/ $u_{max}$  T) (Dia/ $u_{max}$  T), than given in the plots would correspond to larger bottom clearances and pipe diameters relative to the maximum velocities, wave periods, and water particle excursions. For these conditions, the values of k and  $\phi$  would remain at 1 and 90°, respectively, while the lift affect would eventually diminish to zero with increasing values of this parameter. These trends are evident in the data taken at the largest bottom clearances (1 and 2 inches), although these data were not included in the shove plots.

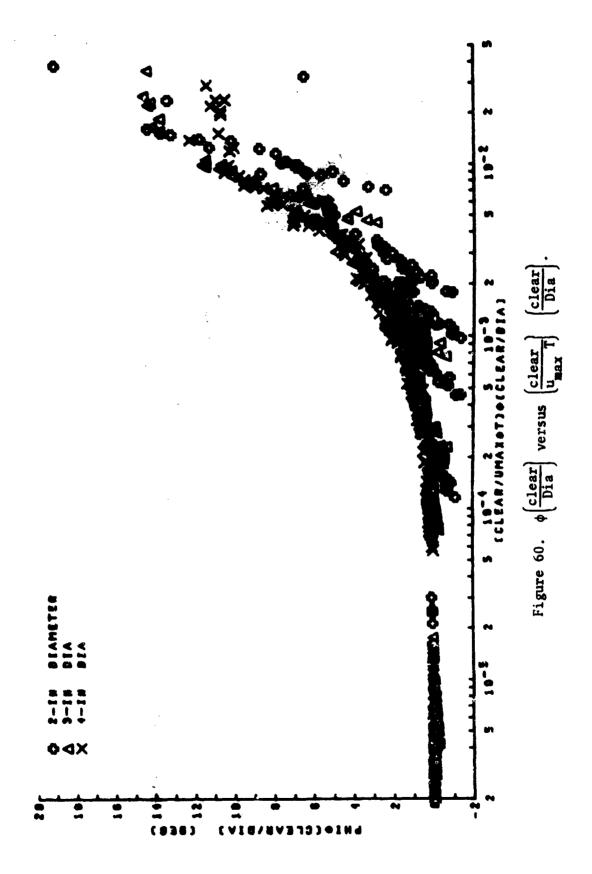
Similarly, lower values of the dimensionless parameter than given in the plots would correspond to higher maximum velocities, wave periods, and water particle excursions relative to the smallest bettom clearances and pipe diameters. So for lower values of this parameter, both k and  $\phi$  should remain at their defined minimum values of 0 and 0°, respectively, corresponding to lift forces acting in the upward direction only, with very little or no flow possible under the pipe section.

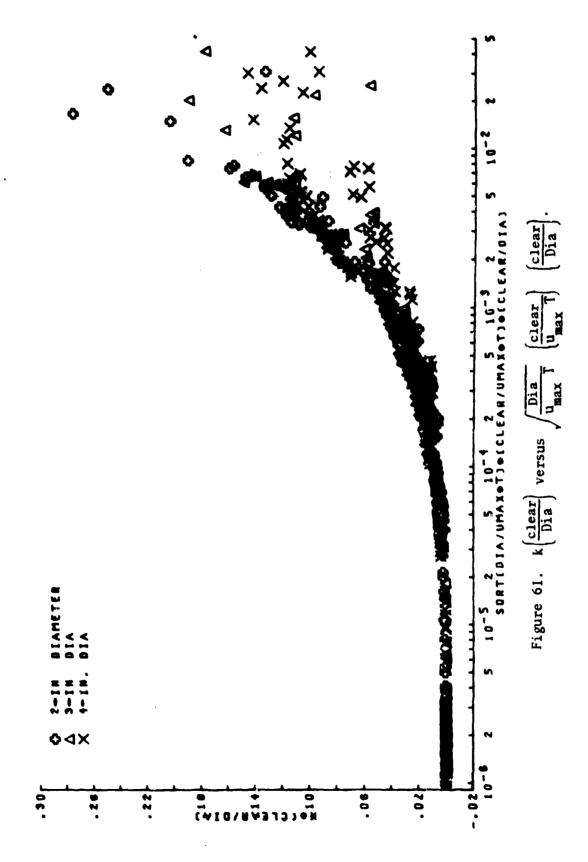
Although  $\phi$  was defined as varying from  $0^{\circ}$  to  $90^{\circ}$  only, negative values of  $\phi$  are exhibited in the data for the lowest values of the dimensionless parameters plotted. However, since most of these data points correspond to the smallest diameter pipeline model tested (2) inches), this could be partly due to experimental error, since the measured forces were smallest for the smallest model. Also, part of this discrepancy could be due to the difficulty of accurately defining the peak of the wave crest in the experimental wave records. This point was arbitrarily defined as the midpoint of the zero crossings on either side of the wave crest in the digitized data records. However, in some cases, the waves were not perfectly symmetrical, so the maximum elevation of the water surface did not coincide exactly with the midpoint of the zero crossings. This was especially true of the largest waves with the longest periods, which in the plotted relationships would correspond to the minimum values of the dimensionless parameters (at the lowest bottom clearance tested). Thus, the actual kinematics under these waves would be slightly out of phase with the calculated kinematics, resulting in an error in the calculated value of  $\phi$ . However, this source of error should be the same for the large-diameter models as for the smallest models.

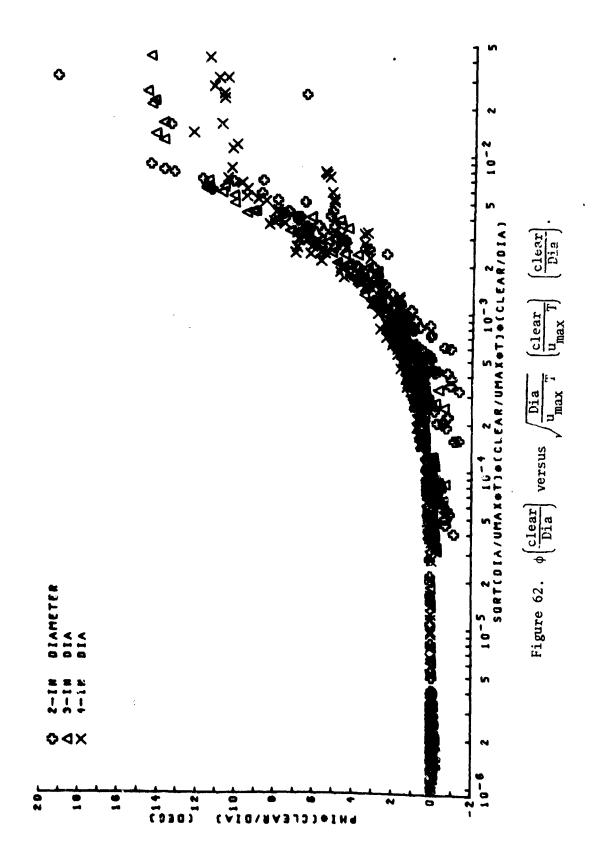
## 5. Relationships Between φ (clear/Dia) and k (clear/Dia) and Parameters Defining the Wave and Pipeline Conditions.

Many other useful relationships were found by multiplying  $\phi$  and k by the relative clearance, (clear/Dia), and plotting these dimensionless products versus various dimensionless parameters defining the wave and pipe conditions. Figures 59 to 62 are examples, although several other parameters also showed good correlation with  $\phi$  (clear/Dia) and k (clear/Dia).









Both  $\varphi$  (clear/Dia) and k (clear/Dia) are correlated with the dimensionless combinations (clear/u\_max T) (clear/Dia) and  $\sqrt{\text{Dia/u}_{\text{max}}}$  T (clear/u\_max T) (clear/Dia). However, k (clear/Dia) appears to be better correlated with the first parameter, while  $\varphi$  (clear/Dia) shows better correlation with the second parameter.

It is clear that for values of the dimensionless parameters lower than those shown on the plots, both  $\phi$  (clear/Dia) and k (clear/Dia) will remain at a value of zero. This would correspond to situations where the clearance was minimal relative to the hori ontal velocities, wave periods, and horizontal excursions of the water particles. Thus, both k and  $\phi$  would be expected to equal zero and 0°, respectively, and the relative clearance would either equal or approach zero.

Large values of the dimensionless parameters correspond to situations where the clearance is large relative to the horizontal velocities, wave periods, and horizontal excursions of the water particles. For these cases, k and  $\phi$  will remain at maximum values of 1 and 90°, respectively, while the relative clearance, (clear/Dia), will increase with increasing values of the dimensionless parameters. But as the relative clearance is increased beyond this point, the lift forces will decrease to zero, so extension of the plotted relationships to much larger values of the dimensionless parameters is of little value.

# 6. Relationships Between the Coefficients of Lift and Parameters Defining the Wave and Pipeline Conditions.

The coefficient of lift,  $C_L$ , the effective positive coefficient of lift,  $C_L(1-k)$ , the effective negative coefficient of lift,  $C_L(k)$ , and the maximum effective coefficient of lift (maximum of  $C_L(1-k)$  or  $C_L(k)$ ) were plotted against various combinations of the dimensionless parameters. The parameter, (clear/ $u_{max}$  T)(Dia/ $u_{max}$ T), which previously gave the best correlations with  $\phi$  and k also demonstrated the best correlation with  $C_L$ ,  $C_L(1-k)$ , and  $C_L(k)$ . However, these relationships exhibited more scatter than the previously discussed interrelationships between the coefficients of lift and the parameters, k and  $\phi$ , so it is suggested that the previously discussed relationships be used for design purposes.

# 7. Relationships Between the Lift Forces and Parameters Defining the Wave and Pipeline Conditions.

As with the coefficient of lift, the total lift force ( $F_L$  = 1/2  $C_L \rho$  A  $u_{max}^2$ ) can be partitioned into the maximum positive lift,  $F_L(1-k)$ , and the maximum negative lift,  $F_L(k)$  (Fig. 6). These three forces, as well as the maximum lift force (maximum of either  $F_L(1-k)$  or  $F_L(k)$ ) were plotted against various combinations of the dimensionless parameters. Only one relationship exhibited good correlations for the data from all three diameters plotted together. This was the Reynolds number,  $u_{max}Dia/\nu$ , versus the maximum lift force (either  $F_L(1-k)$  or  $F_L(k)$ , whichever is greater) (Fig. 63).

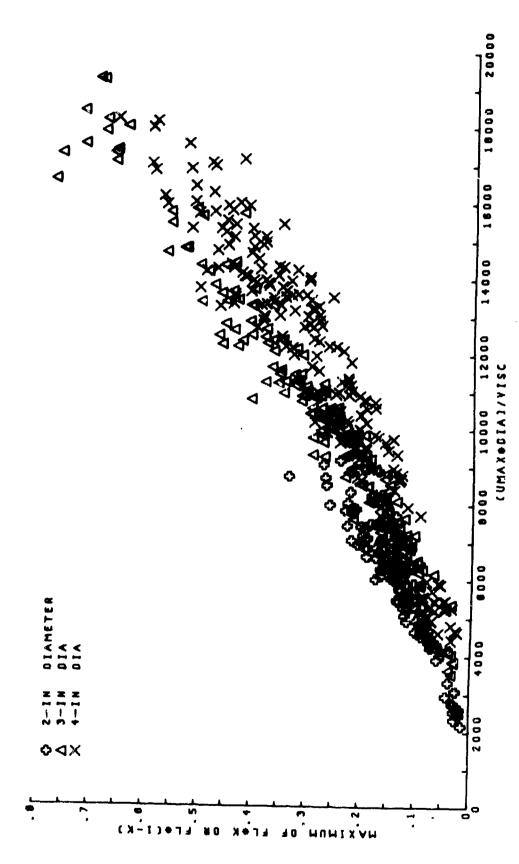


Figure 63. Maximum lift force (positive or negative) versus the Reynolds number.

This relationship shows that for any pipe diameter, orientation angle, or bottom clearance, the maximum lift force increases with the Reynelds number in a regular manner, at least over the range of the data in this investigation. The maximum lift force may occur in either the upward or downward direction, depending on the magnitude of the bottom clearance relative to the wave conditions and pipe size. This relationship does not hold for the maximum upward lift or maximum downward lift alone, but only for the largest of these two forces in any given situation.

8. Relationships Involving the Vertical Coefficients of Mass and Drag and the Vertical Inertial and Drag Forces.

Both the vertical coefficient of mass and the vertical inertial forces were plotted against several dimensionless parameters defining the wave and pipeline conditions, but no useful relationships were found. This is not surprising when considering that the vertical inertial forces are relatively small, and thus subject to error from the transverse eddy-induced forces which were not accounted for in the least squares analysis.

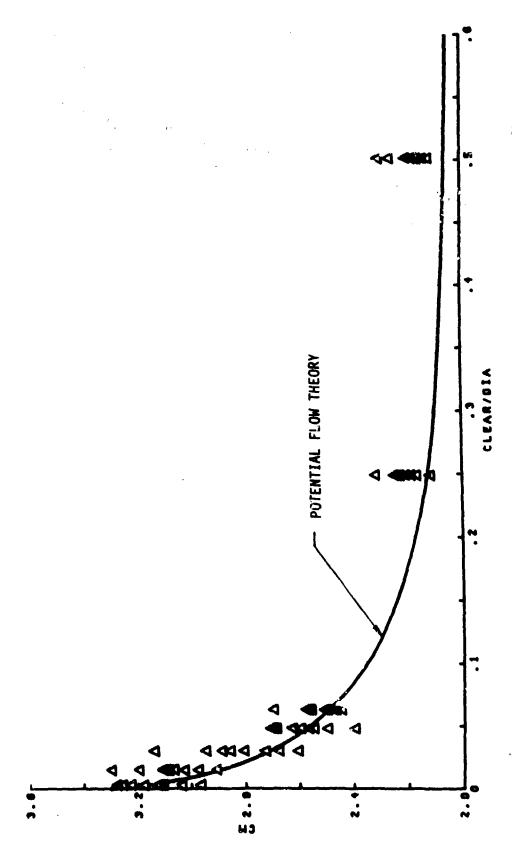
No attempt was made to plot relationships involving the vertical drag forces or drag coefficients, since these forces were negligible.

9. Relationships Between the Horizontal Coefficient of Mass and Parameters Describing the Wave and Pipeline Conditions.

A limited number of horizontal force data were taken using the 4-inch-diameter two-dimensional model. Values of  $C_{\rm M}$  and  $C_{\rm D}$  were calculated from the least squares analysis, and an attempt was made to relate these coefficients to various dimensionless parameters describing the wave and pipeline conditions.

Figure 64 shows the horizontal coefficient of mass plotted versus the relative clearance, clear/Dia, together with the potential flow solution for a circular cylinder in the vicinity of a plane wall subject to a uniform flow with constant acceleration (Grace, 1974). The data follow the potential flow solution reasonably well, although for a given relative clearance, there appears to be some variation in the value of  $C_{\rm M}$  with varying wave conditions. Also, the wave force data give slightly higher values of the coefficient of mass for the highest bottom clearances tested. Although the experimental data are limited, they indicate that the potential flow solution may be very useful in determining a value for the horizontal coefficient of mass, at least for wave conditions where the inertial forces predominate over the drag forces.

However, since there was some variation in the values of  $C_{\underline{M}}$  for different wave conditions for the same relative clearance, an attempt was made to determine relationships between the horizontal coefficient of mass and the various dimensionless parameters defining the wave and pipeline conditions. Reasonably good correlations were found between



Comparison of the horizontal  $C_{\!\!M}$  with potential flow theory for a flow with constant acceleration. Figure 64.

several of the parameters. Figure 65 shows the relationship for  $C_M$  versus clear/ $u_{max}T$ .

## 10. Relationships Involving the Horizontal Coefficient of Drag.

The horizontal coefficient of drag was plotted against several dimensionless parameters, but no useful relationships were found. This was expected since the horizontal drag forces in this investigation were much smaller than the inertial forces, due to the limited horizontal excursions of the water particles relative to the diameter of the pipeline.

### 11. Example Problems.

GIVEN: A design wave with height, H = 10 feet and period, T = 10 seconds acts on a pipeline with a diameter, Dia = 8 feet in a water depth, d = 80 feet. The pipeline is oriented at an angle of 30° with respect to the wave crests. Section A of the peline is in contact with the bottom; section B spans the bottom at a clearance, clear = 6 inches.

FIND: For both sections A and B, find

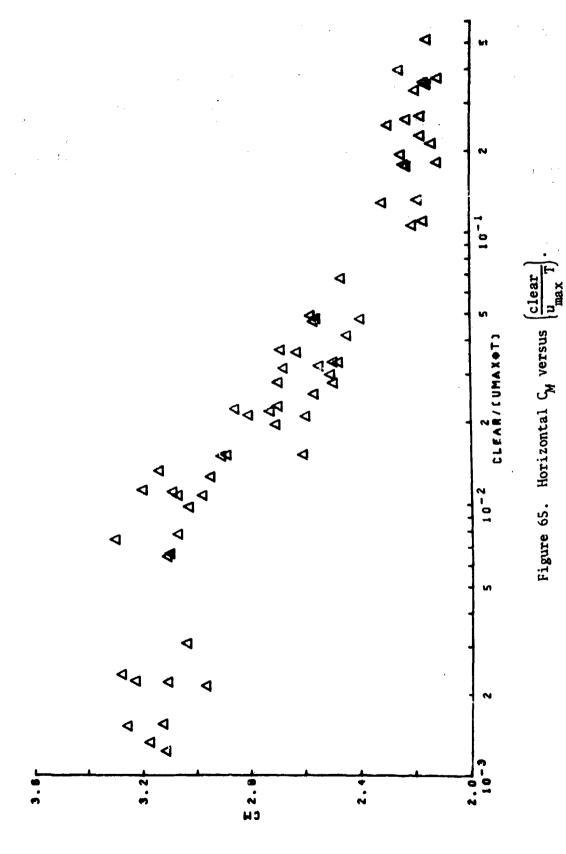
- (a) the values of the lift force parameters  $(C_L, \phi, and k)$ ;
- (b) the maximum positive and negative lift forces;
- (c) the positions of these maximum lift forces in the wave cycle; and
- (d) the lift force at  $\theta = 120^{\circ}$  in the wave cycle.

#### SOLUTION:

$$L_0 = \frac{g}{2} \frac{T^2}{\pi} = 5.12 (10)^2 = 512 \text{ feet}$$
  
 $\frac{d}{L_0} = \frac{80}{512} = 0.1562$ 

Using tables, 
$$\frac{d}{L}$$
 = 0.1885, so L =  $\frac{80}{0.1885}$  = 424 feet 
$$\sinh \frac{2\pi d}{L}$$
 = 1.481

z = distance from bottom to center of pipe
sections.



For section A (clear = 0)

z = 4 feet

$$\frac{z}{L} = \frac{4}{424} = 0.00943$$

From tables,  $\cosh \frac{2\pi z}{L} = 1.0017$ 

$$u_{max} = \frac{\pi H}{T} \frac{\cosh(\frac{2\pi z}{L})}{\sinh(\frac{2\pi d}{L})} = \frac{\pi(10)(1.0017)}{(10)(1.481)} = 2.12$$
 feet per second

Component of  $u_{max}$  perpendicular to the pipeline axis is

$$u_{max}$$
 (cos 30°) = (2.12)(0.866) = 1.84 feet per second

- (a) Since the pipe is in contact with the bottom, (clear = 0),  $\phi = 0^{\circ}$  and k = 0. From Figure 40,  $C_L = 4.5$ .
- (b) Maximum positive lift (per unit length)

$$F_L(1-k) = \frac{1}{2} C_L \rho A u_{max}^2 (1-k)$$
  
=  $\frac{1}{2} (4.5)(2)(8)(1.84)^2 (1-0)$ 

= 121.9 pounds per foot.

Maximum negative lift (per unit length)

Since k = 0, there is no negative lift, and the lift force is positive throughout the wave cycle.

(c) Since  $\phi = 0^{\circ}$ , the positive lift forces are maximum at  $0^{\circ}$  and  $180^{\circ}$  in the wave cycle (under the crests and troughs), corresponding to the points of maximum horizontal velocities.

The lift does not become negative, but diminishes to zero at 90° and 270°, the positions of horizontal flow reversal in the wave cycle.

(d) At  $\theta = 120^{\circ}$ 

$$F_{L} = \frac{1}{2} C_{L} \rho A u_{max}^{2} [\cos^{2} (\theta - \phi) - k]$$

$$= \frac{1}{2} (4.5)(2)(8)(1.84)^{2} [\cos^{2} (120^{\circ} - 0^{\circ}) - 0]$$

= 30.5 pounds per foot

For section B (clear = 6 inches)

$$z = 4.5$$
 feet

$$\frac{z}{L} = \frac{4.5}{424} = 0.0106$$

From tables,  $\cosh \frac{2\pi z}{L} = 1.0022$ 

$$u_{\text{max}} = \frac{\pi H}{T} \frac{\cosh(\frac{2\pi z}{L})}{\sinh(\frac{2\pi d}{L})} = \frac{\pi(10)(1.0022)}{(10)(1.481)} = 2.13 \text{ feet per second}$$

component of  $u_{max}$  perpendicular to the pipeline axis is  $u_{max}$  (cos 30°) = (2.13)(0.866) = 1.84 feet per second

(a) Use Figure 57 to determine a value for k

$$\left(\frac{\text{clear}}{u_{\text{max}}T}\right) \left(\frac{\text{Dia}}{u_{\text{max}}T}\right) = \frac{(0.5)(8)}{(1.84)(10)(1.84)(10)} = 0.0118$$

so from Figure 57, k = 0.67

and from Figure 58,  $\phi = 45^{\circ}$ .

Alternatively, either  $\phi$  or k could be determined from Fig. 39, once the other is known.

From Figure 46, for k = 0.67,

$$C_L(1-k) = 2.75$$
  
so  $C_L = \frac{2.75}{(1-0.67)} = 8.3$ .

(b) Maximum positive lift (per unit length)

$$F_L(1-k) = \frac{1}{2} C_L \rho A u_{max}^2 (1-k)$$
  
=  $\frac{1}{2} (8.3)(2)(8)(1.84)^2 (1 - 0.67)$ 

= 74.2 pounds per foot

Maximum negative lift (per unit length)

- 
$$F_L(k) = -\frac{1}{2} C_L \rho A u_{max}^2 (k)$$
  
=  $-\frac{1}{2} (8.3)(2)(8)(1.84)^2 (0.67)$   
= - 150.6 poun@ per foot

- (c) Since  $\phi$  = 45°, the positive lift forces are maximum at 0° + 45° = 45° and 180° + 45° = 225° in the wave cycle, and the negative lift forces are maximum at 90° + 45° = 135° and 270° + 45° = 515° in the wave cycle.
- (d) At  $\theta = 120^{\circ}$   $F_{L} = \frac{1}{2} C_{L} \rho A u_{\text{max}}^{2} [\cos^{2} (120^{\circ} 45^{\circ}) 0.67]$   $= \frac{1}{2} (8.3)(2)(8)(1.84)^{2} [\cos^{2} (120^{\circ} 45^{\circ}) 0.67]$

= - 135.6 pounds per foot

Again, it should be stressed that the relationships involving the lift force parameters,  $C_L$ ,  $\phi$ , and k, were determined from model studies conducted at much lower values of the Keulegan-Carpenter parameter and Reynolds number than those encountered in full-scale situations in the ocean. Therefore, caution should be used in extrapolating these results to prototype designs.

Further studies using a larger scale facility are necessary to evaluate the importance of scale effects in these relationships, to determine their limitations, and possibly to extend or modify them so they are valid for any scale.

#### IV. CONCLUSIONS

- 1. The traditional steady-flow lift force model, expressed as  $F_L = 1/2 \; C_L \; \rho \; A \; n^2$ , is not a suitable model for the description of wave-induced lift forces. This model assumes that the lift force acts in one direction only (upward or downward) throughout the entire wave cycle.
- 2. For pip lines located at a small clearance above the bottom, a viscous choking effect limits the maximum velocities through the constriction formed by the bottom clearance. Correspondingly, the pressure drop on the bottom side of the pipe section is also limited.

In contrast, the flow velocities and corresponding pressure drop over the top side of the pipeline are not limited. As the choking effect develops and the flow becomes restricted through the bottom

clearance constriction, more of the flow must be diverted over the top of the pipe section, resulting in a downward shift in the stagnation point, as well as an increase in the flow velocities and associated pressure drop over the top side of the pipeline.

The induced changes in the flow pattern, velocities, and associated pressure distribution over the pipe section due to choking through the bottom clearance constriction result in an upward lift force, rather than the downward lift force predicted by potential flow theory.

3. Thus, for an oscillatory wave-induced flow, the lift force acts downward in those parts of the wave cycle where the horizontal water particle velocities are not high enough to produce choking through the bottom clearance. In this case, the unrestricted flow is faster through the bottom clearance constriction than over the top of the pipe section, so the corresponding pressure distribution results in a negative lift toward the bottom boundary.

However, in those parts of the wave cycle where the horizontal velocities are sufficient to induce choking through the bottom clearance constriction, the lift force acts in an upward direction.

4. For a given pipe diameter and wave condition, as the bottom clearance is increased, higher velocities are necessary to produce the choking effect. Thus, the negative lift force can reach a greater magnitude and occur later into the wave cycle before the choking condition is induced.

Correspondingly, the positive lift that occurs only after the choking condition develops is limited to a smaller part of the wave cycle, and the maximum magnitude of these forces decreases with increasing clearance. In addition, since there is a small timelag involved in the development of the choking phenomenon and the transition from negative to positive lift, the maximum positive lift occurs later into the wave cycle, although its magnitude is diminishing.

5. All major features of the wave-induced lift force phenomenon can be described adequately by a modified lift force equation,  $F_L$  = 1/2  $C_L$   $\rho$  A  $u_{max}^2$  [cos² ( $\theta$  -  $\phi$ ) - k], where  $\phi$  represents a phase shift in the position of the maximum positive (upward) lift force relative to the point of maximum horizontal velocity at the center of the wave crest, and k represents the proportion of the total lift force cycle that acts in the negative (downward) direction. The values of  $\phi$  and k vary from 0° and 0, respectively, for the case of a pipeline touching the bottom, and increase with increasing clearance (for a given pipeline and wave condition) to maximum values of 90° and 1, respectively, when the pipeline is far enough from the bottom so that the choking condition does not develop.  $\phi$  = 0° and k = 0 correspond to lift forces that are positive throughout the wave cycle, with maximums occurring at the points of maximum horizontal velocity under the wave crests and troughs.  $\phi$  = 90° and k = 1 correspond to negative lift forces throughout the wave

cycle, with maximum downward forces occurring under the crests and troughs of the passing waves. These two cases represent the extreme conditions bounding the lift force phenomena. At any intermediate of earance between these limiting cases, both positive and negative aft forces will occur at different parts of the wave cycle, and the positions of the maximum upward and downward lift forces will not coincide with the positions of maximum horizontal velocities in the wave cycle.

In order to use this lift force model, values of the parameters,  $C_L$ ,  $\phi$ , and k, must be determined for the given set of wave and pipeline conditions. A model investigation was carried out to determine relationships between these parameters and various dimensionless parameters defining the wave and pipeline conditions.

6. A direct relationship was found between the lift force parameters,  $\phi$  and k. Relationships were also found between the coefficient of lift,  $C_L$ , and both  $\phi$  and k. In addition,  $C_L$  can be partitioned into the positive effective coefficient of lift,  $C_L$  (1-k), and the effective negative coefficient of lift,  $C_L$ (k). Both of these parameters are also related to both  $\phi$  and k. The correlation is better with k than  $\phi$  for the relationships involving  $C_L$ ,  $C_L$ (1-k), and  $C_L$ (k).

All of these relationships were the same for all pipe diameters, bottom clearances, and wave conditions tested.

- 7. The average value of  $C_L$  at k=0 and  $\phi=0^\circ$  (which corresponds to a pipeline in contact with the bottom with no clearance) is 4.5. This is the same as the potential flow solution for the lift force on a circular cylinder against a plane wall subject to a steady, inviscid flow parallel to the wall.
- 8. Maximum values of  $C_L$  occur at k=1/2 and  $\phi=30^\circ$ , where  $C_L=9$ . In the interval from k=0 to 1/2 and  $\phi=0^\circ$  to  $30^\circ$ , the effective positive coefficient of lift  $C_L(1-k)$  remains at approximately 4.5, while the effective negative coefficient of lift  $C_L(k)$  increases from 0 to 4.5. In the interval from k=1/2 to 1 and  $\phi=30^\circ$  to  $90^\circ$ ,  $C_L(1-k)$  decreases to 0, while  $C_L(k)$  increases to reach a maximum of about 6 or 7 at k=0.75 and  $\phi=45^\circ$ , and then decreases to a maximum of 4.5 at k=1 and  $\phi=90^\circ$ .
- 9. Using the above relationships between  $C_L$ ,  $\phi$ , and k, if either  $\phi$  or k is known, the remaining two parameters can be determined. Therefore, an attempt was made to find relationships between  $\phi$  and k and various dimensionless parameters defining the wave and pipeline, conditions.

The best correlation was found in the relationships between  $\phi$  and k and the parameter clear/ $u_{max}T$  for constant values of the relative clearance, clear/Dia. However, comparison of the data corresponding to the different pipe diameters indicates a slight scale effect is present.

 $\phi$  and k were also related to the parameter  $u_{max} clear/\nu$  for constant values of clear/Dia, although the scale effect was worse for these relationships.  $\phi$  and k showed very good correlation with the Keulegan-Carpenter parameter,  $u_{max}$  T/Dia, although these relationships were the same for a constant absolute clearance, rather than a constant relative clearance. Correlation between  $\phi$  and k and the Reynolds number was poor.

10. Because a scale effect was evident in the above relationships, several of the dimensionless parameters were combined to form new dimensionless parameters that contained all of the important variables (clear, Dia,  $u_{max}$ , and T). An attempt was made to find a single parameter that was related to either  $\phi$  or k for all wave and pipeline conditions tested in this investigation.

Both  $\phi$  and k showed very good correlation with the parameter (clear/u<sub>max</sub>T)(Dia/u<sub>max</sub>T). These relationships were valid for all pipe diameters, bottom clearances, orientation angles, and wave conditions tested.

In addition, the relative clearance was combined with both  $\phi$  and k to form the quantities  $\phi(\text{clear/Dia})$  and k(clear/Dia), both of which exhibited very good correlation with more of the dimensionless combinations than either  $\phi$  or k alone. k(clear/Dia) was best correlated with (clear/u<sub>max</sub>T)(clear/Dia).  $\phi(\text{clear/Dia})$  was also correlated with this parameter, but exhibited better correlation with the parameter  $\sqrt{\text{Dia}/u_{max}T}$  (clear/u<sub>max</sub>T)(clear/Dia).

- ll.  $C_L$ ,  $C_L(1-k)$ , and  $C_L(k)$  were correlated with the same parameter as  $\phi$  and k, (clear/ $u_{max}T$ ) (Dia/ $u_{max}T$ ). However, these correlations were not as good as the previous correlations between the coefficients of lift and k or  $\phi$ .
- 12. For a pipeline that is not parallel to the wave crests, the lift forces are apparently due only to the components of the horizontal water particle velocities perpendicular to the axis of the pipeline. Using this convention, consistent values of the coefficient of lift,  $C_L$ , are obtained for all angles of orientation. In addition, the relationships between the lift force parameters  $C_L$ ,  $\phi$ , and k, as well as relationships between these parameters and various dimensionless parameters defining the wave and pipeline conditions, are identical for all angles of orientation.
- 13. The maximum lift force  $(F_L(1-k))$  or  $F_L(k)$ , whichever is greater) exhibited good correlation with the Reynolds number,  $(u_{max}Dia/v)$ . This relationship did not hold for the maximum positive lift  $(F_L(1-k))$  or the maximum negative lift  $(F_L(k))$  alone, but only for the largest of these two forces in any situation. The relationship was the same for all diameters over the range of conditions tested.

14. The horizontal coefficient of mass,  $C_M$ , showed excellent agreement with the potential flow solution for a circular cylinder in the vicinity of a plane wall subject to a uniform flow with constant acceleration. These results indicate that the potential flow solution may be useful for selecting a value of  $C_M$  for wave-induced forces, at least for situations in which the inertial forces predominate over the drag forces. The horizontal  $C_M$  was also correlated with several of the dimensionless parameters defining the wave and pipeline conditions, such as the parameter clear/ $u_{max}T$ .

#### V. RECOMMENDATIONS FOR FURTHER RESEARCH

- 1. Experiments similar to this investigation should be carried out in a larger wave tank facility. This would allow the testing of larger diameter pipeline models as well as experiments at higher Reynolds numbers and higher values of the Keulegan-Carpenter parameter. Such an investigation is necessary to determine the validity of extrapolating the results of the present study to design situations in the ocean, and to point out any weaknesses or limitations of the proposed lift force model due to scale effects.
- 2. It would be of interest to perform experiments to evaluate the magnitude, phase, and frequency spectra of the vertical transverse lift forces due to eddy shedding for a horizontal cylinder subject to oscillatory horizontal flow velocities. This could be done by oscillating a test cylinder horizontally in still water away from a boundary, or by using a pulsating flume facility. The horizontal flow patterns at the bottom could be simulated, but without the lift force phenomenon due to the boundary. Only the transverse lift forces due to eddy shedding would act in the vertical direction, so the magnitude and time history of these forces could be easily measured.

A thorough analysis of the eddy forces for different pipe diameters and flow conditions would allow an evaluation of their importance relative to the Bernoulli-type lift forces, and at the same time explain some of the variations in the vertical wave force parameters calculated from an analysis which neglected the eddy forces because they could not be separated analytically because of their random nature. Adequate knowledge of the eddy forces would allow the addition of the eddy lift force term,  $F'_L = 1/2$   $C'_L$   $\rho$  A  $u_{max}^2$ , to the Morison equation with appropriate values of the coefficient  $C'_L$  for any given set of wave and pipeline conditions.

It should be noted that evaluation of the eddy forces for a cylinder away from a boundary would only give an approximate estimate of the eddy release phenomenon for a pipe located near the bottom. The presence of the bottom boundary changes the flow pattern, velocities, and pressure distribution around the cylinder, and therefore would be expected to have some effect on the formation and release of eddies.

- 3. Since the restricted flow through the narrow bottom clearance constriction is the critical part of the lift force phenomenon, the effect of pipeline roughness and bottom roughness on the wave-induced lift forces should be studied. This has practical significance, since the ocean floor is not necessarily smooth, and pipelines installed in marine waters may soon become encrusted with marine organisms, thus increasing their surface roughness.
- 4. The effect on the lift force phenomenon of a horizontal bottom current superimposed on the oscillatory motions of the wave action should be investigated.
- 5. The effect of porosity of the bottom on lift forces should also be investigated.

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#### APPENDIX A

### LEAST SQUARES ANALYSIS OF EXPERIMENTAL DATA

Using Morison's method for the calculation of wave forces on a pipeline, the vertical component of the wave-induced force may be expressed as:

$$F_{V} = (F_{I})_{V} + (F_{D})_{V} + F_{L} + F_{L}'$$

$$= C_{M} \rho V \frac{\partial v}{\partial t} + 1/2 C_{D} \rho A v |v|$$

$$+ 1/2 C_{L} \rho A u_{max}^{2} [\cos^{2} (\theta - \phi) - k]$$

$$+ 1/2 C_{L}' \rho A u_{max}^{2}. \tag{A1}$$

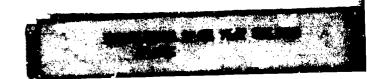
Since the transverse lift force associated with eddy shedding  $(F_L^i)$  is a random phenomenon, there is no way to handle its time history in analyzing a wave force record with several other forces occurring simultaneously. Because the Beinoulli-type lift forces were much larger than the eddy-associated forces for a pipeline located close to the bottom, the eddy-associated lift force term was dropped from the analysis.

The vertical components of the water particle velocities and accelerations near the bottom are small in comparison with the corresponding horizontal components. As a result, the vertical lift forces due to the horizontal components of the water particle velocities are generally much larger than the vertical drag and inertial forces. The drag forces are especially insignificant since the vertical excursions of the water particles near the bottom are smaller than the diameter of the pipeline.

Using linear wave theory, the kinematics of the wave-induced water particle motions with respect to time can be expressed as:

$$u = \frac{\pi H}{T} \frac{\cosh(\frac{2\pi z}{L})}{\sinh(\frac{2\pi d}{L})} \cos \theta$$
 (A2)

$$v = -\frac{\pi H}{T} \frac{\sinh \left(\frac{2\pi z}{L}\right)}{\sinh \left(\frac{2\pi d}{L}\right)} \sin \theta \tag{A3}$$



$$\frac{\partial v}{\partial t} = -\frac{2\pi^2 H}{T^2} \frac{\sinh \left(\frac{2\pi z}{L}\right)}{\sinh \left(\frac{2\pi d}{L}\right)} \cos \theta, \tag{A4}$$

where

H = wave height

T = wave period

L = wavelength

d = stillwater depth

z = vertical distance above the bottom

 $\theta = \frac{2\pi t}{T}$  = position of the wave cycle with respect to time.

Substituting these expressions into the vertical force equation yields:

$$F_{V} = -C_{M} \left[ \frac{\rho V 2\pi^{2} H}{T^{2}} \frac{\sinh \left(\frac{2\pi z}{L}\right)}{\sinh \left(\frac{2\pi d}{L}\right)} \right] \cos \theta$$

$$-C_{D} \left[ \frac{\rho A\pi^{2} H^{2}}{2T^{2}} \frac{\sinh^{2} \left(\frac{2\pi z}{L}\right)}{\sinh^{2} \left(\frac{2\pi d}{L}\right)} \sin \theta \right] \sin \theta$$

$$+C_{L} \left[ \frac{\rho A\pi^{2} H^{2}}{2T^{2}} \frac{\cosh^{2} \left(\frac{2\pi z}{L}\right)}{\sinh^{2} \left(\frac{2\pi d}{L}\right)} \right] \left[ \cos^{2} \left(\theta - \phi\right) - k \right]$$
(A5)

or

$$F_{V} = -C_{M}F_{MV}\cos\theta - C_{D}F_{DV}\sin\theta |\sin\theta| + C_{L}F_{L,v} [\cos^{2}(\theta - \phi) - k], \qquad (A6)$$

where

$$F_{M_V} = \frac{\rho V 2\pi^2 H}{T^2} \frac{\sinh \left(\frac{2\pi z}{L}\right)}{\sinh \left(\frac{2\pi d}{L}\right)}$$
(A7)

$$F_{DV} = \frac{\rho A \pi^2 H^2}{2T^2} \frac{\sinh^2(\frac{2\pi z}{L})}{\sinh^2(\frac{2\pi d}{L})}$$
 (A8)

$$F_{Lv} = \frac{\rho A \pi^2 H^2}{2T} \frac{\cosh^2 \left(\frac{2\pi z}{L}\right)}{\sinh^2 \left(\frac{2\pi d}{L}\right)}.$$
 (A9)

The expressions,  $F_{MV}$ ,  $F_{DV}$ , and  $F_{LV}$ , are constant for a given set of wave and pipeline conditions.

Linear wave theory was used in the analysis because, as discussed previously, there seems to be no obvious way of accurately describing the lift force phenomenon mathematically using higher order theories. Since the lift forces are much larger than the vertical drag or inertial forces, with the drag forces being almost completely insignificant, there was no point in using higher theories to express the vertical components of the drag and inertial forces.

For any vertical wave force record in which the corresponding wave and pipeline conditions are known, a least squares analysis can be performed on the data to determine the values of the unknown parameters  $C_L$ ,  $\phi$ , k,  $C_M$ , and  $C_D$  in the vertical wave force equation. The least squares analysis yields the values of these five parameters which best fit the force data throughout the entire wave cycle. This is accomplished by determining the values of these parameters which minimizes the sum of squares of the difference between the observed force data and the corresponding forces calculated with the mathematical model throughout a complete wave cycle.

Using the appropriate trigonometric identities,

$$\cos^{2} (\theta - \phi) = 1/2 + 1/2 \cos 2 (\theta - \phi)$$

$$= 1/2 + 1/2 (\cos 2\theta \cos 2\phi + \sin 2\theta \sin 2\phi), \quad (A10)$$

so the lift force equation can be expressed as:

$$F_L = 1/2C_L \rho Au_{max}^2 [1/2\cos 2\phi \cos 2\theta + 1/2\sin 2\phi \sin 2\theta + 1/2 - k]$$
 (A11)

or 
$$F_L = A_1 \cos 2\theta + B_1 \sin 2\theta + C_1$$
 (A12)

where 
$$A_1 = 1/4 C_L \rho A u_{max}^2 \cos 2\phi = 1/2 C_L F_{LV} \cos 2\phi$$
 (A13)

$$B_1 = 1/4 C_L \rho A u_{max}^2 \sin 2\phi = 1/2 C_L F_{LV} \sin 2\phi$$
 (A14)

$$C_1 = 1/2 C_L \rho A u_{max}^2 (1/2 - k) = C_L F_{LV} (1/2 - k).$$
 (A15)

In an analogous manner, the vertical components of the inertial and drag forces can be written as:

$$(F_{\bar{I}})_{V} = C_{M} \rho \left(\frac{\partial v}{\partial t}\right)_{max} \cos \theta = D_{1} \cos \theta$$
 (A16)

and 
$$(F_D)_v = 1/2C_D \rho A v_{max} \sin \theta | v_{max} \sin \theta | = E_1 \sin \theta | \sin \theta |$$
, (A17)

where 
$$D_1 = C_M \rho \left(\frac{\partial v}{\partial t}\right)_{max} = -C_M F_{Mv}$$
 (A18)

$$\left(\frac{\partial v}{\partial t}\right)_{\text{max}} = -\frac{2\pi^2 H}{T^2} \frac{\sinh\left(\frac{2\pi z}{L}\right)}{\sinh\left(\frac{2\pi d}{L}\right)} \tag{A19}$$

$$E_1 = 1/2 C_D \rho A v_{max} |v_{max}| = - C_D F_{Dv}$$
 (A20)

$$v_{\text{max}} = -\frac{\pi H}{T} \frac{\sinh \left(\frac{2\pi z}{L}\right)}{\sinh \left(\frac{2\pi d}{L}\right)}.$$
 (A21)

The total vertical wave force at any position  $\theta_{\underline{i}}$  in the wave cycle can then be written as:

$$F_{v}(\theta_{i}) = F_{L} + (F_{I})_{v} + (F_{D})_{v} = A_{1} \cos 2\theta_{i} + B_{1} \sin 2\theta_{i} + C_{1}$$

$$+ D_{1} \cos \theta_{i} + E_{1} \sin \theta_{i} | \sin \theta_{i} | . (A22)$$

The parameters  $A_1$ ,  $B_1$ ,  $C_1$ ,  $D_1$ , and  $E_1$  are constant for any given values of  $C_L$ ,  $\phi$ , k,  $C_M$ , and  $C_D$ , corresponding to the particular wave and pipeline conditions under consideration.

The sum of squares of the differences between the observed vertical forces,  $F_{ov}(\theta_i)$ , and the corresponding calculated forces,  $F_v(\theta_i)$ , is written as:

$$\sum_{i=1}^{n} [F_{v}(\theta_{i}) - F_{ov}(\theta_{i})]^{2} = \sum_{i=1}^{n} [A_{1} \cos 2\theta_{i} + B_{1} \sin 2\theta_{i} + C_{1} + D_{1} \cos \theta_{i} + C_{1} + D_{2} \cos \theta_{i}] + E_{1} \sin \theta_{i} |\sin \theta_{i}| - F_{ov}(\theta_{i})]^{2}.$$
(A23)

To minimize the sum of squares of the differences, the derivative of this expression is taken separately with respect to each of the five unknown parameters  $A_1$ ,  $B_1$ ,  $C_1$ ,  $D_1$ , and  $E_1$ , and the resulting expressions are set equal to zero, yielding a system of five simultaneous equations with five unknowns. The system of equations is then summed for each interval, i, over a complete wave cycle, and the resulting expressions are solved for the values of the unknown parameters  $A_1$ ,  $B_1$ ,  $C_1$ ,  $D_1$ , and  $E_1$  which thus minimize the sum of squares or the differences. The derivatives are:

$$\frac{\partial \{F_{\mathbf{v}}(\theta_{\mathbf{i}}) - F_{\mathbf{o}\mathbf{v}}(\theta_{\mathbf{i}})\}^{2}}{\partial A_{1}} = 2A_{1}\cos^{2} 2\theta_{\mathbf{i}} + 2B_{1}\sin 2\theta_{\mathbf{i}} \cos 2\theta_{\mathbf{i}}$$

$$+ 2C_{1}\cos 2\theta_{\mathbf{i}} + 2D_{1}\cos \theta_{\mathbf{i}} \cos 2\theta_{\mathbf{i}}$$

$$+ 2E_{1}\sin\theta_{\mathbf{i}} |\sin \theta_{\mathbf{i}}| \cos 2\theta_{\mathbf{i}}$$

$$- 2 F_{\mathbf{o}\mathbf{v}}(\theta_{\mathbf{i}}) \cos 2\theta_{\mathbf{i}} = 0 \qquad (A24)$$

$$\frac{\partial \left[F_{\mathbf{v}}(\theta_{\mathbf{i}}) - F_{\mathbf{o}\mathbf{v}}(\theta_{\mathbf{i}})\right]^{2}}{\partial E_{\mathbf{i}}} = 2A_{\mathbf{i}}\cos 2\theta_{\mathbf{i}} \sin 2\theta_{\mathbf{i}} + 2B_{\mathbf{i}}\sin^{2} 2\theta_{\mathbf{i}}$$

$$+ 2C_{\mathbf{i}}\sin 2\theta_{\mathbf{i}} + 2D_{\mathbf{i}}\cos \theta_{\mathbf{i}} \sin 2\theta_{\mathbf{i}}$$

$$+ 2E_{\mathbf{i}}\sin \theta_{\mathbf{i}} |\sin \theta_{\mathbf{i}}| \sin 2\theta_{\mathbf{i}}$$

$$- 2F_{\mathbf{o}\mathbf{v}}(\theta_{\mathbf{i}}) \sin 2\theta_{\mathbf{i}} = 0 \qquad (A25)$$

$$\frac{\partial \left[F_{\mathbf{v}}(\theta_{\mathbf{i}}) - F_{\mathbf{o}v}(\theta_{\mathbf{i}})\right]^{2}}{\partial C_{1}} = 2A_{1}\cos 2\theta_{\mathbf{i}} + 2B_{1}\sin 2\theta_{\mathbf{i}}$$

$$+ 2C_{1} + 2D_{1}\cos \theta_{\mathbf{i}}$$

$$+ 2E_{1}\sin \theta_{\mathbf{i}} |\sin \theta_{\mathbf{i}}|$$

$$- 2F_{\mathbf{o}v}(\theta_{\mathbf{i}}) = 0$$
(A26)

$$\frac{\partial [F_{\mathbf{v}}(\theta_{\mathbf{i}}) - F_{\mathbf{o}\mathbf{v}}(\theta_{\mathbf{i}})]^{2}}{\partial D_{\mathbf{i}}} = 2A_{\mathbf{i}}\cos 2\theta_{\mathbf{i}}\cos \theta_{\mathbf{i}}$$

$$+ 2B_{\mathbf{i}}\sin 2\theta_{\mathbf{i}}\cos \theta_{\mathbf{i}} + 2C_{\mathbf{i}}\cos \theta_{\mathbf{i}}$$

$$+ 2D_{\mathbf{i}}\cos^{2}\theta_{\mathbf{i}} + 2E_{\mathbf{i}}\sin \theta_{\mathbf{i}} | \sin \theta_{\mathbf{i}} | \cos \theta_{\mathbf{i}}$$

$$- 2F_{\mathbf{o}\mathbf{v}}(\theta_{\mathbf{i}})\cos \theta_{\mathbf{i}} = 0 \qquad (A27)$$

$$\frac{\partial [F_{\mathbf{v}}(\theta_{\mathbf{i}}) - F_{\mathbf{o}\mathbf{v}}(\theta_{\mathbf{i}})]^{2}}{\partial E_{\mathbf{i}}} = 2A_{\mathbf{i}}\cos 2\theta_{\mathbf{i}}\sin \theta_{\mathbf{i}} | \sin \theta_{\mathbf{i}} |$$

$$+ 2B_{\mathbf{i}}\sin 2\theta_{\mathbf{i}}\sin \theta_{\mathbf{i}} | \sin \theta_{\mathbf{i}} |$$

$$+ 2C_{\mathbf{i}}\sin \theta_{\mathbf{i}} | \sin \theta_{\mathbf{i}} |$$

$$+ 2D_{\mathbf{i}}\cos \theta_{\mathbf{i}}\sin \theta_{\mathbf{i}} | \sin \theta_{\mathbf{i}} |$$

$$+ 2E_{\mathbf{i}}(\sin \theta_{\mathbf{i}} | \sin \theta_{\mathbf{i}} |)^{2}$$

$$- 2F_{\mathbf{o}\mathbf{v}}(\theta_{\mathbf{i}})\sin \theta_{\mathbf{i}} | \sin \theta_{\mathbf{i}} | = 0 \qquad (A28)$$

For an even number of equally spaced time intervals,  $\theta_i$ , summed over a complete wave cycle, many of the terms cancel out due to the symmetry of these sinusoidal functions. Thus,

$$\sum_{i=1}^{n} \cos \theta_i = 0 \tag{A29}$$

$$\sum_{i=1}^{n} \cos 2\theta_i = 0 \tag{A30}$$

$$\sum_{i=1}^{n} \sin 2\theta_i = 0 \tag{A31}$$

$$\sum_{i=1}^{n} \sin \theta_{i} |\sin \theta_{i}| = 0 \tag{A32}$$

$$\sum_{i=1}^{n} \cos \theta_i \cos 2\theta_i = 0 \tag{A33}$$

$$\sum_{i=1}^{n} \cos \theta_{i} \sin 2\theta_{i} = 0 \tag{A34}$$

$$\sum_{i=1}^{n} \cos \theta_{i} \sin \theta_{i} |\sin \theta_{i}| = 0$$
 (A35)

$$\sum_{i=1}^{n} \cos 2\theta_{i} \sin 2\theta_{i} = 0$$
 (A36)

$$\sum_{i=1}^{n} \cos 2\theta_{i} \sin \theta_{i} |\sin \theta_{i}| = 0$$
 (A37)

$$\sum_{i=1}^{n} \sin 2\theta_{i} \sin \theta_{i} |\sin \theta_{i}| = 0$$
 (A38)

when taken over a complete wave cycle, As a result, only the squared terms, and the terms involving the observed forces,  $F_0$  ( $\theta_i$ ), remain in these equations. The resulting expressions are:

$$A_{1_{i=1}}^{n} \cos^{2} 2\theta_{i} - \sum_{i=1}^{n} F_{ov}(\theta_{i}) \cos 2\theta_{i} = 0$$
 (A39)

$$B_{1_{i=1}}^{n} \sin^{2} 2\theta_{i} - \sum_{i=1}^{n} F_{ov}(\theta_{i}) \sin 2\theta_{i} = 0$$
 (A40)

$$C_{1_{i=1}}^{n} i - \sum_{i=1}^{n} F_{ov}(\theta_{i}) = nC_{1} - \sum_{i=1}^{n} F_{ov}(\theta_{i}) = 0$$
 (A41)

$$D_{1_{i=1}} \sum_{i=1}^{n} \cos^{2} \theta_{i} - \sum_{i=1}^{n} F_{ov}(\theta_{i}) \cos \theta_{i} = 0$$
 (A42)

$$E_{1_{i=1}}^{n} \left( \sin \theta_{i} | \sin \theta_{i} | \right)^{2}$$

$$- \sum_{i=1}^{n} F_{ov}(\theta_{i}) \sin \theta_{i} | \sin \theta_{i} | = 0$$
(A43)

where n is the total number of values taken from the vertical wave force record (from an even number of equally spaced intervals per wave cycle, and over any number of complete wave cycles), and i is the number of the interval.

These expressions are easily solved for the unknown parameter  $^{\prime}$   $A_1$ ,  $B_1$ ,  $C_1$ ,  $D_1$ , and  $E_1$ , yielding:

$$A_{1} = \frac{\sum_{i=1}^{n} F_{ov}(\theta_{i}) \cos 2\theta_{i}}{\sum_{i=1}^{n} \cos^{2} 2\theta_{i}}$$
(A44)

$$B_{1} = \frac{\sum_{i=1}^{n} F_{ov}(\theta_{i}) \sin 2\theta_{i}}{\sum_{i=1}^{n} \sin^{2} 2\theta_{i}}$$
 (A45)

$$C_1 = \frac{\sum_{i=1}^{n} F_{ov}(\theta_i)}{n}$$
 (A46)

$$D_{1} = \frac{\sum_{i=1}^{n} F_{ov}(\theta_{i}) \cos \theta_{i}}{\sum_{i=1}^{n} \cos^{2} \theta_{i}}$$
(A47)

$$E_{1} = \frac{\sum_{i=1}^{n} F_{c}(\theta_{i}) \sin \theta_{i} |\sin \theta_{i}|}{\sum_{i=1}^{n} (\sin \theta_{i} |\sin \theta_{i}|)^{2}}.$$
 (A48)

With these relationships, the corresponding values of the parameters  $C_L$ ,  $\phi$ , k,  $C_M$ , and  $C_D$  in the vertical wave force equation which best fit the data throughout the complete wave cycle can be obtained.

The coefficients of mass and drag,  $C_M$  and  $C_D$ , are obtained directly from the parameters  $\mathcal{D}_1$  and  $E_1$ , since

$$C_{M} = -\frac{D_{\hat{i}}}{F_{MV}} = -\frac{\sum_{i=1}^{n} F_{ov}(\theta_{i}) \cos \theta_{i}}{F_{MV} \sum_{i=1}^{n} \cos^{2} \theta_{i}}$$
 (A49)

$$C_{D} = -\frac{E_{1}}{F_{DV}} = -\frac{\sum_{i=1}^{n} F_{OV}(\theta_{i}) \sin \theta_{i} |\sin \theta_{i}|}{F_{DV} \sum_{i=1}^{n} (\sin \theta_{i} |\sin \theta_{i}|)^{2}}. \quad (A50)$$

Since  $A_1 = 1/2$   $C_L$   $F_{Lv}$  cos 2 $\phi$  and  $B_1 = 1/2$   $C_L$   $F_{Lv}$  sin 2 $\phi$ , the phase shift parameter  $\phi$  can be obtained from:

$$\phi = 1/2 \ (2\phi) = 1/2 \ \tan^{-1} \left( \frac{\sin \frac{2\phi}{2\phi}}{\cos \frac{2\phi}{2\phi}} \right) = 1/2 \ \tan^{-1} \left( \frac{B_1}{A_1} \right)$$
 (A51)

since 1/2 C<sub>L</sub> F<sub>LV</sub> cancels out of the expression  $\left(\frac{B_1}{A_1}\right)$ . Thus,

$$\phi = 1/2 \tan^{-1} \begin{cases} \sum_{i=1}^{n} F_{ov}(\theta_{i}) \sin 2\theta_{i} / \sum_{i=1}^{n} \sin^{2} 2\theta_{i} \\ \sum_{i=1}^{n} F_{ov}(\theta_{i}) \cos 2\theta_{i} / \sum_{i=1}^{n} \cos^{2} 2\theta_{i} \end{cases} . \quad (A52)$$

After  $\varphi$  is known, the coefficient of lift,  $C_{\underline{L}},$  can be obtained from either  $A_1$  or  $B_1,$  since

$$C_{L} = \frac{A_{1}}{1/2 F_{LV} \cos 2\phi} = \frac{\sum_{i=1}^{n} F_{oV} (\theta_{i}) \cos 2\theta_{i}}{1/2 F_{LV} \cos 2\phi \sum_{i=1}^{n} \cos^{2} 2\theta_{i}}$$
(A53)

or

$$C_{L} = \frac{B_{1}}{1/2 F_{Lv} \sin 2\phi} = \frac{\sum_{i=1}^{n} F_{ov} (\theta_{i}) \sin 2\theta_{i}}{1/2 F_{Lv} \sin 2\phi \sum_{i=1}^{n} \sin^{2} 2\theta_{i}}.$$
 (A54)

Alternatively,  $C_L$  could be obtained from  $A_1$  and  $B_1$  directly without first solving for  $\phi$ , since

$$\sqrt{A_1^2 + B_1^2} = \sqrt{(1/2 C_L F_{LV} \cos 2\phi)^2 + (1/2 C_L F_{LV} \sin 2\phi)^2}$$

$$= \sqrt{(1/2 C_L F_{LV})^2 (\cos^2 2\phi + \sin^2 2\phi)}$$

$$= 1/2 C_L F_{LV}. \tag{A55}$$

Thus,

$$C_{L} = \frac{2 \sqrt{A_{1}^{2} + B_{1}^{2}}}{F_{Lv}}$$

$$\frac{2}{\sqrt{\left[\frac{\sum_{i=1}^{n} F_{ov}(\theta_{i}) \cos 2\theta_{i}}{\sum_{i=1}^{n} \cos^{2} 2\theta_{i}}\right]^{2} + \left[\frac{\sum_{i=1}^{n} F_{ov}(\theta_{i}) \sin 2\theta_{i}}{\sum_{i=1}^{n} \sin^{2} 2\theta_{i}}\right]^{2}}}{F_{Lv}} . (A56)$$

Finally, the parameter k can be obtained from  $C_1$  knowing the value of  $C_L$ , since

$$k = 1/2 - \frac{C_1}{C_L F_{LV}} = 1/2 - \frac{\sum_{i=1}^{n} F_{ov}(\theta_i) / n}{C_L F_{LV}}$$
 (AS7)

Thus, once the vertical wave forces on a pipeline are measured experimentally, the values of the parameters  $C_L$ ,  $\phi$ , k,  $C_M$ , and  $C_D$  of the vertical wave force equation which best fit the data throughout the entire wave cycle can be determined for the particular set of wave and pipeline conditions tested.

In an analogous manner, the least squares analysis car be applied to the horizontal wave force data. Omitting the horizontal force associated with eddy shedding, the horizontal component of the wave-induced force can be expressed as equation (2):

$$F_h = (F_I)_h + (F_D)_h = C_M \rho V \frac{\partial u}{\partial t} + 1/2 C_D \rho A u |u|.$$
 (2)

The data from the horizontal force measurements show that the horizontal eddy forces are insignificant in comparison to the horizontal drag and inertial forces for the experimental conditions tested.

Using linear wave theory, the horizontal components of the wave kinematics with respect to time can be expressed as:

$$u = \frac{\pi H}{T} \frac{\cosh \left(\frac{2\pi z}{L}\right)}{\sinh \left(\frac{2\pi d}{L}\right)} \cos \theta$$

$$\frac{\partial u}{\partial t} = -\frac{2\pi^2 H}{T^2} \frac{\cosh \left(\frac{2\pi z}{L}\right)}{\sinh \left(\frac{2\pi d}{L}\right)} \sin \theta. \tag{A58}$$

Substituting these expressions into the horizontal wave force equation yields:

$$F_{h} = C_{D} \left| \frac{\rho A \pi^{2} H^{2}}{2T^{2}} \frac{\cosh^{2} \left(\frac{2\pi z}{L}\right)}{\sinh^{2} \left(\frac{2\pi d}{L}\right)} \right| \cos \theta \left| \cos \theta \right|$$

$$-C_{\mathbf{M}} \left| \begin{array}{c} \frac{\rho V 2\pi^{2}H}{T^{2}} & \frac{\cosh \left(\frac{2\pi z}{L}\right)}{\sinh \left(\frac{2\pi d}{L}\right)} \\ \end{array} \right| \sin \theta \tag{A59}$$

or 
$$F_h = C_D F_{Dh} \cos \theta |\cos \theta| - C_M F_{Mh} \sin \theta$$
 (A60)

where

$$F_{Dh} = \frac{\rho A \pi^2 H^2}{2T^2} \frac{\cosh^2(\frac{2\pi z}{L})}{\sinh^2(\frac{2\pi d}{L})}$$
 (A61)

$$F_{Mh} = \frac{\rho V 2\pi^2 H}{T^2} \frac{\cosh\left(\frac{2\pi z}{L}\right)}{\sinh\left(\frac{2\pi d}{L}\right)}.$$
 (A62)

The expressions  $F_{\mbox{\scriptsize Dh}}$  and  $F_{\mbox{\scriptsize Mh}}$  are constant for a given set of wave and pipeline conditions.

The horizontal component of the wave-induced force can also be written as:

$$F_h = A_2 \cos \theta |\cos \theta| + B_2 \sin \theta$$
 (A63)

where

$$A_2 = C_D F_{Dh} = 1/2 C_D \rho A u_{max} |u_{max}|$$
 (A64)

$$u_{\text{max}} = \frac{\pi H}{T} \frac{\cosh \left(\frac{2\pi z}{L}\right)}{\sinh \left(\frac{2\pi d}{L}\right)}$$
 (A65)

$$B_2 = -C_M F_{Mh} = C_M \rho V \left(\frac{\partial u}{\partial t}\right)_{max}$$
 (A66)

$$\left(\frac{\partial \mathbf{u}}{\partial \mathbf{t}}\right)_{\text{max}} = -\frac{2\pi^2 \mathbf{H}}{\mathbf{T}^2} \frac{\cosh\left(\frac{2\pi z}{\mathbf{L}}\right)}{\sinh\left(\frac{2\pi d}{\mathbf{L}}\right)}.$$
 (A67)

Thus, the total horizontal wave force at any position  $\theta_{\boldsymbol{i}}$  in the wave cycle can be expressed as:

$$F_{h} (\theta_{i}) = (F_{D})_{h} + (F_{I})_{h}$$

$$= A_{2} \cos \theta_{i} |\cos \theta_{i}| + B_{2} \sin \theta_{i} \qquad (A68)$$

where the parameters  $A_2$  and  $B_2$  are constant for any given values of  $C_D$  and  $C_M$ , corresponding to the particular wave and pipeline conditions under consideration.

The sum of squares of the differences between the observed horizontal forces,  $F_{oh}$  ( $\theta_i$ ), and the corresponding calculated forces,  $F_h$  ( $\theta_i$ ), is written as:

$$\sum_{i=1}^{n} [F_h (\theta_i) - F_{oh} (\theta_i)]^2 = \sum_{i=1}^{n} [A_2 \cos \theta_i | \cos \theta_i |$$

$$+ B_2 \sin \theta_i - F_{oh} (\theta_i)]^2. \quad (A69)$$

The derivatives of this expression taken with respect to the unknown parameters  $A_2$  and  $B_2$  and set equal to zero give the following equations:

$$\frac{\partial \left[F_{h} \left(\theta_{i}\right) - F_{oh} \left(\theta_{i}\right)\right]^{2}}{\partial A_{2}} = 2 A_{2} \left(\cos \theta_{i} \left|\cos \theta_{i}\right|\right)^{2}$$

$$+ 2B_{2} \sin \theta_{i} \cos \theta_{i} |\cos \theta_{i}|$$

$$- 2 F_{oh} (\theta_{i}) \cos \theta_{i} |\cos \theta_{i}|$$

$$= 0$$
(A70)

$$\frac{\partial \{F_h (\theta_i) - F_{oh} (\theta_i)\}^2}{\partial B_2} = 2 A_2 \cos \theta_i |\cos \theta_i| \sin \theta_i$$

$$+ 2 B_2 \sin^2 \theta_i$$

$$- 2 F_{oh} (\theta_i) \sin \theta_i$$

$$= 0. \tag{A71}$$

Since  $\sum_{i=1}^{n} \sin \theta_i \cos \theta_i |\cos \theta_i| = 0$  for an even number of equally spaced intervals  $\theta_i$  summed over a complete wave cycle, the resulting summed expressions for the derivatives set equal to zero are

$$A_2 \sum_{i=1}^{n} (\cos \theta_i | \cos \theta_i |)^2 - \sum_{i=1}^{n} F_{oh} (\theta_i) \cos \theta_i | \cos \theta_i | = 0$$
 (A72)

$$B_{2} \sum_{i=1}^{n} \sin^{2} \theta_{i} - \sum_{i=1}^{n} F_{oh} (\theta_{i}) \sin \theta_{i} = 0.$$
 (A73)

These expressions are easily solved for the unknown parameters  ${\bf P}_2$  and  ${\bf B}_2$ , yielding:

$$A_{2} = \frac{\sum_{i=1}^{n} F_{oh}(\theta_{i}) \cos \theta_{i} |\cos \theta_{i}|}{\sum_{i=1}^{n} (\cos \theta_{i} |\cos \theta_{i}|)^{2}}$$
(A74)

$$B_{2} = \frac{\sum_{i=1}^{n} F_{oh} (\theta_{i}) \sin \theta_{i}}{\sum_{i=1}^{n} \sin^{2} \theta_{i}}.$$
 (A/5)

The coefficients of mass and drag which best fit the horizontal wave force data throughout the entire wave cycle can thus be obtained directly from the parameters  $\mathbf{A}_2$  and  $\mathbf{B}_2$  since

$$c_{D} = \frac{A_{2}}{F_{Dh}} = \frac{\sum_{i=1}^{n} F_{oh} (\theta_{i}) \cos \theta_{i} |\cos \theta_{i}|}{F_{Dh} \sum_{i=1}^{n} (\cos \theta_{i} |\cos \theta_{i}|)^{2}}$$
(A76)

$$C_{M} = -\frac{B_{2}}{F_{Mh}} = -\frac{\sum_{i=1}^{n} F_{oh} (\theta_{i}) \sin \theta_{i}}{\sum_{i=1}^{n} \sin^{2} \theta_{i}}.$$
 (A77)

## APPENDIX B

COMPUTER PROGRAM FOR VERTICAL LEAST SQUARES ANALYSIS (TWO-DIMENSIONAL DATA)

```
PROGRAM SMLDIA(INPUT, OUTPUT, PUNCH)
DIMENSION X(41), Y(41), Z(41), YS(41), YI(41), FI(41), FP(41), F(41),
IFY(41), RES(41), G(101), P(41), O(41), HI(4), HX(4)
TIEST CONDITIONS
PROGRAM SMLDIA(INPUT, JUTPUT, PUNCH)
DIMENSION X(41), Y(41), Z(41), Y(41), Y(41), FI(4)
IFV(41), RES(41), G(101), P(41), Q(41), HI(41, HX(4))
READ IN DIGITIZED DATA
B FEAD 1, UP, DN, UF, DF, DIA, XC
IF(UP) JO, 10, 9
PEAD 2, CL, T, N, XW, XF, C, WJ, FO, (HI(1), I=1, 4)
PEAD 3, FI(1), I=1, 40)
PEAD 3, FI(1), I=1, 40
PEAD 3, I=1
```

```
CALCULATE SUMS OF SQUARES AND PRODUCTS

SFF=0.

SFX=0.

SFY=0.

SFZ=0.

SFZ=0.

DT=.31415916536

A=-DT

DO 15 !=1,40

A=A+DT

X(1)=COS (A)

B=2.*A

Y(1)=COS(B)

Q(1)=SIN(B)

C=SIN(A)

Z(1)=C+ABS (C)

F(1)=C+ABS (C)

F(1)=FF+F(1)+F(1)

SFX=SFX+F(1)+X(1)

SFX=SFX+F(1)+X(1)

SFX=SFX+F(1)+X(1)

SFZ=SFZ+F(1)+X(1)

SFZ=SFZ+F(1)+X(1)

SFZ=SFZ+F(1)+X(1)

SFZ=SFZ+F(1)+X(1)

AX=SFX/20.

AY=SFY/20.

AQ=SFQ/20.

A7=SFY/20.
AX=SFX/20.
AY=SFY/20.
AY=SFY/20.
AQ=SFQ/20.
AZ=SFZ/15.
VX=AY=SFX
VY=AY=SFY
VQ=AQ=SFQ
VZ=AZ=SFZ
VR=SFF-VX-VY-VZ-VQ
CALCULATE COEFFICIENTS AND PARAMETERS PHI AND K
PHI=28.64789$ATAN2(AQ,AY)
IF (PHI=LT=-45.)7999,8999

7999 PHI=PHI+180.
8999 CONTINUE
YA=SQRT(AJ$AQ+AY$AY)
CLV=2.*YA/FLV
ANG=(ANGLE*PI)/180.
CLVA=CLVA/COS(ANG)
CLV=CLVA/COS(ANG)
CMV=-AX/FNV
CDV=-AZ/FDV
XK=-5-(SF/(CLV*FLV))
PPINT RESULTS OF ANALYSIS
PRINT 200
200 FGRHAT(1H1)
PRINT 4
FDRHAT(10X,6HT(SEC).7X,6HHT(FT).5X,9HWAVEL(FT).6X,7HDEP(FT).4X,9HU
IMAX(FPS).4X,9HCLEAR(FT).6X,7HDIA(FT).3X,12HCVL LGTH(FT).4X,8HANG(D
PRINT 300,T,H,XL,D,U,CL,DIA,XC,ANGLE
                                   IMAX(PPS),4X,9HCLERX(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FROTER(FI),DA,FR
```

```
DO 25 1=1.40

FV(I)=AxbX(I)+AQ+Q(I)+AY+V(I)+AZ+Z(I)+SF

RES(I)+FP(I)-FV(I)

CONTINUE
    RES(1)=FP(1)-FV(1)
25 CONTINUE
E=ABS(FP(1))
DO 59 1=2.4G
C=ABS(FV(1))
A=ABS(FP(1))
IF(C-B)57,57,56

86 BC
S7 IF(A-B)59,59,58
88 B2A
59 CONTINUE
B8=49./B
IF(B8-400.)62,44,44
62 IF(B8-400.)62,44,44
62 IF(B8-200.)63,45,45
63 IF(B8-100.)64,46,46
64 B8=50.
GO TO 47
4 B8=400.
GO TO 47
45 B8=200.
GO TO 47
46 B8=100.
A7 CONTINUE
         47 CONTINUE
DO 32 I=1,40
G(51)=1HI
                   G(SI)=1HI

J=SI++3B#FP(I)

K=SI++3B#RES(I)

G(J)=1H#

G(K)=1H+

PRINT 100.FP(I).FV(I).RES(I):6

FORMAT(IMZ,JF10-S,101AI/)

G(J)=1H
                     6(K)=1H
          G(L)=1H
32 CONTINUE
60 TO 8
10 CONTINUE
                 CONTINUE
END
SUBRCUTIME WAVEL(T.D.XL)
0=32.24747/6.283185
TPD=6.2831854D
IF(8-TPD) 2:23
DEEP WATER INITIAL ESTIMATE FOR WAVELENGTH
XL=8
GC TO 4
ALTON MATER INITIAL ESTIMATE FOR WAVELENGTH
          SC TO 4
SHALLOW WATER INITIAL ESTIMATE FOR WAVELENGTH
3 XL-T+SQRT(D+32-2)
4 XL-XL
XL-ROTANM(TPD/XLX)
IF(ABS (XLX-XL)-.005)5.4.4
5 RETURN
                      END
```

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#### APPENDIX C

COMPUTER PROGRAM FOR VERTICAL LEAST SQUARES ANALYSIS (THREE-DIMENSIONAL DATA)

```
PROGRAM WYPORC 3 (INPUT, TAPE1, QUYPUT, PUMB)

DIMENSION X (150), Y (150), Z (150), P (130), P (130),
```

```
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```
INA XIPP 31, 0X, SMCLEAR(PT), 0X, TMDIS(PT) AX, TMOSP(PT), 0X, SMMLE

JOST POSMAT (22, PT) 30, 2, PT) 30, 3, PT) 30, PT) 30, PT)

SOT POSMAT (22, PT) 30, 2, PT) 30, SMMLE

PRINT 107

SOT POSMAT (AX, SMTOTAL SAM SO, 0X, SMCGSEA, 10X, SMGCMEA, 11X, 4MCGGA, 0X, 10M

LETMA/S(MA) 6X, SMTARIANCE, VA

301 POSMAT (AX, SMTARIANCE), VA

301 POSMAT (AX, SMTARIANCE, VA

301 POSMAT (AX, SMTARIANCE, VA

301 POSMAT (AX, SMTARIANCE, VA

302 POSMAT (AX, SMTARIANCE, VA

303 POSMAT (AX, SMTARIANCE, VA

304 POSMAT (AX, SMTARIANCE, VA

305 POSMAT (AX, SMTARIANCE, VA)

306 POSMAT (AX, SMTARIANCE, VA)

307 POSMAT (AX, SMTARIANCE, VA)

308 POSMAT (STIR 3), PT 30, PT 30
                  601 [F(T.GT.R.4)603,602
600 JS=2
60 TO 606
402 JS=3
60 TO 606
603 JS=4
606 C=ABS(FMAX)
--ARAS(FMIH)
S=ARAX(A,C)
50=35./8
CC=30./MAX(1)
PRINT 26
26 POMAT(4X,7M FP(LB),3X,7M FY(LB),2X,8M RES(LB)/)
00 31 L=1:101
                              26 FQAMAT(4x,7M FP(LB),3x,7M FV(LB),2x,8M RI

OG 3[ L=1,101

5(L)=1,4

LI CONTINUE

DO 38 I=1,3,38

5(3)=1MI

MP(I)=MI(1017M1-1)

FV(I)=ARAK(I1+AQOQ(I)+AYOY(I)+AZOR(I)+SP

RES(I)=PP(I)-FV(I)

JJ=81,0880FP(I)

KK-82,0800FP(I)
```

## APPENDIX D

COMPUTER PROGRAM FOR HORIZONTAL LEAST SQUARES ANALYSIS (TWO-DIMENSIONAL DATA)

```
### PRIGRAM HORIZL (IMPUT, OUTPUT, PUNCH)

| DIMENSION 2(41), P(41), Y$(41), YI(41), FI(41), FP(41), F
```

```
SFF=0.
SFZ=0.
SFZ=0.
SFZ=0.
SFZ=0.
OT=31415926536
A=-DT
QD 15 [=1,40
A=4+DT
P(1)=3*N(A)
C=CUS(A+BSG (C)
F(1)=FP(1)
SFZ=SFZ+FF(1)*PF(1)
SFZ=SFZ+FF(1)*PF(1)
SFZ=SFZ+FF(1)*PF(1)
SFZ=SFZ+FF(1)*PF(1)
SFZ=SFZ+FF(1)*PF(1)
SFZ=SFZ+FF(1)*PF(1)
SFZ=SFZ-0.
AZ=SFZ/16.
AZ=SZ=Z+FF(1)*AZ=1)
CC CALCULATE COEFFICIENTS
CMM=AZ/FDM
CDM=AZ/FDM
CMM=CDM=AZ/FDM
CDM=AZ/FDM
C
```

KARANIA MANANDA MANAND

```
A=ABS(FMIN)
C=ABS(FMAX)
B=AMAX1(A,C)
BB=35./8
DO 31 L=1,101
G(L)=1H
31 CONTINUE
DO 32 I=1,40
FM(I)=CDM**FDM**Z(I)=CMM**FWH**P(I)
RES(I)=FP(I)-FM(I)
G(S1)=IHI
J=I1+*BB**FP(I)
K=S1-*BB**FH(I)
L=S1-*BB**FH(I)
L=S1-*BB**FH(I)
L=S1-*BB**FH(I)
C=S1-*BB**FH(I)
FM(I)=IH**
G(X)=IH**
G
```

## APPENDIX E

TABULATED VERTICAL FORCE DATA FROM TWO-DIMENSIONAL EXPERIMENTS

CLER	DļA	ANG	Ť	н	L	MAK	CLA	CLVA	CFAR	PHI	K	CMA	CDA
- 90 i - 90 i	140777777777777777777777777777777777777	0.	1:#	:318	7.18 7.003 0.53 0.50 10.70 12.47 12.71 13.01 19.00 7.00 10.23 10.00 10.00 11.0	UMAX 2541 04426 04033 05756 05756 05756 05756 05756 05756 05756 05757 05756 05757 05756 05757 05756 05757 05757 05757 05757 05757 05757 05757 05757 05757 05757 05757 05757 05757 05757 05757	<b>7:</b> 13	1:54	5-8-3 5-8-3 5-17-4 5-8-5 5-50 5-50 5-14-5 5-14-5 5-8-5 7-06 6-87-6 6-87-6 6-87-7	4.34 -74.31 -74.	#2144 #2144 #2144 #1444	2.01 1.02 1.32 2.04 2.22 -1.02 -1.02 -1.01 2.21 2.21 -2.21 -2.3 -2.3 -2.3 -2.3 -2.3 -2.3 -2.3 -2.3	434 - 440 434 - 440 34 - 440 27 - 746 27 - 27 21 - 27
. 021 . 021 . 021 . 021 . 021 . 021 . 021	• 167 • 167 • 167 • 167 • 167 • 167 • 167	0000000000	1.57 1.46 1.59 1.74 1.86 1.80 1.80 1.85 2.55	.507 .330 .401 .457 .244 .349 .368 .262	10-51 10-51 10-70 12-07 12-90 13-16 14-49 19-09	.0766 .4766 .5429 .6668 .3705 .5326 .6119 .2944 .4579	4.57 6.57 6.14 5.14 6.14 6.14 6.14 7	5.06 4.59 6.02 5.07 6.14 5.12 4.00 5.89 4.24	5.09 6.02 5.517 6.14 5.10 5.59 4.24 3.37	40.41 14.57 24.05 20.19 25.48 15.23 12.34 23.05 6.08	.4417 .3687 .8220	-1.71 3.07 .025 015 016 -2.21 .73 1.39 74 .04 2.42	50.55 -4.06 -5.87 -5.65 -29.12 56.52 -7.42 -7.42 -13.86 -29.21
.001 .001 .001 .001 .001 .001 .001 .001	.2008 .2008		1 · 40 1 · 51 1 · 51 1 · 51 1 · 58 1 · 58 1 · 58 1 · 58 2 · 50 1 · 50	.451 .429 .429 .322 .372 .372 .406 .236 .406 .181 .256 .360 .360 .360 .360 .360 .360 .360 .36	777996079226089898989898989898989898989898989898989	**************************************	9208097730217372930120000515989314409466	247 7288973821737293912945118989344099468 27.	9200097302:73729301200013998934409005 	-0.73 -15.03 -45.05 -50.79 -34.04 -21.11 -32.01 -17.12 -38.09 -4.30 -14.70 -14.70 -14.30 -7.40 -	-0349 -7289 -0289 -0180 -0808 -0808 -0808 -0808 -0808 -0809 -0909	-2.704.24 -2.704.25 -2.704	- 0. 00 - 0. 00 - 12. 70 - 32. 14 - 57. 19 - 13. 732 - 25. 19 33. 90 33. 90 33. 90 33. 25. 19 21. 893 - 24. 63 82. 04 82. 04 82. 04 83. 10 11. 23 - 13. 27 - 13. 10 17. 27 - 13. 17 61. 19 17. 23 17. 23
.001 .001 .001 .001	0000770;00;000000000000000000000000000	000000000000000000000000000000000000000	1487565781364859573869023: 4097747070723810023: 4097745048685032445548685032445548685	26 - 4 1 2 6 6 6 7 7 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8	9 - 27 8-18 - 28 - 28 - 28 - 28 - 28 - 28 - 28			4; 13 4: 35 4: 34	4-313582732856133146671452853338533385333385732856478	364189035188890364848769188129924658877912848478384888888888888888888888888888888	-0017410 -011807118711871197119711971197119711971197119	2.96 4.97 7.1 1.1 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1	93558440447782730674977675824584648484848484848484848484848484848484

CLER	DIA	ANG	Ŧ	н	L	XAPU	CL A	CLVA	CFAN	PHI	×	CMA	CDA
	• 33 3 • 33 3 5 5 5 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	000000	1.26	.207 .257 .277 .295 .064 .109	7.28 7.18 7.37 7.37 7.37 7.57 9.39 9.39	.2471 .2647 .2932 .0743 .1341 .2029	7.60 Y.62 6.26 6.63 6.42 3.64 6.01	7.000000000000000000000000000000000000	87.62532241602534552453233556.65356.6536.65	18.777110.201777110.201777110.201777110.201777110.201777110.201777110.201777110.201777110.20177.	.50284 .51747 .51747 .612747 .642747 .38527 .33527 .35527 .35527 .33437 .3347 .33437 .3347	2.9759688808075284721718. 2.975968880807528472718. 2.97522213132247227524272284722847228472284722847228	-75.61 -51.25 -68.12 -38.00
. 0055 . 0055 . 0055 . 0055 . 0055 . 0055 . 0055 . 0055 . 0055 . 0055	. 333 . 323 . 323	0.000	2 < 3 3 3 2 5 4 6 5 6 7 9 9 7 9 1 3 4 3 3 3 4 4 3 5 8 0 9 1 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	• 124 • 169 • 222 • 260 • 290 • 265 • 302 • 057	7.200 7.200	2463 2770 2770 2770 2770 2772 1970 1970 1970 1970 1970 1970 1970 1970	7878-0915275-09178-091776-091779-0915-09178-09178-09178-09178-0917776-09170-09	67867902152798678687878786977766777987888787869977766777766777987887878787	678	99.65 40.84 49.84 38.69 38.59 38.59 43.15 65.11 37.29 55.11 37.29 55.11 37.29 55.11 37.29 26.51 30.27	9925 8249 6089 7558 77023 67031 6723 6723 6725 6725 6726 6726 6726 6726 6726 6726 6726 6726 6726 6726 6726 6726 6726 6726 6726 6726 6727 6726 6727 6726 6727 6726 6727 6726 6727 6726 6727 6726 6727 6726 6727 6726 6727 6726 6727 6726 6727 6726 6727 6726 6727 6726 6727	2-76 2-11 -90 1-89 1-71 3-73 1-27 1-37 1-37 1-39 1-39 1-39	-200.19 -70.38 -77.46 -42.69 -55.45 -10.32 -41.28 -10.97 -92.42 -49.09 -73.91 -20.68 -23.67 -120.68 -11.65 -11.65 -11.65 -11.65 -11.65
.010	. 33 1 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3		4 4 4 5 4 5 4 5 4 5 4 5 4 5 4 5 4 5 4 5		7-67 7-67 9-67 9-78 14-57 12-65 12-6	.3673 .2396 .3924 .0716 .1855 .2658 .2668	2492495097770012950970000000000000000000000000000000000	3555564525559326838161459336692376593268369236836923683692368369237766686777776668697777776668697777777777		84.203 65.203	1 · 00467 · 0467 · 0467 · 8817 · 8817 · 1926 · 817 · 1927 · 19	2303683175362480073152471496842612612626222213222313222231223122312231	-22.17 -4.63 -10.76 -10.3.68 -36.69 -35.52 -9.12 -27.07 -32.81 -13.61 -27.83 -14.63 -27.83 -34.73 -34.74 -4.61 -2.9.36 -2.46.64 -2.9.36 -2.46.61

CLER	DIA	ANG	Ť	н	L	UMAX	CLV	CLVA	CFAN	PHI	κ.	CMA	CDA
.013	• 333 • 333 • 333 • 333 • 333 • 333 • 333 • 333 • 333 • 333	9	1.25 1.26 1.49 1.53 1.61 1.68 2.19 2.19	• 226 • 325 • 309 • 302 • 175 • 175 • 151 • 257 • 142 • 216	7.47 7.76 9.77 10.14 12.71 13.33 16.25 16.27 19.43	• 2218 • 3324 • 2655 • 3944 • 2625 • 2637 • 3977 • 2523 • 2496 • 3797	5.42 5.96 6.99 7.34 7.16 7.19 7.30 7.74 6.72 6.71	5.42 5.96 6.39 7.16 7.10 7.34 7.74 6.72 6.71	7.10 7.34 7.80 7.74 6.72 6.71	49.47 49.49 41.40 54.51 35.08 42.61 29.34	.4607 .7123 .5679 .6703	2.60 2.53 3.72 2.22 .27 .22 1.77	99.19 66.70 1139.26 1139.09 206.49 287.36
- Jibbb - Olde -	- 2333 - 2333	000000000000000000000000000000000000000	432543445564689068890324466458146577987 222222222444544544588888914557585 	0123110597 01271215957 02226105957 02226105959566263315767 022222680226801111107 0222222091111107	7.376877777.3367777.336777.3367777.3367777.33677777.33677777.336777777.336777777.336777777.336777777.336777777.336777777.336777777.3367779.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.3667776.36677777777	.0677 .1156 .26125 .20125 .20125 .20125 .20176 .201	2455767676463976465926597679 345757676767679764336597659776679764536597974667979 445764756597976679	24550674744397443005928312883169718407 2272427597056375647938258097718407 213427444344444455557445656697428859	245506767443974430592831285718407 32727242427597952378647936246977 3374244434444655555344566645466655	83.39 85.78 82.78 82.79 62.63 70.61 68.83 672.47 68.83 77.47 65.79 65.70	.9549 .9367 .9413 .8990 .8990 .8903 .9463 .9463 .9360	2.0100473-10021-001-001-001-001-001-001-001-001-0	-16.38 117.27 146.49
• 021	- 33 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	000000000000000000000000000000000000000	11111111111111111111111111111111111111	662601511394593625254155424514913755554 01122246991016227250259137086434707011102 01122246991016227250001111702011102	7.1384 77.337877777.33788777777777777777777777	.0630 .1175 .2175 .2217 .22127 .22517 .22517 .32737	24222240714563487117769781049764567553	7137166324 ••• 766728 ••• 766728 ••• 766728 ••• 76767 ••• 76778 ••• 76778 •• 76	22.2716663228 	84.65 86.47 81.51 81.75 81.75 81.70 78.05 77.40 85.16 85.16 85.16 85.29 77.29 65.84 66.83	.9808 .9924 .93171 .9534 .9534 .9736	403622462214572423252721506690280215 	-117.43 -04.28 -14.29 -24.75 -21.61 -19.03 -17.73 -18.26 -08.08 -32.74 -21.03 -32.74 -21.03 -21.09 -11.0

CLER	DIA	ANG	T	H	L	XAPU	CLV	CLVA	CFAN	PHI	K	CMV	CDV
	.333 .333 .333 .333 .333 .333 .333 .33	999999999999999999999999999999999999999	114146788472444385577889200513	.1239000000000000000000000000000000000000	7.00 7.00 7.00 7.00 7.02 7.02 7.04 7.04 7.04 7.04 7.04 7.04 7.04 7.04	.1992 .2412 .3037 .3471 .3436 .3791	1.30 .47 .97 .90 1.00 1.16	0483688 042688 0638 0638 0638 0638 0638 0638 0638 0		85.06 95.11 86.06 95.11 93.44 92.08 87.28 87.28 94.78 90.08 89.39 96.40 96.77 87.03 86.55 86.57 87.03 87.03 87.03	. 590 2 . 6 291 . 924 5 . 930 7 . 732 7 . 582 7 . 583 9 . 645 6 . 646 6 . 788 0 . 788 1 . 788 1 . 788 2 . 778 8 . 788 2 . 778 8 . 788 2 . 788 3 . 788	2632169622419344621277522142237966 0009U87097697687518752214223786 22214111111111111111111111111111111111	31370204413253702000000000000000000000000000000000
0.16777777777777777777777777777777777777	. 333 1 333		1.53 1.63 1.68 1.68 1.68 1.69 1.69 1.69 1.69 1.69 1.69 1.69 1.69	30244432667563138880106559031 1448443266756313888 223373737 223374442237032 2233737373737373737373737373737373737	7.037 7.037 7.037 7.037 7.037 9.048 9.056 7.7.99 9.056 9.07 9.089 9.09 9.09 9.09 9 9.09 9 9 9	. 0605 . 1262 . 1263 . 2797 . 3115 . 315 . 315 . 315 . 315 . 315 . 315 . 316 . 316	1.07	436923646200 •226923646170 •2269751641170 •350745170 •3507451 •3507451 •3507451 •3507451 •3507451		87.84 90.43 90.43 90.43 87.83 87.84	4.0400 .7354 .2342 .2342 .3327 .3227 .3227 .3227 .3227 .3227 .3227 .3227 .3227 .3237 .3237 .3237 .3247 .32777 .32777 .32777 .32777 .32777 .3277	1.0785 1.0785 1.0785 1.0795 1.0795 1.0860 1.	- 12.489 - 12.896 - 1.5.96 - 1.5.96 - 1.5.98 - 1.5.98 - 1.5.98 - 1.5.97 - 4.5.99 - 1.5.97

## APPENDIX F

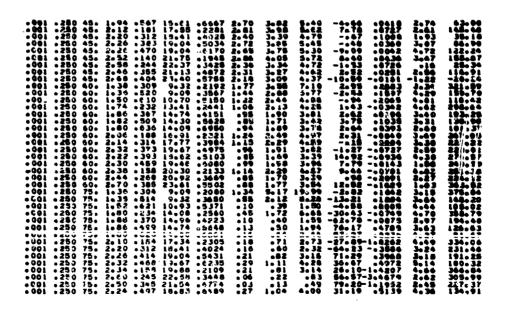
TABULATED VERTICAL FORCE DATA FROM THREE-DIMENSIONAL EXPERIMENTS

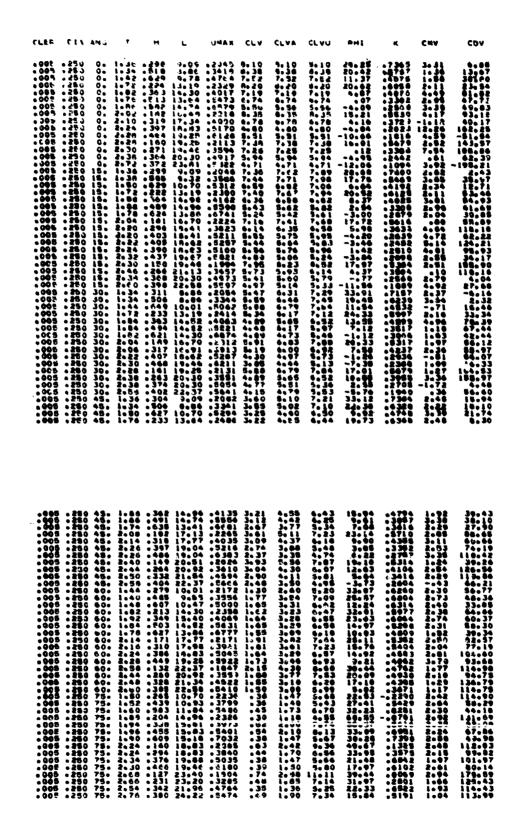
CLEP	DIA	ANG	T	н	Ļ	XAPU	CLV	CLVA	CFAG	DHI	K	CMV	CDV
. 901 . 901		0.00.00.00.00.00.00.00.00.00.00.00.00.0	4.5626020482006288886862240080940884464256626026 4.523083429482651531194307309522744350831194626 211122112221222112221122112211221122	620625518272728120028472862824102244469012881177 0203715462841866084771976172727471747287 0221235546446446465551221222257246446	31-34-7-153-3-14-6-7-7-15-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-1-	.2172 .21953 .21953 .21953 .31925 .31925 .31925 .31925 .31925 .31927 .31929 .31	225144418169868764712066886789367893767171110-56645-555464783334-566886789776717110-5844479	2251441816986478884442142628182960403181507647 -4404610545497964515425252960403181507647 -456544655544646424253425254418899 -456544654544644455455451111111111111111	225144181898999499999999999999999999999999	5.07.07.07.07.07.07.07.07.07.07.07.07.07.	- 1690 - 1890 - 1890 - 1791 - 0749 - 0749 - 0749 - 0749 - 0741 - 0749 -		- 25 2 c 7 1 - 28 2 c 7 1 - 28 2 c 7 1 - 28 3 5 6 6 1 - 74 0 5 6 6 7 0 2
. 005 . 005 . 005 . 005 . 005 . 005 . 005 . 005	• 167 • 167 • 167 • 167 • 167 • 167 • 167	400	2.40 2.22 2.00 1.46 2.42 2.18 1.92 1.36 2.39	. J83 . 402 . 442 . 632 . 200 . 349 . 549 . 195 . 232	12.29 10.51 16.26 10.24 20.72 18.19 15.40 20.90 15.40	4702	2.11 2.40 2.53 1.98 2.64 2.73 2.45 2.49 3.20	4.93 4.93 8.96 5.26 5.73 5.47 4.98 6.56 7.20	34.12.34.44.31.24.44.31.34.44.31.34.44.31.34.34.34.34.34.34.34.34.34.34.34.34.34.	-22.04.016.25.1.74.00.25.26.26.25.26.26.26.26.26.26.26.26.26.26.26.26.26.	.2470 .3090 .2714 .2714 .3257 .4073 .3094 .4422 .7151 .6340	-9.35 21.04 13.10 -4.72 23.39 17.05 -2.61 16.25 13.99	- 1002.442.38 - 153.462 - 244.38 - 28.031 - 144.62 - 28.031 - 144.62 - 124.62 - 124.62

CLER DIA ANG	т н	L U4AX	CFA CFA	4 CLYU PHI	K C	HA CDA
	2.82 .323 .323 .222 .22 .22 .22 .22 .22 .22	20-22 -5192 20-09 -6012 14-76 -4476 21-76 -4476 21-76 -4476 21-76 -4476 21-76 -4723 10-47 -5087 11-46 -3718 11-46 -3718 11-46 -3718 11-46 -3718 11-46 -2336 11-46 -2336 11-46 -2336 11-46 -2336 11-46 -2336 11-46 -3636 11-46	8.25	4 3.86 · 21.71 4 5.04 - 3.87 4 5.04 - 3.87 6 5.26 - 12.32 3 4.03 - 12.34 9 0.04 - 1.00 9 0.34 10	1000 10 1000 10 1000 10 1000 10 1000 10 1	00 '000 02 01 1 100 79 11 3 230 00
016 0167 0. 016 0167 0.	20	18.40	204-2-3-4-6-3-3-4-3-3-3-3-3-3-3-3-3-3-3-3-3-3	56 2.96 -9.6	3 . 1474   1474	12.46 -618.37  0.60 -849.10  7.42 -846.48  7.65 -802.28

CLER	DIA	446	т	H	L	044X	CLA	CLAT	CL AN	PHI	ĸ	C4A	CDA
- Q21 - Q21	.167 .167 .167 .167 .167 .167 .167 .167	0.00.00.00.00.00.00.00.00.00.00.00.00.0	27.07442080808084666600808080808462654 4.150.7450863870448666600080808080808080808 4.10.7450868870848 4.10.745086887086874 4.10.7450868874 4.10.7450868874 4.10.7450868874 4.10.7450868874 4.10.7450888888888888888888888888888888888888	100748170444884252087274984941472100010525384407844058440584405844058440585251505745858440584405845455155150574585846585858585858585858585858585858585	20.81 11.38	.2043 .2181 .2481 .3183 .4022 .4047 .4047 .5130 .5049 .5130 .5130 .5430	7.442441200000000000000000000000000000000	7.04248.11294467824.112174224.11217922.11217924.1122792.11227924.112279227922.112279227922.112279227922.112279227922.1122792279227922792279227922792279227922	7.00 8.24 9.51 9.51 7.39 6.56 7.30 7.56 7.17 7.10 6.17 7.10 7.10 7.10 7.10 7.10 7.10 7.10 7	30.00 30.07 10.00 117.00 117.00 12.00 10.23 70.24 10.23 70.24 10.20	- 1711 -	2. 02 3. 01 3. 01 3. 07 7. 75 7. 75 8. 03 1. 05 1.	20 v  377.60 202.96 144.65 140.69 -24.86 40.61 -25.86 40.61 -35.86 -4.35.86 -26.86 -26.86 -26.86 -26.86 -26.86 -26.86 -26.86 -26.86 -26.86 -26
		100000000000000000000000000000000000000	\$20000404749409074090740942442860020082048044050 5277294206520044296441874204826738428689989888888888888888888888888888888	0209812873496590387659413902110042978085489474641757955914559120471516924062951298292511714	22.17 19.09 119.41 15.41 15.41 15.41 15.41 15.41 16.40	.57Ji .6195 .6195 .5193 .5193 .5193 .4014 .2232 .2364 .2417 .2317 .2317 .2317 .3193 .6134	7.00429 7.00429 7.00429 7.00429 7.00429 7.007 7.	70042#0277828452#00829#8#10#00#7221#8#26#33040#4## 	700478002782848284817950897755678284828481795089891144828488818886544841750889776688888888888888888888888888888888	9. 73 13.30 25.30 25.53 26.53 27.45 31.23 36.25 41.00 60.38 45.46 46 46 46 46 46 46 46 46 46 46 46 46 4	- 3462 - 3462 - 3462 - 3462 - 4472 - 5243 - 5243 - 5717 - 6719 - 6871 -	13-17 9-435 7-625 7-627 9-40 9-40 1-53 9-40 1-53 9-40 1-53 1-53 1-53 1-53 1-53 1-53 1-53 1-53	23590982900982900982900982900982900982900982900982900982900982900982900982900982900982900982900982900982900982900980098

CLER	614	ANG	Ŧ	H	L	MAPU	CFA	CLAN	CF AN	PHI	ĸ	CMA	CDA
	:250		1.33	. 50	7.63	1401	P::	t::	1:::	3:33	:141	2:32	40.27
1651	:350	÷.			10.47	13001		1.11	i.ii	31.43	:6566		
iooi	. 250	ŏ.	1.74	163.	13.41	:3333	2.31	1.34	<b>6.34</b>	1:27	1141	3.11	17.40
	. 250	ō.	1.98	. 376	14.30		1.03	6. 63	6.03	1.47	.0424	2.64	34.31
	. 280	ō.	1.44	. 444	14.62	-5628	4. 04	4.44	4.04	-7.36	:0351	5-18	23.74
	. 540	٠.	1.70	. 6.38	. 13.64	.6771	4.37	4.37	4.37	-8.44	.0140	\$• 3 î	47.63
-001	. 580	Ģ.	5.00	• 104	17.13	. 5560	ş. 00	5.00	E-06	1.23	• • • • • • •	1.13	40.13
	.250	9.	\$ 10	. 316	17.34	· 74 24	20 53	5.25	1:13	11:33	-10464	1:11	25.09
	.250	×.	5.52	- 444	19.25		7:55	4.22	7.41	-14.05	066.5	7:38	414:66
	.250		3.44	1140	21.13	- 2040	2:75	5. 65	1:33	-11.61	0754	1.54	- 67.70
	. 250	- 3.	2.40	271	20.51	13681		4.69	4-49	-4.01	-0 327	-3.55	-146.72
.001	. 253	0.	2.40	. 36 6	27.41	4948	4. 08	4.05	4.08	-1:11	0323	1.01	-25.40
	- 250	0.	2.50	. 377		. 5375	4.40	4.45	4:05 4:48 5:54	- 100 14	1156	3.37	550.01
	- 289	15.	1.34	- 310	7.86	. 2049		5. 36	5.54	9.44	• 750 0	2.70	7.07
• 601	.520	15.	1+34	. 835	4.00	• 7 : 31	4. 22	5-19	2.55	4.52	+1904	4.79	49-72
	. 250	13:	1:31	. 232	10.24			\$.00	1.17	3.70	-1433	1:45	20.90 -1.34
	: 250	15	1:62	:366	14.52	:::::	7: 22	2:76	7.56	7. 20	:1336	1:4:	-1:5
	. 220	iš.	1.82	. 511	14.30	. 5442	31	4.46	4.42	-8.44	. 6761	4.41	27.04
. (0)	-560	15.	1.76	. f 36	13.64	. 4784	4.77	4.94	5.12	-4.96	. 0834	2.66	21.01
	. 253		2.06	. 184	14.91	. 2269	4.76	4.63	8.41	8.14	. 1693	1.47	-21.04
	• 52 3		3.10	• 200				4. 64	4.70	-8.44	.0743	5-48	17.54
. 00 1	. 250		2.24	-419	10.03			3.97	4-11	-7.30	1134	3-13	104-97
	:250		3.40		19.04	.4524	4.00	4. 14	4:47	-13.84	-: 8263	1:21	\$1.72
. 661		iš.	3.50			3404		3.22	4.37	-:31		7,31	31.27
	. 255	15.	2.54	. 353	21.96	14920		4.10	4.33	-7.10	. 6 6 6 6	\$6.39	68.77
. 001	. 250	15.	2.62	-405	22.78	.5722	4.03	4.17	4.38	-17:12	0217	.28	-175.66
-901			1.25	.315	9.63	.1997		5.14	5.93	10.59	•\$019	3.22	7.00
	• 25 0		1.30	. 531	9.86	. 3504	4-11	4.76	9-49	6.17	-4643	4.44	17043
.061	. 250		10.75	. 440	12.97	: 2379	3: ?!	4.5	1.27	2.92	1700	1:11	-43.66
.661			1.43	. 376	14.30	14100	3.41	4.20	4.95	L. 14	11964	<b>5:17</b>	-533
	250		1.04		14.52	3766	I. 37	3.80	4.44	-3.67	10001	1.2	-1.64
- COL	. 250	30.	1.00		14.00	.7042	34 مق	3447	4-30	-3.74	-0040	3.20	-0-01
. 991			2.02	• 1 39	15.40	. 2298	ăa 19	4.43	5.50	4.41	• 1318	1:11	12.34
	- 52 0			. 353	17.34	-4041	3.76	4.35	2.08	-1.68	•1030	2-10	10.00
.001			2.20	+423	14-41	-5462	3-43	3,67	4.55	-0.40	- 9493	1.70	-44-42
. 441	150	30.	<b>**</b> **	491	19.28	.6403	3. 20	3.70	*: 27	-9. <b>60</b> 3. 38	0000	1:17	130.57
	250	10.	3.43	. 284	20.51		3:33	3.74	7:44	7:4			-04.06
	250	10.	2.30	. 345	\$0.30	. 61 96	2. 94	3. 39	3. 42			9.00	-74.66
- 001	. 250	30.	2.65	.407	23.20	. 5784	2.99	Ja 34	3.66	-9.95	0405	-2.29	-71.20
.001	. 250	45,	1.40	. 297	7.55	.2176	2.62	3.71	5,25	7.94	.2793	2.36	\$6.04
	. \$20		1.30	. 506	3.75	. 3564	2.75	3.87	5.50	4. 25	.1989	4.19	\$4.32
	- 250		1.30	.037	10.70	.5360	2.50	3.44	5.15	2-12	-1461	1.91	34.69
.001	.250		1.75	.221	14.94	2386		3. 37 3. 30	4.77	1:41	:0158	3:13	-52:5}
					14.96			3.46	1.93	3. 90	.0114	1.46	42.11
				,,					-300	2000			





CLER GIA	ANG	*	H	L	UMAX	CLV	CLAN	CFAN	PHI	ĸ	CMA	CDA
.010 .250	٥.	1.30	.210	9.32	. 1550	5. 74	5.74	5.74	64.01	1.3744	2.46	-19.91
.010 .250	0.	1.30	.452	4.25	.3510	6.75	6.75	4.75	44.16	.7483	3.94	30.01
.010 .250		1.40	. 620	9.55	.4549	8.01	0.03	6.03	30.63	. 5465	4.10	56.65
.010 .250	0.	1:32	.001	11.44	6067	7.47	7.67	7.67	21.73	1.1315	2.61	23.76
1016 1280		1.76	417	13.46	.4503	7.34	7.36	7.36	26.91	1.3207	1.01	7.02
-010 -250	0.	1.00	. 600	14.00	(55)	7. 15	7.15	7.15	11.06	.3894	1.76	9, 37
.010 .25	• •	1.00	. 774	14.00		4.03	4.03	6.03		- 5661	7.54	5.08
.010 .25	9.	š• i š	• 177	17.55	. 5556	70 15	7.15	7.15 7.23	35.36 16.73	7879	7.03	19.38
.010 .2E		5:12	: 711	17.77	7025	5.60	5, 60	5.60	8.12	:::::	1:41	20.20
.010 .25	. i.	2.16	. 700	17.98	.9014	4:40	4.40	4.40	-10.04	.1623	11.54	-1.65
.010 .250	٠.	2+42	. 154	13.64 13.66 14.06 14.08 17.55 17.77 17.98 20.72	. \$101	5. 97	5. 97	5. 97	25 - 26	. 9338	-1.76	-00-01
-010 -25	9 0.	2.40	• 353	20.81	4 370	2. 17	8.03	5.63	9.82	:3568	1.59	120.13
1010 .25	A A-			20.02			A - A A	4,44	-7.40	. 2000	2.45	28.52
. 010 . 28	0.	2.52 1.36 1.36 1.36	. 479	21.73 9.32 9.32 9.32	. 14 29	30 67	3.47	3.67	-10-08	- 1140		99.40
.010 .25	15.	1 - 30	• \$13	3.35	- 1211	5. 24	6- 15	9. 50	43.07 43.07	.0026 7550	E-21	68.07
.010 .25	0 15. 0 15.	1.34	144	::33	:3333		9.27	0.02	44.34		#:#X	100.37
.010 .25	iš.	1.42	. 207	9.74	.4427	4. 01	8.30	Ba 59	34.44	46136	2.73	70.65
.010 .58	0 130	1.40	.591	9.55	.4338	4.65	4. 57	7.13	32.79	- 6 6 A [	1-17	20.96
.010 .25	0 15. 0 15.	1.42	. 694	9.74	- 5274	7. 33	7.99	7.03	27.88	. 5445	3.47	77.71
.010 .25		1.40	. 701	2. 38	. 2731	7. 78	6.05	0.34	24.77	. 5263		43.72
.310 .26	D 15.	1.72	.172	13.19	1747	4.17	6.37	1.42	46.03	1.1574		85.50
.010 .25	0 15. 0 15.	1.00	* 405	14.08	****	2: 11	5.14	0.34 6.62 6.65 6.17	24. 49		1.41	104.32
.010 .52	0 15.	1.43	783	14-10	-8457	B- 32	5.81	5.70	\$. 69	3102	3. 63	26.10
•010 • 75	ō i5.	2.00	. 170	17.13	.2114	7.56	7.82	8.10	37.46			77.68
• 010 · 15	0 13.	2.14	. 393	17.77	. 4986	6.12	6. 33 5. 31	4. 55	11.75	. 1567	-2.36	73.87
.010 .25	0 15. 0 15.	2.20	. 679	9.78 9.78 9.78 9.78 9.78 13.08 14.08 14.08 17.17 17.78	773	4.41	4.57	3.50 4.73	-7. <b>8</b> 4	1 644		30.10
.010 .25	ŭ iš.	2.40	. 160	18.451 20.551 20.555 9.555 9.558 13.664 14.78 117.94 117.94 117.4	.2165	i. ji	6.58	6.61	32.64	-8437	1.52	178.03
.010 .25	0 15.	2.40	• 330	Z0.51	.4472	•• 02	4.53	4.45	10.84	-4570	1.15	141-01
.010 .25	0 15. 0 15.	2.50	- 544	21013	-0215	3. 72	3.71	3.04 3.09 5.93 6.11	-9.44	.2244	17.17	74.04
.010 .25	5 36.	1.40	: ;;;;	9. 55	.1627	4.45	5. 13	5. 93	74.63		-4.ii	41.61
. 310 . 25	0 30.	1.40	. 407	1.55	. 2900	6.08	7.02	9.11	49.30	.7775	4.00	6.96
-010 -25	30.	1.40	- 547	9.55	. 4310	7.47	7.39	9.97	30.45	-6514	3.66	4 3, 3 3 2 <b>9</b> , 9 <b>9</b>
.010 .25	0 30. 0 30.	1.74	170	13.06	11040	1:11	4.22	6.63	12:31	: 7227	1:11	-19.20
.010 .25	j jo,	1.74	. 373	13.04	.4036	0.64	7.64	4.42	30:17	.6326	3.40	73.67
-010 -25	30.	1.00	504	14.00	.0100	5.05	5.04	6.77	9.54	4801	2.56	3. 10
.010 .25	9 30. 9 30.	2.16	1150	17.00	. 2025	2. 60	1:53	11.47	37. 63	30.79	7.25	120.13
-010 -250	30.	2.16	. 322	17.98	.4112	0.02	9.26	10.60	10.01	.4886	3.14	79.42
.010 .28	30.	5.50	. 493	18-41	. 6 366	6.24	7.20	•• 32		.3782	1.40	-10.91
-010 -25	30.	3.50	: 633	20.30	. 3 1 0 2	2:13	7:39 7:31	1:17	47:02	7319	2:79	72:76
.010 .25	30.	2.40	~ 100	70.31			7.0	0. i o	18.91	.4902	40	40.74
.010 .25		9. 42	. 813	20.73 21.13 9.70 9.32 9.55 9.32 9.78	-1987	5-00	5.00	6.79	-1.46	. 2922	5.30	56.84
.010 .250	30.	2. 16	. 5 4 5	21.17	08	3.45	1:31	4. 59	- 14. 60	122	17:25	-17:31
A010 A250	3 45.	1.45	. 5 85	9.70	*****	4. 4.	5. 57	7. 67	\$5.25	8780	2:44	45.85
010 .25	<b></b> .	1.40	.406	9.55	. 1643	2. 90	4.10	5.80	68.88	1.4170	2.45	59. 18
.010 .25	45.	1.42	. 203	9.32	.2009	3.55	3.61	1.10	43,39	1-1010	3:37	77.47
-010 -25	48.	1.42	. 534	9.78	- 4041	3. 70	5.24	7.41	44.45 35.27	.6292		83.65
.010 .25	) 15. 0 45.	1.42	160	14.30	.1765 .3765 .6176	3. 21	4. 54		47.03	. 4394	1.65	44.09
.010 .25	45.	1.02	. 342	14.30	3786	4. 41		9.62	39.44	• 6 6 2 3	• 44	125.91
.010 ·S.	2 45.	1.84	. 563	14.52 14.55 17.57 18.62 18.62 19.72 21.13	-6175	4.39	4.00	8.79	14.43	.5200	-::;	48.99 \$5.52
.010 .25	0 45. 0 45.	2012	173	17.55	2101	3. 65	5.17	7.31	44.22	. 420 B	2.14	125.63
.010 .2E	45.	2, i 4	. 361	17.77	. 4451	3. 79	8. 34 6. 21	7.54	27.00	. 5761	-1.09	10 30 70
-010 -25	45.	5.50	• • 7	10.41	-6176	3. 72	5.26	7.43	18.32	3402	2.37	76.49
.010 .25	0 45. 0 45.	2022	184	20.51	13110	3. 95	5.14	7. 90	43.67	-6661	-50	154.94
.010 .25	45.	2.34	. 295	19.00	. 3943	4.48	44 44	0.03	25.41	3947	:11	267.01
.010 .25	0 45.	2.42	. 522	20.72	.7110	3- 27	4.68	6.58 5.73	10.03	2074	5.07	134.10
.010 .25	0 45. 0 40.	1.40	310	6:12	1584	1.47	2.94	5.87	60.25	.6163	2.44	-19.07
-010 -25	ŏ 60.	1.40	. 400	9.55	. 29 38	1.50	3.15	4.25	44.67	• 656 6	5.30	17.55
.010 .25	0 60-	1.36	·ērš	9.32	.4394	1.62	3.64 3.72	7.20	45.15	. 8034 . 7310	1:13	20.70
.010 .25	0 69. 0 60.	1.74	170	13.86	1634	1.09	2. 16	4. 34	44. 77	. 8 34 3	2.26	4. 90
-010 -25	ŭ <b>40.</b>	1,62	341	20.72 21.13 9.78 9.55 9.32 10.01 13.86	. 394 7	1.75	3.50	7.00	31:47	.7760	1:32	29.77
. 210 . 25	0 60.	1.62	. 544	14.74	6048	5. 31	4.62	9.25	35.53 24.10	. 5274	23	92. 14 73. 30
.010 .25	0 60. 0 60.	2.12	161	18-19	2070	1.62	3. 24	6.49	59. 74	.7701	.75	-49.94
.010 .25	ō 60.	2.10	. 338	10.19	. 4245	1.04	3, 69	7.37	34.29	. 4-15	.54	34.96
*310 *56	0 60.	5.50	. 445	10001		2.17	4.34	•	23.54	. 4363		7.00
.010 .25	0 60. 0 60.		149	19.86	1990	1 - 32	2.66	S. 30	41.01	1.2342	.51	-27.34
.310 .25	0 60.	2.44		21.13	. 3897	2.02	4.03	7.37	35.08	.7261	1.01	31.10
	0 60.	2.46	.510	21.13	.6948	1.63	3.46	7.33	16.68	.4993	1.33	-2.15
.010 .25	J 60. 9 75.	2093	.072	9.55	. 1505	. 30	3.32	4.45	13.49	-6703	1.33	-72.67
.010 .25	) /Š.	1.40	.364	9.55	.2031	51	1.50	3.70	62.43	: \$ 92 7	1.41	- 25. JB
.010 .25	0 75.	1.44	. 599	10.01	.2031 .4672 .5677		. 47	3.76	44.07	.6767	1079	-15.68
.010 .25	? <b>75</b> •	1.44	. 794	10.01	1829	.00	1.77	1.40	-64.79	-1.9050		-76.14
.010 .25	0 75.	1.45	. 362	14.30	.4003	• 4 7	1.55	6.00	57.02	. 4843	1.47	-27.94
	V /5.	1.000		140/4	-6151		1.04	7.31	31077	EVLE.	4013	-27095
-310 -25	) <u>75</u> .	1.00	4 7 35	14.74	.0319	.57	1.54	4,79	43.23	3332	1.30	-13.54 -43.44
.010 .25	0 75. U 75.	2.19	329	14.19	.4230	. 55	2.11	3.14	46.11	1.2034	1.65	-33.09
.010 .75	0 75.	2.22	. 482	19.62	.6267	. 53	2.03	7.60	33.91	. 6719	1.64	-39.00
-010 -25	2 75.	2.24	. + 36	18.63	1066	.49	1.89	7.31	25.27	.7361 2.1899	1.60	-19.66 -22,79
.010 .25	0 75.	2.44	150	20.92	4047	. 44	1.71	6.63	46.13	1-0257	1.47	-21.47
-010 -25	0 75.	2.48	. * 02	21.34	.0415	. 44	1.69	6.51	32.16	.7160	1.61	-20.18
.010 .25	0 75.	2.52	. * 31	21.75	. 8767	. 48	1.05	7.14	25.07	. 6296	1.79	~21.35

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45.
2.25
.479 .120 .208
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1.32
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9. 69 7. 30 10. 34 10. 34 10. 38
29.78 56.38 36.44 25.94 16.07 81.72 29.47
.4974 .0364 .5763 .4198 .6773 .4465
5; 4.37 1.66 6.14 4.23 1.99
60.32 206.82 194.96 118.19 50.08 160.77

CLER	DIA	AN i	T		L.	JMAK	CLV	CLVA	CFAN	PHI	ĸ	CMA	CDV
27:11 07:21 07:21 07:21 07:22 07		0. 0. 0.	1.38	• 22 ÷	9.32 9.32 9.78	.1606 .3190	2.63 4.40 7.75	2.63 4.40 7.78	2478484642115103585557967489	81.077 48.140 71.20 71.20 57.20 57.21 57.2	1.5948 .9247 .6012 .6013 .6014 .6014 .6014 .7303 .4148 .7303 .4148 .2974 .5241 .7303	2220-7-031731040337198247105174240-7-045371982471051745-7-054145-7	-1.32 3.78 11.10 -24.05 -24.05 -25.57 -11.40 -32.09 -40.15 -40.27 -10.25 -40.27
.021 .021 .021 .021 .021 .021 .021 .021	• • • • • • • • • • • • • • • • • • •				9.32				5-664736657366557-061973565552807756419-13565553280775665557-0619755666655774656666666666666666666666666	4317952-770A97094-4121-517-13145	*2769 *5082 *5082 *5082 *17867 *6487 *64814 *64814 *64814 *5187 *5212 *64814 *64814 *64814 *64814 *64814 *64814 *64814 *64814 *64814 *64814 *64814 *64814 *64814	4.61 .33 1.56 4.01	- 10
	. 25500000000000000000000000000000000000	945660066000755755755755755755755755755	11112230946600482428883259660530546666040466834243141429683424688322445356605446683224433444335466662446883222242242443554666644443444434444344443	06342780443127.7783329886600;98824271;6399457135624562014271;6399467233149457135624567234571356245672456724571356245	13-0-037-1-02995. 1-10-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-	-4313042349397469737041.83744153744 -431304934974976069707171473737874173744 -631304744174173747874476187474 -6313047447618737748761876777 -63130476876767774 -631304768767774 -631304768767774 -631304768767774 -631304768767774 -63130476876774 -63130476876774 -6313047687676774 -631304767676774 -6313047676774 -631304767676774 -631304767676774 -631304767676774 -631304767676774 -631304767676774 -6313047676774 -6313047676774 -6313047676774 -6313047676774 -6313047676774 -63130476774 -63130476774 -63130476774 -63130476774 -63130476774 -63130476774 -63130476774 -63130476774 -63130476774 -63130476774 -63130476774 -631304774 -631304774 -631304774 -631304774 -631304774 -631304774 -631304774 -631304774 -631304774 -631304774 -631304774 -631304774 -631304774 -631304774 -631304 -631304 -631304 -631304 -631304	95089905017059080515438944968271722724405265051997855747276031955710111102117227244410314642308747414112211722724445754441031414141414141414141414141414141414	3.4227465779880777772886766666679886779777728867697777728867677677777788677677777788677677777788677677	4.996 4.997 5.996 5.699 5.699 5.699 5.699 5.699 5.699 5.699 5.699 6.	59.92 19.92 19.92 19.92 19.72 11.53 19.72 11.53 17.04 11.53 17.04 11.53 17.04 11.53 17.04 11.53 17.04		10299	-14.037 21.037 15.69 21.07 15.69 21.07 12.07 12.07 15.06 15.06 16.06 16.06

	<b>n</b>			, J31	•	• .		1.06	1.05	PH1		2.50	CDV 25.79
	.250 .250	٥. و.	1+4: 1+4: 1+24	.2A0	7:737 7:737 7:757 9:457 9:47 10:47 1	.212A .3151	1.04	1.02	1.02	66.7/ 65.54 69.92	1.0815 1.1516 .9885 .7646	2.17 3.13 3.51	22.70 36.94 45.59
	.250	0.	1.65	.215	14.74	12434	1.20	1.26	2.27	82.74	1-1215	1.65	17.43
	250	ç.	1-72	• • • •	14,74	.5658 .5799	7.06	7.00 7.99 1.85	7.06	34.93	.6033 .5497 L.0232	1.57	-,01 13.03 32.21
:	250	ŏ.	2.22	295 296	13.62	3936	5. L7 5. 51	5-17	5,17	77.69 55.59 57.38	.6957	3.11	65.14 -29.01
:5	50	0.	2,18	414	14.19	.5J22 .6464	7.53	7.53 7.04	7.63	37.20 26.21	.5842 .4771	3.02 6.93	93.37 173.01
	250	0.	2.54	.378	22.37	.5873	7.17	7.17	7.17	32.69 17.37	.5308 .4362	0.59 3.43	100 ~ 30 176 • 93
:	250	15.	1.32	527	6.63 9.78	13.2	1.16	1.20 2.79	1.24	35.98 63.22 64.31	. 3936 .6763 .8013	2:06 3:63 -:81	28.57 36.64 11.48
•	250	15.	1.78	371	13.19	4012	2.67	1-10	2.66	03.24 69.27	.9201 .7740	3.63	24.96 62.55
:	250	15.	1.75	.622	13.86	6726	7.31 6.56	7.57 6.79 1.53	7.64 7.03 1.58	41.82 36:57 80:10	•5 22 •5277 •8602	7.04	70.42 67.37
	250	15.	2.06	320	16.91	.395) .5294	3.68	3.91	3.64	61.11 35.00	.0965 .5565	3.70	32.65 114.80 118.51
•	250	15.	2.56	135	18.63	.6522	1.62	1.66	1.74	26.39 70.26	.4969	1.55	113.89
:	250 250 250	15.	2,30	369	20.39	4364	7.31	4.65 7.57 6.99	7.84	34.74 37.96	.664 9 .5377	3.04	178-58
:	250	30.	1.40	29A	9.55	2193	74	• F 6	?24 99	20.50 69.12 83.20	-4886 -3470 -6860	2.60	266.57 21.39
:	25C 250	30.	1.48	.E41	10-47	.52HB	1.27	1.46	1.01	70.08 86.31	. 661 7 . 7309	3.44	19.14 36.25 18.29
-	250 250 250	30.	1.78	. 350	16.03	.5492	4.66	1 - 65	2.13 C.21 7.73	49.20	-4819	2.70	20.97 -34.85 36.25
•	250 250	30. 30.	1.96	198	13,63	2352	1.07	6.70 1.24 J.25	1.43	30.64 62.61	6755 7352 •7362	5.90 2.57	36.26 39.60 26.57
	250 250 250	30.	2.16	429	17.98	.5473	5.11	5.90	7.02	65.01 39.76 31.41	•5783	1.57	83.05
•	250 250	30.	2.34	174	19.68	.7250 .3615	3.07	3.54	1.21	31.41 63.13 61.29	.5326 .7360 .6985	1:27	23.94
•	250 250 250	30.	2.55	1413	22.37	.5801	36 72	6.43	7.43 7.63	23.60 86.04	•5461 •6171 •5027	-,49 -76	221.52 25.30
:	250 250	45.	1.32	. 29 . 659	10.01	3367	- 56 - 78	1.10	1.16	86.73	.5027 -5073	2.74	50.69 50.69
	250 250 250	45.	1.52	.235	136.091111000310000000000000000000000000000	.2336	• 55 • 94	1.32	1.87	71.14	-49E8	2.87	38.02 24.14
•	250	45.	2.00	. 456	10.26	• 50 36	2.40	3. 39	4.79	59.16	.6978	.40	41.27
	250			410		4.000							
•	250	45	2.02	194	16.48	996	20 73	3+67 +67	5.47 1.22 2.65	55. 12 91.26	.6274 .9327 .7612	3.71 2.00 3.31	54.84 82.51 70.75
	250 250	45.	2.24	. 505	14.30 16.48 17.34 18.43 19.04	.5.104 .66 le	2.57 3.84	3.63	5-14 7-67	71:65 60:36	.6547	1.35	87.09 122.14
•	\$20 \$20	45.	2.44	· 163	19.46 19.49	· £160	• 76 1• 39	1.97	1.51 2.78	83.52 47.98	. 8280 . 7943	1.35 5.33 2.35 2.11	49.38 75.36
	750 750	46.	2.64	.401	35.60	5676	7.94	3.65 3.57 3.7	5.21 7.88 .75	40.27			24.47 104.22 107.55
	250	60.	1.45	623	H-26	3320 5005	40	• 38 • 79	75 1.58	103.17	-48249 -7457 -1443 -1443 -9463 -9357	2.55	70.59 48.82
:	250	60.	1.36	229	12.97	.2345	• 50	.02	1.05	102-69	9605	2.78	03.66
:	250	60.	1.75	637	13.86	684'3	80	1.59				40.0	70.26 52.19
•	250	60.	2.20	321	17.13	# 3997	57	1.15	2.30 3.38	79.22	47474	3.04	98,65 85.52 97.27
•	250	50.	2.26	60	19.04	• 2107	• 40	2.49	1.92	90.10	10127	2.49	86, 18 95, 41
:	250	60.	2.32	. 366	20.30	3700	. 53	1:10	3.76	77.(1) 79.64	.0239	2:32	72.49 82.74
•	250 250 250	75.	1.44	207	23.43 10.01	+2241 +370	1-31 +54 +25	2.62 2.09	5.25 8.07 3.72	-4.48 -20.18	.7423	2.50	130.61
	250	75. 75. 75.	1.50	.520 .591	10.70	.5700 .5001	- 13	.68	2.63	109+20	.5433 .9/46 .0401	2.00	44.59 71.55
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	.250 .230	75. 75.	2-02	-010	15.86	. 65° 3	•33	1.26	4.92	-59.42	.9975	2.02 3.20 2.79	34.46 128.39
	.250 .230	75.	7.2h	- 400 - 400	19.04	.3957 .5260	- 12	1.25 1.61	4.81	90.56 78.53 89.20	7361 •746.7 •8037	£.52	77.50 66.91 62.61
2	250	75	2.25	26)	19.88	2076	95.	1.15	4, 24	<b>₽1≥6</b> €	0231	3.16	137.64

. ORS	25500000000000000000000000000000000000	000000000000000000000000000000000000000	1.8246 1.076 1.076 1.076 2.022 2.032 2.032 2.032 2.032 1.032 1.032 1.032 1.032 1.032 1.032	- 3961 - 3961 - 3961 - 3961 - 3961 - 3961 - 3961 - 3971 - 3971 - 3971 - 3971 - 3971 - 3971 - 3971 - 3971	11.89 14.30 15.61 13.86 13.86 14.03 14.62 14.62 14.62 14.62	15260 1570616549716633389 4401726209886633389 4401729286633389 4401729287449 44049445	.51 .71 1.83 2.30 1.02 2.30 1.02 4.40 .49 .49 .49 .49	• 42314 • 42314 • 42314 • 62314 • 6	4514412004207000528400946074844941200422079404548452974449	71-94 61-146 61-	- 7.36 - 73.6 - 73.6		2.77.22. - 17.22. - 17.22. - 17.57. - 21.39. - 21.3
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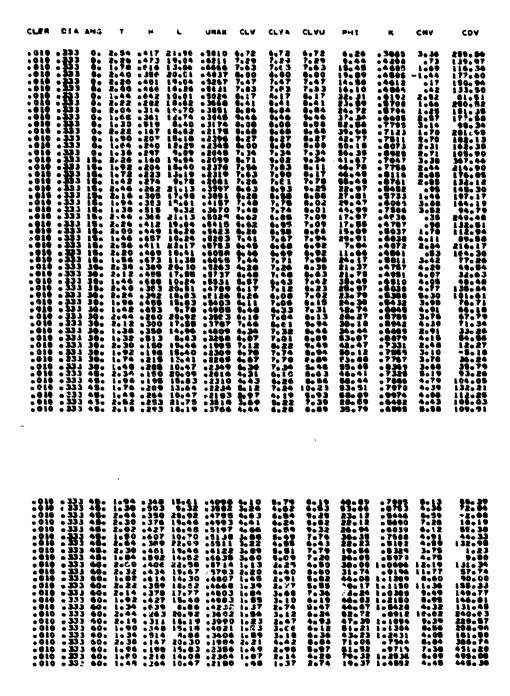
CLER	DIA	ANG	Ť	н	L	KANU	Cr.v	CLVA	CEAN	PHI	¥	CHY	CDA
147	. 250			.236 .479	11.64	· 2246 • 3679	.19	•19	• 19 • 20	-02.33-		1:22	-0.01
147	250	5.	1.44	-542	10.24	1370	:23	.23	.23	-44.44	0307	1.60	5.44
. 167	. 250	ě.	1.45		10.47	.6431	.20	. 2 .	.26	66.47	48240		-1.00
. 147	. 250	0.	1.90	. 190	15.14	. 2300	-21	.21	.21	-64.87 78.32	-+9711	1.77	-5.37
. 167	. 250		1.96		15.63	-4066	. 37	• 37	• 37	75.32		.41	16
- 107	. 260	9.	2.04	.440	16.70	. 5423	1.65	1755	1675	-47.21	-4042	5.65	22.04
- 167	- 250 - 250		1779	+ + 24	19.04	.2072	:69	17	17	127.37	-c2364	1341	-17.56
167	250	<u>.</u>	2.24	2762	14.43	3770	iii	iii	ii	70.34	3701		- 68
167	280				20.72	1010		ie i	. 61	70.34			18.88
0 167	. 280	ŏ.	2.20	440	17.25	.5093	.20	. 70	759	71.70	1.1025	36	1.94
107	. Z8 0			. 122	24.01	01702	. 43	. e 3	. 43	-47.01		1.24	-32-70
- 167	250	9.	2.44	. 229	23,20	. 3261	-21	.21	• 21	64.45	1014	77	-4.01
- 167	• 26¢					. 50 79	. 86	- 54	. 56	44.67	.0639	70	1.17
• 167	- 570				\$5.99	.5635	3.05	3.02	3.65	49.10	• • • • • • • • • • • • • • • • • • • •	70	45-01
:167	- 520				17-36	. 2262	.25	. 20	:44	-00.70	2500	2.04	-2.76
iii	:20	39.	1:30		19.70	. 6286		.26	:37	30.90	11045	74	67
iii	146	36.	1.64		11.16	337A	.23	. 37	:31	37.34	:::533	. ii	-2.67
147	- 230	30-	1.44		14.94	2377	121	- 24	. 24	70.60		1.67	-14-84
iet	256	35.		343	18.03	4049	. 10	.2.	. <del>2</del> 5	72.47	.2044	•2•	-6.69
-167	. 880	30.	2.84	.431	16.70	.F 306	. 23	. 26	. 30	-57.00	. 2603	-61	-1.10
. 147	. 260			. 444	16.40	.5425	.50	•58	- 67	-83.76	.5170	. 63	-1.09
. 107	. 250		2.32		10.30	• 1426	. 70	•91	• 94	-48.18	+ 3495	1.19	6.13
• 14Y	- 250		1.06				. 42	• 94	1-05		-3000	1.55	2.04
. 167				.163	17.00	.2012 .3055	.29	.33	: 34	-22.60	-50164	1.62	-24.65
	.250	30.	5: 12	:373			23	.32	. 37	64.30	.784	-: 12	-0-73
ier	Há	17.	2.24	400		5245	:43	.44	. 57	43:55	76167		- 40
1147	250			,453			1.14	1.32	1.52	44.19			- 3.71
. 167	- 260	30.	2.50	. 135	22.37	. 1897	44	. 50	. 59	78.07		1.33	-31.01
- 10 f			2.54	- 246	22.17	. 3474	. 16	.20	.24	77.55	.5149	-0.	-22.59
- 167			3.40	369	20.81	+4563	1.03	1 - 19	1.30	50.69	-4727	• 88	3.20
- 147	+ 280						. \$6	-67	77	64.50		-1.45	-3-36
. 167	. 250			-260			. 26 . 07	. 53		- 13.31	-1.5137 -3.6000	1.55	43.52
:107			1034				:17	. 36	12	97.57		1.74	14.13
iar	.250	14.		. 21	14.30	2375	. 26	52	1.62	-00.92	3376	2.62	¥.23
147	. 250		12.47	. 350	15.61	4235	.23		. 91	-78.4J	.3170	1.72	22.63
107	. 250		2.04	442	18.40		. 23	.44	. 73		. 6396	. 06	20.13
. 107	. 250	40.	1.04	-600	14,52	-4016	. 45	. 90	1.79	75.95	-	.70	17.09
-147	. 243		2.14	. 164	17.77	.2150	• • 5	• 94	1572	- 80.09	4833	2.33	31.30
. 107	.250			-310			.07	• 1.5	• 30	-60.43		1.45	29.25
- 167			S- 30	- 374			• 12	-24		55.73		• 1 •	15.14
- 107				+455	21.34	.1969	. 54	1.09	2.15	80.78 -73.99		2.02	25.15
147	.250				22.50			110	38	19.72		1.06	37.44
	280	40.	5.44	197	20.92	4467		76	1.52	80.23			32.07
- 1147	250	40.	3.67	307	22.99	- BO - L	. 24	4.7	7.35	87.34		-:::	37.97

		- 4.0	•	••		V-44	C. V		CC 10		•		CLV
. 00	049070000000000000000000000000000000000	00000000000000000000000000000000000000	022488820244428826406826406826406826406286406666662864	00704542401397929; 0023451326235300138788271146071030061374413242; 00234513423423423423441334224566922	11.030 14.30 14.30 14.00 17.12.00 17.12.00 17.12.00 17.12.00 17.13.10 18.44 22.90 18.45 18	-40176 -40176 -1760 -27605 -27605 -376091 -376	46314130025677000298974437739814763845000973084 453878300256784000973084 46446448473444000973084 12 136 136 136 128 128 128 128 136 136 136 136 136 136 136 136 136 136	10314120025070000117186899556928660402765349354697115 2013794016456446273627502750411288957641333 2013794015455445411288957641133 1201379401545555555555555555555555555555555555	345314130025070700053830074554267514004338384452944449	90.84 83.19 58.19 58.44 74.44 44.32 91.44 45.63 93.64 45.63 93.64 65.85 66.81 46.96 86.81 66.81 66.81 87.96 88.85 88 88.85 88.85 88.85 88.85 88.85 88.85 88.85 88.85 88.	.8219 .5476 .7145 .7145 .9340 .9465 .6426 .7643 .5404 .9743 .6431 .6131 .6131 .6431 .6131		1572255788511512121866166782619866782619866716573668786576231866716777868767878787878787878787878787
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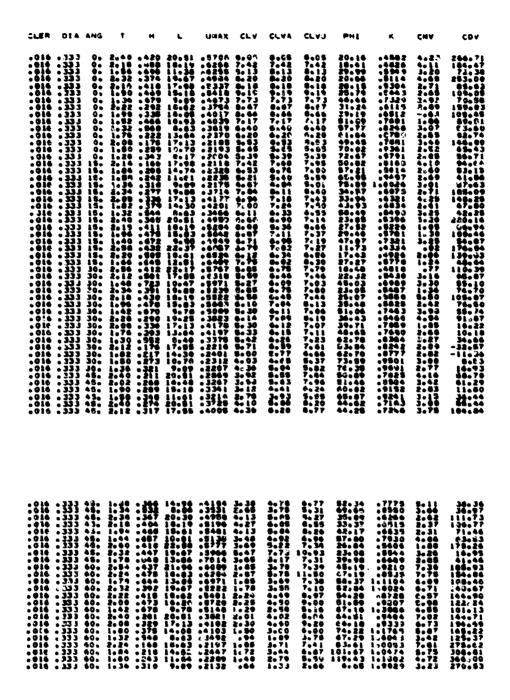
CLER	214	ANG	Ŧ	H		WAR	CLV	CLVA	CLAN	PHI	ĸ	CMA	COV
	. 250				11.04	.2245	.19	•19	.19	-62 - 36-		1.92	-8.01
- 147	. 550	٠.	1.43		10.24	3670	.20	.20	:20	-66.44	111	2.01	3.44
0 167	. 250	9.	1.44	• 662	10.47	. 5431	.23	:::	:::	45.47	*****	1.22	-1.09
. 167	250		1.90		15.10	.2300	:3:	.71	:21		0711	1.77	-3.37
167	. 250	<b>.</b>	1.04	340	15.63	.4056	:37	:37	.37	75.32	i fé E .	ièi	7.16
. 107	. 260	ě.		.440	14.70	. 5423	1.65	1.68	1.65	-47.21	4042	2.52	22.04
. 167	. 250			. 624	13.86	.4790			.69	127.37	. 8763	12	10.72
. 167	. 250	••			19.04	.2072	.17	.17	. 17		2 362	1.41	-17.56
. 167	* \$20				10.63			• 1 •	. 18	70. 34	- 3761	.27	.=•••
. 167	. 250				20.72	-5010	• • • •	•€1	• • • •	70-36	-4614	• • • •	15.85
- 147	. 250			.460	19.25	-6093	. 20	. 20			1 - 1 - 5	7 - 36	3. 24
: 167	:250				23.20	.1762	.63	• • • 3	:53	-17:01	1016	1:37	-32.70 -4.01
167	250				14.62	5079	. 54	:54	.56	44.67	-1011	70	7:17
167	. 250	ŏ		.367		1635	3.02	3.02	3.02	49.20	-6196		46.61
. 147	. 250	30.	1.56	. 248	11.30	. 224 2	. 25	. 29	. 34		2500	2.00	-0.90
167	250	30.	1.36		9.32	- 36 1 8	.37	.43	.40	-80.74		1.74	-2.76
. 167	. 260	36.	1.50		10.70	. 52 04	. 24	.20	. 32	80.70	. 2060	•74	07
. 167			1.54	.601	11.14	.5376	. 23	.27	. 31	89. 64	.4533	• 51	-2.67
<u>• 167</u>			1.00		14.96	.2377	.21	• 24	. 24	70-48	4464	1.47	-10.04
. 167				• 343		.4009	+ 10	.21	. 25	72.47	. 2044	• • • •	-5.59
- 147	. 25 0		3.04		16.70	•£ 306	• 53	• 26	• 39	-67.00	-5663	• • • •	-3-10
167				• 444	16-40	.5425	.50	.50	• • 7	-63.76	-6170		-1-66
: 167			2.02	:444	16.48	.426 .4905	. 70	.91	1.09	-12: 38	.3000	1:18	8.13 2.96
. 167	. 250		3.32	. 153		. 2012	: 29	:::		07.22	:3476	1.62	-24.45
167	250		2.14	. 30 î	17.90	.3655	. 1 7	120	. 23	-42.60	0164	-:-	-1.11
107			2, 34		19.00	.5274	.20	.32	. 37	40 - 30	. 7214	12	-0.73
107		30.	2.24	.400	19.04	. 5245	,43	.49	. 57	65.00	. 6 1 0 1	.29	4.45
. 167				.453		.6060	1.14	1.32	1.52	40.19	. 4 5 4 5	.13	-3.71
. 167			2.50	. 135		-1697	. 44	• 50	• 50	76.07		1.33	-21-61
. 107	. 260		2554	. 246	22.17	.3454	1.0	•20	24	77.85	. 8149	-0	-55-59
- 147				. 34 5		.4563	1.03	1.19	1.36	50.69	-6727		3. 50
• 147				- 367			. 50	.67	1.06	64.50	-7020	-1.45	-1-36
- 147				.260		.2224	.26	.53	1.27	-15.51-		2.55	43.52
: 137		10.			10.93		. 14	. 36	72	97.57	7:22:3	1:11	16.13
: 167	250	10.		.213		.2375	. 24	.52	1.03			1.43	39.23
. 167				. 350		.4235	.23	.46	. 41	-70.43	.3170	1.72	22.63
147				452		.5520	. 23	.46	.93		. 6396	. 06	20.15
101			1.04		14.52	.4010	. 45	. 90	1.79	75. 85	. 9407	.70	17.47
. 167	250	66.	2.14	.169	17.77	.2158	.43	. 56	1.72		6632	2.33	31.39
. 107	.250			•310			.07	• 15	. 30			1.45	24.25
- 167					19.43		• 12	. 24	. 49	55.73	• 5203	•1•	32.39
- 167	. 250		2.34	•455	19.60	-6114	• • • •	. • 9 1	1.51	80 - 76	. 5667	54	\$5. i \$
• 167				• 142		-1969		1.09	5.10	-73.09 19.72	4834	2.02	28.11 37.89
. 167	0 250	60.		- 248	22.54	.3502 .4897	.10	.76	1.52	40.23	5674		32.07
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## APPENDIX G

TABULATED HORIZONTAL FORCE DATA FROM TWO-DIMENSIONAL EXPERIMENTS

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Bowle, George L.  Porces exerted by waves on a pipeline at or near the ocean bottom / by George L. Bowle Fort Belveir, Va.: U.S. Coastal Engineering Research Center; Springileld, Va.: available from Mational Technical Information Service, 1977.  177 p.: 111. (Technical paper - U.S. Coastal Engineering Besearch Center; no. 77-11) A; o (Contract - U.S. Coastal Engineering Research Center; no. 77-11) A; o (Contract - U.S. Coastal Engineering Research Center; no. 77-11) A; o (Contract - U.S. Coastal Engineering Research Center; DAGN72-74-C. Apr.)  Biblingtaphy: p. iii  This report presentation analysis of wave-induced forces on a submarine pipeline near the ocean Floor. The wave-induced forces; consist of several components—instial forces, days forces, lift forces, and under some conditions, eddy-iduced forces.  1. Wave forces. 2. Underwater pipelines. 1. Title, II. Series: U.S. Coastal Engineering Research Center. Contract DAGN72-74-C-00u)4.	Bowie, George L.  Forces exerted by waves on a pipeline at or near the ocean bottom / by George L.  Forces exerted by waves on a pipeline at or near the ocean bottom / by George L. Bowie Fort Belvoir, Va.: U.S. Coastal Engineering Research Center; Springfield, Va. : aval.able from National Technical Information Service, 1977.  177 p.: ill. (Technical paper - U.S. Coastal Engineering Research Center; no. 77-11) Also (Contract - U.S. Coastal Engineering Research Center; no. 77-12-74-C-0004)  Bibliography: p. 123.  This report presents an analysis of wave-induced forces on a submarine pipeline near the ocean floor. The wave-induced forces consist of several components-inertial forces, drag forces, lift forces, and under aome conditions, addy-induced forces.  1. Mave forces. 2. Underwater pipelines. 1. Title, II. Series: U.S. Coastal Engineering Research Center. Contract DACH72-74-C-0004.	TC203 .U581tp no. 77-11 627
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