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ASYMPTOTIC AREA AND PERIMETER OF SUMS OF RANDOM PLANE CONVEX SE--ETC(U)

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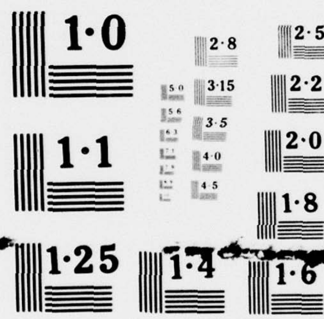

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ASYMPTOTIC AREA AND PERIMETER OF SUMS OF RANDOM PLANE CONVEX SETS

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ABSTRACT

We consider the asymptotic distribution of functionals associated with normed sums of random plane convex sets. Methods involving symmetric statistics and weak convergence of stochastic processes are used to examine area and perimeter in particular.

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Key Words and Phrases: Random sets, Convex sets, Minkowski sum, Support function, Area, Perimeter

Work Unit Number 4 (Probability, Statistics, and Combinatorics)

EXPLANATION

Minkowski addition and scalar multiplication of subsets of $\mathbb{R}^d$ are defined respectively by

$$K_1 + K_2 = \{k_1 + k_2 : k_1 \in K_1, k_2 \in K_2\}$$
$$\alpha K_1 = \{\alpha k_1 : k_1 \in K_1\} .$$

If $X_1, X_2, \ldots$ are <u>random</u> subsets of $\mathbb{R}^d$ then it is of interest to study the behavior of their averages

$$\bar{X}_n = \frac{1}{n} [X_1 + \cdots + X_n] .$$

Following on earlier work, we consider the behavior of area and perimeter of $\bar{X}_n$ when the X_i lie in the plane.

Potential applications for this work include the modelling of biological growth phenomena and techniques for two-dimensional image processing.

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ASYMPTOTIC AREA AND PERIMETER OF SUMS OF RANDOM PLANE CONVEX SETS

Richard A. Vitale

1. Introduction.

If $X_1, X_2, \ldots$ are random subsets of $\mathbb{R}^d$, then we may form their successive averages, $X_n = \frac{1}{n}[X_1 + \cdots + X_n]$, under Minkowski addition and inquire into their asymptotic behavior. A first step in this direction was taken in [1] where a strong law of large numbers for sets was derived. Our purpose here is to look at some distributional considerations associated with this convergence in the case when the X_i are valued in the compact, convex subsets of the plane. Certain of our results evidently have extensions to higher dimensions but we have chosen the restriction $d = 2$ for uniformity of exposition. The assumption of convexity, on the other hand, is central to most of the discussion.

The paper is divided as follows. In Section 2 we set down some notation and preliminaries including a statement of the strong law. Section 3 presents a central limit theorem which follows from the consideration of the support function of $\bar{X}_n$ as a stochastic process. The asymptotic behavior of linear functionals such as perimeter then follows directly. In Section 4 the area functional is considered from the point of view of a symmetric statistic.

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2. Preliminaries.

Let $\mathcal{K}$ denote the collection of non-empty, compact, and convex subsets of $\mathbb{R}^2$. With the introduction of the Hausdorff metric

$$d(K_1,K_2) = \inf\{\varepsilon > 0 : K_1 \subseteq K_2 + \varepsilon B,\ K_2 \subseteq K_1 + \varepsilon B\} ,$$

$\mathcal{K}$ may be regarded as a separable, locally compact metric space. Here B is the closed unit disc and scalar multiplication and Minkowski addition are defined as usual:

$$\varepsilon K = \{\varepsilon k : k \in K\}$$

$$K_1 + K_2 = \{k_1 + k_2 : k_1 \in K_1,\ k_2 \in K_2\} .$$

The norm of K is defined by

$$\|K\| = \max\{ \|k\| : k \in K\}$$

or, equivalently, $d(\{0\},K)$.

To each K, we assign a support function given by

$$s(\theta,K) = \max\{k\cdot e(\theta) : k \in K\} \qquad e(\theta) = (\cos\theta,\ \sin\theta) \qquad \theta \in [0,2\pi] . \tag{2.1}$$

The map $K \to s(\cdot,K)$ uniquely embeds $\mathcal{K}$ in the space $C[0,2\pi]$. Linear structure as well as distance are preserved:

$$s(\cdot,\alpha K) = \alpha s(\cdot,K) \qquad \alpha \geq 0 \tag{2.2}$$

$$s(\cdot,K_1 + K_2) = s(\cdot,K_1) + s(\cdot,K_2) \tag{2.3}$$

$$\begin{aligned} d(K_1,K_2) &= \|s(\cdot,K_1) - s(\cdot,K_2)\| \\ &= \max\{|s(\theta,K_1) - s(\theta,K_2| : \theta \in [0,2\pi]\} \\ \|K\| &= \|s(\cdot,K)\| \end{aligned} \tag{2.4}$$

We shall regard a random set X as map, measurable in the Borel sense, from an abstract probability space into $\mathcal{K}$. In order to formulate the strong law of large numbers, let us first define the expectation of a random set. By virtue of the inequality

$$|s(\theta,X)| \leq \|X\|$$

$Es(\theta,X)$ is well-defined if $E\|X\| < \infty$.

Definition. Let $E\|X\| < \infty$. Then the expectation of X, written EX, is that element of $\mathcal{K}$ which uniquely satisfies

$$s(\theta, EX) = Es(\theta, X) \qquad \theta \in [0, 2\pi] .$$

Strong Law of Large Numbers ([1]). Let $X_1, X_2, \ldots$ be iid random sets with $E\|X_i\| < \infty$. Then $\bar{X}_n$ converges almost surely to EX_i.

3. Weak Convergence of the Support Function Process

To each random set X is associated its support function $s(\cdot,X)$, which may be regarded as a random element of $C[0,2\pi]$. Using (2.2) and (2.3) we have

$$s(\theta,\bar{X}_n) = \frac{1}{n}\sum_{i=1}^{n} s(\theta,X_i)$$

and the difference process

$$\Delta_n(\theta) = s(\theta,\bar{X}_n) - s(\theta,EX_i) = \frac{1}{n}\sum_{i=1}^{n} [s(\theta,X_i) - Es(\theta,X_i)] \quad (3.1)$$

Theorem 1. Let $X_1,X_2,\ldots$ be iid random sets with $E\|X_i\|^2 < \infty$. Then, as $n \to \infty$, $\sqrt{n}\,\Delta_n(\theta)$ converges weakly to the $C[0,2\pi]$-valued Gaussian process $\Delta(\theta)$ with

$$E\Delta(\theta) = 0$$

$$\mathrm{Cov}(\Delta(\theta_1),\Delta(\theta_2)) = \mathrm{Cov}(s(\theta_1,X_i),s(\theta_2,X_i)) .$$

Proof. The existence of EX_i is certainly ensured by the condition in $E\|X_i\|^2 < \infty$. In addition, this requirement together with the representation (3.1) of $\Delta_n(\theta)$ as a sum of iid random values implies that the finite dimensional distributions of $\sqrt{n}\,\Delta_n(\theta)$ converge to the appropriate Gaussian limits.

It remains to show tightness of the measures on $C[0,2\pi]$ associated with the $\sqrt{n}\,\Delta_n(\cdot)$. Accordingly, we verify the bound

$$E[\sqrt{n}\,\Delta_n(\theta_2) - \sqrt{n}\,\Delta_n(\theta_1)]^2 \le E\|X_i\|^2 (\theta_2 - \theta_1)^2 \quad (3.2)$$

([2, p. 95]). We have

$$\begin{aligned} E[\sqrt{n}\,\Delta_n(\theta_2) - \sqrt{n}\,\Delta_n(\theta_1)]^2 &= E[s(\theta_2,X_i) - s(\theta_2,EX_i) - s(\theta_1,X_i) + s(\theta_1,EX_i)]^2 \\ &\le E[s(\theta_2,X_i) - s(\theta_1,X_i)]^2 . \end{aligned}$$

Now an easy application of the Cauchy-Schwarz inequality together with (2.1) provides the general bound

$$|s(\theta_2,K) - s(\theta_1,K)| \le \|K\|\,|\theta_2 - \theta_1|$$

which yields (3.2).

As an example, let us consider the homogeneous line segment model which is constructed as follows. Let α be uniformly distributed on $[0,2\pi]$ and let L, independent of α, be a non-negative random variable with $EL^2 < \infty$. Then X is taken to be the line segment of length L centered at the origin and inclined at an angle α to the horizontal. It follows that $s(\theta,X) = \frac{L}{2}|\cos(\theta - \alpha)|$ and EX is the disc of radius $\frac{EL}{\pi}$ centered at the origin. The covariance structure of $s(\theta,X)$ and hence of Δ is given by

$$\mathrm{Cov}(s(\theta + h,X),s(\theta,X)) = \frac{EL^2}{4}\left[\frac{\sin|h|}{\pi} + \left(\frac{1}{2} - \frac{|h|}{\pi}\right)\cos|h|\right] - \left(\frac{EL}{\pi}\right)^2 .$$

Theorem 1 and (2.4) imply a rate estimate for the convergence $\bar{X}_n \to EX$.

Corollary. With the assumptions of the theorem, $\sqrt{n}\, d(\bar{X}_n,EX)$ converges in distribution to $||\Delta(\cdot)||$.

Unfortunately at this stage we know of no methods for investigating $||\Delta(\cdot)||$ in even the simplest, non-trivial cases such as the example just given.

The situation for bounded linear functionals is direct. Let T be a map taking $\mathcal{K}$ into $\mathbb{R}^1$ such that $T(\alpha K_1 + \beta K_2) = \alpha T(K_1) + \beta T(K_2)$ $\alpha,\beta \geq 0$ and $|T(K)| \leq C||K||$ for some $C \geq 0$ and all $K \in \mathcal{K}$.

Corollary. With the assumptions of the theorem, a bounded linear functional obeys a central limit theorem: $\sqrt{n}\,[T(\bar{X}_n) - T(EX)]$ converges in distribution to an $N(0,\mathrm{Var}\,T(X_i))$ variable.

The perimeter functional provides an example for this convergence. Here $\mathrm{per}(K) = \int_0^{2\pi} s(\theta,K)d\theta$ so that $\sqrt{n}[\mathrm{per}(\bar{X}_n) - \mathrm{per}(EX)]$ converges in distribution to a normal variable with mean 0 and variance $\mathrm{Var}\,\mathrm{per}(X_i) = \int_0^{2\pi}\int_0^{2\pi} \mathrm{Cov}(s(\theta,X_i),s(\gamma,X_i))d\theta d\gamma$. In the homogeneous line segment model we can evaluate this expression directly by noting that $\mathrm{per}(X_i) \equiv 2L_i$ (the factor 2 enters since a line segment is regarded as a degenerate polygon) and so $\mathrm{Var}\,\mathrm{per}(X_i) = 4\,\mathrm{Var}\,L_i$.

Related linear functionals are $T(K) = s(\theta_0,K)$, the extent of K in the direction θ_0, and $T(K) = s(\theta_0,K) + s(\theta_0 + \pi,K)$, the width of K in the direction θ_0.

4. Area.

Let us first remark that the continuity of the area functional ensures that $A(\bar{X}_n) \to A(EX)$ a.s. under the conditions of the strong law result. Evidently the mode of this convergence must be investigated by means other than those in the preceding section since area is not linear in the Minkowski sum.

Instead we use the representation

$$A(\bar{X}_n) = \frac{1}{n^2} \sum_{j,k} A(X_j;X_k) . \tag{4.1}$$

Here $A(X_j;X_j) = A(X_j)$ while $A(X_j;X_k)$ ($j \neq k$ implicitly) is the symmetric mixed area of X_j and X_k ([4]). We shall assume throughout that $E\|X_j\|^4 < \infty$ which ensures a bounded second moment for each term of (4.1) (note $0 \leq A(K_1;K_2) \leq \pi\|K_1\| \; \|K_2\|$).

The form of (4.1) suggests the use of standard techniques for studying symmetric (or U-) statistics (as discussed, for instance, in von Mises [8], Hoeffding [7], Filippova [5], Rubin and Sethuraman [9]). With some labor in the adaptation this can be done. However, we shall take a different approach which is based on orthogonal expansions (a general discussion of this method will appear in [10]).

Since

$$EA(\bar{X}_n) = \frac{1}{n^2} [(n^2 - n)EA(X_j;X_k) + nEA(X_j)]$$

we have

$$A(EX_1) = \lim EA(\bar{X}_n) = EA(X_1;X_2)$$

so that

$$\begin{aligned} \delta_n &= A(\bar{X}_n) - A(EX_1) \\ &= \frac{1}{n^2} \sum_{j \neq k} [A(X_j;X_k) - EA(X_j;X_k)] \\ &+ \frac{1}{n^2} \sum_j [A(X_j) - EA(X_j)] \\ &+ \frac{1}{n} (EA(X_1) - A(EX_1)) . \end{aligned} \tag{4.2}$$

We define

$$\sigma^2 = E_{X_j}[E_{X_k} A(X_j;X_k) - EA(X_j;X_k)]^2, \quad j \neq k$$

which is finite. The convergence properties of δ_n will depend on whether σ^2 is zero or strictly positive.

Theorem 2. Under the assumptions made above, δ_n can display two types of convergence:

(i) If $\sigma^2 > 0$, then $n^{1/2}\delta_n$ converges in law to a mean 0, variance $4\sigma^2$ normal variable.

(ii) If $\sigma^2 = 0$, then $n\delta_n$ converges in law to a variable of the form

$$\sum_{\nu=1}^{\infty} c_\nu [Z_\nu^2 - 1] + EA(X_1) - A(EX_1)$$

where the Z_ν are independent standard normal variables and the c_ν form a square summable sequence described below.

Proof. Before considering the two cases separately, we make the following observations. Under either normalization the contribution of the second term in (4.2) is asymptotically negligible. Moreover the bias effect of the third term persists only in the second case. Accordingly, we focus on the first term

$$S_n = \frac{1}{n^2} \sum_{j\neq k} [A(X_j;X_k) - EA(X_j;X_k)] \;. \tag{4.3}$$

Each term of this expression is symmetric in its arguments and has bounded second moment. This is sufficient to conclude a denumerable expansion (convergent in mean square) of the form

$$A(X_j;X_k) - EA(X_j;X_k) = \sum_\nu c_\nu \varphi_\nu(X_j)\varphi_\nu(X_k)$$

where $E\varphi_\nu(X_1)\varphi_\mu(X_1) = \delta_{\nu\mu}$ ([3, p. 1087]). We then have

$$0 = E[A(X_j;X_k) - EA(X_j;X_k)] = \sum_\nu c_\nu \bar{\varphi}_\nu^2 \tag{4.4}$$

$$\sigma^2 = \sum_\nu c_\nu^2 \bar{\varphi}_\nu^2 \tag{4.5}$$

where $\bar{\varphi}_\nu = E\varphi_\nu(X_1)$. Finally we have

$$S_n = \frac{1}{n^2} \sum_{j\neq k} \sum_{\nu} c_\nu \varphi_\nu(X_j)\varphi_\nu(X_k)$$

$$= \frac{1}{n^2} \sum_{\nu} c_\nu \sum_{j\neq k} \varphi_\nu(X_j)\varphi_\nu(X_k) .$$

Case (i). With the insertion of the null term $\sum_\nu c_\nu \bar{\varphi}_\nu^2$ ((4.4)) we have

$$n^{1/2}S_n = n^{-3/2} \sum_\nu c_\nu \left\{ [\sum_{j\neq k} (\varphi_\nu(X_j) - \bar{\varphi}_\nu)(\varphi_\nu(X_k) - \bar{\varphi}_\nu)] + 2(n-1)\bar{\varphi}_\nu \sum_j (\varphi_\nu(X_j) - \bar{\varphi}_\nu) \right\}$$

$$= n^{-3/2} \sum_\nu c_\nu [\sum_{j\neq k} (\varphi_\nu(X_j) - \bar{\varphi}_\nu)(\varphi_\nu(X_k) - \bar{\varphi}_\nu)]$$

$$+ 2(n-1)n^{-3/2} \sum_\nu c_\nu \bar{\varphi}_\nu \sum_j (\varphi_\nu(X_j) - \bar{\varphi}_\nu) .$$

Using the orthonormality of the φ_ν, one can verify directly that the variance of the first term tends to zero with increasing n (it is in fact bounded above by $4n^{-3}\binom{n}{2} \sum c_\nu^2$). The second term is essentially

$$V_n = 2n^{-1/2} \sum_\nu c_\nu \bar{\varphi}_\nu \sum_j (\varphi_\nu(X_j) - \bar{\varphi}_\nu)$$

for which we consider a finite truncation

$$V_{nN} = 2n^{-1/2} \sum_{\nu=1}^{N} c_\nu \bar{\varphi}_\nu \sum_j (\varphi_\nu(X_j) - \varphi_\nu) . \tag{4.6}$$

Again using the properties of the φ_ν, one can find

$$EV_n = EV_{nN} = 0$$

$$E[V_n - V_{nN}]^2 \leq 4 \sum_{\nu=N+1}^{\infty} c_\nu^2 \bar{\varphi}_\nu^2 \tag{4.7}$$

$$EV_{nN}^2 = 4[\sum_{\nu=1}^{N} c_\nu^2 \bar{\varphi}_\nu^2 - (\sum_{\nu=1}^{N} c_\nu \bar{\varphi}_\nu^2)] . \tag{4.8}$$

Hence $V_{nN} \to V_n$ in mean square uniformly in n. Since the inner sum of (4.6) is over iid elements with zero mean and finite variance, we conclude that, as $n \to \infty$, V_{nN}

converges in law to a zero mean normal variable with variance given by (4.8). With increasing N, (4.8) converges to $4\sigma^2$. This is sufficient to conclude the result (see, for instance, [2, Theorem 4.2]).

Case (ii). Here $\sigma^2 = 0$ so that by (4.5) $\bar{\varphi}_\nu = 0$ for all ν. Again we use a truncation argument. Setting

$$V_n = nS_n = \frac{1}{n}\sum_\nu c_\nu \sum_{j\neq k} \varphi_\nu(X_j)\varphi_\nu(X_k)$$

and

$$V_{nN} = \frac{1}{n}\sum_{\nu=1}^{N} c_\nu \sum_{j\neq k} \varphi_\nu(X_j)\varphi_\nu(X_k)$$

we have

$$EV_n = EV_{nN} = 0$$

and

$$E[V_n - V_{nN}]^2 \leq 4\binom{n}{2}n^{-2}\sum_{\nu=N+1}^{\infty} c_\nu^2 \tag{4.9}$$

As before (4.9) provides a uniform bound on the truncation error (in mean square). Now we may re-write

$$V_{nN} = \sum_{\nu=1}^{N} c_\nu\left[\left(\frac{1}{n^{1/2}}\sum_i \varphi_\nu(X_j)\right)^2 - \frac{1}{n}\sum_j \varphi_\nu^2(X_j)\right]$$

$$= \sum_{\nu=1}^{N} c_\nu\left(\frac{1}{n^{1/2}}\sum_i \varphi_\nu(X_j)\right)^2 - \sum_{\nu=1}^{N} c_\nu \frac{1}{n}\sum_j \varphi_\nu^2(X_j) .$$

By the law of large numbers, the second term converges to $\sum_{\nu=1}^{N} c_\nu$. Setting $Y_{n\nu} = n^{-1/2}\sum_i \varphi_\nu(X_j)$, we have

$$EY_{n\nu} = 0$$

$$EY_{n\nu}Y_{n\mu} = \delta_{\nu\mu}$$

so that $(Y_{n1},\ldots,Y_{nN})$ converges in law to $(Z_1,\ldots,Z_N)$ where the Z_ν are independent $N(0,1)$ variables. Hence, as $n \to \infty$, V_{nN} converges in law to a variable of the form

$$\sum_{\nu=1}^{N} c_\nu[Z_\nu^2 - 1]$$

which, with increasing N, converges to

$$\sum_{\nu=1}^{\infty} c_\nu [Z_\nu^2 - 1]$$

and we are done.

As an example, let us again use the homogeneous line segment model. Here

$$A(X_j;X_k) = \frac{L_j L_k}{2} |\sin(\alpha_j - \alpha_k)| .$$

We have

$$EA(X_j;X_k) = \frac{(EL_k)^2}{2}\frac{2}{\pi} = \frac{(EL_k)^2}{\pi}$$

and

$$E_{X_k} A(X_j;X_k) = \frac{L_j EL_k}{2}\frac{2}{\pi} = \frac{L_j EL_k}{\pi}$$

so that

$$\sigma^2 = \frac{(EL_k)^2}{\pi^2} \, Var\, L_k .$$

If $Var\ L_k > 0$, then $\sigma^2 > 0$ and the central limit theorem holds. That is, $n^{1/2}\left[A(\bar{X}_n) - \frac{(EL_j)^2}{\pi}\right]$ converges in law to a normal variable with zero mean and variance $4\sigma^2$.

In the other case, let us suppose for concreteness that $L \equiv 1$. Then we have the ordinary Fourier orthogonal expansion

$$\begin{aligned} A(X_j;X_k) - EA(X_j;X_k) &= \frac{1}{4}\left[|\sin(\alpha_j - \alpha_k)| - E|\sin(\alpha_j - \alpha_k)|\right] \\ &= \sum_{\nu=2,4,\ldots} \frac{1}{1-\nu^2}\left(\frac{\cos \nu\alpha_j}{\sqrt{\pi}} \cdot \frac{\cos \nu\alpha_k}{\sqrt{\pi}} + \frac{\sin \nu\alpha_j}{\sqrt{\pi}} \cdot \frac{\sin \nu\alpha_k}{\sqrt{\pi}}\right) . \end{aligned}$$

Since $EA(X_j) = 0$, case (ii) provides the convergence in law

$$n[A(\bar{X}_n) - \tfrac{1}{\pi}] \to \sum_{\nu=2,4,\ldots} \frac{1}{1-\nu^2}[Z_\nu^2 - 1 + \tilde{Z}_\nu^2 - 1] - \frac{1}{\pi} = 1 - \frac{1}{\pi} - \sum_{\nu=2,4,\ldots} \frac{1}{\nu^2 - 1} W_\nu$$

where we have used $\sum_{\nu=2,4,\ldots} \frac{1}{\nu^2-1} = \frac{1}{2}$ ([6, #6.1.37]) and indicated by $\{W_\nu = Z_\nu^2 + \tilde{Z}_\nu^2\}$ a collection of independent, mean 2 exponential variables.

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~~We consider~~ the asymptotic distribution, are considered of functionals associated with normed sums of random plane convex sets. Methods involving symmetric statistics and weak convergence of stochastic processes are used to examine area and perimeter in particular.