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ASYMPTOTIC FORMULA FOR THE DERIVATIVES OF ORTHOGONAL POLYNOMIAL--ETC(U)

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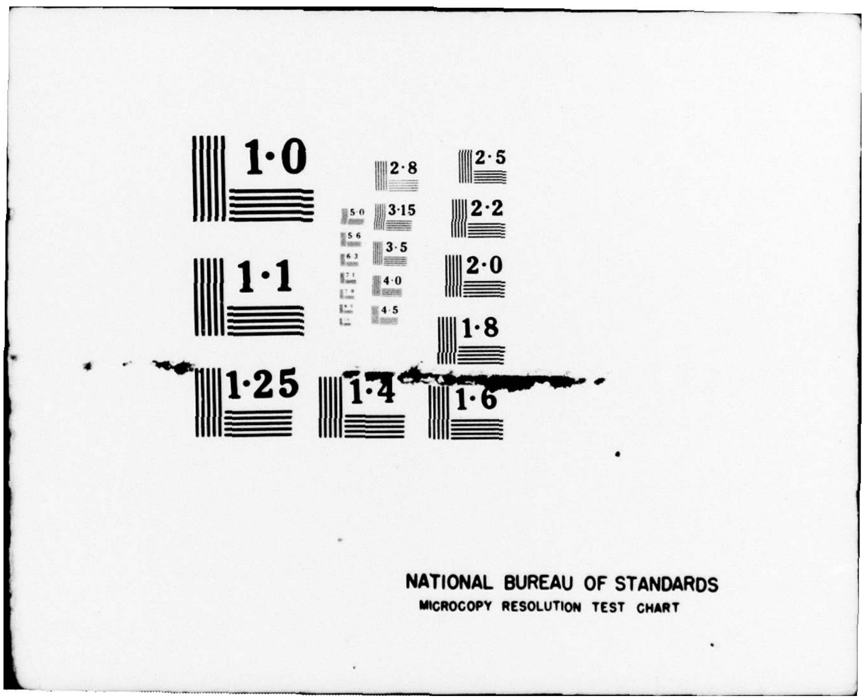
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AN ASYMPTOTIC FORMULA FOR THE DERIVATIVES OF ORTHOGONAL POLYNOMIALS

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ABSTRACT

An asymptotic expression is found for the derivatives of orthogonal polynomials on the unit circle. The condition on the weight function is local and the result is stronger than the previous ones.

SIGNIFICANCE AND EXPLANATION

When expanding functions into orthogonal series and investigating the convergence of the derivatives of these series one is led to consider the asymptotic behavior of the derivatives of orthogonal polynomials. In the paper we show that essentially one can differentiate the asymptotic formula for the orthogonal polynomials in order to get asymptotics for the derivatives of these polynomials.

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AN ASYMPTOTIC FORMULA FOR THE DERIVATIVES OF ORTHOGONAL POLYNOMIALS

Paul G. Nevai

Let σ be a bounded nondecreasing function on $[0,2\pi]$ taking infinitely many values. Then there exists a unique sequence of polynomials $\{\varphi_n(d\sigma)\}_{n=0}^{\infty}$ such that $\varphi_n(d\sigma,z) = \alpha_n(d\sigma)z^n + \ldots$, $\alpha_n(d\sigma) > 0$ and

$$\frac{1}{2\pi} \int_0^{2\pi} \varphi_n(d\sigma,z) \overline{\varphi_m(d\sigma,z)} \, d\sigma(\theta) = \delta_{nm} \qquad (z = e^{i\theta}) .$$

One of the basic problems in the theory of orthogonal polynomials on the unit circle is to find asymptotic expressions for $\varphi_n(d\sigma,z)$ as $n \to \infty$. There is an extensive literature dealing with this question. (See e.g. [2], [3] and [7].) In order to obtain asymptotics one has to assume that σ behaves nice in a certain sense. Usually there are two kinds of assumptions: globally σ must satisfy a growth condition and locally (near θ, $z = e^{i\theta}$) σ has to be smooth. The weakest condition under which one can prove asymptotics for $\varphi_n(d\sigma,z)$ belongs to G. Freud [2].

Let the Szegö function $D(d\sigma,z)$ corresponding to σ be defined by

$$D(d\sigma,z) = \exp\{\frac{1}{4\pi} \int_{-\pi}^{\pi} \log \sigma'(t) \frac{1+ze^{-it}}{1-ze^{-it}} dt\} \qquad (|z| < 1) .$$

If

$$\int_{-\pi}^{\pi} \log \sigma'(t)dt > -\infty \tag{1}$$

then $D(d\sigma) \in H_2(|z| < 1)$, $D(d\sigma,z) \neq 0$ for $|z| < 1$, $D(d\sigma,0) > 0$ and

$$\lim_{r \to 1^-} D(d\sigma,re^{it}) = D(d\sigma,e^{it})$$

exists and $|D(d\sigma,e^{it})|^2 = \sigma'(t)$ for almost every $t \in [-\pi,\pi]$.

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Using the notion of Szegö's function we can formulate Freud's result. Assume that (1) is satisfied and in a neighborhood of θ $(z = e^{i\theta})$ σ is absolutely continuous with $0 < m \le \sigma'(t) \le M < \infty$ for $|\theta-t|$ small and

(2) $$\int_{|\theta-t| \text{ small}} \left(\frac{\sigma'(\theta)-\sigma'(t)}{\theta - t}\right)^2 dt < \infty .$$

Then

(3) $$\lim_{n\to\infty} [\varphi_n(d\sigma,e^{i\theta}) - e^{in\theta}\, \overline{D(d\sigma,e^{i\theta})^{-1}}] = 0 .$$

In the end of L. Geronimus' book [3] fourteen other conditions are given all of which imply the asymptotic relation (3). The problem of finding asymptotics for $\varphi_n^{(k)}(d\sigma,z)$ $(k = 1,2,\dots)$ seems to be more difficult. There are only a very few papers investigating the relationship

(4) $$\lim_{n\to\infty} [n^{-k}\varphi_n^{(k)}(d\sigma,z) - z^{n-k}\,\overline{D(d\sigma,z)^{-1}}] = 0 \qquad (z = e^{i\theta}) .$$

(See [4], [5] and [6].) In all these papers it is assumed that σ satisfies some very restrictive conditions. In particular, σ has to be absolutely continuous and $(\sigma')^{-1} \in L^2$. In [4] and [5] the authors apply strong methods of approximation theory. The purpose of this paper is to show that (4) can be proved under Freud's conditions. Instead of approximation theory our approach is based on the following two observations. First, it is easy to prove (4) provided that σ is very nice locally. Second, the weak asymptotics

(5) $$\lim_{n\to\infty} \frac{1}{2\pi}\int_{-\pi}^{\pi} |\varphi_n(d\sigma,e^{i\theta}) - e^{in\theta}\,\overline{D(d\sigma,e^{i\theta})^{-1}}|^2\, d\sigma(\theta) = 0$$

always holds whenever (1) is satisfied. (See [2], §V.4.)

In the following Δ will denote a closed interval in $[-2\pi,2\pi]$, Δ^0 is its interior and $\widehat{\Delta}$ is the corresponding arc on the unit circle. If P is a polynomial then $\overline{P}$ denotes the polynomial whose coefficients are the complex conjugates of the corresponding coefficients of P. The polynomial $\varphi_n^*(d\sigma)$ is defined by

$$\varphi_n^*(d\sigma,z) = z^n\,\overline{\varphi}_n(d\sigma,z^{-1}) .$$

We have therefore by (5)

(6) $$\lim_{n\to\infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} |\varphi_n^*(d\sigma,e^{i\theta}) - D(d\sigma,e^{i\theta})^{-1}|^2 \, d\sigma(\theta) = 0$$

provided that (1) holds.

Lemma. Let σ be such that (1) is satisfied. Let Δ and $\Delta_1 \subset \Delta^0$ be given. Suppose that σ is absolutely continuous on Δ and $\sigma'(t) = 1$ for $t \in \Delta$. Then for every fixed $k = 0,1,\ldots$ $n^{-k}|\varphi_n^{(k)}(d\sigma,z)|$ and $n^{-k}|\varphi_n^{*(k)}(d\sigma,z)|$ are uniformly bounded for $z \in \widehat{\Delta}_1$. Further (4) holds uniformly for $z \in \widehat{\Delta}_1$.

Proof. Fix Δ_2 so that $\Delta_1 \subset \Delta_2^0 \subset \Delta_2 \subset \Delta^0$. By a result of L. Geronimus [3]

(7) $$\varphi_n^*(d\sigma,e^{i\theta}) = D(d\sigma,e^{i\theta})^{-1} + o(1)$$

uniformly for $\theta \in \Delta_2$. Because of the assumptions $D(d\sigma,e^{i\theta})^{-1}$ is continuous on Δ_2. We have

$$\left|\frac{d^k}{d\theta^k} \varphi_n^*(d\sigma,e^{i\theta})\right| \le \left|\frac{d^k}{d\theta^k} [\varphi_n^*(d\sigma,e^{i\theta}) - \varphi_{[\sqrt{n}]}^*(d\sigma,e^{i\theta})]\right| +$$
$$+ \left|\frac{d^k}{d\theta^k} \varphi_{[\sqrt{n}]}^*(d\sigma,e^{i\theta})\right| .$$

Therefore by the local version of Bernstein's inequality (see [1], p. 896.)

$$\max_{\theta\in\Delta_1} \left|\frac{d^k}{d\theta^k} \varphi_n^*(d\sigma,e^{i\theta})\right| \le$$

$$\text{const}[n^k \cdot \max_{\theta\in\Delta_2} |\varphi_n^*(d\sigma,e^{i\theta}) - \varphi_{[\sqrt{n}]}^*(d\sigma,e^{i\theta})| + n^{\frac{k}{2}} \max_{\theta\in\Delta_2} |\varphi_{[\sqrt{n}]}^*(d\sigma,e^{i\theta})|] .$$

Consequently for $k = 1,2,\ldots$

(8) $$\left|\frac{d^k}{d\theta^k} \varphi_n^*(d\sigma,e^{i\theta})\right| = o(n^k)$$

uniformly for $\theta \in \Delta_1$. Hence $|\varphi_n^{*(k)}(d\sigma,z)| = o(n^k)$ uniformly for $z \in \widehat{\Delta}_1$ if $k \ge 1$ is fixed. Now we have

$$\varphi_n(d\sigma,e^{i\theta}) = e^{in\theta}\, \overline{\varphi_n^*(d\sigma,e^{i\theta})} .$$

Differentiating this identity and using (8) we obtain for $k = 1,2,\ldots$

$$\text{(9)} \qquad \frac{d^k}{d\theta^k}\varphi_n(d\sigma,e^{i\theta}) = (in)^k e^{in\theta}\overline{\varphi_n^*(d\sigma,e^{i\theta})} + o(n^k)$$

uniformly for $\theta \in \Delta_1$. This is also valid when $k = 0$. Replacing differentiation in θ by differentiation in z and using the fact that (9) is valid for every k we obtain (7).

<u>THEOREM</u>. Let σ satisfy (1) and let $z = e^{i\theta}$ be fixed. Assume that σ is absolutely continuous near θ, $0 < m \le \sigma'(t) \le M < \infty$ for $|\theta-t|$ small and (2) holds. Then for every fixed $k = 1,2,\ldots$ the asymptotic relation (4) holds true.

<u>Proof</u>. Pick up a sufficiently small neighborhood Δ of θ and define σ_1 by

$$d\sigma_1(t) = \begin{cases} d\sigma(t) & \text{for } t \notin \Delta \\ dt & \text{for } t \in \Delta . \end{cases}$$

Let the function g be defined by

$$g(t) = \begin{cases} 1 & \text{for } t \notin \Delta \\ \sigma'(t) & \text{for } t \in \Delta . \end{cases}$$

If Δ is small enough then $d\sigma = g\,d\sigma_1$, $0 < m_1 \le g(t) < M_1 < \infty$ and

$$\int_{-\pi}^{\pi}\left(\frac{g(\theta)-g(t)}{\theta-t}\right)^2 dt < \infty .$$

Note that σ_1 satisfies the conditions of the lemma. Let us expand $\varphi_n(d\sigma)$ into Fourier series in $\varphi_\ell(d\sigma_1)$. We have

$$\text{(10)} \qquad \varphi_n(d\sigma,z) = \frac{1}{2\pi}\int_0^{2\pi}\varphi_n(d\sigma,e^{it})\,K_n(d\sigma_1,z,e^{it})\,d\sigma_1(t)$$

where

$$K_n(d\sigma_1,z,y) = \sum_{\ell=0}^{n}\varphi_n(d\sigma_1,z)\,\overline{\varphi_n(d\sigma_1,y)} .$$

Differentiating (10) by z and using the fact that $\varphi_n(d\sigma)$ is orthogonal with respect to $g\,d\sigma_1$ we obtain

$$\varphi_n^{(k)}(d\sigma,z) = \frac{1}{2\pi}\int_0^{\pi} \varphi_n(d\sigma,e^{it})\,\overline{\varphi_n(d\sigma_1,e^{it})}\,d\sigma_1(t)\,\varphi_n^{(k)}(d\sigma_1,z) +$$

$$+ \frac{1}{2\pi}\int_0^{2\pi} \varphi_n(d\sigma,e^{it})\,\frac{d^k}{dz^k}K_{n-1}(d\sigma_1,z,e^{it})\,[1-\frac{g(t)}{g(\theta)}]\,d\sigma_1(t) \quad .$$

Using the Christoffel-Darboux formula

$$(1-z\bar{y})\,K_{n-1}(d\sigma_1,z,y) = \varphi_n^*(d\sigma_1,z)\,\overline{\varphi_n^*(d\sigma_1,y)} - \varphi_n(d\sigma_1,z)\,\overline{\varphi_n(d\sigma_1,y)}$$

(See [2].) we get

$$(1-z\bar{y})\,\frac{d^k}{dz^k}K_{n-1}(d\sigma_1,z,y) - k\bar{y}\,\frac{d^{k-1}}{dz^{k-1}}K_{n-1}(d\sigma_1,z,y) =$$

$$= \varphi_n^{*(k)}(d\sigma_1,z)\,\overline{\varphi_n^*(d\sigma_1,y)} - \varphi_n^{(k)}(d\sigma_1,z)\,\overline{\varphi_n(d\sigma_1,y)} \quad .$$

Therefore $\varphi_n^{(k)}(d\sigma,z)$ $(z = e^{i\theta})$ can be written as

$$\varphi_n^{(k)}(d\sigma,z) = A + B + C$$

where

$$A = \varphi_n^{(k)}(d\sigma_1,z)\cdot\frac{1}{2\pi}\int_0^{2\pi}\varphi_n(d\sigma,e^{it})\,\overline{\varphi_n(d\sigma_1,e^{it})}\cdot[1-\frac{1-\frac{g(t)}{g(\theta)}}{1-e^{i(\theta-t)}}]\,d\sigma_1(t) \quad ,$$

$$B = \varphi_n^{*(k)}(d\sigma_1,z)\,\frac{1}{2\pi}\int_0^{2\pi}\varphi_n(d\sigma,e^{it})\,\overline{\varphi_n^*(d\sigma_1,e^{it})}\cdot\frac{1-\frac{g(t)}{g(\theta)}}{1-e^{i(\theta-t)}}\,d\sigma_1(t)$$

and

$$C = k\cdot\frac{1}{2\pi}\int_0^{2\pi}\varphi_n(d\sigma,e^{it})\,\frac{d^{k-1}}{dz^{k-1}}K_{n-1}(d\sigma_1,z,e^{it})\cdot e^{-it}\cdot\frac{1-\frac{g(t)}{g(\theta)}}{1-e^{i(\theta-t)}}\,d\sigma_1(t) \quad .$$

We will estimate A, B and C separately. First we consider A . We will show that the integral in A converges as $n \to \infty$. Write σ and σ_1 as

$$\sigma = \sigma^a + \sigma^s + \sigma^j, \quad \sigma_1 = \sigma_1^a + \sigma_1^s + \sigma_1^j$$

where a, s and j refer to the absolutely continuous, singular and jump components respectively. It is clear from the construction that $\sigma^s = \sigma_1^s$, $\sigma^j = \sigma_1^j$ and $d\sigma^a = g\,d\sigma_1^a$. Since the function

$$f(t) = 1 - \frac{1 - \frac{g(t)}{g(\theta)}}{1 - e^{i(\theta-t)}}$$

is uniformly bounded on the support of $d(\sigma_1^s + \sigma_1^j)$ we obtain from (5) that

$$\lim_{n\to\infty} \frac{1}{2\pi} \int_0^{2\pi} \varphi_n(d\sigma, e^{it}) \overline{\varphi_n(d\sigma_1, e^{it})} f(t)\, d[\sigma_1^s(t) + \sigma_1^j(t)] = 0 \quad .$$

Now fix $\varepsilon > 0$ and choose $\delta > 0$ so that $\theta \pm \delta \in \Delta_1 \subset \Delta^0$ and

$$\frac{1}{2\pi} \int_{\theta-\delta}^{\theta+\delta} |f(t)|^2 g(t)^{-1} dt < \varepsilon.$$

We have

$$\frac{1}{2\pi} \int_{\theta-\delta}^{\theta+\delta} |\varphi_n(d\sigma, e^{it}) \varphi_n(d\sigma_1, e^{it}) f(t)|\, d\sigma_1^a(t) \le$$

$$[\frac{1}{2\pi} \int_{\theta-\delta}^{\theta+\delta} |\varphi_n(d\sigma, e^{it})|^2 g(t)\, d\sigma_1^a(t)]^{\frac{1}{2}} \cdot [\frac{1}{2\pi} \int_{\theta-\delta}^{\theta+\delta} |f(t)|^2 g(t)^{-1}\, d\sigma_1^a(t)]^{\frac{1}{2}} \cdot$$

$$\cdot \max_{t\in\Delta_1} |\varphi_n(d\sigma_1, e^{it})|$$

which by the lemma is $O(\sqrt{\varepsilon})$. Using (5) we obtain that

$$\lim_{n\to\infty} \frac{1}{2\pi} \int_{|\theta-t|>\delta} \varphi_n(d\sigma, e^{it}) \overline{\varphi_n(d\sigma_1, e^{it})} f(t)\, d\sigma_1^a(t) =$$

$$= \frac{1}{2\pi} \int_{|\theta-t|>\delta} \overline{D(d\sigma, e^{i\theta})^{-1}} D(d\sigma_1, e^{it})^{-1} f(t)\, d\sigma_1^a(t) \quad .$$

Letting $\varepsilon \to 0$ and using the lemma we finally get

(11) $$A = \rho\, \varphi_n^{(k)}(d\sigma_1, z) + o(n^k)$$

as $n \to \infty$ where the number ρ does not depend on k . The expression B can be estimated in a similar way. The only difference is that this time we have to apply both (5) and (6). Because $\varphi_n^*(d\sigma_1)$ weakly converges we obtain

(12) $$B = o(n^k)$$

on $n \to \infty$. In order to show that

(13) $$C = o(n^k)$$

as $n \to \infty$ we use Cauchy's inequality. We get

$$|C|^2 \le k^2 \frac{1}{2\pi} \int_0^{2\pi} \left|\frac{d^{k-1}}{dz^{k-1}} K_{n-1}(d\sigma_1, z, e^{it})\right|^2 \left|\frac{1-\frac{g(t)}{g(\theta)}}{1-e^{i(\theta-t)}}\right|^2 q(t)^{-1}\, d\sigma_1(t) .$$

By the lemma

$$\left|\frac{d^{k-1}}{dz^{k-1}} K_{n-1}(d\sigma_1, z, e^{it})\right|^2 \le \text{const } n^{2k}$$

for $|\theta-t|$ small and by the conditions

$$\lim_{\delta\to 0} \frac{1}{2\pi} \int_{|t-\theta|<\delta} \left|\frac{1-\frac{g(t)}{g(\theta)}}{1-e^{i(\theta-t)}}\right|^2 g(t)^{-1}\, d\sigma_1(t) = 0 .$$

Therefore we have to estimate

$$\frac{1}{2\pi} \int_{|t-\theta|\ge\delta} \left|\frac{d^{k-1}}{dz^{k-1}} K_{n-1}(d\sigma_1, z, e^{it})\right|^2 \left|\frac{1-\frac{g(t)}{g(\theta)}}{1-e^{i(\theta-t)}}\right|^2 g(t)^{-1}\, d\sigma_1(t)$$

for fixed $\delta > 0$. But this is less than

$$\text{const } \frac{1}{2\pi} \int_0^{2\pi} \left|\frac{d^{k-1}}{dz^{k-1}} K_{n-1}(d\sigma_1, z, e^{it})\right|^2 d\sigma_1(t) = \text{const} \sum_{j=0}^{n-1} |\varphi_j^{(k-1)}(d\sigma_1, z)|^2$$

which is $O(n^{2k-1})$ by the lemma. Hence we have proved (13). From (11)-(13) we obtain

$$n^{-k} \varphi_n^{(k)}(d\sigma, z) = \rho\, n^{-k} \varphi_n^{(k)}(d\sigma_1, z) + o(1) \qquad (n \to \infty)$$

for $k = 0,1,\ldots$ fixed where ρ is independent of k. By the lemma

$$n^{-k} \varphi_n^{(k)}(d\sigma_1, z) = z^{-k} \varphi_n(d\sigma_1, z) + o(1)$$

as $n \to \infty$. Therefore

$$n^{-k} \varphi_n^{(k)}(d\sigma, z) = z^{-k} \varphi_n(d\sigma, z) + o(1)$$

and the theorem follows from Freud's result which was formulated in the beginning.

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