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ELECTROFLUID DYNAMIC GENERATORS AND CYCLE PERFORMANCES FOR OPTI--ETC(U)
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ELECTROFLUID DYNAMIC GENERATORS AND CYCLE PERFORMANCES FOR OPTIMALLY MATCHED INJECTORS

JULY 1976

TECHNICAL REPORT AFAPL-TR-76-111
Final Report for Period September 1974 – July 1976

AIR FORCE AERO-PROPULSION LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

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This report contains the results of an effort to optimally match electrofluid dynamic generators and injectors to provide improved performance. The work was performed in the High Power Branch of the Air Force Aero Propulsion Laboratory, Air Force Systems Command, Wright-Patterson AFB, Ohio under Project 7116-01, Task 01. The effort was conducted by Maurice O. Lawson during the period September 1974 to July 1976.

The author wishes to thank Dr. John Minardi of the University of Dayton Research Institute for his many valuable suggestions and comments.

This report has been reviewed by the Information Office (ASD/OIP) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication

Maurice O. Lawson

Maurice O. Lawson
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The Air Force in-house analytical portion of the electrofluid dynamics (EFD) direct energy conversion research is described. The experimenal portion has been given in the final report, AFAPL-TR-76-35. EFD generator performance analyses have been extended to apply to more general injector-generator types and to include effects of friction and mixing losses. Through the definition of an EFD specific work function, a convenient method of optimally matching an injector to a specific generator is described. Application examples are pre-

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sented for high overall efficiency cycles as well as for low capitol cost generators.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

SECTION I

INTRODUCTION

This report presents the in-house analytical portion of the electrofluid dynamic (EFD) direct energy conversion research conducted since the 1974 publication of "Report on Progress in Achieving Direct Conversion of a Major Fraction of Sonic Flow Kinetic Power into Electrical Power by Electrofluid Dynamic (EFD) Processes." [1]

The corresponding period of in-house experimental research is covered in AFAPL-TR-76-35. [2]

At the Air Force Aerospace Research Laboratories (ARL) a research program in electrofluid dynamic direct energy conversion was established in 1963. During the existence of ARL, the program averaged about a five-man effort. Because EFD is a highly interdisciplinary principle, many major research areas had to be advanced before experimental generators began to demonstrate promise. In the early period of research, electrical pressures that were imposed on the flow were low, about 3 $lb/ft.^2$. In 1973, electrical performance as well as direct pressure measurements, demonstrated values about 1000 times greater. Still, this performance expressed as an isentropic pressure ratio was only 1.05, while in comparison a single impulse turbine stage may attain a value of about 10. Because of such low pressure ratio potential of EFD stages, about half of the research groups in the field advocated staging while the rest, including the ARL group, advocated the application of injectors. The kind of injector advocated is not the one with which most engineers are familiar, namely the steam injector which was used to pump water into a boiler (corresponding to a low density fluid pumping a high density fluid). Such an injector on principle has a very low efficiency as can be demonstrated readily using the analysis in this report. But rather, the fluids have the opposite characteristics, a very dense fluid transfers its kinetic energy to a low density fluid.

This type of injector can operate efficiently at isentropic pressure ratios greater than 10, allowing a single EFD stage operating with a small pressure ratio to be applied because of a greatly augmented volume flow.

This report presents a convenient method for optimally matching injectors and EFD generators according to the characteristics of the fluids and performance objectives. Also, some generator performances and potential overall cycle efficiencies are examined.

SECTION II

EFD STAGE SPECIFIC WORK FUNCTION

For an injector-generator analysis it is convenient to apply a generator, or a generator stage, specific work or head function. It is the ideal electric power that a particular channel operating with a specific working fluid and channel field strength produces per unit mass flow of that working fluid. The expression for the generator specific work function for an axisymmetrical channel (see Appendix I for development) is:

$$\phi = 1/2\ \epsilon\ \frac{V_s}{\rho\ \ell}\ E_b\ \frac{\ell}{r},\ \text{j/kg}$$

where ϵ = gas permittivity, c/(V-m)

V_s = sparkover voltage in a uniform field geometry, V

ρ = gas density, kg/m^3

E_b = gas breakdown field strength, V/m

ℓ = sparkover gap or conversion channel length, m

r = channel radius, or channel height for two dimension flow, m

For a two dimensional geometry channel the ideal value of the generator specific work function is just twice as large as the value given in the above relationship for the axisymmetric channel.

Noting that $V_s / \ell = E_b$, the EFD generator specific work function could be written as

$$\phi = (1/2)\epsilon\ \frac{E_b^2}{\rho}\ \frac{\ell}{r}$$

However, for a given gas V_s is a constant over a wide range of gas density for a fixed value of the $\rho\ell$ product, therefore it is useful to maintain $V_s/\rho\ell$

as a constant for the particular gas. Another very good reason for maintaining the separation of the two E_b's is that there are different upper limits of electrical field strengths that can be important. Some of these limits may correspond to insulator breakdown strength, insulator surface breakdown strength and field (cold electron) emission. For example, electric breakdown experiments conducted by TRW Systems [3] as a part of the Air Force EFD program provide a limiting value of E_b for air of 8.13×10^7 volts per meter before $V_s/\rho \ell$ began to depart from its constant value. For SF_6 the value was 5.8×10^7 volts/meter. Both of these values approximate 10^8 volts/meter, the region of field emission. [4]

In Air Force experiments [2] with a generator using air as the working fluid, the corresponding highest value of field strength was about 5×10^7 volts/meter for which the scaling conditions i.e., $v_s/\rho \ell$, still remained constant.

SECTION III

INJECTOR-GENERATOR OPTIMALLY MATCHING SOLUTION AND EXAMPLE APPLICATIONS

The following assumptions are made in the injector analysis (see Appendix 2 for solution):

1. Constant area mixing
2. Incompressible one dimensional flow
3. Frictional losses represented by a loss coefficient equal to $(1 - \eta_D)$

The injector efficiency is defined as

$$\eta_{inj} = (P_{O3} - P_{OS})\, V_2\, A_2 \,/\, [\, \dot{m}_p \frac{V_p^2}{2} - (P_{OS} - P_1) V_p\, A_P \,]$$

see Figure 1 for notation. The denominator of the above equation represents the available primary flow power where the second term in the denominator represents the power that the primary mass flow retains to raise itself from the local pressure inside the channel to the secondary total pressure and therefore is not available for the injector process. The numerator is the injector output: the power that drives the generator.

In Appendix 2, the following Equation is derived for the injector efficiency.

$$\eta_{inj} = \frac{(1 + \bar{V}\bar{A})}{\bar{V}\,\bar{A}\,(\bar{\rho}\bar{V}^2 - 1)} \left[\frac{2(\bar{\rho}\bar{A}\,\bar{V}^2 + 1}{(1' + \bar{A})} - \frac{(2 - \eta_D)\,(\bar{\rho}\,\bar{V}\,\bar{A} + 1)(1 + \bar{V}\bar{A})}{(1 + \bar{A})^2} - 1 \right]$$

where the bars indicate the normalization of the initial primary flow values by the corresponding secondary flow values. As an example, this equation has been plotted in Figure 2 for two values of area ratio, $\bar{A}$, and for a density ratio of 100 corresponding to mercury as the primary fluid and hydrogen (at the same temperature) as the secondary fluid. For comparison,

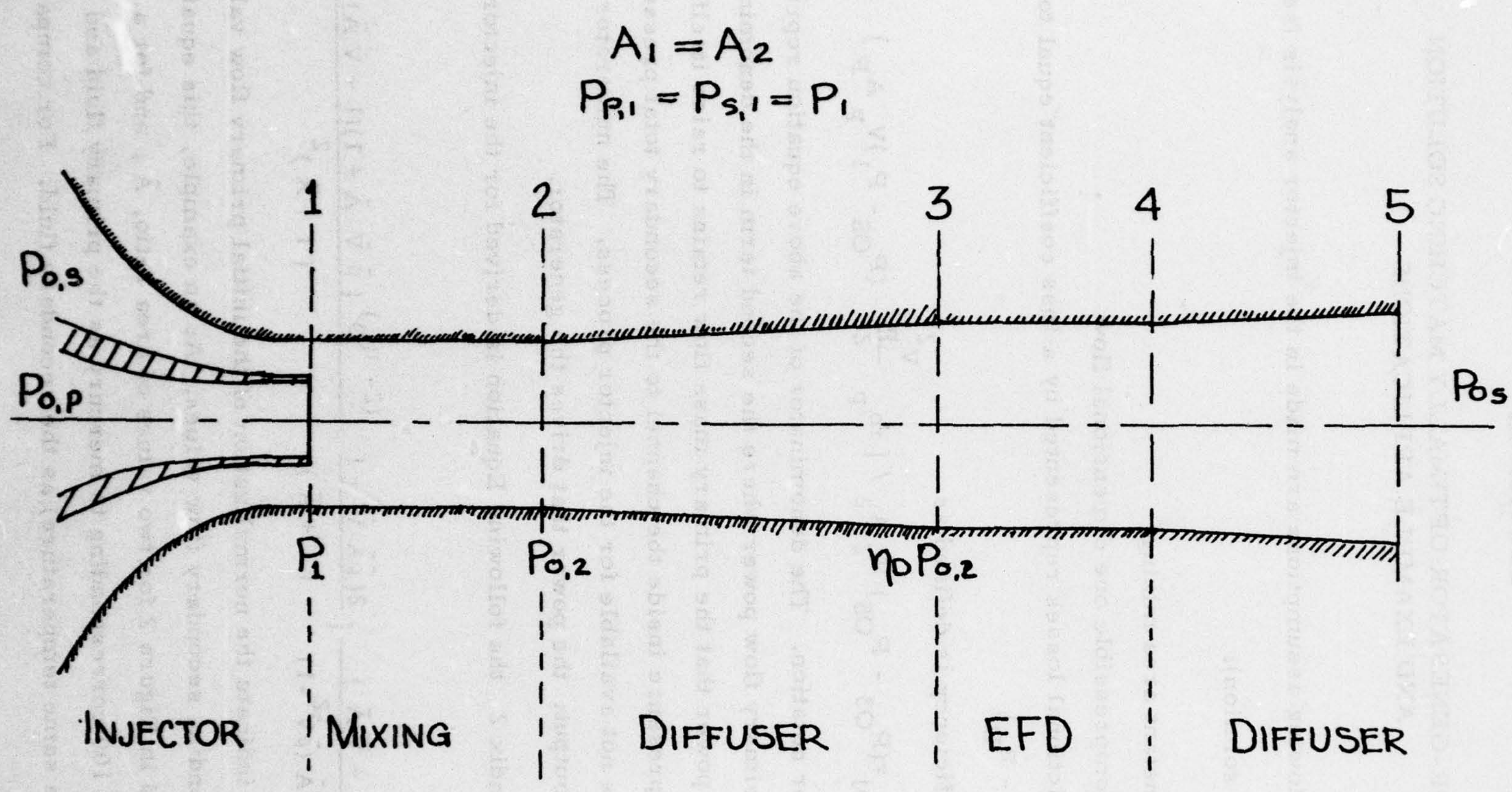

Figure 1. INJECTOR-EFD GENERATOR SCHEMATIC

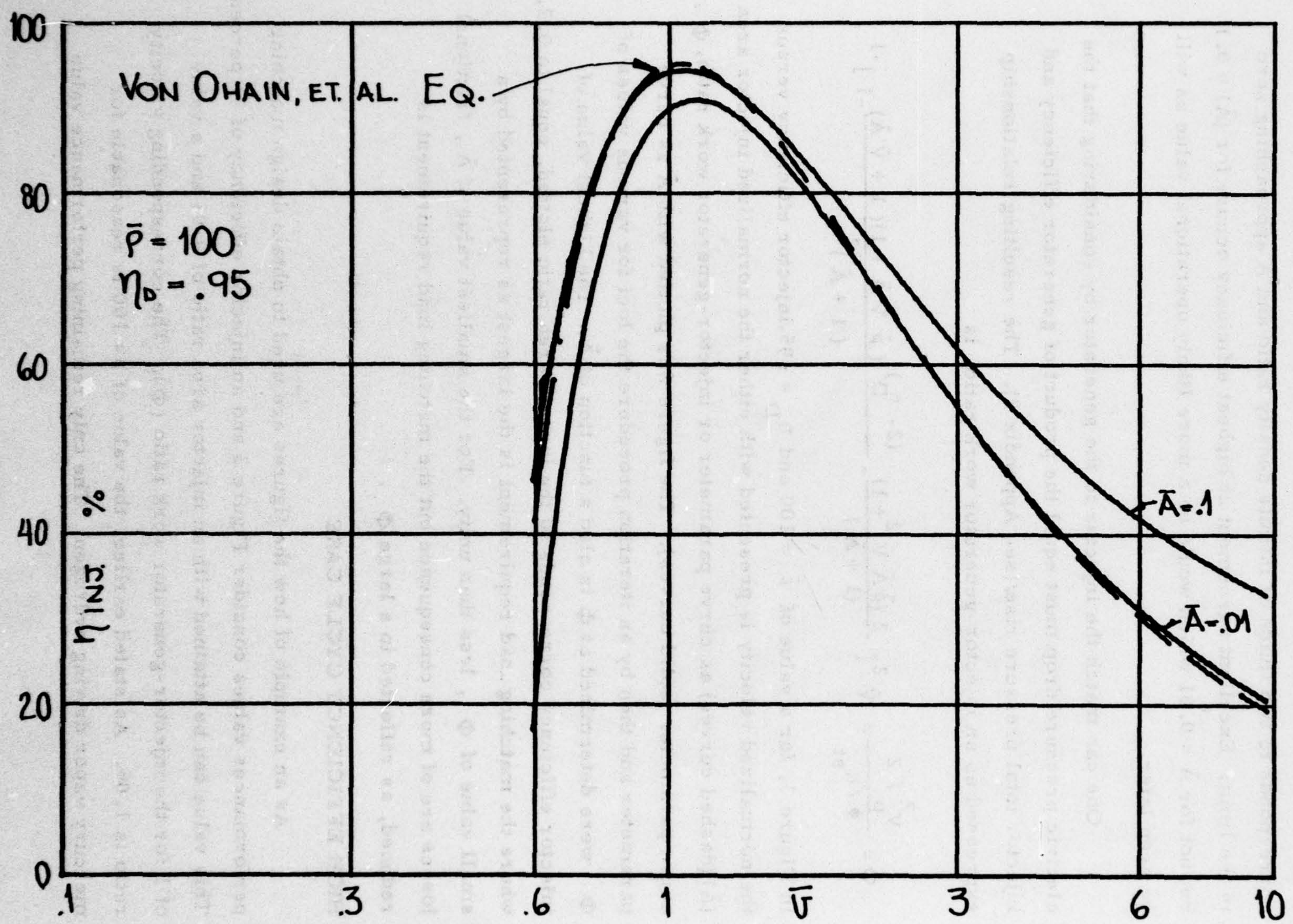

Figure 2. Injector Efficiency Versus Primary to Secondary Velocity Ratio

another injector solution[5,6] plot (dashed curve) is also given which corresponds to two fluids of infinite density ratio and $\bar{A}$ approaching zero in the limit. Excellent agreement at highest efficiency occurs for $(\bar{A}) = 0.1$ but not for $\bar{A} = 0.01$ which would be a more likely operational value as will be seen later.

One can match the injector to the generator by considering that the electric pressure drop must equal the product of generator efficiency and injector total pressure rise (see Appendix 2). The resulting relationship expressed as an injector-generator work ratio, is

$$\Phi = \frac{V_p^2 / 2}{\phi / \eta_{st}} = \bar{V}^2 \left[\frac{2(\bar{\rho}\bar{A}\bar{V}^2 + 1)}{(1 + \bar{A})} - \frac{(2 - \eta_D)(\bar{\rho}\bar{V}\bar{A} + 1)(1 + \bar{V}\bar{A})}{(1 + \bar{A})^2} - 1 \right]^{-1}$$

In Figure 3, for a value of $\bar{\rho} = 100$ and $\eta_D = .95$, injector efficiency versus the normalized velocity is presented with either the normalized injector area $(\bar{A})$ (dashed curves) as curve parameter or injector-generator work ratio, Φ, curve parameter (solid curves). The figure was plotted with $\bar{A}$ as curve parameter and then by an iteration procedure the loci for various values of Φ were determined as Φ is also a function of $\bar{A}$. The highest value of injector efficiency corresponds to the largest area ratio plotted, equal to 0.2, where the matching load requirement is the largest as represented by a small value of Φ, less than unity. For the smallest value of $\bar{A}$, frictional losses are of more consequence but the matching load requirement is reduced, as reflected in a large Φ.

HIGH EFFICIENCY CYCLE CASE

As an example of how the figures are used to obtain design matching performances values consider Figure 3 and an injector efficiency of 91 percent. This value can be attained with an injector area ratio of 0.01 and a value of 1 for the injector-generator work ratio (Φ). The corresponding velocity ratio is 1.06. As stated earlier, the value of $\bar{\rho} = 100$ is reasonable for mercury vapor driving hydrogen. The only remaining performance value

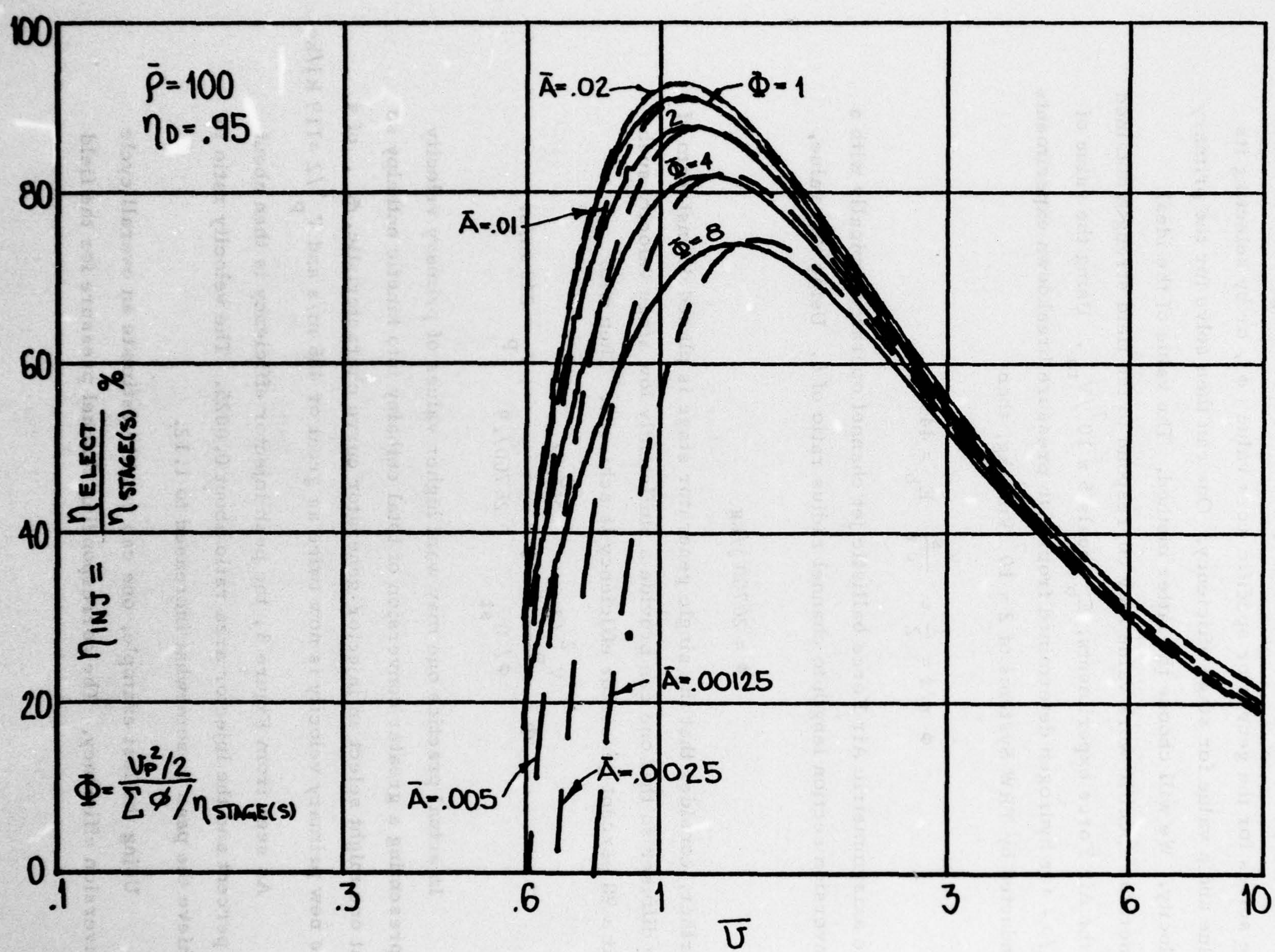

Figure 3. Injector-Generator Matching Curves for Optimizing Generator Efficiency

to be identified is the primary velocity either by assuming a value so that one solves for the generator specific work value (ϕ), or by selecting its value and a value for stage efficiency. One can then solve for the primary velocity. We will choose the latter method. The value of the ideal generator specific work value may correspond to the field strength obtained in the Air Force experiments, E_b equals $5 \times 10^7 / V_m$. Using the value of $V_s/\rho \ell$ for hydrogen determined from high pressure breakdown experiments conducted by TRW Systems of 2×10^7 Vm^2/kg, then

$$\phi \ r/\ell = \frac{1}{2} \epsilon \quad \frac{V_s}{\rho \ell} \quad E_b = 4450$$

The axisymmetric Air Force ballistic jet channel operated typically with a conversion section length to channel radius ratio of 6. Using this value,

$$\phi = 26700 \text{ j/kg}$$

Further, consider that the single generator stage is placed downstream of the diffuser so that one can provide a sufficiently low stage velocity such that a 90 percent generator efficiency is achieved. Then with

$$\Phi = \frac{V_p^2/2}{\phi/\eta_{st}} = 1 = \frac{V_p^2/2}{26700/.9} ; V_p = 243 \text{ m/s.}$$

In actual practice one may want higher values of primary velocity representing a greater conversion of total enthalpy into kinetic enthalpy so that one might select an injector-generator curve characteristic, Φ , of 4. The new primary velocity is now twice as great or 486 m/s and $V_p^2/2$ =119 kj/kg.

As seen from Figure 3 , the peak injector efficiency is then about 82 percent and the injector area ratio about 0.0025. The velocity ratio to achieve the peak is somewhat increased to 1.12.

Using the last example, one can readily estimate an overall cycle conversion efficiency. The corresponding channel pressure for the field strength of 5×10^7 v/m for hydrogen is about 40 atm. Mercury vapor at this pressure and at saturation condition has an enthalpy of 430 kj/kg. The mercury velocity head ($V_p^2/2$) was 119 kj/kg, so that the total enthalpy is

$$h_o = 430 + 119 = 549 \text{ kj/kg}$$

and

$$\eta_{thermal} = \frac{119}{549} = 22 \text{ percent.}$$

The overall efficiency assuming a nozzle efficiency of 96 percent is,

$$\eta_{overall} = \eta_{thermal} \times \eta_{inj} \times \eta_{st} \times .96$$

$$= .22 \times .82 \times .90 \times .96$$

$$= 16 \text{ percent.}$$

Although, at the present time, research has not been done in staging, we will assume that one applies four stages of the above value of generator specific work function, then $V_p^2/2$ is four times higher, or 476 kj/kg and

$$h_o = 430 + 476 = 906$$

and

$$\eta_{thermal} = \frac{476}{906} = .53$$

so that

$$\eta_{overall} = .53 \times .82 \times .90 \times .96$$

$$= 38 \text{ percent.}$$

From the above considerations, a great deal of flexibility is shown to exist such as trade offs in injector efficiency for increased values of primary flow velocity for given generator specific work values and increased thermal (and cycle) efficiency by the application of only several stages.

LOW CAPITAL COST EFD SYSTEM

At the opposite extreme of the above two fluid cases is the one-fluid injector, open cycle EFD generator case having a low overall efficiency but very low capital costs. Specific applications would be for short runs and

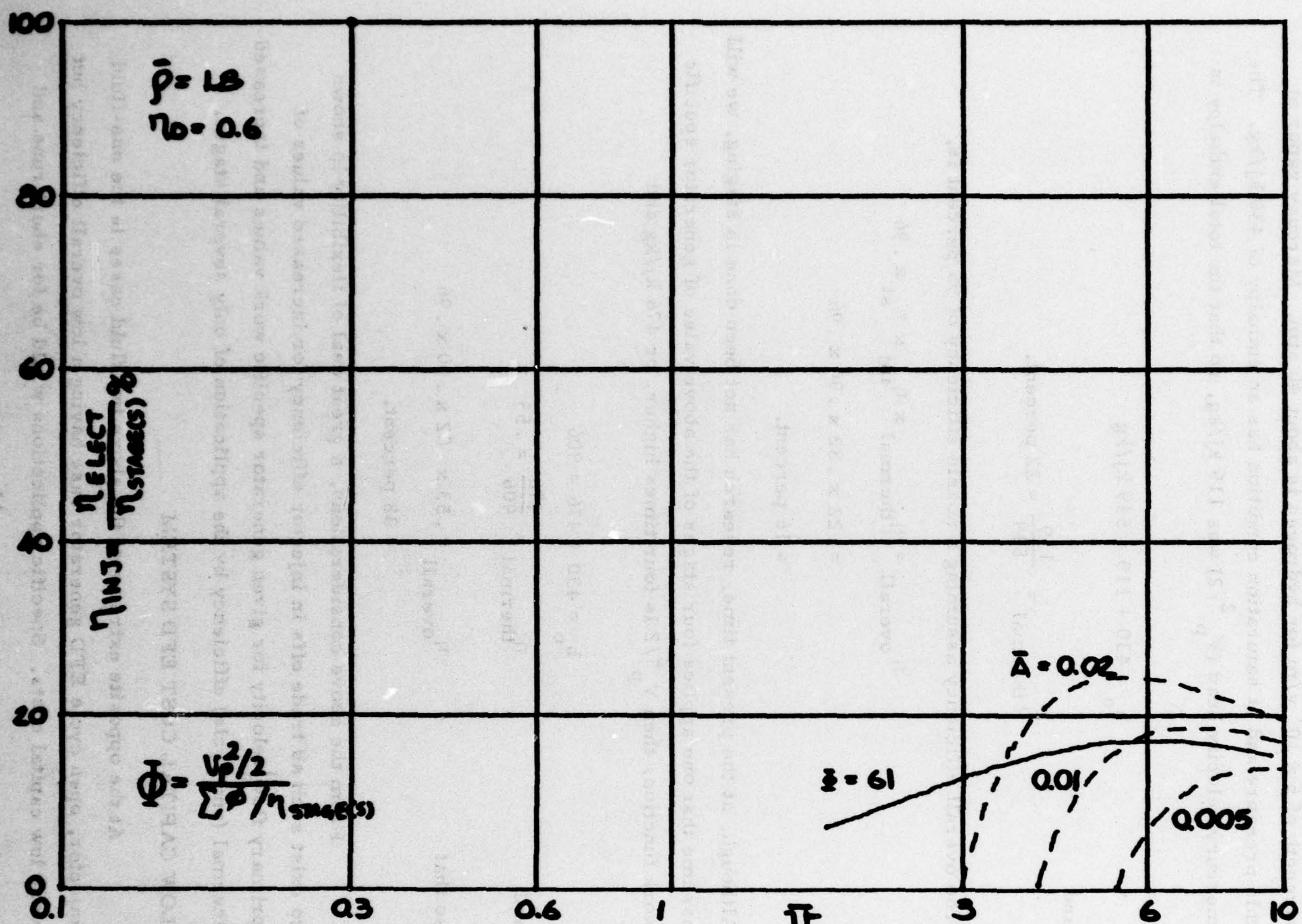

Figure 4. Single Fluid Injector-Generator Case for a Specific Value of Injector-Generator Work Ratio

very high power. This case is represented in Figure 4 where figure parameters are density ratio of 1.8 and "diffuser" efficiency of 60 percent. The density ratio corresponds to a primary Mach number of 2 for a specific heat ratio of 1.4 (e.g., air). For air as the working fluid at $700^{o}K$, the primary flow velocity is 780 m/s so that the injector specific work ($V_p^2/2$) is 305 kj/kg. For this case a good value of generator specific work is 5 kj/kg which corresponds to an ideal two dimensional geometry with a channel field strength of about 5×10^7 v/m. Applying the ballistic jet geometry(1) where the EFD generator is integrated with the injector, the generator stage efficiency may be taken as unity. For the above values,

$$\Phi = \frac{305}{5} = 61$$

From Figure 4, the peak value of this characteristic curve is about 17 percent, occurring for an injector area ratio of about 0.01. The total enthalpy converted to kinetic enthalpy is 45 percent for Mach 2 primary flow and to electrical about 8 percent. Considering that series staging is not being applied and that the generator channel and injector channel are combined in one, the 8 percent conversion of total enthalpy may be quite acceptable for a high power, short period operating generator of low capital cost. In Reference 2, an experimental value of 5 percent is reported for a Mach number of 1.5 and an axisymmetric channel geometry of lower specific work function (ϕ). This was in good agreement with predictions of the present theory.

OTHER COMMENTS

In the theoretical investigation a constant pressure case was also studied but was not presented herein. In this investigation the constant area injector analysis was compared with the constant pressure case for the two fluid case and small injector area ratios which are applicable in injector-generator matching. Results were very closely the same for the two cases so that the constant pressure case is not presented here. In Reference 3, Dailey presents a constant area injector analysis and a constant pressure generator channel case. Still other injector solutions are available in the literature.

Also, a study of Figure 3 reveals that a small value of injector generator work ratio characteristic is necessary to achieve a very high injector efficiency. This corresponds to increasing injector area ratio (or momentum ratio) values so that channel wall friction loss effects are less significant. Channel wall friction can be diminished by applying multiple nozzles for small injector area ratios of about 0.01. The multiple nozzles produce total mixing in a shorter distance than could be produced by a single nozzle with the same total area, which could result in the channel length being less than one diameter long. The latter possibility was taken into consideration in choosing as figure parameter a diffuser efficiency of 95 percent for Figure 3.

SUMMARY

A particularly useful EFD generator parameter is its power output per unit mass flow of EFD fluid which we call the generator specific work, ϕ . In turbine and fan technology this parameter is called head. This generator specific work parameter is shown to be a most useful tool for optimally matching EFD generator and injector characteristic design operating points.

Examples of two types of generator systems were discussed: one a closed cycle of high efficiency using mercury and hydrogen and the second an open cycle or low efficiency but of low capital costs, applying air as the working fluid.

In addition, some representative potential upper values of EFD generator stage specific work values have been presented. For some applications where a high overall cycle efficiency (about 40 percent) is sought, multistaging capability is shown to be desirable. In order to achieve multistaging, research is needed in stage geometry, charge particle production, and charge collection.

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APPENDIX I

The power of an EFD channel is the product of collector current, I, and voltage, V. The current for an axisymmetric channel is

$$I = \rho_e \cdot v \cdot \frac{A}{4} = \frac{amp \times s}{m^3} \times \frac{m}{s} \cdot m^2$$

where the initial charge cloud radius is taken as one-half the channel radius to allow for charge cloud enlargement along the channel while maximizing the power[7]. For the cylindrical column of charge and the anchoring of the space charge field lines essentially radially,

$$E_{radial} = E_r = \frac{\rho_e r/2}{2\epsilon}$$

solving for the charge density,

$$\rho_e = \frac{4\epsilon}{r} E_r$$

substituting into the above eq. for the current provides

$I = \frac{\epsilon}{r} E_r \cdot v \cdot A$, where the transport velocity of the charge equals the flow velocity. For maximum power[7],

$$E_r = E_{axial} = \frac{\sqrt{2}}{2} E_b$$

so that

$V = \frac{\sqrt{2}}{2} E_b \cdot \ell$, where E_b equals the sparkover field strength of the gas in a uniform field geometry and ℓ is the length of the sparkover gap. Then,

$$I \cdot V = \frac{\epsilon}{r} \frac{\sqrt{2}}{2} E_b \cdot v \cdot A \cdot \frac{\sqrt{2}}{2} E_b \cdot \ell$$

$$= \frac{1}{2} \epsilon E_b^2 \cdot \frac{\ell}{r} vA$$

multiplying the numerator and denominator of the right hand side by the gas density, ρ,

$$I \cdot v = \frac{1}{2} \epsilon \, E_b^2 \cdot \frac{\ell}{r} \quad \frac{\dot{m}}{\rho}$$

and the power per unit mass flow is

$$\frac{I \cdot V}{\dot{m}} = \frac{1}{2} \epsilon \frac{E_b^2}{\rho} \cdot \frac{\ell}{r}$$

Re-arranging,

$$\frac{I \cdot V}{\dot{m}} = \frac{1}{2} \epsilon \frac{E_b}{\rho} E_b \frac{\ell}{r}$$

where $\quad E_b = \dfrac{v_s}{\ell}$

then,

$$\frac{I \cdot V}{\dot{m}} = \frac{1}{2} \epsilon \left(\frac{v_s}{\rho \ell}\right) E_b \frac{\ell}{r} = \phi$$

This is the EFD ideal work function. Its value depends on the type of working fluid, the channel's field strength and its length to radius ratio. Experiments have shown that the Law of Similitude holds so that in scaling, holding $\rho \times \ell$ constant, $v_s / \rho \ell$ is constant, while the channel field strength, E_b, increases with gas density. Thus, the EFD work function increases linearly with gas density or inversely with the channel's length. Additionally, one would like to make ℓ / r as large as possible but in practice, turbulent spreading of the charged colloids place an upper limit on its value; in Air Force experiments typical best operation values were 6 for an open jet channel. Larger values than 6 resulted in the charged colloids escaping the main flow region of the channel and providing an additional space charge field strength contribution, detrimental to channel performance.

EFD work function values achieved for air as the working fluid at 20 at. pressure were about 2700 j/kg. The same channel for hydrogen as the working fluid and the same field strength would be 23,500.

It is important in the design of a channel that adequate cross-sectional area of metal is provided for anchoring the space charge field lines of an axisymmetric column of charge, a two dimensional slab of charge, or even that corresponding to the one-dimensional *EFD* case.

APPENDIX 2

INJECTOR-EFD GENERATOR MATCHING ANALYSIS

The injector performance solved for here is the efficiency of conversion of the primary flow kinetic power into total flow power of the mixed flow. The injector geometry considered is constant area and the flows are assumed to be incompressible.

Referring to Figure 1 for notation

$$P_{O3} = P_2 + \eta_D \quad \rho_2 \frac{v_2^2}{2}$$

$$P_{OS} = P_1 + \rho_s \frac{v_s^2}{2}$$

The injector efficiency is defined as

$$\eta_{inj} = \frac{(P_{O3} - P_{O,S})\, v_2\, A_2}{\dot{m}p \dfrac{v_p^2}{2} - (P_{OS} - P_1)\, v_p\, A_p}$$

where the numerator represents the power of the mixed flow available through expansion to the lower total pressure of the secondary fluid. The denominator is the kinetic power of the primary (expanded to the local pressure inside the entering portion of the channel) less the power the primary flow needs to rise up to the total pressure of the secondary. Utilizing the above equations and the impulse eq., which follows,

$$P_2 A_2 + (\dot{m}_p + \dot{m}_S) v_2 = P_1 A_1 + \dot{m}_p v_p + \dot{m} v_S$$

one finds

$$\eta_{inj} = \frac{(1 + \bar{V}\bar{A})}{\bar{V}\bar{A}(\bar{\rho}\bar{V}^2 - 1)} \left[\frac{2(\bar{\rho}\bar{A}\bar{V}^2 + 1)}{(1 + \bar{A})} - \frac{(2 - \eta_D)\, (\bar{\rho}\bar{V}\bar{A} + 1)(1 + \bar{V}\bar{A})}{(1 + \bar{A})^2} - 1 \right]$$

where $\bar{A} = A_P/A_S$, $\bar{v} = v_p/v_S$, $\bar{\rho} = \rho_P/\rho_S$, and η_D equals diffuser efficiency.

The matching of the EFD generator to the injector requires that

$$\Delta P_{gen.} = (\Delta P_{elect} + \Delta P_{drag})_{gen} = (P_{O3} - P_{O,S})$$

where

$$\frac{\Delta P_{elect}}{(\Delta P_{elect} + \Delta P_{drag})_{gen}} = \eta_{stage\ (s)} = \eta_{St}$$

then

$$\Delta P_{elect} = \eta_{St} (P_{O3} - P_{O,S})$$

dividing both sides by ρ_S

$$\frac{\Delta P_{elect}}{\rho_S} = \phi = \eta_{St} \frac{(P_{O3} - P_{OS})}{\rho_S}$$

solving

$$(P_{O3} - P_{OS}) = \rho_S \frac{\phi}{\eta_{St}}$$

and substituting into the earliest expression for the injector efficiency,

$$n_{inj} = \frac{\frac{\phi}{\eta_{St}} \rho_S v_2 A_2}{\dot{m}_p \frac{v_p^2}{2} - (P_{OS} - P_1) v_p A_p}$$

with $\quad v_p A_p + v_S A_S = v_2 A_2$

then $\quad \bar{v}\bar{A} + 1 = \bar{v}_2 \bar{A}_2$

and with $\quad \frac{\rho_S v_S^2}{2} = (P_{OS} - P_1)$

and dividing the last expression, numerator and denominator by $\rho_S \, v_S \, A_S$ and making the above substitutions one has

$$\eta_i \eta_j = \frac{\phi/\eta_{St}}{v_p^2/2} \; \frac{(1 + \bar{v}\bar{A})}{\bar{A}\,(\bar{\rho}\bar{v}^2 - 1)} \; \bar{v}$$

$$= \frac{1}{\Phi} \; \frac{(1 + \bar{v}\bar{A})}{\bar{v}\bar{A}\,(\rho v^2 - 1)} \; \bar{v}^2$$

where

$$\frac{v_p^2/2}{\phi/\eta_S} = \Phi$$

one can find

$$\Phi = \frac{v_p^2/2}{\phi/\eta_{st}} = \bar{v}^2 \left[\frac{2\,(\bar{\rho}\,\bar{A}\,\bar{v}^2 + 1)}{(1 + \bar{A})} - \frac{(2 - \eta_D)\;(\bar{\rho}\bar{v}\,\bar{A} + 1)\,(1 + \bar{v}\,\bar{A})}{(1 + \bar{A})^2} - 1 \right]^{-1}$$

The latter is the primary flow specific work divided by the EFD stage (s) specific work and serves in the plots as the injector-generator matching parameter.

A study of the figures reveals that the locus of peak values of the Φ curves is to the left of the locus of peak values of injector area ratio, $\bar{A}$, curves. Thus, when one matches the injector area ratio, $\bar{A}$, to the peak of the injector-generator work ratio curve, Φ, the injector has some reserve pressure rise capability which is desirable. However, it is probably not as essential as it is in matching loads to a compressor.

Considering start-up conditions for the injector-generator, one should bring the electric load on gradually to avoid cutting through (above) the injector area ratio, $\bar{A}$, curve. With zero electric load there exists only the drag loss of the generator so that the velocity ratio, $\bar{v}$, is a minimum.

*U.S.Government Printing Office: 1977 — 757-001/110