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ON THE INFLUENCE OF ICE COVER ON INTERNAL WAVES (O VLIYANII LED--ETC(U)
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ON THE INFLUENCE OF ICE COVER ON INTERNAL WAVES

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[Abstract. This study deals with influence of the ice cover floating on the surface of a two-layer fluid of finite depth, on waves generated by periodic temporal atmospheric disturbances.] [53

This paper is a study of the influence of a thin elastic plate floating on the surface of a two layer fluid of finite depth on waves generated by periodic normal stresses. As a special case one can consider the ice cover to be a plate. Analogous problems in the absence of a plate in a homogeneous fluid were considered in [1], and in a two-layer fluid, in [2,3]. The influence of the plate on waves in a uniform fluid were investigated in [4] to [7] and in a two-layer, infinitely deep fluid, in [8].

1. Let an ice cover float on the surface of an incompressible two-layer fluid. The thickness of the upper layer of the fluid and its density will be denoted by H_1 and ρ_1, respectively and the thickness of the lower layer and its density, by H_2 and ρ_2, respectively. We shall investigate the influence of the ice cover, considered to be a thin elastic plate, on waves generated by a periodic atmospheric disturbance

$$P_0 = af(x,y)\cos\sigma t. \tag{1.1}$$

It is assumed that the motion of the fluid is potential and the velocity potentials for the motion of the upper and lower layers of the fluid are designated $\varphi_{1,2}(x,y,z,t)$. Assuming the velocity and downwarp of the ice to be small, one obtains the following equations for $\varphi_{1,2}$

$$\Delta\varphi_1 = 0 \quad (0 < z < H_1), \qquad \Delta\varphi_2 = 0 \quad (-H_2 < z < 0), \tag{1.2}$$

subject to the following boundary conditions: [54

$$d\Delta^4\xi_1 + \alpha\frac{\partial^2\xi_1}{\partial t^2} + \rho g\xi_1 + \rho_1\frac{\partial\varphi_1}{\partial t} = -P_0(x,y,t),$$

$$\frac{\partial\xi_1}{\partial t} = \frac{\partial\varphi_1}{\partial z} \quad \text{when } z = H_1,$$

Numbers in right margins indicate pagination in original text.

$$\rho_2 \frac{\partial^2\varphi_2}{\partial t^2} - \rho_1 \frac{\partial^2\varphi_1}{\partial t^2} + g(\rho_2 - \rho_1)\frac{\partial\varphi}{\partial z} = 0, \tag{1.3}$$

$$\frac{\partial\xi_2}{\partial t} = \frac{\partial\varphi_2}{\partial z}, \qquad \frac{\partial\varphi_1}{\partial z} = \frac{\partial\varphi_2}{\partial z} \quad \text{when } z = 0,$$

$$\frac{\partial\varphi_2}{\partial z} = 0 \quad \text{when } z = -H,$$

where $d = Eh^3[12(1 - \nu^2)]^{-1}$ is the cylindrical rigidity of ice, E and ν are the Young's modulus and Poisson's ratio of ice, h and ξ_1 are thickness and downwarp of the ice, α is the mass per unit area of the ice surface, ξ_2 is the deviation of the fluid interface from the equilibrium position. Let the z axis be directed vertically upward and the origin of the coordinate system lie along the undisturbed interface separating layers with different densities.

Applying the two dimensional Fourier transform in variables x and y to Eqs. (1.2) and the boundary conditions (1.3) we obtain the following expression for ξ_1 and ξ_2:

$$\xi_{1,2} = \frac{a}{2\pi\rho_1 g}\iint_{-\infty}^{\infty} r\psi_{1,2}\Delta^{-1}(r)\bar{f}(m,n)\exp[i(mx + ny - \sigma t)]dmdn, \tag{1.4}$$

where

$$\Delta = [d_1 r^5 + r(1 - \alpha_1)][\eta \tanh(rH_1) + \sigma^2\alpha\tanh(rH_2)] - \frac{\sigma^2}{y}[\eta + \sigma^2\alpha\tanh(rH_1)\tanh(rH_2)],$$

$$\psi_1 = -\eta\tanh(rH_1) - \sigma^2\alpha\tanh(rH_2), \quad \psi_2 = -\sigma^2\alpha\tanh(rH_2)\cosh^{-1}(rH_1),$$

$$\eta = \sigma^2 - rg\epsilon\tanh(rH_2), \quad r = (m^2 + n^2)^{\frac{1}{2}},$$

$$\epsilon = 1 - \alpha, \quad \alpha = \frac{\rho_1}{\rho_2}, \quad d_1 = \frac{d}{\rho_1 g}, \quad \alpha_1 = \frac{\sigma^2\alpha}{\rho_1 g},$$

$\bar{f}(m,n)$ is the Fourier transform of $f(x,y)$.

In the plane and axi-symmetrical cases, $\xi_{1,2}$ assumes the form

$$\xi_{1,2} = \frac{a}{(2\pi)^{\frac{1}{2}}\rho_1 g}\int_{-\infty}^{\infty} r\psi_{1,2}\Delta^{-1}(r)\bar{f}(r)\exp[i(rx - \sigma t)]dr, \tag{1.5}$$

$$\bar{f}(r) = \frac{1}{(2\pi)^{\frac{1}{2}}} \int_{-\infty}^{\infty} f(x) \exp(- irx)\, dx,$$ [55

$$\xi_{1,2} = \frac{a}{\rho_1 g} \int_0^{\infty} r^2 \psi_{1,2} \Delta^{-1}(r) J_o(rR) \bar{f}(r)\, dr \exp(- i\sigma t),$$

$$\bar{f}(r) = \int_0^{\infty} R f(r) J_o(rR) dR, \qquad R = (x^2 + y^2)^{\frac{1}{2}}. \tag{1.6}$$

For large values of R, Eq.(1.6) will assume the form

$$\xi_{1,2} = \frac{a}{\rho g} \left(\frac{2}{\pi R}\right)^{\frac{1}{2}} \int_0^{\infty} r^{3/2} \psi_{1,2} \Delta^{-1}(r) \bar{f}(r) \cos (rR - \pi/4) dr \exp(- i\sigma t). \tag{1.7}$$

The error introduced in rewriting Eq. (1.6) in the form given by (1.7) is on the order $R^{-3/2}$.

Analysis of the roots of the equation

$$\Delta (r) = 0 \tag{1.8}$$

has shown that when $d_1 > 0$, $\alpha_1 < 1$ the function under the integral sign in Eq. (1.7) has first order poles at points r_1, r_2 along the positive axis. Satisfying the radiation condition, let's deform the initial integration path into the path *L along the positive* axis passing around points $r_{1,2}$ along small semicircles in the lower half space. At $d = 0$, $\alpha_1 \geq 1$ Eq. (1.8) has no real roots and integration can be performed along the positive axis. Results of the integration of Eq. (1.7) show that at $d_1 > 0$, $\alpha_1 < 1$ the final expressions for the type of waves generated at the ice-water surface and the interface separating layers with different densities

$$\xi_1 = \eta_1 + \eta_2, \qquad \xi_2 = \eta_3 + \eta_4 \tag{1.9}$$

$$\eta_{1,2} = B_1(r_{1,2}) \sin \alpha(r_{1,2}), \qquad \eta_{3,4} = B_2(r_{1,2}) \sin \alpha(r_{1,2}),$$

where

$$B_{1,2} = \frac{a}{\rho_1 g} \left(\frac{2\pi}{R}\right)^{\frac{1}{2}} \frac{\psi_{1,2}}{\Delta(r)} r^{3/2} \bar{f}(r), \qquad \alpha = rR - \sigma t - \pi/4 \tag{1.10}$$

$\psi_{1,2}$ and Δ (r) are the same as in Eq. (1.4) and a prime indicates differentiation with respect to r.

In the two dimensional case analogous method can be used to solve Eq. (1.5) to give formula (1.9) for $\xi_{1,2}$, where $B_{1,2}$ and α are given by the following expressions

$$B_{1,2} = \frac{a(2\pi)^{\frac{1}{2}}}{\rho_1 g \Delta'(r)} \, r\,\psi_{1,2}\bar{f}(r), \qquad \alpha = rx - \sigma t. \tag{1.11}$$

From Eqs. (1.9) to (1.11) it follows that two systems of waves appear on the ice-water surface and the interface separating layers with different densities when $d_1 > 0$, $\alpha_1 < 1$. The first system of waves corresponding to the pole $r = r_1$ represents the usual surface waves, while the second one, corresponding to the pole $r = r_2$ ($r_2 > r_1$) corresponds to pure internal waves. The only difference between the waves arising at the interface separating layers with different densities and the ice water surface are their amplitudes. When $d_1 = 0$, $\alpha_1 \geq 1$, the amplitude of surface waves decays exponentially with increasing R. The velocity and wave length of the first and second systems of waves have the following form: [56

$$V_{1,2} = \frac{\sigma}{r_{1,2}}, \qquad \lambda_{1,2} = \frac{2\pi}{r_{1,2}}. \tag{1.12}$$

An analytical expression for the roots $r_{1,2}$ can be found only in special cases.

Short Waves $\frac{\sigma^2 H_1}{g} \gg 1$

From equation (1.8) we find

$$r_2 = (1 + \alpha)\sigma^2/\epsilon g. \tag{1.13}$$

Then, $$V_2 = \frac{\epsilon g}{(1+\alpha)\sigma}, \quad \lambda_2 = \frac{2\pi\epsilon g}{(1+\alpha)\sigma^2}, \quad \frac{B_2(r_2)}{B_1(r_1)} = -\frac{1}{\epsilon}(1+\alpha)\exp(r_2 H_1) \tag{1.14}$$

From this it can be seen that the velocity and length of internal waves and also the ratio of the amplitude of the internal wave at the layer interface with the amplitude of the same wave at the ice-water surface are independent of the properties fo the ice cover. In the case of broken ice ($E = 0$) we find that for all values of $\alpha_1 < 1$

$$r_1 = \frac{\sigma^2}{g(1-\alpha_1)}, \quad V_1 = g\sigma^{-1}(1-\alpha_1), \quad \lambda_1 = 2\pi g\sigma^{-2}(1-\alpha_1),$$

$$\frac{B_1(r_1)}{B_2(r_2)} = (1-\alpha_1)^{-1} - \frac{\alpha_1(1-\alpha)}{2\sigma(1-\alpha_1)} \exp(r_1 H_1). \tag{1.15}$$

In the case of a continuous ice cover ($E = 0$) and for small values of d_2

and α_1 $(d_2 = d_1\sigma^3 g^{-4})$ we get

$$r_1 = \sigma^2 g^{-1}(1 + \alpha_1 - d_2),\quad V_1 = g\sigma^{-1}(1 - \alpha_1 + d_2),$$

$$\lambda_1 = 2\pi g\sigma^{-2}(1 - \alpha_1 + d_2),\quad \frac{B_1(r_1)}{B_2(r_1)} = \left[1 + \frac{\epsilon(d_2 - \alpha_1)}{2\alpha}\right]\exp(r_1 H_1). \tag{1.16}$$

From formulas (1.18) it can be seen that an increase in Young's modulus of ice increases the velocity and length of surface waves, while on the other hand large forces decrease V_1 and λ_1. The ratio of the amplitude of the surface wave at the ice-water surface with the amplitude of this wave at the layer interface also increases with increasing E but decreases with increasing large forces. However, at small values of ϵ this change is of the order of ϵ^2. [57

Long waves $\left(\sigma\left(\frac{H}{g}\right)^{\frac{1}{2}}\right) \ll 1$.

After replacing tangents with their arguments, equation (1.8) can be written in the following form

$$(1 - \alpha_1 + d_1 r^4)\left[r^4 g\epsilon H_1 H_2 - r^2\sigma^2(H_1 + \alpha H)\right] + \frac{\sigma^4}{g} = 0. \tag{1.17}$$

From this it follows that at d = 0 and for any values of $\alpha_1 < 1$ we have

$$r_1 = \frac{\sigma}{[gH(1 - \alpha_1)]},\quad V_1 = [gH(1 - \alpha_1)]^{\frac{1}{2}},\quad \lambda_1 = \frac{2\pi}{\sigma}[gH(1 - \alpha_1)]^{\frac{1}{2}},$$

$$\frac{B_1(r_1)}{B_2(r_2)} = \frac{H}{\alpha H_2}. \tag{1.18}$$

When $d_2 = d_1\left(\frac{\sigma}{(gH)^{\frac{1}{2}}}\right)^4 \ll 1$ and $\alpha_1 \ll 1$ we obtain

$$r_1 = \frac{\sigma}{(gH)^{\frac{1}{2}}}\left[1 + \tfrac{1}{2}(\alpha_1 - d_2)\right],\quad V_1 = (gH)^{\frac{1}{2}}\left[1 - \tfrac{1}{2}(\alpha_1 - d_2)\right],$$

$$\lambda_1 = 2\pi\sigma^{-1}(gH)^{\frac{1}{2}}\left[1 - \tfrac{1}{2}(\alpha_1 + d_2)\right],\quad \frac{B_1(r_1)}{B_2(r_1)} = \frac{H}{\alpha H_1}\ . \tag{1.19}$$

From Eqs. (1.18) and (1.19) it follows that in the case of long waves the broken ice decreases velocity and length of the surface waves. Continuous ice at $d_2 > \alpha_1$ increases, and at $d_2 < \alpha_1$ decreases V_1 and λ_1. The ratio of amplitudes of surface waves at layer interfaces is independent of the parameters of the ice cover.

For the internal waves both in the case of broken ice at arbitrary values of $\alpha_1 < 1$ and continuous ice at small values of d_2 and α_1 we have

$$r_2 = \frac{\sigma}{(\eta H)^{\frac{1}{2}}} \frac{H}{(\epsilon H_1 H_2)^{\frac{1}{2}}}, \qquad V_2 = (gH)^{\frac{1}{2}} H^{-1} (\epsilon H_1 H_2)^{\frac{1}{2}};$$

$$\lambda_2 = 2\pi\sigma^{-1} (gH)^{\frac{1}{2}} H^{-1} (\epsilon H_1 H_2)^{\frac{1}{2}}, \qquad \frac{B_2(r_2)}{B_1(r_2)} = -\frac{\alpha H}{\epsilon H_1}. \tag{1.20}$$

From this it can be seen that the velocity V_2, the length λ_2 and the ratio $\frac{B_2(r_2)}{B_1(r_2)}$ are independent of d_2 and α_1.

Thus, in the case of short period and long period disturbances, the velocity, length, and the ratio of amplitudes of internal waves generated by the pressure of the form given by (1.1) at the ice-water surface and at the layer interface are independent of the parameters of the ice cover. [58

Numerical calculations were made in order to determine the influence of the ice cover on the surface and internal waves. Calculations were made using formulas (1.9), (1.11), (1.12) for $f(x)$ in the form

$$f(x) = \begin{cases} 1 & |x| \le \ell \\ 0 & |x| > \ell \end{cases} \tag{1.21}$$

for two values of ϵ (2×10^{-3} and 10^{-2}), $\ell = 10^3$ m and the values of parameters h(m), $H_{1,2}$(M), $\sigma(s^{-1})$, varying between the limits

$$0 \le h \le 3, \quad 50 \le H_1 \le 500, \quad 500 \le H_2 \le 4 \times 10^3, \quad 5 \times 10^{-3} \le \sigma \le 7 \times 10^{-1}. \tag{1.22}$$

The Young's modulus, specific weight, and Poisson's ratio of ice were taken to be

$$E = 3 \times 10^7 \text{ n/m}^2, \qquad \rho = 870 \text{ kg/m}^3 \qquad \nu = 0.34. \tag{1.23}$$

Analysis of the results of the calculations has shown that the ice cover has practically no effect on the velocity and length of the internal waves (V_2, λ_2). The velocity and length of the surface waves (V_1, λ_1) when $\sigma > \sigma_1$, where

$$\sigma_1 = \left[\frac{12(1 - \nu^2)\, \rho_1 g^4}{Eh^2} \right]^{1/6} , \qquad (1.24)$$

increase, and when $\sigma < \sigma_1$ decrease with increasing thickness of the continuous ice cover. Broken ice decreases V_1 and λ_1 in comparison with the velocity and length of surface waves in a two layer fluid in the absence of ice. When $\lambda_1 (H_1 + H_2)^{-1} < 1$, the influence of the ice cover on parameters of surface waves is less than 1 percent.

The influence of the fluid nonuniformity on the velocity and length of surface waves is the same as that in the absence of ice. During short-period oscillations, V_1 and λ_1 are practically indepent of ϵ. In the case of long-period oscillations, an increase in ϵ results in a decrease in the velocity and length of the surface waves. However, this change for parameter values given by (1.22) does not exceed 0.1 percent. The velocity and length of internal waves increase considerably with increasing ϵ. Table 1 shows the numerical values of V_1, λ_1, and V_2, λ_2 for $H_1 = 50$ m and $H_2 = 10^3$ m. From this it can be seen that an increase in ϵ from 2×10^{-3} to 10^{-2} at $\sigma = 5 \times 10^{-1}$ s^{-1} and at $\sigma = 3 \times 10^{-1}$ s^{-1} resulted in an approximately five fold increase in V_2 and λ_2. This increase is the same in the absence of ice ($h = 0$) and both in the presence of continuous ($E \neq 0$) and broken ($E = 0$) ice. As the thickness of continuous ice increases from $h = 0$ to $h = 3$ m, the velocity and length of surface [59
waves at $\sigma = 5 \times 10^{-1}$ s^{-1} for both values of ϵ (2×10^{-3} and 10^{-2}) increased 13 percent for $E \neq 0$ and decreased 8.6 percent for $E = 0$. For $\sigma = 3 \times 10^{-1}$ s^{-1} and the same values of h and ϵ, continuous ice increased V_1 and λ_1 2.4 percent.

Table 2 shows the numerical values of amplitude of waves η_i $i = 1,2,3,4$ (see (1.8)), denoted correspondingly a_i, for $f(x)$ of the form given by (1.21), $H_1 = 50$ m, $H_2 = 500$ m, and other parameter values of the problem shown in the table for the case of continuous ice. The values of a_i in Table 2 are given with an accuracy up to a factor $\frac{a}{10 \rho_1 g}$. In the case of broken ice, the dependence is approximately the same. From Table 2 it can be seen that an increase in ϵ leads to a decrease in the numerical values of amplitudes a_1, a_3 of surface waves η_1, η_3. However, the decrease in amplitude is practically independent of the thickness of ice. At $\sigma = 10^{-2}$ s^{-1} and $\sigma = 8 \times 10^{-3}$ s^{-1} the amplitudes of internal waves a_2, a_4 increased 5.2 and 6.9 times, respectively with increasing ϵ. This increase is the same both for $h = 0$ and for $h = 3$m. For $\sigma = 3 \times 10^{-1}$ s^{-1} the amplitudes of a_2, a_4 are small in comparison with the surface wave amplitudes.

An increase in the thickness of ice from $h = 0$ to $h = 3$ at $\sigma = 3 \times 10^{-1}$s [60
resulted in an approximately 3.2 times decrease of the amplitude of a_1 for both $\epsilon = 2 \times 10^{-3}$ and $\epsilon = 10^{-2}$. At $\sigma = 10^{-2}$ s^{-1} and $\sigma = 5 \times 10^{-3}$ s^{-1} and the same values of ϵ, the ice cover has practically no influence on

Table 1

The Influence of Ice Properties and Fluid Inhomogeneity on Velocity and Wavelength of Waves

V_1, m/s	λ_1, m	V_2, m/s	λ_2, m	h, m	E, N/m^2	ϵ	σ, s^{-1}
19.61	246.3	0.019	0.246	0	3×10^7		
22.27	278.5	0.019	0.246	3		2×10^{-3}	
18.30	229.9	0.019	0.246	3	0		
19.61	246.3	0.098	1.238	0			5×10^{-1}
22.17	278.5	0.098	1.238	3	3×10^7	10^{-2}	
18.30	229.9	0.098	1.238		3	0	
26.68	684.1	0.033	0.685	0			
32.08	671.6	0.033	0.685	3	3×10^7	2×10^{-3}	
31.88	667.4	0.033	0.685	3	0		3×10^{-1}
32.[illegible]	684.1	0.164	3.438	0			
[illegible]	671.6	0.164	3.438	3	3×10^7	10^{-2}	
[illegible]	668.1	0.164	3.438	3	0		

Table 2

The Influence of Ice Properties and Fluid Inhomogeneity of the Wave Amplitude

a_1	a_2	a_3	a_4	h	σ, s^{-1}	ϵ
4.838	8.46×10^{-5}	3.057	0	0	3×10^{-1}	
1.484	8.34×10^{-14}	0.930	0	0		
1.359	1.43×10^{-2}	1.235	6.154	0	10^{-2}	2×10^{-3}
1.359	1.39×10^{-2}	1.235	5.994	3		
0.680	6.89×10^{-1}	0.618	3.997	0	5×10^{-3}	
0.680	6.88×10^{-1}	0.618	3.993	3		
4.823	4.29×10^{-3}	3.047	0	0	3×10^{-1}	
1.479	7.55×10^{-10}	0.927	0	3		
1.351	7.45×10^{-2}	1.228	7.983	0	1×10^{-2}	10^{-2}
1.351	7.44×10^{-2}	1.228	7.979	3		
0.676	1.00×10^{-1}	0.615	5.837	0	5×10^{-3}	
0.676	1.00×10^{-1}	0.615	5.837	3		

the amplitudes a_1 and a_3 of surface waves, while the amplitudes of internal waves decrease somewhat with increasing h from 0 to 3 m. At $\epsilon = 2 \times 10^{-3}$ and $\sigma = 10^{-2}\ s^{-1}$, the decrease in a_2 and a_4 is 2.8 and 3.1 percent, respectively. However, the influence of ice on a_2 and a_4 decreases with decreasing σ so that the properties of ice, exert no noticable influenc on the amplitudes of internal waves and on their velocity and length at $\sigma = 5 \times 10^{-3}$.

The amplitude a_4 of the internal wave η_4 arising at the layer interface can exceed many times the amplitude a_1 of the surface wave η_1. For example, for $\tau = 2\pi/\sigma = 6$ hr, $H_1 = 50$ m, $H_2 = 10^3$ m, and $\epsilon = 2 \times 10^3$, $a_4/a_1 = 100$. It should be noted, that proper selection of the width (ℓ) of the pressure region may lead to zero values of amplitudes a_1 and a_3 of the surface waves. However, the amplitude a_4 of the internal wave will [61
not be equal to zero and, consequently, we will have a clearly defined case of the "dead water" phenomenon. In an analogous manner, only surface waves can appear in a two-layer fluid under the influence of atmospheric disturbances. For (1.21), this will occur at $\ell = \frac{n\pi}{r_2}$, n = 1,2,...

2. Let periodically varying pressure be applied to the surface of the ice cover. If this pressure is of the form

$$P_o = af(x) \cos (Ky - \sigma t), \tag{2.1}$$

we can obtain the following expression for deviation of the ice-water interface ξ_1 and deviation of the density separating-interface ξ_2 from the undisturbed state

$$\xi_{1,2} = \frac{a}{(2\pi)^{\frac{1}{2}}\rho_1 g}\int_{-\infty}^{\infty} r\psi_{1,2}\Delta^{-1}(r)\bar{f}(m) \exp\left[i(mx + ky - \sigma t)\right]dm, \tag{2.2}$$

where $r = (m^2 + K^2)^{\frac{1}{2}}$, $\psi_{1,2}$ and $\Delta(r)$ are the same as in formula (1.4), $\bar{f}(m)$ is the Fourier transform of f(x). At large x calculation of the integral in Eq. (2.2) leads to the following expressions for $\eta_{1,2}$ and $\eta_{3,4}$ in formulas (1.9) for $\xi_{1,2}$:

$$\eta_{1,2} = B_1(m_{1,2}) \sin\alpha_{1,2}, \quad \eta_{3,4} = B_2(m_{1,2}) \sin\alpha_{1,2},$$

$$B_{1,2} = \frac{a(2\pi)^{\frac{1}{2}}r^2}{\rho_1 g\Delta'(r)m} \cdot \psi_{1,2}\bar{f}(m), \tag{2.3}$$

$$m_{1,2} = (r_{1,2}^2 - K^2)^{\frac{1}{2}}, \qquad \alpha_{1,2} = r_{1,2}x_{1,2} - \sigma t,$$

$r_{1,2}$ are the positive roots of Eq. (1.8), $K < r_1 < r_2$, the direction of

$ox_{1,2}$ forms an angle $\beta_{1,2}$ ($\tan \beta_{1,2} = K/m_{1,2}$) with the x axis, and prime indicates differentiation with respect to r.

From this it can be seen that two systems of waves arise at the ice-water surface and the layer interface. The first system of waves, η_1 and η_3, are surface waves, while the second system, η_2 and η_4, are pure internal waves. The velocity and length of surface and internal waves is determined by Eqs. (1.12). The influence of heterogeneity of the fluid and the properties of the ice cover on $V_{1,2}$, $\lambda_{1,2}$ and also on the ratio of amplitudes $a_i (i = 1,2,3,4)$ of waves η_i is the same as in the case of pressures of the form given by (1.1).

The direction of motion of internal waves η_2 and η_4 varies substantially with ϵ and is practically independent of the thickness of ice. For example, for $H_1 = 50$ m, $H_2 = 800$ m, $K = 2 \times 10^{-4}$ m^{-1}, and $\sigma = 10^{-3}$ s^{-1} a change in ϵ from 2×10^{-3} to 10^{-2} resulted in an increase in the angle β_2 from 11°24' to 25°15'. For $\sigma > \sigma_1$ (see (1.24)), continuous ice increases the angle β_1, while at $\sigma < \sigma_1$ it decreases β_1. Broken ice decreases the [62
angle β_1 for all values of σ. For example, for $\sigma = 7 \times 10^{-1}$s^{-1}, $K = 2 \times 10^{-2}$m^{-1}, $H_1 = 50$ m, $H_2 = 500$ m, $\epsilon = 10^{-2}$, a change in the thicknessof ice from $h = 0$ to $h = 3$ m results in an increase in the angle β_1 from 24°10' to 40°10', in the case of continuous ice, and a decrease of β_1 from 24° 10' to 20° 20' in the case of broken ice. Variation of ϵ from 2×10^{-3} to 10^{-2} has negligible effect on the angle β_1.

Since the amplitudes of surface waves η_1, η_3 are proportional to $\left[1 - \left(\frac{\lambda_1}{\bar{\lambda}}\right)^2\right]^{-\frac{1}{2}}$, $(\bar{\lambda} > \lambda_1)$, they can be quite large when the length of the pressure wave $\bar{\lambda}$ is close to the length λ_1 of the surface waves. When $r_1 < K < r_2$, surface waves η_1 and η_3 will decay exponentially with increasing distance from the pressure region. In this case, disturbances of the ice-water surface and layer interface at a certain distance from the pressure region will represent pure internal waves

$$\xi_1 = \eta_2, \qquad \xi_2 = \eta_4, \tag{2.4}$$

where η_2 and η_4 are determined by formulas (2.3). In this case, resonance can occur at length of the pressure wave $\bar{\lambda}$ close to the wavelength of internal waves $\lambda_2 = \frac{2\pi}{r_2}$.

Since, when $\sigma > \sigma_1$ the continuous ice cover decreases the numerical value of the root r_1 in comparison with its value r_1^0 in the absence of ice, wave numbers K in the expression for the pressure (2.1) exist such that $r_1 < K < r_1^0$. This indicates the existence of lengths for pressure waves $\bar{\lambda}$ satisfying the condition $\lambda_1^0 < \bar{\lambda} < \lambda_1$ ($\lambda_1^0 = \frac{2\pi}{r_1^0}$, $\lambda_1 = \frac{2\pi}{r_1}$) at which the two systems of waves (surface and internal) arise in a two-layer

with a free surface under the influence of pressure of the form given by equation (2.1). Assuming all conditions remain the same, it also indicates that only internal waves (η_2, η_4) arise in a fluid covered with continuous ice. What's more, if surface waves (η_1, η_3) were not generated in the absence of ice, they also can not appear in the absence of ice. Broken ice increases the value of r_1 in comparison with the value r_1^0. Consequently, if in the absence of ice the pressure given by (2.1) with a fixed wavelength λ generates surface and internal waves in a fluid in the absence of ice, then the waves η_i ($i = 1,2,3,4$) in a fluid covered with broken ice at this λ will not be attenuated. However, it is possible to have wavelengths of pressure wave ($\lambda_1 < \bar{\lambda} < \lambda_0$), for which only internal waves (η_2, η_4) arise in a fluid with a free surface, while [63
both surface and internal waves can exist in a fluid covered with broken ice. At $r_2 < K$, unattenuated waves will not form at the ice-water interface and the layer interface.

3. Let a periodic pressure system of the form

$$P_o = a \cos (rx - \sigma t) \tag{3.1}$$

propagate along the ice cover floating on a free surface of a two layer fluid. In this case, a single system of waves of the form

$$\xi_{1,2} = \frac{a}{\rho_1 g} r \psi_{1,2} \Delta^{-1}(r) \cos (rx - \sigma t)$$

arises at the ice-water surface and the layer interface. In this case, $\psi_{1,2}$, and $\Delta(r)$ are determined by formula (1.4). The velocity and length of the wave generated are equal to velocity and length of the pressure wave

$$V = \frac{\sigma}{r}, \qquad \lambda = \frac{2\pi}{r}$$

Since $\xi_{1,2}$ is proportional to $\Delta^{-1}(r)$, resonance will take place when the wavelength of the pressure wave coincides with the wavelength of natural oscillations of the fluid. In this case, there are only two such resonance wave lengths for pressure waves. Both correspond to positive roots $r_{1,2}$ of Eq. (1.8). The influence of the ice cover and inhomogeneity of the fluid on the values of resonance wave lengths of pressure waves is analogous to the influence on the wave lengths of waves created under the influence of pressure in the form given by (1.1) and (2.1) (Table 1).

REFERENCES

1. Sratenskiy, L.N., "On Directed Radiation of Waves from a Region Subjected to an External Pressure," Prikladnaya matematika i mekhanika, Vol. 20, No. 3, 1958.

2. Voyt, S.S., "Waves at the Interface Between Two Fluids, Generated by a Moving Periodic Pressure System." Trudy MGI, AN SSSR, 17, 1969.

3. Cherkesov, L.V.,"On the Development of Waves on the Free Surface and the Interface Between Two Fluids, Under the Influence of Moving Pressures," Prikladnaya matematika i mekhanika, Vol. 28, No. 3, 1962.

4. Krylov, Yu. M., "Propagation of Long Waves Under the Ice Field," Trudy GOIN, 8(20), 1948.

5. Kheysin, D.N., "Dynamics of the Ice Cover, " Publishing house Gidrometeoisdat, Leningrad, 1967.

6. Bukatov, A.Ye. and L.V. Cherkesov, "Non-Steady State Oscillations of an Elastic Plate Floating on the Surface of a Fluid," Prikladnaya Mekhanika, Vol. 6, No. 8, 1970.

7. Bukatov, A.Ye., "On the Influence of the Ice Cover on the Non-Steady State Waves," Morskiye Gidrofizicheskiye Issledoviniya, 3(49), Sevastopol', Publishing house "MGI AN UkrSSR", 1970.

8. Bukatov, A.Ye., "On the Influence of an Elastic Plate Floating on a Free Surface of a Fluid on the Internal Waves," Morskiye Gidrofizicheskiye Issledoviniya, 1(43), Publishing house "MGI AN UkrSSR," Sevastopol', 1969.