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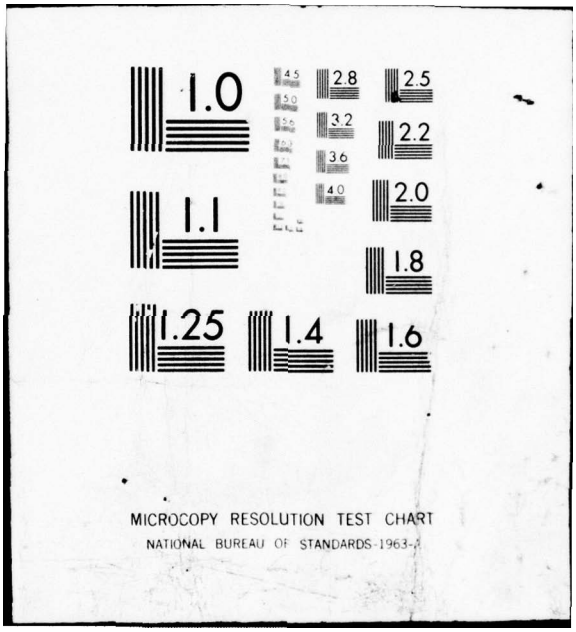
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AFGL-TR-77-0202

CALIBRATION OF THE SSJ/3 SENSOR ON THE DMSP SATELLITES

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SSJ/3 SENSOR
CHANNELTRON
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ELECTROSTATIC ANALYZER

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The SSJ/3 sensor is designed to measure electrons from 50 eV to 20 keV. This is accomplished by using two head assemblies with common signal processing and voltage sources. The low-energy head assembly selects 50.0 to 1,000 eV electrons over eight channels with a normalization factor H = 4.3 x 10^{-5} cm^2-ster. The high-energy assembly similarly selects 1.00 to

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20.0 keV electrons over eight channels with an H-factor equal to 1.30×10^{-5} cm^2-ster.

Energy resolution is approximately 10% for an isotropic incident flux. An electron beam was used to determine angular and energy response. These measurements were compared with results from a Monte-Carlo computer code and approximate analytic methods to determine the final normalizations.

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I. INTRODUCTION

The calibration of the SSJ/3 sensor was carried out using both experimental and theoretical techniques. Experimental measurements were compared with predictions from a Monte Carlo particle tracing code and from analytic approximations.

In Section II instrument design is discussed. Section III described techniques used in calibration while Section IV describes the logic used to obtain the final normalization values.

II. INSTRUMENT DESIGN

Introduction. The SSJ/3 instrument package is shown in Figures 1 and 2. Figure 1 shows the instrument case and the associated electronic boards. The apertures on the case are directly in front of the particle collimators that are shown in the bottom of Figure 2. Behind the collimators are curved cylindrical plates. The plate separation and the applied voltage determine the energy band of the transmitted electrons. The electrons, after passing through the exit collimator, are detected by the C-shaped Channeltron Electron Multiplier detectors that are also shown in Figure 2.

Head Assembly. Electrostatic analyzers (ESA's) select charged particles by applying a voltage between two concentric plates. The SSJ/3 instrument uses two sets of cylindrical concentric plates as shown in Figure 2. Charged particles of the proper sign and energy have trajectories that are almost parallel to the plate surfaces. Particles of greater or lesser energy impinge on the plate surfaces. The mean energy E can be easily derived by requiring a balance between electrostatic and centrifugal accelerations. For plates of radii r_1 and r_2 ($r_2 > r_1$) and an applied voltage difference of V, E is given by

$$E = \frac{eV}{2 \ln(r_2/r_1)} \qquad (1)$$

If V is fixed then the detected energy is determined by the ratio of r_2 to r_1. This explains the larger plate separation, ΔR, for the lower energy detector assembly as tabulated in Figure 3. The sector angle of the plates and their separations, together with aperture and exit slits, determine the

transmission efficiency. Additional baffles are placed in the aperture assembly to eliminate sunlight (UV) scatter into the detectors.

Behind the exit aperture, the two channeltrons are mounted as shown in the lower righthand corner of Figure 2. Electrons impinging on the cone-shaped area produce secondaries which cascade through the channeltron producing a detectable pulse at the end. This pulse is then processed by the instrument's internal electronics. Channeltron operating lifetime is extended by shutting off the accelerating potential whenever the instrument is looking at the sun. This is done by a small phototransistor mounted adjacent to the instrument apertures. See Figure 1. In Figure 3 is a schematic representation of the SSJ/3 head assemblies and gives the relevant dimensional values.

Electronic Logic. Figure 4 shows the electronic logic. The channeltron outputs (DET L and DET H) are amplified and then counted in nine-bit logarithmic counters. A complete 16-channel energy spectrum is read out every second. Appendix A describes in detail the conversion of the logarithmic counter to decimal form. The same programmable power supply is used for biasing both sets of plates. The voltage difference for each set of plates is obtained by applying $\pm V/2$ to each plate.

III. TECHNIQUES USED IN CALIBRATION

Experimental. Experimental calibrations were made using accelerated electrons from a Tritium source. After acceleration by an applied voltage, the beam was collimated so that a mono-energetic, unidirectional source was incident on the instrument apertures. A turntable enclosed in a vacuum system provided mobility for angular response measurements. The beam was found to have an energy dispersion which was negligible in comparison with the instrument's response. By changing the applied voltage various energies could be selected.

Theoretical Approximations. Geometric considerations lead to the following results: 1) If a and b are the heights of the entrance and exit apertures of the collimator and ℓ is the separation then between them such that $\ell >> a + b$, then the angular resolution, $\Delta\beta$, is to a good approximation given by $\Delta\beta \approx (a + b)/\ell$. This approximation works well for the direction parallel to the plates, but not for the orthogonal component. 2) The geometric factor (cm^2-sec) for a long rectangular collimator is approximated by

$$G = \frac{A_1 \times A_2}{\ell^2}$$

A_1 = area of collimator entrance aperture

A_2 = area of collimator exit aperture

ℓ = distance between apertures

This formula provides a rapid means of obtaining a "ballpark" estimate for the geometric factor of a rectangular collimator. We used, however, the exact expression given in Appendix B in deriving our final results.

Monte Carlo Program. In Appendix C is a listing of the Monte Carlo computer code used to analyze instrument response. The basic idea is to trace electrons through the instrument to determine if they hit the channeltron. In this manner the

effect of design can immediately be recognized.

Input data includes aperture geometry, plate radii, arc length of plates, plate voltage, channeltron size, and the position of the aperture and channeltrons relative to the plates. A retarding potential can also be inputed which impedes secondary electrons produced in the plates from hitting the channeltrons. Provision has also been made to examine elastic scattering off the plate surfaces. This feature will not be used.

Let us now consider an isotropic flux, J, incident on the outside aperture. The number of incident particles per unit area is given by

$$dN = J \sin\theta \cos\theta \, d\theta \qquad (1)$$

or if R is a random number over the interval 0-1 then the relation

$$\theta = \tfrac{1}{2}\cos^{-1}(1-2R) \qquad (2)$$

generates the appropriate angular distribution of incident particles.

As a test we generated 6×10^5 particles through the high-energy collimator using equation (2). The theoretically calculated geometric factor for this collimator (1.8 cm high x 0.2 cm wide x 1.15 cm long) is 0.0622 cm^2/sr (Appendix B). The "measured" geometrical factor using particle tracing was found to be 0.0624 cm^2/sr which agrees with the theoretical value to within 0.32%. This result confirms that the computer code was properly simulating an incident isotropic flux through the collimator.

IV. RESULTS

The Method. Instrument calibration is determined by comparing the predicted (Monte Carlo) response curves with the measured ones. First, we look at the energy resolution for a normally incident beam. Then angular response is examined in directions parallel and perpendicular to the plates. After showing substantial agreement with experimental data, the Monte Carlo code is used to calculate the final normalizations for an incident, isotropic flux. Comparisons are made with results from approximate analytic techniques. Channeltron detection efficiency as a function of incident electron energy is also discussed and included.

Throughout this section we use energy channels 8 and 9 to determine the response properties of the two head assemblies.

Energy Resolution. In Figure 5 the Monte-Carlo and experimental results are shown for a normally incident electron beam. The beam energy was systematically scanned across the channel to obtain the energy response curve. It is seen from Figure 5 that the Monte Carlo and experimental results are in excellent agreement.

Angular Response. Figure 6 shows the angular response (α) of the high energy plates in the plane perpendicular to the plates (R-plane). In this case the measured resolution is somewhat wider than that predicted by Monte Carlo. This effect is probably due to scattering off the plate surface. Figure 7 shows a similar response curve for the low-energy plates. Observe that there are fewer measured particles than predicted at large positive angles. This effect is probably from fringe electric field resulting from larger plate separation. See Figure 2. The triangular shape of the theoretical response curve indicates that the front collimator

is the dominant determinant of the angular response.

Now we look at the angular response (β) in the plane parallel to the plate surfaces. All angular response curves are determined at the central (peak) channel energy. Figure 8 shows the appropriate curves for the high energy (narrowly separated) plates. Note that while the resolution is in excellent agreement, there is a systematic shift of about 2.5^{o}. Because the resolution is about 8.0^{o}, we do not consider this error as being serious. Figure 9 shows similar curves for the low energy (widely separated) plates. Here there is also a systematic shift of about 1.25^{o}. Note that for both angular scans the measured resolutions for the low energy plates are less than the predicted ones. This is opposite to what one would normally expect and indicates a slight systematic error.

Final Normalizations. The final normalizations were determined using the Monte Carlo program. As described previously, an isotropic flux is taken incident on the front aperture. One thousand electrons are traced through the plates at each energy. Figure 10 and 11 show the results for channels 8 and 9 respectively. The ordinate represents the percentage of electrons that are detected after leaving the collimator (i.e. the transmission efficiency through the plates). The integral of this curve times the aperture G-factor gives the final normalizations.

For an incident flux J the count rate (CR) is given by

$$JG = CR$$

where G is the normalization factor in $cm^2/sr/ev$. The G-factor can be approximated by

$$G \simeq A\Omega \ \Delta E$$

A = area of entrance aperture

Ω = acceptance solid-angle

E = energy bandwidth

Dividing by the center energy E we have

$$H = G/E \simeq A\Omega \frac{\Delta E}{E} .$$

Now since A, Ω and $\Delta E/E$ are constant for each set of plates, then this one number H characterizes the normalization for all channels. For the high-energy (narrowly separated plates) channels $H = 1.30 \times 10^{-4}$ cm^2/sr. For the low-energy (widely-separated plates) $H = 4.3 \times 10^{-5}$ cm^2/sr.

We now compare this result with an approximate analytic approach. In this approach it is assumed that the energy and angular response curves are independent gaussian distributions. The final normalization is the integral product of these distributions. The result is

$$H = 1.20 \ A\varepsilon \ R_o \ \Delta\alpha\Delta\beta$$

where A is the aperture area, R_o is the channeltron detection efficiency (at E $\sim$ 1 keV, $\varepsilon = 1$), R_o is $\Delta E/E$, $\Delta\alpha$ and $\Delta\beta$ are the FWHM (full-width half maximum) of the angular response curves.

Using the measured results tabulated in Table 1, this approximation gives us $H = 6.8 \times 10^{-5}$ cm^2/sr for the high energy plates and $H = 5.55 \times 10^{-5}$ cm^2/sr for the low energy plates. Note, however, that energy and angular resolutions are not independent for this particular electrostatic analyzer, contrary to the assumptions in the approximate approach.

A wide collimator allows more electrons at various energies to reach the channeltrons. A narrow collimator allows fewer electrons to reach the detectors over a narrower energy bandwidth. This effect is seen by comparing Figure 5 with Figures 10 and 11. The energy resolution of the high-energy plates is 2.3 times greater for an isotropic flux than for a normally incident flux. For the low energy plates it is 1.5 times greater. We interpret this difference to be due to the collimator in front of the high energy plates having a G-factor 16 times that of the one in front of the low-energy plates. It is for this reason we consider the Monte Carlo normalization as more closely reflecting realistic fluxes.

The final normalizatin values for each energy channel as given in Table 2. Dividing the count rate by the approximate normalization gives the equivalent flux in electrons per $cm^2/sr/ev/sec$. The channeltron efficiencies were obtained from Archuleter and DeForest (1971). For 1 keV $\leq$ E $\leq$ 50 MeV, $\varepsilon = 1.0 - 2.0/(3.0 + 6.5/(E - 0.5) + 30.0/(E - 0.5)^3)$ where E is expressed in keV. For 10 ev $\leq$ E $\leq$ 70 eV ε was taken to be

$$\varepsilon = 0.10\ E^{0.515} \quad (E \text{ in ev})$$

Note that the midpoint energies in Figures 10 and 11 ar not the same as in Table 2, which are the final calibrated values. This does not affect H, however, which is energy-independent. The stated errors in Table 2 for the various energy channels are a best estimate from electronic and experimental uncertainties.

TABLE 1

Measured Energy and Angular Resolutions With a Monoenergetic and Unidirectional Electron Beam

	Channels 1-8	Channels 9-16
$\Delta\alpha$	1.6±0.2°	3.7±0.7°
$\Delta\beta$	8.0±0.8°	4.75±0.08°
$\Delta E/E(R_o)$	4.0±0.4%	7.2±0.4%
G (Aperture)	6.22×10^{-2} cm^2-sr	3.929×10^{-3} cm^2-sr

TABLE 2

SSJ/3 Normalizations

$H = 1.30 \times 10^{-4} cm^2\text{-sr}$

Channel Number	Center Energy (eV)	Channeltron Efficiency ε	$G = HE\varepsilon$ Normalization (G) $HE\varepsilon (cm^2\text{-sr-eV})$
1	20,000±3%	0.405	1.05×10^{0}
2	13,700±3%	0.43	7.66×10^{-1}
3	8,990±3%	0.48	5.59×10^{-1}
4	5.500±3%	0.56	4.00×10^{-1}
5	3.790±3%	0.66	3.25×10^{-1}
6	2,290±3%	0.83	2.47×10^{-1}
7	1,590±5%	0.94	1.94×10^{-1}
8	1,060±6%	0.99	1.36×10^{-1}
	$H = 4.3 \times 10^{-5} cm^2\text{-sr}$		
9	1,045±3%	1.00	4.49×10^{-2}
10	661±3%	1.00	2.84×10^{-2}
11	434±3%	1.00	1.86×10^{-2}
12	264±3%	1.00	1.13×10^{-2}
13	183±3%	1.00	7.86×10^{-3}
14	110±3%	1.00	4.73×10^{-3}
15	77±5%	1.00	3.31×10^{-3}
16	51±6%	.76	1.67×10^{-3}

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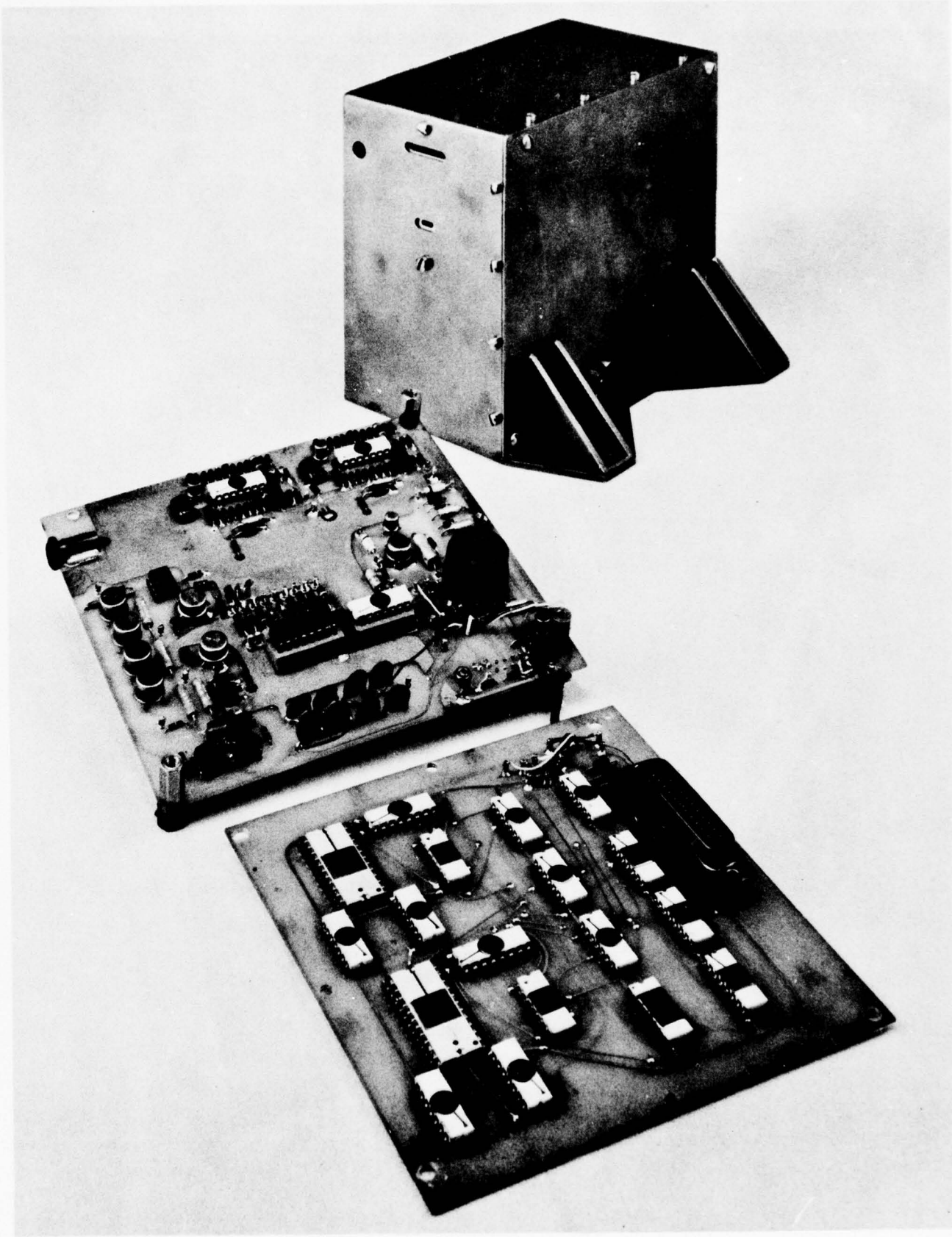


Fig. 1

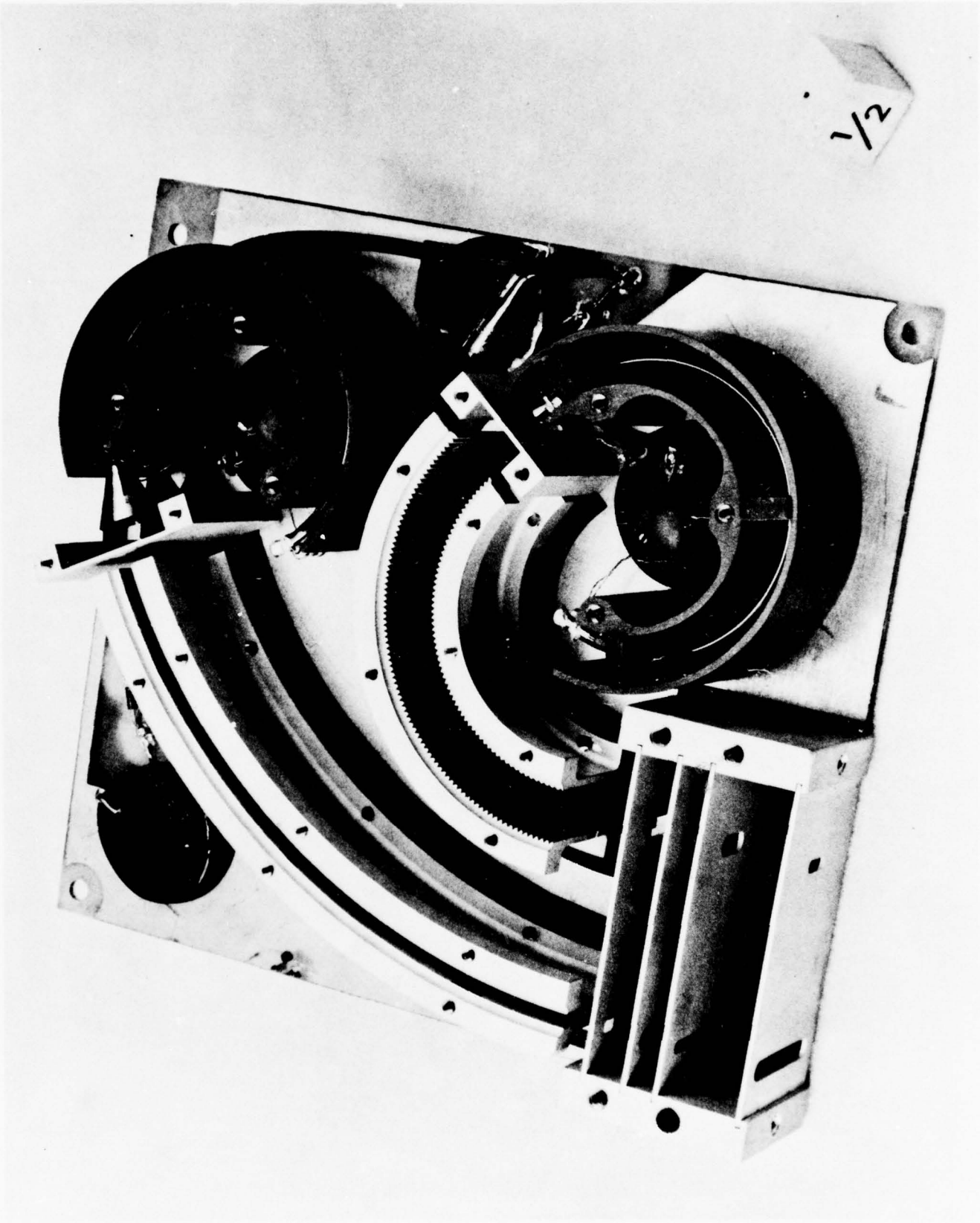
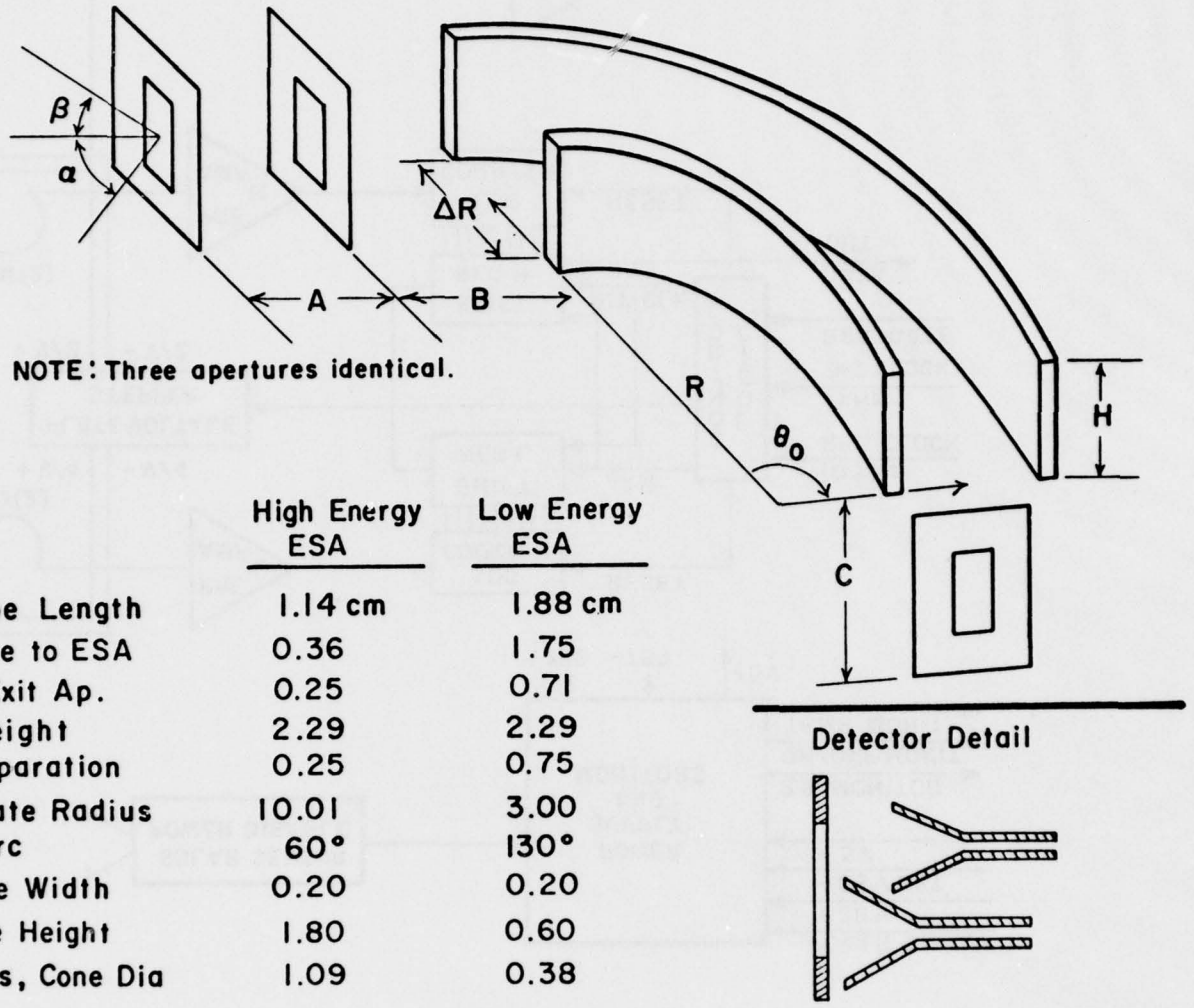


Fig. 2

SSJ/3 ESA ASSEMBLY



	High Energy ESA	Low Energy ESA
A – Telescope Length	1.14 cm	1.88 cm
B – Telescope to ESA	0.36	1.75
C – ESA to Exit Ap.	0.25	0.71
H – Plate Height	2.29	2.29
ΔR – Plate Separation	0.25	0.75
R – Mean Plate Radius	10.01	3.00
θ_0 – Plate Arc	60°	130°
Aperture Width	0.20	0.20
Aperture Height	1.80	0.60
Detectors, Cone Dia	1.09	0.38

Fig. 3

SSJ/3 BLOCK DIAGRAM

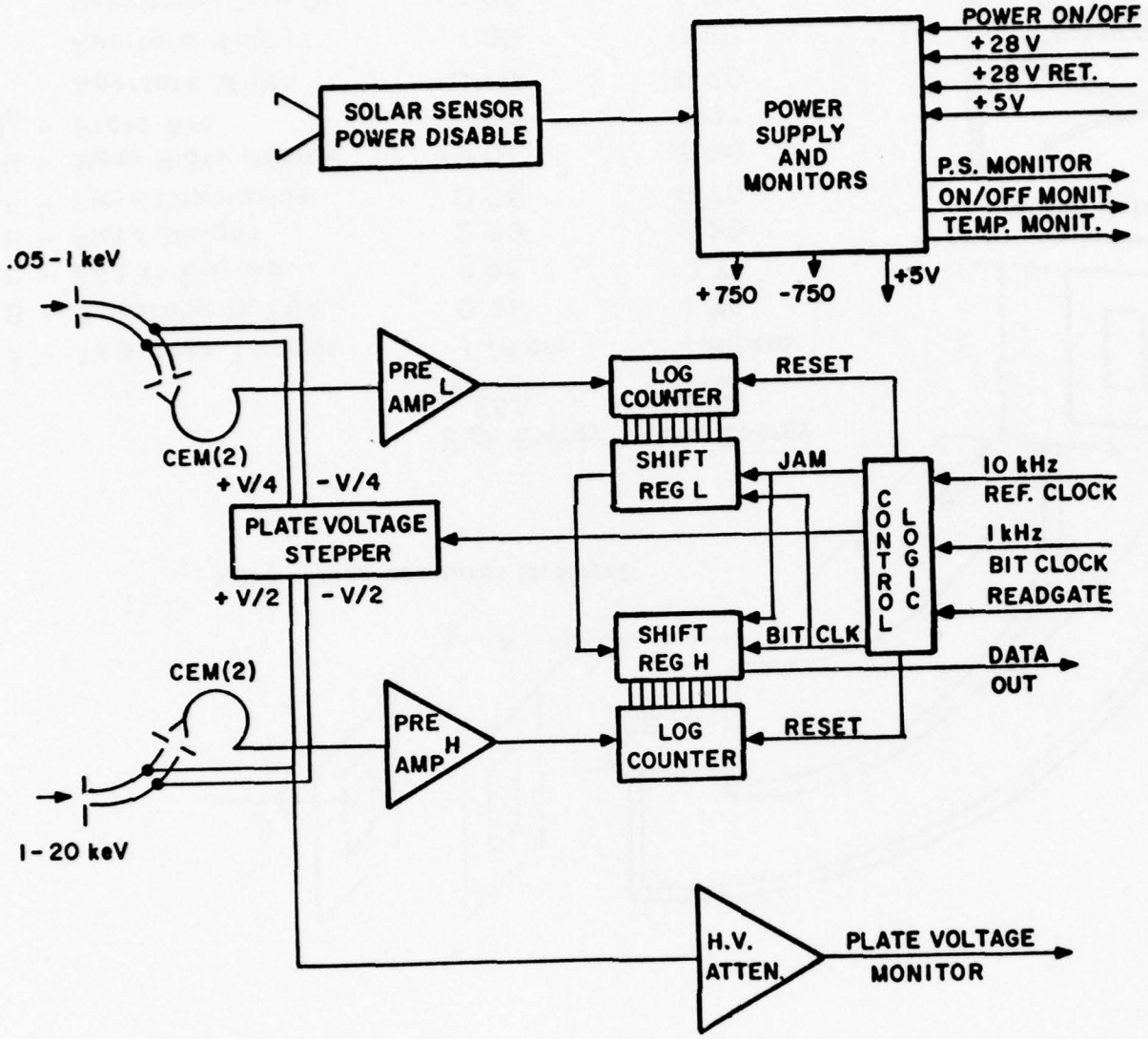


Fig. 4

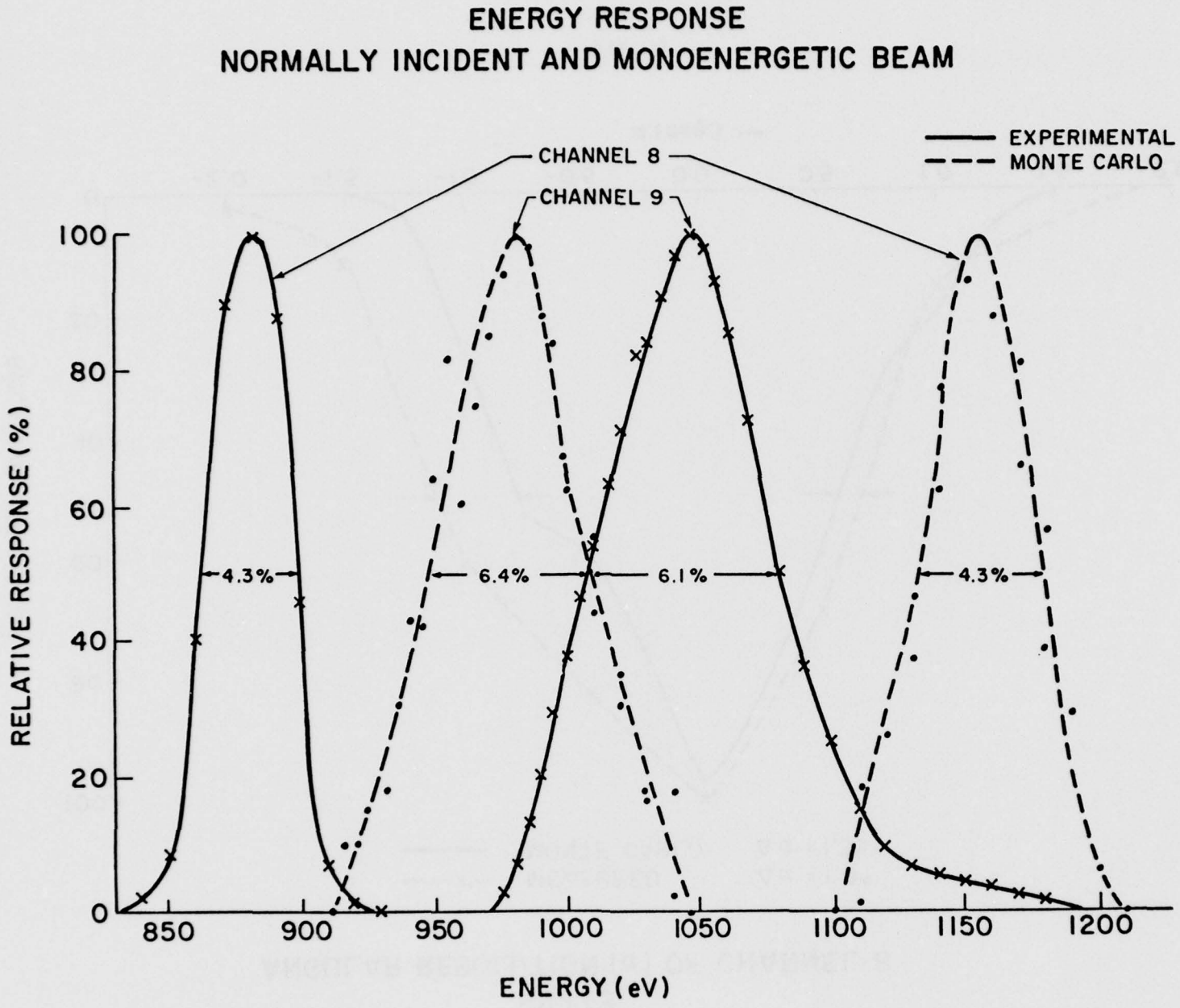

ENERGY RESPONSE
NORMALLY INCIDENT AND MONOENERGETIC BEAM
EXPERIMENTAL
MONTE CARLO
CHANNEL 8
CHANNEL 9
RELATIVE RESPONSE (%)
100
80
60
40
20
0
4.3%
6.4%
6.1%
4.3%
850
900
950
1000
1050
1100
1150
1200
ENERGY (eV)

Fig. 5

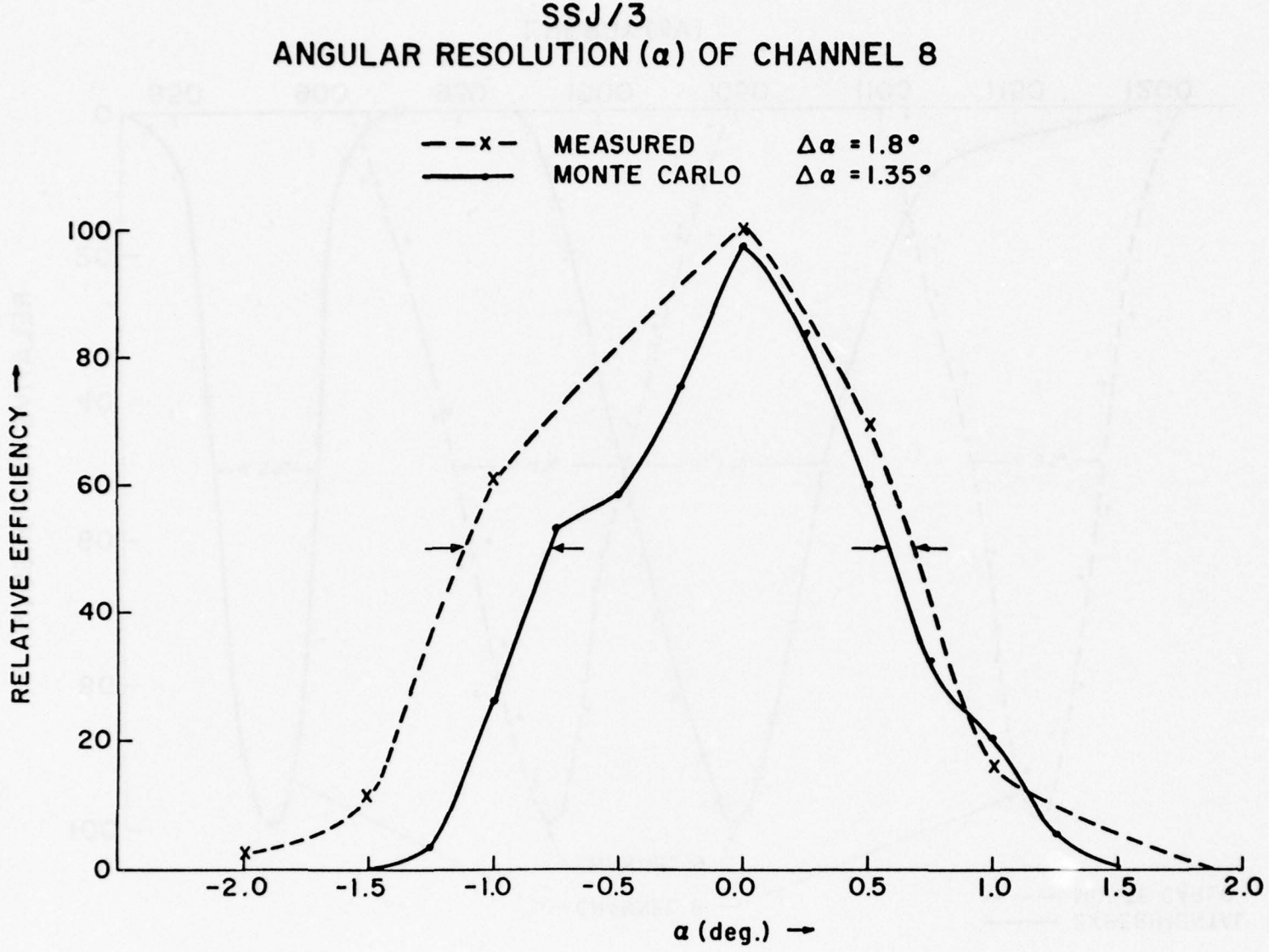

SSJ/3
ANGULAR RESOLUTION (α) OF CHANNEL 8
MEASURED Δα = 1.8°
MONTE CARLO Δα = 1.35°
RELATIVE EFFICIENCY →
100
80
60
40
20
0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
α (deg.) →

Fig. 6

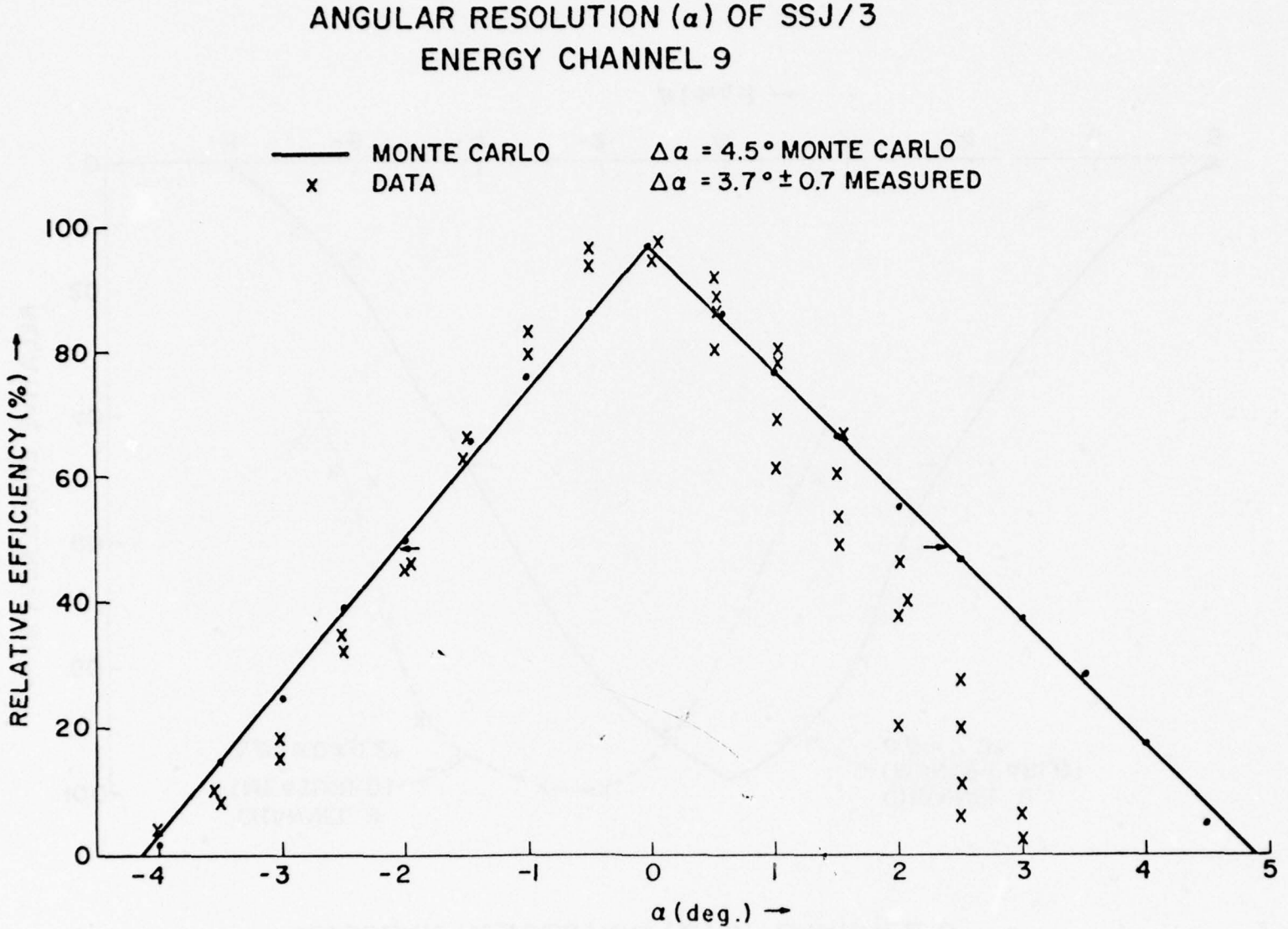

ANGULAR RESOLUTION (α) OF SSJ/3
ENERGY CHANNEL 9
MONTE CARLO
DATA
Δα = 4.5° MONTE CARLO
Δα = 3.7° ± 0.7 MEASURED
RELATIVE EFFICIENCY (%) →
100
80
60
40
20
0
-4
-3
-2
-1
0
1
2
3
4
5
α (deg.) →

Fig. 7

SSJ / 3

ANGULAR RESOLUTION (β) OF CHANNEL 8

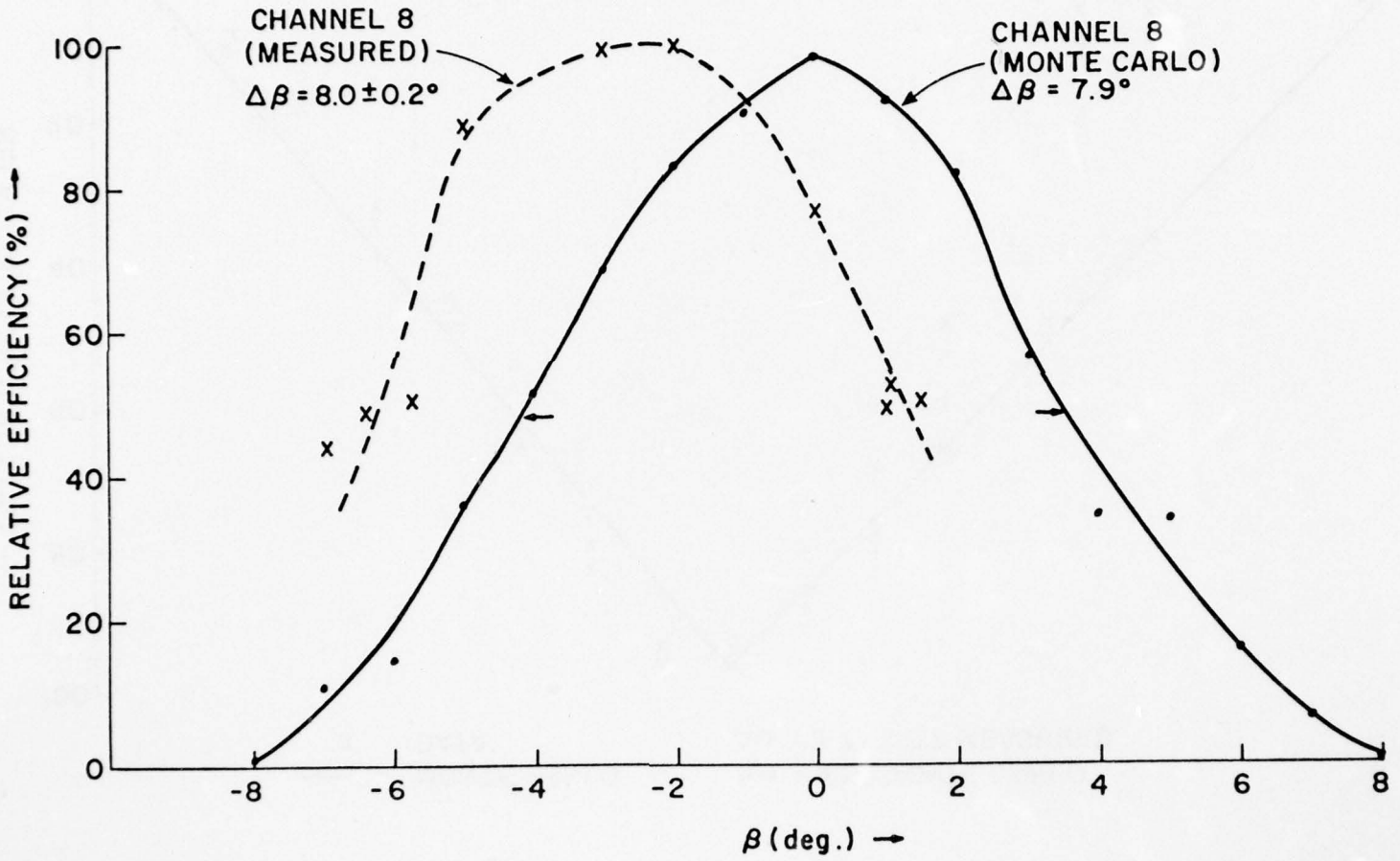


Fig. 8

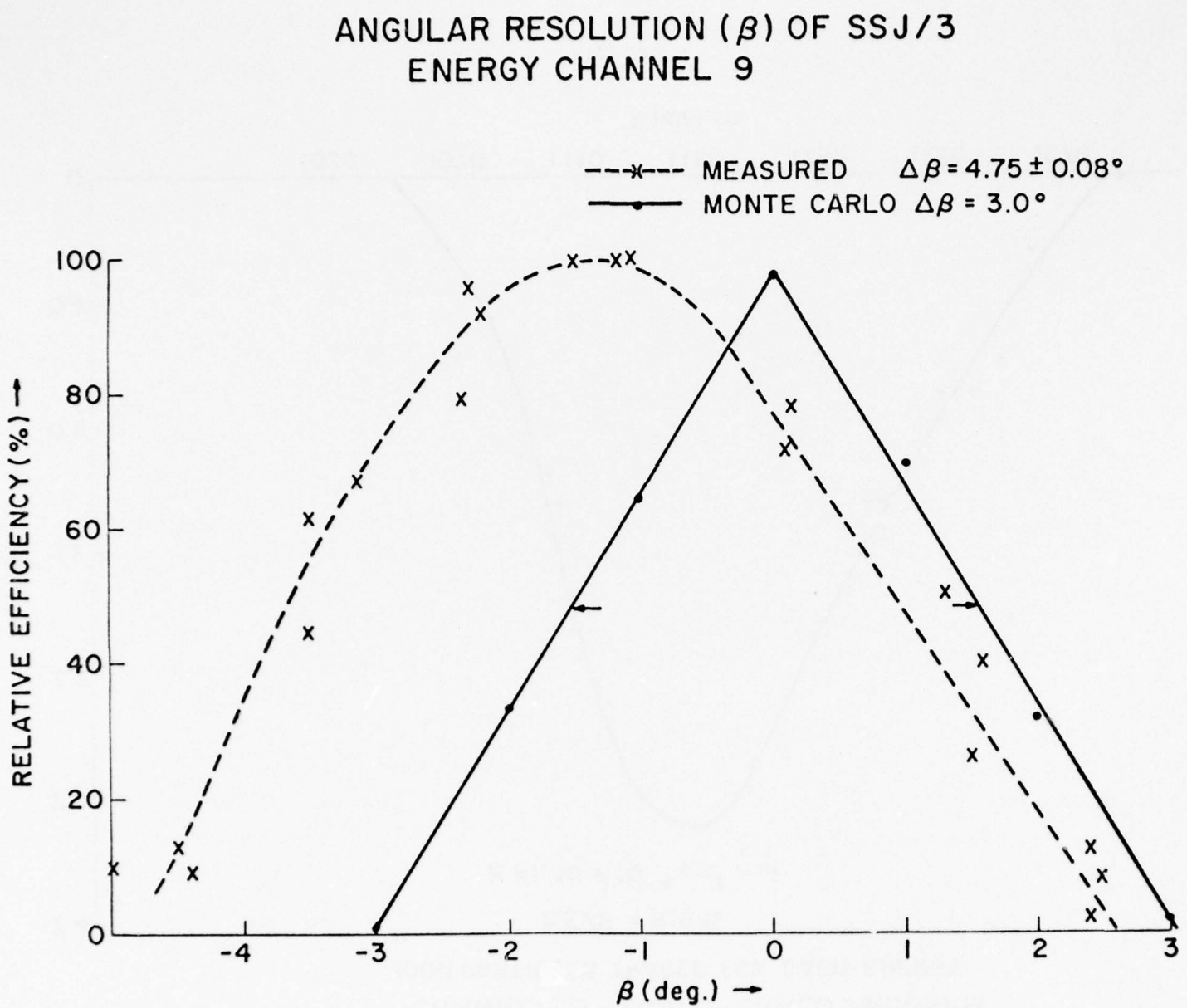

ANGULAR RESOLUTION (β) OF SSJ/3
ENERGY CHANNEL 9
MEASURED Δβ = 4.75 ± 0.08°
MONTE CARLO Δβ = 3.0°
RELATIVE EFFICIENCY (%) →
100
80
60
40
20
0
-4
-3
-2
-1
0
1
2
3
β (deg.) →

Fig. 9

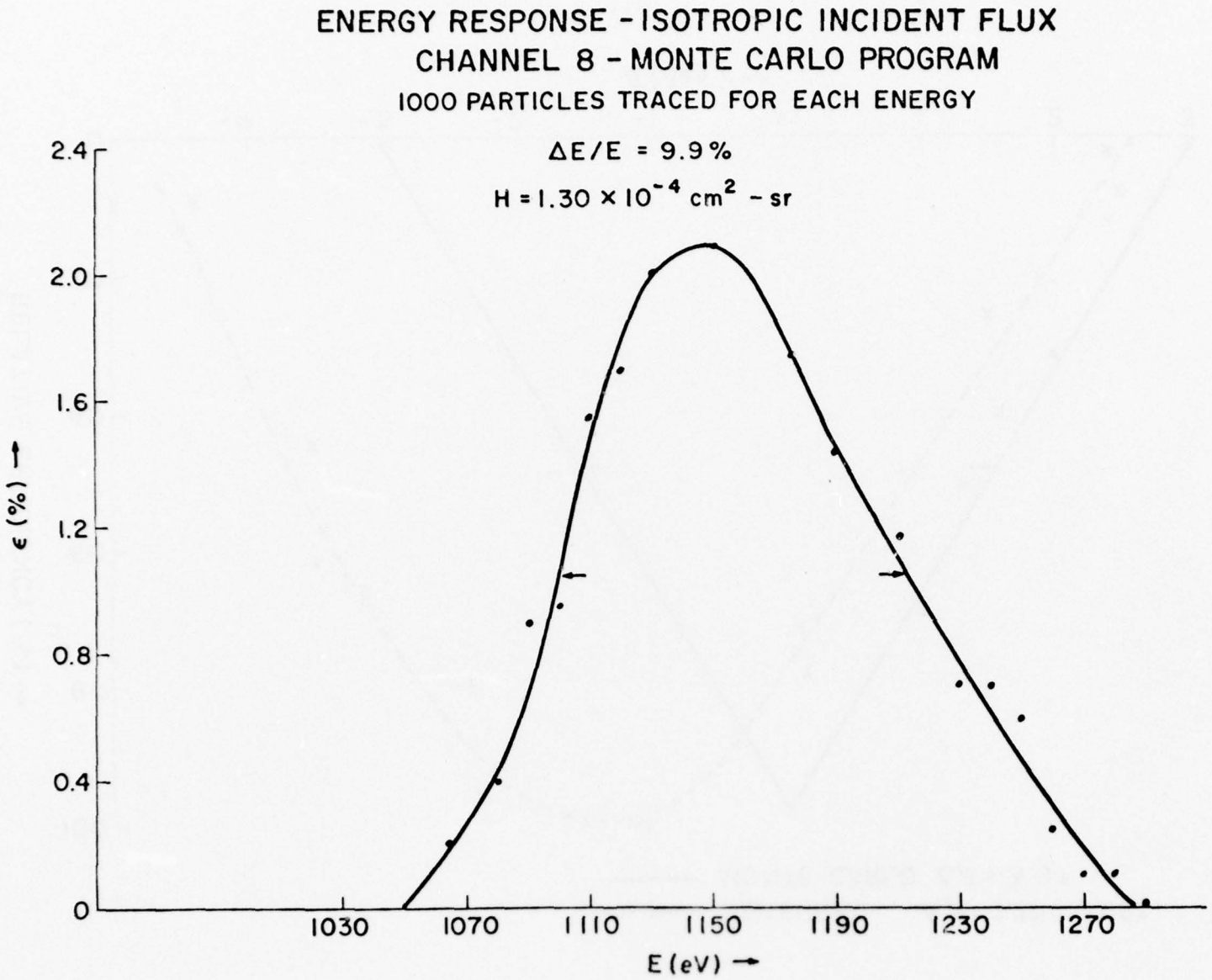

ENERGY RESPONSE - ISOTROPIC INCIDENT FLUX
CHANNEL 8 - MONTE CARLO PROGRAM
1000 PARTICLES TRACED FOR EACH ENERGY
ΔE/E = 9.9%
H = 1.30 × 10⁻⁴ cm² - sr
ε (%) →
2.4
2.0
1.6
1.2
0.8
0.4
0
1030
1070
1110
1150
1190
1230
1270
E (eV) →

Fig. 10

ENERGY RESPONSE - ISOTROPIC INCIDENT FLUX
CHANNEL 9 - MONTE CARLO PROGRAM

1000 PARTICLES TRACED FOR EACH ENERGY

$\Delta E/E = 9.2\%$

$H = 4.3 \times 10^{-5}\ cm^2 - sr$

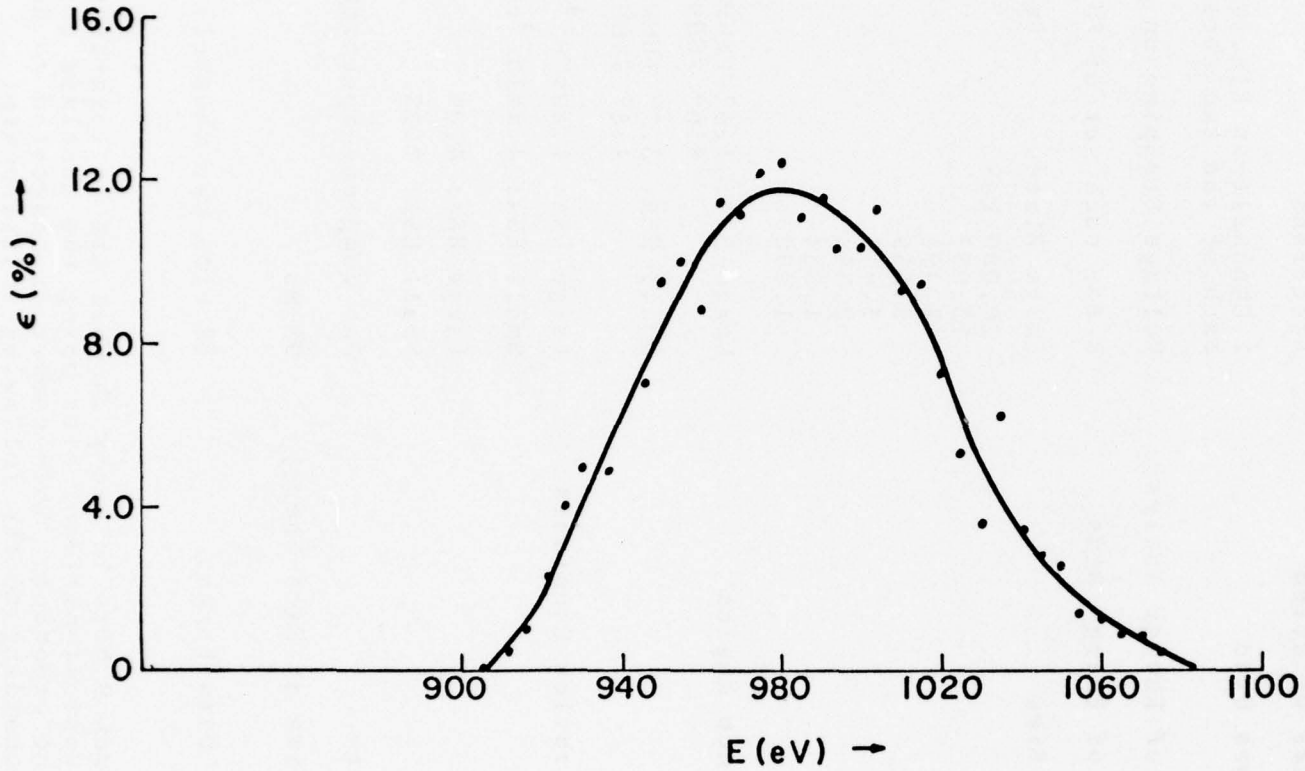


Fig. 11

APPENDIX A

ELECTROSTATIC ANALYZER (ESA)

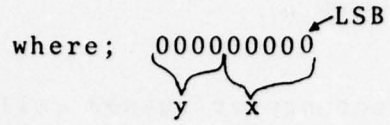
SSJ/3 OR GFE3

Summary of Characteristics

Particles Detected	Electrons
Detectors Used	2 Channeltron Electron Multipliers for each of the two sets of ESA plates
Method of Energy Analysis	Voltage Stepping on ESA plates
Number of Energy Bins	8 for each set of ESA plates; total of 16
Energy Bins	Large Plates: 20.000 keV, 13.037, 8.498, 5.539, 3.611, 2.354, 1.534, 1.000 Small Plates: 1.000 keV, .652, .425, .277, .181, .118, .077, .050
Acceptance Angles	Large ESA: 1.6° FWHM across the apertures 8.0° FWHM along the apertures Small ESA: 3.7° FWHM across the apertures 4.8° FWHM along the apertures
Normalization Constants	Large ESA: 1.30×10^{-4} cm^2-ster. Small ESA: 4.3×10^{-3} cm^2-ster.
$\frac{\Delta E}{E}$	Large ESA: 4.0% Small ESA: 7.2%
Data Rate	One complete Spectrum per second
Dwell Time at Each Energy Level	98 ms.
Digital Data Format	(9 bits per channel) x (16 channels) = 144 bits

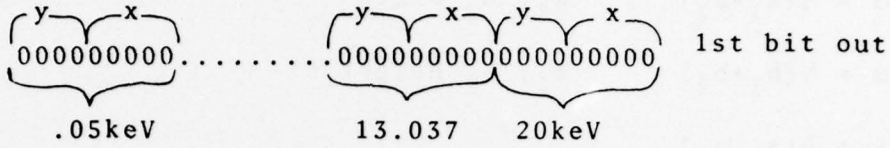
Note: Each 9 bits in every channel are in logarithmic form, the five least significant bits being the mantissa and the remaining four the exponent. This number is converted to decimal form according to the following relationship:

$$N = 2^{y}(x+32) - 33$$



Data Readout

The first bit to be read out is the least significant bit (LSB) of the highest channel followed by the next to the highest one etc., i.e.:



[.....144 bits.....]

read out in one group
at the end of each second

Analog Monitors	Plate voltages: 5.0 volts to .25v
	Power supply : 2.5v
	Temperature : 2.5v at room temperature
Size	5.50in X 3.39in X 5.10in
Weight	3.046 lbs
Power Dissipation	.125 watts

APPENDIX B

The G-factor for a rectangular shaped collimator can be calculated exactly (Rothwell and Moomey, 1972). With the front and rear apertures having the dimensions (a_1, b_1) and (a_2, b_2) respectively, one can define

$$\alpha = \tfrac{1}{2}(a_1+a_2) \qquad a_1,\ a_2 \text{ width}$$

$$\beta = \tfrac{1}{2}(b_1+b_2) \qquad b_1,\ b_2 \text{ height}$$

$$\gamma = \tfrac{1}{2}(a_1-a_2)$$

$$\delta = \tfrac{1}{2}(b_1-b_2)$$

If the collimator has a length L then

$$G = L^2 \ln\left[\frac{L^2+\alpha^2+\delta^2}{L^2+\alpha^2+\beta^2} \cdot \frac{L^2+\gamma^2+\beta^2}{L^2+\gamma^2+\delta^2}\right]$$

$$+ F(\alpha,\beta) + F(\gamma,\delta) - F(\alpha,\delta) - F(\gamma,\beta)$$

where $F(\alpha,\beta) = F(\alpha,\beta) + F(\beta,\alpha)$

$$F(\alpha,\beta) = 2\beta(L^2+\alpha^2)^{\frac{1}{2}} \tan^{-1}\left[\frac{\beta}{(L^2+\alpha^2)^{\frac{1}{2}}}\right]$$

This expression is consistent with that given by Willis and Thomas (1972) and by Sullivan (1971) (with a corrected typographical error).

APPENDIX C

Listing of computer code that was used for the Monte-Carlo computations.

```
      PROGRAM ESAPLTS(INPUT,OUTPUT)                                      10
       DIMENSION DAT(24),EFF(200),EE(200),EG(200),AMAT(10,10),S          11
     +UM1(10)                                                            11
       DIMENSION THH(100),AX(100),THM(100)                               12
       COMMON/F1/EFF,EG,DELE,N5,GINT                                     13
       R2(X)=A*X+B                                                       14
       F(X,Y,AL)=2.0*Y*SQRT(AL**2+X**2)*ATAN(Y/SQRT(AL**2+X**2))         15
       N5=60                                                             16
       N7=N5/6                                                           17
       N9=5                                                              18
        N11=30                                                           19
        N12=N11/2                                                        20
       AMP=1.66E-27                                                      21
       QOVMP=9.638E 07                                                   22
        AME=9.1E-31                                                      23
       QOME=1.7582E 11                                                   24
       AA=180./3.141596                                                  25
        CNNTA=0.0                                                        26
       ELEC=8HELECTRON                                                   27
 1     CONTINUE                                                          28
        CALL RANSET(TIME(DUM))                                           29
        CNNTA=0.0                                                        30
C IF II=1 DO BACKGROUND CALCULATION BASE ON "LINE OF SIGHT"              31
C    IJ.NE. 0,  AUTOMATICALLY ASSUMES PARAMETERS FOR L. FRANKS E         32
     +SA                                                                 32
C IJ=2, SIMULATES ELECTRON GUN SCAN OF FRANKS ESA AS SHOWN IN F          33
     +IG. 2 OF                                                           33
C HIS PAPER                                                              34
C II=1, DOES LINE OF SIGHT BACKGROUND  CALCULATION                       35
C K1=0, NO ELASTIC SCATTERING OFF OF ESA PLATES                          36
C K1=1, MORE THAN 1 SCATTERING OFF OF ESA PLATES PER ELECTRON A          37
     +S DEFINED                                                          37
C  BY K2 (INNER PLATE)                                                  38
C K1=2, 1/K2X100 PER CENT SCATTERING OFF OF ESA PLATES (INNER P          39
     +LATES)                                                             39
C K1=3, SCATTERING OFF OF OUTER ESA PLATE AS DEFINED BY K9               40
C K1=4,SCATTERING OFF OF OUTER ESA PLATE AS DEFINED BY 1/K9X100          41
C  K1=5,SCATTERING OFF OF BOTH PLATES,K2 TIMES INNER PLATE,K9 T         42
     +IMES OFF                                                           42
C  ATE                                                                   43
C K1=6,CATTERING OFF OF BOTH PLATES1/K2 INNER,1/K9 TIMES OUTER          44
     +PLATE                                                              44
C IF IK=0 INCIDENT FLUX IS ISOTROPIC                                     45
C IK=1 INCIDENT FLUX AT INCIDENT ANGLE ANGE                              46
C IF IK=2 SCAN IN R-PLANE IN STEPS OF ANGE DEGREES                       47
C IF IK=3 SCAN IN Z-PLANE IN STEPS OF ANGE DEGREES                       48
C NOTE-- IF IK GE 2 INPUT DATA PACK CHANGES                              49
       READ 4,II,IJ,K1,K2,K9,ANGA,ANG2,ANGD,IK,ANGE                     50
 4     FORMAT(5I5,3F7.2,I5,F7.2)                                         51
        PRINT 1900,IK,ANGE                                               52
 1900  FORMAT(1X,I5,F7.2)                                                53
```

```
      IF(K1.EQ.0) K7=1                                                  54
      IF(K1.EQ.1) K7=1                                                  55
      IF(K1.EQ.2) K7=1                                                  56
      IF(K1.EQ.3) K7=2                                                  57
      IF(K1.EQ.4) K7=3                                                  58
      IF(K1.EQ.5) K7=2                                                  59
      IF(K1.EQ.6) K7=3                                                  60
      IF(K1.EQ.0) K8=1                                                  61
      IF(K1.EQ.1) K8=2                                                  62
      IF(K1.EQ.2) K8=3                                                  63
      IF(K1.EQ.3) K8=1                                                  64
      IF(K1.EQ.4) K8=1                                                  65
      IF(K1.EQ.5) K8=2                                                  66
      IF(K1.EQ.6) K8=3                                                  67
      IF(K1.EQ.7)  K7=2                                                 68
      IF(K1.EQ.7)  K8=3                                                 69
      IF(K1.EQ.8) K7=3                                                  70
      IF(K1.EQ.8) K8=2                                                  71
      READ 3,(DAT(I),I=1,24)                                            72
3     FORMAT((12A6))                                                    73
      READ2,PART,POST                                                   74
2     FORMAT(A8,F8.0)                                                   75
      PRINT 5,(DAT(I),I=1,24)                                           76
 5    FORMAT(1H1,12A6/12A6)                                             77
      IF(II.EQ.1) PRINT 10                                              78
10    FORMAT(1X,*BACKGROUND CALCULATION*)                               79
       IF(IK.EQ.1) PRINT 12,ANGE                                        80
12     FORMAT(1X,* CONST. ANGLE SORT,ANGLE=*,F7.2,*  DEG.*)             81
      IF(PART.EQ.ELEC)  AMP=AME                                         82
      IF(PART.EQ.ELEC)  QOVMP =-QOME                                    83
      PRINT 6,PART                                                      84
    6  FORMAT(1X,*TRACING*,3X,A8/)                                      85
      IF(PART.EQ.ELEC.AND.IJ.NE.0) POST=160.                            86
      IF(IJ.NE.0) PRINT 16                                              87
16    FORMAT(1X,*  FRANKS ESA    *)                                     88
      IF(IJ.EQ.2) PRINT 18                                              89
18    FORMAT(1X,*ELECTRON GUN FLUX=2.0E 06 ELEC/CMSQ/SEC*)              90
      IF(K1.NE.0) PRINT 17,K1,K2,K9                                     91
 17    FORMAT(1X,*ELASTIC SCATTERING OFF OF ESA PLATES      ,*,         92
     +*K1=*,I5,                                                         92
     A *    K2=*,I5/1X,*IF K1=1 K2 SCATTERINGS PER PART., IFK1=2       93
     + EVERY OT                                                         93
     BHER K2 PART. MAY SCATTER*,*K9=*,I5)                               94
       IF(K1.NE.0)                          PRINT 21,ANG2              95
       IF(K1.NE.0)                          PRINT 19,ANGA              96
       IF(K1.NE.0) PRINT 23,ANGD                                        97
 19   FORMAT(1X,* MAX SCATTERING ANGLE OFF OF OUTER PLATE=*,F7.          98
     +2,* DEG*)                                                         98
21    FORMAT(1X,* MAX SCATTERING ANGLE OFF OF INNER PLATE=*,F7.          99
     +2,* DEG*)                                                         99
 23    FORMAT(1X,*MAX. ANGLE FOR INELASTIC SCATTERING OFF OF OU        100
     +TER PLATE                                                        100
```

```
     A=*,F7.2)                                                          101
C SET-UP APERTURE AND CALCULATE ITS G-FACTOR                            102
      READ 8,A1,B1,A2,B2,AL1,AL2, AL3,VOLT1,R5                          103
  8     FORMAT(7F7.3,2F7.3)                                             104
      PRINT 11,A1,B1,A2,B2,AL1,AL2,AL3,VOLT1,R5,POST                    105
 11    FORMAT(1X,*APERTUREDIMENSIONS*/1X,*FRONT WIDTH=*,F7.3,* C        106
     +M*,3X,*FR                                                         106
     1ONTHT=*,F7.3,* CM*,3X, *BACK WIDTH=*,F7.3,* CM*,3X,*BACK          107
     +HT=*,F7.3                                                         107
     2,*CM*/1X,*LENGTH OF APER.=*,F7.3,* CM*,3X, *DIST BTWN APE        108
     +R AND PLA                                                         108
     3TES=*,F7.3,* CM*/1X,*DIST BTWN PLATES AND CHANN.=*,F7.3,*         109
     + CM*,3X,*                                                         109
     4BIAS VOLTAGE IN FRONT OF DETECT.=*,F7.1,* VOLTS*/1X,*CHAN        110
     +NELTRON R                                                         110
     1ADIUS=*,F7.2,*  CM*,*  POST ACCELERATION=*,F8.0,*VOLTS*/)         111
      ALPHA=(A1+A2)/2.0                                                 112
      BETA=(B1+B2)/2.0                                                  113
      GAM=(A1-A2)/2.0                                                   114
      DELTA=(B1-B2)/2.0                                                 115
      G1=AL1**2*ALOG((AL1**2+ALPHA**2+DELTA**2)*(AL1**2+GAM**2+         116
     +BETA**2)/                                                         116
     1(AL1**2+ALPHA**2+BETA**2)/(AL1**2 +GAM**2+DELTA**2))              117
      G2=F(ALPHA,BETA,AL1)+F(BETA,ALPHA,AL1)+F(GAM,DELTA,AL1)+F         118
     +(DELTA,GA                                                         118
     1M,AL1)-F(ALPHA,DELTA,AL1)-F(DELTA,ALPHA,AL1)-F(GAM,BETA,A        119
     +L1)-F(BET                                                         119
     2A,GAM,AL1)                                                        120
      G=G1+G2                                                           121
      PRINT 13,G                                                        122
 13    FORMAT(1X,*G-FACTOR FOR APERTURE=*,E10.4,* CMSQ-SR*/)            123
      PRINT 15                                                          124
 15    FORMAT(1X,*ESA PLATE DIMENSIONS*)                                125
C R2A=RADIUS AT APERTURE                                                126
C R2B=RADIUS AT EXIT                                                    127
      READ 7,R1,R2A,R2B,THE,VOLT,H,TOT,R9                               128
 7     FORMAT(4F7.3,  E10.2,  F7.3,E10.4,F7.2)                          129
      PRINT 9,R1,R2A,R2B,THE,VOLT,H,TOT,R9                              130
 9     FORMAT(1X,*ZERO POTENTIAL RADIUS=*,F7.3,*CM*,3X,*RADIUS A        131
     +T POTENTI                                                         131
     1AL V AT APERTURE=*,F7.3,* CM*,3X,*RADIUS AT POTENTIAL V A        132
     +T EXIT=*,                                                         132
     2F7.3,* CM*/3X,*MAX. ANGLE OF PLATES=*,F7.2,* DEGS*,3X,*VO        133
     +LTAGE ON                                                          133
     3 PLATES=*,1PE10.2,* VOLTS* /3X,*PLATE HEIGHT=*,0PF7.2,*          134
     +CM*, *  N                                                        134
     4UMBER OF PARTICLES TRACED FOR EACH ENERGY=*,1PE10.2/1X,*D        135
     +ISTANCE O                                                         135
     5F CHANNELTRON FROM CENTER OF ESA AXIS=*,1PE10.2,*  CM*/)         136
      IF(PART.EQ.ELEC) VOLT =-VOLT                                      137
      THE=THE/AA                                                        138
```

```
      H=H*1.0E-02                                                        139
      ANGB=ANGA/AA                                                       140
      ANGC=ANG2/AA                                                       141
      ANGD=ANGD/AA                                                       142
       ANGE=ANGE/AA                                                      143
      R1=R1*1.0E-02                                                      144
      R2A=R2A*1.0E-02                                                    145
      R2B=R2B*1.0E-02                                                    146
       R5=R5*1.0E-02                                                     147
      R9=R9*1.0E-02                                                      148
      B=R2A                                                              149
      A=    (R2B-R2A)/THE                                                150
      WID1=ABS(R2A-R1)                                                   151
      RR=(R1+R2B)/2.0                                                    152
      A1=A1*1.0E-02                                                      153
      B1=B1*1.0E-02                                                      154
      A2=A2*1.0E-02                                                      155
      B2=B2*1.0E-02                                                      156
      AL1=AL1*1.0E-02                                                    157
      AL2=AL2*1.0E-02                                                    158
      AL3=AL3*1.0E-02                                                    159
      R6=RR-A1/2.0                                                       160
      R7=RR-A2/2.0                                                       161
      R8=RR+A2/2.0                                                       162
      IF(IJ.NE.0) R6=1.24E-01                                            163
      IF(IJ.NE.0) R7=R6                                                  164
      IF(IJ.NE.0)  R8=R6+A2                                              165
      IF(IJ.EQ.2)  THY=-5.0/AA                                           166
      DELY=1.0/AA                                                        167
      THW=THY-DELY                                                       168
      THX=THE/2.0                                                        169
      R3=P2(THX)                                                         170
       GO TO 14                                                          171
  24   IF(IK.GE.2) READ 26,VOLT                                           172
       IF(IK.GE.2) PRINT 29,VOLT                                         173
 26    FORMAT(1PE10.2)                                                   174
  29   FORMAT(1X,* VOLTAGE ON PLATES=*,1PE10.2,* VOLTS*)                 175
      IF(PART.EQ.ELEC) VOLT =-VOLT                                       176
 14    CONTINUE                                                          177
C USING MKS SYSTEM--ENERGY IN JOULES                                     178
      EE(1)=VOLT/2.0/ALOG(R3/R1)*1.6E-19                                 179
      EE(1)=ABS(EE(1))                                                   180
      CONST=QOVMP*VOLT                                                   181
      EA=EE(1)/1.6E-19                                                   182
      PRINT 31,EA                                                        183
 31    FORMAT(1X,*MEAN ENERGY=*,1PE10.2,*EV*)                            184
      IF(IK.LT.2) GO TO 34                                               185
       THH(1)=-ANGE*N12                                                  186
       DO 33 I=1,N11                                                     187
       THH(I+1)=THH(I)+ANGE                                              188
       THM(I)=THH(I)*AA                                                  189
```

```
      AX(I)=0.0                                                          19
 33   CONTINUE                                                           191
      THM(N11+1)=THH(N11+1)*AA                                           192
      GO TO 36                                                           193
 34    CONTINUE                                                          194
      DELE=0.005*EE(1)                                                   195
      EE(1)=0.9*EE(1)                                                    196
      DO 35 K=1,N5                                                       197
      EE(K+1)=EE(K)+DELE                                                 198
 35    CONTINUE                                                          199
 36    CONTINUE                                                          200
      DO 22 J=1,10                                                       201
      DO 22 K=1,10                                                       202
      AMAT(K,J)=0.0                                                      203
 22   CONTINUE                                                           204
 25    CONTINUE                                                          205
      THW=THW+DELY                                                       206
      THW=-4.0/AA                                                        207
      DO 600 K=1,N5                                                      208
       K13=1                                                             209
 37     CONTINUE                                                         210
      K4=0                                                               211
      N4=0                                                               212
        N6=0                                                             213
      K5=0                                                               214
      CNT=0.0                                                            215
      COUNT=0.0                                                          216
       CNTA=0.0                                                          217
C DELT=TIME INTERVAL FOR EACH STEP                                       218
       GO TO 40                                                          219
 38    IF(IK.GE.2) GO TO 202                                             220
 40    CONTINUE                                                          221
      K3=0                                                               222
      N3=0                                                               223
        K11=0                                                            224
        K12=0                                                            225
      EX=EE(K)                                                           226
       IF(IK.GE.2)    EE(K)=EA*1.6E-19                                   227
      V=SQRT(2.*EE(K)/AMP)                                               228
C PULL OUT DEFOREST CARDS                                                229
      R=R6+A1*RANF(DUM)                                                  230
      Z1=B1*RANF(DUM)                                                    231
C CHOOSE T1 SUCH THAT FOR J(FLUX) = CONST. T1 IS PICKED SIN(T1*          232
     +COS(T1)                                                            232
C TIMES,,  SIN FOR SOLID ANGLE    COS FOR PROJECTED AREA                 233
       T1=ACOS(1.0-2.0*RANF(DUM))/2.0                                    234
      T2=6.2831*RANF(DUM)                                                235
       IF(IK.EQ.1) T1=ANGE                                               236
      IF(IK.EQ.1) T2=0.0                                                 237
       IF(IK.EQ.2) T1=THH(K13)                                           238
       IF(IK.EQ.2) T2=0.0                                                239
```

```
      IF(IK.EQ.3) T1=THH(K13)                                          240
      IF(IK.EQ.3) T2=1.5707                                            241
     IF(IJ.EQ.2) T1=ABS(THW)                                           242
     IF(IJ.EQ.2.AND.THW.GE.0.0) T2=180./AA                             243
     IF(IJ.EQ.2.AND.THW.LT.0.0) T2=0.0                                 244
     Y1=0                                                              245
     VR=V*SIN(T1)*COS(T2)                                              246
     VTT=V*COS(T1)                                                     247
     VZ=V*SIN(T1)*SIN(T2)                                              248
     DT=AL1/VTT                                                        249
     R=R+VR*DT                                                         250
     Z1=Z1+VZ*DT                                                       251
     IF(Z1.GT.B2.OR.Z1.LT.0.0) GO TO 38                                252
     IF(R.GT.R8.OR.R.LT.R7) GO TO 38                                   253
      CNTA=CNTA+1.0                                                    254
     DT=AL2/VTT                                                        255
     R=R+VR*DT                                                         256
      Z1=Z1+VZ*DT                                                      257
     IF(R.GE.R2A.OR.R.LE.R1) GO TO 202                                 258
       ANGM=VTT*R                                                      259
     TT1=0.0                                                           260
     R3=R2(TT1)                                                        261
     CONTT=CONST/ALOG(R3/R1)                                           262
     CONSS=CONST                                                       263
     IF(II.EQ.1) CONSS=0.0                                             264
     DELTT=CONTT/(R1+R3)*2                                             265
     DELTT=3.6E-01*V/DELTT                                             266
     DELT=3.1416*THE*(R1+R3)/2./V/2.0E 02                              267
     IF(DELTT.LE.DELT) DELT=DELTT                                      268
     GO TO 20                                                          269
 27  CONTINUE                                                          270
     IF(K12.GT.0.OR.K11.GT.0)   R=R+VR*DELT                            271
     GOTO 28                                                           272
20    CONTINUE                                                         273
     R3=R2(TT1)                                                        274
     CONS2=(VTT**2-CONSS/ALOG(R3/R1))/R                                275
      R=R+VR*DELT+CONS2*DELT**2/2.0                                    276
      VR=VR+ CONS2*DELT                                                277
 28   CONTINUE                                                          278
      VTT=ANGM/R                                                       279
     TT1=TT1+VTT*DELT/R                                                280
     Z1=Z1+    DELT*VZ                                                 281
      IF (Z1.LE.0.0.OR.Z1.GE.H) GOTO 202                               282
     IF(TT1.GE.THE) GO TO 400                                          283
     IF(R.GT.R1.AND. R.LT.R3) GO TO 1700                               284
     GO TO 1800                                                        285
1700  CONTINUE                                                         286
     K11=0                                                             287
     K12=0                                                             288
     GO TO 20                                                          289
1800 CONTINUE                                                          290
```

```
C    K1=1,2-----SCATTER OFF OF INNER SURFACE ONLY                    291
C    K1=3,4----SCATTER OFF OF OUTER SURFACE ONLY                     292
       IF(R.GE.R3) GO TO 1300                                        293
       IF(R.LE.R1) GO TO 1400                                        294
       GO TO 202                                                     295
 1300 CONTINUE                                                       296
       IF(K11.GT.0) GO TO 27                                         297
       K11=K11+1                                                     298
       GO TO (202,1100,1200),K7                                      299
 1400 CONTINUE                                                       300
       IF(K12.GT.0) GO TO 27                                         301
       K12=K12+1                                                     302
       GO TO (202,900,1000),K8                                       303
  900 CONTINUE                                                       304
       N3=N3+1                                                       305
       IF(N3.GT.K9) GO TO 202                                        306
       VD=ABS(VR)                                                    307
       THP=ATAN(VD/VTT)                                              308
       IF(ABS(THP).GT.ANGC) GO TO 1500                               309
        VR=-VR                                                       310
       GO TO 27                                                      311
 1000 CONTINUE                                                       312
       VD=ABS(VR)                                                    313
       THP=ATAN(VD/VTT)                                              314
        N6=N6+1                                                      315
       IF(ABS(THP).GT.ANGC) GO TO 1500                               316
        VR=-VR                                                       317
        GO TO 201                                                    318
 1100 CONTINUE                                                       319
       VD=ABS(VR)                                                    320
        K3=K3+1                                                      321
       IF(K3.GT.K2) GO TO 202                                        322
       THP= ATAN(VR/VTT)                                             323
       IF(ABS(THP).GT.ANGC) GO TO 202                                324
       IF(ABS(THP).GT.ANGB) GO TO 1600                               325
        VR=-VR                                                       326
       GO TO 27                                                      327
 1200 CONTINUE                                                       328
       VD=ABS(VR)                                                    329
       K4=K4+1                                                       330
       THP= ATAN(VR/VTT)                                             331
       IF(ABS(THP).GT.ANGC) GO TO 202                                332
       IF(ABS(THP).GT.ANGB) GO TO 1600                               333
        VR=-VR                                                       334
       GO TO 200                                                     335
 1500 CONTINUE                                                       336
       EX=EX    *(1.0-RANF(DUM))**(1.0/1.35)                         337
       VA=SQRT(2.0*EX/AMP)                                           338
       VR=VA*SIN(THP)                                                339
       VTT=VA*COS(THP)                                               340
       IF(K8.EQ.3) GO TO 201                                         341
```

```
      GO TO 27                                                          342
 1600  CONTINUE                                                         343
      EX=EX    *(1.0-RANF(DUM))**(1.0/1.35)                             344
      VA=SQRT(2.0*EX/AMP)                                               345
      VR=-VA*SIN(THP)                                                   346
       VTT=VA*COS(THP)                                                  347
      IF(K7.EQ.3) GO TO 200                                             348
      GO TO 27                                                          349
 200   CONTINUE                                                         350
      IAB=MOD(K4,K2)                                                    351
      IF(IAB.EQ.0) K4=0                                                 352
      K5=K4                                                             353
      IF(IAB.EQ.0) GO TO 27                                             354
      GO TO 202                                                         355
 201  CONTINUE                                                          356
      IAC=MOD(N6,K9)                                                    357
      IF(IAC.EQ.0) N6=0                                                 358
      N4=N6                                                             359
      IF(IAC.EQ.0) GO TO 27                                             360
 202   CONTINUE                                                         361
      CNT=CNT+1                                                         362
      IF(CNT.GE.TOT) GO TO 500                                          363
      GO TO 40                                                          364
 400  CONTINUE                                                          365
C SET UP MATRIX AT EXIT APERTURE OF PLATES                              366
      DO 420 N=1,N9                                                     367
      IF(Z1.GE.(N-1)*H/N9.AND.Z1.LT.N*H/N9) N1=N                        368
 420  CONTINUE                                                          369
      DO 430 N=1,N9                                                     370
      RRR1=R1+(N-1)*WID1/N9                                             371
      RRR2=R1+N*WID1/N9                                                 372
      IF(R.GE.RRR1.AND.R.LT.RRR2) N2=N                                  373
 430   CONTINUE                                                         374
      AMAT(N1,N2)=AMAT(N1,N2)+1.0                                       375
C   DO WE HIT THE CHANNELTRON?                                          376
      RR=(R1+R2B)/2.0                                                   377
C FULL DEFOREST ESA CARDS                                               378
      DIS2=AL3                                                          379
      DT=DIS2/VTT                                                       380
      XX=0.0                                                            381
       VXX=VTT                                                          382
      VT=VOLT1                                                          383
      DTT=0.1*DT                                                        384
      TT=0.0                                                            385
      ACC=QC/MP*VT/DIS2                                                 386
 460   CONTINUE                                                         387
      TT=TT+DTT                                                         388
       XX=XX+VXX*DTT-ACC*DTT**2/2.0                                     389
      VXX=VXX-ACC*DTT                                                   390
C   DOES LOW-ENERGY ELECTRON STOP                                       391
      IF(VXX.LE.0.0) GO TO 202                                          392
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      IF(XX.LT.DIS2) GO TO 460                                          393
      R=R+VR*TT                                                         394
      Z1=Z1+VZ*TT                                                       395
      IF(IJ.NE.0) R5=1.77E-03                                           396
      IF(IJ.NE.0) R9=-R5                                                397
      R4=ABS(RR-R9-R)                                                   398
C PULL OUT DEFOREST CARDS                                               399
      IF(R.LT.(RR-A2/2.0).OR.R.GT.(RR+A2/2.0)) GO TO 202                400
      IF(Z1.GT.B2.OR.Z1.LT.0.0) GO TO 202                               401
      R4A=SQRT(R4**2+(Z1-B2/2.0-R5)**2)                                 402
      R4B=SQRT(R4**2+(Z1-B2/2.0+R5)**2)                                 403
      IF(R4A.GT.R5.AND.R4B.GT.R5) GO TO 202                             404
      IF(IK.GE.2) AX(K13)=AX(K13)+1.0                                   405
      COUNT=COUNT+1                                                     406
      GO TO 202                                                         407
  500 CONTINUE                                                          408
      EFF(K)=COUNT/CNT                                                  409
      IF(IK.GE.2) AX(K13)=AX(K13)/CNT                                   410
      EG(K)=EE(K)/1.6E-19                                               411
      CNNTA=CNNTA+CNTA                                                  412
      PRINT 507,EG(K),CNTA,CNNTA                                        413
  507 FORMAT(1X,*  AT ENERGY =  *,1PE10.2,* EV    NUMBER OF PA          414
     +RTICLES                                                           414
     1 THROUGH APERTURE= *,0PF10.2, * RUNNING TOTAL= *,0PF10.2)         415
      IF(II.EQ.1) GO TO 630                                             416
      E99=EG(K)/1.0E 03                                                 417
      EFI=1.0                                                           418
      TEMP=EG(K)                                                        419
      EG(K)=EG(K)-VT+POST                                               420
      IF(PART.EQ.ELEC.AND.EG(K).GE.1.0E 03.AND.EG(K).LE.5.0E 0          421
     +6)                                                                421
     1  EFI=1.0-(2.0/(3.0+6.5/(E99  -0.5)+30./(E99  -0.5)**3))         422
      IF(PART.EQ.ELEC.AND.EG(K).GE.10.0.AND.EG(K).LE.70.) EFI=0         423
     +.10*EG(K)                                                         423
     1**0.515                                                           424
      IF(PART.EQ.ELEC.AND.EG(K).GE.200.AND.IJ.NE.0)EFI=8.199/EG         425
     +(K)**0.41                                                         425
     A7                                                                 426
      IF(PART.EQ.ELEC.AND.EG(K).LT.200.AND.IJ.NE.0)EFI=5.9E-04*         427
     +EG(K)**1.                                                         427
     A38                                                                428
      EG(K)=TEMP                                                        429
      IF(IK.GE.2) AX(K13)=AX(K13)*EFI                                   430
      IF(IK.GE.2) GO TO 440                                             431
      EFF(K)=EFF(K)*EFI                                                 432
      IF(IJ.EQ.2) EFF(K)=EFF(K)*2.0E 06                                 433
      IF( MCD(K,N7).NE.0  ) GO TO 440                                   434
      E1=EE(K-N7+1)/1.6E-19                                             435
      E2=EE(K)/1.6E-19                                                  436
      AN7=N7                                                            437
      DO 435 N=1,N9                                                     438
```

```
      SUM1(N)=0.0                                                       439
      DO 435 I=1,N9                                                     440
      SUM1(N)=SUM1(N)+AMAT(I,N)                                         441
  435 CONTINUE                                                          442
       AN8=N7*TOT                                                       443
      PRINT 433,E1,E2,((AMAT(N1,N2),N2=1,N9),N1=1,N9),(SUM1(N),         444
     +N=1,N9),                                                          444
     1AN8                                                               445
  433  FORMAT(1X,*EFFICIENCY MATRIX FOR EXIT APERTURE OF ESA FLA         446
     +TES FROM                                                          446
     1ENERGY=*,1PE10.2,*EV*,*TO*,1PE10.2,*EV*/0P5F10.2/5F10.2/5         447
     +F10.2/5F1                                                         447
     Z0.2/5F10.2/1X,*TOTALS*/5F10.2,*PARTICLES TRACED=*,1PE10.2         448
     +//)                                                               448
      DO 438 I=1,10                                                     449
      DO 438 N=1,10                                                     450
      AMAT(I,N)=0.0                                                     451
  438   CONTINUE                                                        452
  440 CONTINUE                                                          453
       IF(IK.GE.2) K13=K13+1                                            454
       IF(IK.GE.2.AND.K13.LT.N11) GO TO 37                              455
       IF(IK.GE.2) GO TO 610                                            456
      GO TO 600                                                         457
  600 CONTINUE                                                          458
      CALL SIMP                                                         459
  610  CONTINUE                                                         460
       IF(IK.EQ.3) PRINT 613                                            461
   613 FORMAT(1X,*ANGULAR SCAN IN Z-PLANE*)                             462
       IF(IK.EQ.2) PRINT 609                                            463
   609 FORMAT(1X,*ANGULAR SCAN IN R-PLANE*)                             464
       IF(IK.GE.2) PRINT 611, (THM(K),AX(K),K=1,N11)                    465
  611  FORMAT((1P12E10.2))                                              466
       IF(IK.GE.2) GO TO 24                                             467
  630 CONTINUE                                                          468
      THR=THW*AA                                                        469
      IF(IJ.EQ.2) PRINT 740,THR                                         470
  740 FORMAT(1X,*ELECTRONS FROM GUN INCIDENT ON APERTURE AT AN          471
     +ANGLE OF                                                          471
  740 A    *,F7.2,*   DEG*)                                             472
      PRINT 51                                                          473
   51 FORMAT(1X,*EFFICIENCY   OF CHANNELTRON TO SEE PARTICLES*)         474
      PRINT 55,(EG(K),EFF(K),K=1,N5)                                    475
   55 FORMAT(1X,*ENERGY-EFFICIENCY*/(1P10E12.2))                        476
      G1=G*GINT                                                         477
      PRINT 63,GINT,G1                                                  478
   63 FORMAT(1X,*INTEGRAL OF CHANN. EFFIC.=*,1PE10.4,* EV*,3X,*         479
     +NORMALIZA                                                         479
     1TION FACTOR FOR ENTIRE ESA=*,1PE10.4,* CMSQ-SR-EV*)               480
      HH=G1/EA                                                          481
       PRINT 67,HH                                                      482
   67 FORMAT(1X,*DEFOREST H-FACTOR=*,1PE10.4,* CMSQ-SR*)                483
      GO TO 1                                                           484
      END                                                               485
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      SUBROUTINE SIMP                                                   466
      DIMENSION EFF(200),EG(200)                                        487
      COMMON/F1/EFF,EG,DELE,N5,GINT                                     488
C SIMPSON RULE                                                          489
      DEL=DELE/1.6E-19                                                  490
      N6=N5-3                                                           491
      N7=N5-2                                                           492
      SUM1=0.0                                                          493
      DO 10 J=3,N6,2                                                    494
      SUM1=SUM1+EFF(J)                                                  495
 10   CONTINUE                                                          496
      SUM2=0.0                                                          497
      DO 20 J=2,N7,2                                                    498
       SUM2=SUM2+EFF(J)                                                 499
 20    CONTINUE                                                         500
      GINT=(EFF(1)+EFF(N5-1)+4.0*SUM2+2.0*SUM1)*DEL/3.0                 501
      RETURN                                                            502
      END                                                               503
```

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