

1 URITY CLASSIFICATION OF THIS PAGE (When Data Entered) READ INSTRUCTIONS BEFORE COMPLETING FORM **REPORT DOCUMENTATION PAGE** REPORT NUMBER 2. GOVT ACCESSIO 3. RECIPIENT'S CATALOG NUMBER FOSR TR-77-1216 E (and Subtitie) TYPE OF REPORT & PERIOD COVERED NONLINEAR PROGRAMMING GLOBAL OPTIMIZATION TECHNIQUES Interim 6. PERFORMING ORG. REPORT NUMBER 7. AUTHOR(S) 8. CONTRACT OR GRANT NUMBER(S) AF05R-73-2504 James E. /Falk, Anthony V. /Fiacco 15 Garth P. /McCormick A0458 9. PERFORMING ORGANIZATION NAME AND ADDRESS 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS The George Washington University Institute for Management Science & Engineering Washington, DC 20052 61102F 11. CONTROLLING OFFICE NAME AND ADDRESS 12. REPORT DAT Air Force Office of Scientific Research/NM 1977 Bolling AFB, DC 20332 13. NUMBER OF PAGES A 14. MONITORING AGENCY NAME & ADDRESS(II different from Controlling Office) 15. SECURITY CLASS. (of this report) 12 UNCLASSIFIED 15a. DECLASSIFICATION DOWNGRADING SCHEDULE 16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited. 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, il different from Report, 18. SUPPLEMENTARY NOTES 0 19. KEY WORDS (Continue on reverse side if necessary and identify by block number) BSTRACT (Continue on reverse side if necessary and identify by block number) Several areas of research in nonconvex programming are being pursued, both at the research and directly applied levels. Works continues on extending the verification procedures for factorable optimization to equality and inequality constrained problems. Algorithms for finding second order optima for unconstrained problems are investigated. Problems have been constructed which have multiple equilibrium points. Work continues on solving varieties of max-min problems which arise in weapons exchange models DD 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE UNCLASSIFIED 406 744 SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

AFOSR-TR- 77-1216

THE GEORGE WASHINGTON UNIVERSITY Institute for Management Science and Engineering

INTERIM SCIENTIFIC REPORT FOR GRANT AFOSR-73-2504

Covering the Time Period 1 May 1976 - 30 April 1977

TITLE: "Nonlinear Programming Global Optimization Techniques"

PRINCIPAL INVESTIGATORS: James E. Falk

Anthony V. Fiacco Garth P. McCormick

ABSTRACT

Several areas of research in nonconvex programming are being pursued, both at the research and directly applied levels. Work continues on extending the verification procedures for factorable optimization to equality and inequality constrained problems. Algorithms for finding second order optima for unconstrained problems are investigated. Problems have been constructed which have multiple equilibrium points. Work continues on solving varieties of max-min problems which arise in weapons exchange models. ACC SS V or



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DETAILED WORK OF PRINCIPAL INVESTIGATORS

1. In the paper (Reference [6], Interim Report 1 May 75 - 30 April 76) "Bounding Global Minima", Mancini and McCormick established the theoretical framework for a global verification scheme. They showed conditions under which a local (strict) unconstrained minimizer in a hyperrectangle is a global unconstrained minimizer there. Further byproducts of that research were formulas which could provide upper and lower bounds on the global minimizer. In this time period, research has been directed to apply these theoretical results. This research led to considerations of the numerical techniques embodied in the field known as "Interval Arithmetic." The particular approach taken was to do an interval extension version of Newton's method. The research has culminated in a paper [1] which explores the numerical aspects of bounding global minima. The algorithms presented therein are illustrated with the hypothetical design of a chemical plant.

Further research in this area is concentrating on extending meoretical and computational results to equality and inequality constrained optimization problems. This turns out to be much more difficult. The equality constrained problem has been completed and the basic idea which allows for handling the inequality constrained case has been found.

2. An important part of an overall algorithm for obtaining global solutions is the obtaining of a "good" local solution. Both the convex underestimating scheme and the global verification procedure benefit from having an objective function value which is attainable. It is important to have nonlinear programming algorithms which obtain a point which satisfies not only the first order Kuhn-Tucker conditions, but the second order optimality conditions. Most algorithms do not do this. For the unconstrained case there has been much work recently in developing second order algorithms to guarantee convergence to a second order point. In [2] is a survey of techniques for doing this and inclusion of some new ideas. Current research effort is being expended in generalizing these results to the equality and inequality cases.

3. A paper dealing with infinitely constrained optimization problems was published [4].

4. In problems dealing with economic equilibria, one is interested in locating a point where demand prices equal supply prices for several commodities, when the prices of these commodities are interdependent. It is often (whether explicitly or implicitly) assumed that the equilibrium point is unique.

We have constructed a three-commodity example which has three distinct equilibrium points. Each of these points is the (global) maximizer of a distinct objective function: consumer surplus, producer surplus, or social surplus.

Nonconvex programming methods developed at The George Washington University under AFOSR sponsorship were used to solve separable economic equilibria problems.

The results of this work are available as IDA Report P-1208 [3] and have been submitted for open literature publication in <u>Management Science</u> (with J. Bracken).

5. Work has progressed on max-min and min-max strategies in multi-stage games and ATACM. Two types of sequential games are considered: adaptive games and non-adaptive games. The max-min and min-max values of the non-adaptive

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games are shown to bound the same values for the adaptive game. Therefore, a saddle point in a non-adaptive game will insure a saddle point in an adaptive game.

An example is provided to show this in the report [5]. The example shows that the adaptive game can have a saddle point without the non-adaptive game possessing one.

These results are part of a recent study of ATACM: ACDA Tactical vs. Air Campaign Model by Ketron, Inc.

6. The paper "Strategic Weapons Exchange Allocation Model" is at the printers and will shortly appear in <u>Operations Research</u>. This work describes a two-strike weapons exchange scenario in terms of a non-convex max-min problem, whose solution is obtained by global optimization techniques developed under this study.

Results of this work lead to the operational package IDASNEM developed at IDA by Jeff Grotte, This work was done jointly with J. Bracken and F. Miercourt.

7. Work was initiated into the area of exact penalty functions. For the last several years investigators have felt that solution of nonlinear nonconvex programming problems could be done by the creation of a penalty function which was exact in that local/global minimizers of the unconstrained problem were local/global unconstrained minimizers of the constrained problem. Work on constructing a natural exact penalty function is in progress.

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