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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The following theories were systematically developed: (1) Harmonic classification theory of Riemannian manifolds, (2) Quasiharmonic classification theory of Riemannian manifolds, (3) Theory of bounded biharmonic functions on Riemannian manifolds, (4) Dirichlet finite biharmonic functions, (5) Bounded Dirichlet finite biharmonic functions, (6) Riesz representation of biharmonic functions, (7) Green's functions of simply supported bodies, (8) Green's functions of clamped bodies. These theories were published in the 83 papers listed in Sections II-IV and the research monograph described in Section V.

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Name of institution: University of California, Los Angeles.

Author of report: L. Sario.

This report is divided into six sections:

I. Main achievements.

II. Papers published.

III. Papers accepted.

IV. Papers submitted.

V. Research monograph published.

VI. Ph.D. training.

I. Main achievements

A. Research. The following theories were systematically developed:

(1) Harmonic classification theory of Riemannian manifolds.

(2) Quasiharmonic classification theory of Riemannian manifolds.

(3) Theory of bounded biharmonic functions on Riemannian manifolds.

(4) Dirichlet finite biharmonic functions.

(5) Bounded Dirichlet finite biharmonic functions.

(6) Riesz representation of biharmonic functions.

(7) Green's functions of simply supported bodies.

(8) Green's functions of clamped bodies.

These theories were published in the 83 papers listed in Sections II - IV, and the research monograph described in Section V. (The totals of our publications under all our Army grants is 153 papers and 6 research monographs.)

B. Training. The doctoral dissertations of 7 students, listed in Section VI, were directed. (The total number of [illegible]ral dissertations directed under all our Army grants in 35.)

II. Papers published

(1) "A maximum principle for bounded harmonic functions on Riemannian spaces" by Y. K. Kwon and L. Sario, Canad. J. Math. 22 (1970), 847-854.

(2) "A maximum principle for Dirichlet-finite harmonic functions on Riemannian spaces" by Y. K. Kwon and L. Sario, Canad. J. Math. 22 (1970), 855-862.

(3) "A remark on Royden's compactification of Riemannian spaces" by I. Lin, Kōdai Math. Sem. Rep. 22 (1970), 338-340.

(4) "A new operator for elliptic equations, and the P-compactification for $\Delta u = Pu$" by M. Nakai and L. Sario, Math. Ann. 189 (1970), 242-256.

(5) "Continuity of mappings of vector lattices with norms and seminorms" by M. Nakai and L. Sario, Kōdai Math. Sem. Rep. 22 (1970), 473-479.

(6) "Green's lines on Royden's compactification of a Riemannian space" by I. Lin and L. Sario, J. Indian Math. Soc. 34 (1970), 159-174.

(7) "Infinite boundary value problems for second order elliptic partial differential equations" by M. Nakai, J. Fac. Sci. Univ. Tokyo 17 (1970), 101-121.

(8) "Bounded energy-finite solutions of $\Delta u = Pu$ on a Riemannian manifold" by Y. K. Kwon, L. Sario, and J. Schiff, Nagoya Math. J. 42 (1971), 95-108.

(9) "The P-harmonic boundary and energy-finite solutions of $\Delta u = Pu$" by Y. K. Kwon, L. Sario, and J. Schiff, Nagoya Math. J. 42 (1971), 31-41.

(10) "The P-singular point of the P-compactification for $\Delta u = Pu$" by Y. K. Kwon and L. Sario, Bull. Amer. Math. Soc. 77 (1971), 128-133.

(11) "Biharmonic classification of Riemannian manifolds" by M. Nakai and L. Sario, Bull. Amer. Math. Soc. 77 (1971), 432-436.

(12) "Dirichlet finite solutions of $\Delta u = Pu$, and the classification of Riemann surfaces" by M. Nakai, Bull. Amer. Math. Soc. 77 (1971), 381-385.

(13) "A remark on classification of Riemannian manifolds with respect to $\Delta u = Pu$" by M. Glasner, R. Katz, and M. Nakai, Bull. Amer. Math. Soc. 77 (1971), 425-428.

(14) "A remark on classification of Riemann surfaces with respect to $\Delta u = Pu$" by M. Nakai, Bull. Amer. Math. Soc. 77 (1971), 527-530.

(15) "Behavior of Green lines at the Kuramochi boundary of a Riemann surface" by M. Nakai and L. Sario, Pacific J. Math. 36 (1971), 447-455.

(16) "Harmonic functions on a subregion of a Riemannian manifold" by Y. K. Kwon and L. Sario, J. Indian Math. Soc. 35 (1971), 135-161.

(17) "Biharmonic projection and decomposition" by L. Sario, C. Wang, and M. Range, Ann. Acad. Sci. Fenn. 494 (1971), 432-436.

(18) "Dirichlet finite biharmonic functions with Dirichlet finite Lapacians" by M. Nakai and L. Sario, Math. Z. 122 (1971), 203-216.

(19) "Extremal solutions of $\Delta u = Pu$" by K. Kawai and L. Sario, Kōdai Math. Sem. Rep. 23 (1971), 276-289.

(20) "Extremal properties of quasiharmonic forms and functions" by K. Kawai and L. Sario, Kōdai Math. Sem. Rep. 23 (1971), 267-275.

(21) "$\tilde{HD}$-functions on Royden's compactification of a Riemannian space" by I. Lin and L. Sario, J. Indian Math. Soc. 35 (1971), 135-161.

(22) "Royden's algebra on Riemannian spaces" by J. Chang and L. Sario, Math. Scand. 28 (1971), 139-158.

(23) "Behavior of biharmonic functions on Wiener's and Royden's compactifications" by Y. K. Kwon, L. Sario, and B. Walsh, Ann. Inst. Fourier (Grenoble) 21 (1971), 217-226.

(24) "A property of biharmonic functions with Dirichlet finite Laplacians" by M. Nakai and L. Sario, Math. Scand. 29 (1971), 307-316.

(25) "The equation $\Delta u = Pu$ on E^m with almost rotation free $P \geq 0$" by M. Nakai, Tôhoku Math. J. 23 (1971), 413-431.

(26) "Royden compactification of harmonic spaces" by Y. K. Kwon, Kyungpook Math. J. 11 (1971), 1-8.

(27) "Examples in the classification theory of Riemannian manifolds and the equation $\Delta u = Pu$" by M. Glasner, R. Katz, and M. Nakai, Math. Z. 121 (1971), 233-238.

(28) "Dirichlet finite solutions of $\Delta u = Pu$ on open Riemann surfaces" by M. Nakai, Nagoya Math. J. 23 (1971), 385-397.

(29) "Relations between Wiener's and Royden's P-compactifications of a Riemannian manifold" by C. Wang, Kyungpook Math. J. 11 (1971), 189-194.

(30) "Radon-Nikodym densities and Jacobians" by M. Nakai, Pacific J. Math. 40 (1972), 375-395.

(31) "Royden algebras and quasi-isometrics of Riemannian manifolds" by M. Nakai, Pacific J. Math. 40 (1972), 397-414.

(32) "Quasiharmonic classification of Riemannian manifolds" by M. Nakai and L. Sario, Proc. Amer. Math. Soc. 31 (1972), 165-169.

(33) "The equation $\Delta u = Pu$ on the unit disk with almost rotation free $P \geq 0$" by M. Nakai, J. Diff. Eqs. 11 (1972), 307-320.

(34) "Biharmonic functions on Riemannian manifolds" by M. Nakai and L. Sario, Tbilisi volume honoring 80th birthday of Muskhelishvili, Continuum Mechanics and Related Problems of Analysis, Nauka, 1972, 329-355.

(35) "The class of (p,q)-biharmonic functions" by C. Wang and L. Sario, Pacific J. Math. 41 (1972), 799-808.

(36) "Dirichlet finite solutions of $\Delta u = Pu$" by I. Singer, Proc. Amer. Math. Soc. 32 (1972), 464-468.

(37) "Existence of bounded Dirichlet finite biharmonic functions" by M. Nakai and L. Sario, Ann. Acad. Sci. Fenn. 505 (1972), 1-12.

(38) "Polyharmonic classification of Riemannian manifolds" by C. Wang and L. Sario, J. Math. Kyoto Univ. 12 (1972), 129-140.

(39) "Parabolicity and existence of bounded biharmonic functions" by L. Sario and C. Wang, Comment. Math. Helv. 47 (1972), 341-347.

(40) "Positiveness of the reproducing kernel in the space PD(R)" by I. Singer, Nagoya Math. J. 48 (1972), 67-72.

(41) "Gererators of the space of bounded biharmonic functions" by L. Sario and C. Wang, Math. Z. 127 (1972), 273-280.

(42) "Dirichlet finite biharmonic functions on the plane with distorted metrics" by M. Nakai, Nagoya Math. J. 51 (1973), 131-135.

(43) "Existence of Dirichlet finite biharmonic functions" by M. Nakai and L. Sario, Ann. Acad. Sci. Fenn. A. I. 532 (1973), 1-33.

(44) "Positive harmonic functions and biharmonic degeneracy" by L. Sario and C. Wang, Bull. Amer. Math. Soc. 79 (1973), 182-187.

(45) "Radial quasiharmonic functions" by L. Sario and C. Wang, Pacific J. Math. 46 (1973), 515-522.

(46) "Bounded polyharmonic functions and the dimension of the manifold" by N. Mirsky, L. Sario, and C. Wang, J. Math. Kyoto Univ. 13 (1973), 529-535.

(47) "Existence of bounded biharmonic functions" by M. Nakai and L. Sario, J. Reine Angew. Math. 259 (1973), 147-156.

(48) "Harmonic and biharmonic degeneracy" by L. Sario and C. Wang, Kōdai Math. Sem. Rep. 25 (1973), 393-396.

(49) "Existence of Dirichlet finite biharmonic functions on the Poincaré 3-ball" by L. Sario and C. Wang, Pacific J. Math. 48 (1973), 267-274.

(50) "Quasiharmonic functions on the Poincaré N-ball" by L. Sario and C. Wang, Rend. Mat. (4) 6 (1973), 1-14.

(51) "Riemannian manifolds with bounded Dirichlet finite polyharmonic functions" by L. Chung, L. Sario, and C. Wang, Ann. Scuola Norm. Sup. Pisa 27 (1973), 1-6.

(52) "Riemannian manifolds of dimension $N \geq 4$ without bounded biharmonic function" by L. Sario and C. Wang, J. London Math. Soc. (2) 7 (1974), 635-644.

(53) "N-manifolds carrying bounded but no Dirichlet finite harmonic functions" by D. Hada, L. Sario, and C. Wang, Nagoya Math. J. 54 (1974), 1-6.

(54) "Quasiharmonic degeneracy of Riemannian N-manifolds" by L. Sario, Kōdai Math. Sem. Rep. 26 (1974), 53-57.

(55) "Parabolicity and existence of Dirichlet finite biharmonic functions" by L. Sario and C. Wang, J. London Math. Soc. 8 (1974), 145-148.

(56) "Negative quasiharmonic functions" by L. Sario and C. Wang Tôkohu Math. J. 26 (1974), 85-93.

(57) "Completeness and existence of bounded biharmonic functions on a Riemannian manifold" by L. Sario, Ann. Inst. Fourier (Grenoble) 24 (1974), 311-317.

(58) "Parabolicity and existence of bounded or Dirichlet finite polyharmonic functions" by N. Mirsky, L. Sario, and C. Wang, Rend. Ist. Mat. Univ. Trieste 6 (1974), 1-9.

(59) "Biharmonic measure" by L. Sario, Ann. Acad. Sci. Fenn. 587 (1974), 3-18.

(60) "Counterexamples in the biharmonic classifications of Riemannian manifolds" by L. Sario and C. Wang, Pacific J. Math. 50 (1974), 159-162.

(61) "Bounded biharmonic functions on the Poincaré N-ball" by D. Hada, L. Sario and C. Wang, Kodai Math. Sem. Rep. 26 (1975), 327-342.

(62) "Harmonic and quasiharmonic degeneracy of Riemannian manifolds" by L. Chung and L. Sario, Tôhoku Math. J. 27 (1975), 487-496.

(63) "Quasiharmonic L^p-functions on Riemannian manifolds" by L. Chung, L. Sario, and C. Wang, Ann. Scuola Norm. Sup. Pisa 2 (1975), 460-478.

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(65) "Biharmonic Green's functions and harmonic degeneracy" by L. Sario, J. Math. Kyoto Univ. 15 (1975) 351-362.

(66) "Dirichlet finite biharmonic functions on the Poincaré N-ball" by D. Hada, L. Sario and C. Wang, J. Reine Angew. Math. 272 (1975), 92-101.

(67) "Harmonic L^p functions on Riemannian manifolds" by L. Sario and C. Wang, Kodai Math. Sem. Rep. 26 (1975), 204-209.

(68) "A criterion for the existence of biharmonic Green's functions" by L. Sario, J. Austral. Math. Soc. 21 (1976), 155-165.

(69) "A relation between biharmonic Green's functions of simply supported and clamped bodies" by J. Ralston and L. Sario, Nagoya Math. J. 61 (1976), 59-71.

(70) "Parabolic Riemannian planes carrying biharmonic Green's functions of the clamped plate" by M. Nakai and L. Sario, J. Analyse Math. 30 (1976), 372-389.

(71) "Harmonic functions on a Riemannian ball" by M. Nakai and L. Sario, Math. Proc. Cambridge Philos. Soc. 80 (1976), 277-282.

(72) "Manifolds with strong harmonic boundaries but without Green's functions of clamped bodies" by M. Nakai and L. Sario, Ann. Scuola Norm. Sup. Pisa 3 (1976), 665-670.

III. Papers accepted

(73) "A nonexistence test for biharmonic Green's functions of clamped bodies" by M. Nakai and L. Sario, Math. Scand.

(74) "Existence of negative quasiharmonic functions" by M. Nakai and L. Sario, Jubilee volume dedicated to the 75th birthday of Academician I. Vekua.

(75) "On Hadamard's problem for higher dimensions" by M. Nakai and L. Sario, J. Reine Angew. Math.

(76) "A strict inclusion related to biharmonic Green's functions of clamped and simply supported bodies" by M. Nakai and L. Sario, Ann. Acad. Sci. Fenn.

(77) "Existence relations between harmonic and biharmonic Green's functions" by M. Nakai and L. Sario, Rend. Ist. Mat. Univ. Trieste.

(78) "Existence of biharmonic Green's functions" by M. Nakai and L. Sario, Proc. London Math. Soc.

(79) "Duffin's function and Hadamard's conjecture" by M. Nakai and L. Sario, Pacific J. Math.

(80) "Biharmonic and quasiharmonic degeneracy" by L. Chung, L. Sario, and C. Wang, Kodai Math. Sem. Rep.

(81) "Harmonic and polyharmonic degeneracy" by L. Chung, L. Sario, and C. Wang, Math. Scand.

IV. Papers submitted

(82) "Green's functions of the clamped punctured disk" by M. Nakai and L. Sario, J. Austral. Math. Soc.

(83) "One point clamping and supporting" by M. Nakai and L. Sario, Rend. Mat.

V. Research monograph published

"CLASSIFICATION THEORY OF RIEMANNIAN MANIFOLDS, Harmonic, quasiharmonic, and biharmonic functions" by L. Sario, M. Nakai, C. Wang, and L. Chung, Lecture Notes in Mathematics No. 605, Springer-Verlag, August, 1977, 518 pages.

VI. Ph.D. training

The doctoral dissertations of the following students were directed:

1. D. Martinez, June 1971.
2. K. Kawai, June 1971.
3. J. Rader, June 1971.
4. I. Singer, June 1971.
5. N. Mirsky, November 1972.
6. D. Hada, June 1973.
7. L. Chung, August 1974.