AD-A045 387 CIVIL AND ENVIRONMENTAL ENGINEERING DEVELOPMENT OFFIC--ETC F/G 13/2

A THEORY ON WATER FILTRATION. PART I. BACKGROUND.(U)

JAN 77 S P SHELTON

UNCLASSIFIED CEEDO-TR-77-1 NL

1 OF 1

AD
A045387

END

DATE

FILMED

11 -77

DDC

CEEDO-TR-77-1

6

B.S.

A THEORY ON WATER FILTRATION
PART I BACKGROUND

DIRECTORATE OF ENVIRONICS

JANUARY 1977

FINAL REPORT FOR PERIOD
JUNE 1973-DECEMBER 1976

CEEDO

Approved for public release; distribution unlimited

CIVIL AND ENVIRONMENTAL ENGINEERING DEVELOPMENT OFFICE

(AIR FORCE SYSTEMS COMMAND)
TYNDALL AIR FORCE BASE
FLORIDA 32403

393250

UNCLASSIFIED (9) Final rept. 1 Jun 73-31 Dec 76,

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (14) CEEDO-TR-77-1	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) A THEORY ON WATER FILTRATION - PART I, BACKGROUND,		5. TYPE OF REPORT & PERIOD COVERED Final - 1 June 1973 - 31 December 1976
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Stephen P./Shelton, Capt, USAF, BSC		8. CONTRACT OR GRANT NUMBER(s) 63723F
9. PERFORMING ORGANIZATION NAME AND ADDRESS Det 1 (CEEDO) HQ ADTC Water and Solid Waste Resources Division Tyndall Air Force Base, FL 32403		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 21036W45
11. CONTROLLING OFFICE NAME AND ADDRESS Det 1 (CEEDO) HQ ADTC Tyndall Air Force Base, FL 32403		12. REPORT DATE (11) Jan 1977
		13. NUMBER OF PAGES (12) 42p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (16) 2103 (17) 6W		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Available in DDC.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Environmental engineering, Filtration, Mass transfer, Diffusion, Pressure drop

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The specific objective of this investigation was to apply existing theoretical concepts used in aerosol mechanics to various water filtration systems. Once developed, these equations were used to describe the water filtration processes of concern as a function of the characteristics of the fluid, suspended particles, and filter media. It was concluded that the proposed model had the potential to predict the relationship between flow, pressure, over

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

393250

20. Abstract (continued).

time, and efficiency for the data evaluated. In addition, the model was found to have advantages over current water filtration models since, unlike current models, it considers raw water quality and predicts filtration efficiency.

PREFACE

This report summarizes work done between 1 July 1973 and 31 December 1976. Stephen P. Shelton, Capt, USAF, BSC, was the project engineer; however, a portion of the work was performed while Capt Shelton was a PhD candidate at the University of Tennessee, Knoxville, as a part of a university-sponsored research and development program.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

STEPHEN P. SHELTON, Capt, USAF, BSC
Water Quality Research Engineer

PETER A. CROWLEY, Maj, USAF, BSC
Director of Environics

DONALD G. SILVA, Lt Col, USAF, BSC
Commander

(The reverse of this page is blank)

LIST OF ABBREVIATIONS, ACRONYMS AND SYMBOLS

C_1 = influent suspended particle concentration

C_2 = effluent suspended particle concentration

C_I = influent suspended particle concentration

$\mathcal{D}$ = diffusivity

K_1 = clean filter drag constant

K_2 = filter drag constant with filter cake

K_D = fluid/media interaction constant

L = total filter thickness

L_c = filter cake thickness

L_e = streamline distance

L_f = fiber bed thickness

N_{pe} = Peclet number

N_{sc} = Schmidt number

ΔP = pressure drop

ΔP_c = pressure drop across filter cake

ΔP_f = pressure drop across filter media

ΔP_t = total pressure drop across a filter

R_c = interception parameter

R_e = Reynolds number

S_f = Solidarity factor for filter media

S_f = Solidarity factor for filter cake

S_p = particle surface to volume ratio

T = temperature

LIST OF ABBREVIATIONS, ACRONYMS
AND SYMBOLS (Continued)

V_A = van der Waals forces of attraction

V_R = Electrical potential between double layers

W_f = filter weight per unit area

$\bar{d}$ = mean particle diameter

$\bar{d}_A$ = arithmetic mean particle diameter

$\bar{d}_c$ = effective particle diameter in a filter cake

$\bar{d}_f$ = effective fiber diameter as a collector

$\bar{d}_{mm}$ = mass mean particle diameter

$\bar{d}_{sm}$ = surface mean particle diameter

$\bar{d}_{43}$ = 43 percent particle size in a log-normal distribution

d_f = discrete fiber diameter

d_o = minimum collection efficiency particle diameter

d_p = particle diameter

$f(x_s)$ = function of separation distance X_s

g = acceleration due to gravity

k = Boltzmann's constant

k_c = Carman-Kozeny coefficient

k_o = Carman shape factor

t = time

u = approach velocity

X_s = separation distance

π = the natural function

α_c = solids fraction of a filter cake

α_f = solids fraction of a fiber bed

LIST OF ABBREVIATIONS, ACRONYMS AND SYMBOLS (Concluded)

ϕ = collision efficiency - theoretical

ϕ' = collision efficiency - empirical

η_c = filter cake efficiency

η_{CL} = initial cake thickness efficiency

η_f = filter media efficiency

η_{ICD} = Friedlander single fiber efficiency

$\eta_{ICD\phi}$ = modified form of η_{ICD} that considers ϕ

$\eta'_{ICD\phi}$ = modified form of $\eta_{ICD\phi}$ for the filter cake

η_t = filter efficiency

μ_g = dynamic viscosity

ρ_{BC} = bulk density of the filter cake

ρ_f = discrete filter density

ρ_{fB} = bulk density of a fiber filter

ρ_g = fluid density

ρ_p = discrete suspended particle density

σ_g = log-normal standard deviation

ν_g = kinematic viscosity

SECTION I

INTRODUCTION

The primary goal of this investigation was to derive a rational water filtration design concept by generalizing filtration theories employed in aerosol mechanics, and incorporating them with current water filtration theories. This rational design concept considers the fluid, media, and particles suspended in the fluid, predicting the relationship between flow, pressure drop, time, and efficiency.

Modification of aerosol mechanics theory to provide application for water filtration entails an evaluation of the theoretical concepts upon which air filtration processes are based. These concepts must be merged and modified before they can be adapted to describe water-oriented systems.

SECTION II

SMALL PARTICLE COLLECTION

The potential for collection of small particles is related to the size distribution of the particles suspended in the fluid, as well as physical/chemical properties of those particles. A log-normal distribution of particle sizes is assumed for filtration considerations contained herein. The validity of this assumption has been substantiated by previous investigations (References 1-10).

Chen (Reference 11) has suggested that an analogy between average fiber diameter in a fiber bed and average particle diameter in the filter cake can be made for the one-dimensional case. In this system the flow streamlines about a fiber or a sphere are the same; thus, inferences can be made for sphere-on-sphere collection based upon sphere-cylinder collection theory. Furthermore, Chen suggests that the effective diameter of a collector in a log-normal distribution of particle or cylinder collectors can be approximated by the ratio of the surface mean diameter squared to the arithmetic mean diameter ($\bar{d}_{sm}^{2}/\bar{d}_{A}$). Chen, Fuchs (Reference 10), and others (References 2-9), suggest that, in the Stokes' law range, the arithmetic mean particle size can effectively be used to predict the behavior of that portion of particles in the distribution influenced by inertial forces and that the geometric mean particle size can be used to effectively predict the behavior of particles in the sub-Stokes' law range, where diffusion forces have predominant effects.

The small particle forces that normally exert significant influences upon collection efficiency are interception, inertial impaction, diffusion, surface properties, and electrical properties (References 10-15). The first three of these forces greatly influence the determination of collision efficiency; the latter two properties are most influential from the standpoint of collision success. For this reason, one would expect that a combination of these systems would provide the basis for small particle collection theory.

It is essential to consider suspended particle removal in the filter media as involving at least two separate and distinct steps: a transport step and an attachment step

(References 10, 11). Particle transport is a physical-hydraulic process; thus, it is affected by those parameters which govern mass transfer (References 8, 9). Particle attachment is normally a surface property of the particles involved; thus, it is influenced by both chemical and physical parameters (References 15, 16). The two primary mechanisms involved in attatchment success are particle electrical and inelasticity (stickiness).

Most theoretical investigations into water filtration processes have considered only the physical transport (Reference 17-20) parameters, such as filter media characteristics and flow rate, to have significant influence upon filtration operations. The results of these investigations, as discussed by O'Melia and Stumm (Reference 21), with respect to filter performance, disagree in terms of the relationship between flow, time, pressure drop, and efficiency. Their ability to predict design parameters for unique systems is subject to question.

It is proposed herein that disagreement among current water filtration theories lies in the empirical base upon which they are founded. This permits contradiction because it does not allow theoretical prediction of the relationships between flow, time, efficiency, and pressure. Thus, when these concepts are transposed to new cases, different particle size distributions or particle surface characteristics may cause wide variations between the true relationships and the prediction made by the different models. It seems plausible that the apparent inconsistencies among water filtration models may originate for two reasons: (1) two or more transport mechanisms may be simultaneously effective but not considered and (2) significant surface properties either unknown or assumed to be insignificant are not considered and thus are not controlled.

In discussion of the first item, the transport mechanisms, it is felt that the concepts from aerosol mechanics may be the best available. These concepts are those of Stokes' law and mass transfer used to predict particle collection efficiency in air filtration systems. It is felt that these concepts can provide insight into the transport mechanisms of water filtration. Friedlander (References 8, 9) has successfully correlated data on aerosol filtration by fibrous filters operated at low flows, with the following efficiency equation:

$$\eta_{ICD} = 6N_{SC}^{-2/3}R_e^{-1/2} + 3R_c^{\,2}R_e^{\,1/2} \qquad (1)$$

where η_{ICD} is Friedlander's single element collection efficiency, N_{SC} is the Schmidt number, R_e is the Reynolds number, and R_C is the interception parameter (the ratio of particle-to-collector diameter). The use of collection efficiency here indicates that all collisions are successful. This, however, may not always be the case and a more complete discussion will ensue in the next section. For this reason, η_{ICD} will be redefined as the efficiency of collision (transport from the fluid to the collection surface). The Schmidt number, used in Equation 1, is a measure of the ratio of transport by convection forces to the transport caused by molecular diffusion. The term is equal to ν_g/D where ν_g is the kinematic viscosity of the fluid and D is particle diffusivity. The Reynolds number is equal to $\rho_g d_f v/\mu_g$ where ρ_g is fluid density, d_f is the fiber diameter, v is fluid velocity, and μ_g is the dynamic viscosity of the fluid. The product of the Reynolds number and the Schmidt number is defined as the Peclet number. This parameter expresses both influences simultaneously.

In a fiber bed of depth L_f, the single fiber collision efficiency, η_{ICD} can also be described:

$$\eta_{ICD} = \frac{\pi d_f}{4\alpha_f L_f} \ln \frac{C_1}{C_2} \qquad (2)$$

where η_{ICD} is the single fiber collision efficiency, d_f is the fiber diameter, L_f is the fiber bed thickness or bed depth, C_1 is the inlet particle concentration, C_2 is the effluent particle concentration, and α_f is the solids fraction of the fiber bed (the ratio of the fiber bed bulk density, ρ_{fB}, to the discrete fiber density ρ_p). This equation relates small particle collection systems to overall filtration collection efficiency since C_1 and C_2 are, respectively, the gross filter influent and effluent particle loadings.

It is necessary to evaluate the assumptions employed by Friedlander (References 8, 9) in Equation 1. The equation is composed of a rational base with empirical coefficients. Broad application to experimental data has confirmed the merit of the expression and the coefficient. The most important assumptions made by the Friedlander equation are:

1. The primary transport mechanism described is diffusion. Interception is introduced as a boundary condition on the differential equation. Other transport mechanisms, such as sedimentation and inertial impaction, are not directly considered; however, these mechanisms, because of their nature, fall within the boundary conditions described for interception.

2. N_{pe}, the Peclet number, is much greater than one. This assumes that the transport by convection forces is large when compared to diffusion in the bulk flow. Molecular diffusion is considered to be predominant in the boundary layer near the surface of the filter media. Thus, diffusion of small particles near the media surface controls the overall rate of transfer.

3. R_c is less than one. Lamb's (Reference 22) solution for the velocity distribution around a sphere in one dimension is assumed. This allows comparison of both spherical and cylindrical collectors with the same set of equations when the effective collector diameter is used.

Equation 1 expresses two collision components, one from diffusion and one from interception. The

$6N_{SC}^{-2/3}R_e^{-1/2}$ term represents the contact efficiency

for small particles (R_e approaches zero) where the molecular diffusion controls. The subsequent term, $3R_c^{2}R_e^{1/2}$, controls the boundary condition for inertial contact. The minimum contact particle is described by the first derivative of Equation 1. This derivation requires a substitution for diffusivity:

$$\mathcal{D} = \frac{KT}{3\pi\mu_g d_p} \qquad (3)$$

where $\mathcal{D}$ is diffusivity, K is Boltzmann's constant, T is temperature in degrees Kelvin, μ_g is the dynamic viscosity of the fluid, and d_p is the particle diameter. If Equation 1 is differentiated with respect to particle size (particle size is contained in all three terms), and set equal to zero, the minimum contact efficiency particle may be predicted:

$$d_o = \frac{0.855 d_f^{3/8}}{u^{3/8} \mu_g^{1/8} \rho_g^{1/8}} \left[\frac{kT}{3\pi}\right]^{1/4} \qquad (4)$$

where d_o is the particle that is least likely to be collected, d_f is the fiber (collector) diameter, u is the approach velocity, and ρ_g is the fluid density. If the particle size calculated in Equation 4 is used in Equation 1, the calculated efficiency will be the minimum for any particle in the population. Thus, a filter designed to collect minimum efficiency particles, at the desired efficiency, would yield a high confidence design.

Equations 1, 2, and 4 have many assumptions that appear to preclude their use in cake filtration systems. The most apparent of these assumptions is the sphere/cylinder relationship. Sand grains or particles collected from a solution do not greatly resemble cylinders in shape. Furthermore, the porosity of filter cakes is normally much lower than the fiber mats typically used in aerosol filtration (Reference 6). Despite these, and perhaps other limitations in the filtration analogy, it will be shown that the merits of models such as Friedlander's far outweigh these inconsistencies for use in the prediction of filter performance.

The second property of importance to the filtration process occurs after the transport step. This is the attachment of the suspended particle to the filter at the solid-fluid interface. This interface is presented either by a sand grain or particle previously collected; it is controlled by the surface properties of the particle and/or the filter media (References 6, 23, 24, 25). Particle attachment, like particle transport, can be produced by a number of different mechanisms. The two major models, which have both theoretical and practical interest, will be discussed here.

The most simplistic colloid-chemical model, that can be used to describe interactions between suspended particles and the filter cake, is based upon the theory of electrical double-layer interactions (Reference 25). Application of this theory assumes that the net interaction between a suspended particle and the filter cake surface can be described by the quantitative combination of van der Waals forces of attraction with the coulombic repulsion or attraction of the two double layers. Although the theory of the double layer has been developed primarily for water- and sludge-oriented systems, application to air systems, especially when collected particles are liquid phase, should be analogous.

An electrical double layer exists at every interface between a solid and fluid phase. The solid side assumes an electrostatic charge, the primary charge, which may be either positive or negative. The origin of the primary charge is a function of the chemistry of the material. An equivalent number of counter ions forms a diffuse layer in the fluid phase. When a particle approaches the surface of a filter cake, the two diffuse layers begin to interact. If both layers are charged in the same polarity, this interaction will yield a repulsive energy potential, V_R, whose intensity is inversely proportional (to an exponential power) to the distance separating the two particle surfaces. The van der Waals attractive forces also increase as particles approach each other. For large particles, where G, the gravitational constant becomes significant, the potential energy of attraction, V_A, is inversely proportional to the square of the separation distance. If these force potentials are added, the net interaction energy can be expressed as a function of separation distance:

$$f(X_S) = V_R - V_A \tag{5}$$

where $f(X_S)$ is a function of the separation distance, V_R is the electrical potential between the double layers, and V_A is the van der Waals forces of attraction. If the V_R force is sufficiently strong that it overcomes V_A, particle attachment will be prevented. Conversely, if the V_A force is weak or negative, particle attachment will be improved and the bond formed, once attached, will increase in strength as V_A becomes more negative.

The van der Waals attractive force is relatively independent of the composition of the fluid phase. The coulombic potential, however, may be controlled by characteristics of both solid and fluid phases. This aspect of the coulombic forces contributes to the success of most air systems. In most instances, when air is the supporting system, coulombic forces do not inhibit collision success, since their supporting fluid is less amenable to charge conduction than are water-oriented systems. Coulombic forces in air filtration can cause filter cleaning problems because the bonding force, at the particle surface, may be very strong.

The complexity of interrelationships between particle dynamics and the collecting systems has inhibited a truly quantitative evaluation of collection potential. For the purpose of this investigation, collision success will be expressed:

$$\phi = f(u, \nu_g, d_p, V_R, V_A) \tag{6}$$

where ϕ is the collision efficiency, u is particle approach velocity, ν_g is kinematic viscosity, d_p is particle diameter, V_R is coulombic force, and V_A is van der Waals force. At this point, it is assumed that ϕ will be an empirically determined value.

SECTION III

AIR FILTRATION EFFICIENCY

The overall filter efficiency for a cake-type filter regardless of fluid can be written:

$$\eta_t = 1 - (1-\eta_f)(1-\eta_c) \tag{7}$$

where η_t is the filter efficiency, η_f is the filter media efficiency, and η_c is the filter cake (composed of particles) efficiency. New expressions for collection efficiency are not within the scope of this investigation; however, existing efficiency expressions, used in air filtration theory, will be re-evaluated and modified to facilitate application to water-oriented systems.

Current theories relative to cake-type air filtration have been reviewed by Noll et al (Reference 26). This review indicates that collection efficiency is high (99+ percent); however, quantitative methods for the determination of cake efficiency have not been made. Several investigators (References 8-10, 27-30) have developed efficiency expressions to predict the single particle upon collector efficiency. These expressions have been generalized to predict the efficiency of a homogeneous media such as a fibrous mat filter.

The general form of the current air filtration efficiency equation can be written:

$$\eta_f = 1 - \exp^{-S_f \eta_{ICD\phi}} \tag{8}$$

where η_f is the filter cloth or fabric collection, S_f is the solidarity factor which describes the filter fabric, and $\eta_{ICD\phi}$ is Friedlander's single fiber collection efficiency modified to include ϕ, the probability of successful collection if collision occurs. The mathematics of the modified Friedlander equation will be defined subsequently.

The solidarity factor, S_f, describes a characteristic of the filter media, the ratio of the projected fiber surface area to the filter volume. In the case of the graded media, it describes the sand grain characteristics:

$$S_f = \frac{4W_f}{\pi \rho_f \bar{d}_f} \qquad (9)$$

where S_f is the solidarity factor for the filter fabric, W_f is the filter weight per unit area, ρ_f is the density of a discrete fiber, and $\bar{d}_f$ is the effective fiber diameter as a collector (i.e., $\bar{d}_{sm}{}^2/\bar{d}_A$) assuming a log-normal fiber size distribution in the fabric.

Friedlander's modified equation for the single fiber collection efficiency can be expressed for sphere-on-cylinder collision:

$$\eta_{ICD\phi} = 6N_{SC}{}^{-2/3}R_e{}^{-1/2} + 3R_c{}^2R_e{}^{1/2}\ \phi \qquad (10)$$

where $\eta_{ICD\phi}$ is the modified value for single fiber efficiency, N_{SC} is the Schmidt number, R_e is the Reynolds number, R_c is the particle/collector ratio, and ϕ is the empirical success coefficient determined as a function of Equation 6.

The Schmidt number is defined:

$$N_{SC} = \frac{\nu_g}{\mathcal{D}} \qquad (11)$$

where N_{SC} is the Schmidt number, ν_g is the kinematic viscosity of the fluid, and $\mathcal{D}$ is the diffusivity for the suspended particles in the fluid. The Reynolds number is defined:

$$R_e = \frac{u\bar{d}_f}{\nu_g} \qquad (12)$$

where R_e is the Reynolds number and u is the fluid velocity. The particle/collector ratio is defined:

$$R_c = \frac{\bar{d}_A}{\bar{d}_f} \qquad (13)$$

where R_c is the particle/collector ratio and $\bar{d}_A$ is the arithmetic mean particle size.

Although the efficiency of the filter fabric is important when cake-type filtration is used, the fabric efficiency becomes far less significant (References 6, 10, 12, 15). The cake efficiency term is the parameter of interest in cake filtration theory.

The basis for a cake collection efficiency expression was described by Equation 3. If this expression is written to consider only cake filtration, it may be expressed:

$$\eta_c = 1-\frac{C_2}{C_1} = 1-\exp\left[-\eta'_{ICD\phi}\left(\frac{6L_c\alpha_c}{\pi\bar{d}_c}\right)\right] \tag{14}$$

where η_c is the cake collection efficiency for the conditions described by the right hand side of the equation, C_1 and C_2 are the influent and effluent suspended solids, respectively, $\eta'_{ICD\phi}$ is Friedlander's modified collection efficiency equation using effective collector diameter and arithmetic mean suspended particle diameter with the correction for the probability of a successful collision, L_c is the cake thickness, α_c is the cake/discrete particle density ratio, and $\bar{d}_c$ is the effective particle diameter as a collector ($\bar{d}_{sm}{}^2/\bar{d}_A$) assuming a log-normal particle size distribution.

The first term in Equation 14 that requires further explanation is $\eta_{ICD\phi}$ the Friedlander (References 8, 9) modified efficiency prediction for particle interaction. This is the same equation used for the fabric with the collector size redefined as the effective cake particle size:

$$\eta'_{ICD\phi} = \left[6N_{SC}^{-2/3}\,R_e^{-1/2} + 3R_c^2\,R_e^{1/2}\right]\phi' \tag{15}$$

where $\eta'_{ICD\phi}$ is the Friedlander modified collection efficiency, N_{SC} is the Schmidt number, R_e is the Reynolds number, and R_c is the ratio of the arithmetic mean suspended particle size to the effective suspended particle size acting as the filter cake. Where the log-normal standard deviation, σ_g, is reasonably low (σ_g less than 1.8), R_c approaches 1.0 because the arithmetic mean and effective particle size are nearly equal. This is one of the reasons that cake-type filters are so efficient. When R_c approaches unity, collection by inertial forces is a function only of the Reynolds number. The probability of successful particle collision, ϕ, is an empirically determined value. This term, and its associated variables, was discussed in the section on small particle collection. Methods to evaluate this factor and the determination and statistical mathematics required to estimate the coefficient were not within the scope of this investigation. It is felt, however, that recognition of the existence of the term and identification of it as a problem area may stimulate research toward definition of the parameters involved in particle attachment.

The remaining terms in Equation 14 reflect the solidarity relationship between the collector and the fluid/particle suspension as filtration occurs. This factor is expressed for the filter cake:

$$S_f' = \frac{6L_c\alpha_c}{\pi\bar{d}_c} \qquad (16)$$

The values of L_c, α_c, and $\bar{d}_c$ are calculated from particle and flow data. The cake thickness, L_c, is a function of the suspended solids concentration, the unit area flow, and time. It can be expressed as a rate function:

$$\frac{dL_c}{dt} = \frac{C_I u \eta_c}{\rho_{BC}} \qquad (17)$$

where dL_c/dt is the rate of increase in cake thickness, L_c, with respect to time, t; C_I is the unit volume concentration of suspended particles in the bulk flow of the fluid; u is flow per unit area or approach velocity; η_c is cake collection efficiency; and ρ_{BC} is the bulk density of the filter cake. Because the object of this calculation is determination of the rate of change in cake thickness to ascertain the rate of efficiency increase, it would be difficult to include efficiency as a variable. Fortunately, through use of numerical integration techniques on the digital computer, the efficiency can be included by successive evaluation. With this in mind, the change in cake thickness over a given time increment can be expressed:

$$\frac{dL_c}{dt} = \frac{C_I u}{\rho_{BC}} \eta_{CL} \quad (18)$$

where η_{CL} is the efficiency for the initial cake thickness. This equation can be integrated with respect to time between t_1 and t_2:

$$\Delta L_c = \int_{t_1}^{t_2} \frac{C_I u}{\rho_{BC}} \eta_{CL} dt \quad (19)$$

where L_c is the change in cake thickness over the time increment. If Equation 19 is evaluated over very small time increments, it can be numerically integrated. Cake thickness can thus be expressed as a function of time, (in the form of an infinite series as Δt approaches zero), fluid velocity, and suspended particle concentration.

The other parameter used in Equation 16 that may be non-constant is α_c, the ratio of the cake bulk density (mass per unit volume of filter cake) to the density of a discrete suspended particle. In most instances this parameter is assumed to be constant for air and water filtration processes. The significance of cake compression in these systems is normally small; the converse is true, however, in some liquid filtration systems. The affect of cake compression can be very significant; this is most commonly observed at the high pressure differentials encountered in sludge filtration (References 31-33). Cake compression plays an important role in the relationship between pressure, flow, and time; however, fortunately, Equation 19 can be manipulated so that α_c is eliminated and thus the solidarity factor is not affected. This manipulation can be accomplished by combining Equations 16 and 19 and substituting ρ_{BC}/ρ_p (ρ_{BC} is cake bulk density and ρ_p is discrete particle density) for α_c:

$$S_f' = \frac{6C_I ut}{\pi \bar{d}_c} \frac{\rho_{BC}}{\rho_p \rho_{BC}}$$

the bulk density terms fall out to yield:

$$S_f' = \frac{6C_I ut}{\pi \bar{d}_c \rho_p} \tag{20}$$

Thus, as with fiber beds, cakes are dependent upon particle size and continuous density for description of their filtration efficiency properties.

The remaining term in Equations 16 and 20 that requires description is $\bar{d}_c$, the effective particle size. This is Chen's (Reference 11) sphere/cylinder collector approximation as discussed in the section on small particle collection. This term, which is determined from analysis of suspended particle size distribution data, may be expressed:

$$\bar{d}_c = \frac{\bar{d}_{sm}^2}{\bar{d}_A} \tag{21}$$

where $\bar{d}_c$ is the effective particle size as a collector, $\bar{d}_{sm}$ is the surface mean particle size, and $\bar{d}_A$ is the arithmetic mean particle size. The values of $\bar{d}_{sm}$ and $\bar{d}_A$ are determined from particle size distribution laboratory data.

Many statistical methods can be used to manipulate log-normal probability distributions; however, the most straight-forward method for engineering application is the graphical solution. If the raw particle size data is plotted as a function of percent by mass less than that size on log-probability graph paper, a graphical solution for many different particle means may be accomplished (Reference 1). A sample plot of this nature is shown by Figure 1.

The first procedural step, after data is plotted, is estimation of the mass mean particle size, $\bar{d}_{mm}$, and the geometric standard deviation, σ_g. The value of $\bar{d}_{mm}$ can be estimated directly from the plot at the 50.00 percent particle size. The standard geometric deviation can be estimated:

$$\sigma_g = 0.5\left[\frac{\bar{d}@84.13\%}{\bar{d}_{mm}} + \frac{\bar{d}_{mm}}{\bar{d}@15.87\%}\right] \tag{22}$$

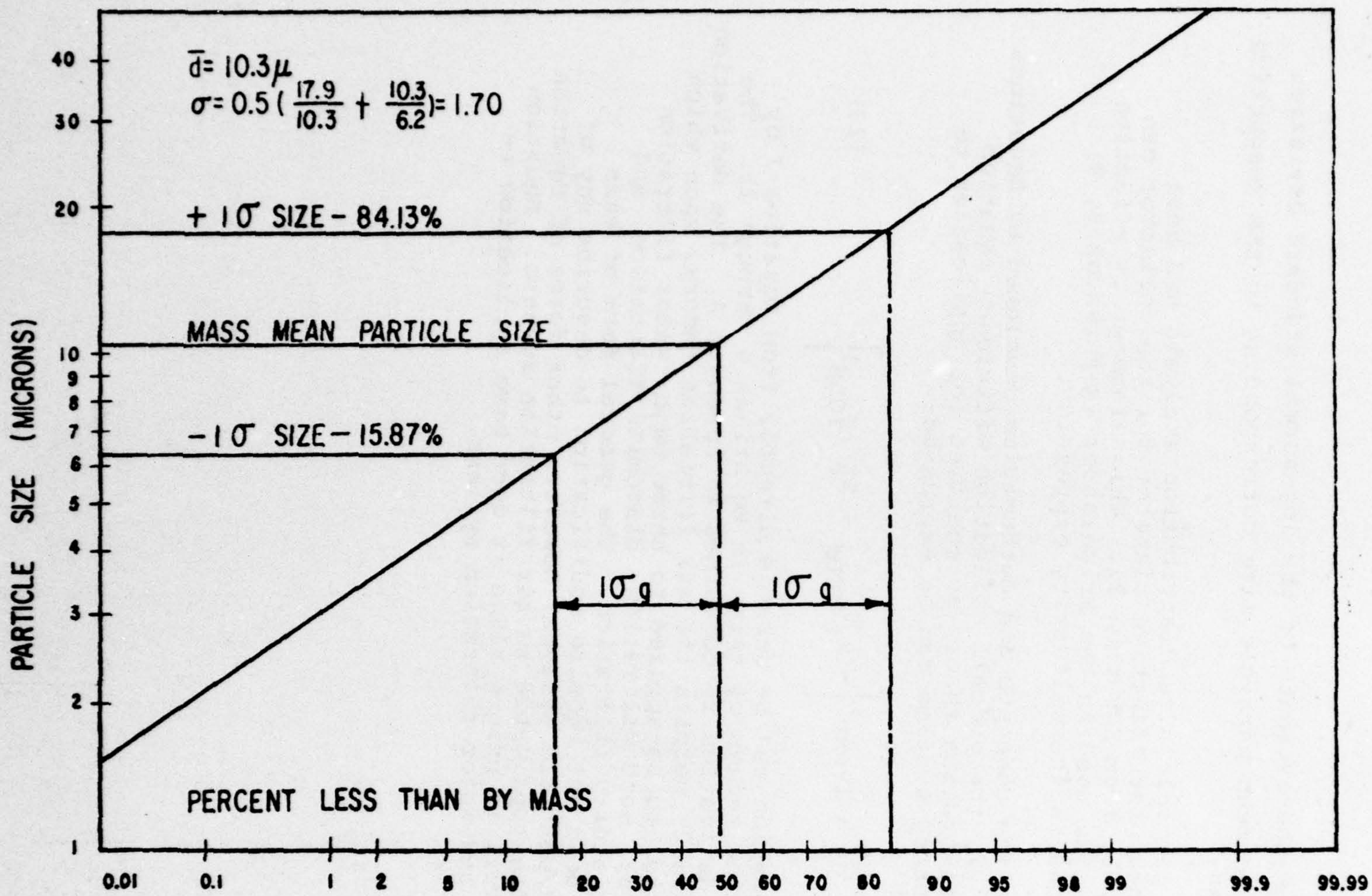

Figure 1. The Log-Normal Distribution

where σ_g is the estimate of the log-normal standard deviation and $\bar{d}$ is the mean particle size corresponding to the respective percentages.

Once particle size distribution analysis has been accomplished, the effective diameter for the collector can be determined from Equation 21. This diameter is effective collector size used in the Friedlander (References 8, 9) equation and in the solidarity expression.

Using the concepts and mathematics developed by Equations 7 through 22, the overall collection efficiency equation from air filtration theory as modified for application to water-oriented systems can be expressed:

$$\eta_t = 1\text{-exp}\left[-(S_f \eta_{ICD\emptyset} + S_f' \eta_{ICD\emptyset}')\right] \qquad (23)$$

This expression can be derived directly from Equation 7 by substituting values of terms in Equations 6 through 22. The complete derivation is contained in Appendix A. The derivation of Equation 23 permits the air filtration theory, upon which it is based, to be applied to three major water filtration operations: sand filtration, diatomite filtration, and sludge or slurry filtration. The general form of this equation does not require modification to describe any of the fluid/particle systems; however, other forms of Equation 23 are more applicable to air filtration systems. Equation 23, however, is unique since it does have application to both air and water filtration systems.

SECTION IV

FLOW, PRESSURE, TIME, AND EFFICIENCY

Energy loss across an ideal filter can be described by Darcy's law (References 10, 26, 34, 35):

$$\Delta P = K_D L u \tag{24}$$

where ΔP is the energy loss expressed as pressure drop, L is the filter thickness, u is unit flow or approach velocity, and K_D is a constant that describes the interaction of the fluid suspension and the filter media under given physical/chemical conditions. Using the relationship given by Equation 24, the constant, K_D, can be evaluated as a function of pressure drop, filtration velocity, and filter thickness:

$$K_D = \frac{\Delta P}{uL} \tag{25}$$

thus, for the ideal case, an increase in pressure drop must be accompanied with an increase in the product of filter thickness and flow velocity. If the filter thickness is contained by the thickness of a fiber bed, the pressure and velocity are directly proportional. This ideal case holds only for a clean (particle free) fluid since the entrapment of particles by the filter (either fabric or cake) would modify the value of the filter constant. For this reason K_D should be considered as a time dependent function rather than a constant for the fluid/particle separation process.

The pressure drop across a fiber mat caused by particles entrapped by the mat can be expressed:

$$\Delta P_f = \int_0^t K_2 C_I u^2 dt$$

or integrating:

$$\Delta P_f = K_2 C_I u^2 t \qquad (26)$$

where ΔP_f is the pressure drop across the filter cloth, K_2 is a constant that describes the filter drag caused by interaction by the filter cloth and particles, C_I is the unit volume particle concentration, u is the unit area flow or approach velocity, and t is time. The pressure drop across an air filtration system may thus be described:

$$\Delta P_t = K_1 u + K_2 C_I u^2 t \qquad (27)$$

where ΔP_t is the total pressure drop across the filter and K_1 is the drag constant for the "conditioned" fiber bed.

The only parameter that required further clarification in Equation 27 is K_2. The constant is expressed in the literature (References 5, 36, 37, 38):

$$K_2 = \frac{k_c}{g} \frac{\mu_g}{\rho_g} S_p^2 \frac{(1-\varepsilon_p)}{\varepsilon_p^3} \frac{1}{\rho_p} \qquad (28)$$

where K_2 is the drag term (pressure drop per unit area weight per unit velocity), k_c is the Carman-Kozeny (References 37, 38) coefficient, g is acceleration due to gravity, μ_g is the dynamic viscosity of the fluid, ρ_g is the fluid density, S_p is the particle surface-to-volume ratio, ε_p is the voids ratio of the particles in bulk, and ρ_p is the density of the discrete suspended particles.

This investigation was not primarily interested in fibrous filtration; however, much insight into cake performance can be gained from the theory presented with respect to fiber beds. For example, if Chen's (Reference 11) effective collector size is assumed to be correct, pressure drop and flow can be considered in a manner analogous to efficiency. With this analogy in mind, the generalized pressure loss equation for air filtration systems may be written:

$$\Delta P_t = \Delta P_f + \Delta P_c \qquad (29)$$

where ΔP_t is the total system pressure drop, ΔP_f is the pressure loss across the fiber bed, and ΔP_c is the pressure loss across tne filter cake.

If the filter fabric is assumed to be only a supporting system for the filter cake, as is the case with most cake filtration systems, then the initial pressure drop caused by the fabric is a function of only the conditioned cloth characteristics, thus ΔP_c goes to zero for the start of each filtration cycle:

$$\Delta P_t = K_1 u + K_2 C_I u^2 \cancelto{0}{t} \qquad (30)$$

This can be assumed since the conditioned cloth is cleaned so that only the cake is removed and the entrained particles remain in the fabric. The value of K_1 is defined:

$$K_1 = W_f K_2 \qquad (31)$$

where K_1 is the filter cloth drag constant and W_f is the filter cloth weight per unit area.

The value of K_2 remains to be defined. This drag term is analogous to the filter resistance terms (empirically determined) in water filtration systems. In fact, the K_2 expression can be derived directly from the Carman-Kozeny (References 37, 38) relationship (see Appendix B).

The advantage of air filtration theory, however, does not lie in the ability of Equation 30 to predict pressure drop for a clean filter bed. The advantage realized incorporates basic water filtration theory with efficiency concepts developed in aerosol mechanics to permit semi-theoretical prediction of the pressure, time, flow, and efficiency interrelationships for water-oriented systems. Since relationships in water filtration to predict headloss are generally modificationsor empirical improvements upon the Carman-Kozeny relationship, this common ground permits comparison of the different headloss relationships in water filtration to the headloss relationships in air filtration. The importance of this relationship cannot be overemphasized since efficiency and pressure drop theories in water filtration are primarily empirical (References 35, 39, 40, 43-48) and do not consider the characteristics of the fluid/particle suspension to the necessary degree.

The most commonly used headloss equations for water filtration systems are shown in Table 1 along with the air filtration headloss equation. It should be noticed in Table 1 that all of the water-oriented equations are modifications of the Carman-Kozeny relationship. Using the equations shown in Table 1, headloss ratios (the ratio of headloss at any time to the initital clean bed headloss) were plotted as a function of specific deposit, σ, in Figure 2. Because specific deposit is an indirect measurement of filter efficiency in terms of the total mass of suspended particles per unit volume of filter, Figure 2 indicates that filter efficiency changes through a filter run as the slope on the curves change.

All of these expressions attempt to describe the filter bed (or cake) during the filter cycle as a two-variable system. The primary variable is the change in porosity due to clogging by the suspended particles. The secondary variable deals with the change in the surface area of the matrix grains due to deposition. This second consideration is related to the first in that the change in bed porosity is a function of the amount of material collected. Unfortunately, most researchers (References 35, 39, 46-48) in water filtration have concluded that the effect of increased surface area is too complex for mathematical modeling and thus infer that it is related to the solids loading and mode of deposition or the hydradynamic characteristics of flow. These two variables are major considerations in Friedlander's equation for collection efficiency in air filtration systems.

Two additional variables are considered by some of the equations in Table 1. These are the tortousity factor (L_e/L^2) and the Carman (Reference 49) shape factor, k_o. The tortousity factor is related to the streamline distance increase and it, like the change in porosity and the change in σ, is an attempt to relate the effect of material collected in the bed upon the pressure drop. The shape factor, k_o, changes as a function of the material collected in the filter section; however, this change is second order. These

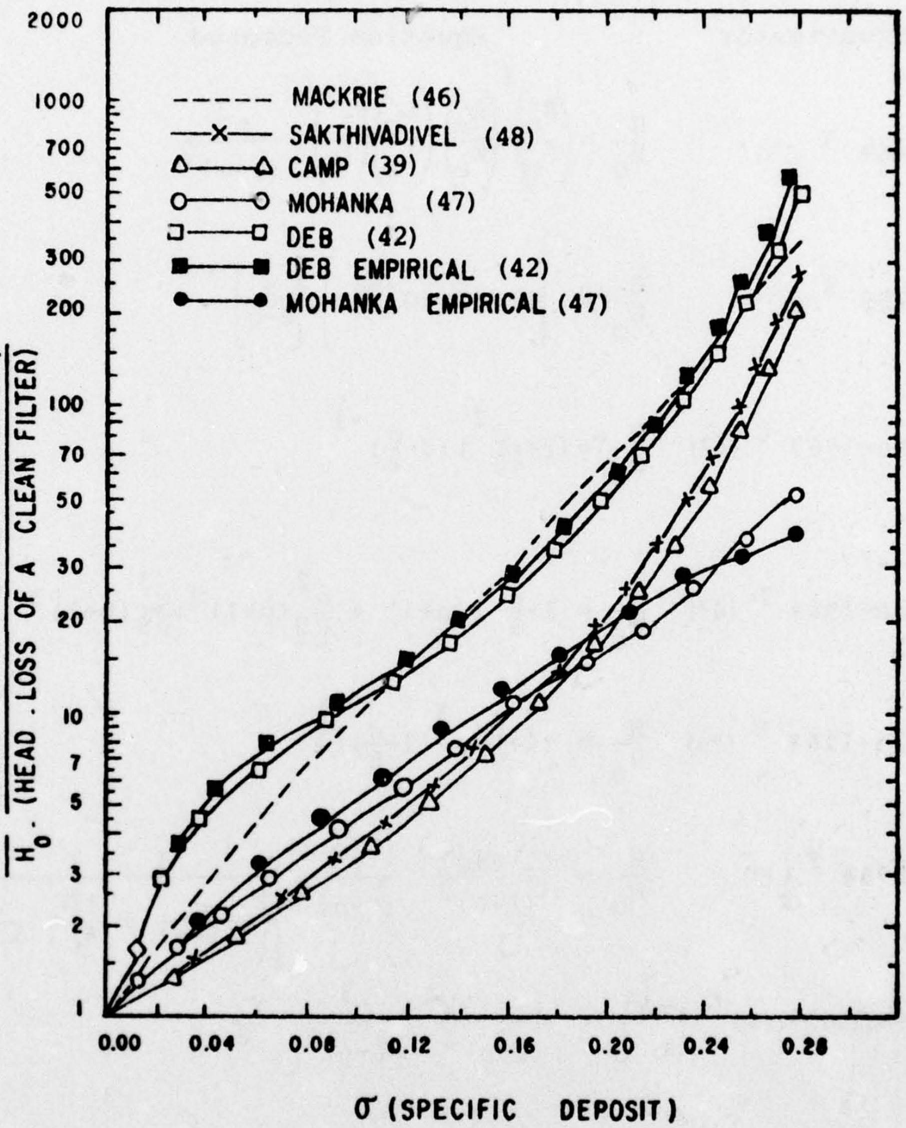

Figure 2. Comparison of Headloss Equations in Water Filtration during the Filtration Cycle

TABLE 1. SUMMARY OF HEADLOSS EQUATIONS IN WATER FILTRATION

Investigator	Equation Proposed
Deb-1969 [A] (35)	$\frac{H}{H_o} = \left(\frac{S'_o}{S_o}\right)^2 \left(\frac{K'_o}{K_o}\right) \left(\frac{L'_o}{L_o}\right)^2 \left(\frac{L}{L_o}\right)^2 \frac{\theta^3}{(\theta-\sigma)^2}$
Deb-1969 [B] (35)	$\frac{H}{H_o} = \left[1+G(1-10^{-k\sigma})\right] \left[\frac{\theta}{\theta-\sigma}\right]^3$
Mohanka-1969 [C] (47)	$\frac{H}{H_o} = (1+p\frac{\sigma}{\theta})^2 (1-\frac{\sigma}{\theta})^{-1}$
Mohanka-1969 [D] (47)	$\frac{H}{H_o} = 1+\frac{\sigma}{\theta}(2p+1) + \frac{\sigma^2}{\theta^3}(p+1)^2 + \frac{\sigma^3}{\theta^3}(p+1)^3$
Macrkle-1963 [E] (46)	$\frac{H}{H_o} = (1+\frac{p\sigma}{\theta})^3 (1-\frac{\sigma}{\theta})^{-\frac{1}{2}}$
Camp-1964 [F] (39)	$\frac{H}{H_o} = \frac{(1-\theta+\sigma)^3}{(1-\theta)^2} \frac{\theta^3}{(\theta-\sigma)^2} \left[\frac{1}{\left(\frac{\sigma}{3(1-\theta)} + \frac{1}{4}\right)^{\frac{1}{2}} + \frac{\sigma}{3(1-\theta)} + \frac{1}{2}}\right]$
Sakthivadivel-1966 [G] (48)	$\frac{H}{H_o} = \frac{(1-\theta+\sigma)^2}{(\theta-\sigma)^3} \frac{\theta^3}{(1-\theta)^2} \frac{1}{\xi^2}$
Ives-1960 [H] (43)	$\frac{H}{H_o} = \frac{K'}{K} \frac{r_1}{r_o} \frac{(k-\theta+\sigma)^2 \theta^2}{(\theta-\sigma)^3 (1-\theta)^2}$
This investigation [I]	$\frac{H}{H_o} = \frac{k_{2p} C_I u t \eta_c}{k_{2f} L_f \alpha_f}$

TABLE 1 (Concluded)

[A]The factor $\left(\frac{S_o'}{S_o}\right)\left(\frac{K_o'}{K_o}\right)\left(\frac{L_o'}{L}\right)^2\left(\frac{L}{L_o}\right)^2$ is designated as $\frac{J\psi^2}{J_o\psi^2}$ and is determined experimentally. $J_o\psi_o{}^2$ is called overall fiber-medium characteristics.

[B]$G = 3.2$, $K = 13.3$. Primarily an empirical equation. G and K are empirical constants.

[C]p depends on surface area. It is derived from Mackrle's mathematical model, assuming $x = y = 1$.

[D]This is a simplified version of the above equation.

[E]This equation is obtained by using the experimentally determined values of $x = 1.5$, $y = 0.75$, and $p = (29/S')^{8.65}$ by Mohanka.

[F]Based on Carman-Kozeny equation. This ratio $\left(\frac{K_o'}{K_o}\right)\left(\frac{L_o'}{L}\right)^2\left(\frac{L}{L_o}\right)^2$ is assumed constant and equal to I.

[G]Equation takes into account only porosity explicitly. All other variables are combined and denoted ξ^2, where $\xi^2 = \left(\frac{K_o'}{K_o}\right)\left(\frac{L_o'}{L}\right)^2\left(\frac{L}{L_o}\right)^2\left(\frac{S_o'}{S_o}\right)^2$

[H]$r_1 = \frac{\text{Area}}{\text{volume ratio of coated filter grains}}$ suggested empirical determination of $K \cdot r_1{}^2$.

[I]The factors K_{2p} and K_{2f} are the particle and fabric unit thickness drag coefficients, C_1 is the suspended solids load, t is the filtration time, L_f is the thickness of the filter, and f is the solids ratio for the media.

last two variables, if combined, yield the Carman-Kozeny constant:

$$k_c = k_o \frac{L_e}{L^2}$$

where k_c is the Carman-Kozeny constant. If the two factors are analyzed in terms of their performance during the filtration cycle, the rationale for the constant can become more evident. The shape factor is reduced during the filter run as particles collect so as to yield the minimum resistance to flow; conversely, the tortousity factor increases with time during the filter run since the streamline distance increases. As particles are collected these two variables are complementary during the filtration cycle; hence, the value of k_c remains approximately 5.

This discussion recognizes that a generalized equation for headloss during filtration should consider all four variables. The inability to determine these variables during filtration has often lead to approximation, idealization, and simplifying assumptions, which in turn give rise to different headloss equations dependent upon the assumptions made and the filtration system used.

Thus, we find that the headloss expression in Table 1 used in air filtration theory has no true advantage over its counterparts in water filtration; however, the air filtration equation is supported by expressions for efficiency, allowing description of the change in filter bed characteristics as a function of time and suspended particle concentration. Therefore, the advantage of air filtration theory lies not in the headloss equation itself but in the ability to describe the change in filter characteristics and subsequent change in headloss as a function of raw water characteristics.

REFERENCES

1. Aitchison, J. and Brown, J. The Lognormal Distribution. Great Britain: Cambridge at the University Press, 1969.

2. Davies, C. "Technical Note," Proceedings of Physics Society of London, 57:259, 1950.

3. Davies, C. "Technical Note," Proceedings of Physics Society of London, 63B:288, 1950.

4. Davies, C. "Technical Note," Proceedings of Institute of Mechanical Engineers of London, 1B:185, 1952.

5. Davies, C. "Problems in Aerosol Filtration," Filtration and Separation (Great Britain), 7:6:692, 1970.

6. Davies, C. Air Filtration. London: Academic Press, Inc., 1973.

7. Davies, G. and Jeffreys, G. "Separation of Droplet Dispersions, Part 1. Coalescence of Liquid Droplets," Filtration and Separation (Great Britain), 7:5:546, 1970.

8. Friedlander, S. "Theory of Aerosol Filtration," Jour. Industrial and Engineering Chemistry, 50:8:1161, 1958.

9. Friedlander, S. and Pascari, R. "The Efficiency of Fibrous Aerosol Filters," The Canadian Jour. of Chemical Engineers, p. 212, Dec 1960.

10. Fuchs, N. A. The Mechanics of Aerosols. New York: Pergamon Press, 1964.

11. Chen, C. "Filtration of Aerosols by Fibrous Media," Chemical Review, 55:595, 1955.

12. Stern, S., Zeller, H., and Schekman, A. "The Aerosol Efficiency and Pressure Drop of a Fibrous Filter at Reduced Temperature," Jour. of Colloid Science, 15:549, 1960.

13. Strauss, W. Industrial Gas Cleaning. New York: Pergamon Press, 1966.

14. Strauss, W. and Lancaster, B. "Prediction of Effectiveness of Gas Cleaning Methods at High Temperatures and Pressures," Jour. Atmospheric Envr. Pergamon Press, 2:135, 1968.

15. Strauss, W. Air Pollution Control - Part I. New York: Wiley-Interscience Publishing Co., 1971.

16. Stumm, W. and Lee, G. "The Chemistry of Aqueous Iron," Schweiz. z. Hydrol, 22:295, 1960, according to Stumm, Ref. No. 244.

17. Baylis, J. "Experiences in Filtration," Jour. Am. Water Works Association, 48:3:585, 1956.

18. Baylis, J. "Seven Years of High-Rate Filtration," Jour. Am. Water Works Association, 48:3:585, 1956.

19. Baylis, J. "Nature and Effects of Filter Backwashing," Jour. Am. Water Works Association, 51:1:129, 1959.

20. Baylis, J. "Design Criteria for Rapid Sand Filters," Jour. Am. Water Works Association, 51:11:1443, 1959.

21. O'Melia, C. and Stumm, W. "Theory of Water Filtration," Jour. Am. Water Works Association, 59:11:1393, 1967.

22. Lamb, H. Hydrodynamics. Sixth Edition. London: Cambridge University Press, 1932.

23. Gruner, P. "Experimental Investigations on the Relationship between the Separation Efficiency of a Nonclogging Fiber Filter and Its Porosity,: STAUB, 28:9:5, 1968.

24. O'Melia, C. and Crapps, D. "Some Chemical Aspects of Rapid Sand Filtration," Jour. Am. Water Works Association, 56:11:1326, 1964.

25. Stumm, W. and Morgan, J. "Chemical Aspects of Coagulation," Jour. Am. Water Works Association, 54:8:971, 1962.

26. Noll, K., Davis, W., and Shelton, S. "New Criteria for the Selection of Fabric Filters for Industrial Application," presented at the 66 Annual Am. Pollution Control Association Meeting in Chicago, Illinois, June 1973.

27. Ranz, W. and Wong, J. "Technical Notes," Jour. Industrial Engineering Chemistry, 44:1371, 1952.

28. Ranz, W. "Technical Report No. 8," January 1, 1953, Univ. of Illinois, Experimental Station.

29. Ranz, W. and Johnstone, H. "Technical Notes," Jour. Applied Physics, 26:244, 1956.

30. Wong, J. Ranz, W., and Johnstone, H. "Collection Efficiency of Aerosol Particles and Resistance Flow Through Fiber Mats," Jour. of Applied Physics, 27:2:161, 1956.

31. Grace, H. "Resistance and Compressibility of Filter Cakes," Chemical Engineering Progress, 49:6:303, 1953.

32. Grace, H. "Structure and Performance of Filter Media," Jour. Am. Institute Chemical Engineers, 2:3:307, 1956.

33. Grace, H. "Resistance and Compressibility of Filter Cakes - Part II," Chemical Engineering Progress, 49: 7:367, 1953.

34. Calvert, J. "Sludge Dewatering - Trends in Great Britain," Water Pollution Control Federation - Deeds and Data, Dec 1972.

35. Deb, A. "Theory of Sand Filtration," Am. Soc. Civil Engineers, Sanitary Engineering Journal, 95:SA3:399, 1969.

36. Billings, C. and Wilder, J. "Engineering Analysis of the Field Performance of Fabric Filter System," Paper No. 70-129, Air Pollution Control Association 63 Annual Meeting, 1970.

37. Carman, P. "Fluid Flow Through Granular Beds," Transactions of the Institute of Chemical Engineers (London, England), 15:150, 1937.

38. Carman, P. "Fundamental Principles of Industrial Filtration," Transactions of the Institution of Chemical Engineers, 16:168, 1938.

39. Camp, I. "Theory of Water Filtration," Am. Society of Civil Engineers - Sanitary Engineering Division Jour. 90:SA4:1, 1964.

40. Deb, A. "Discussion," Proc. Am. Soc. Civil Engineers - Sanitary Engineering Division, 91:SA2 Proc. 4281:84, 1965.

41. Deb, A. "Discussion," Proc. Am. Soc. Civil Engineers - Sanitary Engineering Division, 93:SA1 Proc. 4637:321, 1966.

42. Deb, A. "Discussion," Proc. Am. Soc. Civil Engineers - Sanitary Engineering Division, 93:SA4 Proc. 5358:124, 1967.

43. Ives, K. "Rational Design of Filters," Proc. Institute of Civil Engineers (London), 16:180, 1960.

44. Ives, K. "New Concepts in Filtration," Jour. of Water and Wastewater Engineering, 65:307, 1961.

45. Ives, K. "Simplified Rational Analysis of Filter Behavior," Proc. Institute of Civil Engineers (London) 25:345, 1963.

46. Mackrle, V. and Mackrle, S. "Adhesion in Filters," Am. Soc. Civil Engineers - Sanitary Engineering Division Jour., 87:SA5:7, 1961.

47. Mohanka, S. "Theory of Multilayer Filtration," Am. Soc. Civil Engineers - Sanitary Engineering Division Jour., 95:SA6:1079, 1969.

48. Sakthivadival, R., Thanikachalam, V., and Seetharaman, S. "Headloss Theories in Filtration," Jour. Am. Water Works Association, 64:4:253, 1972.

49. Carman, P. "In Symposium on New Methods for Particle Size Determination in the Subsieve Range," Am. Soc. Test. Mat., Philadelphia, Pa., 1951.

APPENDIX A

DERIVATION OF EQUATION 23

Given: Equation 2:

$$\eta_{ICD\emptyset} = \frac{\pi \bar{d}_f}{4\alpha_f L_f} \ln \frac{C_1}{C_2} \qquad (A.1)$$

where $\eta_{ICD\emptyset}$ is the Friedlander (References 8, 9) simple fiber collection efficiency corrected for attachment success, $\bar{d}_f$ is the effective diameter as a collector of the fibers in the filter, α_f is the ratio of the bulk density, ρ_{Bf}, to the discrete fiber density, ρ_f, L_f is the fabric thickness, C_1 is the inlet suspended particle loading and C_2 is the effluent suspended particle loading.

Given: Equation 9:

$$S_f = \frac{4W_f}{\pi \rho_f \bar{d}_f} \qquad (A.2)$$

where S_f is the solidarity factor for the cloth and W_f is the unit area weight of the cloth (equal to the bulk density, ρ_{Bf}, times the depth of the cloth, L_f, times unit area).

Step 1 Derive: Equation 8:

$$\eta_f = 1. - \exp(-S_f \eta_{ICD\emptyset}) \qquad (A.3)$$

where η_f is the gross efficiency of the filter cloth, S_f is the solidarity factor, and $\eta_{ICD\emptyset}$ is the modified Friedlander single fiber collection efficiency.

Solution of Step 1:

$$W_f = \rho_{Bf} L_f$$

$$\alpha_f = \rho_{Bf} / \rho_f$$

thus, Equation A.2 may be written:

$$S_f = \frac{4W_f}{\pi \rho_f \bar{d}_f} = \frac{4\rho_{Bf} L_f}{\pi \rho_f \bar{d}_f} = \frac{4\alpha_f L_f}{\pi \bar{d}_f} \tag{A.4}$$

Solve Equation A.1 for ln expression:

$$\ln \frac{C_1}{C_2} = \frac{4\alpha_f L_f}{\pi \bar{d}_f} \eta_{ICD\emptyset} \tag{A.5}$$

Substitute Equation A.4 into A.5:

$$\ln \frac{C_1}{C_2} = S_f \eta_{ICD\emptyset} \tag{A.6}$$

since ln y = x can be expressed as $y = e^x$, re-expresses A.6 inverted:

$$\frac{C_2}{C_1} = \exp(-S_f \eta_{ICD\emptyset}) \tag{A.7}$$

Efficiency can be expressed in terms of C_1 and C_2:

$$\eta_f = \frac{C_1 - C_2}{C_1} \tag{A.8}$$

using Equation A.8, re-express A.7 in terms of efficiency (add C_1/C_1 to both sides of the equation and multiply through by -1):

$$\eta_f = \frac{C_1 - C_2}{C_1} = 1 - \exp(-S_f \eta_{ICD\phi}) \qquad \text{(A.9)}$$

Given: Equations 14 and 20:

$$\eta_c = 1 - \exp(-S_f' \eta_{ICD\phi}') \qquad \text{(A.10)}$$

η_c, the gross cake efficiency can be derived in a sequence parallel to the derivation of Equation A.9.

Given: Equations A.9 and A.10 and Equation 7:

$$\eta_t = 1 - (1 - \eta_f)(1 - \eta_c) \qquad \text{(A.11)}$$

where η_t is total filter efficiency, η_f is gross fabric efficiency, and η_c is gross cake efficiency.

Derive: Equation 23:

$$\eta_t = 1 - \exp -(S_f \eta_{ICD\phi} + S_f' \eta_{ICD\phi}') \qquad \text{(A.12)}$$

Solution: Restating Equation A.11 by combining terms:

$$\eta_t = 1 - (1 - \eta_c - \eta_f + \eta_f \eta_c) \qquad \text{(A.13)}$$

or

$$\eta_t = \eta_c + \eta_f - \eta_f \eta_c \qquad \text{(A.14)}$$

substituting Equations A.9 and A.10 for η_f and η_c:

$$\eta_t = \left[1 - \exp(-S_f' \eta_{ICD\phi}')\right] + \left[1 - \exp(-S_f \eta_{ICD\phi})\right]$$

$$- \left[1 - \exp(-S_f \eta_{ICD\phi})\right]\left[1 - \exp(-S_f' \eta_{ICD\phi}')\right] \qquad \text{(A.15)}$$

multiplying and combining terms:

$$\eta_t = 1-\exp(-S_f'\eta_{ICD\phi}') - \exp(S_f\eta_{ICD\phi}) + \exp(-S_f'\eta_{ICD\phi}')$$

$$+ \exp(-S_f\eta_{ICD\phi}) - \exp\left[-(S_f\eta_{ICD\phi} + S_f'\eta_{ICD\phi}')\right] \qquad (A.16)$$

or

$$\eta_t = 1-\exp\left[-(S_f\eta_{ICD\phi} + S_f'\eta_{ICD\phi}')\right] \qquad (A.17)$$

APPENDIX B

DERIVATION OF K_2 FROM THE CARMAN-KOZENY EQUATION

Given: The Carman-Kozeny Equation (9,10):

$$u = \frac{\varepsilon^3}{k_c S^2} \frac{\Delta P_g}{\mu_g L} = \frac{\varepsilon^3}{(1-\varepsilon)^2} \frac{\Delta P}{k \mu_g L S_o^2} \qquad \text{(B.1)}$$

where:

- u - unit area flow (cm/sec)
- ε - porosity or voids ratio; $1 - \rho_B/\rho_p$; (dimensionless)
- ρ_B - bulk density of the particle bed (g/cm^3)
- ρ_p - density of the discrete particles in the bed (g/cm^3)
- k_c - Carman-Kozeny constant for particle beds, approx 5, (dimensionless)
- S - surface/unit volume ratio for particle beds (1/cm). This value is defined:
- S_o - specific surface of particle defined (1/cm) This value is defined:
- μ_g - the dynamic viscosity of the fluid (poise - g/sec.cm)
- L - particle bed thickness (cm)
- g - acceleration due to gravity (980 cm/sec^2)
- ΔP - pressure loss across the particle bed (g/cm^2)

Derive: Equation 28 for K_2:

$$K_2 = \frac{k_c}{g} \mu_g S_o^2 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{1}{\rho_p} \qquad \text{(B.2)}$$

Solution: Solve B.1 for ΔP:

$$\Delta P = \frac{u k_c \mu_g L S_o^2}{g} \frac{(1-\varepsilon)^2}{\varepsilon^3} \quad \text{(B.3)}$$

Using Equation 13:

$$L = \frac{C_I u t}{\rho_B} \quad \eta_c \simeq 1.0 \quad \text{(B.4)}$$

where L is cake thickness (cm), C_I is the unit volume particle concentration (g/cm^3), u is the unit area flow rate (cm/sec), t is time (sec), ρ_B is the bulk density of the particle bed (g/cm^3), and η_c is the gross collection efficiency of the filter cake.

If Equation B.4 is substituted into B.3, pressure drop can be re-expressed:

$$\Delta P = \frac{k_c}{g} \mu_g S_o^2 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{C_I u^2 t}{\rho_B} \quad \text{(B.5)}$$

if K_2 is defined:

$$K_2 = \frac{\Delta P}{C_I u^2 t} \quad \text{(B.6)}$$

then:

$$K_2 = \frac{\Delta P}{C_I u^2 t} = \frac{k_c}{g} \mu_g S_o^2 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{1.}{\rho_B} \quad \text{(B.7)}$$

since $1-\varepsilon = \rho_B/\rho_p$, B.7 can be re-expressed:

$$K_2 = \frac{k_c}{g} \mu_g S_o^2 \frac{1-\varepsilon}{\varepsilon^3} \frac{1}{\rho_p} \quad \text{(B.8)}$$

this is the same as Equation B.2.

INITIAL DISTRIBUTION

HQ USAF/PREE	1
HQ USAF/PREVP	1
HQ USAF/RDPS	2
HQ USAF/SAFOI	1
HQ USAF/SGPA	2
CINCAD/SGPAP	1
AFLC/SGB	1
AFSC/DE	1
AFSC/SD	1
AFSC/SGB	1
AFSC/SGPE	1
AFSC/DLCAM	2
ATC/SGPAP	1
AAC/DEV	1
AAC/SGB	1
MAC/DEEE	1
CINCPACAG/DEMU	2
CINCPACAF/SGPE	1
CINCSAC/DEPV	1
CINCSAC/SGPA	1
TAC/DEEV	1
TAC/SGPB	1
USAFSS/DEMM	1
CINCUSAFE/SG	1
CINCUSAFE/DEPV	2
USAFA/DEV	1
AFIT/DEM	1
AUL	1
AFOSR	1
AFFDL/TST	1
AFML/DO (Library)	1
OEHL/CC	3
OEHL/OL-AA	1
OEHL/OL-AB	1
AFWL/SUL	1
USAFSAM/EDE	2
AFRPL/Library	1
FTD/LGM	1
AMRL/THE	1
ASD/ENAMC	1
ASD/DEP	1
RADC/DOT	1
AEDC/DEE	1
SAMTEC/SEH	1
SAMSO/DEC	1
SAMSO/SG	1
AMD/RDU	1
ADTC/DLOSL (Tech Library)	1
AFFTC/DE	1
ESD/DE	1
SAMMA/MAGCB	1
USAFSO/DEE	1
1 Med Service Wg/SGB	1
DDC/TCA	12
ARPA	1
Defense Research & Engrg/AD (E&LS)	1
OASD/(I&L)ES	1
USA Environ Hygn Agency	1
Ch of Engrg/ENGMC-RD	1
Dir, USA WW Exp Sta	1
USA CERL	1
Dir, USA Eng R&D Lab/MERDC	1
Ch of R&D	
Dept of the Army/DARD-ARE-E	1
AFRCE (ER)	1
AFRCE (CR)	1
AFRCE (WR)	1
Ch of Naval Op	1
Naval Air Dev Ctr/MAE	1
Technical Applications Center (UNM)	1
Technology Transfer Staff (EPA)	1
Office of R&D (EPA)	1
National Science Foundation	1
US Army Med Bioengrg R&D Lab	1
Det 1 HQ ADTC/EC	1
Det 1 HQ ADTC/ED	1
Det 1 HQ ADTC/PRL	1

(The reverse of this page is blank)