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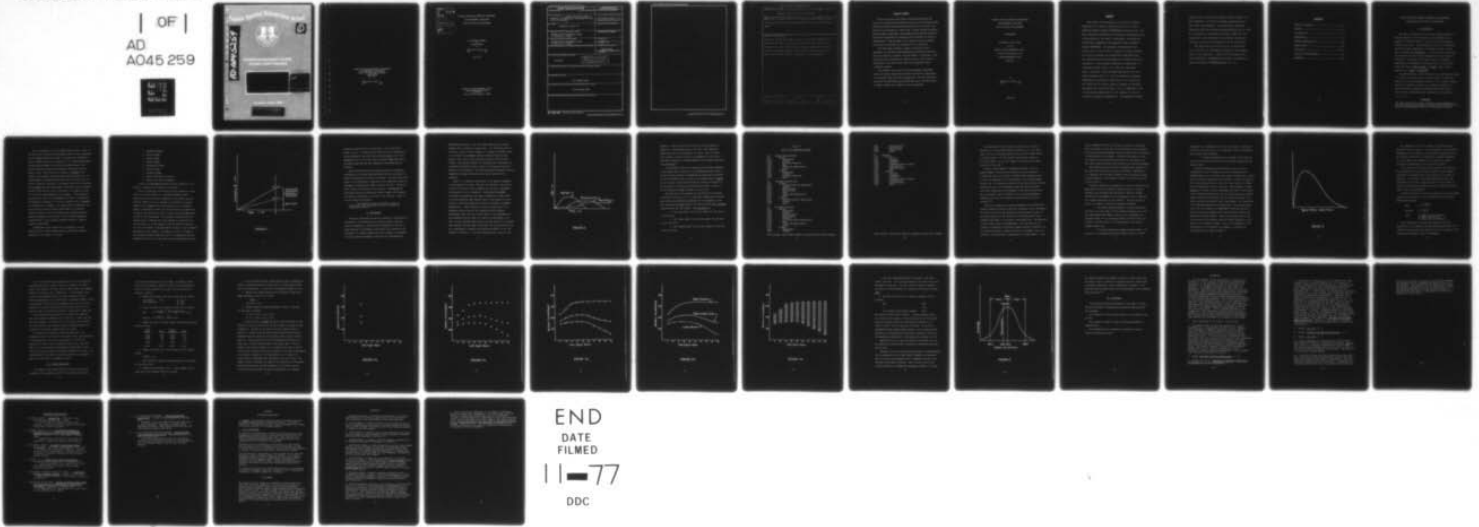
DEFENSE SYSTEMS MANAGEMENT SCHOOL FORT BELVOIR VA  
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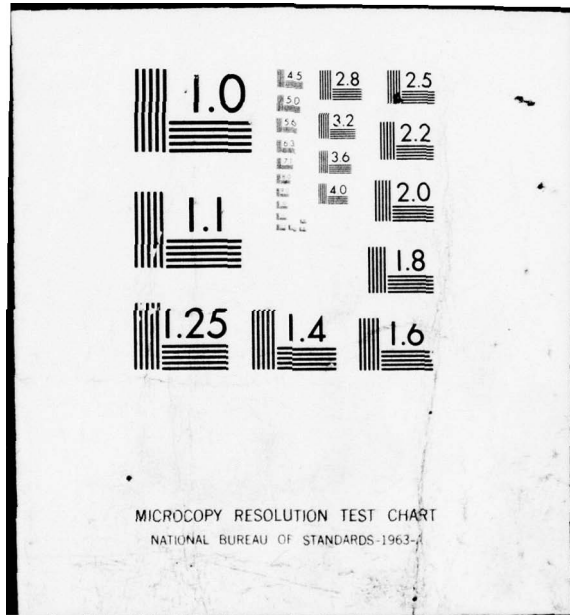
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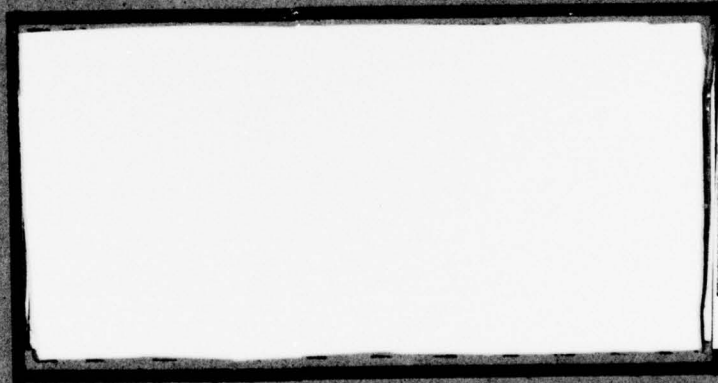
# Defense Systems Management School



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## PROGRAM MANAGEMENT COURSE STUDENT STUDY PROGRAM



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A SIMPLE STATISTICAL METHOD OF PRESENTING  
THE UNCERTAINTY ASSOCIATED  
WITH LIFE CYCLE COST ESTIMATES  
STUDY REPORT  
PMC 73-1

Warfield M. Lewis, Jr.  
LTC USA



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A SIMPLE STATISTICAL METHOD OF PRESENTING  
 THE UNCERTAINTY ASSOCIATED  
 WITH LIFE CYCLE COST ESTIMATES

An Executive Summary  
 of a  
 Student Report  
 by

Warfield M. Lewis, Jr.  
 LTC USA

May 1973

Defense Systems Management School  
 Program Management Course  
 Class 73-1  
 Fort Belvoir, Virginia 22060

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DEFENSE SYSTEMS MANAGEMENT SCHOOL

STUDY TITLE: A SIMPLE STATISTICAL METHOD OF PRESENTING THE UNCERTAINTY  
ASSOCIATED WITH LIFE CYCLE COST ESTIMATES

STUDY TOPIC: To develop and present a simple, statistically valid method of displaying the uncertainty associated with a Life Cycle Cost Estimate (LCCE).

STUDENT REPORT ABSTRACT:

DoD and the Services have shown a growing awareness of the importance of considering the uncertainty associated with LCCE's during the decision-making process. This report develops a simple technique that may be used by system acquisition managers to display this uncertainty as a band or range of possible costs with a "most-likely" cost also specified.

Student, Rank, Service	Class	Date
Warfield M. Lewis, Jr., LTC, USA	73-1	May 1973

## EXECUTIVE SUMMARY

DoD and the Services have shown an increasing awareness that there is an uncertainty associated with Life Cycle Cost Estimates (LCCE) and that the further system costs are projected into the future, the greater is the uncertainty of those costs. Current systems acquisition policy documents (DoDD 5000.1 and implementing instructions) emphasize the need for displaying and considering this uncertainty at decision-making points. These documents do not, however, specify techniques to be used in the display and evaluation of cost uncertainty.

This Study Report develops a simple, statistically valid, methodology for evaluating and displaying the uncertainty associated with cost estimates, in general, and with LCCE's, in particular. This uncertainty can be displayed as a band of costs on either side of a "most-likely" cost and the total cost can be displayed within a confidence interval.

The technique relies upon the most knowledgeable individuals within the project organization for basic input and the computations are relatively simple and easily computerized. The basic philosophy upon which the methodology was developed was that it should be based on simple concepts and be simple to use and understand.



A SIMPLE STATISTICAL METHOD OF PRESENTING  
THE UNCERTAINTY ASSOCIATED  
WITH LIFE CYCLE COST ESTIMATES

STUDY REPORT

Presented to the Faculty  
of the  
Defense Systems Management School  
in Partial Fulfillment of the  
Program Management Course  
Class 73-1

by

Warfield M. Lewis, Jr.  
LTC USA

May 1973

## PREFACE

The intent of this paper is to provide a display technique to be used in conjunction with U. S. Army Safeguard System Command (USASAFSCOM) Regulation No. 11-1. This regulation prescribes the policies, responsibilities, and procedures to be used in developing, coordinating, validating, presenting, and approving cost estimates within USASAFSCOM. As presently published, the regulation has a void in that it does not contain guidance as to how to present the uncertainty associated with the estimates that are developed in accordance with the regulation. The technique presented is applicable to the base estimate (para. 7a), "What If" exercises (para. 7d(1)(g)), cost estimates associated with economic analyses (para 7i), and the parametric estimates (Appendix D) developed as specified in the regulation. The technique can also be used to display the estimate confidence as required by para. 1c(5) of Appendix J and is particularly applicable to the display of cost uncertainty required by Appendix K. It expands the MICOM

technique for a total cost estimate found on page 6 of that appendix to the dollar streams associated with a Life Cycle Cost Estimate. The uncertainty displays resulting from the technique presented herein can be used by an analyst in structuring realistic ranges for the significant cost parameters in performing sensitivity analyses as required by Appendix M of the regulation.

The paper has been written so as to stand alone since this was the philosophy followed in the preparation of the regulation. Copies of this document have been forwarded to USASAFSCOM for possible inclusion in future revisions of USASAFSCOM Reg. No. 11-1.

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A SIMPLE STATISTICAL METHOD OF PRESENTING THE UNCERTAINTY  
ASSOCIATED WITH LIFE CYCLE COST ESTIMATES\*

I. Introduction

The task of developing a Life Cycle Cost Estimate (LCCE) for a proposed program (or, for that matter, for the remainder of any program at a particular point in its life cycle) is an extremely difficult job at best. The forecasting of costs for a system based on historical data, regardless of the volume or validity of that data, is fraught with uncertainty. It is precisely this uncertainty and the manner in which it can be displayed that is the subject of this report. The purpose of this paper, then, is to develop a methodology for displaying cost uncertainty that is statistically valid, but based on simple concepts, is simple to use, and most importantly, is simple to understand.<sup>1</sup>

The primary need for a cost estimate in the decision-making process is to assist the decision maker in forming a basis for comparison between two or more alternatives--whether or not to acquire a system, which of two systems to acquire, etc. Uncertainty analysis can provide the decision maker with information which, when taken in proper context, should improve the rationality of his decisions.

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\*ABSTAINER

This study represents the views, conclusions, and recommendations of the author and does not necessarily reflect the official opinion of the Defense Systems Management School nor the Department of Defense.



What is uncertainty in a cost estimate and how does it arise in the first place? From the decision-maker's point of view, uncertainty is that nagging feeling in the back of his mind that, although the array of numbers before him is impressive in its seeming completeness, and although the numbers in the rows and columns add up as they should, something is missing that prevents the whole picture from being in focus. That thing that is missing is a measure of the validity of the numbers that the decision maker has before him. The decision-maker's troubled feeling should stay with him until he can relate a measure of confidence to the numbers he has in his possession. He must always associate some measure of uncertainty with his estimate and never fall into the trap of considering it as a fact as he uses it in the day-to-day management of his program. If the uncertainty associated with LCCE's is not addressed, but rather is assumed away by regarding the estimate as a concrete statement of fact, management cost problems do not disappear, but rather multiply dramatically and can include such things as annual underfunding of the program, repeated deferrals of work to succeeding years requiring repeated program realignments, cost overruns, and the ultimate management disaster-threat of early program termination because of repeated financial difficulties.

A memorandum, dated 5 August 1970, and signed by the then Deputy Secretary of Defense David Packard, defined nine mandatory categories of cost growth, as follows:

1. Engineering Change
2. Quantity Change
3. Support Change
4. Schedule Change
5. Unpredictable Change
6. Economic Change
7. Estimating Change
8. Contract Performance Incentives
9. Contract Cost Overrun (Underrun)

A copy of the memorandum defining the above categories of cost growth is contained in the Appendix to this paper.

Since, by definition, any deviations from an estimate to actual costs, or from one estimate to a succeeding estimate, must be identified to one of the above categories of cost growth, they and inflation (which can also be included in cost growth as "economic change") are the total source of uncertainty in a cost estimate. Figure 1 illustrates the effect that the uncertainty associated with cost growth and inflation can have on actual costs when compared with an LCCE for the same program. The uncertainty associated with the estimate tends to compound with time (although not necessarily as a linear function, as in the figure) so that the further one projects cost data, the greater is the magnitude (in dollars) of the uncertainty associated with the estimate. For example, at point t in Figure 1, the originally estimated cumulative cost for a program may have been represented by point a, but the actual inflation experienced and cost

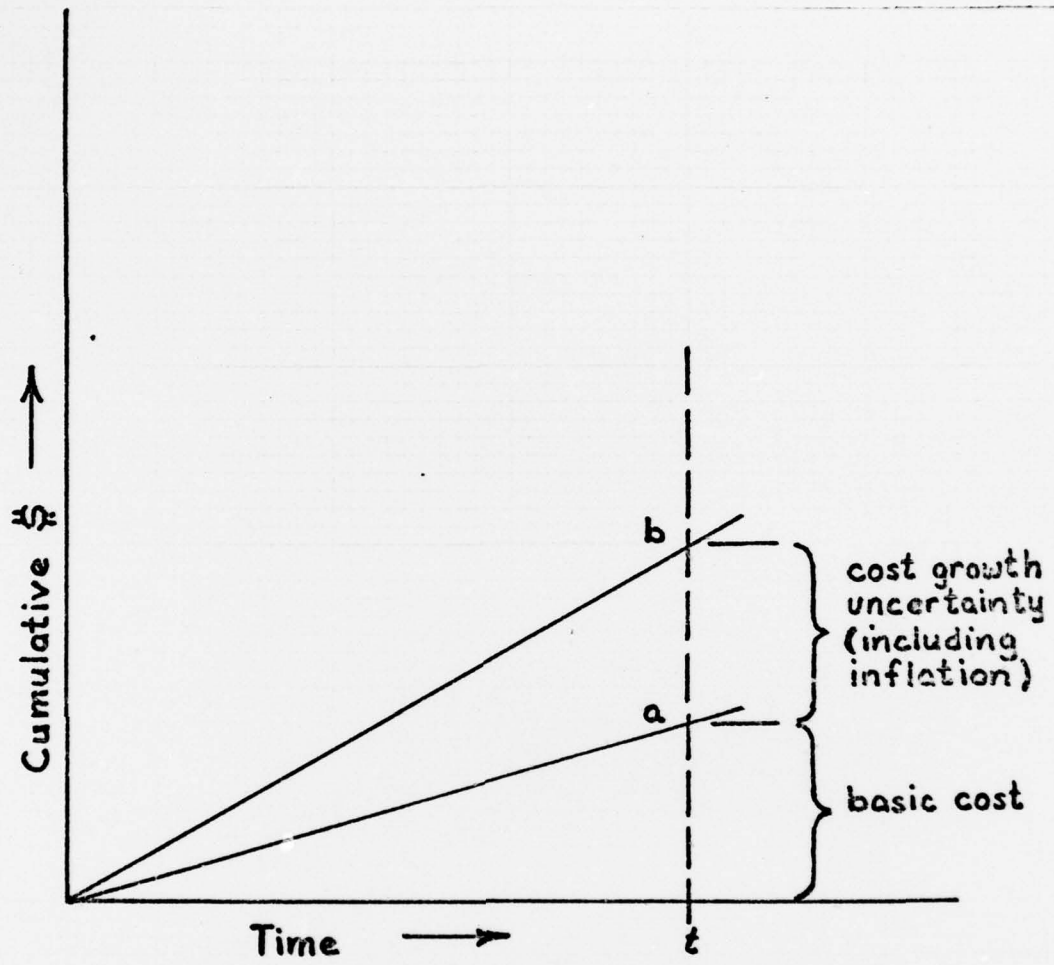


FIGURE 1.

growth may cause the costs to reach point b, or any other point between a and b. (It should also be noted that if the estimate was unduly pessimistic, the actual cost could fall below a, but from a management point of view, this result is equally undesirable, since perishable assets may have been committed to the program and not utilized.)

DoD and the Services have recognized the need for including a consideration of the uncertainty associated with cost estimates as a means of assisting in the control of resources. The basic policy with respect to the utilization of resources in the acquisition process is promulgated in DoD Directive 5000.1, dated 13 July 1971. Within the Army, a Letter of Instructions (LOI), subject: Implementing the New Materiel Acquisition Guidelines, dated 23 August 1972,<sup>2</sup> contains more specific instructions on the use of cost estimates. Annex L of this LOI states the following:

... Cost estimates should be expressed in terms of limits within which the ultimate cost of the program may be expected to fall....<sup>3</sup>

## II. Methodology

Having now established the need for including a consideration of uncertainty in the formulation of an LCCE and briefly stated the possible consequences of omitting this consideration in the use of these estimates, the remainder of this paper will be devoted to the development of a simple technique that can be applied by individuals with only a general knowledge of statistics for establishing and



displaying uncertainty so that this display might serve to remind managers that an estimate is exactly that. At a particular point in the future, costs are better thought of as "ranges of possible costs" or "cost bands" and management decisions should be based on that premise. That, then, is the theme that we follow from this point forward. The proposed technique for displaying cost uncertainty will be described in general terms as a means of providing the rationale leading to its development. Following this general discussion, specific computational examples illustrating the salient points will be presented.

Figure 2 is a graphical presentation of the dominant categories of costs present in an LCCE. The total cost estimate at any point in time will be the sum of these categories at that point (considering the categories to be all inclusive). Characteristically RDT&E, Procurement, and Operating costs phase in generally as shown (there are certain Operating funds required early in the program to pay the salaries of Government employees who cannot be charged to the RDT&E appropriation). These categories of costs are widely used in the management process (they are the categories by which Congress appropriates funds) and form a logical basis for the development of a Cost Breakdown Structure (CBS) to be used in the development of an LCCE. The CBS developed should also be directly related to the program Work Breakdown Structure (WBS) in the sense that costs associated with the accomplishment of elements of the WBS should sum to, or be, the elements of the CBS, i.e., the costs associated with a specific work



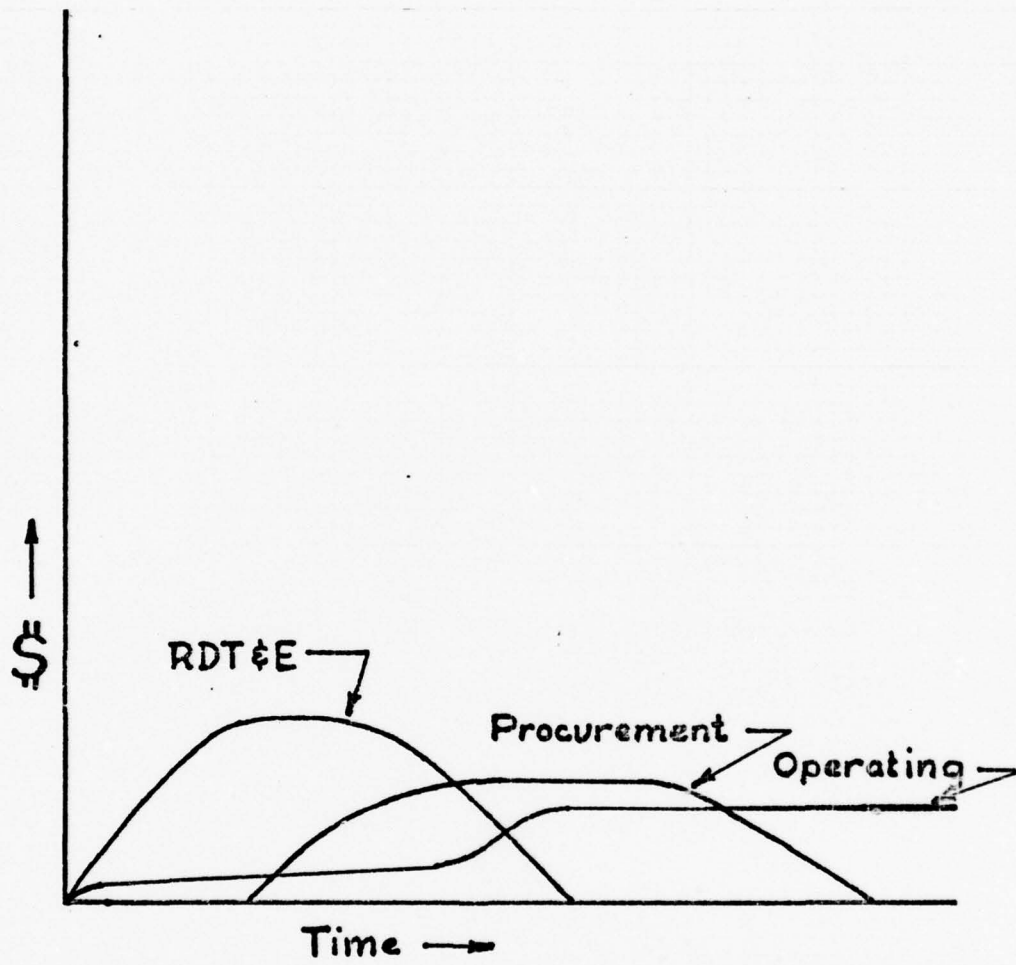


FIGURE 2.

package at a particular WBS level should not be split among cost elements at a corresponding level in the CBS. Table 1<sup>4</sup> depicts a typical CBS used in the development of an LCCE. (For the purpose of this paper, the life cycle will be considered to be ten years.) This CBS is described in a USAMC handbook of Cost Analysis definitions (see bibliography).

In developing the CBS for an LCCE using the technique presented in this paper, and, in particular, in determining how many levels of the CBS one must penetrate in assembling and presenting the estimate, it must be considered that at least 30 cost elements must be present (i.e., non-zero) in each yearly column for statistical validity.<sup>5</sup> In the third-level of the typical CBS presented in Table 1, there are 49 cost elements that may be present in a given year.

The next step in the preparation of the LCCE by this technique is the most critical, by far. The individual assembling the estimate must contact the individuals responsible for the management of the work associated with each element in the CBS and obtain **three decisions** associated with each CBS element. These decisions are:

1. The "most-likely" cost of that element for each year in the life cycle.
2. The "lowest-likely" cost of that element for each year in the life cycle.
3. The "highest-likely" cost of that element for each year in the life cycle.

Table 1<sup>4</sup>

TYPICAL COST BREAKDOWN STRUCTURE

1.0	Research and Development
1.01	Contract
1.011	Engineering*
1.012	Tooling*
1.013	Prototype Production*
1.014	Other
1.015	General and Administrative
1.016	Profit
1.02	In-House
1.021	Engineering*
1.022	Tooling*
1.023	Prototype Production*
1.024	Other
2.0	Investment Non-Recurring
2.01	Contract
2.011	Advanced Production Engineering*
2.012	Tooling*
2.013	Manufacturing*
2.014	Quality Control
2.015	Other
2.016	General and Administrative
2.017	Profit
2.02	In-House
2.021	Advanced Production Engineering*
2.022	Tooling*
2.023	Other
3.0	Investment Recurring
3.01	Contract
3.011	Engineering*
3.012	Tooling*
3.013	Quality Control*
3.014	Manufacturing*
3.015	Leasehold
3.016	First Destination Transportation
3.017	Other
3.018	General and Administrative
3.019	Profit
3.02	In-House
3.021	Engineering*
3.022	Tooling*

\*Cost includes: Direct Labor, Material, Overhead and Other Direct Charges.

3.023	Quality Control*
3.024	Manufacturing*
3.025	Transportation
3.026	Other
4.0	Operation
4.01	Contract
4.011	Personnel
4.012	Consumption
4.013	Leasehold
4.014	Integrated Logistic Support
4.015	Transportation
4.016	Depot Maintenance
4.017	Other
4.02	In-House
4.021	Personnel
4.022	Consumption
4.023	Leasehold
4.024	Integrated Logistic Support
4.025	Transportation
4.026	Depot Maintenance
4.027	Other

\*Cost includes: Direct Labor, Material, Overhead and Other Direct Charges.

These decisions could be stated in another way as a matter of convenience to the individual making the decision, i.e., the "most-likely" cost and a percentage range of that cost from "lowest-likely" to "highest-likely." The cost decision then would be stated as, "The 'most-likely' cost for this element is X dollars with a range from 90 to 115 per cent."

There is a large number of techniques available to the cost element manager to assist him in arriving at his cost decisions. It is beyond the scope of this paper to address these techniques since, in fact, our requirement is for the products of the decision-making process only. The uncertainty present in these cost decisions naturally reflects the bias of the individuals making the various decisions. Since these individuals are the managers of the work associated with the cost elements, they are presumed to be the most knowledgeable individuals for those cost elements and should provide the "best estimates."<sup>6</sup>

Note that throughout the preceding paragraphs the individual CBS element managers were asked to make cost "decisions" and not "estimates." The choice of this word was deliberate and was intended to convey to the reader some idea of the intensity of the reasoning which those managers should apply to the process of developing costs. The importance of this concept cannot be overemphasized. From this point on, the estimate is assembled by universally accepted statistical methods. But no statistical methods, no matter how valid, can assemble a good cost decision if the basic input is developed in an offhand manner. It must



also be remembered that this is the point at which the uncertainty about cost growth is input into the cost decision--by the individual with the most detailed knowledge of the cost element and its likelihood of being affected by cost growth. Statistical procedures to follow can "smooth" the inconsistencies which may be in the estimate, but they cannot improve the basic data provided by the individual managers.

While making his cost decisions, the manager must consider also the time frame in which the costs will be incurred. From Figure 1, it is apparent that his range of possible costs (uncertainty) should be much broader for a cost element in year 10 of the life cycle than for one in year 2.

The basic computational techniques that follow are based upon the application of the beta probability density function to the data obtained in the preceding step (See Wilton's "Notes on the Beta Distribution," bibliography, or any comprehensive text on statistics for a further description of this function). The beta function is chosen for a number of reasons, including the following:

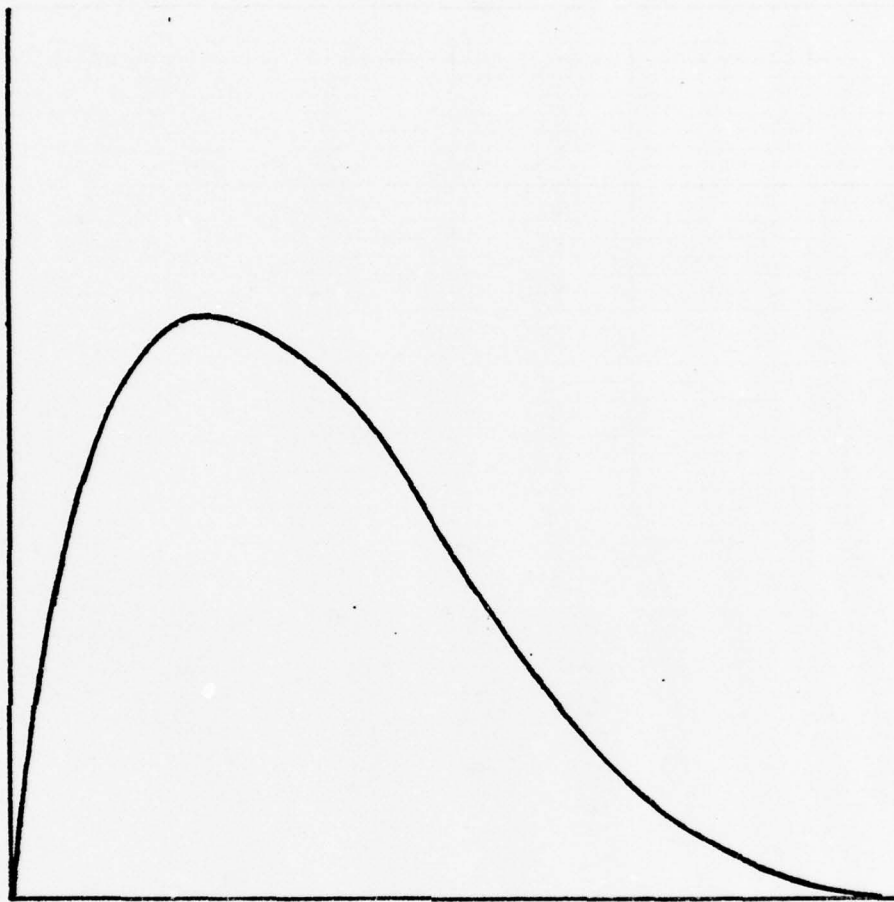
1. With proper shaping parameters, the beta distribution fits what intuitively seems to be the probable distribution of costs. The common skewed-right shape of the beta curve accommodates the optimism inherent in cost estimating wherein the "most-likely" cost estimate is usually closer to the "lowest-likely" cost than the "highest-likely" cost.

2. The beta distribution has upper and lower bounds. The selection of an unbounded distribution probably would be of small

consequence in a methodology such as this, but bounds or constraints do exist in the real world so it is appealing to select a distribution that reflects these conditions.

3. Accurate approximations are available for the mean and variance of the beta distribution that will greatly simplify calculations to follow.

The major disadvantage associated with the use of the beta distribution is that it assumes that costs are distributed in an unimodal manner (or, in a special case, along a straight line). Unless there are obvious indications to the contrary in a particular application, this limitation does not seem to be overly severe or to render the total methodology invalid, in general. The requirement to model exactly (or at least to closely approximate) the actual cost distribution is further mitigated since later in the methodology the extremely powerful "Central Limit Theorem" will be invoked with regard to the yearly cost data (See Murphy's report, bibliography, for a description of the rationale supporting this application). This theorem implies that if the cost elements are independently distributed and have a finite variance and arithmetic mean, then the yearly variances and means may be readily computed. Further, the distributions of costs for the yearly totals approach nearer the normal distribution as the number of cost elements increases. This is true regardless of the distribution of the individual cost elements. (A typical beta distribution plot is shown in Figure 3.)



Typical Shape - Beta Curve

FIGURE 3.

The independence of the cost elements in the CBS previously described is open to question. In the development of this technique, independence will be assumed and the individual considering its use must determine whether his cost elements are actually independent, or, if not, whether the dependence is of a sufficient degree to invalidate the methodology in that particular situation.

(The true power of the Central Limit Theorem is not evident in this discussion since we have limited ourselves to cost elements distributed in accordance with the beta function for the sake of simplicity. Any function closely approximating the actual cost distribution can, in fact, actually be used for a cost element as long as the variance and arithmetic mean can be computed and a "most-likely" cost can be determined.)<sup>7</sup>

The mean and variance of each cost element in each of the columns (yearly and total costs) can now be computed using the following approximating formulas:

$$\text{Mean} \quad M = \frac{A + 4M + B}{6}$$

$$\text{Variance} \quad \sigma^2 = \left(\frac{B - A}{6}\right)^2$$

Where  
A = Lower bound of distribution  
M = Mode of distribution  
B = Upper bound of distribution

Exact expressions for the mean and variance of the beta distribution are available<sup>8</sup> but the approximations given above are of sufficient accuracy (and much simpler to use) for the purpose of this methodology (See Murphy's report, bibliography).



The rationale behind the requirement for the cost decisions that the manager was earlier tasked to make is now obvious. The "most-likely" cost becomes M, the mode, and the "lowest-likely" and "highest-likely" costs become A and B, respectively. (If the range of costs is given in percentages, they can be easily converted to dollar values corresponding to the "lowest-likely" and "highest-likely" costs.) In this manner we have provided the managers with a set of intuitive guidelines to follow as they make their cost decisions that do not require a depth of statistical knowledge to understand or communicate.

After the means and variances for each element are computed, they and the modes are summed for each column (i.e., yearly cost data and total estimate). The square roots of the sums of the variances for each column are then extracted, giving a standard deviation (in dollars) for each yearly column and the total column.

The Central Limit Theorem is invoked at this point in the estimating process. By the use of this powerful tool, the distribution of the column total costs can be considered to be normal.<sup>9</sup> The implication of this is that a confidence interval (i.e., 90%, 95%, 99%, etc.) can be selected and standardized normal random variable tables can be used to display the probable distribution of costs. The data developed to this point can then be displayed in a meaningful manner as we will see in an example to follow.

### III. Sample Computations

The example in this section will be structured using the CBS developed in the preceding section.<sup>10</sup> To avoid repetitive numerics,



only typical calculations will be included. To simplify the CBS array, the index numbers from Table 1 rather than the descriptive titles of the cost elements will be used. All cost data are in millions of dollars.

A. Display the necessary cost data for each cell as follows:

Cost Element:	1.011	A:	10.1
Life Cycle Year:	3	B:	11.9
		M:	10.7

B. Compute the means and variances for each cell as follows:

$$\text{Mean } M = \frac{A + 4M + B}{6} = \frac{10.1 + 42.8 + 11.9}{6} = 10.8$$

$$\text{Variance } \sigma^2 = \left(\frac{B - A}{6}\right)^2 = \left(\frac{1.8}{6}\right)^2 = 0.09$$

C. Compute the sums of the means, modes, and variances for each column as follows:

CBS Element	Mean	Year 3 Variance	Mode
1.011	10.8	0.09	10.7
1.012	5.1	0.48	5.0
1.013	8.2	0.69	8.1
...	...	...	...
4.027	1.7	0.01	1.6
Totals	128.7	65.61	123.2

D. Compute the square root of the variance for each column as follows:

$$\sqrt{65.61} = 8.1$$

This calculation yields the standard deviation (in dollars) for this yearly data.

E. Establish the confidence level. In this example, we will assume that a 95% confidence level will be used.

F. Using standardized normal random variable tables, determine the number of standard deviations on either side of the mean which include 95% of the distribution. In this case, the appropriate number is 1.96.

G. Multiply the standard deviation for each yearly column by the number determined in step F, as follows:

Year 3

$$1.96 \times 8.1 = 16.2$$

H. Add and subtract the number developed in step G to the mean for each year, as follows:

$$M + 16.2 = 128.7 + 16.2 = 144.9$$

$$M - 16.2 = 128.7 - 16.2 = 112.5$$

I. Plot the sum of the modes and the two values developed in step H above for each year (but not for the total column) on a graph as shown in Figure 4a. The data for all years can then be added as shown in Figure 4b. A smooth curve can then be drawn through the upper points, through the sums of the modes, and through the lower points as shown in Figure 4c. The data points can then be de-emphasized as shown in the final chart, 4d, which would be used as the display chart. The only purpose of the curves in Figure 4d is to emphasize the range of the uncertainty and to provide some visual indication of the relative probability of an overrun or underrun of the "most-likely" cost. Figure 4e is another method of displaying this uncertainty by the use of bars. The length of the bar represents the range of the uncertainty, the horizontal hatching represents the relative probability of an overrun, and the vertical hatching represents the relative probability of an underrun.

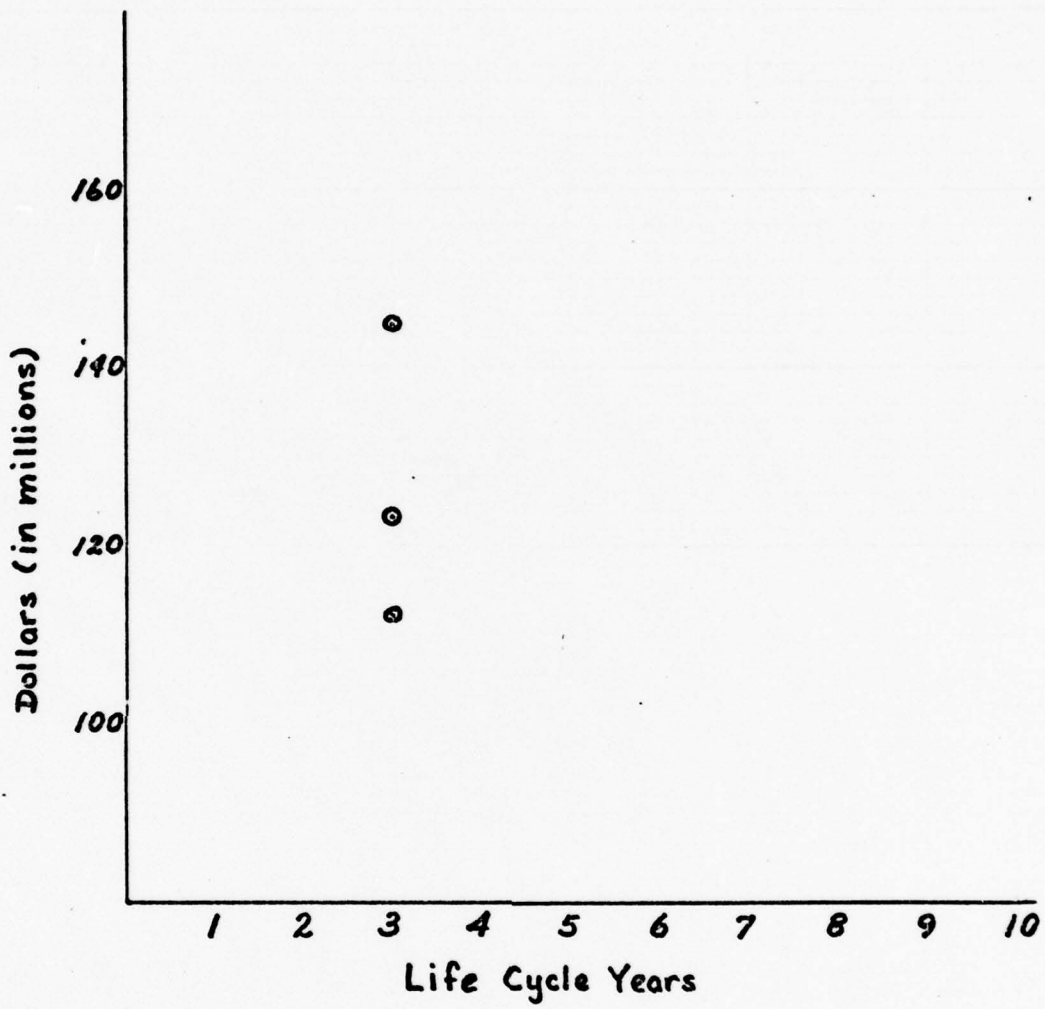


FIGURE 4a.

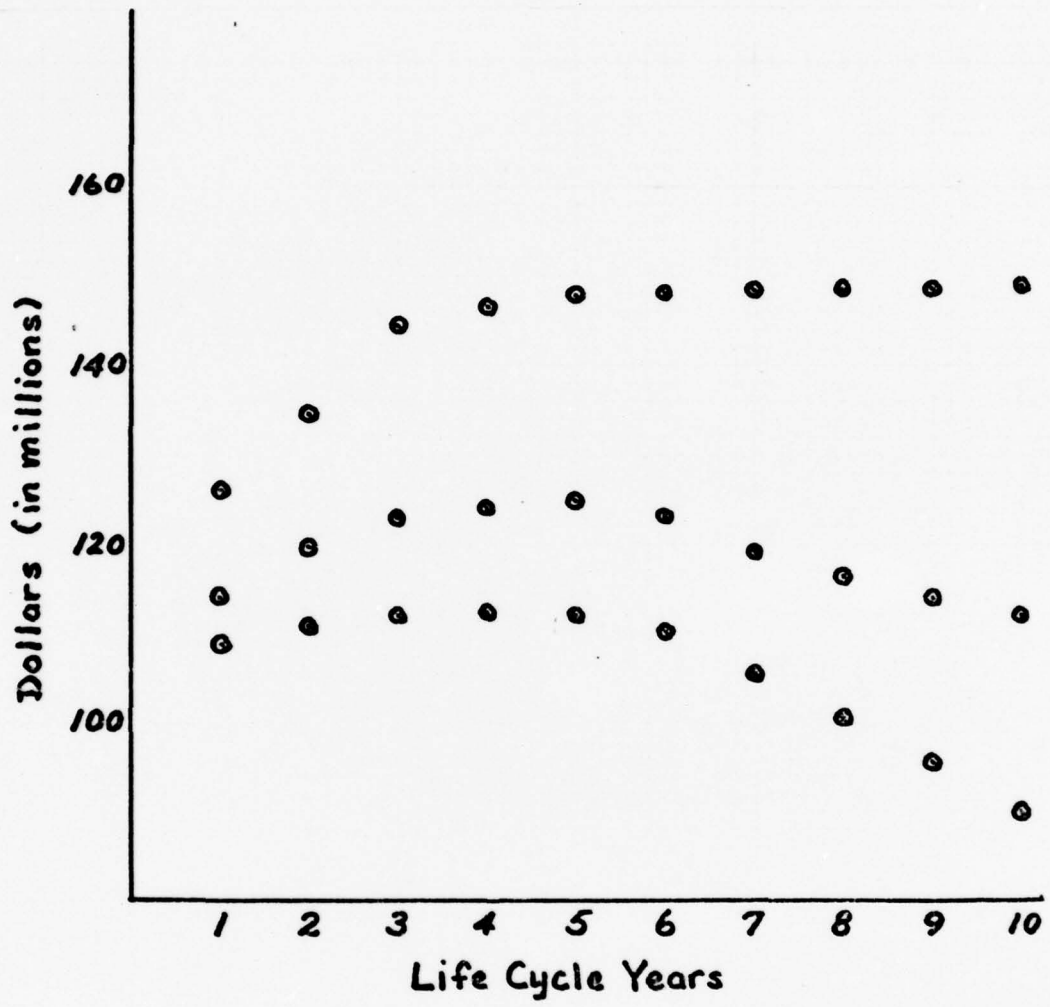


FIGURE 4b.

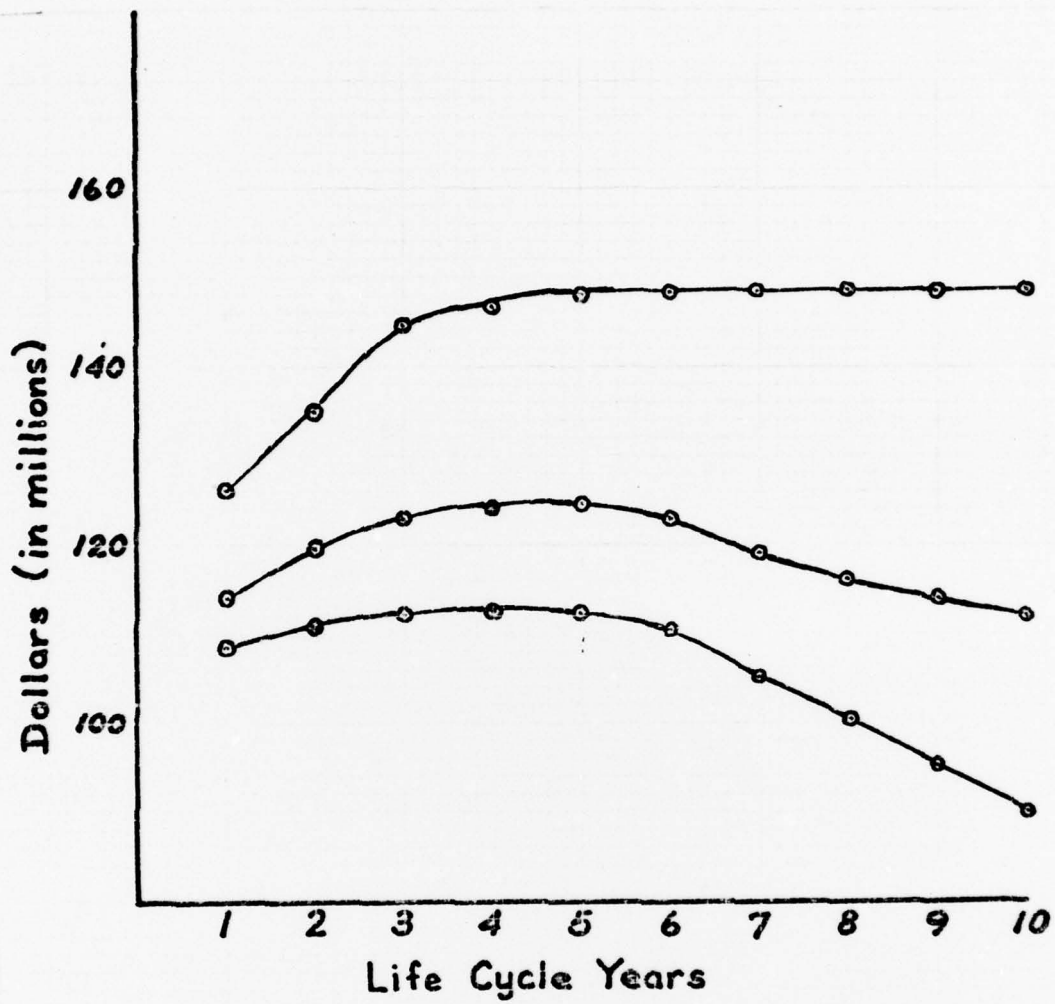


FIGURE 4c.



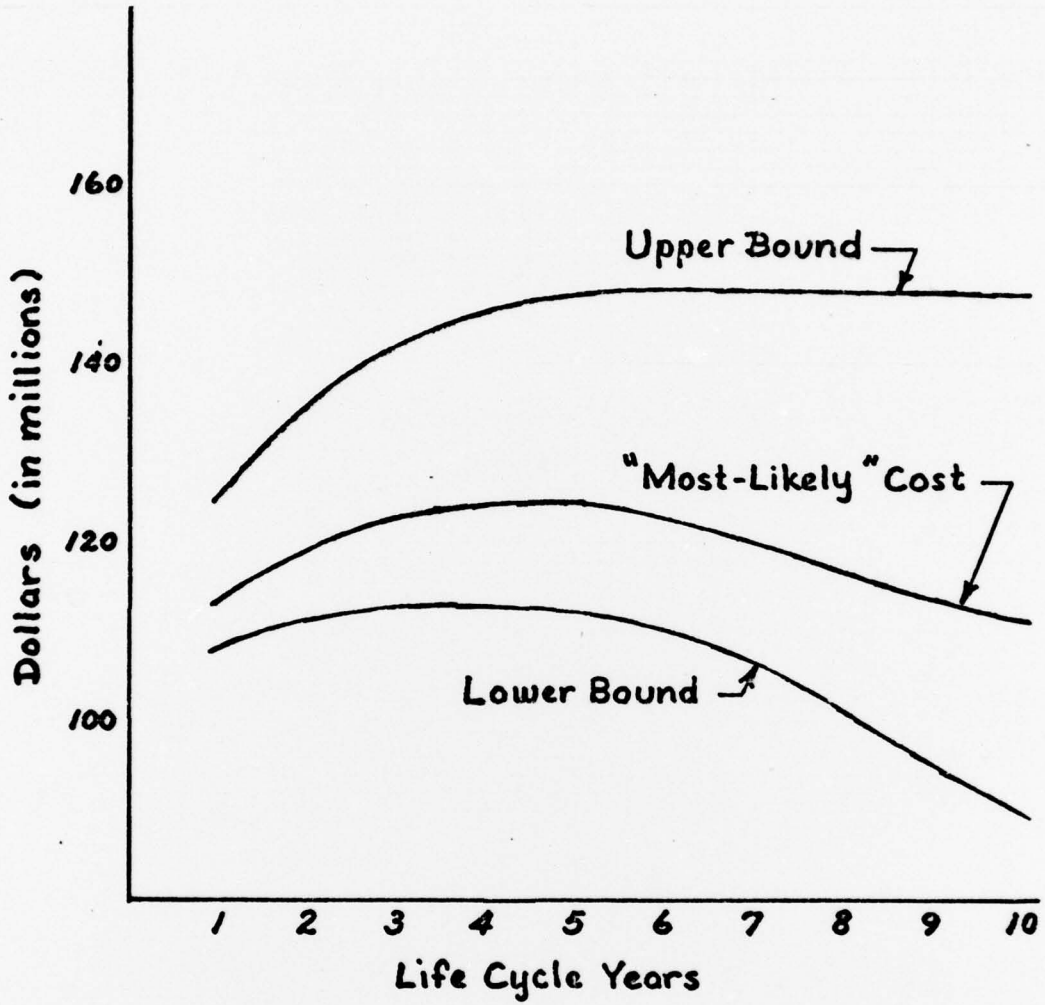


FIGURE 4d.

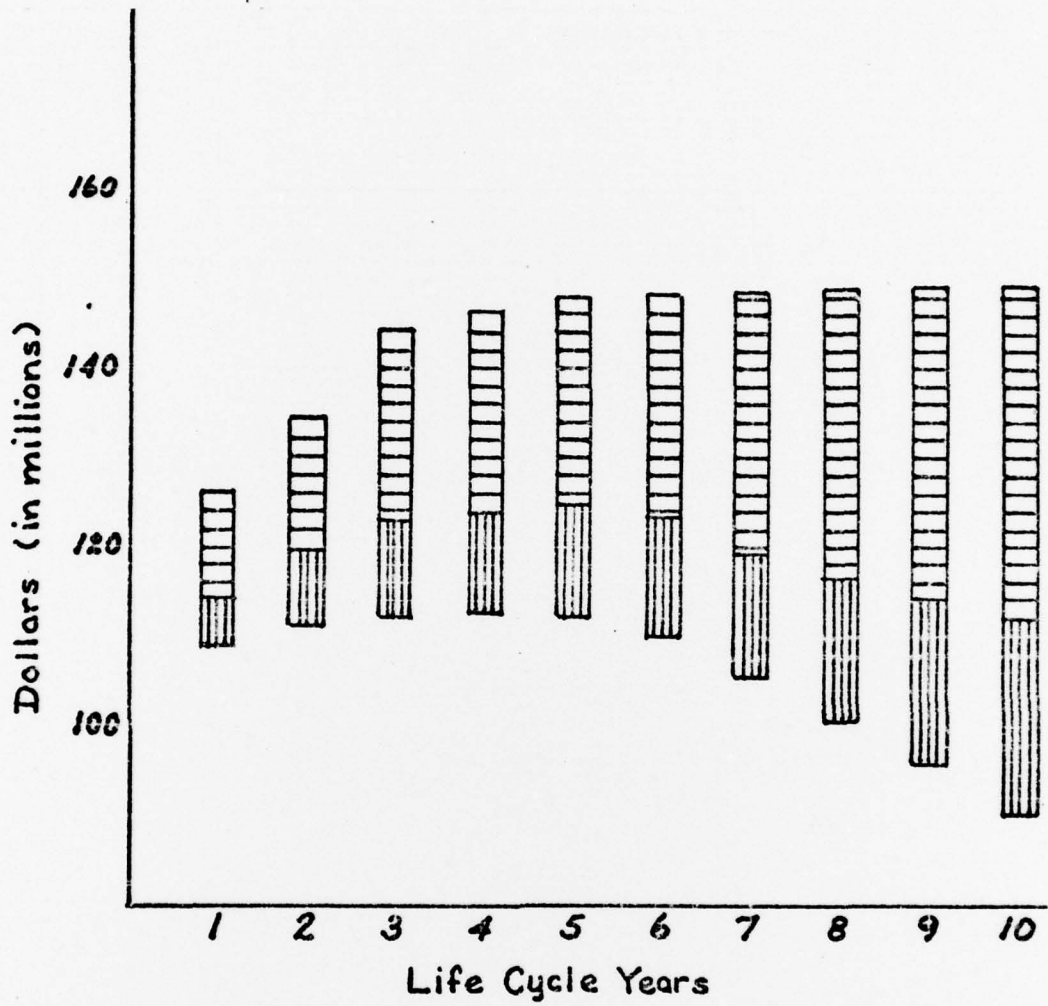


FIGURE 4e.

J. The curve connecting the sums of the modes is the "most-likely" cost curve. (In a one-point estimate, this would be the cost developed for each year.) The outer curves of Figure 4d represent the limits of a 95% probability (or confidence) band about the yearly means.

K. The data for the total cost column is assumed to be the following:

Mean	539.0
$\sigma$	26.6
Sum of modes ("most-likely" estimate)	525.0

This data has been plotted in Figure 5. The 95% confidence limits are as shown on that figure. We also note that the sum of the modes (which we have been calling the "most-likely" estimate) is  $.53 \sigma$   $((539.0 - 525.0) \div 26.6)$  to the left of the mean. By the use of standardized normal random variable tables, it can be shown that the probability of underrunning the "most-likely" estimate is 29.9% and the complementary probability of overrunning this estimate is 70.1%.

Figures 4d (or 4e, if that type display is preferred) and 5 are the displays that would be used to present the uncertainty associated with the LCCE.

The mode of any of the distributions developed in this methodology can be considered to be the "most-likely" estimate for a particular year (or for the total estimate) since it represents the sum of the individual "most-likely" estimates. This, in fact, would be the estimate provided in a single-point estimating technique. The means

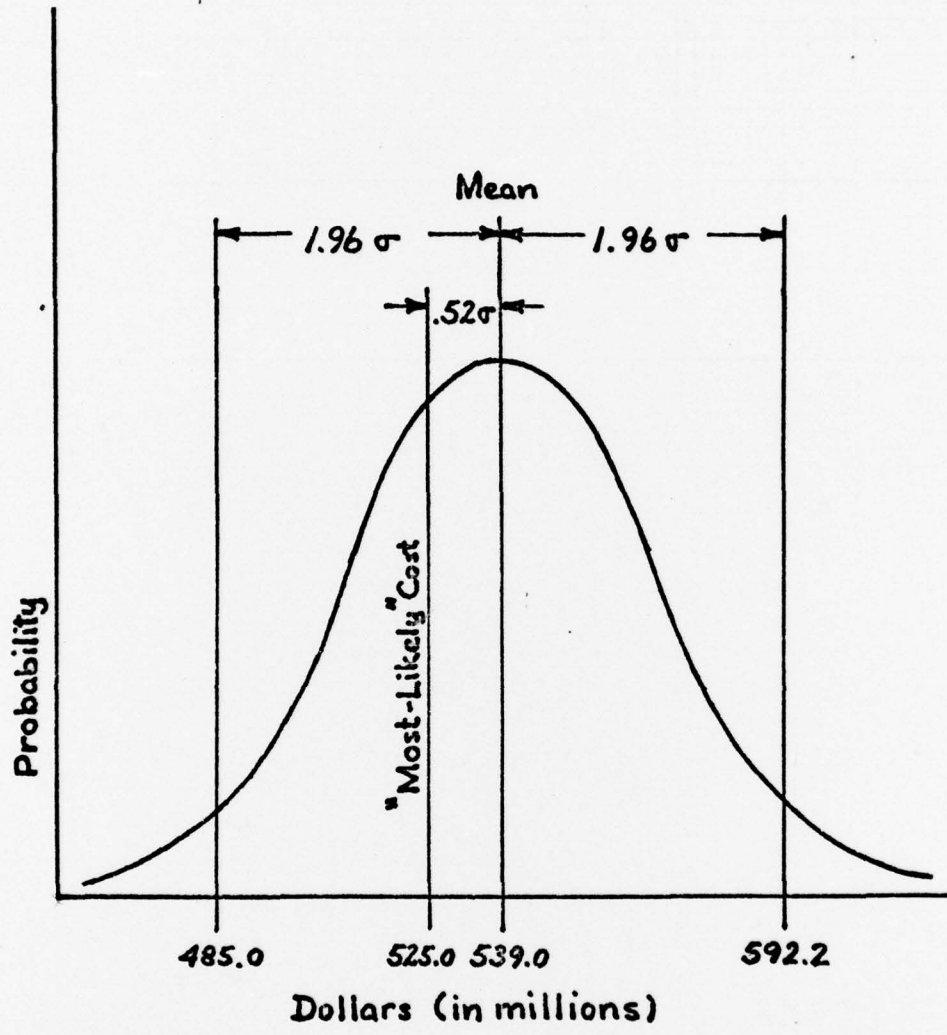


FIGURE 5.

are normal distributed and represent the point at which actual costs are equally likely to overrun as to underrun since the column totals are normally distributed. This is demonstrated in Figure 5. The means are thus the logical basis for the establishment of the confidence bands and limits.<sup>11</sup>

#### IV. Conclusions

The technique developed and presented in this paper is a valid and effective method of presenting the uncertainty associated with cost estimates.

The technique is valid for other types of cost estimates as well as LCCE.

The technique is simple to apply and obviously adaptable to computerization.

The information that is displayed is presented in terms of intuitively appealing concepts.



## FOOTNOTES

1. The development of any methodology to display the results of a cost estimate necessarily involves some discussion of the procedures used in developing that estimate. It is not the purpose of this paper to develop an estimating technique for Life Cycle Cost Estimates. The estimating technique used in this paper for illustration of the display methodology is derived from the standard USAMICOM three-point estimating procedure described in the second-referenced document in the attached bibliography. This estimating technique was originally developed only for a total systems cost estimate. The concept embodied in this technique has been expanded within this paper for use with a LCCE and applied to a standard USAMC Cost Breakdown Structure for Life Cycle Costs. It will become apparent to the reader that other estimating techniques are equally adaptable to the display methodology developed herein. The conditions that these other estimating techniques must satisfy are contained in the body of this paper.
2. Since published as AR 1000-1, 30 June 1972.
3. Language similar to this has appeared in policy documents for a considerable period of time. However, cost estimates in general, and LCCE's in particular, have not commonly been expressed in terms of limits within which the ultimate cost of the program may be expected to fall. It is the author's contention that the reason for this is that the majority of the techniques previously developed have been too complex and time-consuming to be easily applied by the program management personnel who most frequently assemble these estimates. This paper attempts to solve this management problem and fill an existing management void by extending a simple, previously-developed, total systems cost estimating technique to LCCE's.
4. USAMC, Key Cost Analysis Definitions, p. 12.
5. Murphy, E. L., Jr., Statistical Methods of Measuring the Uncertainty of Cost Estimates, p. 2.

6. The process of developing a cost estimate is often thought of as a discrete operation. In actuality it is most frequently an iterative process - estimates are continually revised as new information becomes available, additional constraints are imposed upon the estimate, etc. In this sense, the uncertainty originally associated with a cost element by the estimator may be obscured by subsequent requirements. Care should be taken to avoid this circumstance, if possible, or to highlight it in an accompanying narrative, if not. It must also be remembered that the basic cause of uncertainty in a cost estimate is not the result of the statistical process under which it was assembled but rather is due to the fact that, in the final analysis, the estimate is simply a judgement made by an ordinary human being. It is for the managers who use the information to decide upon the validity of the judgement given a particular expert's knowledge and the cost element being assessed. See Schlaifer, Analysis of Decisions Under Uncertainty, Chapter 6, for a short (17 pages) exposition on the use and interpretation of data gotten from experts.

7. Murphy, op. cit., p. 2.

8. Wildon, Notes on the Beta Distribution, p. 2.

9. Murphy, op. cit., p. 2.

10. This technique for calculating the yearly means and standard deviations is developed in Murphy, op. cit. Since this is a standard technique used within USAMC, its use has been extended within this paper to apply to the dollar streams associated with LCCE's. The notation in this paper follows Murphy.

11. Once the estimate has been completed and the confidence interval has been established, several important uses can be made of the result in addition to the basic purpose of displaying the uncertainty. Thresholds can be established by selecting the appropriate probability

that one is willing to accept of their being exceeded and determining the corresponding dollar value from the display. The probability of exceeding an already established threshold can also be determined by using the standardized normal random variable tables to determine the probability associated with exceeding a particular dollar value.

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5. Department of Defense Directive 5000.1. Acquisition of Major Defense Systems. DDR&E, 13 July 1971.  
Basic policy guidance on all phases of materiel acquisition.
6. Department of the Army. Letter of Instructions (LOI) for Implementing the New Materiel Acquisition Guidelines. DAFD-SDY, 23 August 1972.  
DA implementation instructions for DODD 5000.1. Since published as AR 1000-1.



7. U. S. Army Materiel Command. Key Cost Analysis Definitions. Office of the Comptroller, October 1972.

Pamphlet containing definitions of terms used in cost estimates. Includes a standard Work Breakdown Structure (WBS) based on Mil Std 881 and Cost Elements based on AR 37-18.

8. U. S. Army Safeguard System Command. Army Programs Cost Estimating Discipline, SAFSCOM Regulation No. 11-1, 27 July 1972.

Regulation prescribing policies, responsibilities, and procedures relative to cost estimates for the Safeguard System and the Site Defense Program.



## APPENDIX

### COST GROWTH DEFINITIONS

1. General. This appendix contains the text of the Deputy Secretary of Defense Memorandum, dated 5 August 1970, subject: Cost Growth Definitions. These definitions will be used in documenting cost changes from one cost estimate to a succeeding estimate.

2. Text of Memorandum.

As indicated in my memorandum of November 26, 1969, the views of each addressee were obtained relative to the tentative definition of "cost growth" which was distributed at that time. These views have been carefully considered and changes made to improve the clarity or the categorization of the reasons for "cost growth."

Distributed with this memorandum is the definition of "cost growth." This will apply to the net increased cost to the Government of items or services procured or to be procured. There are nine listed categories of reasons for cost growth which provide the visibility required.

This definition for "cost growth" or "cost decrease" will be used when necessary to explain programs, budgets or contracts. For internal management purposes, any of the categories may be grouped or further stratified to serve management needs. However, any grouping of categories thus used must be capable of being identified by the nine individual categories if this is later required for reconciliation purposes.

It is expected that this "cost growth" definition will be used wherever appropriate in management reporting, testimony, official correspondence or speeches, to explain instances of cost growth.

### COST GROWTH

Cost Growth is the net change of an estimated or actual amount from a base figure previously established. The base must be relatable to a program, project or contract and be clearly identified including source, approval authority, specific items included, specific assumptions made, date and amount. The events causing "Cost Growth" must then be identified by one or more of the following categories and the appropriate amount of each shown as "estimated" or "actual." These categories do not necessarily determine whether the cost growth could have been avoided by the Government or contractor or both. They provide the essential visibility and information required to determine the cause of the cost growth.

## CATEGORIES

1. Engineering Change - An alteration in the physical or functional characteristics of a system or item delivered, to be delivered, or under development, after establishment of such characteristics.
2. Quantity Change - A change in quantity to be procured, the cost of which is computed using the original cost-quantity estimating relationships, thereby excluding that portion of the current price attributable to changes in any other category.
3. Support Change - A change in support item requirements (e.g., spare parts, training, ancillary equipment, warranty provisions, Government furnished property/equipment, testing, etc.).
4. Schedule Change - A change in a delivery schedule, completion date or intermediate milestone of development or production.
5. Unpredictable Change - A change caused by Acts of God, work stoppage, Federal or State Law changes or other similar unforeseeable events. Unforeseeable events include extraordinary contractual actions under the authority of PL 85-804 except that formalization of informal commitments should be reflected under the other categories, as appropriate and not included under this category.
6. Economic Change - A change due to the operation of one or more factors of the economy. This includes specific contract changes related to economic escalation and the economic impact portion of contract quantity changes computed using the original contract cost-quantity relationship. This also includes changing real dollar amounts in program estimates to reflect (1) revised economic impact or (2) defined contract amounts.
7. Estimating Change - A change in program or project cost due to refinements of the base estimate. These include mathematical or other errors in estimating, changing the base year of the constant dollars, revised estimating relationships, changing from constant dollars to real dollars, etc.
8. Contract Performance Incentives - A net change in contractual amount due to the contractor's actual performance being **different** than was predicted by performance (including delivery) incentive targets; as differentiated from cost incentive targets; established in an FPI or CPIF contract. This category also includes any changes in amounts paid or to be paid a contractor due to (1) award fee for performance accomplishments under a cost plus award fee contract or (2) the sharing provisions of a value engineering incentive clause included in any type of contract.

9. Contract Cost Overrun (Underrun) - A net change in contractual amount over (under) that contemplated by a contract target price (FPI contract), estimated cost plus fee (any type cost reimbursement contract) or redeterminable price (FPR contract), due to the contractor's actual contract costs being over (under) target or anticipated contract costs, but not attributable to any other cause of cost growth previously defined. Offsetting profit or fee adjustments attributable to cost incentive provisions, if any, shall be considered in determining the net contract cost overrun (underrun).