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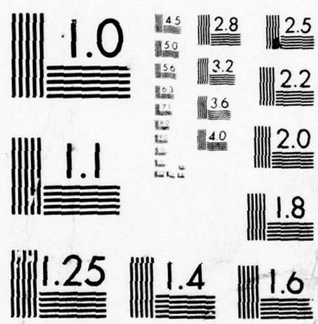
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TECHNICAL REPORT NO. 6-780

STUDY OF VIBRATION IN CONCRETE

Report 3

MECHANICS OF MOTION OF FRESH CONCRETE

by

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Mechanical impedance tests on fresh concrete revealed the properties that resist motion. It was found that there were two distinct mechanical impedance curves depending on whether the dynamic stress applied was above or below a threshold level. At low stress levels, the impedance of the mixture was almost flat, possessing high damping, stiffness, and mass characteristics. No resonant frequency was found, but on the contrary, a wide spectrum of frequencies would produce sufficient motion for consolidation.

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20. ABSTRACT (Continued)

At higher stress levels, the impedance drops sharply by a factor of 5 to 10 at the threshold level. The motion then is mass controlled with little or no effect from stiffness or damping. Because inertia is the primary hindrance to motion, Newton's second law of motion, F = MA, describes the motion. Acceleration then is the best indicator of consolidation rather than velocity or displacement.

It is likely that fresh concrete requires a continuous spectrum of energy over a wide frequency range to produce consolidation rather than discrete or optimum frequencies. Since concrete has a continuous spectrum of particle dimensions, it is probable that vibrators should supply energy over a spectrum of frequencies. Tests are planned to study the consolidation of concrete with impulses rather than a sinusoidal motion as impulses contain energy over a wide band of frequencies.

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SUMMARY

This research was conducted to study the nature of motion of fresh concrete during vibration, in an effort to improve consolidation. Field vibrators could be designed, if the motion were understood, on a basis of theory rather than empirically. The method of measurement used was the mechanical impedance technique. A model sinusoidal wave form was derived from an electromagnetic vibrator. Variables of motion, such as force, frequency, and power, could be controlled; the response of the material could be measured in terms of displacement, velocity, and acceleration. Various combinations of force and frequency were found to cause liquefaction of the concrete and a simultaneous sharp drop in impedance by a factor of 5 to 10. It is believed that the critical or threshold stimulus required to produce this impedance breakdown is the physical phenomenon impulse, a product of force and time. Before breakdown the motion of the concrete was a function of mass, stiffness, and damping. After breakdown those other restrictions to motion vanished, and the impedance was a function solely of mass.

A high-density concrete will have a higher impedance than low-density concrete, if all other things are equal. The most reliable indicator of the desired motion in fresh concrete was the response to acceleration. Velocity and displacement were less important in describing the ideal motion. The impedance of concrete is like that of water, indicating the possibility of using hydrodynamic principles to describe motion of fresh concrete. There is also evidence that the compressional wave velocity increases sharply and simultaneously at the impedance breakdown by the same factor that the impedance decreases. Furthermore, fresh concrete does not have a resonance before breakdown because of high damping or after breakdown because there is no stiffness. Since field vibrators do not match the concrete in its mechanical impedance, energy can only move from vibrator to concrete, and not vice versa. A study is being made to incorporate stiffness characteristics in a field vibrator so that the concrete-vibrator system is matched in impedance and becomes resonant. This could conserve large amounts of

energy. Tests are being planned to determine the impedance of the concrete from controlled impulses or impacts. Variables that influence the effect of an impulse are the height of the pulse in force units, the width of the pulse in time, the rise time of the pulse, and the repetition rate.

PREFACE

This study of the behavior of fresh concrete during vibration forms a part of Work Unit 31251, "Determination of Adequacy of Consolidation of Concrete," and was authorized by the Office, Chief of Engineers (OCE), U. S. Army. The OCE Technical Monitor was Mr. J. A. Rhodes, DAEN-CWE-DC.

The research was conducted by Mr. A. M. Alexander, Research Physicist, Concrete Laboratory, U. S. Army Engineer Waterways Experiment Station (WES), during the period 1972-1975 under the general supervision of Messrs. Bryant Mather, Chief of the Concrete Laboratory; R. V. Tye, Jr., and Leonard Pepper, former Chiefs of the Engineering Sciences Division (ESD); and Mrs. Katharine Mather, Chief of ESD. Mr. B. R. Sullivan, Chief of the Engineering Physics Branch, was the immediate supervisor; he had made previous studies of the dynamics of motion of fresh concrete using the mechanical impedance technique. Mr. S. W. Guy, under the direction of Mr. H. C. Greer, Instrumentation Services Division, was in charge of the mechanical impedance equipment. The report was written by Mr. A. M. Alexander.

Directors of the WES during the study and the preparation and publication of this report were BG E. D. Peixotto, CE: COL G. H. Hilt, CE: and COL J. L. Cannon, CE. Technical Director was Mr. F. R. Brown.

CONTENTS

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
degrees (angular)	0.01745329	radians (rad)
degrees (angular) per second	0.01745329	radians per second (rad/sec)
feet	0.304800*	metres (m)
feet per second	0.304800*	metres per second (m/sec)
inches	0.025400*	metres (m)
inches per second	0.025400*	metres per second (m/sec)
inches per second squared	0.025400*	metres per second squared (m/sec^2)
picocoulombs per inch per second squared	39.37007874	picocoulombs per metre per second squared ($pC/m/sec^2$)
picocoulombs per pound (force)	0.2248089237	picocoulombs per newton (pC/N)
pounds (force)	4.448222	newtons (N)
pounds (force) per inch	175.1268504	newtons per metre (N/m)
pounds (force)-seconds squared per inch	175.1268504	newtons-seconds squared per metre ($N \cdot sec^2/m$)
pounds (mass)	0.4535924	kilograms (kg)
pounds (mass) per cubic foot	16.01846	kilograms per cubic metre (kg/m^3)

* The asterisk after the sixth decimal place indicates that the conversion factor is exact and that all subsequent digits are zero.

STUDY OF VIBRATION IN CONCRETE

MECHANICS OF MOTION OF FRESH CONCRETE

PART I: INTRODUCTION

Purpose

1. Mechanics is the branch of physics that deals with the motion of bodies and with the forces that bring about that motion. Understanding the nature of movement of fresh concrete under vibratory forces will allow consolidation by vibration to be specified mathematically rather than empirically. This should result in efficient vibration for consolidation. This investigation seeks to understand the mechanics of motion of fresh concrete under vibration to improve the consolidation of concrete.

Background

2. Proportioned concrete is usually consolidated by vibration.[1] Because the motion of fresh concrete under vibration is not understood, much more energy may be used in consolidation than is necessary. A. G. A. Saul[2] estimates that internal vibrators lose up to 90 percent of their energy through friction. Reduction of this loss to 80 percent would double the energy available for consolidation. A review of literature on concrete vibration by Sugiuchi[3] shows that explaining the behavior of fresh concrete under vibration has defied theoretical analysis and that no viscoelastic model has been developed to simulate the motion of fresh concrete. The empirical approaches have oversimplified the relationship of all the parameters of motion, and experiments give data that are restricted in application. The literature shows that research investigators try to study simultaneously the mechanics of motion, consolidation, and the field vibrator. Consolidation and vibrators can only be fully understood after the mechanics of motion of fresh concrete is understood.

3. Measurements of motion are complicated in fresh concrete. The

variables change with the frequency; all may influence consolidation. Variables include power, velocity, force, acceleration, and displacement; all are important and vary simultaneously with frequency, unless the system producing the motion is able to control chosen variables. Therefore, field vibrators should not be used to study the mechanics of consolidation.

a. Field vibrators do not control variables such as force and power; a variable cannot be held constant while the response of the others is observed. It is likely that the wrong variable will be interpreted as the cause of a significant motion in the concrete.

b. Field vibrators do not produce a clean repetitive wave form. The wave form is neither sinusoidal, triangular, nor rectangular; it differs from all the standard wave forms and is a complex type of motion unsuitable for accurate measurements. Much effort and sophisticated electronic equipment would be needed to derive the velocity and displacement signals from the acceleration signals; this operation is necessary in measuring motion.

c. It is unnecessary to understand the field vibrator to study the mechanics of motion of fresh concrete. The properties that determine the motion of fresh concrete are elasticity, mass, and damping; each of these or all three together can be altered only over a limited range, and each is fixed for a given concrete at a given instant. The field vibrator can be redesigned to be efficient in this range or in a particular region of it. Therefore, the concrete should be studied rather than the vibrator or the vibrator-concrete system at this stage in our knowledge.

4. Because of the three considerations described in the paragraph above, the mechanical impedance technique was used to study the motion of concrete. Most of the difficulties of measurement are circumvented by this technique. It can use simple harmonic motion that is the simplest of all motions to describe for the vibratory excitation. The technique allows control over the force level as well as of frequency and power. Because a concrete mixture has many variables that might influence the mechanics of vibration, Sullivan[4] decided on a systems-science approach. In this technique, he measured the mechanical impedance of the mixture. Although the information obtained was incomplete, the difficulty was caused by the lack of adequately sophisticated equipment and not the method of test.

5. The study reported herein deals with the mechanical impedance technique applied to fresh concrete. It can be seen intuitively that the consolidating effect is increased when the motion increases, so the idea of an impedance measurement is a natural conclusion. Mechanical impedance of a system containing damping is a complex number giving the ratio of an applied force to a resultant velocity at a given frequency:[4]

$$\bar{Z}^{*} \text{ (Mechanical impedance at frequency } \omega) = \frac{\text{Force } (\bar{F}) \text{ at } \omega}{\text{Velocity } (\bar{V}) \text{ at } \omega}$$

where ω is angular frequency ($2\pi f$). The quantity measured is velocity. Mechanical impedance represents the degree to which the concrete resists motion. If a material is excited at some fixed frequency with a given force level and the velocity measured is low, the material is said to have a high impedance. If the material responds with a high velocity at the same frequency driven by the same force level, the impedance is low. If the increase in velocity is significant, the acceleration and displacement will also increase; hence less power is required to maintain a given motion. Resonant frequencies or frequencies of low impedance require a minimum input of energy to maintain very large magnitudes of motion. Mechanical impedance techniques appear to provide an ideal means to study the mechanics of motion of fresh concrete.

6. One concrete mixture was used for most of the tests, since it was desired to hold variables to a minimum until some phenomena could be consistently measured with repeats of the concrete mixture. Furthermore, if a significant feature that might be used to improve future consolidation existed, it would be in any single regular concrete mixture. After the properties of concretes of this concrete mixture were understood, other mixtures were tested to confirm the results on the standard mixture.

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* For convenience, symbols are listed and defined in the Notation (Appendix F).

PART II: THEORETICAL CONSIDERATIONS

7. Traditionally, mechanical impedance measurements have been made on various materials, foundations, structures, etc., to locate fundamental resonant frequencies. At these frequencies material destruction of the material, foundation, or structure can occur because small forces cause large magnitudes of motion. Once these frequencies are located, dampers or stiffeners as needed are incorporated into the mechanical system to eliminate the resonant characteristics. Contrary to the traditional use of mechanical impedance measurements, efforts have been made to locate and use low impedance or resonant frequencies in fresh concrete to obtain the most motion for the least force to produce more consolidation with a minimum amount of energy input.

8. Materials including fresh concrete can be modeled mathematically by three ideal elements in various series and parallel configurations. They are mass elements, stiffness or spring elements, and dashpots or dampers. An ideal mathematical mass element is infinitely rigid or stiff and possesses no damping characteristics. An ideal spring is without mass and has no damping characteristics. An ideal dashpot is without mass and has no stiffness. Real elements can be constructed and found in nature that closely approximate the ideal elements. Often a material or system that would require an infinite number of mass elements, dampers, and stiffness elements to define it can be described by an operation called lumping. One or a limited number of mass elements, stiffness elements, and dampers lumped together can describe the motion found in a material designated in vibration textbooks as a continuous medium. Fresh concrete belongs in the class of continuous media as mass, elasticity, and damping are distributed uniformly over the whole system rather than being discrete and separable elements.

9. Consolidation falls under the subject of dynamics, which deals with motion as well as the forces that bring about that motion. The consolidation of fresh concrete falls under the subject of forced vibration as opposed to free vibration, since consolidation stops immediately when the vibratory force stops. Mechanical impedance measurements allow

studies of the dynamics of a continuous medium such as fresh concrete under forced damped vibration.

10. The influential parameter in the motion of an ideal mass element is acceleration. The motion is described by Newton's second law,

$$F = MA \ .$$

where

F = peak force

M = mass being vibrated

A = peak acceleration

The magnitude of the impedance of a mass element is $Z = \omega M$ (see Appendix A for the derivation). Plate 1 shows the variation of impedance with frequency. In the sinusoidal motion, the force will precede the velocity by 90 deg.*

11. For a stiffness element, displacement is the parameter of interest. Displacement is directly proportional to the force and related to it by Hooke's law,

$$F = KX$$

where K = stiffness coefficient and X = peak displacement. The impedance of a stiffness element is $Z = \frac{K}{\omega}$. The impedance curve will be parallel with one of the stiffness lines as shown in Plate 1. The force will lag the velocity by 90 deg on a phase plot.

12. The influential parameter of motion in describing a viscous damper is velocity. The motion is described by

$$F = CV$$

where C = damping coefficient and V = peak velocity. The impedance is a constant with frequency, $Z = CV$ (Plate 1). The phase angle is equal to 0 deg. The velocity is in phase with the force as can be seen by the equation $F = CV$. An infinite number of impedance-frequency curves can be obtained with various models constructed from the three elements, mass, stiffness, and dampers.

13. The technique of mechanical impedance measurements is unique in that a model of the physical properties of the fresh concrete is not

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* A table of factors for converting U. S. customary units of measurement to metric (SI) units is presented on page 6.

needed. A known force is applied to the concrete and the resulting response is measured, without any knowledge of the mass, stiffness, or damping characteristics. Although fresh concrete is a continuous medium, by the operation of lumping, a model can be constructed of one or a few of the rheological elements that can simulate the impedance of the fresh concrete exactly. An infinite number of elements that have some total characteristic impedance can then be reduced to a model with a finite number of elements exhibiting the same total impedance. The dynamic behavior or nature of the motion under vibratory forces will be the same in the concrete as the motion seen by the model.

14. For a system to vibrate freely, with the source of excitation removed, it must contain conservative elements, i.e. both stiffness and mass elements. Both kinds of elements can store energy and that energy can be recovered. If a system lacks stiffness, it will not vibrate freely; if a system lacks mass characteristics, it, too, will not vibrate except under forced excitation. If a system contains too much damping, it becomes critically damped, or overdamped, and again refuses to vibrate freely. A damper is not a conservative element but a dissipative element, for any energy given to it is lost; the damper absorbs the energy and converts it to heat. This energy cannot be shifted back and forth between elements and cannot be recovered. In summary, a resonant condition cannot exist unless a system possesses both mass and elastic qualities and the damping is less than some critical value. Mechanical impedance measurements reveal the existence of and values for these three elements. The mechanical impedance of a mass element and the frequency of a resonant system are derived in Appendix A.

15. Mechanical impedance is defined as the ratio of the force to the resulting velocity.[5] It is a measure of the property of a material to resist motion from a given force. Because both force and velocity have a phase angle as well as a magnitude, parameters cannot be divided, multiplied, added, and subtracted by algebraic means. The mathematics of complex notation must be employed to describe the impedance of a material. On a complex-plane coordinate system such as that shown in Figure 1, the impedance of a mass element is plotted on the imaginary

axis in the +J direction. The impedance of the stiffness element is plotted on the imaginary axis in the -J direction. The impedance from the damping characteristics is plotted along the + real axis. Since the impedance from a mass element cancels the impedance of a stiffness element in a resonant circuit because of a 180-deg phase difference between the two elements, the impedance of both kinds of elements is termed reactance. The impedance of a damper is termed resistance and is the only element that requires real power to vibrate (as distinguished from reactive power). Although the other two elements exhibit an opposition to motion from an applied force when alone, their reactances can vanish in a resonant circuit. The power required to vibrate reactive elements is termed *reactive* power. These concepts are necessary to understand the cause of the absorption of energy in the consolidation of fresh concrete by vibration.

16. The mechanical impedance of a system is described in Figure 1 and by the equations that follow.

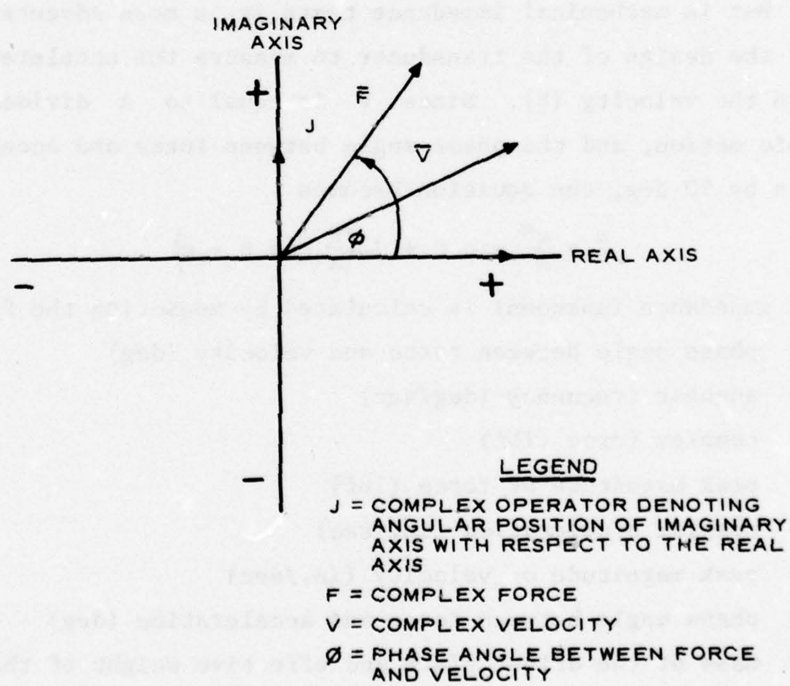


Figure 1. Diagram of the mechanical impedance of a system

Consider that the velocity has an angle of 0 deg with the positive real axis at the *starting time*.

Then $\bar{V} = V \cos 0° + JV \sin 0°$

and since $\cos 0° = 1$ and $\sin 0° = 0$

then $\bar{V} = V$.

The complex impedance is expressed as

$$\bar{Z} = \frac{\bar{F}}{\bar{V}}$$

but $\bar{V} = V$

and $\bar{F} = F \cos \phi + JF \sin \phi$

then by substituting $\bar{Z} = \dfrac{F \cos \phi + JF \sin \phi}{V}$

and rearranging $\bar{Z} = \dfrac{F}{V} \cos \phi + J\left(\dfrac{F}{V}\right) \sin \phi$

Subtracting the constant mass impedance of the fixture used results in

$$\bar{Z} = \frac{F}{V} \cos \phi + J\left(\frac{F}{V} \sin \phi - \omega m\right)$$

where ωm = impedance of fixture.

17. But in mechanical impedance tests it is more advantageous because of the design of the transducer to measure the acceleration (A) rather than the velocity (V). Since V is equal to A divided by ω for harmonic motion, and the phase angle between force and acceleration (θ) differs by 90 deg, the equation becomes

$$\bar{Z} = \frac{\omega F}{A} \sin \theta + J\omega\left(\frac{F}{A} \cos \theta - m\right)$$

$\bar{Z}$, complex impedance (unknown) is calculated by measuring the following:

- ϕ = phase angle between force and velocity (deg)
- ω = angular frequency (deg/sec)
- $\bar{F}$ = complex force (lbf)
- F = peak magnitude of force (lbf)
- $\bar{V}$ = complex acceleration (in./sec)
- V = peak magnitude of velocity (in./sec)
- θ = phase angle between force and acceleration (deg)
- m = mass of the driver plate and effective weight of the force and acceleration transducer (lbm)

J = complex operator denoting angular position of the imaginary term with respect to the real term.

Since the particular mechanical impedance system used in this program calculates the magnitude and the phase angle and plots both expressions against frequency, the impedance equation can be reduced to

$$\bar{Z} = Z \sin \theta + J (Z \cos \theta - \omega m)$$

where

Z = magnitude of impedance (lbf-sec/in.)

m = mass of test fixture ($\text{lbf-sec}^2/\text{in.}$) where acceleration due to gravity is expressed in in./sec^2

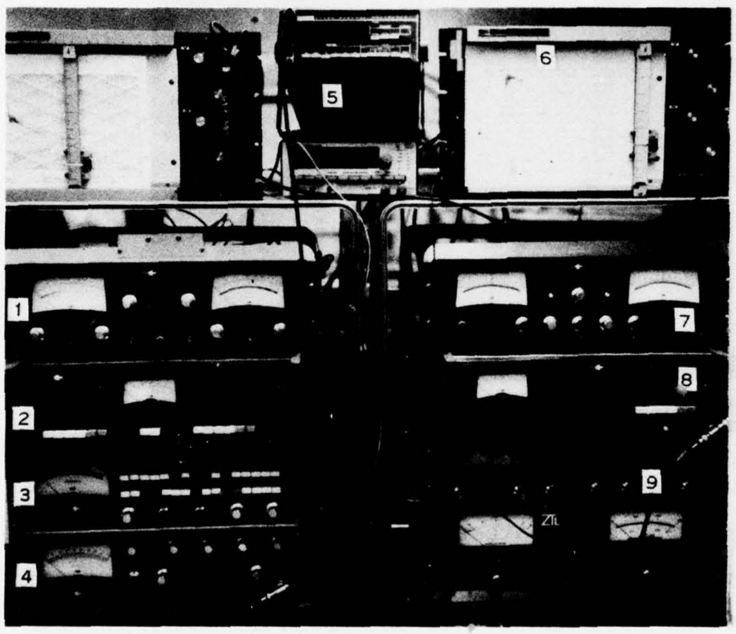
Traditionally, in vibration problems it has been common to isolate materials and structures from destruction due to shock and motion. However, in the case of fresh concrete, vibration is a beneficial operation and the longer and larger the agitation generally the benefits are better. In structures and various systems, dampers and stiffness are incorporated to isolate either machinery or supports against failure. It is known that impulses or shocks are more damaging to systems than vibration. The reason that impulses are more severe is that the energy is supplied to a system so rapidly in comparison to the natural period of the system that the system cannot dissipate the energy. Vibrations are periodic pulses that are supplied at a rate compatible with the natural period of the system. As the energy is being supplied, it is also being dissipated by internal damping in the system. The system must dissipate energy, or energy would accumulate and finally destroy the system afterwards. But with impulses the energy is supplied so rapidly that the system is unable to respond quickly enough to dissipate the energy, hence the energy is stored. If the energy stored is excessive, it breaks down the material properties--damping, stiffness, and mass--that are resisting movement. If the energy supplied is below the level of isolation, it will merely vibrate at the natural frequency of the system until the energy is dissipated. If the system is not shock isolated as is the suspension system of a car, the electronics equipment on an airplane, or a combustion motor resting on a foundation, dampers and/or stiffness must be added at strategic points between the source of the impulse and the part being affected.

These dampers then provide isolation of the system from large motions or large forces. The reverse situation is being dealt with in concrete; large motions are desired and the natural shock isolation existing in fresh concrete must be overcome in vibration, or better yet, removed.

PART III: DESCRIPTION OF TESTS

Test Equipment

18. Much emphasis has been placed on the measurement technique for these tests. Because of the many variables that could potentially influence the consolidation of concrete, accurate and controlled measurements must be made to prevent crediting the wrong variable for some measured response. Various automatic and continuous features have been incorporated in the system used for these tests. This system (Figure 2) has several features that overcome various problems in measuring the impedance of materials. Manually, a measurement of the impedance of fresh



LEGEND

1 VOLTMETER/FREQUENCY LOG CONVERTER
2 TRACKING FILTER
3 MODE CONTROL UNIT
4 SWEEP OSCILLATOR
5 DIGITAL VOLTMETERS
6 X-Y RECORDERS
7 CO/QUAD ANALYZER
8 TRACKING FILTER
9 AMPLITUDE SERVO/ MONITOR

Figure 2. Mechanical impedance system

concrete over the complete frequency range of interest would require hours, and then all frequencies cannot be recorded because of the tedious work of readjusting the system and recording the values. These problems were overcome with a sweep oscillator, which sweeps through all the frequencies of interest in less than 5 min. This is a must for fresh concrete as the physical characteristics are continually changing with time of hydration. A test should not exceed 10 min as a maximum.

19. As mechanical impedance measurements are normally made with accelerometers rather than with velocity pickups and the velocity must be known to calculate the complex impedance, the acceleration signal must be integrated to obtain the velocity. By means of a feature built into the system, the velocity can be obtained by an artificial integration and the displacement by double artificial integration. No calculations are required by the engineer before the curves can be plotted. Figures 3 and 4 show typical plots of magnitude and phase measurements, respectively.

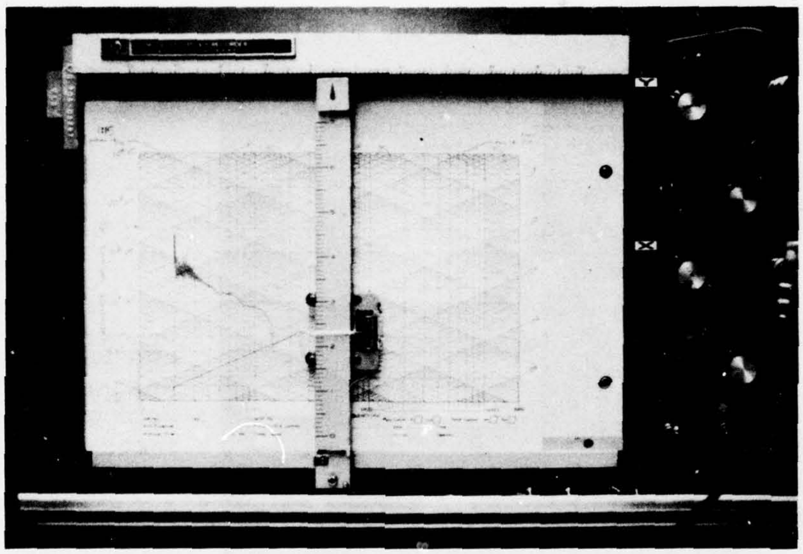


Figure 3. Magnitude of impedance plot

Because the various parameters of interest undergo drastic dynamic range changes when impedance measurements are made, the system contains logarithmic voltmeter converters that can detect an amplitude change of 70 decibels (db), corresponding to a ratio of 3160 to 1. Frequency and impedance are plotted on log-log scales to allow large ranges to be seen so that a whole range of frequencies of a concrete mixture can be tested in a much shorter time than previously. Time is shortened for each test. Another important feature incorporated into the system is the ability of the system to hold some parameter constant, while others are left free to vary depending on the nature of the material being

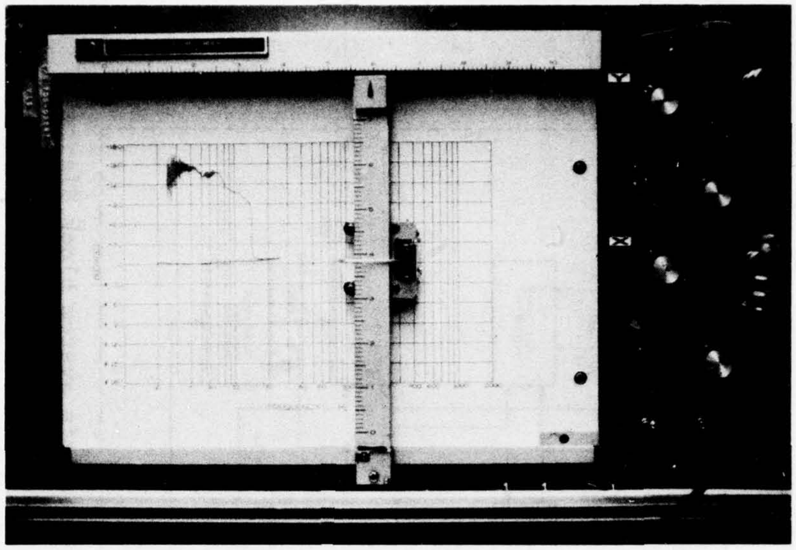


Figure 4. Phase plot: force leads acceleration

vibrated. An example of these is force. Since force is a stimulus to motion, it would have to be held fixed as the frequency changed to test the response of fresh concrete to another parameter that is varying. Also, acceleration, velocity, displacement, and frequency can be controlled; that may be the most significant feature in testing fresh concrete. Figure 5 presents a block diagram of the system. Tens of tests can be run in a day, and magnitude and phase curves can be obtained throughout the complete frequency range of interest. Manually, it requires a good half day to measure, to calculate useful parameters, and to plot both magnitude and phase curves. Again, the changing characteristics of fresh concrete with time make the ability to test rapidly a necessity.

20. The test fixture is critical to the measurement system. Both force and acceleration transducers are mounted on a plastic driver plate that is in contact with the fresh concrete in an actual measurement. The plate is 3 in. in diameter and 3/8 in. thick. The force gage is mounted directly in the center of the plate with the accelerometer close to the center to ensure a driving point measurement. Figure 6 shows the

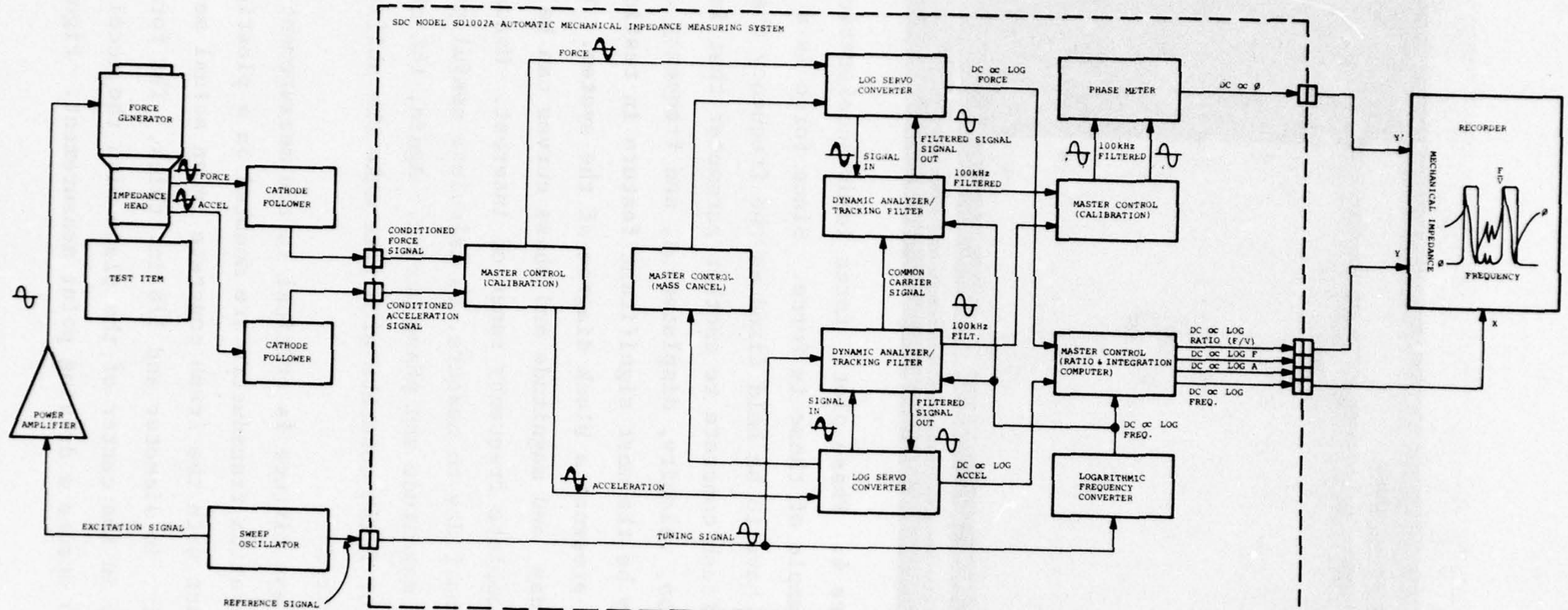


Figure 5. Complete mechanical impedance test and analysis system block diagram

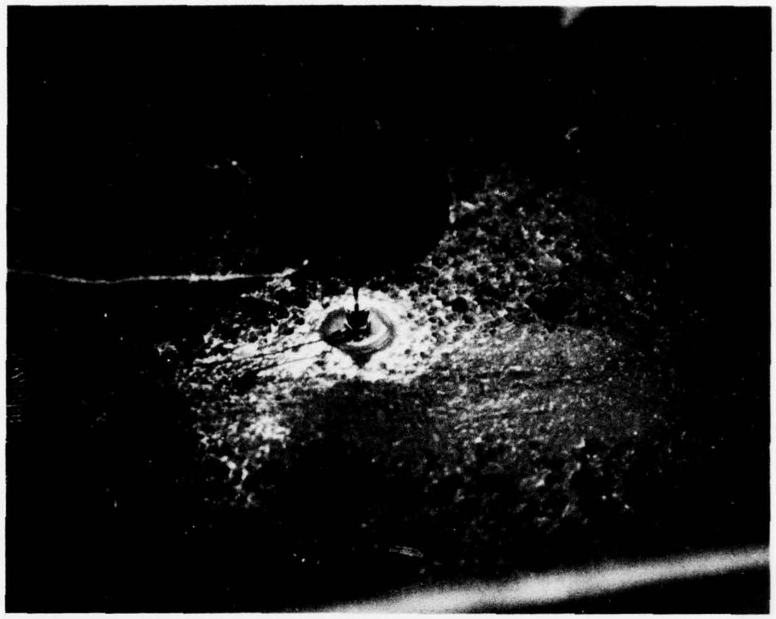


Figure 6. Test fixture in contact with concrete

test fixture, which is in contact with the fresh concrete, and the electromagnetic vibrator.

21. Various considerations arose in the construction of the test fixture. The least rigid part of the fixture must be much stiffer than the stiffness of the concrete being tested. In order that the glue used for mounting both transducers would not introduce a low stiffness, the bases of the transducers and the surface of the driver plate were lapped flat to ensure that a very thin layer of adhesive would be satisfactory for a bond. The driver plate must also be thick enough to prevent flexing. Various modes of vibration can be set up in thin plates that could mask actual measurements of the fresh concrete. Phase measurements verified that these were not a problem. Another problem can arise in a test fixture if the mass of the fixture is excessive. Since the impedance of the fixture must be kept below the impedance of the material being measured, the fixture must be made of low-density material. The diameter must be large enough to cover a representative area to ensure that measured impedance is a function of the fresh concrete and not of

the matrix or the aggregate alone. With increasing diameter thickness must increase to retain rigidity. Furthermore, the fixture doing the measuring should alter the motion of the specimen being measured by a minimum. Another important problem to be concerned about is a driver that does not give an equal response for a constant amount of stimulus (constant voltage level) over the range of consideration. This particular driver held the antiresonance peak at about 2000 Hz by keeping the mass of the test fixture low. The higher the mass of the test fixture the lower the frequency peak will appear. Accurate measurements were possible from 20 to 1000 Hz, which more than covered the frequency range of a field vibrator (50 to 300 Hz).

22. Finally, the maximum force desired from the driver should be considered. This driver had an upper limit of 30 lbf, which was sufficient for the measurements taken. Occasionally, the 100-w nominal capacity of the power amplifier was exceeded. A much larger driver and power amplifier would be necessary to reach levels of force and acceleration corresponding to a field vibrator.

23. Several operations are performed in the calibration of the system. First, it is important that there is no leakage of charge from the piezoelectric transducers. All connections must be cleaned with a fluid, such as freon, to ensure that possible leakage paths are of the order of 1000-Mohm resistance. The slightest contamination can decrease the output of the crystals to their charge amplifiers. Because the driver plate is immersed in the wet concrete, a latex cover was needed to keep the transducers dry. Rarely were any problems experienced with this cover except when sharp aggregate tore the latex cover, allowing moisture to reach the transducers and connectors.

24. The accelerometer had a factory calibration of 13.9 picocoulombs (pC)/g. It was also calibrated on another driver and found to be 13.8 pC/g. The force gage had a factory calibration of 101 pC/peak lbf. The charge amplifiers were adjusted to that value temporarily until known impedances could be measured to verify its correctness. A constant mass test was then made in air separated from any test specimen. The dynamic mass measured was 0.25 lb. The impedance is shown in Plate 2.

A small test cylinder having a mass of 0.701 lb was added to the fixture. Another constant mass test was conducted and the results yielded a dynamic mass of about 1 lb, which verified that the calibration was satisfactory. Also in Plate 2, the magnitude of the impedance of the fixture and cylinder closely tracked a dynamic mass line, while the phase angle in Plate 3 approximated 0 deg very closely. Since very little deviation from a straight line occurred for the magnitude curve in the frequency range of 20 to 1000 Hz, the fixture should yield accurate data in that range.

25. Early measurements showed a problem of the driver plate losing contact with the fresh concrete as it settled under the load. Although the test was begun by applying a static force against the surface mixture, the concrete would relax the load by slowly deforming under the static and dynamic load until the total mass of the shaker hung entirely on the suspension wires. This problem was solved by using a seismic support. A set of four soft screen door springs was used to support the 75-lb shaker. The springs were allowed to extend to their full length of about 15 in. with the total mass resting on the springs. The shaker and driver plate were then lowered by a pulley system supported by a frame until the fresh concrete supported about 5 lb of mass. Since the total spring constant was 5 lbf/in., the concrete would have to deform 1 in. before the static force would diminish to zero. No more problems were experienced as the concrete settled only about 1/4 in. or less during a normal run.

26. The force and acceleration signals are calibrated by first estimating the maximum levels expected, and these are set on the charge amplifiers. Both the maximum force and acceleration signals expected can be simulated by a built-in test signal covering the range of interest. The impedance axis can be scaled by making the following calculation of the magnitude of the force to velocity value:

$$Z = \frac{F}{V} = \frac{F}{A/\omega} = \frac{2\pi fF}{gn}$$

where

f = frequency

g = local acceleration of free-fall

n = number of g's

The impedance paper is preprinted, and the powers of 10 on each axis must be chosen to correctly scale the plot. The frequency axis can be scaled over only a desired range spanning no more than three decades of change. The range for all of our tests was from 20 to 1000 Hz. The stiffness lines are scaled by calculating the spring constant, K, from the following equation:

$$Z = \frac{K}{\omega}$$

The mass lines are scaled by using the equation:

$$Z = \omega M$$

The impedance of a mass changes with frequency as a straight line with a positive slope of 20 db per octave. The impedance or stiffness element also changes as a straight line on a log-log plot with a negative slope of 20 db per octave. The resistance lines are flat with frequency, or resistance does not change with frequency.

Test Concrete

27. Most of the mathematical impedance tests were made on a standard concrete mixture with 2-in. slump, 3/4-in. maximum size aggregate; the proportions are given in Table 1. To understand the consolidation, this mixture was studied before adding complicated data from other mixtures. Later, an extremely dry and one extremely wet mixture were tested to verify that the phenomenon discovered occurred in other mixtures. It cannot be overemphasized that variables must be controlled or known so that the correct variable is regarded as the cause of the response obtained.

PART IV: RESULTS

28. Two basic impedance curves were obtained dependent on the level of the applied dynamic stress. Plates 4 and 5 show the behavior of fresh concrete at lower and higher stresses, respectively.

29. In Plate 4, freshly mixed concrete has a broad resonance band with a minimum impedance at 600 Hz. The impedance at that frequency is 6 lbf-sec/in. At frequencies below this point, the response is primarily stiffness-controlled. In this range, the main opposition to movement from the applied stress results from the elastic properties in the mixture. The stiffness value is approximately 5000 lbf/in. At 600 Hz damping controls the motion, and from 600 Hz and higher mass properties control the motion. The state of the fresh concrete in terms of its mechanical properties is similar to that of a solid; it has rigidity and shear strength. Plate 5 shows that at about 60 Hz a critical point was reached at which the impedance of the concrete made a significant drop. The drop in impedance was by a factor of 10. The force level was maintained constant throughout the test at 2.1 lbf. The breakdown of impedance in the fresh concrete is seen as liquefaction; it is also noted that concrete is shear-thinning under vibration. Once the impedance breakdown occurred, the motion was mass-controlled, regardless of whether the frequency was increased or decreased. The elastic and damping properties that had existed just before the breakdown vanished. The only remaining forces were the inertia forces of the mass limiting the degree of movement. In Plate 5, the response of the fresh concrete at breakdown is identical to the response of an ideal mass element.

30. The fresh concrete appeared to become liquid at the point of the impedance breakdown. Thus, it was decided to test whether the impedance of water is similar to that of the fresh concrete after breakdown. The results of the test showed the similarity to exist and the impedance also to follow a mass line as frequency was increased and decreased. Consequently, the ratio of the impedance of the water and the fresh concrete should be in inverse proportion to the densities of the water and the concrete, as the impedance to movement was due only

to mass. This idea was verified by calculation. Possibly much of the nature of fresh concrete can be explained by the concepts and equations of hydrodynamics.

31. Plate 6 illustrates a phase plot obtained simultaneously with the impedance plot. As the frequency is swept from 20 Hz up, the phase angle begins at 150 deg and goes to 0 deg. An ideal stiffness element would give a phase angle of 180 deg proving that motion is controlled by stiffness early in the test. At about 60 Hz, the phase angle drops sharply through 90 deg, the damping angle. At breakdown, the phase angle is 0 deg, which indicates mass control. This means that the force is in phase with the acceleration and that the motion is described by Newton's second law,

$$F = MA$$

32. Either force or frequency can be varied to produce a breakdown in impedance. With constant force and increased frequency or constant frequency and increased force, breakdown can be reached. The threshold level of vibration required to cause the fresh concrete to change state is a function of the frequency-force combination. The drop in the impedance was measured in the tests made as a factor of 5 to 10; it usually occurs within a few seconds as a very sharp drop, once the particular critical force-frequency value is reached.

Mechanical Properties of Fresh Concrete Before Breakdown

33. Fresh concrete has mechanical properties like a solid at low applied dynamic stresses; it has mass, damping, and stiffness characteristics. At higher stress levels, above a critical level of vibration, its mechanical properties are similar to a liquid, in that the resistance to movement is only a function of mass. Fresh concrete does not have a resonant frequency before breakdown, because the damping is too high. There is a minimum impedance for each mixture of fresh concrete, but the frequency at which the impedance is a minimum is a function of the mixture. The impedance minimum is neither sharp nor consequential.

34. After breakdown, fresh concrete still does not have a resonant

frequency, because the stiffness has vanished. If the findings reported herein can be confirmed by other investigators, speculations about the resonant frequency of fresh concrete can be ended.

35. Limited evidence indicates that the compressional wave velocity increases by the same factor that the impedance decreases at the time of breakdown or liquefaction (Appendix A). In the past, it has been a mystery why the compressional wave velocity of fresh concrete was only about 400 ft/sec, although all of the components of the mixtures have velocities of a few thousand feet per second. Water has a velocity of 5000 ft/sec. When the damping and stiffness vanish at breakdown, the velocity of the compressional wave increases significantly. Apparently, the compressional wave velocity could be used to determine the time at which consolidation is complete. Appendix B describes this discovery. Work is continuing to make use of that principle in developing an instrument that will detect completed consolidation.

36. Probably, vibrators should be matched in mechanical impedance to fresh concrete. Because vibrators lack sufficient elasticity to balance the mass properties in fresh concrete, energy is not transferred efficiently between the vibrator and the concrete. Energy is lost that should be conserved in the elastic-mass system of a vibrator matched to the concrete. Appendix C presents an illustration of the energy necessary to drive a mass system lacking stiffness. The addition of stiffness in a vibrator can produce a resonant system requiring a minimum input of energy to maintain vibration. This feature should be incorporated in the design of a vibrator.

37. In Appendix D the following equation for the work consumed per cycle in vibrating concrete while it is in the liquid state is derived:

$$W = \frac{2F^2}{(2\pi f)^2 M}$$

Velocity and displacement were not found to be influential variables in producing breakdown. Acceleration and frequency were important variables, and force was especially important, since force and acceleration are directly related when the concrete changes to the liquid state. More precisely, force and frequency alone are the only influential variables. The other variables, such as velocity, displacement, and acceleration, are only responses or effects caused by force. Because the system is mass-controlled and is described by $F = MA$, acceleration will obviously be always proportional to the force. The differential motion between heavy and light particles causes the shear resistance to vanish and the mixture to break down, to liquefy, or to shear-thin.

Producing Liquefaction

38. If a high acceleration is present in a mixture, high forces are evidently available to produce breakdown. At a given force, lighter particles are accelerated farther than heavier particles, thus producing shearing and differential motion between particles that cause the friction and cohesive forces to vanish.

39. Higher frequency means more repetitions of peak force in unit time that make the force more effective; 200 Hz produce a more continuous force than 100 Hz. At higher frequencies, the material cannot respond at the rates produced.

Modeling Fresh Concrete

40. The physical properties of fresh concrete have been modeled mathematically using mass, stiffness, and damper elements. Appendix E describes this mathematical model in more detail.

PART V: DISCUSSION OF RESULTS

41. The motion of fresh concrete after breakdown or liquefaction is controlled by mass, with the resistance to motion directly related to the density or mass of the concrete. Since the system is a mass system, its motion can be described by Newton's second law; this means that an impulse delivered to a body is directly related to the change in the momentum of the body. An impulse is defined as a force multiplied by the time it exists. If it is a shock impulse, it will occur suddenly and will have a high force. The time will be so short that the maximum acceleration will be delivered before displacement can occur. This is very important, as noted in the following paragraphs.

42. The measurements revealed that the force required to cause the static concrete to break down in impedance was much higher than that needed to maintain motion once breakdown occurred. Obviously, fresh concrete has a threshold level of resistance to movement and does not move easily until the delivered force exceeds the critical shearing and frictional forces of resistance that tend to keep the particles locked together.

43. If a force large enough to exceed the forces of shear and friction is delivered to the concrete, light particles are displaced farther than heavier particles, according to F = MA. Also, if impulses are continually applied, motion will be maintained, but the forces that restrict motion will practically vanish with the exception of the concrete resisting acceleration due to its mass or density.

44. The results of the mechanical impedance tests showed that the mechanical properties of the fresh concrete are experiencing a breakdown. The properties of damping and stiffness practically vanished leaving only mass properties. As explained in Part II, this represents the phenomenon of exceeding the shock isolation properties. This breakdown is desirable and necessary if the concrete is to be consolidated. The question then is, What is the best means to precipitate a material breakdown? The answer, as explained earlier, is with shocks or impulses, and not vibration. The key idea is to overcome damping characteristics

as soon as possible. During continuous vibration, much energy can be dissipated and lost if there is any delay in producing breakdown. Impulses can instantly exceed the threshold isolation properties of damping and stiffness and prevent giving time for the energy-dissipating mechanism to consume energy. Continuous vibration with simple harmonic motion serves only to maintain breakdown once it is produced. Therefore, a field vibrator should develop two types of mechanical wave forms, impulses and vibration. The vibrator would continually generate harmonic motion but would incorporate a governor system like combustion motors. When the vibrator encountered a heavy load or high impedance, this would trigger the generation of a series of shocks or impulses. As the material broke down and the impedance dropped, this would signal the shock generator to stop the shock motion. Simple harmonic motion would keep the mixture fluid for the time necessary for the air to be expelled and the concrete to densify.

45. As previously mentioned, the mechanical impedance measurements prior to liquefaction did not reveal an optimum frequency. On the contrary, the impedance curve was fairly flat or constant with frequency (Plate 4). This means that all frequencies within the range tested develop about the same motion for a given force. It also confirms that there is a wide range of frequencies capable of consolidating concrete rather than the ideal frequency that causes the fresh concrete to liquefy in an instant.

46. This suggests the theory that the continuous spectrum of particle dimensions in a mixture needs a continuous spectrum of excitation frequencies to cover all the vibrational modes. It is a known fact that particles respond with the most motion when the wavelength of the exciting stress is comparable in dimensions to the particles being vibrated. In other words, if there is a wide range of particle sizes in a system, then there must be a wide range of consolidating frequencies to excite all the possible modes of vibration.

47. Consequently, plans are underway to test the mechanical impedance of fresh concrete with impact loading rather than sinusoidal motion. Obviously, a sine wave contains energy at only one discrete frequency.

However, impact wave forms contain energy over a whole range of frequencies depending on the shape of the wave form. Parameters that describe an impact wave form are rise time, pulse duration, stress level, and repetition rate. The results should be forthcoming in fiscal year 1978. The results from such tests should enable specifications to be written to describe the construction of an efficient field vibrator for fresh concrete.

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3. Sugiuchi, H., "Study of Vibration of Concrete; Review of Literature," Technical Report No. 6-780, Report 1, Jun 1967, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss.

4. Sullivan, B. R., "Study of Vibration of Concrete; Mechanical Impedence Measurements," Technical Report No. 6-780, Report 2, Aug 1969, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss.

5. Kerfoot, R. E., "Solutions for Mechanical Impedance Measurement Problems," Technical Publication No. M-1, Sep 1966, Spectral Dynamics Corporation of San Diego, San Diego, Calif.

Table 1

Concrete Mixture Proportions

JOB NAME	CONCRETE MIXTURE PROPORTIONS (WORK SHEET) (CRD-C 3)	DATE
VIBRATION STUDIES		11 June 1968
JOB. NO. / MIXTURE SER. NO.		INITIALS: F.S.

PORTLAND CEMENT TYPE 2	POZZOLAN SER. NO.	A. E. ADMIX: SER. NO.
SER. NO. RC 602 ADDITION	TYPE	NAME NONE
BRAND AND MILL	SOURCE	AMOUNT % ML

OTHER CEMENT SER. NO.	CHEMICAL ADMIX SER. NO. % ML
BRAND AND MILL	NAME WRA 89 ml/bag*

FINE AGGREGATE		COARSE AGGREGATE	
TYPE LIMESTONE	SER. NO.	TYPE LIMESTONE	SER. NO.
SOURCE CRDMS-17(5)		SOURCE CRDG-31(3)	SIZE 3/4-in.

MATERIALS

MATERIAL	SIZE RANGE	BULK SPECIFIC GRAVITY	UNIT WEIGHT (SOLID), LB/CU FT	ABSORPTION, PERCENT	TOTAL MOISTURE CONTENT, PERCENT	NET MOISTURE CONTENT, PERCENT
CEMENT		3.15	196.24			
F. AGGREGATE		2.67	166.34			2.1
C. AGGREGATE (A)		2.69	167.59	0.6		0.4
C. AGGREGATE (B)						
C. AGGREGATE (C)						
C. AGGREGATE (D)						
POZZ/OTHER CEMENT						

PROPORTIONS

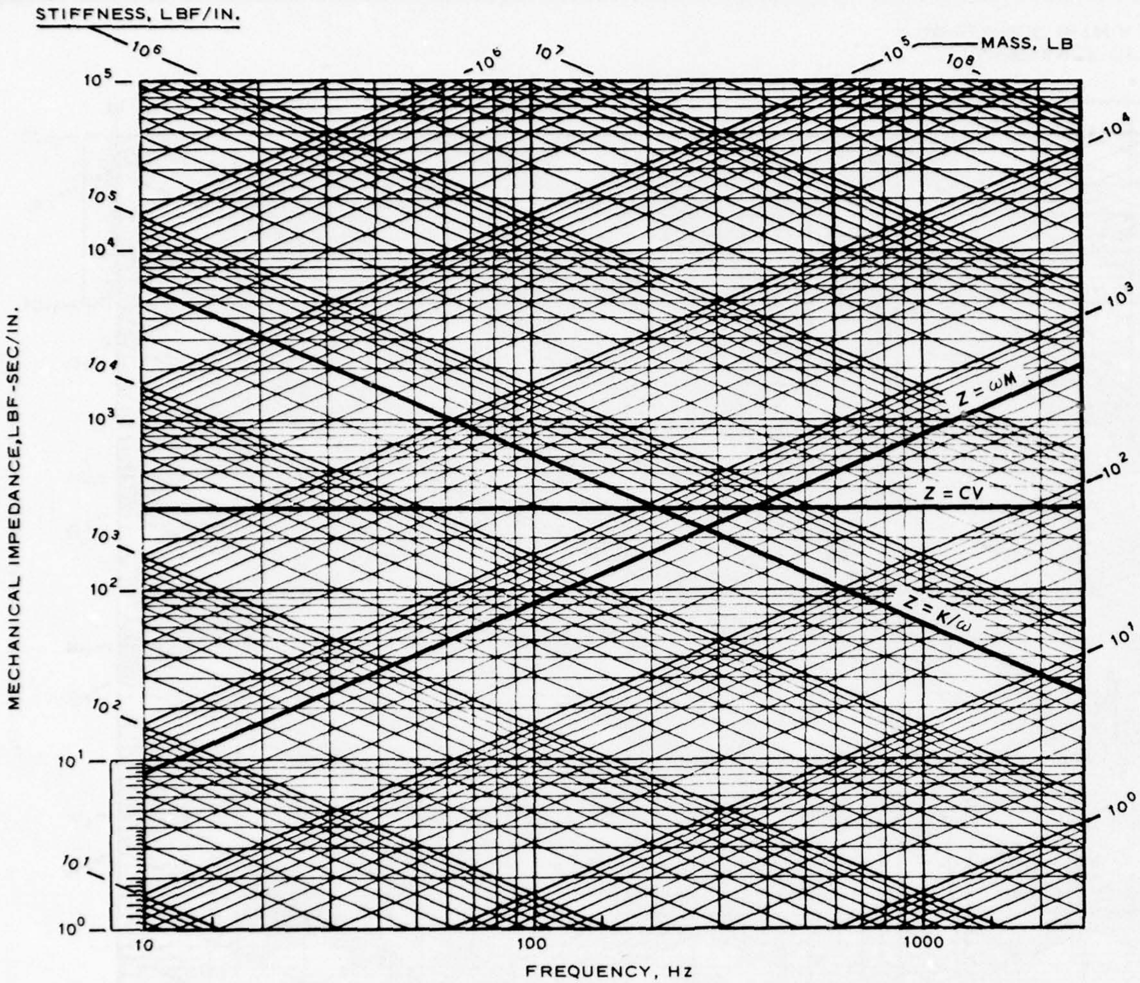
		CALCULATED BATCH DATA (1 CU YD)			ACTUAL BATCH DATA CU FT		
MATERIAL		SOLID VOLUME CU FT/BATCH	SAT. SURF DRY BATCH WT, LB	FACTOR	SAT. SURF DRY BATCH WT, LB	WATER CORRECTION, LB	ACTUAL BATCH WT
CEMENT		2.395	470.60 (13)	0.44	206.8		206.8
F. AGGREGATE		8.906	1481.42		651.8	+14.0	665.4
C. AGGREGATE (A)		10.886	1824.38		802.7	-3.2	799.5
C. AGGREGATE (B)							
C. AGGREGATE (C)		(11)					
C. AGGREGATE (D)		(12)					
XXXXXXXXXXX WRA			445.0 ml		196 ml		196 ml
WATER		4.813	299.88 (3)		131.9 (1)	-10.8	121.1
AIR							
TOTAL	AIR FREE	27.000 (2)	4075.68 (4)				
	YIELD	(14)					

MIXTURE DATA

SLUMP 2", 2-1/4" IN.	AIR CONTENT (D) %	MIXING WATER F	TH CF LB/CU YD
REMOLD EFF DROPS	AIR CONTENT (E) %	AMBIENT F	ACT CF LB/CU YD
TH UW 151 LB CU FT	AIR CONTENT (F) %	CONCRETE F	W/C
ACT UW LB CU FT	BLEEDING %	S/A 45 PERCENT VOL.	

WES FORM NO. 476, REV MAR 1972 (OVER)

* One bag = 94 lb of cement = 0.4535924 kg (mass).



LEGEND

MASS ELEMENT ($Z = \omega M$)

F = MA DESCRIBES MOTION.
IF FORCE IS MAINTAINED CONSTANT OVER FREQUENCY RANGE, THE ACCELERATION WILL BE CONSTANT ALSO.

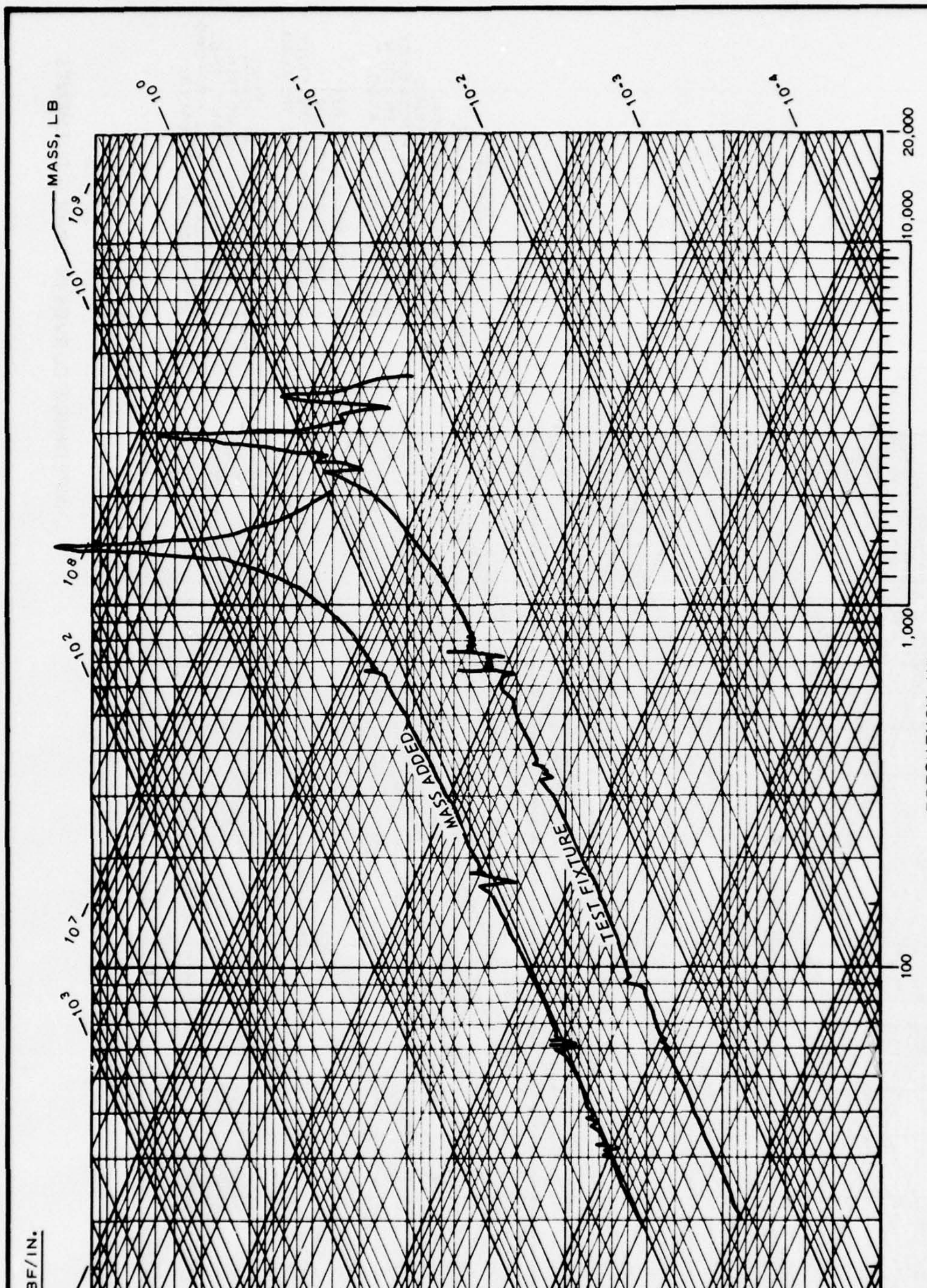
DAMPING ELEMENT ($Z = CV$)

F = CV DESCRIBES MOTION.
IF FORCE IS KEPT CONSTANT, THE VELOCITY WILL BE ALSO.

STIFFNESS ELEMENT ($Z = K/\omega$)

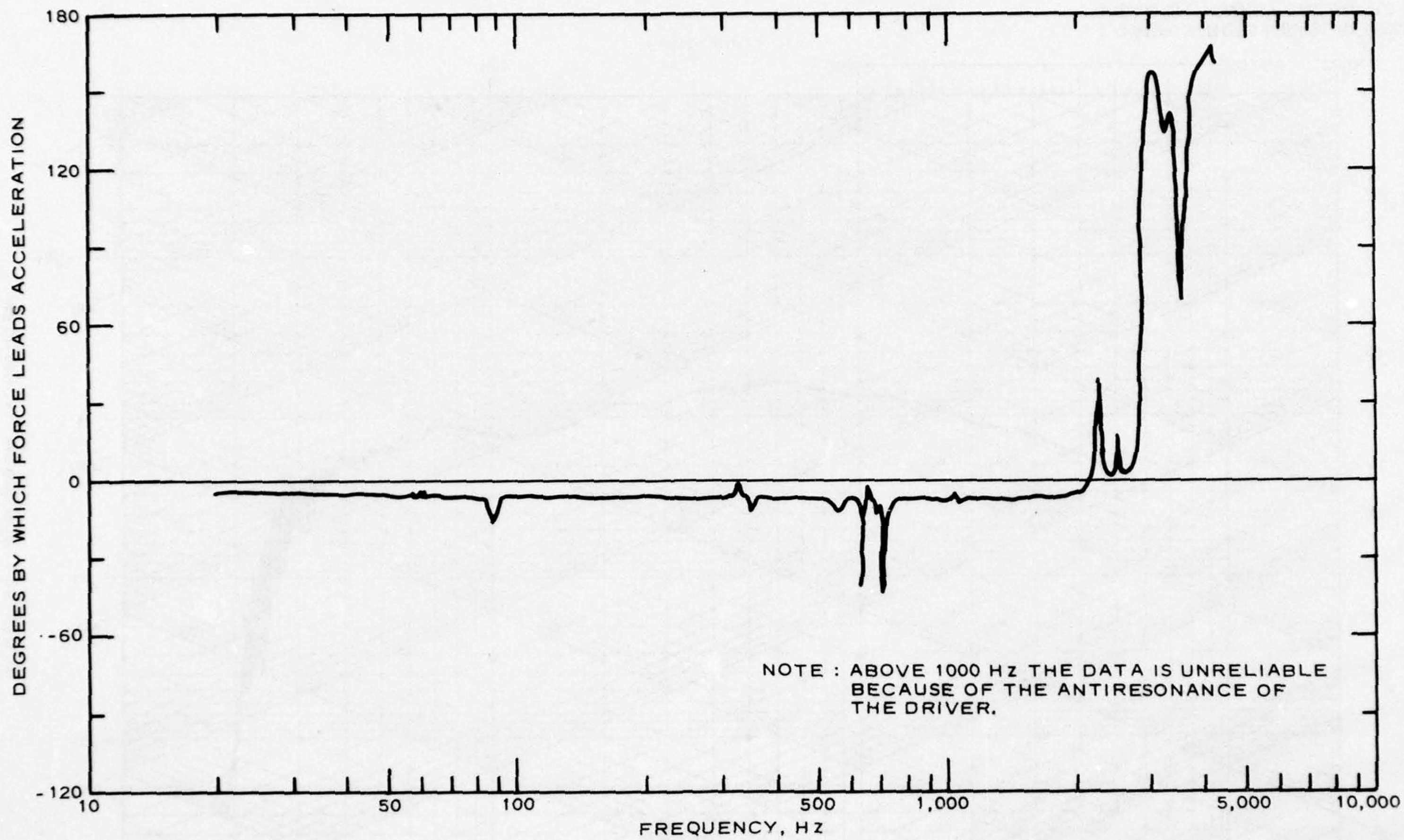
F = KX DESCRIBES THE MOTION.
IF FORCE IS CONSTANT, THE DISPLACEMENT WILL RESPOND WITH A CONSTANT VALUE.

IMPEDANCE CURVES OF SINGLE ELEMENTS



IMPEDANCE OF TEST FIXTURE IN AIR. AND IMPEDANCE WITH A 0.7-LB MASS ADDED TO FIXTURE

PLATE 2



TEST OF PHASE ANGLE OF CONSTANT MASS

PLATE 3

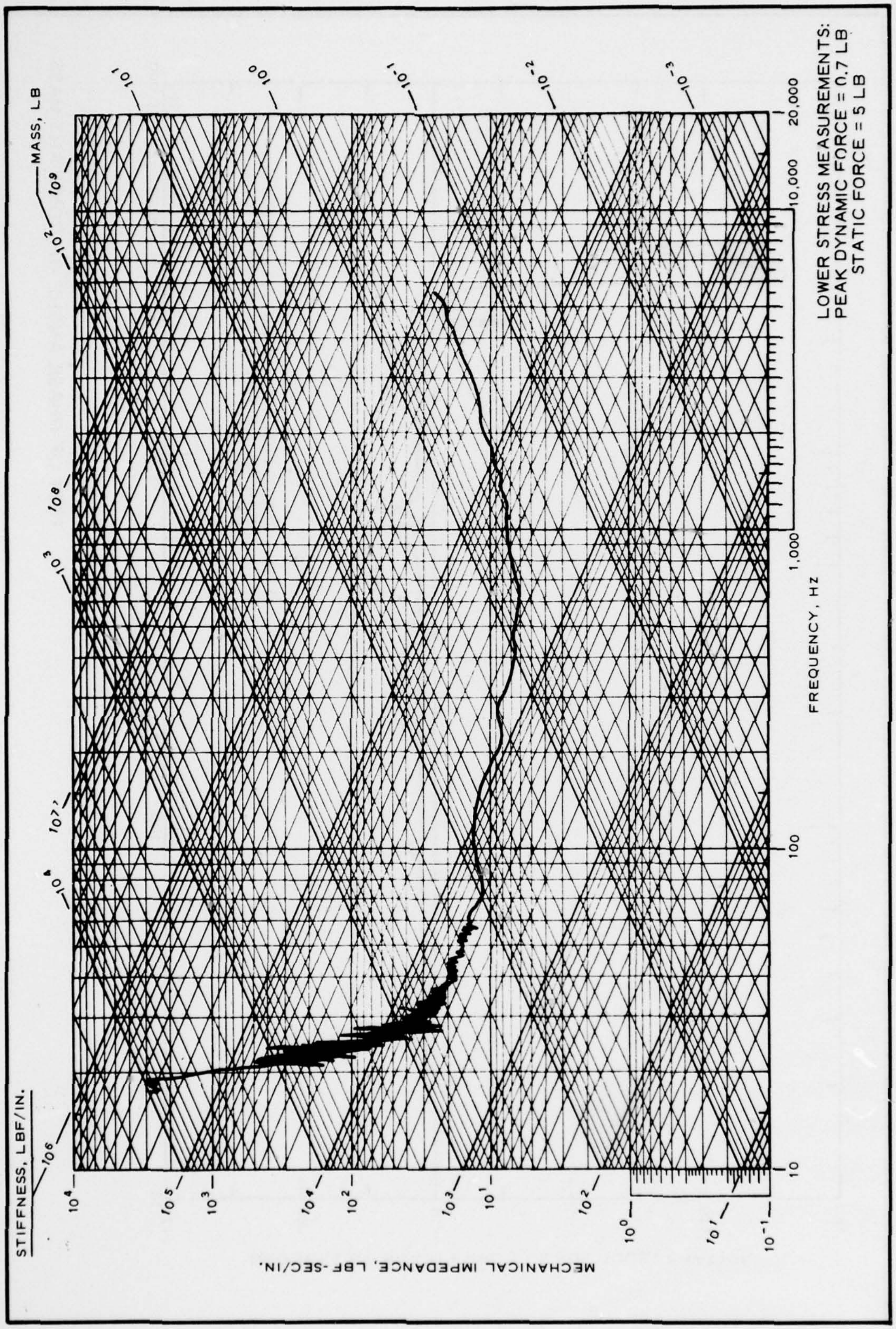
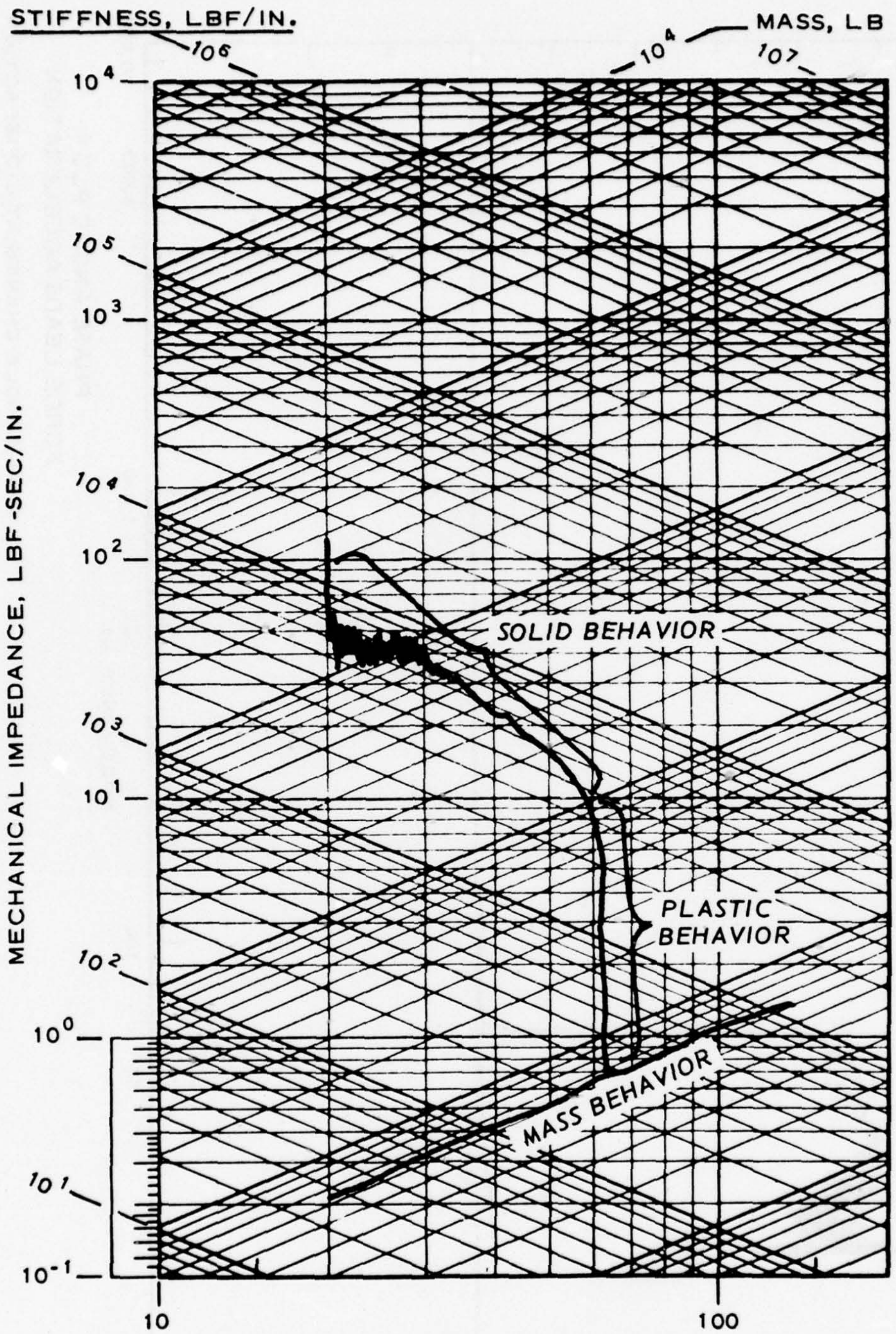
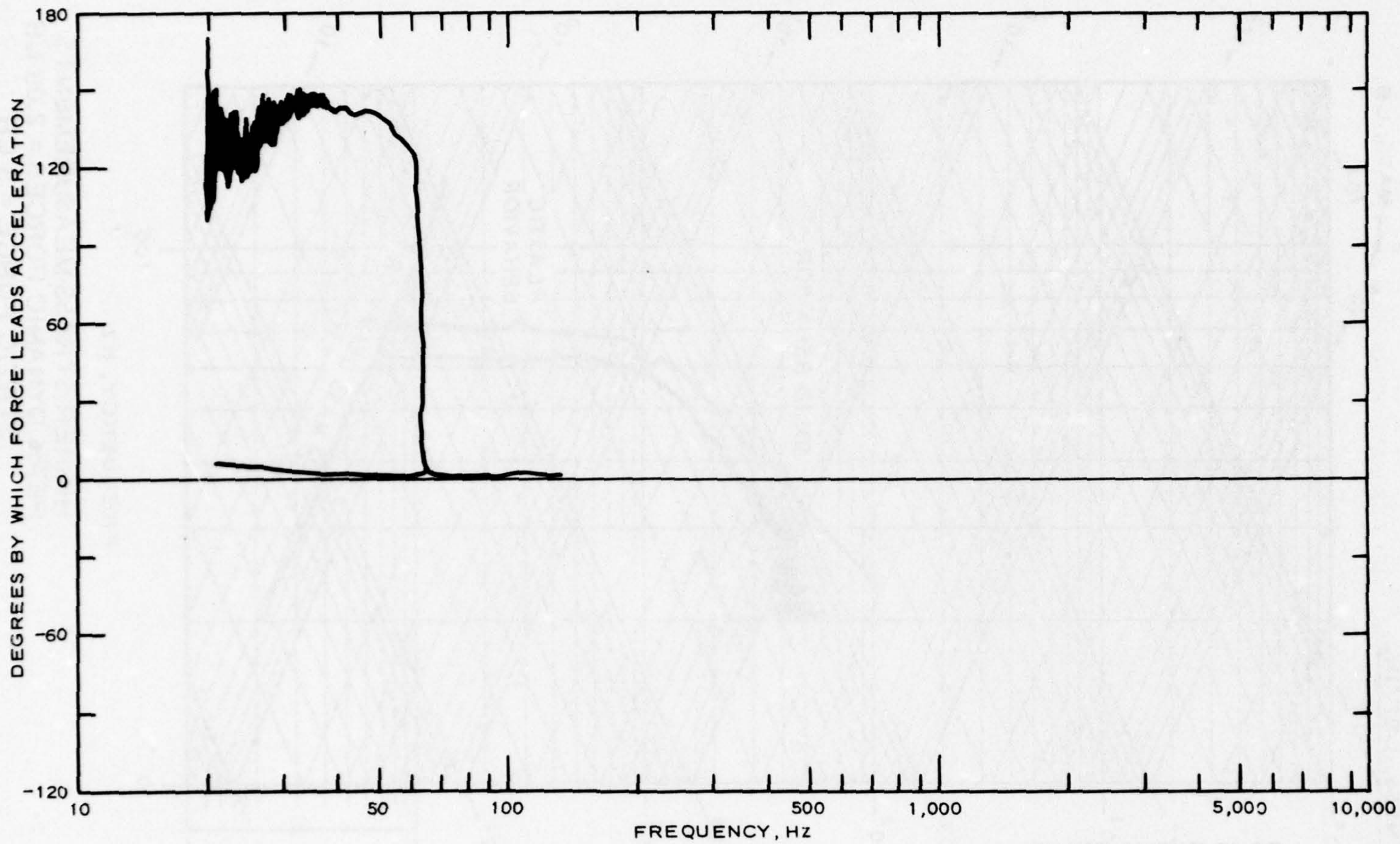

STIFFNESS, LBF/IN.
MASS, LB
MECHANICAL IMPEDANCE, LBF-SEC/IN.
FREQUENCY, HZ
10
100
1,000
10,000
20,000
LOWER STRESS MEASUREMENTS:
PEAK DYNAMIC FORCE = 0.7 LB
STATIC FORCE = 5 LB

PLATE 4



HIGHER STRESS MEASUREMENTS:
PEAK DYNAMIC FORCES = 2.09 LB
STATIC FORCE = 5 LB



PHASE ANGLE PLOT:
FORCE LEADS ACCELERATION

PHASE ANGLE CHANGE AT LIQUEFACTION

PLATE 6

APPENDIX A: DERIVATION FOR IMPEDANCE OF A MASS AND RESONANT FREQUENCY OF A SPRING AND MASS COMBINATION

1. To explain $Z = \omega M$ where $\omega = 2\pi f$, impedance is defined in electrical terms as the ratio of the voltage, or stimulus, to the current or response, and in mechanical terms as the ratio of the force, or stimulus, to the velocity or response.

2. According to equation $Z = \frac{F}{V}$, if the force is maintained constant as frequency is increased and the velocity decreases, then it follows that the impedance becomes larger, yet the stimulus (force) has not changed. This is the case if the element being vibrated is a pure mass with no damping or stiffness.

3. The impedance curves (Plate 1) are obtained by vibrating an ideal mass infinitely rigid without damping, an ideal damper without mass or stiffness, and an ideal spring without mass or damping, respectively. Each curve represents only one element. Thousands of curves can exist for various series and parallel connections of the three elements in combination. As noted in Plate 1, the impedance of a stiffness element decreases with frequency; the impedance of a damper is constant with frequency; and the impedance of a mass increases with frequency.

a. When a stiffness element is being vibrated by a sinusoidal force, the displacement wave is exactly in phase with the force wave:

$$F = KX \text{ (Hooke's law)}$$

b. When a viscous damper is vibrated, the velocity of the damper is in phase with the force:

$$F = CV$$

c. When a mass is vibrated, the acceleration is in phase with the force:

$$F = MA \text{ (Newton's second law)}$$

d. Therefore, if a material is vibrated, the phase angle obtained indicates whether the response of the material is predominantly a mass element, a stiffness element, or a damper element. Fresh concrete responds as a pure mass element. The acceleration is exactly in phase with the force.

4. Consider the relation of displacement, velocity, and acceleration for harmonic motion as expressed in the following equations:

instantaneous displacement $x = X \cos \omega t$

$$\frac{dx}{dt} = v = \frac{d}{dt}(X \cos \omega t)$$

instantaneous velocity $v = -X\omega \sin \omega t$

Acceleration, the time rate of change of the velocity, is written as

$$\frac{dv}{dt} = a = \frac{d}{dt}(-X\omega \sin \omega t)$$

instantaneous acceleration $a = -X\omega^2 \cos \omega t$

where t represents the time.

If these three functions are plotted, the maximum positive acceleration leads the maximum positive velocity by 90 deg, and the velocity leads the maximum positive displacement by 90 deg. Therefore, if the force is in phase with the acceleration, then the force will lead the velocity by 90 deg and the displacement by 180 deg.

5. By substituting in the equations for each parameter of motion, the maximum displacement when $\cos \omega t = 1$ is

(1) $x = X$ (peak displacement);

the maximum velocity when $\sin \omega t = 1$ is

(2) $v = V = X\omega$ (peak velocity);

the maximum acceleration during a cycle of motion is

(3) $a = A = X\omega^2$ (peak acceleration).

This means that A is equal to the following:

(2) $V = X\omega$

(3) $A = X\omega^2$

Rearranging (2) yields $X = \frac{V}{\omega}$

Combining (2) and (3) results in

$$A = \left(\frac{V}{\omega}\right)\omega^2$$

$$A = V\omega$$

The maximum acceleration is equal to the maximum velocity multiplied by the angular frequency. Actually, the impedance is a complex number involving magnitude and phase. Now $Z = \omega M$ is derived as follows:

by definition $\bar{Z} = \frac{\bar{F}}{\bar{V}}$ (complex values)

or $Z = \frac{F}{V}$ (magnitude values)

but for a mass element (Newton's second law)

$$F = MA$$

$$Z = \frac{MA}{V}$$

and for simple harmonic motion, with the maximum acceleration $A = V\omega$

$$Z = \frac{MV\omega}{V}$$

$$Z = \omega M \text{ (magnitude of the impedance)}$$

6. The equations above show the magnitude one would obtain for a pure mass. Similar equations can be written to describe the magnitude of the impedance of a stiffness element.

$$Z = \frac{K}{\omega}$$

A resonant condition is produced when the impedance of the stiffness element becomes equal to the impedance of the mass element.

$$\omega M = \frac{K}{\omega}$$

$$\omega^2 = \frac{K}{M}$$

Substituting $2\pi f$ for ω , $(2\pi f)^2 = \frac{K}{M}$

$$2\pi f = \sqrt{K/M}$$

$$f = \frac{1}{2\pi} \sqrt{K/M} = \text{resonant frequency.}$$

At this frequency, the impedance of the mass cancels the impedance of the stiffness element; the resulting impedance is due solely to damping, and a small force can produce large motion in the fresh concrete because of the minimum impedance. Resonant frequencies require a minimum of power input since the material offers little opposition to movement.

7. The equations that describe the impedance of an electrical circuit are analogous to the equations that describe the mechanical impedance. The three elements that also determine impedance in electrical terms are resistors, capicitors, and inductors. Only resistors

absorb energy; capacitors and inductors store energy. The energy given to these elements can be recovered if certain conditions are met. It is a common practice in the electrical power industry to add capacitors in parallel with inductors in circuits where there is an excess of inductance. High reactive currents are set up in the circuits, resulting in a loss of power that can be eliminated by balancing the circuits with the proper capacitance.

8. The same thing should be possible in the mechanical sense. In mechanical terms, only dampers dissipate energy. Stiffness elements and mass elements are conservative elements and store energy. Therefore, the energy is recoverable in a mass and stiffness combination.

APPENDIX B: COMPRESSIONAL WAVE VELOCITIES IN FRESH CONCRETE

1. Tests conducted in August 1972 indicated that pulse velocities might be used to determine degree of optimum consolidation. Because the attenuation characteristics of fresh concrete are so high, a shock generator was constructed that would impart a larger pressure pulse than is possible with a conventional ultrasonic system.

2. Commercial firecrackers were waterproofed and used for the shock generator. The waterproofed firecrackers were immersed in the fresh concrete and ignited by a Nichrome wire wrapped around the fuse. A high current power supply and switch controlled the power that heated the wire and ignited the fuse.

3. The response from the stimulus just mentioned was detected by an accelerometer in the mixture and recorded on a magnetic tape recorder. Initially, water was used as the medium to test the technique. The pulse reverberated from the walls of the 1-ft square box for more than 2 sec before the oscillations damped out. The frequency of the oscillation was 9600 Hz. The velocity of sound (ν) in water is about 5000 ft/sec. From the equation $\nu = f\lambda$, the wavelength (λ) calculated to be 0.5 ft. This meant that the standing waves set up in the mixture were two wavelengths in length.

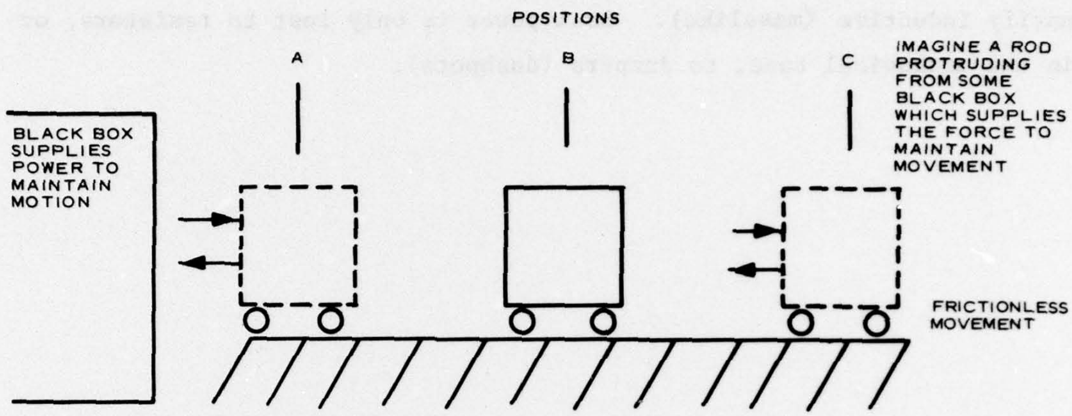
4. The results from the concrete were markedly different. A test was made on the fresh concrete before it was consolidated. The frequency obtained was only 120 Hz, indicating that the velocity was lower than water by a factor of 80. The mixture was vibrated for 10 sec, and another test showed that the frequency had increased to 192 Hz. After vibrating for 10 sec more, the frequency went to 281 Hz. With an additional 2 min of vibration the frequency increased to about 400 Hz, representing an increase in velocity of more than 300 percent due to consolidation.

5. Futhermore, the oscillations recorded on the tape damped out after a few milliseconds rather than seconds as observed in the water. This showed the high degree of damping that fresh concrete has in

relation to water.

6. Further tests are necessary to establish that these results hold for all mixtures. This test demonstrates that the pulse velocity increases rapidly with degree of consolidation.

APPENDIX C: STEPS OF POWER CONSUMPTION IN VIBRATING AN IDEAL MASS



STEPS IN ONE CYCLE OF VIBRATION

1. Accelerate mass from A to B. The source supplies energy to the mass. At point B, the mass has stored energy as kinetic energy of motion. The velocity is zero at A and maximum at B.

2. Decelerate mass from B to a full stop at C. Regardless of the type of energy source the mass lost all of its stored kinetic energy of motion when it came to a stop. If a stiffness element were connected to the mass, it would have accepted the energy from the mass and stored it as potential energy. Since the system lacked elasticity, the source was forced to do work to stop the mass. Although the mass had available energy to do work, it was dissipated rather than restored.

3. Accelerate mass from C to B. As the mass has zero velocity at C (stored energy is zero) and will have maximum velocity at B, energy must be delivered to the mass from the source.

4. Decelerate mass from B to A. This ends one cycle of motion. Here again, the mass has kinetic energy of motion stored that could be released to a spring or stiffness element if one were present. If not, the source must apply a force in an opposite direction to stop the mass at A. Once more, the source must do the work to stop the mass, and another opportunity to conserve energy is lost.

5. This is exactly analogous to the electrical problems electrical power companies have with reactive power. It is power that is lost because capacitors (springs) have not been added to a circuit that is primarily inductive (masslike). Real power is only lost to resistors, or in the mechanical case, to dampers (dashpots).

APPENDIX D: POWER CONSUMED IN VIBRATING FRESH CONCRETE DURING LIQUEFACTION

1. After breakdown, fresh concrete responds as an ideal mass to a stimulus. The motion has no restriction to movement from damping or stiffness. The only restriction to movement is the inertia of the mass. The derivation of work required to vibrate an ideal mass is as follows:

<u>a</u>. The expression for instantaneous force is

$$f_{in} = F \sin \omega t$$

<u>b</u>. For an ideal mass, this force will always be in phase with acceleration. For simple harmonic motion, the acceleration leads the displacement by 180 deg. Therefore, the force leads the displacement by 180 deg, which is expressed as

$$x = X \sin (\omega t - 180°)$$

<u>c</u>. Assume that the source can only supply energy to the mass and cannot accept the stored energy of kinetic motion in the mass. To eliminate the problem of negative power (mass-feeding source) so that all the power is positive (source-feeding mass), the work should be calculated for one-fourth cycle by the equation

$$1/4W = \int_0^{\pi/2} f_{in}\, dx$$

where W equals work done in one cycle.

By differentiating the displacement equation (<u>b</u>)

$$dx = X\omega \cos (\omega t - 180°)\, dt$$

and substituting into the work equation (<u>c</u>)

$$1/4W = \int_0^{\pi/2} F \sin \omega t\, [X\omega (\cos (\omega t - 180°))\, dt]$$

Since $\cos (\omega t - 180°) = -\cos \omega t$

then $1/4W = -FX\left(\int_0^{\pi/2} \sin \omega t \cos \omega t\right) \omega dt$

but $\sin \omega t \cos \omega t = \dfrac{\sin 2\omega t}{2}$

so $1/4W = \dfrac{-FX}{2}\left(\int_0^{\pi/2} \sin 2\omega t\right) \omega dt$

Next, by multiplying and dividing by 2

$$1/4W = \frac{-FX}{4}\left(\int_0^{\pi/2} \sin 2\omega t\right) 2\omega dt$$

and afterwards multiplying both sides by 4 to have a complete cycle

$$W = -FX\left(\int_o^{\pi/2} \sin 2\omega t\right) 2\omega dt$$

By integrating and then collecting terms

$$W = -FX \left|\cos 2\omega t\right|_o^{\pi/2}$$

$$W = -FX\ (-1-1)$$

$$W = 2FX$$

Then, inserting $X = \frac{A}{\omega^2}$, which is derived from simple harmonic motion, results in

$$W = \frac{2FA}{\omega^2}$$

According to Newton's second law

$$F = MA$$

or $A = \frac{F}{M}$

Thus, by substituting the value of A

$$W = \frac{2F^2}{\omega^2 M} = \text{work/cycle}$$

Using the equation for W, power (P) or work per unit time is derived as follows:

$$P = W\omega$$

$$P = \left(\frac{2F^2}{\omega^2 M}\right)\omega$$

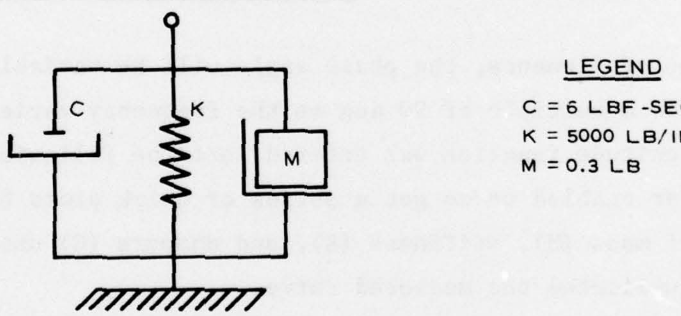
$$P = \frac{2F^2}{\omega M}$$

APPENDIX E: MATHEMATICAL MODEL OF FRESH CONCRETE

1. A mathematical model of fresh concrete describing the impedance or resistance to movement can be developed. This is simplified by a process known as the lumping of elements. A material such as fresh concrete is termed a continuous medium, because the elements within the mixture are numerous and inseparable. By the process of lumping, all of the mass, stiffness, and damping elements can be lumped into one element each or a few elements at most. Such a model would respond with the same impedance as the fresh concrete in the range of frequency of interest.

2. Various configurations can be made with the lumped elements by variations of parallel and series arrangements of the elements. Combinations were tried with three elements until the correct one was found that duplicated the impedance measured. This was accomplished by the following steps.

3. First, the impedance function was derived for the model shown below.



The impedance of elements in a parallel configuration is obtained by adding the individual elements.

$$Z = C + J\omega M - \frac{JK}{\omega}$$

$$Z = C + J\left(\omega M - \frac{K}{\omega}\right)$$

The magnitude of the impedance is the hypotenuse of a right triangle where C lies along the real or positive X axis and $\omega M - \frac{K}{\omega}$ lies along the imaginary or Y axis.

$$Z^2 = C^2 + \left(\omega M - \frac{K}{\omega}\right)^2$$

$$Z = \sqrt{C^2 + \left(\omega M - \frac{K}{\omega}\right)^2}$$

The phase angle relationship is the following:

$$\phi = \arctan \frac{\omega M - K/\omega}{C}$$

This represents the angle between the force and the resulting velocity. For single ideal elements, the following phase angles exist:

	MASS	DASHPOT	SPRING
Force leads acceleration or Acceleration lags force	0°	-90°	-180°
Force leads velocity or Velocity lags force	90°	0°	-90°
Force leads displacement or Displacement lags force	180°	90°	0°

For a combination of elements, the phase angle will be variable and will not necessarily be a multiple of 90 deg as the frequency varies.

4. The magnitude function was entered into the following program. An on-line plotter enabled us to get a series of quick plots by testing various values of mass (M), stiffness (K), and dampers (C) until the computed curve duplicated the measured curve.

5. The values that simulated the fresh concrete were found to be the following:

C = 6 lbf-sec/in.

K = 5000 lbf/in.

M = 0.3 lb

6. Program listing:

```
1000 120 PRINT,"VALUE OF WT.,K,C ?"
1010     READ,WT,K,C
1020     IF(WT.LT.0)GO TO 130
1040     F=20
1050     INC=5
1060     PRINT,"PLTL"
1070     DO 100 J=1,100
1080     Z=(C↑2+(.01625*F*WT-K/(6.28*F))↑2)↑.5
1090     IF(F.GT.99)INC=50
1100     IF(F.GT.999)INC=500
1110     IF(F.GT.5000)GO TO 140
1120     X=ALOG10(F)
1130     X=(X-ALOG10(19.99))*9999/(ALOG10(5000)-ALOG10(19.99))
1140     Y=ALOG10(Z)
1150     Y=(Y-0)*9999/4
1160     PRINT 110,X,Y
1170 110 FORMAT(F6.0,1X,F6.0)
1180     F=F+INC
1190 100 CONTINUE
1200 140 PRINT,"PLTT"
1210      GO TO 120
1220 130  STOP;END
```

7. After the fresh concrete transforms to the fluid state, the impedance can be modeled with a single mass unit. Stiffness and damping properties vanish to insignificant levels. Both of the impedance curves are shown in Figure E1 as the plotter would plot them for the solid and fluid condition of the fresh concrete. The plastic or breakdown condition is drawn in by hand rather than computed.

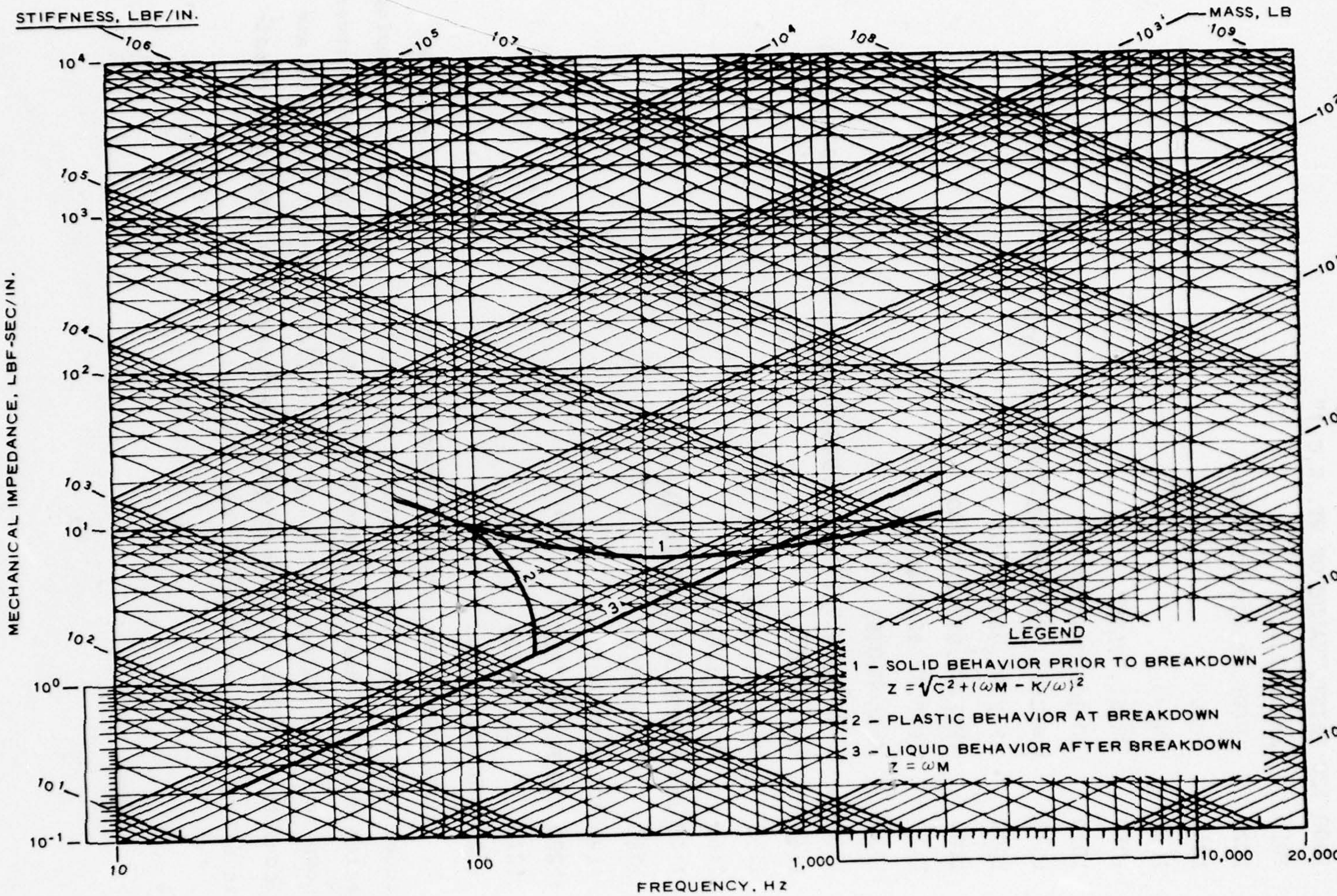


Figure E1. Three types of behavior of fresh concrete

APPENDIX F: NOTATION

a	Instantaneous acceleration
A	Peak acceleration
C	Damping coefficient
f	Frequency
f_{in}	Instantaneous force
F	Peak force
$\bar{F}$	Complex force
g	Local acceleration of free-fall
J	Complex operator denoting angular position of the imaginary term with respect to the real term
K	Stiffness coefficient
m	Mass of the driver plate and effective weight of the force and acceleration transducer (lbm); mass of test fixture (lbf-sec^2/in.) where acceleration due to gravity is expressed in in./sec^2
M	Mass being vibrated
n	Number of g's
P	Power or work per unit time
t	Time
v	Instantaneous velocity
V	Peak velocity
$\bar{V}$	Complex velocity
W	Work done in one cycle
x	Instantaneous displacement
X	Peak displacement
Z	Magnitude of impedance
$\bar{Z}$	Complex impedance
θ	Phase angle between force and acceleration
λ	Wavelength
υ	Velocity of sound
π	3.1416
ϕ	Phase angle between force and velocity
ω	Angular frequency ($2\pi f$)

In accordance with letter from DAEN-RDC, DAEN-ASI dated 22 July 1977, Subject: Facsimile Catalog Cards for Laboratory Technical Publications, a facsimile catalog card in Library of Congress MARC format is reproduced below.

Alexander, A Michel
Study of vibration in concrete; Report 3: Mechanics of motion of fresh concrete / by A. Michel Alexander. Vicksburg, Miss. : U. S. Waterways Experiment Station, 1977.
32, [16] p., 6 leaves of plates : ill. ; 27 cm. (Technical report - U. S. Army Engineer Waterways Experiment Station ; 6-780, Report 3)
Prepared for Office, Chief of Engineers, U. S. Army, Washington, D. C., under Work Unit 31251.
References: p. 32.

1. Concrete vibration. 2. Consolidation (Concrete). 3. Fresh concretes. 4. Mechanical impedance. I. United States. Army. Corps of Engineers. II. Series: United States. Waterways Experiment Station, Vicksburg, Miss. Technical report ; 6-780, Report 3.
TA7.W34 no.6-780 Report 3