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THE USE OF GRAVITY ANOMALIES ON A BOUNDING SPHERE TO IMPROVE POTENTIAL COEFFICIENT DETERMINATIONS.

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The precise determination of potential coefficients from terrestrial gravity data requires that, among other things, the topography of the earth's surface must be considered. This paper first formulates a procedure where the potential coefficients can be determined using anomalies determined on a sphere that encloses the mass of the earth. The resultant equations can also

be formulated to compute correction terms to potential coefficients derived from uncorrected surface free-air anomalies. In order to obtain the anomalies on the bounding sphere the method of least squares collocation was investigated. The computation of anomaly correction terms for 1654 5° equal area blocks was carried out with the largest correction being 5.1 mgals with the root mean square value being ±0.9 mgals. Using these correction terms and previously derived potential coefficients from terrestrial gravity data, an improved set was derived. From degree 5 to degree 20, the improved set showed slightly better agreement with the GEM 9 (satellite derived) potential coefficients than the original coefficients. The correction of the original coefficients was small, however, being 1.8% of the original coefficients at degree 2, rising to 7.5% at degree 40.

Finally, the anomaly correction terms were used to obtain an improved comparison of satellite derived 5° anomalies and terrestrial data. This was done by using the satellite determined potential coefficients to derive anomalies on a bounding sphere which were then downward continued to the surface. These anomalies showed a better agreement with the observed anomalies than did anomalies computed directly on the surface (mean square difference: 109 $mgal^2$ vs. 91 $mgal^2$).

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Foreword

This report was prepared by Richard H. Rapp, Professor, Department of Geodetic Science, The Ohio State University, under Air Force Contract No. F19628-76-C-0010, The Ohio State University Research Foundation Project No. 4214 B1. The contract covering this research is administered by the Air Force Geophysics Laboratory, L. G. Hanscom Air Force Base, Massachusetts, with Mr. Bela Szabo, Contract Monitor.

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Introduction

The determination of potential coefficients from the analysis of satellite data, gravity data, or a combination of both data types is an active area of research in geodesy. These potential coefficients are useful in improved orbit prediction and estimation; in the computation of long wave length features of quantities dependent on the gravity field such as geoid undulations and deflections of the vertical; and in the information such coefficients imply about the internal structures of the earth.

In these determinations from two different data types, satellite data and terrestrial gravity data, it is very important to be sure that we are determining the same quantity (the potential coefficients) through consistent models. In this paper we examine the model problem for the determination of potential coefficients from terrestrial data. A recent discussion with numerical results for the potential coefficients can be found in Rapp (1977). In that report a problem was found with certain corrections to be applied to the terrestrial anomaly that were needed to determine potential coefficients. To define this further we express the fully normalized potential coefficients ($\overline{C}_{\ell m}$, $\overline{S}_{\ell m}$) in terms of gravity anomalies as follows (Ostach and Pellinen, 1970):

$$(1) \qquad \left\{ \begin{matrix} \overline{C}_{\ell m}{}^{*} \\ \overline{S}_{\ell m}{}^{*} \end{matrix} \right\} = \frac{1}{4\pi G(\ell-1)} \iint_{\sigma} (\Delta g + G_1 + \ldots) \left\{ \begin{matrix} \cos m\lambda \\ \sin m\lambda \end{matrix} \right\} \overline{P}_{nm}(\sin\overline{\varphi})\, d\sigma$$

where G is an average value of gravity, $\overline{\varphi}$ is the geocentric latitude, $\overline{P}_{nm}$ are the fully normalized associated Legendre functions, Δg is the mean surface free air anomaly given in block $d\sigma$, and G_1 is the Molodensky G_1 term. The * indicates that the coefficients are given with respect to the reference field used for defining the Δg values. Pellinen (1962) has indicated that the neglect of the G_1 term in (1) can cause errors in the low degree coefficients of 10 to 20%. Numerical studies described in Rapp (1977) agree with this estimate but the results are based on a number of assumptions relating the G_1 term to the terrain correction.

In practice , the computation of the G_1 term for mean anomalies (such as 1° and 5°) can be difficult in areas of rapidly varying topography. Consequently, it is of interest to develop a procedure for the estimation of the potential coefficients from terrestrial data that does not require the computation of the G_1 term, yet retains the rigor implied by that procedure. In this way we can determine these coefficients in a manner that such coefficients can be directly compared to satellite derived values. In addition, our results should indicate a proper anomaly (which is easily computed) to use in the combination of satellite and gravity data for potential coefficient determinations.

Procedures

The gravitational potential of this earth is usually expressed in terms of the potential coefficients as:

$$(2) \qquad V(r,\overline{\varphi},\lambda) = \frac{kM}{r}\left[1 + \sum_{\ell=2}^{\infty}\left(\frac{a}{r}\right)^{\ell}\sum_{m=0}^{\ell}(\overline{C}_{\ell m}\cos m\lambda + \overline{S}_{\ell m}\sin m\lambda)\,\overline{P}_{\ell m}(\sin\overline{\varphi})\right]$$

where kM is the geocentric gravitational constant and a is the equatorial radius of an ellipsoid approximating the geoid. In fact, a is arbitrary, but traditional use has been for it to be taken as an ellipsoid equatorial radius. Equation (2) is considered convergent on and outside the Brillouin sphere $\overline{R}$ that just encloses all the mass of the earth. (In this discussion we will ignore the mass of the atmosphere.)

Adopting a reference potential (usually that implied by an equipotential ellipsoid), a disturbing potential, T, can be found from (2). Using the basic boundary condition of gravimetric geodesy (Heiskanen and Moritz, 1967, p. 86, eq. (2-147c)), we find for the gravity anomaly:

$$(3) \qquad \Delta g(r,\theta,\lambda) = \frac{kM}{r^2}\sum_{\ell=2}^{\infty}(\ell-1)\left(\frac{a}{r}\right)^{\ell}\sum_{m=0}^{\ell}(\overline{C}_{\ell m}{}^{*}\cos m\lambda + \overline{S}_{\ell m}{}^{*}\sin m\lambda)\,\overline{P}_{\ell m}(\sin\overline{\varphi})$$

Equation (3) is regarded as formally convergent on and outside the boundary sphere. [It should be noted here that in practice the summation in (3) is not to ∞ but to some small finite degree. The resultant finite series has often been evaluated at the surface of the earth.] On the bounding sphere the gravity anomaly is:

$$(4) \qquad \Delta g(\overline{R},\overline{\varphi},\lambda) = \frac{kM}{\overline{R}^2}\sum_{\ell=2}^{\infty}(\ell-1)\left(\frac{a}{\overline{R}}\right)^{\ell}\sum_{m=0}^{\ell}(\overline{C}_{\ell m}{}^{*}\cos m\lambda + \overline{S}_{\ell m}{}^{*}\sin m\lambda)\,\overline{P}_{\ell m}(\sin\overline{\varphi})$$

We now apply the usual orthogonal relationships of spherical harmonics to derive the potential coefficients from anomaly values given on the boundary sphere. We have:

$$(5) \qquad \left\{\begin{matrix}\overline{C}_{\ell m}{}^{*}\\ \overline{S}_{\ell m}{}^{*}\end{matrix}\right\} = \frac{1}{4\pi\left(\frac{kM}{\overline{R}^2}\right)(\ell-1)\left(\frac{a}{\overline{R}}\right)^{\ell}}\iint_{\sigma}\Delta g(\overline{R},\overline{\varphi},\lambda)\left\{\begin{matrix}\cos m\lambda\\ \sin m\lambda\end{matrix}\right\}\overline{P}_{\ell m}(\sin\overline{\varphi})\,d\sigma$$

This equation can be compared to (1) and the differences seen. Let $(\bar{C}_{\ell m}^*, \bar{S}_{\ell m}^*)_A$ be the potential coefficients computed from (1) setting G_1 and higher terms to zero. In addition we let:

$$\Delta g\,(\bar{R},\bar{\varphi},\lambda) = \Delta g\,(R,\bar{\varphi},\lambda) + \delta \tag{6}$$

where $\Delta g\,(R,\varphi,\lambda)$ are the anomalies assumed to refer to a spherical approximation of the geoid where R is the radius of the sphere. If we now substitute (6) into (5) and using (1) we have:

$$\left\{\begin{matrix}\bar{C}_{\ell m}^*\\ \bar{S}_{\ell m}^*\end{matrix}\right\} = \left(\frac{\bar{R}}{R}\right)^2\left(\frac{\bar{R}}{a}\right)^\ell \left\{\begin{matrix}\bar{C}_{\ell m}^*\\ \bar{S}_{\ell m}^*\end{matrix}\right\}_A + \frac{1}{4\pi G(\ell-1)}\iint_\sigma \delta \left\{\begin{matrix}\cos m\lambda\\ \sin m\lambda\end{matrix}\right\} \bar{P}_{\ell m}(\sin\bar{\varphi})\, d\sigma \tag{7}$$

where in the coefficient of the integral we have set $(kM/\bar{R}^2)$ to an average value of gravity and $(a/\bar{R})^\ell$ has been set to 1. These approximations are justified as the δ term is expected to be small. The coefficient of $(\bar{C}_{\ell m}^*, \bar{S}_{\ell m}^*)$ in (7) is 1.0039 at degree 2 and reaches 1.0628 at degree 60 letting a = 6378140 m, R = 6371000 m, and $\bar{R}$ = 6384405 m. Computations related to (7) will be described in a later section.

We now turn to the determination of the δ term introduced in equation (6). This term may be computed in several ways. An upward continuation procedure using 5° mean anomalies has been described by Rapp (1969). However, a more convenient procedure is possibly the use of least squares collocation. For its application we consider that we are given a set of surface anomalies which are to be extended to the boundary sphere. We can represent this computation in the following general form:

$$\overline{\Delta g}_{\bar{R}} = C_{\bar{R},s}\,(C_{ij}^* + D_{ij})^{-1}\,[\overline{\Delta g}_s] \tag{8}$$

where $\overline{\Delta g}_{\bar{R}}$ is the mean anomaly on the bounding sphere, $\overline{\Delta g}_s$ is the mean free-air surface anomaly, $C_{\bar{R},s}$ represents the covariance between $\overline{\Delta g}_{\bar{R}}$ and $\overline{\Delta g}_s$, C_{ij}^* is the covariance between the known surface anomalies $\overline{\Delta g}_s$ and D_{ij} is the standard deviation of the known anomalies. A similar procedure has been used in Rapp (1977) to estimate 5° mean free-air surface anomalies from 1° x 1° mean free-air anomalies. However, in the application of (8) we need to use a spatial covariance function as opposed to a surface covariance function used in Rapp (1977).

In the past few years a number of spatial covariance functions have been described. We can write:

$$C\,(P,Q) = \sum_{\ell=2}^{\infty} c_\ell \left(\frac{R_B^2}{r_P\, r_Q}\right)^{\ell+2} P_\ell(\cos\psi) \tag{9}$$

where: C is the covariance between two anomalies located at a distance r_P and r_Q from the center of the earth, and separated by a spherical distance ψ. c_ℓ are anomaly degree variances and R_B is the radius of a sphere totally imbedded within the earth. This sphere is often called the Bjerhammar sphere. Anomaly covariance functions may also be derived from disturbing potential degree variances. The infinite series in (9) may be expressed in closed form provided a model is chosen for the anomaly degree variances. Examples of these models and the resultant closed expression for C (P, Q) are described in Tscherning and Rapp (1974). In this report a subroutine COVA was given that computed covariances for the following c_ℓ model:

$$c_\ell = \frac{A\,(\ell-1)}{(\ell-2)(\ell-B)} \tag{10}$$

where $A = 425.28\ \text{mgal}^2$, $B = 24$ with a R_B value of 6369780 m. This particular model fits quite well information that exists concerning point anomaly variances, mean anomaly variances, and anomaly degree variances of low degree as implied from satellite derived potential coefficients. However, the computation of the covariances needed in (8) requires an excessive amount of computer time so that we have investigated the use of a simplified c_ℓ model that yields simplified covariance expressions. For such a case we used the following model suggested by Tscherning:

$$c_\ell = \frac{A'\,(\ell-1)}{\ell} \tag{11}$$

which implies the following anomaly covariance function:

$$C\,(P,Q) = A' s^2 \left(\frac{1}{L} - 1 - \ln\frac{2}{N}\right) \tag{12}$$

where

$$s^2 = R_B^2/(r_P\, r_Q);$$

$$L = [1 - 2st + s^2]^{\frac{1}{2}}$$

$$N = 1 + L - st$$

$$t = \cos \psi$$

For the application of this model appropriate values of A' and s or R_B need to be chosen. How this was done and results from (8) for test cases are described in the next section.

Computations of the δ term

We first propose to evaluate (8) for 5° mean anomalies. In doing this we will consider three different covariance functions. The first spatial covariance function is defined through equation (10) and associated constants and is obtained from subroutine COVA given in Tscherning and Rapp (1974). We then considered the covariance function described in (12) with values of A' determined for two different R_B values. To do this we first defined R_B and evaluated (12) with $\psi = 0^\circ$ and $r_P = r_Q = R + 13.405$ km. The anomaly variance from COVA was also evaluated for this case and the value of A' computed so that the anomaly variance at the given $r_P = r_Q$ values would be the same from COVA and the simplified model of equation (12). The two R_B values were based on a value given by Lauritzen (1973, p. 84) and on a value for R_B given by Tscherning (1972, p. 58). We summarize in Table 1 various constants to be used in computations to follow.

Table 1

Parameters Associated with Covariance Computations

Quantity	Value	Description
R	6371000 m	Mean Earth Radius
$\overline{R}$	6384405 m	Radius of Bounding Sphere
R_B	6369780 m	Implied by Constants of COVA
R_B	6335960 m	R_B from Lauritzen with A'
A'	16.95 $mgal^2$	computed for use in (12)
R_B	6348827 m	R_B from Tscherning with A'
A'	10.16 $mgal^2$	computed for use in (12)

For future referencing we designate the covariances from (12) using $A' = 16.95$ mgal^2 and $R_B = 6335960$ m as Model B, and with $A' = 10.16$ mgal^2 and $R_B = 6348827$ m as Model C.

For these initial computations we chose 4 5° anomalies of upward continuation to the bounding sphere using (8). In each case the anomaly on the bounding sphere was computed using the 5 closest known 5° anomalies (including the surface value itself). The mean anomaly covariances needed in (8) were computed by the numerical integration of the point covariance function as described in Heiskanen and Moritz (1967, p. 277) except our application is to the computation of the covariance between mean anomalies. This integration is the most time consuming aspect of the computation. Various tests were made with different grid spacings in this numerical integration procedure. For a given grid spacing computations for a single 5° block took 4 times longer using subroutine COVA than when the simplified model represented by equation (12). However, we also found that a smaller grid interval was required with the use of COVA to achieve a given accuracy than was required when using the simplified model. Considering this fact, the use of COVA in the upward continuation process would take 29 times longer than the simplified model when accuracies on the order of ±0.2 mgals are sought in the process. An initial description of the 4 test blocks is given in Table 2.

Table 2

Description of 5° Test Blocks

Block No.*	Mean Surface FA Anomaly	Accuracy	φ°	λ°	$\bar{h}$
208	-18.1 mgals	±3.4	45-50	96-103	2011 m
310	- 4.3	±4.3	35-40	82- 88	2967
408	21.5	±3.7	30-35	319-325	-3192
562	-32.1	±3.7	15-20	83- 89	-1581

* Rapp (1977)

As can be seen from Table 2, two of the blocks have a large mean elevation, while two blocks are ocean blocks with their depths given .

Computations were made to determine δ as defined by equation (6) using (8) with three different covariance functions. In (8) the D_{ij} values were set to zero as we wished to find solely the effect of the upward continuation and not a filtered anomaly value. The r_P value was determined by adding to the mean earth radius the average elevation of the block (or zero if we had an oceanic block). The r_Q value was taken as the radius of the bounding sphere defined in Table 1.

The procedure for computing r_P can be regarded as a spherical approximation. An improved ellipsoidal approximation can be found by adding the mean elevation of the block to the ellipsoidal geocentric distance (plus geoid undulation). We found, however, that the ellipsoidal computations changed our final results on the order of 0.2 mgals. In the results given in Table 3 the spherical approximation has been used.

Table 3

Estimates of $\delta = \Delta g\,(\bar{R},\varphi,\lambda) - \Delta g\,(R,\varphi,\lambda)$
for 4 5° equal area blocks

Block No.	δ (mgals)	Covariance Used
208	0.5	Original COVA
208	0.6	Model B
208	0.5	Model C
310	0.4	Original COVA
310	0.4	Model B
310	0.4	Model C
408	-1.1	Original COVA
408	-1.1	Model B
408	-0.9	Model C
562	1.6	Original COVA
562	1.6	Model B
562	1.3	Model C

It is immediately clear that the results computed from the three different covariance functions differ only slightly. For computations dealing with the complete 1654 anomaly field we choose to use the constants of Model B defined previously.

From Table 3 we see that the influence of the covariances on the upward continued anomaly is small. We next examine two additional quantities that must be specified before a global computation can be carried out. The first is the numerical integration subdivision interval and the second is the number of known 5° anomalies to use in the upward continuation procedure. We first look at the integration interval. This arises when a point covariance function is used to compute the covariance between two quantities that represent area averages. Consider the two blocks A_1 and A_2 shown in Figure One for which the covariance $C_{A_1A_2}$ is designed for some unspecified type of variable.

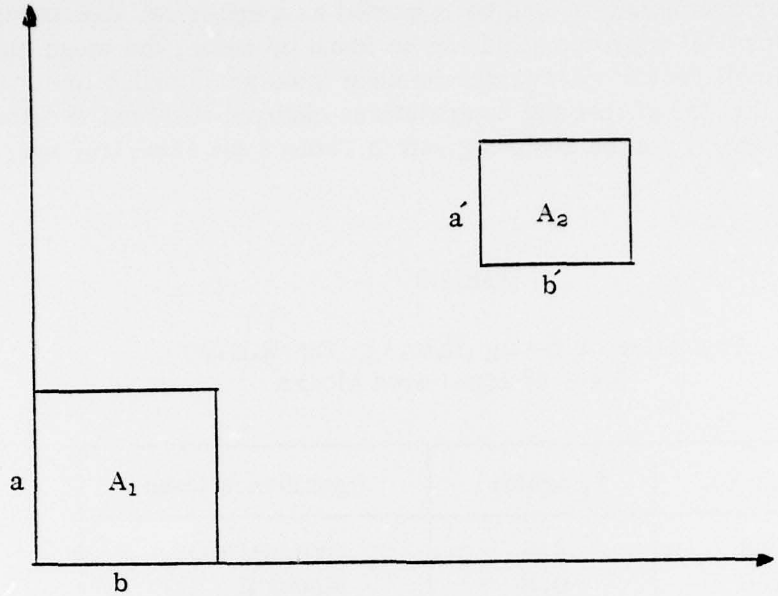


Figure One

Location of Mean Blocks for which
Covariances are to be Derived

Let C (P, Q) be a point covariance function between points P (in A_1) and Q (in A_2). Then we can write (following from equation (7-82) in Heiskanen and Moritz, 1967):

(13) $$C_{A_1A_2} = \frac{1}{(ab)(a'b')} \int_{x=0}^{a} \int_{y=0}^{b} \int_{x'=0}^{a'} \int_{y'=0}^{b'} C(x, y, x', y') \, dx \, dy \, dx' \, dy'$$

If we now let:

$a = n\,dy$
$b = n\,dx$
$a' = n\,dy'$
$b' = n\,dx'$

we can express (13) in the following numerical integration form:

(14) $$C_{A_1A_2} = \frac{1}{n^4} \sum_{i}^{n} \sum_{j}^{n} \sum_{i'}^{n} \sum_{j'}^{n} C(x_i\, y_j\, x'_{i'}\, y'_{j'})$$

The accuracy of this computation will improve as n increases, but so will the computation time. Our interest is to choose the smallest n consistent with the final accuracies desired. To do this we computed the upward continued anomalies for the four test blocks given in Table 2 using different n values, and the 5 closest 5° anomalies, and the ellipsoidal reference model ($a = 6378140$m, $f = 1/298.25$). The results are shown in Table 4.

Table 4

Effect of Grid Size on Upward Continuation of 5° Anomalies

	n			
Block No.	4	6	8	10
208	-17.27 mgals	-17.40	-17.43	-17.43
310	- 3.83	- 3.90	- 3.91	
408	20.49	20.64	20.67	20.67
562	-31.20	-31.33	-31.35	

We see from this only small changes in the upward continued anomalies. For our purposes we select for future use an n of 5 which falls between two tested values. Such a choice would appear to yield accuracies in this part of the upward continuation of about ± 0.1 mgals.

We next examined the effect of the number of known blocks to be used in the upward continuation. We carried out computations with 1, 5, and 9 5° blocks closest to the block being upward continued to the bounding sphere. These results are shown in Table 5.

Table 5

Effect of Number of Known Blocks on Upward Continuation of 5° Anomalies

	Number of Known Blocks		
Block	1	5	9
208	-17.01 mgals	-17.40	-17.32
310	- 4.12	- 3.90	- 3.87
408	20.52	20.64	20.86
562	-31.18	-31.33	-31.34

We see that the difference between using 5 and 9 blocks is small. Because of symmetry, however, we choose to use for further computations, the 9 closest 5° anomalies for the global upward continuation.

With the above information available, the upward continuation of the 1654 surface 5° anomalies given in Rapp (1977) was carried out under the following specifications:

Covariances: Model B
Grid Division: 5
Number of Known Blocks: 9

In addition, the geocentric distance to the surface anomaly block was computed by adding the height of the block to the geocentric radius of an ellipsoid whose equatorial radius was 6378140 m with a flattening of 1/298.25.

The mean difference between the surface anomalies and the bounding sphere anomalies was 0.0 mgals while the root mean square difference was ± 0.9 mgals. The maximum difference was 5.1 mgals which occured for a block whose surface anomaly was 67 mgals. A complete listing of the surface anomalies and the bounding sphere anomalies can be obtained from the author.

Application of Upward Continued Anomalies to Potential Coefficient Determinations

We now wish to use the anomalies on the bounding sphere to derive potential coefficients as shown in equation (7). In doing this it is convenient to write (7) in the following form:

$$\begin{Bmatrix} \bar{C}_{\ell m}^{*} \\ \bar{S}_{\ell m}^{*} \end{Bmatrix} = \begin{Bmatrix} \bar{C}_{\ell m}^{*} \\ \bar{S}_{\ell m}^{*} \end{Bmatrix}_{A} + \begin{Bmatrix} \Delta\bar{C}_{\ell m} \\ \Delta\bar{S}_{\ell m} \end{Bmatrix}_{I} + \begin{Bmatrix} \Delta\bar{C}_{\ell m} \\ \Delta\bar{S}_{\ell m} \end{Bmatrix}_{II} \tag{15}$$

where

$$\begin{Bmatrix} \Delta\bar{C}_{\ell m} \\ \Delta\bar{S}_{\ell m} \end{Bmatrix}_{I} = \left[\left(\frac{\bar{R}}{R}\right)^{2}\left(\frac{\bar{R}}{a}\right)^{\ell} - 1\right] \begin{Bmatrix} \bar{C}_{\ell m} \\ \bar{S}_{\ell m} \end{Bmatrix}_{A}$$

$$\begin{Bmatrix} \Delta\bar{C}_{\ell m} \\ \Delta\bar{S}_{\ell m} \end{Bmatrix}_{II} = \frac{1}{4\pi G(\ell - 1)} \iint_{\sigma} \delta \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} \bar{P}_{\ell m}(\sin\bar{\varphi})\, d\sigma$$

Using the potential coefficients given by Rapp (1977) and the values of R, $\bar{R}$, and a (6378140 m) given previously the above correction terms were computed and used to obtain the corrected coefficients given on the left hand side of (15). In order to judge the magnitude of these correction terms, root mean square values by degree, of the total correction, correction (I), and correction (II) are given in Table 6. In addition, the ratio (in percent) of these corrections to the magnitude of the coefficients is shown in Figure 2.

Table 6

Root Mean Square Value of Potential Coefficient Correction Terms By Degree ($\times 10^{11}$)

ℓ	Total Correction	Correction (I)	Correction (II)
2	717	796	1346
3	981	797	1682
4	375	357	635
5	470	234	605
6	234	266	443
7	312	200	472
8	151	113	214
9	222	147	344
10	114	103	195
11	132	93	206
12	93	53	132
13	91	66	140
14	59	50	99
15	82	57	132
16	71	54	117
17	66	47	104
18	66	42	100
19	52	39	81
20	59	31	86
21	57	34	85
22	56	35	85
23	52	30	78
24	51	28	74
25	54	29	79
26	58	27	80
27	55	25	74
28	60	27	82

29	56	30	82
30	56	26	76
31	50	22	67
32	53	27	75
33	50	24	70
34	54	27	77
35	44	25	65
36	51	27	74
37	42	24	61
38	46	26	67
39	42	22	60
40	43	26	64

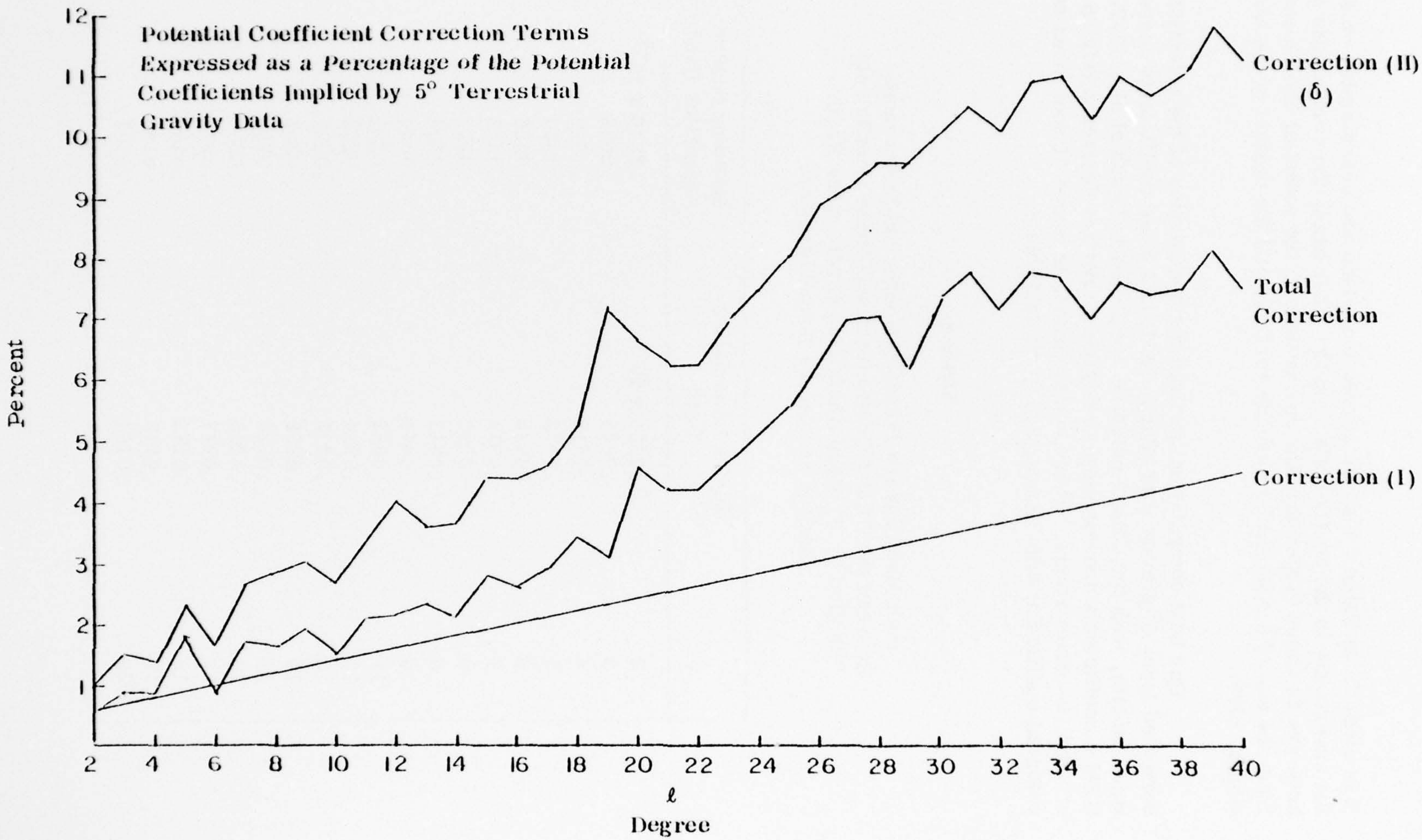
From Figure 2 we see that the correction terms, relative to the coefficients, increase with degree. The first correction term shows almost a linear dependence on ℓ while correction term (II) shows a more rapid increase than (I). The total correction is less (on the average) than correction term (II) as there is some cancellation between the two correction terms. We see then that the correction terms have larger effects at high degrees than at the lower degrees. This is opposite to what was found in Rapp (1977) when techniques for estimating the G_1 term in equation (1) were investigated. The results of this paper seem to be more realistic.

We should point out here that the correction terms are small, not only with respect to the potential coefficients, but also with respect to the estimated standard deviations of the potential coefficients as determined from the standard deviations of the anomalies used in the computation. Such comparisons, in terms of %, are given in Table 7.

Table 7

Total Correction Term, and Standard Deviation of, Potential Coefficients Expressed as a Percent of the Coefficients

ℓ	Correction	Std. Dev.	ℓ	Correction	Std. Dev.
5	1.8%	18.7%	25	5.5%	80.5%
10	1.6	29.1	30	3.4	86.0
15	2.7	45.5	35	7.4	87.2
20	4.6	77.7	40	7.5	83.4


Potential Coefficient Correction Terms
Expressed as a Percentage of the Potential
Coefficients Implied by 5° Terrestrial
Gravity Data
Percent
12
11
10
9
8
7
6
5
4
3
2
1
2
4
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ℓ
Degree
Correction (II)
(δ)
Total
Correction
Correction (I)

<u>Figure 2</u>

It is clear from Table 7 that the correction terms are considerably smaller than the uncertainty in the coefficients. On the other hand, the corrections computed here are 20 times larger than the corrections to the potential coefficients arising from the use of a spherical reference surface, and the neglect of the atmosphere (Rapp, 1977).

Our next computation involved the comparison of the potential coefficients computed from 5° gravity data (Rapp, 1977) and those coefficients corrected using equation (15), with the GEM 9 potential coefficients (Lerch et al., 1977). The GEM 9 coefficients are complete to degree 20 and are derived solely on the basis of satellite observations. These comparisons, in terms of root mean square potential coefficient differences, are given in Table 8.

Table 8

Root Mean Square Potential Coefficient Differences Between the GEM 9 Coefficients and Those Coefficients Implied by 5° Mean Anomalies Given on the Earth's Surface and On the Bounding Sphere

ℓ	Surface Anomalies Used	Bounding Sphere Anomalies Used
2	$.2934 \times 10^{-6}$	$.2946 \times 10^{-6}$
3	.2625	.2638
4	.1612	.1619
5	.1473	.1494
6	.0935	.0926
7	.0574	.0573
8	.0592	.0593
9	.0612	.0604
10	.0444	.0444
11	.0332	.0331
12	.0254	.0252
13	.0222	.0222
14	.0252	.0251
15	.0228	.0224
16	.0205	.0202
17	.0217	.0214
18	.0172	.0169
19	.0145	.0143
20	.0164	.0161

From this table we see a slight degradation in the agreement when the correction terms are applied at the lower degrees. However, from degree 6 and beyond, the corrected coefficients agree with the GEM 9 coefficients better than the uncorrected coefficients. However, the improvement is small and could be considered negligable when the accuracy of the potential coefficient determinations is considered. Nevertheless the results are encouraging.

Application of Upward Continued Anomalies to Anomaly Comparisons with Anomalies Implied by Potential Coefficients

For a number of years potential coefficient solutions have been judged by computing anomalies from such coefficients and comparing such anomalies with surface anomalies. In this computation, equation (3) or a spherical approximation has been used to compute the anomalies from the coefficients. Such a procedure has been done ignoring the convergence question as it has been considered to have negligable influence on the final results. We now turn our attention to such comparisons using the results of this paper.

To do this we first computed anomalies in 5° blocks as implied by the GEM 9 coefficients to degree 20 using equation (3) where r was evaluated as the geocentric distance to the block whose anomaly was being determined. Let this anomaly be designated Δg_{SA}. Next compute the 5° anomaly on the bounding sphere using equation (4). Using this value, compute from equation (6) the anomaly on the surface using the δ value implied by the upward continuation process. This anomaly, which we will designate Δg_{SB}, can be rigerously compared to the surface anomaly values, as given in Rapp (1977). Such comparisons between the observed surface values Δg_S, Δg_{SA} and Δg_{SB} are shown in Table 9 in terms of root mean square differences (when the mean difference has been excluded).

Table 9

Root Mean Square Difference Between Potential Coefficient Anomalies and Surface 5° Anomalies

	m (max) (mgals)		
	5	10	15
$E((\Delta g_S - \Delta g_{SA})^2)$	107	109	123
$E((\Delta g_S - \Delta g_{SB})^2)$	88	91	101

In this table m is the maximum standard error of the terrestrial anomaly used in the comparison. The number of such anomalies is 1003 ($m \le 5$ mgals); 1343 ($m \le 10$ mgals) and 1483 ($m \le 15$ mgals).

It is clear that the better agreement between the anomalies occurs when the potential coefficients are used to evaluate anomalies on the bounding sphere which are then downward continued to the surface, instead of just evaluating the anomalies on the surface using an expression of questionable converging properties.

Summary and Conclusions

This paper has been written to show how potential coefficients can be determined from terrestrial gravity data rigorously considering the topography by upward continuing the surface free-air anomalies to a sphere that completely surrounds the earth. This upward continuation can be performed by least squares collocation. We found that the use of sophisticated covariance functions was too expensive in terms of computer costs so that it was necessary to choose simplified equations. However, tests showed that results from the simplified equations agreed quite well with values obtained from the more realistic covariance functions. Using these simplified models, a set of 1654 5° equal area anomalies on the bounding sphere have been computed. The maximum difference between the surface anomaly and the anomaly on the bounding sphere was 5.1 mgals with the root mean square difference being ±0.9 mgals.

The correction term δ, and the potential coefficients implied by the surface data, were used in equation (15) to derive correction terms to the surface data potential coefficients. The correction terms were small, being about 2% of the actual coefficients at degree 5 rising in a roughly linear way (percentage wise) to about 7% at degree 40.

The corrected coefficients were compared to the GEM 9 potential coefficients, as were the coefficients implied by the uncorrected surface anomalies. We found that above degree 5, the corrected coefficients agreed slightly better with GEM 9 than did the coefficients implied by the uncorrected surface anomaly data.

A final test was made by computing surface anomalies directly from a spherical harmonic expansion evaluated on the surface of the earth (and thus of questionable accuracy), and also by first computing the anomalies on the bounding sphere and subsequently downward continuing them to the earth's surface using the δ term derived through collocation. These two anomaly sets were then compared to the surface anomalies given in Rapp (1977). We found that the better agreement was obtained when the corrected bounding sphere anomalies were used (mean square difference = 109 mgals), than when directly computed anomalies on the surface from potential coefficients were used (mean square difference = 91 mgals).

This paper has shown that the problems raised by the convergence question are real at the $\pm$ 1 mgal level for 5° anomalies. Thus for the most accurate determination of potential coefficients from surface gravity material, equation (7) or (15) should be used.

Finally, an important implication of these studies exists for those computations that combine surface gravity and satellite data for potential coefficient determinations. In most current applications, equation (2) is used to relate anomalies and potential coefficients. However, this approach is in error with the more correct procedure being the use of equation (4) with the needed anomaly being found from equation (6).

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