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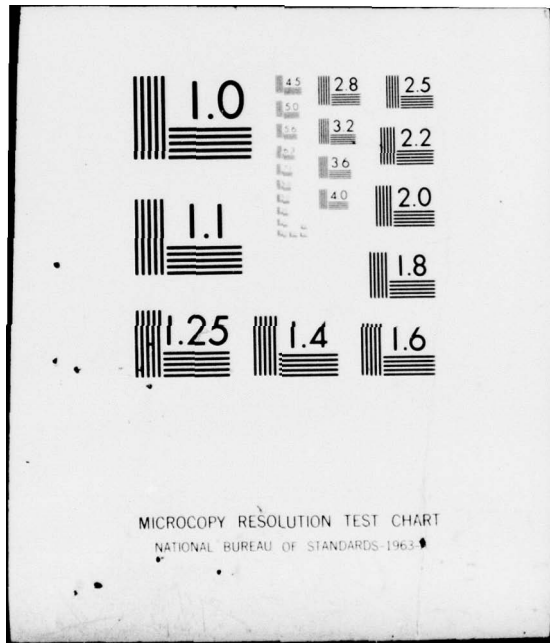
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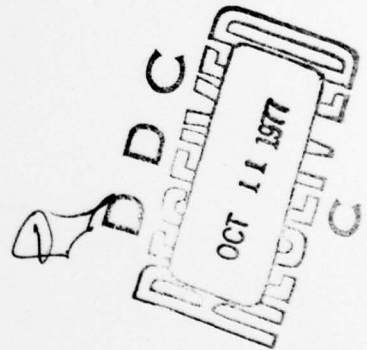
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THE COMPLEX COMPLIANCES OF QUARTZ

Arthur Ballato  
Electronics Technology & Devices Laboratory

September 1977

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The imaginary part of the complex compliance matrix, denoted as the fluency, is defined, and the elements are computed for alpha-quartz. Its application to low frequency vibrators is discussed. The role of motional time constants in characterizing resonator behavior is also described and applied to the minus 18.5 degree bar.			

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## INTRODUCTION

A great deal of current interest in piezoelectric vibrators focuses at the low end of the frequency spectrum, primarily for timing applications. In the solution of the vibration problem for the bars and forks used at low frequencies, it is usually convenient to employ the elastic compliance matrix  $s$  instead of the stiffness matrix  $c$ .<sup>1,2</sup>

In those situations where the stiffnesses  $c$  are used, e.g., the high frequency thickness modes of plates, the inclusion of acoustic loss is accomplished by allowing  $c$  to become complex.<sup>3</sup> The material coefficients appearing in the imaginary part are the elements of the viscosity matrix  $\eta$ . In the sequel we give the corresponding fluency matrix  $\zeta$ , arising from the complex compliance, and compute the elements for quartz.

## FLUENCY MATRIX

The influence of internal friction upon the elastic constants  $c$  and  $s$  was discussed briefly by Voigt.<sup>4</sup> Under suitable conditions of approximation Voigt gives, in our notation, the Hooke's law relations in the presence of loss:

$$T = c*S = (c + \eta\partial/\partial t)S, \quad (1)$$

$$S = s*T = (s - \zeta\partial/\partial t)T. \quad (2)$$

The compliance matrix may be developed from these equations as a power series in  $c$  and  $\eta$ . For harmonic time dependence one obtains

$$\begin{aligned} s &= c^{-1} \left[ \sum_{n=0}^{\infty} (-)^n \omega^{2n} (\eta c^{-1})^{2n} \right] \\ &= \left[ \sum_{n=0}^{\infty} (-)^n \omega^{2n} (c^{-1}\eta)^{2n} \right] c^{-1}, \end{aligned} \quad (3)$$

and

$$\zeta = c^{-1}\eta s. \quad (4)$$

- 
1. W. G. Cady, Piezoelectricity. New York: McGraw-Hill, 1946, and Dover, 1964.
  2. "Standards on Piezoelectric Crystals, 1949." Proc. IRE, Vol. 37, pp. 1378-1395, Dec. 1949. (IEEE Standard No. 176.)
  3. J. Lamb and J. Richter, "Anisotropic Acoustic Attenuation with New Measurements for Quartz at Room Temperature," Proc. Roy. Soc. (London), Vol. 293A, pp. 479-492, 1966.
  4. W. Voigt, Lehrbuch der Kristallphysik. Leipzig: Teubner, 2nd ed., 1928, pp. 792-796.



The analogous inverse relations are obvious.

Alternatively, one may write

$$c^*s^* = (c + j\omega\eta)(s - j\omega\zeta) = I, \quad (5)$$

multiply out, equate real and imaginary parts, and solve to get:

$$s = (c + \omega^2\eta c^{-1}\eta)^{-1}, \quad (6)$$

$$\zeta = c^{-1}\eta s, \quad (7)$$

with corresponding inverse relations. In the limit of zero frequency, Equations (3) and (6) become

$$s = c^{-1}, \quad (8)$$

and Equations (4) and (7) are

$$\zeta = c^{-1}\eta c^{-1} = \eta s. \quad (9)$$

#### APPLICATION TO QUARTZ

Using the room temperature values for  $c^E$  from Bechmann<sup>5</sup> with the  $\eta$  values of Lamb and Richter<sup>3</sup> in Equation 9 gives the values for the elements of the fluency matrix as shown in Table 1. The  $\zeta$  matrix has the same symmetry as  $s$  for all crystal classes. Recalculating  $\zeta$  with  $\omega = 10^{12}$  changes the entries by a few units in the third decimal place. Table 2 contains the fluency matrix elements for  $\omega = 10^{14}$ .

TABLE 1.  
FLUENCY MATRIX FOR QUARTZ AT  $\omega = 0$

$\zeta_{11}$	18.56	$\zeta_{44}$	17.65
$\zeta_{33}$	6.21	$\zeta_{66}$	30.70
$\zeta_{12}$	3.21	$\zeta_{14}$	7.87
$\zeta_{13}$	3.80	(all in $10^{-26}$ s/Pa)	

5. R. Bechmann, "Elastic and Piezoelectric Constants of Alpha-Quartz," Phys. Rev., Vol. 110, pp. 1060-1061, June 1958.

TABLE 2  
FLUENCY MATRIX FOR QUARTZ AT  $\omega = 10^{14}$

$\zeta_{11}$	5.30	$\zeta_{44}$	9.01
$\zeta_{33}$	3.42	$\zeta_{66}$	13.52
$\zeta_{12}$	-1.46	$\zeta_{14}$	2.56
$\zeta_{13}$	-0.46	(all in $10^{-26}$ s/Pa)	

#### MOTIONAL TIME CONSTANTS

From the solution to a particular vibration problem, the eigenvalue  $s_m^*$  or  $c_m^*$  for the mode  $m$  in question will arise. Motional time constants  $\tau_1^{(m)}$  and  $\bar{\tau}_1^{(m)}$  are then defined as

$$\tau_1^{(m)} = \lim_{\omega \rightarrow 0} \frac{\text{Im}(c_m^*)}{\omega \text{Re}(c_m^*)} = \frac{\eta_m}{c_m} \quad (10)$$

and

$$\bar{\tau}_1^{(m)} = \lim_{\omega \rightarrow 0} \frac{-\text{Im}(s_m^*)}{\omega \text{Re}(s_m^*)} = \frac{\zeta_m}{s_m} \quad (11)$$

Application of Equation (10) to doubly rotated quartz plates has recently been made,<sup>6</sup> where it is shown how  $\eta_m$  may be obtained from  $\eta$  and from the solution to the lossless problem.

$\tau_1^{(m)}$  is equivalently expressed in terms of the constants of the equivalent circuit as

$$\tau_1^{(m)} = R_{11} C_{11} = \frac{1}{\omega_m Q_m} \quad (12)$$

Additional relations may be given in terms of acoustic attenuation or of logarithmic decrement.

Equation (12) permits calculation of the intrinsic  $Q$  of a mode, exclusive of extrinsic factors such as mounting loss and ambient loading. It also pertains to  $\bar{\tau}_1^{(m)}$  for bar and fork resonators.

6. A Ballato, "Doubly Rotated Thickness Mode Plate Vibrators," in Physical Acoustics: Principles and Methods (W. P. Mason and R. N. Thurston, eds.) New York: Academic, 1977, Vol. 13, pp. 115-181.

For quartz at room temperature,  $\tau_1$  and  $\bar{\tau}_1$  run about 10 fs, with corresponding relaxation frequencies falling<sup>1</sup> in the<sup>1</sup> infrared region. It is no accident, therefore, that the IR absorption spectrum is used as a quality control criterion for inspection of cultured quartz<sup>7, 8, 9</sup>; the absorption is found to be correlated with the intrinsic Q of resonators made from the material.

Very recently, a new procedure has been proposed for evaluating the quality of cultured quartz samples.<sup>10</sup> The method requires resonator Q measurements made on extensional mode quartz bars. The data lead to experimental values of  $\bar{\tau}_1$  for comparison with Equation (11). The ratio of measured  $\bar{\tau}_1$  to calculated  $\bar{\tau}_1$  furnishes the condition of acceptability.

A constraint on the method is the Q-limiting effect of the mounting supports of the resonator bar. Sherman<sup>11</sup> has given an approximate relation that prescribes limits on the dimensions of the mounting supports and on the accuracy of their placement about the bar nodal positions to ensure the dominance of the intrinsic Q.

#### THE -18.5° QUARTZ BAR

Mason introduced the -18.5° quartz bar.<sup>12</sup> Its orientation is denoted<sup>2</sup> as (XYt) $\psi = -18.5^\circ$ , and is shown in Figure 1. This cut vibrates in an extensional mode along its length. Its angle of cut is determined by location of the zero of the elastic compliance  $s'_{24}$ :

- 
7. D. M. Dodd and D. B. Fraser, "The 3000-3900  $\text{cm}^{-1}$  Absorption Bands and Anelasticity in Crystalline  $\alpha$ -Quartz," J. Phys. Chem. Solids, Vol. 26, pp. 673-686, 1965.
  8. D. B. Fraser, D. M. Dodd, D. W. Rudd, and W. J. Carroll, "Using Infrared to Find the Mechanical Q of  $\alpha$ -Quartz," Frequency, Vol. 4, pp. 18-21, Jan.-Feb. 1966.
  9. J. Asahara and S. Taki, "Physical Properties of Synthetic Quartz and Its Electrical Characteristics," in Proc. 26th Annu. Frequency Control Symp., US Army Electronics Command, Fort Monmouth, NJ, June 1972, pp. 93-103.
  10. H. Fukuyo, N. Oura, and F. Shishido, "A New Quality Evaluation Method of Raw Quartz by Measuring the Q-Value of Y-Bar Resonator," in Proc. 31st Annu. Frequency Control Symp., US Army Electronics Command, Fort Monmouth, NJ, June 1977, in press.
  11. J. H. Sherman, Jr., private communication, Aug. 1977.
  12. W. P. Mason, "Electrical Wave Filters Employing Quartz Crystals as Elements," Bell Syst. Tech. J., Vol. 13, pp. 405-452, 1934.

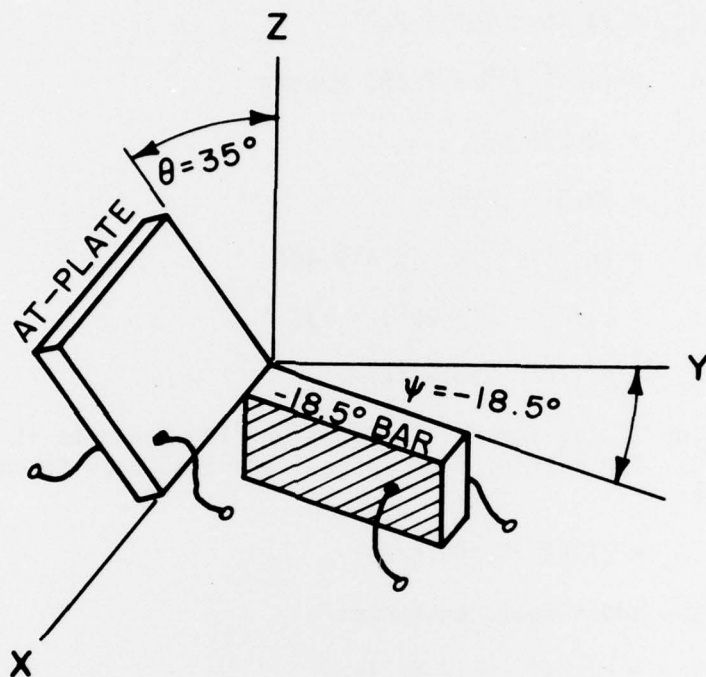


Figure 1. Orientations of AT-plate and  $-18.5^\circ$  Quartz Bar.

$$\begin{aligned}
 s'_{24} = & \{-2s_{11} \cos^2\psi \sin\psi + 2s_{33} \sin^3\psi \\
 & + (s_{44} + 2s_{13}) \sin\psi (\cos^2\psi - \sin^2\psi) \\
 & + s_{14} \cos\psi (3 \sin^2\psi - \cos^2\psi)\} \cdot \cos\psi.
 \end{aligned} \tag{13}$$

The compliance governing the frequency is  $s'_{22}$ :

$$\begin{aligned}
 s'_{22} = & s_{11} \cos^4\psi + s_{33} \sin^4\psi + (s_{44} + 2s_{13}) \cos^2\psi \sin^2\psi \\
 & - 2s_{14} \cos^3\psi \sin\psi.
 \end{aligned} \tag{14}$$

The bar is driven by an x-directed electric field; the piezoelectric constant is:

$$d'_{12} = -d_{11} \cos^2\psi + d_{14} \cos\psi \sin\psi, \tag{15}$$

and the dielectric permittivity is  $\epsilon_{11}$ .

Although the vibrator is referred to as the "-18.5° quartz bar," the more recent elastic values of Bechmann<sup>5</sup> show that  $s'_{24} = 0$  at  $\psi = -19.70^\circ$ . At this angle, the appropriate numerical constants describing the resonator are:

- $s'_{22} = 14.46 \times 10^{-12} \text{ Pa}^{-1}$
- $N = (4\rho s'_{22})^{-1/2} = 2.555 \text{ MHz-mm}$
- $d'_{12} = -2.278 \text{ pC/N}$
- $\epsilon'_{22} = 39.97 \text{ pF/m}$
- $k = (d'_{12}{}^2 / \epsilon'_{22} s'_{22})^{1/2} = 9.48\%$
- $r = C_0 / C_1 = \pi^2 / (8k^2) = 137.4$
- $\Gamma_1 = \epsilon'_{22} / r = 290.9 \text{ fF/m}$ .

Substitution of  $\zeta_{ij}$  for  $s_{ij}$  in Equation (14) converts it into the formula for  $\zeta'_{22}$ . The entries from Table 1 are then used to compute the effective value:

- $\zeta'_{22} = 21.63 \times 10^{-26} \text{ s/Pa}$ .

This leads to the additional constants:

- $\bar{\tau}_1 = \zeta'_{22} / s'_{22} = 14.96 \text{ fs}$
- $P_1 = \bar{\tau}_1 / \Gamma_1 = 5.14 \times 10^{-2} \text{ } \Omega\text{-m}$ .

A relaxation time constant appears from other considerations in a letter by Cook and Wasilik<sup>13</sup> on the -18.5° quartz bar.

In order to put the size of  $\bar{\tau}_1$  into perspective, it is instructive to observe that in the time  $\bar{\tau}_1$  the longitudinal elastic waves traveling along the bar length cover a distance of 0.76 Å, or only a little in excess of a Bohr radius (0.529 Å). The remaining quantities appearing above are related to the parameters of the lumped electrical equivalent circuit, and are defined in Bechmann's paper.<sup>14</sup>

Figure 2 shows the analog electrical equivalent circuit superimposed on a sketch of the bar. Piezoelectric excitation takes place, according to

- 
13. R. K. Cook and J. H. Wasilik, "Anelasticity and Dielectric Loss of Quartz," *J. Appl. Phys.*, Vol 27, pp. 836-837, 1956.
  14. R. Bechmann, "Schwingkristalle für Siebschaltungen," *Arch. d. Elektr. Übertrag.*, Vol. 18, pp. 129-136, Feb. 1964.

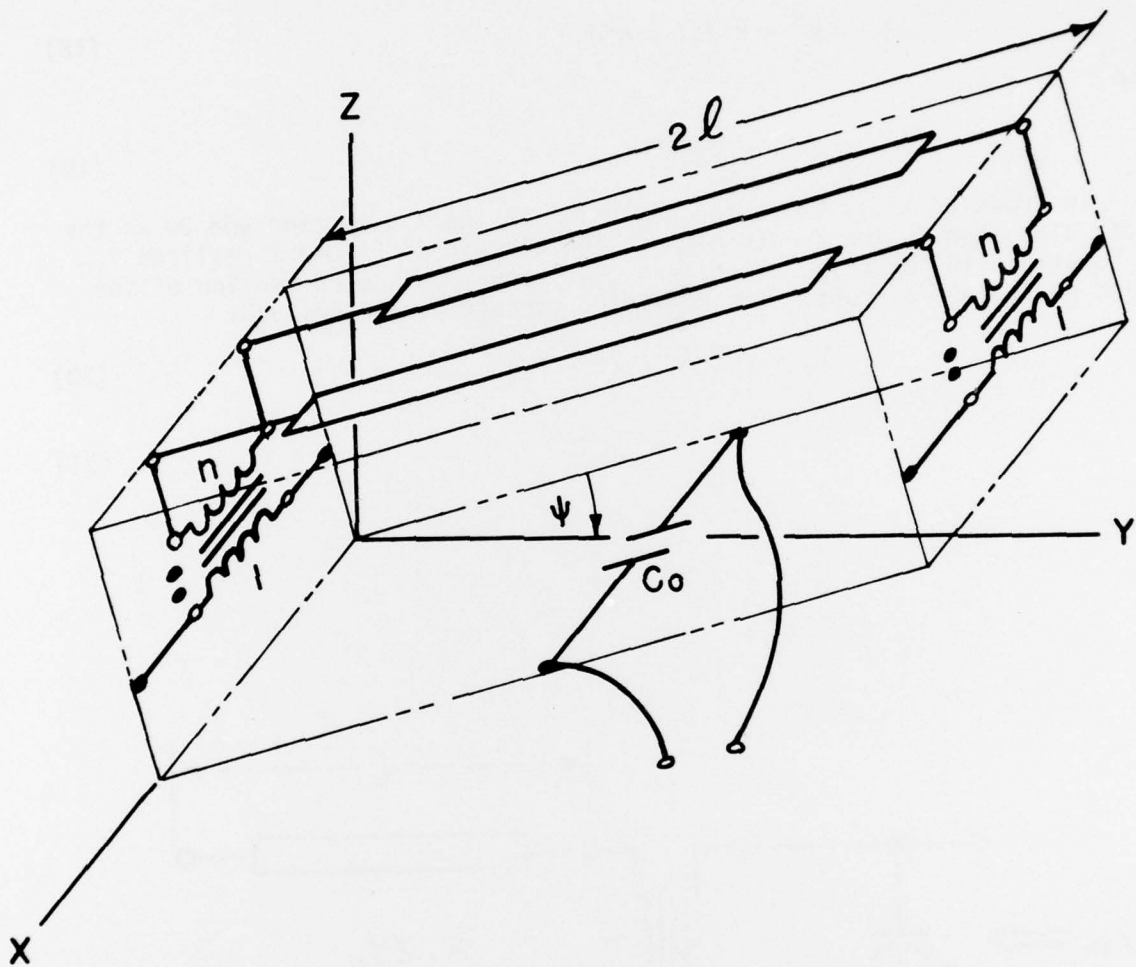


Figure 2. Analog Equivalent Circuit of Extensional Mode Bar.

this scheme, at the bar ends. The transformers are accordingly placed there. The input admittance is<sup>15</sup>

$$Y_{in} = Z_{in}^{-1} = j\omega C_0 \left\{ 1 + \frac{k^2}{\kappa l} \frac{\tan \kappa l}{\kappa l} \right\}, \quad (16)$$

15. D. A. Berlincourt, D. R. Curran, and H. Jaffe, "Piezoelectric and Piezomagnetic Materials and Their Function in Transducers," in Physical Acoustics: Principles and Methods (W. P. Mason, ed.) New York: Academic, 1964, Vol. 1A, pp. 169-270.

where

$$C_0 = \frac{2\ell w}{t} \epsilon'_{22} (1 - k^2), \quad (17)$$

$$\underline{k}^2 = k^2 / (1 - k^2), \quad (18)$$

and

$$\kappa = \omega \sqrt{\rho s'_{22}}. \quad (19)$$

In Equation (17),  $2t$  is the dimension in the X direction and  $2w$  is the width dimension in the Z direction. The circuit of Figure 2 realizes  $Y_{in}$  of Equation (16) exactly. In Figure 3 is shown a bisected version of the exact equivalent circuit. The remaining circuit parameters are:

$$Y_0 = \sqrt{s'_{22}/\rho} / (2t \cdot 2w) \quad (20)$$

$$n = 2w \cdot d'_{12} / s'_{22}. \quad (21)$$

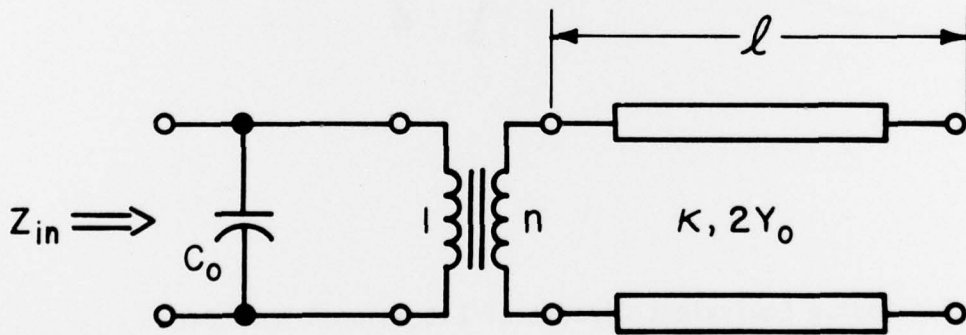


Figure 3. Bisected Exact Equivalent Circuit of Bar.

## CONCLUSIONS

We have shown how the acoustic loss at high frequency may be used to derive an associated set of constants for low frequency use. These have been denoted as the fluencies, have been calculated for quartz, and have been applied to the vibrations of bars. The low frequency time constant  $\tau_1$  has been introduced, as well as the analog equivalent circuit for low frequency resonators. A useful future area of work consists in measuring the temperature coefficients of the viscosities and/or fluencies.