SENSITIVITY APPROACH TO THE DUAL CONTROL PROBLEM. (U)

AUG 77 C S PADILLA, J B CRUZ

AFOSR-TR-77-1171
Sensitivity Approach to the Dual Control Problem

Abstract

A stochastic adaptive control problem that incorporates estimation cost considerations is formulated. A sensitivity index is introduced to represent the estimation error. The estimation error on estimation is distributed according to the accuracy required to achieve a given control objective when there is a bound on the estimation cost. The effect of the control on the sensitivity functions is used to influence the future uncertainties of the system. A dynamic feedback control algorithm is proposed that explicitly takes into account the accuracy of estimation and distributes the estimation effort in an optimal fashion.

1. Introduction

When a control law is to be designed for an uncertain system with unknown parameters, two roles of the control signal have to be considered: one is the effect of the control signal on the estimation of the unknown plant parameters and/or state variables, and another is the attainment of the control objective. In general, these two roles of the control are conflicting. The optimal solution to the dual control problem is achieved through the use of a closed-loop controller. The problem of getting the closed-loop control law leads in general to a nonlinear problem and only approximate solutions can be obtained. Much effort has been made to obtain tractable methods of solution [1,2,10,12]; some are approximations to the optimal solution [10] and others use the conflicting characteristic of the dual property to constrain the control so as to achieve a short-term control objective without exceeding a bound on the future covariances of the unknown quantities [1,13].

We propose a way of formulating a stochastic adaptive control problem that incorporates a cost assignment for the estimation effort and that automatically distributes the estimation budget in a rational way. In this approach a fixed budget for estimation is considered. The estimation effort is distributed according to the accuracy required to achieve a given control objective. This relationship between estimation and control objective is based on the assumption that greater accuracy in the estimation of an unknown quantity implies a greater cost. Parameters to which the state of the system is more sensitive require more accurate estimation than those whose effect on the state is less significant. We will represent the fixed budget for estimation by a sensitivity constraint which is related to the estimation cost in the following fashion. It has been shown [7] that for time-invariant systems the maximization of a sensitivity criterion [7,8] is equivalent to the maximization of the Fisher information matrix. We will use this fact to explicitly modify the control to affect the future uncertainties of the system. When we look at the estimation problem, the Cramer-Rao inequality [11] gives us a lower bound on the covariance of the estimate of the unknown parameters. We will use the influence of the control on the sensitivity of the system to decrease this lower bound.

The exact computation of the state sensitivity functions represents an infinite dimensional system when the feedback includes the sensitivity terms; due to this fact a dynamic system whose state closely approximates the sensitivity vector is used instead. This approximation to the sensitivity function has been used by Kreindler [6] and has been shown to give good performance for deterministic systems.

We consider the problem of designing feedback control laws for a class of multi-input-multi-output discrete time stochastic systems with unknown parameters. The performance index to be minimized is quadratic in the state and the control. The optimization is to be performed so that a measure of the energy the control spends in estimation does not exceed the budget available for estimation. We model cost of estimation as a quadratic function of the state sensitivity functions. We seek a feedback solution for the control that explicitly takes
into account the accuracy of the estimation and that distributes the estimation effort in an optimal fashion.

2. Problem Formulation

Consider the discrete linear system

\[ x_{k+1} = A x_k + B u_k + w_k \]
\[ y_k = s_k \]

where \( x_k \in \mathbb{R}^n \), \( u_k \in \mathbb{R}^m \), \( y_k \in \mathbb{R}^m \), and \( w_k \) is normal with zero mean and covariance \( V_w \). \( A \) and \( B \) are unknown random matrices of appropriate dimension and might be time varying. The state and the disturbance at the same instant are statistically independent. The entries of \( A \) and \( B \) are considered independent. The performance index to be minimized is:

\[ J_1 = E \left( \sum_{k=1}^{N_o} N_o x_k^T Q x_k + u_k^T R u_k \right) \]

subject to

\[ N_o + y \]

where

\[ c(k) = (c_1(k), \ldots, c_d(k))^T \]

and \( c_i \) which is designed to closely approximate the state sensitivity, satisfies the following equation:

\[ c_j(k+1) = (A_{N_o} - B_{N_o} E_j(k)) c_j(k) + (\tilde{A} - \tilde{B} E_j(k)) c_j(k) \]

\[ + \frac{d}{2} \sum_{i=1}^{d} B_{2i} E_{2i}(k) c_i(k) \]

where \( A_{N_o} \) and \( B_{N_o} \) are the derivatives of \( A \) and \( B \) with respect to \( E_j \), \( E_j(k) \) and \( E_{2i}(k) \) are matrices to be found in the algorithm and \( \tilde{A} \) and \( \tilde{B} \) are the latest estimates of \( A \) and \( B \) respectively. \( c_i \) is a component of the vector \( c \) which is formed by the unknown entries of the matrices \( A \) and \( B \) taken by rows. \( N_o \) is an unknown diagonal matrix to be chosen by the designer at time \( N_o \) and satisfies \( N_o \geq 0 \) for \( i = 1, \ldots, d \) and \( \text{tr} N_o = 1 \). The nonnegative numbers \( \varepsilon_i \) are the specified minimum relative weights where \( \sum_{i=1}^{d} \varepsilon_i = 1 \). \( Q \) is an \( m \times m \) positive definite matrix and \( R \) is an \( n \times n \) positive definite matrix.\( \text{Both} \ Q \text{and} \ R \text{may vary with} \ k \).

\( N_o \)

The choice of the matrix \( W \) allows us to distribute our estimation effort. According to the Cramer-Rao inequality for better estimation a maximization of the sensitivity with respect to the unknown parameters to be estimated \( N_o \) is required. Then we choose the matrix \( W \) such that the large state sensitivities are kept large in order to estimate those parameters more accurately. The design of \( W_o \) will be accomplished as follows: The sensitivity constraint (3) is appended through a Lagrange multiplier to the cost functional \( J_1 \) and the cost function is minimized with respect to the control law \( u_k \) and the weighting matrix \( W \). Through the bound \( r \) and the design of the weighting matrix \( W_o \) as indicated, the following goals can be achieved:

i) The larger sensitivity terms in (3) will receive less weight so that they will be less affected by the control than the smaller sensitivity terms which will receive more weight. Thus the control will increase the accuracy to which the parameters that affect the state were at the expense of a decrease in the accuracy of the parameters that affect the system state less.

ii) The estimation effort of the control will be rationally distributed to estimate better the parameters that have greater influence on the state.

We restate our original problem as follows: consider the augmented system

\[ x_{k+1} = \tilde{A} x_{k} + \tilde{B} u_{k} + \tilde{w}_{k} \]

with perfect state information. We want to minimize with respect to \( u_k \) and \( W_o \) the performance index:

\[ J = E \left( \sum_{k=1}^{N_o} N_o x_k^T Q x_k + u_k^T R u_k \right) \]

where

\[ \tilde{A} = \begin{bmatrix} A & 0 & \cdots & 0 \\ B_{11} & \tilde{A} - \tilde{B} E_{11} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ B_{d1} & \cdots & \cdots & \tilde{A} - \tilde{B} E_{d1} \end{bmatrix} \]

\[ \tilde{B} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I \end{bmatrix} \]

\[ \tilde{w}_{k} = \begin{bmatrix} w_k \\ \vdots \\ w_k \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} \]

\[ W_o = \begin{bmatrix} \tilde{P} & 0 & \cdots & 0 \\ 0 & \tilde{P} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{P} \end{bmatrix} \]

\[ A \text{ and } \tilde{Q} \text{ are } [(d+1)n]^{2} \text{ matrices, } \tilde{B} \text{ is a } (d+1)n \times m \text{ matrix, } \tilde{w} \text{ is a } (d+1)m \times 1 \text{ matrix and } w_k \text{ is of dimension } n. \]

The problem formulation in (3) and (6) was obtained by appending \( J_3 \) through a Lagrange multiplier \( \lambda \) to the cost function \( J_1 \). The Lagrange multiplier \( \lambda \) is chosen so that (3) is satisfied.
1. Solution

The optimal solution of problem (6) subject to (5) is extremely involved and complicated; to seek a closed loop control law would be impractical. A suboptimal solution which is of the feedback form is sought. First we will find a control law for a fixed $W_0$. Next we will optimize with respect to $W_0$.

3.1. Computation of $u_{w0}$

To find the control $u_{w0}$ given the information up to $N_0$ we proceed according to the following steps:

a) Compute the estimates $\hat{A}_{w0}$ and $\hat{B}_{w0}$ based on the information received up to $N_0$.

b) Apply the certainty equivalence principle to the augmented system. The plant matrices are assumed to be $A_{w0}$ and $B_{w0}$. Although this will lead to a certainty equivalent controller in the augmented state space, the error in the estimates is incorporated through the sensitivity constraint and the controller will be cautious. The application of the certainty equivalence principle to the augmented system will result in a better control performance than the certainty equivalent controller for the original problem because the uncertainty of the parameters can be influenced through the sensitivity constraint. The solution will differ from the original certainty equivalent controller because the system matrix $A$ contains unknown feedback gains.

The suboptimal feedback solution outlined in the above steps is given in Theorem 1.

Theorem 1: For a fixed $W_0$ and given $A_{w0}$ and $B_{w0}$, if equation (9) admits a solution, the control $u_{w0}$ that minimizes (6) is:

$$u_{w0} = - (R + B_{w0} A_{w0} B_{w0})^{-1} (A_{w0} B_{w0} + R)$$

where $F_{i}$ is the solution to

$$F_{i} = \hat{A}_{w0} F_{i} + B_{w0} u_{w0} - 1 = 0, \ldots, N_{0} \mu + 2$$

and

$$K_{i} = (R + B_{w0} A_{w0} B_{w0})^{-1} (A_{w0} B_{w0} + R)$$

where

$$K_{i} = [K_{1}(i) : K_{2}(i) : \ldots : K_{2d}(i)]$$

and

$$\hat{A}_{w0} = \hat{A}_{w0} - \hat{B}_{w0} K_{2d}(k)$$

$$\hat{B}_{w0} = \hat{B}_{w0} - \hat{B}_{w0} K_{2d}(k)$$

$\hat{A}_{w0}$ and $\hat{B}_{w0}$ are the estimates of $A$ and $B$. The application of the certainty equivalence principle to the augmented system is found.

Proof: To find $u_{w0}$, $F_{i}$, and $K_{i}$ given the information up to $N_0$. $F_{i}$ in equation (5) depends on the matrices $K_{i}(k)$ and $K_{2}(k)$ which are found through the design of the control $u_{w0}$.

The existence of the solution of (7) will depend on whether (9) which is linear in $F_{i}$ admits a solution. This question is addressed in Theorem 2.

Theorem 2: The rank condition:

$$\text{rank}(F_{i} F_{j}^{T}) = \text{rank}(F_{i} F_{j}^{T} F_{i} F_{j}^{T}) = d$$

$$\text{rank}(F_{i} F_{j}^{T}) = \text{rank}(F_{i} F_{j}^{T} F_{i} F_{j}^{T}) = d$$

where $f(F_{i} F_{j}^{T})$ are the columns of the matrix $F_{i} F_{j}^{T}$ written as a long vector, is necessary and sufficient for the existence of a solution to equation (9), where

$$F_{i} = (R + B_{w0} A_{w0} B_{w0})^{-1} (A_{w0} B_{w0} + R)$$

and

$$K_{i} = (R + B_{w0} A_{w0} B_{w0})^{-1} (A_{w0} B_{w0} + R)$$
\[
\begin{align*}
S_j &= \begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix} + j + 1, \\
T_j &= \begin{bmatrix}
\cdots \\
\cdots
\end{bmatrix} 0 \cdots 0,
\end{align*}
\]

where

\[
\begin{bmatrix}
\tilde{A}_{NO} & 0 \\
0 & \tilde{A}_{No} \\
& \ddots \end{bmatrix}
\]

\[
N_{\omega} = \begin{bmatrix}
\tilde{A}_{N0} & \tilde{A}_{N1} \\
\tilde{A}_{N0} & \ddots
\end{bmatrix}
\]

Proof:

1) Rewrite equation (9) as the equivalent equation:

\[
(1-F_j)K_1 \cdot \begin{bmatrix}
\tilde{A}_{NO} & 0 \\
0 & \tilde{A}_{No} \\
& \ddots
\end{bmatrix} F_j = \begin{bmatrix}
\tilde{A}_{NO} & 0 \\
0 & \tilde{A}_{No} \\
& \ddots
\end{bmatrix}
\]

Equation (11) is obtained after manipulating equation (9) and identifying terms.

2) Write equation (11) as a vector equation in the columns of \(K_1\). This results in the vector equation:

\[
(F-1-F_j)K_1 + \sum_{j=1}^{d} \begin{bmatrix}
\tilde{A}_{NO} & 0 \\
0 & \tilde{A}_{No} \\
& \ddots
\end{bmatrix} F_j = \begin{bmatrix}
\tilde{A}_{NO} & 0 \\
0 & \tilde{A}_{No} \\
& \ddots
\end{bmatrix}
\]

where \(k_i\) are the columns of \(K_1\) written as a long column vector.

From equation (12) the rank condition given in Theorem 2 follows.

3.2. Computation of \(\omega_{N0}\)

In the subsection above we found the feedback solution \(u_{\omega}\) for a given matrix \(\omega_{N0}\), for a given control sequence \(u_i\), \(i = N_0, \ldots, N_0 + v - 1\).

Given the information up to \(N_0\), compute the estimate \(\hat{A}_{N0}\) and \(\hat{B}_{N0}\). We choose a fixed sequence \(u_i\), \(i = N_0, \ldots, N_0 + v - 1\) by the method of Subsection 3.1,

\[
u_i = \begin{bmatrix}
\tilde{A}_{N0} & 0 & \tilde{A}_{N1} \\
0 & \tilde{A}_{N0} & \ddots
\end{bmatrix}^{-1} \begin{bmatrix}
\tilde{A}_{N0} & 0 \\
0 & \tilde{A}_{N0} \\
& \ddots
\end{bmatrix} \begin{bmatrix}
\hat{A}_{N0} \\
\hat{B}_{N0}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
\vdots
\end{bmatrix}
\]

where \(P_i\) is given by equations (8) and (9). For a given control sequence \(J\) in (6) is linear in \(\omega_{N0}\). We minimize this linear function subject to the linear constraints

\[
\begin{align*}
\omega_{N0} & \geq \epsilon_i, i = 1, \ldots, nd \\
0 & \leq \omega_{N0} \\
0 & \leq \omega_{N0}
\end{align*}
\]

(14)

where \(\epsilon_i \geq 0\) and these numbers are given.

This simple linear programming problem is easily solved by examining \(J\) at the \(nd\) vertices of the feasible space

\[
\omega_{N0} = \epsilon_i, i = 1, \ldots, nd
\]

(15)

and we choose a vertex at which \(J\) is a minimum. If this vertex is the same as that assumed to obtain the control sequence \(u_i\), we have the optimum \(\omega_{N0}\). If not, we try another vertex, choose another control sequence, and repeat the cycle. If a solution exists, we obtain it in at most \(nd\) steps.

After substituting the value of \(u_i\) from (13) into (6), the value of \(J\) can be written as

\[
J_{N0} = \epsilon_i \frac{N_{\omega}}{N_{\omega}} 0_{N_{\omega}} + \begin{bmatrix}
\hat{A}_{N1} & 0 \\
0 & \hat{A}_{N1} \\
& \ddots
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
\vdots
\end{bmatrix}
\]

(16)

where \(P_{N}\) is the covariance of \(\epsilon_i\) given the information up to time \(N_{\omega}\). It satisfies the propagation equation

\[
\begin{bmatrix}
\hat{A}_{N1} & 0 \\
0 & \hat{A}_{N1} \\
& \ddots
\end{bmatrix} P_{j+1} = \begin{bmatrix}
\hat{A}_{N1} & 0 \\
0 & \hat{A}_{N1} \\
& \ddots
\end{bmatrix} P_{j} + \begin{bmatrix}
\hat{A}_{N1} & 0 \\
0 & \hat{A}_{N1} \\
& \ddots
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
\vdots
\end{bmatrix}
\]

(17)

where \(\hat{A}_{N1} = 0\).

The matrix \(\hat{A}_{N1}\) is the same as \(\hat{A}_{N1}\) except that instead of the first row

\[
\begin{bmatrix}
\hat{A}_{N1} & 0 \\
0 & \hat{A}_{N1} \\
& \ddots
\end{bmatrix}
\]

we have

\[
\hat{A}_{N1} = \begin{bmatrix}
\hat{A}_{N1} & 0 \\
0 & \hat{A}_{N1} \\
& \ddots
\end{bmatrix}
\]

and \(A_{k1}\) and \(B_{k1}\) are evaluated as \(\hat{A}_{N1}\) and \(\hat{B}_{N1}\). For details see [9]. For a fixed control sequence, \(\omega_{N0}\) is fixed and in minimizing \(J_{N0}\) in (16) with respect to \(\omega_{N0}\) only \(\hat{A}_{N1}\) and \(\hat{B}_{N1}\) vary with \(\omega_{N0}\), and they are linear in \(\omega_{N0}\). The dependence of \(P_{j+1}\) on \(\omega_{N0}\) in \(J_{N0}\) is only through \(\hat{A}_{N1}\). That is, \(\hat{A}_{N1}\) and \(\hat{B}_{N1}\) are fixed when (8) is used to generate \(P_{j+1}\) and \(\omega_{N0}\).
The solution obtained above is open loop optimal for the augmented system for a given estimate $\hat{x}_N$ and $\hat{N}_N$. We use the feedback gains $K_{1N}$ and $K_{2N}$ for implementation in feedback form at time $N_0$. As new information is received one time unit later, the value of $N_0$ is increased by one and the process is repeated to obtain the next feedback gain values. This is essentially an open loop optimal feedback solution.

The algorithm presented above will automatically assign more weight to the small state sensitivity functions and less weight to the large state sensitivity functions; this is so because the estimation budget is fixed and the optimality condition for $W$ has to be satisfied. Since $W$ is the weighting matrix of $\sigma$ which is designed to closely approximate the state sensitivity, and $W_N$ is found to minimize the performance index $J$ in equation (6), then the components of $W$ will be found such that less weight will be assigned to large state sensitivity functions and more weight will be assigned to small state sensitivity functions.

### 3.3. Sensitivity Approximations

In this section we will analyze the sensitivity approximation used in the development of the previous section. Consider the approximate feedback sensitivity [6] given by (4) and reproduced here:

$$\sigma_j(k+1) = (A_0 - B_0 K_1(k))\sigma_j(k) + \sum_{i=1}^{d} (B_0 K_{2i}(k))\sigma_i(k)$$

(18)

where $\sigma_j$ depends on $K_1$ and $K_{2i}$, which are the feedback gain values that define feedback control law obtained from the algorithm.

The exact sensitivity functions for the feedback system are given by

$$\sigma_j(k+1) = (\hat{\Lambda} - \hat{B}_0 K_1(k))\sigma_j(k) + \sum_{i=1}^{d} (B_0 K_{2i}(k))\sigma_i(k)$$

(19)

and

$$\frac{d}{dN} \sigma_j(k+1) = (\hat{\Lambda} - B_0 K_1(k))\frac{d}{dN} \sigma_j(k) + \sum_{i=1}^{d} (B_0 K_{2i}(k))\frac{d}{dN} \sigma_i(k)$$

(20)

The true feedback sensitivity $\sigma$ will be the solution of four sets of simultaneous equations, (1), (18), (19), and (20) using the mean values of $A$ and $B$ in equation (1).

If we compare (18) with (19) by writing an error equation, we see that $\sigma$ will approximate $\sigma$ closely if the first derivatives of $\sigma$ with respect to $\sigma_j$ are very small and if $\hat{\Lambda} - \hat{B}_0 K$ is stable. This can be seen through the following error equation:

$$e_j(k+1) = (\hat{\Lambda} - B_0 K_1(k))e_j(k) + \sum_{i=1}^{d} (B_0 K_{2i}(k))e_i(k)$$

(21)

### 4. Example

We consider the system

$$x_{k+1} = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_k$$

(22)

$$y_k = x_k$$

where $a = (a_1, a_2)$ are random variables with prior statistics, $E\{w_k w_k^T\} = 0.5$, $\Sigma_w = \text{Diag}(0.0001, 0.0001)$. The noise $w_k$ is a random sequence with zero mean and variance 1.0. The initial state $x_{N_0}$ is normal with mean $x_{N_0} = (5, 5)'$ and variance $\Sigma_{x_{N_0}} = \text{Diag}(0.0001, 0.0001)$.

We seek a control law $u_k$, and matrix $N_0$ such that the following performance index is minimized,

$$J_{N_0} = E\{\sum_{k=0}^{N_0} x_k^T Q x_k + u_k^T R u_k\}$$

(23)

subject to

$$N_0 \geq 1, \quad y_k = x_k$$

(24)

where $\Sigma_{w_k}$ are the components of the diagonal $\Sigma_w$.

In this example we take $\Sigma_w = \text{Diag}(1.0, 1.0)$, $R = 1.0$. The enlarged system $\Sigma_{w_k}$ is given by

$$\Sigma_{w_k} = \begin{bmatrix} \Sigma_{w_k} & 0 \\ 0 & \Sigma_{w_k} \end{bmatrix}$$

(25)

where $A$ with $a_1$ and $a_2$ replaced by their means at $N_0$, and $K$ is the feedback control gain which is designed to achieve a close approximation of the state sensitivity by $\sigma_j$.

Applying the certainty equivalence principle to system (22) we obtain $\Sigma_{w_k} = 0.809$. Using the sensitivity approach, $J = 52.71$, which is a good improvement in the cost. From Table 1 we see that the assignment of the weighting matrix $W$ is such that the smallest sensitivity functions receive weight 1.0 and the others weight zero. Thus the theoretical goal is achieved. This is so because we wanted to keep the large sensitivity functions as large as possible so that the parameters whose
effect on the state of the system is large are estimated as accurately as possible. Also the control $u$ is affected by the choice of $W$, and by the feedback of the sensitivity functions in such a way that the control performance is improved.

6. Conclusion

In this paper we described a feedback control law that explicitly takes into account the accuracy of estimation and which distributes the estimation effort in an optimal fashion through the design of the weighting matrix $W$. This is achieved by incorporating the estimation cost $J_e$ and designing the weighting matrix $W$ for the sensitivity terms in $J_e$ in such a way that all the energy available for estimation is used. Moreover through the constrained minimization with respect to $W$ the available estimation energy is distributed so that the larger sensitivity terms receive less weight and the smaller sensitivity terms more weight, thereby forcing the control to affect the accuracy of estimation in such a way that the more crucial parameters are estimated more accurately.

7. References


Table 1. Average Trajectory of the feedback gains $K$, control law, state, approximate sensitivity and weighting matrix $W$.

<table>
<thead>
<tr>
<th>$N=1$</th>
<th>$N=2$</th>
<th>$N=3$</th>
<th>$N=4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{11}$</td>
<td>0.38170</td>
<td>0.3416</td>
<td>0.38567</td>
</tr>
<tr>
<td>$K_{12}$</td>
<td>-0.22596</td>
<td>-0.23683</td>
<td>-0.26391</td>
</tr>
<tr>
<td>$K_{21}$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$K_{22}$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$K_{31}$</td>
<td>-0.00818</td>
<td>-0.01534</td>
<td>-0.01540</td>
</tr>
<tr>
<td>$K_{32}$</td>
<td>0.0218</td>
<td>0.01760</td>
<td>0.01767</td>
</tr>
<tr>
<td>$u$</td>
<td>-0.80140</td>
<td>-1.30834</td>
<td>-0.3591</td>
</tr>
<tr>
<td>$w_{0}$</td>
<td>$\text{diag}(0.0,0,1.0,0)$</td>
<td>$\text{diag}(0.0,0,0,0)$</td>
<td>$\text{diag}(0.0,0,0,0)$</td>
</tr>
<tr>
<td>$x_{1}$</td>
<td>5.0016435</td>
<td>0.60718</td>
<td>0.6284</td>
</tr>
<tr>
<td>$x_{2}$</td>
<td>0.60718</td>
<td>0.6284</td>
<td>-0.06291</td>
</tr>
<tr>
<td>$x_{3}$</td>
<td>0.0</td>
<td>4.99672</td>
<td>5.32945</td>
</tr>
<tr>
<td>$x_{4}$</td>
<td>4.99672</td>
<td>5.32945</td>
<td>1.53338</td>
</tr>
<tr>
<td>$x_{5}$</td>
<td>0.0</td>
<td>5.0016435</td>
<td>1.47897</td>
</tr>
<tr>
<td>$x_{6}$</td>
<td>5.0016435</td>
<td>0.36938</td>
<td>1.98969</td>
</tr>
</tbody>
</table>

Average cost $J = 52.7053$.
REPORT DOCUMENTATION PAGE

REPORT NUMBER
AFOSR-TR-77-1171

TITLE (and Subtitle)
SENSITIVITY APPROACH TO THE DUAL CONTROL PROBLEM

AUTHORS
Consuelo S./Padilla and J. B./Cruz, Jr.

PERFORMING ORGANIZATION NAME AND ADDRESS
University of Illinois at Urbana-Champaign
Coordinated Science Laboratory
Urbana, Illinois 61801

CONTRACT OR GRANT NUMBER(S)
AFOSR-73-2570

REPORT DATE
August, 1977

MONITORING AGENCY NAME AND ADDRESS (if different from Controlling Office)
Air Force Office of Scientific Research/MD
Boiling Air Force Base, D. C.

DISTRIBUTION STATEMENT (of this Report)
Approved for public release, distribution unlimited.

KEY WORDS (Continue on reverse side if necessary and identify by block number)
Stochastic Adaptive Control
Dynamic Feedback Control Algorithm

ABSTRACT (Continue on reverse side if necessary and identify by block number)
A stochastic adaptive control problem that incorporates estimation cost considerations is formulated. A sensitivity index is introduced to represent the estimation cost. The control effort on estimation is distributed according to the accuracy required to achieve a given control objective when there is a bound on the estimation cost. The effect of the control on the sensitivity functions is used to influence the future uncertainties of the system. A dynamic feedback control algorithm is proposed that explicitly takes into account the accuracy of estimation and distributes the estimation effort in an optimal fashion.