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5 October 1971 - 5 January 1972

NAVAL SHIP SYSTEMS COMMAND
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ABSTRACT

The analysis presented here is a continuation of the research discussed in the preceding progress report.* Thus, for purposes of continuity, this report begins with Chapter V, which is a short introduction that provides the necessary link to the previous report.

The covariance functions and associated intensity spectra of the fluctuating envelope functions of reverberation processes generated by cw and FM transmissions are derived here. Using these second-order statistics, the corresponding statistics are obtained for the time-averaged ("smoothed") envelope. A discussion of the relative merits of transmitting cw or FM signals is included. Concluding remarks about the goals of the next research effort are given.

*Quarterly Progress Report No. 7 under Contract N00024-70-C-1184

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THE COVARIANCE FUNCTION AND INTENSITY SPECTRUM
OF THE ENVELOPE OF A NARROWBAND REVERBERATION PROCESS

by

T. D. Plemons

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V. INTRODUCTION

In the last progress report the derivations of the covariance function and intensity spectrum of the envelope function of a narrowband reverberation process were presented. Thus, we obtained the following expression for the covariance function of the fluctuating component, $y(t)$, of the envelope,

$$K_y(\tau) = \frac{\pi\psi}{8} k_o^2(\tau) \quad , \quad \tau = t_2 - t_1 \quad , \qquad (3.9)^*$$

where ψ is the variance of $X(t)$, the narrowband reverberation process, and $k_o(\tau)$ is the normalized envelope of the covariance of $X(t)$. The intensity spectrum of $y(t)$ is the cosine transform of $K_y(\tau)$ (W-K theorem),

$$W_y(f) = 4 \int_0^\infty K_y(\tau) \cos \omega\tau d\tau, \quad \omega = 2\pi f \quad , \qquad (4.1)$$

or, using Eq. (3.9),

$$W_y(f) = \frac{\psi\pi}{2} \int_0^\infty k_o^2(\tau) \cos \omega\tau d\tau \quad . \qquad (4.2)$$

*Equation numbers in Chapter V reference equations in QPR No. 7, under Contract N00024-70-C-1184.

If the transmitted signal is $S(t)$ with autocorrelation function

$$C(\tau) \equiv \int_{-\infty}^{\infty} S(t)S(t+\tau)dt \tag{4.3}$$

$$= C_o(\tau)\cos\left[2\pi f_o\tau + \xi_o(\tau)\right] \quad , \tag{4.5}$$

then under certain conditions

$$k_o(\tau) = C_o(\tau) \quad , \tag{4.6}$$

and the covariance $K_y(\tau)$ (Eq. (3.9)) and intensity spectrum (Eq. (4.2)) can be computed.

We now proceed in Chapter VI to compute $K_y(\tau)$ and $W_y(f)$ corresponding to cw and FM transmitted signals. In Chapter VII these results are used to compute some statistical properties of the time-averaged envelope of the reverberation.

VI. THE ENVELOPE FUNCTIONS OF REVERBERATION PROCESSES GENERATED BY FM AND cw TRANSMITTED PULSES

The types of reverberation processes that are of interest in this study are those generated by an active sonar transmitting either pulsed cw or frequency modulated (FM) signals. As mentioned in Chapter IV of the preceding progress report, the autocovariance of the reverberation is approximately equivalent, within a scale factor, to the (time-) autocorrelation function of the transmitted signal. This is true if certain critical factors, the motion of the reverberation producing scatterers (Doppler) and frequency selectivities of the transmitting and receiving apertures for example, are included in the expression for the autocorrelation function of the transmitted signal. For now, however, we shall assume that the covariance of the reverberation can be expressed as

$$K_X(\tau) = \psi C(\tau) \quad , \tag{6.1}$$

where $C(\tau)$ is the normalized autocorrelation function of the transmitted signal (Eq. (4.3)). Since we arbitrarily set $C(0) = 1$, ψ is the mean intensity of the reverberation process. In a practical situation ψ is a function of time (range) which means that the process is not stationary. For analytical conveniences, however, we assume ψ is relatively constant over the range interval of interest; i.e., we are assuming that the process is locally stationary.

The covariance of the fluctuating component of the envelope of the reverberation is given by Eq. (3.9),

$$K_y(t_1,t_2) = K_y(\tau) = \frac{\pi}{8}\,\psi\, k_o^2(\tau) \quad , \quad \tau = t_2 - t_1 \quad , \tag{6.2}$$

where $k_o(\tau)$ is now the envelope of the normalized autocorrelation of the transmitted signal. The functional form of $k_o(\tau)$ is determined by the nature of the transmitted signal waveform. Two cases, corresponding to cw and FM transmissions, will be considered in the following sections.

A. <u>The Transmission of cw Signals</u>

When the transmitted signal is a pulsed cw waveform defined by

$$\left.\begin{aligned} S_{cw}(t) &\equiv A\cos 2\pi f_o t \quad , & -\frac{T}{2} \leqq t \leqq \frac{T}{2} \\ &\equiv 0 \; , & \text{elsewhere} \end{aligned}\right\} \; , \tag{6.3}$$

where A is the constant amplitude (defined to give unit energy in S(t)), f_o is the center frequency, and T is the pulse duration, then the autocorrelation is, from Eq. (4.3),

$$\left.\begin{aligned} C_{cw}(\tau) &= A^2\left(1 - \frac{|\tau|}{T}\right)\cos 2\pi f_o\tau \quad , & |\tau| \leqq T \\ &= 0 \quad , & \text{elsewhere} \end{aligned}\right\} \; . \tag{6.4}$$

Comparing this with Eqs. (4.5) and (4.6), we see that

$$\left.\begin{aligned} k_o(\tau)_{cw} &= 1 - \frac{|\tau|}{T} \quad , & |\tau| \leqq T \\ &= 0 \quad , & \text{elsewhere} \end{aligned}\right\} \; . \tag{6.5}$$

Using this result in Eq. (6.2), we obtain the covariance of the fluctuating component, y(t), of the envelope,

$$\left.\begin{aligned} K_y(\tau) &= \frac{\pi}{8}\,\psi_{cw}\left(1 - \frac{|\tau|}{T}\right)^2 , & |\tau| \leqq T \\ &= 0 \quad , & \text{elsewhere} \end{aligned}\right\} , \qquad (6.6)$$

where ψ_{cw} is the mean intensity of the narrowband reverberation process generated by transmitting cw signals.

Using Eq. (6.6) in Eq. (4.1), or Eq. (6.5) in Eq. (4.2), we find that the intensity spectrum of y(t) is

$$W_y(f)_{cw} = \frac{\pi\psi_{cw}}{2}\int_0^T \left(1 - \frac{\tau}{T}\right)^2 \cos(2\pi f\tau)\, d\tau \quad . \qquad (6.7)$$

The integration is straightforward and we obtain

$$W_y(f)_{cw} = T\psi_{cw}\,\frac{\pi}{(\omega T)^2}\left[1 - \frac{\sin\omega T}{\omega T}\right] \quad , \quad \omega = 2\pi f \quad . \qquad (6.8)$$

It is interesting to compare $W_y(f)_{cw}$ with the intensity spectrum of X(t), the narrowband reverberation process. It is (cf. Eq. (4.1))

$$W_X(f)_{cw} = 4\int_0^\infty K_X(\tau)\cos(\omega\tau)\,d\tau \tag{6.9}$$

$$= 4\psi_{cw}\int_0^T \left(1 - \frac{\tau}{T}\right)\cos(\omega_o\tau)\cos(\omega\tau)\,d\tau$$

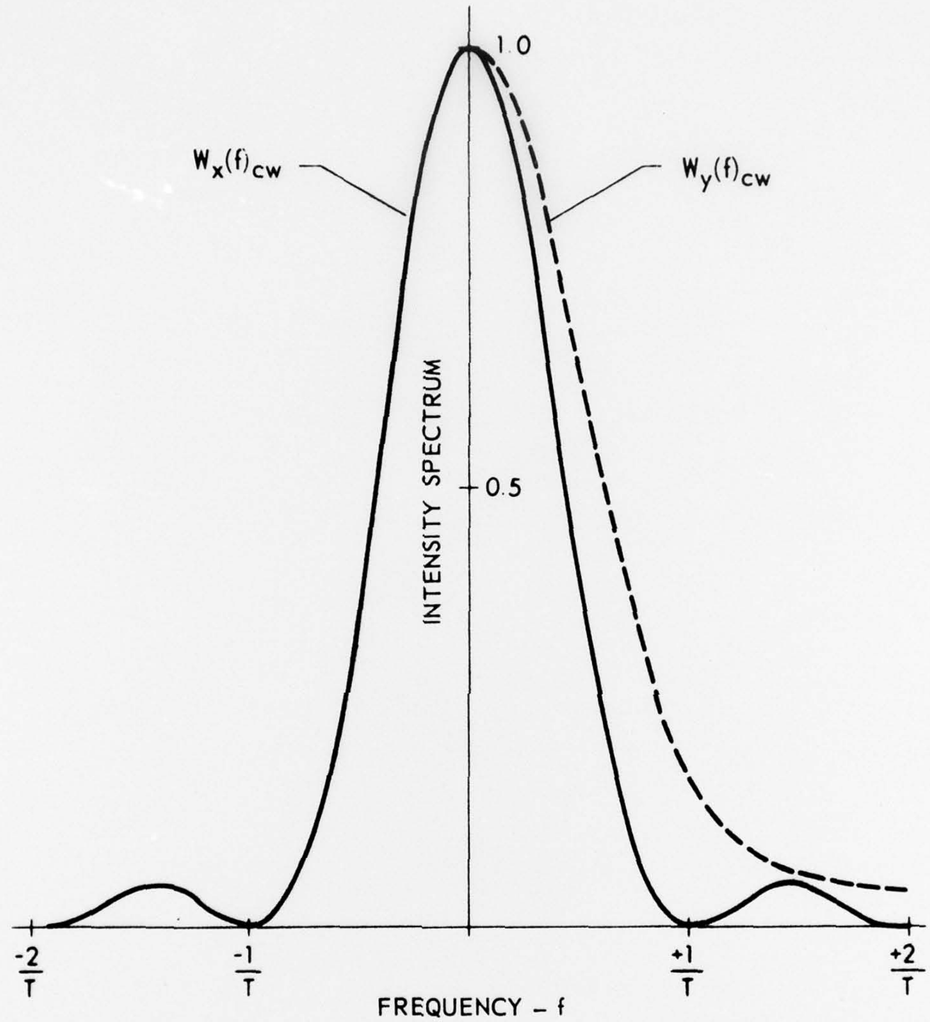
$$= \psi_{cw}T\,\frac{\sin^2\left[\left(\omega-\omega_o\right)T/2\right]}{\left[\left(\omega-\omega_o\right)T/2\right]^2} \quad , \tag{6-10}$$

where $\omega_o = 2\pi f_o$. Notice that the envelope $y(t)$ is a lowpass process, whereas $X(t)$ is narrowband centered about the frequency f_o. A comparison of the shapes of the two intensity spectra can be seen in Fig. 6.1, where the narrowband spectrum, $W_X(f)_{cw}$, has been translated to the origin. The maximum values of these two spectra have been set to unity in Fig. 6.1. The bandwidth of the narrowband reverberation is approximately T^{-1}, which is the length of the frequency interval between the half-power points of $W_X(f)_{cw}$. The corresponding bandwidth of the envelope process is seen to be about $0.6T^{-1}$; i.e., the bandwidth of the envelope is considerably less than that of the narrowband process.

B. The Transmission of FM Signals

The frequency modulated (FM) transmitted signal considered here is

$$\left.\begin{aligned} S_{FM}(t) &\equiv A\cos\left[2\pi\left(f_o t + \frac{W}{2T}t^2\right)\right] \quad , \quad && -\frac{T}{2} \leqq t \leqq \frac{T}{2} \\ &\equiv 0 \quad , && \text{elsewhere} \end{aligned}\right\} , \tag{6.11}$$



——— INTENSITY SPECTRUM (TRANSLATED TO ORIGIN OF FREQUENCY AXIS) OF NARROWBAND REVERBERATION

– – – – INTENSITY SPECTRUM OF THE FLUCTUATING COMPONENT OF THE REVERBERATION ENVELOPE

FIGURE 6.1
INTENSITY SPECTRA OF A NARROWBAND REVERBERATION PROCESS AND ITS ENVELOPE

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where W is the bandwidth induced into the signal by the phase modulation $Wt^2/2T$. When $S_{FM}(t)$ is narrowband (W, $1/T \ll f_o$), the autocorrelation of $S_{FM}(t)$ is, from Eq. (4.3), approximately (see Sec. V.E, Ref. 1)

$$\left.\begin{aligned} C_{FM}(\tau) &= \frac{TA^2}{\pi W|\tau|} \sin\left(\pi W|\tau| - \frac{\pi W\tau^2}{T}\right)\cos 2\pi f_o\tau \quad , \quad |\tau| \leqq T \\ &= 0 \quad , \qquad \text{elsewhere} \end{aligned}\right\} . \tag{6.12}$$

In practical situations TW is usually much larger than unity and the term $\pi W\tau^2/T$ can be neglected. In this case the normalized envelope of $C_{FM}(\tau)$ is

$$k_o(\tau)_{FM} = \frac{\sin\pi W\tau}{\pi W\tau} \quad , \quad |\tau| \leqq T \quad . \tag{6.13}$$

Combining this result with Eq. (3.9) yields the covariance of the fluctuating component of the reverberation envelope,

$$K_y(\tau)_{FM} = \frac{\pi\psi_{FM}}{8} \frac{\sin^2\pi W\tau}{(\pi W\tau)^2} \quad , \quad |\tau| \leqq T \quad , \tag{6.14}$$

where ψ_{FM} is the mean intensity of the narrowband reverberation process generated by the transmitted FM signals (Eq. (6.11)).

The corresponding intensity spectrum is obtained by combining Eqs. (6.13) and (4.2);

$$W_y(f)_{FM} = \frac{\pi\psi}{2}\int_0^T \frac{\sin^2\pi W\tau}{(\pi W\tau)^2} \cos 2\pi f\tau d\tau \quad . \tag{6.15}$$

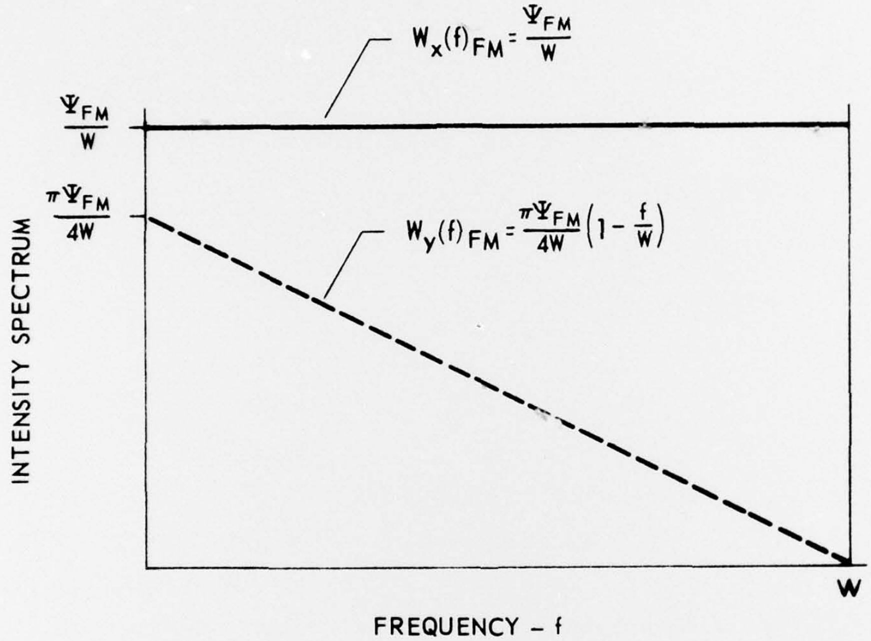
Since the significant values of $\sin^2 \pi W\tau/(\pi W\tau)^2$ correspond to the interval $(-W^{-1} \leqq \tau \leqq W^{-1})$ and $TW >> 1$, Eq. (6.15) can be approximated as

$$W_y(f)_{FM} = \frac{\pi\psi}{2} \int_0^\infty \frac{\sin^2 \pi W\tau}{(\pi W\tau)^2} \cos 2\pi f\tau d\tau \quad . \tag{6.16}$$

Using Eq. (8), Sec. 1.6 of Ref. (2) to evaluate this integral, we obtain the intensity spectrum of the fluctuating component of the reverberation envelope,

$$\left.\begin{aligned} W_y(f)_{FM} &= \frac{\pi\psi_{FM}}{4W}\left(1 - \frac{f}{W}\right) \quad , \quad 0 \leqq f \leqq W \\ &= 0 \quad , \quad \text{elsewhere} \end{aligned}\right\} \quad . \tag{6.17}$$

In Fig. 6.2, a comparison is made between $W_y(f)_{FM}$ and $W_X(f)_{FM}$, the intensity spectrum of the narrowband reverberation generated by FM transmitted signals. As in Fig. 6.1, the narrowband spectrum, $W_X(f)_{FM}$, has been translated to the vicinity of the origin. Again, as in the cw case, the bandwidth of the envelope is approximately one-half the bandwidth of the narrowband process.



——— INTENSITY SPECTRUM (TRANSLATED TO ORIGIN OF FREQUENCY AXIS) OF NARROWBAND REVERBERATION

– – – – INTENSITY SPECTRUM OF THE FLUCTUATING COMPONENT OF THE REVERBERATION ENVELOPE

FIGURE 6.2
INTENSITY SPECTRA OF A NARROWBAND REVERBERATION PROCESS AND ITS ENVELOPE

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VII. STATISTICAL PROPERTIES OF THE TIME-AVERAGED ENVELOPE

In many sonar receivers the envelope of the received signal, which may consist of reverberation only, or reverberation plus the echo from a target, is detected and then time-averaged ("smoothed") before detection and/or classification of target echoes is attempted. The time-averaged output is defined as

$$Y(t) = \frac{1}{T_0} \int_{t-T_0/2}^{t+T_0/2} y(t)dt \quad , \tag{7.1}$$

where $y(t)$ is the fluctuating component of the detected envelope and T_0 is the averaging time. In this chapter we investigate covariance and intensity spectra of $Y(t)$ corresponding to cw and FM transmitted signals.

As in the case of the fluctuating component, $y(t)$, the covariance and intensity spectrum of $Y(t)$ are the second-order statistics of primary interest here. These two quantities, given in general forms by Papoulis (Ref. 3, p. 351), are, respectively,

$$K_Y(\tau) = \frac{1}{T_0} \int_{-T_0}^{T_0} \left(1 - \frac{|\alpha|}{T_0}\right) K_y(\tau-\alpha)\, d\alpha \quad , \tag{7.2}$$

and

$$W_Y(f) = W_y(f) \frac{\sin^2 \pi T_0 f}{(\pi T_0 f)^2} \quad . \tag{7.3}$$

The variance of the smoothed output is obtained from Eq. (7.2) by setting $\tau = 0$,

$$\sigma_Y = K_Y(o) = \frac{1}{T_0} \int_{-T_0}^{T_0} \left(1 - \frac{|\alpha|}{T_0}\right) K_y(-\alpha)\, d\alpha \quad . \tag{7.4}$$

Since $K_y(\alpha) = 0$ for $|\alpha| > T$, the result of increasing T_0 is that σ_Y decreases. Simultaneously the bandwidth of $Y(t)$ decreases with increasing T_0 since the width of $[\sin^2 \pi T_0 f/(\pi T_0 f)^2]$ decreases with increasing T_0. In the following two sections, the time-averaged envelope processes corresponding to cw and FM transmissions are considered.

A. The Time-Averaged Envelope of Reverberation Generated by cw Transmitted Signals

Whenever the reverberation is produced by cw transmitted signals, the covariance of $y(t)$, the fluctuating component of the envelope, is given by Eq. (6.6). With this result substituted for $K_y(\tau)$ in Eq. (7.2), we obtain the covariance of $Y(t)$, the time-averaged envelope,

$$K_Y(\tau)_{cw} = \frac{\pi \psi_{cw}}{8T_0} \int_{-T_0}^{T_0} \left(1 - \frac{|\alpha|}{T_0}\right)\left(1 - \frac{|\tau - \alpha|}{T}\right)^2 d\alpha,\ |\tau| \leqq T \quad . \tag{7.5}$$

At the time this report was written this integral had not been evaluated. Setting $\tau = 0$ we have the variance

$$\sigma_{Y,cw} = K_Y(0)_{cw} = \frac{\pi \psi_{cw}}{4T_0} \int_{0}^{T_0} \left(1 - \frac{\alpha}{T_0}\right)\left(1 - \frac{\alpha}{T}\right)^2 d\alpha,\ \alpha \leqq T \quad . \tag{7.6}$$

To determine $\sigma_{Y,cw}$ we must consider two cases corresponding to $T_0 \leqq T$ or $T_0 \geqq T$.

Case 1: $T_0 \leqq T$

The averaging time, T_0, is less than the length, T, of the transmitted signal and, using Eq. (7.6),

$$\sigma^2_{Y,cw} = \frac{\pi \psi_{cw}}{4T^2 T_0^{\ 2}} \int_0^{T_0} (T_0 - \alpha)(T-\alpha)^2 \, d\alpha \quad . \tag{7.7}$$

The integration is straightforward and

$$\sigma^2_{Y,cw} = \frac{\pi \psi_{cw}}{8} \left[1 - \frac{1}{3} \frac{T_0}{T} + \frac{1}{12} \left(\frac{T_0}{T} \right)^2 \right] \quad . \tag{7.8}$$

Case 2: $T_0 \geqq T$

Now the averaging time T_0 is greater than or equal to the transmitted signal length T and Eq. (7.6) must be written as

$$\sigma^2_{Y,cw} = \frac{\pi \psi_{cw}}{4T^2 T_0^{\ 2}} \int_0^{T} (T_0 - \alpha)(T-\alpha)^2 \, d\alpha \quad , \tag{7.9}$$

which gives

$$\sigma^2_{Y,cw} = \frac{\pi \psi_{cw}}{12} \frac{T}{T_0} \left[1 - \frac{1}{4} \frac{T}{T_0} \right] \quad . \tag{7.10}$$

Whenever $T = T_0$, then either Eq. (7.8) or (7.7) gives

$$\sigma^2_{Y,cw} = \frac{3\pi}{32}\,\psi_{cw} \quad . \tag{7.11}$$

B. <u>The Time-Averaged Envelope of Reverberation Generated by FM Transmitted Signals</u>

Whenever the reverberation is produced by FM signals, then the variance of the time-averaged envelope is computed by substituting the expression for $K_Y(\tau)_{FM}$, Eq. (6.14), in Eq. (7.2). This gives

$$\sigma^2_{Y,FM} = \frac{\pi\psi_{FM}}{4T_0}\int_0^{T_0}\left(1 - \frac{\tau}{T_0}\right)\frac{\sin^2\pi W\tau}{(\pi W\tau)^2}\,d\tau \quad . \tag{7.12}$$

In practical situations T, $T_0 >> W^{-1}$ and the term $(\sin\pi W\tau)^2/(\pi W\tau)^2$ decreases rapidly relative to $(1 - \tau/T_0)$. This allows us to write Eq. (7.12) approximately as

$$\sigma^2_{Y,FM} = \frac{\pi\psi_{FM}}{4T_0}\int_0^{\infty}\frac{\sin^2\pi W\tau}{(\pi W\tau)^2}\,d\tau \quad , \tag{7.13}$$

which readily gives

$$\sigma^2_{Y,FM} = \frac{\pi\psi_{FM}}{8T_0W} \tag{7.14}$$

$$= \frac{\sigma^2_{y,FM}}{T_0W} \quad .$$

C. A Comparison of the Time-Averaged Envelopes Corresponding to cw and FM Transmissions

As a special case let us now assume that the pulselengths and amplitudes of the cw and FM signals are identical; i.e., the corresponding transmitted energies are equal. When this condition is met, the intensities of the corresponding reverberation processes are equal and $\psi_{cw} = \psi_{FM}$. Let us also assume that the averaging time T_0 is the same for the cw and FM generated envelopes and that $T_0 \geqq T$. Forming the ratio of Eq. (7.10) and Eq. (7.14) we find, under these assumptions, that

$$\frac{\sigma^2_{Y,cw}}{\sigma^2_{Y,FM}} = \frac{2}{3}\left(1 - \frac{1}{4}\frac{T}{T_0}\right) TW \quad . \tag{7.15}$$

For a given T, T_0 this ratio is proportional to the time-bandwidth product, TW, of the FM transmitted signal. If, for example, $T = T_0$, then

$$\frac{\sigma^2_{Y,cw}}{\sigma^2_{Y,FM}} = \frac{1}{2} TW \quad . \tag{7.16}$$

Equation (7.16) illustrates the advantage of transmitting FM signals when the target echo is reverberation limited and the sonar receiver consists of an envelope detector followed by a time averager. This apparent advantage, however, is available only if $TW \gg 1$ and the time averaging does not degrade the signal-to-noise ratio of the acoustic target. Such a degradation will occur if the averaging time T_0 is made larger than the duration of the target echo.

VIII. CONCLUDING REMARKS

Until now the research effort in this project has been entirely theoretical. Our next task is to obtain a collection of experimental data that will provide a test of these analytical results. Therefore, the covariance functions and intensity spectra of cw and FM generated reverberation envelopes need to be estimated and compared to the corresponding theoretical quantities derived and presented in this progress report.

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