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A STUDY OF THE EFFECT OF ELEMENT SPACING ON PERFORMANCE OF LINE--ETC(U)
JAN 65 D E BENNETT, R P KEMPFF

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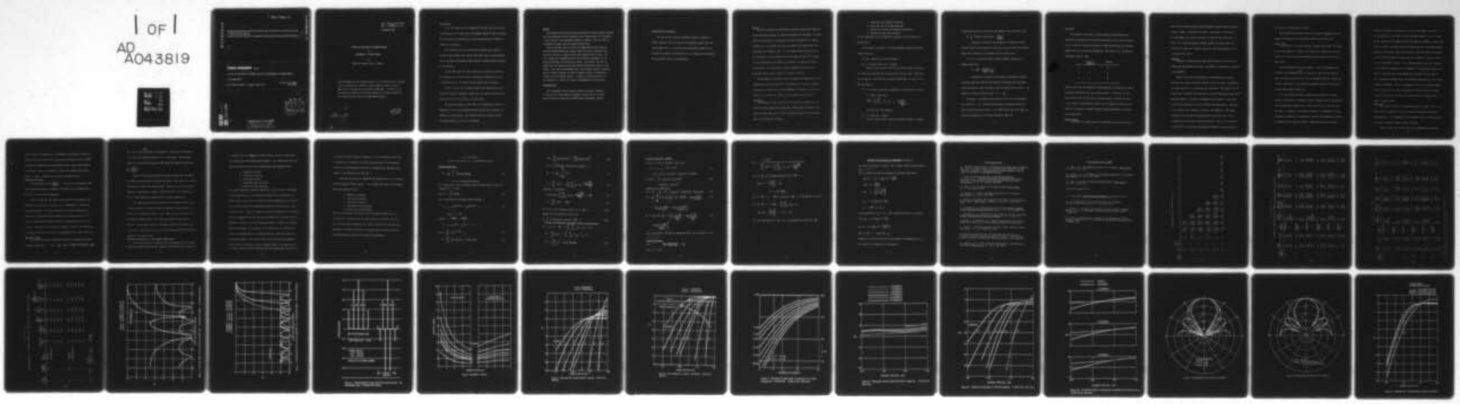

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TECHNICAL MEMORANDUM TM-759

NEL-TM-759

A STUDY OF THE EFFECT OF ELEMENT SPACING ON PERFORMANCE OF LINEAR ARRAYS

15 January 1965

D. E. Bennett and R. P. Kempff (Code 3130)

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A STUDY OF THE EFFECT OF ELEMENT SPACING

on

PERFORMANCE OF LINEAR ARRAYS.

by

Donald E. Bennett and R. P. Kempff

This Memorandum has been prepared because it is believed that the information it contains may be useful to others working in allied fields at NEL, and to a few persons or activities outside NEL. It should not be construed as a report since its only function is to present information on a small portion of the work on NEL Problem L30251.

Task 8051
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NEL L30251

THE PROBLEM

It is well known that the beamwidth of the main lobe of the directivity pattern of a linear array with elements spaced at half wavelengths may be narrowed by lengthening the array, and increasing the number of elements in proportion.

The beamwidth can also be decreased by placing more elements in the given array length, thus spacing them closer than a half-wavelength, and at the same time applying shading factors, including phase reversals, to the elements.

On the other hand, the array length can be shortened by spacing the elements more closely, without increasing the beamwidth more than a prescribed amount. Of course, shading factors must be applied.

Closer spacing of the elements, together with shading, affects the directivity pattern, beamwidth, signal gain, noise gain, signal-to-noise gain, and the directivity index, in various ways.

The principal purpose of this study is to determine the degree of degradation of the array performance which results from a reduction of beamwidth or array length. The unshaded array with elements at half-wavelength spacing is used as the reference.

RESULTS

The beamwidth can be reduced and the directivity index increased, compared with that obtainable with an unshaded array of equal length, if the spacing is less than $3/8\ \lambda$ and appropriate shading is applied. The price paid is a reduction in signal gain and signal-to-noise gain.

Tables and graphs show the effect of element spacing and shading on array performance parameters, namely, minor lobe level, beamwidth, signal gain, noise gain, signal-to-noise gain, and directivity index. The price payable for "superdirectivity" can be readily determined for the arrays considered, and estimated for others. For example, the loss of signal gain and signal-to-noise gain appears prohibitively high for long arrays. Also, Yaru has determined (Ref. 13) the large currents necessary, and the accuracy required, to achieve superdirectivity in transmitting arrays with close element spacing. It would be equally difficult, if not impossible, to design and operate corresponding receiving arrays.

RECOMMENDATIONS

This information should be made available to persons interested in arrays with a small number of elements, and who desire to achieve greater directivity than can be obtained with conventional designs.

ADMINISTRATIVE INFORMATION

This problem was initially assigned to Donald E. Bennett, a student employee. The work which he had completed between June 1961 and September 1962, on a part-time basis under NEL Problem L302 was extended and compiled for publication by R. P. Kempff between December 1963 and October 1964, as time permitted.

PREFACE

Articles concerning the possibility of greatly reducing the beamwidth of the directivity pattern of a linear array have been published. In places where the practicality of using such an array is considered, for example, reference 13, only extreme cases are used, and it then appears that such techniques are of limited value. In this report enough different cases are considered so that interpolations can be made, and the rate of change of array performance with respect to the number of elements and spacing can be noted. From these data, it may be determined whether or not a particular array would be useful in a specific situation.

The mathematical techniques used to determine the shading factors and beamwidth are those outlined by R. L. Pritchard in his article, "Optimum Directivity of Linear Arrays," which appeared in the Journal of the Acoustical Society of America, volume 25, No. 5, September 1953. (Ref. 6.)

DEFINITIONS

Some confusion seems to exist in the definitions of array gain, directivity index, signal gain, noise gain, coherent gain, incoherent gain, signal-to-noise ratio, etc., and the following terms are used interchangeably in the technical literature.

a. Signal gain and coherent array gain

b. Noise gain and incoherent array gain

c. Signal-to-noise gain and signal-to-noise ratio

d. Directivity index and array gain

In this memorandum, the first member of each pair will be used and is defined below:

(1) M+1=number of elements in a linear broadside array with uniform element spacing d.

(2) E_k = shading factor of kth element.

(3) S = maximum signal gain = $(\Sigma E_k)^2$.

Ratio of array output to that of the center element in response to a plane wave arriving from the direction of the main lobe. Same phase at all elements. This quantity is called "signal gain" for short in the text and figures.

(4) $\overline{S}$ = average signal gain, averaged over all directions in space.

$\overline{S} = \Sigma(E_k)^2 + f(\frac{d}{\lambda})$, where

$$f(\frac{d}{\lambda}) = 2 \sum_{i=1}^{M} \sum_{k=i+1}^{M+1} E_{k-i} E_k \cdot \frac{\sin(\frac{i\pi d}{\lambda})}{(\frac{i\pi d}{\lambda})}$$

For derivation, See Appendix.

(5) N = noise gain = $\Sigma(E_k)^2$.

Ratio of array output to that of the center element in response to randomly phased noise arriving at all elements, such as circuit noise.

(6) $\frac{S}{N}$ = signal to noise gain = $\frac{(\Sigma E_k)^2}{\Sigma(E_k)^2}$

A measure of the ability of the array to discriminate between a desired signal arriving from the direction of the main lobe, and randomly phased noise arriving at all elements, such as circuit noise.

(7) D.I. = directivity index = ratio of maximum signal gain to average signal gain.

$$D.I. = \frac{S}{\overline{S}} = \frac{(\Sigma E_k)^2}{\Sigma(E_k)^2 + f(\frac{d}{\lambda})}$$

A measure of the ability of the array to discriminate between a desired signal arriving from the direction of the main lobe, and plane waves arriving from other directions, such as isotropic ambient noise. (See Appendix for certain conditions when D. I. = $\frac{S}{N}$) .

Array gain is a measured quantity and is often used interchangeably with directivity. In a 100% efficient antenna, the measured array gain equals the calculated directivity. For actual arrays, the gain equals the directivity multiplied by the array efficiency. (Ref. 14)

DISCUSSION

Most emphasis in articles on linear arrays has been placed on the mathematical representation of directivity patterns such as shown in Figures 1 and 2, where the directivity patterns of three linear arrays are presented respectively non-normalized and normalized. Each array is 2λ long and has equal minor lobes of -25db:

	Number of Elements	Spacing
a.	5	$\frac{\lambda}{2}$
b.	9	$\frac{\lambda}{4}$
c.	17	$\frac{\lambda}{8}$

Figure 2 shows that the beamwidth is indeed reduced by increasing the number of elements and keeping the array length constant. Conversely, the beamwidth could be held constant while the array length is reduced, by spacing a given number of elements more closely. In the subsequent discussion the effect of a reduction of element spacing on array performance is investigated in some detail.

SHADING FACTORS

In general, the shading factors of the elements are less than unity, and for close element spacing, alternate elements require negative shading factors. Figure 3 illustrates this point. The length of both arrays is two wavelengths, and a minor lobe level of -25db has been prescribed. In Table I are listed Tschebyscheff shading factors for linear arrays with from five to twenty-five elements, spaced from $\frac{1}{8}$ to $\frac{1}{2}$ wavelength, with a -25db minor lobe level.

BEAMWIDTH

Beamwidth is defined as the total width in degrees of the main lobe between half-power (3db down) points. The method of calculation is outlined in the Appendix.

Table II shows that the beamwidth of an unshaded array increases rapidly as the element spacing is reduced, because the array is approaching the characteristics of an omnidirectional hydrophone. The beamwidth of the shaded array is somewhat wider than that of the corresponding unshaded array. The same material is presented in graphical form in Figure 4, where beamwidths can be predicted for various unshaded and shaded arrays. Returning to the example used in Figures 1 through 3, the beamwidth of the shaded 5-element array with half wavelength spacing is 25°, while that of the 9-element array with quarter wavelength spacing is $19^\circ.1$, i.e. a reduction of about 25%. But as we have already seen in Figure 3, one price paid is the reduction of signal gain by 32.4db.

What additional price must be paid in the form of degradation of the signal-to-noise gain and the directivity index will be shown later.

SIGNAL GAIN

"Signal gain," or strictly speaking, "maximum signal gain," is defined in this memorandum as $S = (\Sigma E_k)^2$. This must not be confused with the term "array gain" often seen in the literature which is denoted as "directivity index" here.

When some shading factors are negative as in the 9-element array in Figure 3, the algebraic sum ΣE_k is less than if they were all positive. Thus, the signal gains of the five and nine element arrays are +10.6db and -21.8db, respectively. The reduction of beamwidth achieved in this manner is about 25%, as shown in Figure 2.

In a more complete fashion, signal gain as a function of element spacing and the number of elements is shown in Figures 5 and 6 (not normalized and normalized, respectively). A minor lobe level of -25db has been prescribed. In Figure 7, the material of Figure 6 is rearranged by using the number of elements as an independent variable, and showing the decrease of signal gain as a family of curves. Observe that at half wavelength spacing, the decrease of signal gain is zero for any number of elements. It is important that a reduction from half wavelength spacing results in a far greater decrease of the signal gain when the number of elements is large, than when it is small. How much reduction in signal gain can be tolerated depends largely on the required signal-to-noise gain. (See below under that heading) In Table II the dependence of signal gain on the number of elements, element spacing and shading is shown. For the unshaded array, signal gain increases with the number of elements but is independent of the spacing, as might be expected. The signal gain of the shaded array is always less than that of the corresponding unshaded array, and decreases rather rapidly as the element spacing is decreased. This is a consequence of the use of negative shading factors (Ref. Table I). When the elements are closely spaced, the signal "gain" actually becomes a signal loss, compared with the signal output of a single unshaded element.

NOISE GAIN

Noise gain is defined in this memorandum as $N = \Sigma(E_k)^2$. This is the array output in response to noise arriving at all elements at completely random phases. N is independent of the algebraic signs of the shading factors, and is therefore always a positive quantity.

Table II shows that the noise gain of the unshaded array increases with the number of elements but is independent of the spacing as might be expected. The noise gain of the shaded array is always less than that of the corresponding unshaded array, and decreases slowly as the element spacing is decreased. This is a consequence of the use of shading factors (Ref. Table I). Figure 8 presents this material in graphical form.

SIGNAL-TO-NOISE GAIN.

S/N is defined here as $\frac{(\Sigma E_k)^2}{\Sigma(E_k)^2}$. This is not in general the same as the directivity index, but equals it only when $\frac{d}{\lambda}$ is an integral multiple of $\frac{1}{2}$, or very large (see Appendix).

Table II shows that the signal-to-noise gain of the unshaded array increases with the number of elements, but is independent of the spacing. In the shaded array it is always less than that of the corresponding unshaded array, and decreases rather rapidly as the element spacing is decreased. This results from the sharp drop in signal gain and the slow decrease in noise gain, as the spacing is reduced. Figure 9 shows these data in graphical form, and can be used for configurations not listed in Table II.

DIRECTIVITY INDEX

The directivity index is defined as the ratio of maximum signal gain to average signal gain. D.I. = $\frac{S}{\overline{S}}$, where $S=(\Sigma E_k)^2$ and $\overline{S}=\Sigma(E_k)^2 + f(\frac{d}{\lambda})$.

The last quantity $f(\frac{d}{\lambda})$ is defined in the appendix, equation (9), and becomes zero if $\frac{d}{\lambda}$ is an integral multiple of $\frac{1}{2}$, or very large. Only in those cases is $\overline{S} = N$, and the directivity index equals the signal-to-noise ratio

$$\frac{S}{N} = \frac{(\Sigma E_k)^2}{\Sigma (E_k)^2} \quad .$$

Table II shows that the directivity index increases with the number of elements, and decreases with decreasing spacing. The decrease is, however, less rapid for shaded than unshaded arrays. This is, of course, the result desired in "superdirective" arrays. Figure 10 presents the directivity indices of three shaded and unshaded arrays for direct comparisons.

The computation of directivity indices for the arrays with many closely spaced elements is critical because the numerator is small and the two terms in the denominator, namely N and $f(\frac{d}{\lambda})$, are very nearly equal and of opposite algebraic signs. Hence they were omitted in such cases in Table II. Yaru pointed this out in reference 13 where in an extreme case coefficient errors must be less than 10^{-9} to achieve current distributions accurate to one decimal place.

SPECIFIC APPLICATIONS TO A FIVE-ELEMENT LINEAR ARRAY.

Let us now restrict our discussion to a five-element linear broadside array with uniform element spacing. Figure 11 shows directivity patterns of a 5-element array with quarter wavelength spacing, and with various minor lobe suppressions using Tschebyscheff shading. It is readily seen from this figure and Table III that with increasing minor lobe suppression that:

a. Beamwidth increases,

b. Signal gain increases,

c. Noise gain decreases,

d. Signal/Noise gain increases,

e. Directivity index decreases.

If a narrow beamwidth is desired, higher minor lobe levels and a low signal-to-noise gain (actually a loss) must be accepted; on the other hand, if a higher signal-to-noise gain is desired, a greater beamwidth must be accepted. It is interesting that the accompanying variation of the directivity index is less that 1db. Figure 12 compares the directivity patterns of a 5-element array with quarter wavelength spacing, when unshaded and with 14db minor lobe suppression (similar to the 15db suppression pattern in Figure 11). Somewhat unexpectedly perhaps, the beamwidth of the shaded array is narrower than that of the unshaded one, but the shaded array has a much lower signal-to-noise gain (actually a loss of 8db versus a gain of 7db for the unshaded array), while its directivity index in somewhat higher. The signal gains of a 5-element array with three different minor lobe suppressions are shown as a function of element spacing in Figure 13. It is of interest to note that the signal gain is greatest with -15db minor lobe levels if the spacing is between $\frac{3}{8}$ and $\frac{1}{2}$ wavelength, whereas it is greatest with -35db minor lobe levels, if the spacing is less than $\frac{3}{8}\lambda$.

Table III also shows for comparison the characteristics of a 5-element array with half wavelength spacing. It is readily seen that with increasing minor lobe suppression that:

a. Beamwidth increases,

b. Signal gain decreases,

c. Noise gain decreases,

d. Signal/Noise gain decreases,

e. Directivity index decreases.

The last two quantities are equal at half wavelength spacing, and vary less than 1db. Therefore, here the primary choice lies between low minor lobe level and narrow beamwidth, which cannot be achieved at the same time. This cannot be achieved with quarter wavelength spacing either, but there the signal-to-noise gain may be the deciding consideration.

APPENDIX

DERIVATION OF EXPRESSIONS

<u>Average Signal Gain</u>

$$\bar{S} = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} P(\theta,\varphi)\sin\theta d\theta d\varphi \qquad (1)$$

$P = R^2$; R - directivity function.

In a linear array, there is symmetry about the array axis, so that P is a function of θ only,

and $$\bar{S} = \frac{1}{2} \int_0^{\pi} R^2 \sin\theta d\theta. \qquad (2)$$

For a linear array with uniform element spacing d,

$$R = E_1 + E_2 e^{j\beta d\cos\theta} + E_3 e^{j2\beta d\cos\theta} + - - - - - - \qquad (3)$$

where $\beta = \frac{2\pi}{\lambda}$.

Letting $\psi = \frac{\beta d\cos\theta}{\pi} = \frac{2d\cos\theta}{\lambda}$

$$R = E_1 + E_2 e^{j\pi\psi} + E_3 e^{j2\pi\psi} + - - - - -$$

and $$R = \sum_{k=1}^{M+1} E_k e^{j(k-1)\pi\psi} \quad \text{or} \qquad (4)$$

$$R = \sum_{k=1}^{M+1} E_k \left[\cos(k-1)\pi\psi + j\sin(k-1)\pi\psi\right] \qquad (5)$$

$$R^2 = \left[\sum_{k=1}^{M+1} E_k \cos(k-1)\pi\psi\right]^2 + \left[\sum_{k=1}^{M+1} E_k \sin(k-1)\pi\psi\right]^2 \tag{6}$$

$\bar{S} = \frac{1}{2}\int_0^{\pi} R^2 \sin\theta d\theta$, and with the variable ψ,

$$\bar{S} = \frac{1}{2} \cdot \frac{\lambda}{2d} \int_{\psi=-\frac{2d}{\lambda}}^{\frac{2d}{\lambda}} R^2 \, d\psi \tag{7}$$

$$\bar{S} = \sum_{k=1}^{M+1} (E_k^{\,2}) + 2\sum_{i=1}^{M} \sum_{k=i+1}^{M+1} E_{k-i} E_k \cdot \frac{\sin\left(\frac{i\pi 2d}{\lambda}\right)}{\left(\frac{i\pi 2d}{\lambda}\right)} \tag{8}$$

(Reference 8, Appendix)

By placing $2\sum_{i=1}^{M} \sum_{k=i+1}^{M+1} E_{k-i} E_k \cdot \frac{\sin\left(\frac{i\pi 2d}{\lambda}\right)}{\left(\frac{i\pi 2d}{\lambda}\right)} = f\left(\frac{d}{\lambda}\right)$ (9)

$$\bar{S} = \sum_{k=1}^{M+1} (E_k^{\,2}) + f\left(\frac{d}{\lambda}\right) \tag{10}$$

(a) If $\frac{d}{\lambda}$ is an integral multiple of $\frac{1}{2}$, $f\left(\frac{d}{\lambda}\right) = 0$ (10a)

because the sine functions become zero.

(b) If $\frac{d}{\lambda}$ approaches infinity, $f\left(\frac{d}{\lambda}\right) \to 0$ (10b)

because the denominator approaches infinity (wide spacing).

(c) If $\frac{d}{\lambda} \to 0$, $f\left(\frac{d}{\lambda}\right) \to 2\sum_{i=1}^{M} \sum_{k=i+1}^{M+1} E_{k-i} \cdot E_k$, and

$$\bar{S} = \sum_{k=1}^{M+1} (E_k^{\,2}) + 2\sum_{i=1}^{M} \sum_{k=i+1}^{M+1} E_{k-i} \cdot E_k \text{, or}$$

$$\bar{S} = \left(\sum_{k=1}^{M+1} E_k\right)^2 \text{ (close spacing)} \tag{10c}$$

Average Signal Gain. Example

Let M + 1 = 3 for a 3 element linear array.

$$R = E_1 + E_2 \cdot e^{j\pi\psi} + e^{j2\pi\psi} \tag{11}$$

$$= E_1 + E_2\cos\pi\psi + E_3\cos2\pi\psi + j(E_2\sin\pi\psi + E_3\sin2\pi\psi)$$

$$R^2 = R\cdot R^* = (E_1 + E_2\cos\pi\psi + E_3\cos2\pi\psi)^2 + (E_2\sin\pi\psi + E_3\sin2\pi\psi)^2 \tag{12}$$

Expanding and simplifying,

$$R^2 = E_1^2 + E_2^2 + E_3^2 + 2E_1E_2\cos\pi\psi + 2E_2E_3\cos\pi\psi + 2E_1E_3\cos2\pi\psi \tag{13}$$

$$\overline{S} = \frac{\lambda}{4d}\int_{\frac{-2d}{\lambda}}^{\frac{2d}{\lambda}} R^2 d\psi = \frac{\lambda}{4d}\left\{\left(E_1^2 + E_2^2 + E_3^2\right)\frac{4d}{\lambda} + 4E_1E_2\frac{\sin\left(\pi\frac{2d}{\lambda}\right)}{\pi} + 4E_2E_3\frac{\sin\left(\pi\frac{2d}{\lambda}\right)}{\pi} + 4E_1E_3\frac{\sin\left(2\pi\frac{2d}{\lambda}\right)}{2\pi}\right\} \tag{14}$$

$$\overline{S} = (E_1^2 + E_2^2 + E_3^2) + 2\left[E_1E_2\frac{\sin\left(\pi\frac{2d}{\lambda}\right)}{\pi\frac{2d}{\lambda}} + E_2E_3\frac{\sin\left(\pi\frac{2d}{\lambda}\right)}{\pi\frac{2d}{\lambda}} + E_1E_3\frac{\sin\left(2\pi\frac{2d}{\lambda}\right)}{\left(2\pi\frac{2d}{\lambda}\right)}\right] \tag{15}$$

This corresponds to the general expression for $\overline{S}$ (Eq. 8) with M + 1 = 3, i.e. M = 2.

Directivity Index

$$\text{D.I.} = \frac{\text{Max. signal gain}}{\text{Ave. signal gain}} = \frac{S}{\overline{S}}$$

(Ref. 9, P. 580)

$$\text{D.I.} = \frac{(\Sigma E_k)^2}{\sum_{k=1}^{M+1} (E_k^2) + 2 \sum_{i=1}^{M} \sum_{k=i+1}^{M+1} E_{k-i} E_k \cdot \frac{\sin(i\pi\frac{2d}{\lambda})}{i\pi\frac{2d}{\lambda}}}$$

(a) If $\frac{d}{\lambda}$ is an integral multiple of $\frac{1}{2}$, $f(\frac{d}{\lambda}) = 0$

and $\text{D.I.} = \frac{(\Sigma E_k)^2}{\Sigma (E_k^2)} = \frac{S}{N}$

or $10 \log (\frac{S}{N})$db.

(b) If $\frac{d}{\lambda} \to \infty$, $f(\frac{d}{\lambda}) \to 0$, and D.I. $\to \frac{S}{N}$ or $10 \log (\frac{S}{N})$db, as in (a).

(c) If $\frac{d}{\lambda} \to 0$, $f(\frac{d}{\lambda}) \to 2 \sum_{i=1}^{M} \sum_{k=i+1}^{M+1} E_{k-i} \cdot E_k$

and $\text{D.I.} \to \frac{\left(\Sigma E_k\right)^2}{\left(\Sigma E_k\right)^2} = 1$ or 0 db.

(d) For other values of $\frac{d}{\lambda}$, D.I. can be greater or less than $\frac{S}{N}$.

OUTLINE OF CALCULATIONS FOR BEAMWIDTH (See Ref. 6)

For a given odd number of elements $2m+1$ choose a minor lobe amplitude L

where $0 < L < 1$

Let θ_L designate one-half the beamwidth at the minor lobe level -

$$\text{Then } \theta_L = \sin^{-1}\left\{\frac{1}{kd}\cos^{-1}\left[\frac{1-B}{A}\right]\right\}$$

$$\text{where } A = \frac{1+Z_{om}}{1+X_e}$$

$$\text{and } B = -\left[\frac{1+X_e Z_{om}}{1-X_e}\right]$$

$$Z_{om} = \cosh\left[\frac{1}{m}\cosh^{-1}\left(\frac{1}{L}\right)\right].$$

$$X_e = \frac{2\pi d}{\lambda} \text{ for } \frac{d}{\lambda} \leq \frac{1}{2}$$

If the beamwidth at level (ϵ) other than minor lobe level is desired

$$\text{then } Z_{\epsilon m} = \cosh\left[\frac{1}{m}\cosh^{-1}\left(\frac{\epsilon}{L}\right)\right]$$

$$\text{and } \theta_\epsilon \ \sin^{-1}\left\{\frac{1}{kd}\cos^{-1}\left(\frac{Z_{\epsilon m}-B}{A}\right)\right\}$$

Same A and B computed with Z_{om}.

Formulae for currents for arrays of odd numbers of elements from 5 to 13 are listed in the Appendix A of reference 6.

R E F E R E N C E S

1. NEL Report 1108, entitled: "Simultaneous Multibeam Phase Compensation XII," by R. P. Kempff, 7 May 1962, contains a good introduction to the technique of shading to control the array response patterns. This inclues the application of Tschebyscheff shading.

On the theory and application of Tschebyscheff shading:

2. Dolph, C. L., "A Current Distribution for Broadside Arrays which Optimizes the Relationship Between Beamwidth and Side Lobe Level;" Proc. I.R.E., Vol. 34, pp. 335-348, June 1946.

3. Riblet, H. J., Discussion on the paper by Dolph; Proc. I.R.E., Vol. 35, pp. 489-492; 1947.

4. Riblet, H. J., "Note on the Maximum Directivity of an Antenna;" Proc. I.R.E., Vol. 36, pp. 620; 1948.

5. Bloch, A. and others, "New Approach to the Design of Super Directive Aerial Arrays;" Proc. I.E.E., (London) Vol. 100, pt. 3, pp. 303-314; 1953.

6. Pritchard, R. L., "Optimum Directivity of Linear Arrays;" Journal of the Acoustical Society of America, 25, No. 5, p. 879; September 1953.

7. Pritchard, R. L., "Maximum Directivity Index of a Linear Point Array;" Journal of the Acoustical Society of America; Vol. 26, p. 1034; 1954.

8. Brown, L. B. and Sharp, G. A., "Tschebyscheff Antenna Distribution, Beamwidth and Gain Tables;" Naval Ordnance Laboratory, Corona, Calif.

9. Silver, S., "Microwave Antenna Theory and Design," Radiation Laboratory Series. Vol. 12. McGraw Hill, 1949.

The preceding articles go into the theory very well with only one, A. Bloch, being much concerned with practical results. The following articles are mainly concerned with the extreme cases of the limitations.

10. Wilmotte, R. M., "Note on Practical Limitations in the Directivity of Antennas;" Proc. I.R.E., Vol. 36, p. 878, 1948.

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12. Taylor, T. T., "A Discussion of the Maximum Directivity of an Antenna," Proc. I.R.E., Vol. 36, p. 1135, 1948.

13. Yaru, N., "A Note on Super-Gain Antenna Arrays," Proc. I.R.E., Vol. 39, p. 1081, 1951.

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14. Jasek, Henry, Antenna Engineering Handbook, McGraw Hill, 1961.

15. G. Broussard et E. Spitz, "Superdirectivité. Supergain," Annales de Radio Electricité, Vol. 15, p. 289, 1960.

16. Spitz, E., "Supergain and Volumetric Antennas," France, Compagnie Générale de T. S. F., Contract AF 61(052)-102, Final Report, part 2, 15 June 1959.

17. Spitz, E., "Superdirective Antennas at Low Frequencies," France, Compagnie Générale de T. S. F., Contract AF 61(052)-387; Technical Report, 31 January 1961.

TABLE I

TSCHEBYSCHEFF SHADING FACTORS. MINOR LOBE LEVEL - 25 db

NUMBER ELEMENTS	SPACING d/λ	MINOR LOBE LEVEL db	E_o	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}
5	1/2	-25	1.0	.7975	.3925										
	3/8		1.0	.58381	.50238										
	1/4		1.0	- .46411	+ .47145										
	1/8		1.0	- .70602	+ .22832										
7	1/2	-25	1.0	.8939	.6264	.3667									
	3/8		.48681	1.0	.29619	.50446									
	1/4		1.0	- .94906	+ .48965	- .22917									
	1/8		1.0	- .77864	+ .34880	- .07165									
9	1/2	-25	1.0	.9364	.7639	.5310	.3783								
	3/8		1.0	.00814	.75946	.00402	.38891								
	1/4		1.0	- .85648	+ .57581	- .25660	+ .07804								
	1/8		1.0	- .82003	+ .44202	- .14360	+ .02169								
11	1/2	-25	1.0	.9580	.8400	.6683	.4737	.4035							
	3/8		+ .43597	-1.0	+ .34283	- .68528	+ .14868	- .30606							
	1/4		1.0	- .90520	+ .62533	- .33677	+ .12580	- .01428							
	1/8		1.0	- .84890	+ .51303	- .21138	+ .05375	- .00643							
13	1/2	-25	1.0	.9703	.8853	.7564	.6003	.4363	.4351						
	3/8		1.0	- .67306	+ .84538	- .44794	+ .49526	- .17090	+ .18571						
	1/4		1.0	- .90480	+ .66791	- .39340	+ .17830	- .05657	+ .01030						
	1/8		1.0	- .87001	+ .56913	- .27308	+ .09132	- .01913	+ .00190						
25	1/2	-25	1.0	.9923	.9697	.9327	.8829	.8218	.7516	.6746	.5932	.5100	.4275	.3478	.6644
	3/8		1.0	- .97129	+ .89768	- .78100	+ .64465	- .49717	+ .36131	- .24126	+ .14980	- .08215	+ .04074	- .01571	+ .00513

TABLE II

SUMMARY OF CHARACTERISTICS OF ARRAYS OF VARIOUS SIZES AND SPACINGS

NUMBER ELEMENTS	ARRAY LENGTH λ	SPACING d/λ	TYPE SHADING	MINOR LOBE LEVEL db	BEAMWIDTH AT -3db DEGREES	SIGNAL GAIN db	NOISE GAIN db	$\frac{\text{SIGNAL}}{\text{NOISE}}$ GAIN db	DIRECTIVITY INDEX db
5	2.0	1/2	UNSHADED	-12.0	21.0	+14.0	7.0	+ 7.0	7.0
	1.5	3/8	"	-12.0	28.0	+14.0	7.0	+ 7.0	5.8
	1.0	1/4	"	-14.0 *	42.0	+14.0	7.0	+ 7.0	4.3
	0.5	1/8	"	No Minor Lobe	92.0	+14.0	7.0	+ 7.0	1.6
5	2.0	1/2	Tscheby-scheff	-25	25.0	+10.6	4.1	+ 6.5	6.5
	1.5	3/8	"	-25	30.8	+10.0	3.4	+ 6.6	5.7
	1.0	1/4	"	-25	35.2	+ 0.05	2.7	- 2.65	5.0
	0.5	1/8	"	-25	39.2	-27.3	1.6	-28.9	4.7
7	3.0	1/2	UNSHADED	-12.5	14.6	+16.9	8.45	+ 8.45	8.45
	2.25	3/8	"	-12.6	19.5	+16.9	8.45	+ 8.45	7.25
	1.5	1/4	"	-12.6	29.5	+16.9	8.45	+ 8.45	5.6
	0.75	1/8	"	No Minor Lobe	61.2	+16.9	8.45	+ 8.45	2.85
7	3.0	1/2	Tscheby-scheff	-25	17.6	+13.6	5.6	+ 8.0	8.0
	2.25	3/8	"	-25	21.7	+12.2**	4.65**	+ 7.55	7.1
	1.5	1/4	"	-25	24.9	- 8.5	5.3	-13.8	6.6
	0.75	1/8	"	-25	26.9	-50.6	3.9	-54.5	6.2
9	4.0	1/2	UNSHADED	-12.9	11.6	+19.1	9.55	+ 9.55	9.55
	3.0	3/8	"	-12.9	15.2	+19.1	9.55	+ 9.55	8.4
	2.0	1/4	"	-12.9	22.8	+19.1	9.55	+ 9.55	6.5
	1.0	1/8	"	-19.1*	46.7	+19.1	9.55	+ 9.55	3.95
9	4.0	1/2	Tscheby-scheff	-25	13.6	+15.9	6.8	+ 9.3	9.3
	3.0	3/8	"	-25	16.7	+10.4	3.9	+ 6.5	8.15
	2.0	1/4	"	-25	19.1	-21.8	5.1	-26.9	7.45
	1.0	1/8	"	-25	20.6	-75.6	4.4	-80.0	6.75

TABLE II (CONT)

NUMBER ELEMENTS	ARRAY LENGTH λ	SPACING d/λ	TYPE SHADING	MINOR LOBE LEVEL db	BEAMWIDTH AT -3 db DEGREES	SIGNAL GAIN db	NOISE GAIN db	$\frac{\text{SIGNAL}}{\text{NOISE}}$ GAIN db	DIRECTIVITY INDEX db
11	5.0	1/2	UNSHADED	-13.0	9.3	+20.8	10.4	+10.4	10.4
	3.75	3/8	"	-13.0	12.4	+20.8	10.4	+10.4	9.25
	2.5	1/4	"	-13.0	18.7	+20.8	10.4	+10.4	7.55
	1.25	1/8	"	-13.2 *	37.8	+20.8	10.4	+10.4	4.73
11	5.0	1/2	Tscheby-scheff	-25	10.9	+17.75	7.75	+10.0	10.0
	3.75	3/8	"	-25	13.4	+ 8.15**	5.55**	+ 2.6	* * *
	2.5	1/4	"	-25	15.6	-39.8	5.65	-45.45	* * *
	1.25	1/8	"	-25	16.7	-77.4	4.85	-82.25	* * *
13	6.0	1/2	UNSHADED	-13.0	7.9	+22.3	11.15	+11.15	11.15
	4.5	3/8	"	-13.0	10.5	+22.3	11.15	+11.15	9.95
	3.0	1/4	"	-13.0	15.8	+22.3	11.15	+11.15	8.25
	1.5	1/8	"	-13.0	31.8	+22.3	11.15	+11.15	5.4
13	6.0	1/2	Tscheby-scheff	-25	9.15	+19.2	8.5	+10.7	10.7
	4.5	3/8	"	-25	11.25	+ 3.3	6.4	- 3.1	* * *
	3.0	1/4	"	-25	12.9	-49.2	5.9	-55.1	* * *
	1.5	1/8	"	-25	14.0	-71.7	5.2	-76.9	* * *
25	12.0	1/2	UNSHADED	-13.2	4.1	+28.0	14.0	+14.0	14.0
	9.0	3/8	"	-13.2	4.45	+28.0	14.0	+14.0	12.75
	6.0	1/4	"	-13.2	8.2	+28.0	14.0	+14.0	11.03
	3.0	1/8	"	-13.2	17.55	+28.0	14.0	+14.0	8.1
25	12.0	1/2	Tscheby-scheff	-25	4.6	+25.1	11.5	+13.6	13.6
	9.0	3/8	"	-25	5.7	-33.8	8.75	-42.55	* * *
	6.0	1/4	"	-25	6.6	* * *	* * *	* * *	* * *
	3.0	1/8	"	-25	7.1	* * *	* * *	* * *	* * *

* $\theta = 90^{\circ}$

** Normalized to element with maximum shading factor (See TABLE I)

*** Not computed because of extreme accuracy required.

TABLE III

CHARACTERISTICS OF 5-ELEMENT ARRAYS. $\frac{\lambda}{2}$ AND $\frac{\lambda}{4}$ SPACING. UNSHADED AND SHADED

ARRAY LENGTH	ELEMENT SPACING	TYPE SHADING	MINOR LOBE LEVEL db	BEAMWIDTH (-3db) degrees	S SIGNAL GAIN $(\Sigma E_k)^2$ db	N NOISE GAIN $\Sigma(E_k)^2$ db	S/N $\frac{\text{SIGNAL}}{\text{NOISE}}$ GAIN $\frac{(\Sigma E_k)^2}{\Sigma(E_k)^2}$ db	D.I. * DIRECTIVITY INDEX db
2λ	λ/2	UNSHADED	-13	21.0	+14.0	+7.0	+7.0	+7.0
"	"	TSCHEBYSCHEFF	-13	21.2	+13.2	+6.2	+7.0	+7.0
"	"	"	-14	21.4	+12.9	+5.9	+7.0	+7.0
"	"	"	-25	25.0	+10.6	+4.1	+6.5	+6.5
"	"	"	-35	29.0	+ 9.6	+3.5	+6.1	+6.1
1λ	λ/4	UNSHADED	-14	42.0	+14.0	+7.0	+7.0	+4.3
"	"	TSCHEBYSCHEFF	-14	30.2	- 4.55	+3.45	-8.0	+5.6
"	"	"	-15	30.8	- 4.05	+3.40	+7.45	+5.6
"	"	"	-25	35.2	+ 0.05	+2.70	-2.65	+5.0
"	"	"	-35	38.5	+ 2.65	+2.2	+0.45	+4.7

$$*\text{D.I.} = \frac{S}{\bar{S}} = \frac{(\Sigma E_k)^2}{\Sigma(E_k)^2 + f(\frac{d}{\lambda})} = \frac{S}{N + f(\frac{d}{\lambda})}$$

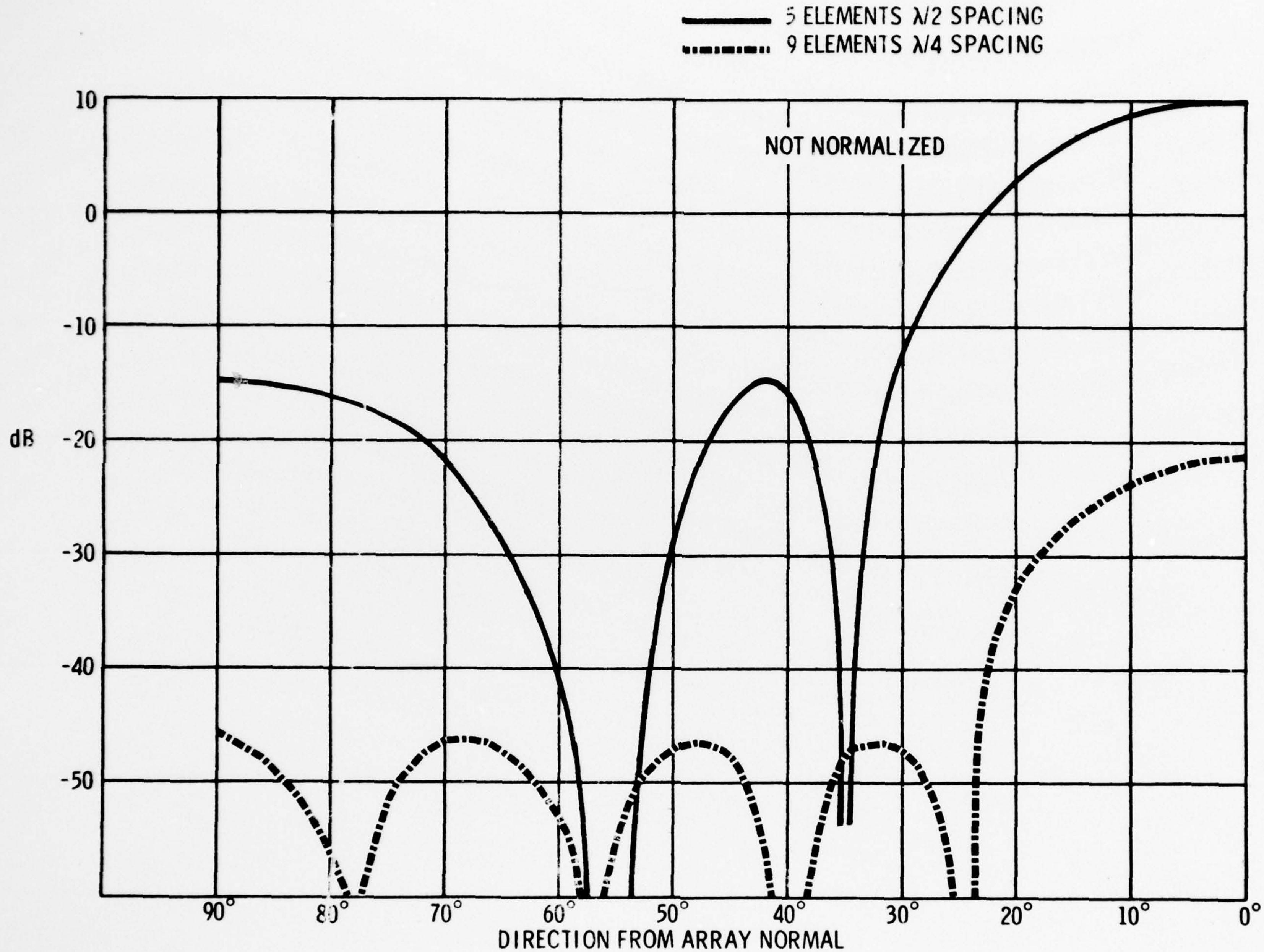

Figure 1. Directivity patterns of arrays two wavelengths long. Tschebyscheff shading. -25 dB minor lobe level.

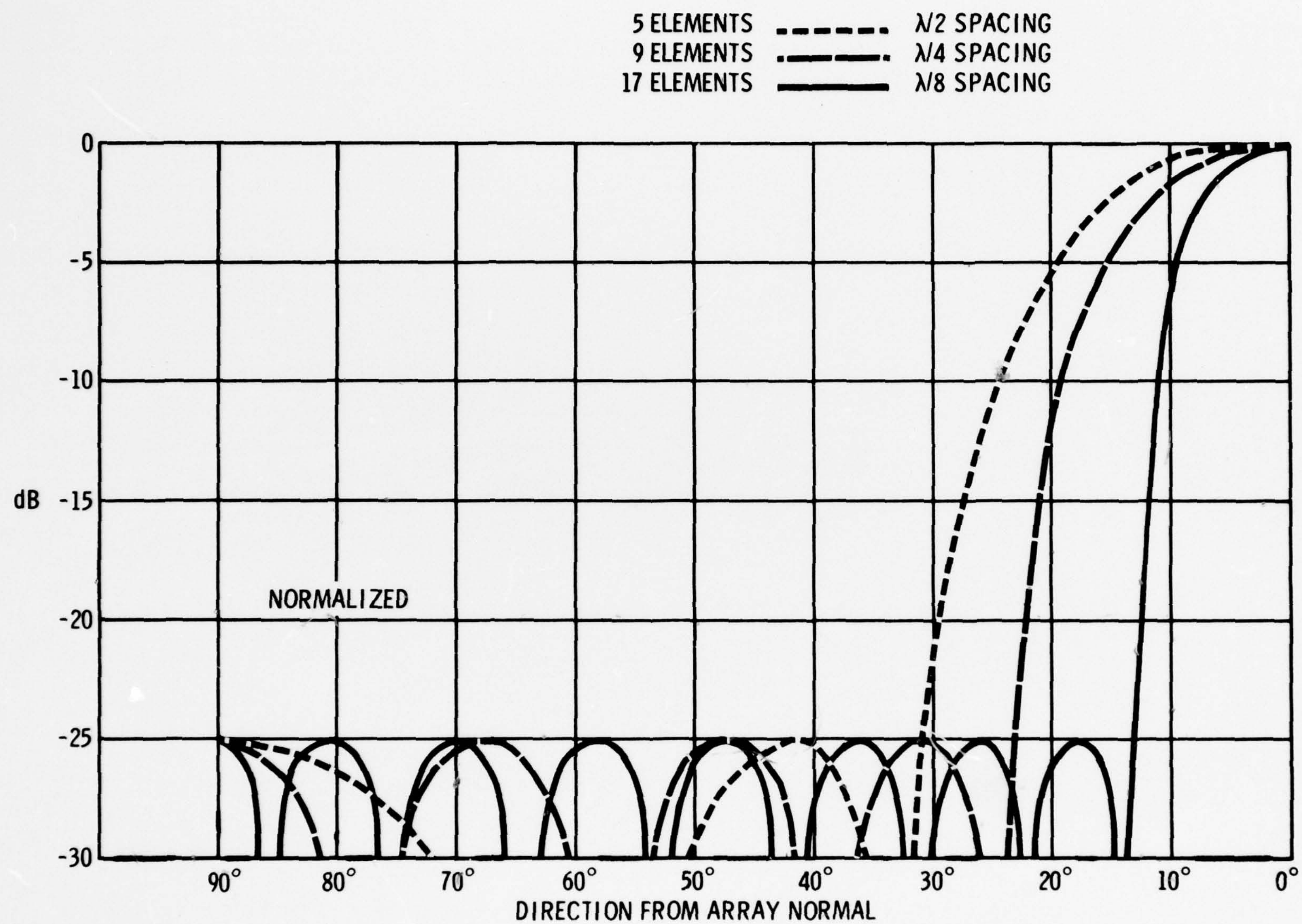

Figure 2. Directivity patterns of arrays two wavelengths long. Tschebyscheff shading. -25 dB minor lobe level.

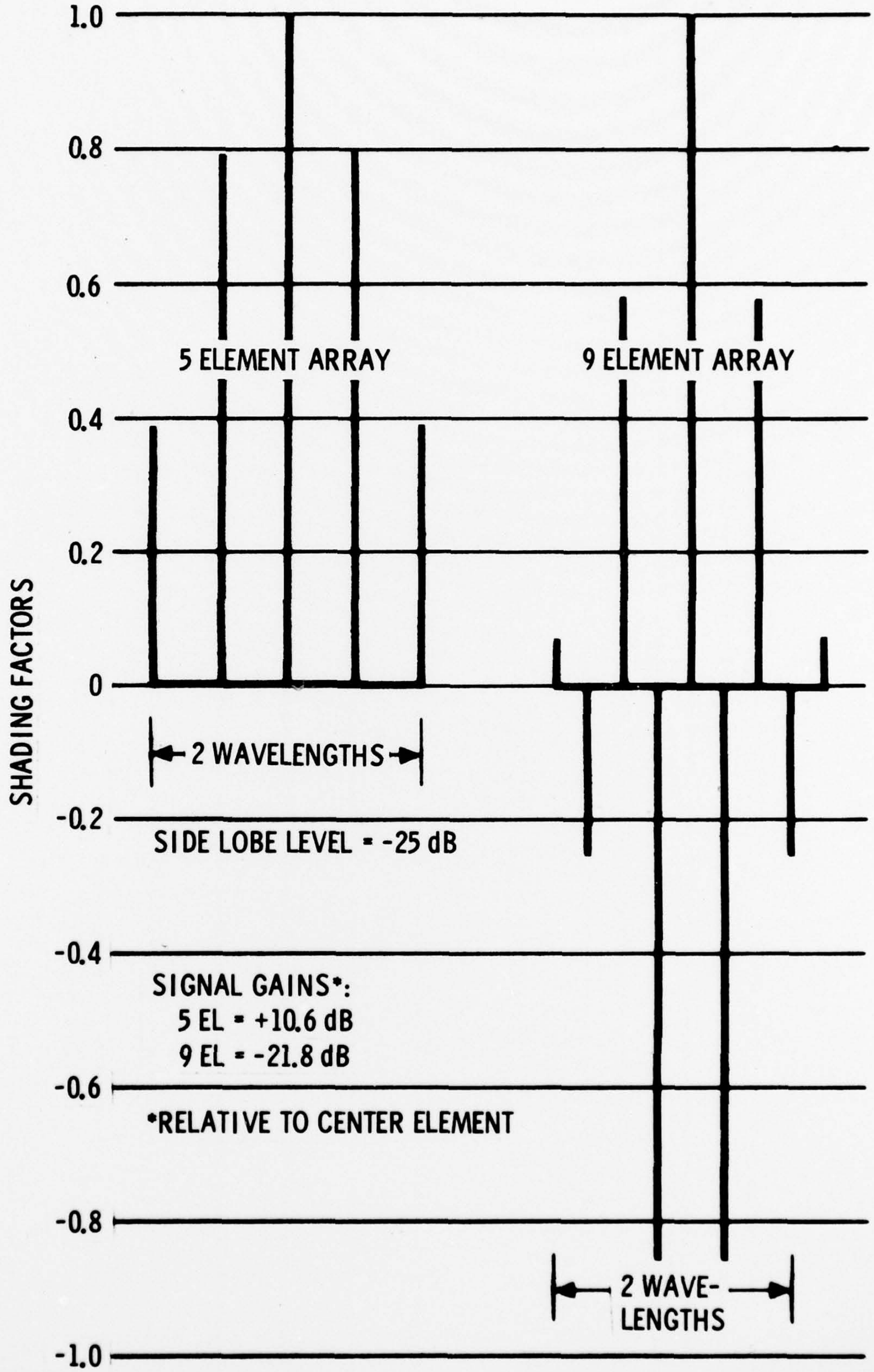

Figure 3. Shading factors for five- and nine-element arrays. Two wavelengths long. Tschebyscheff shading.

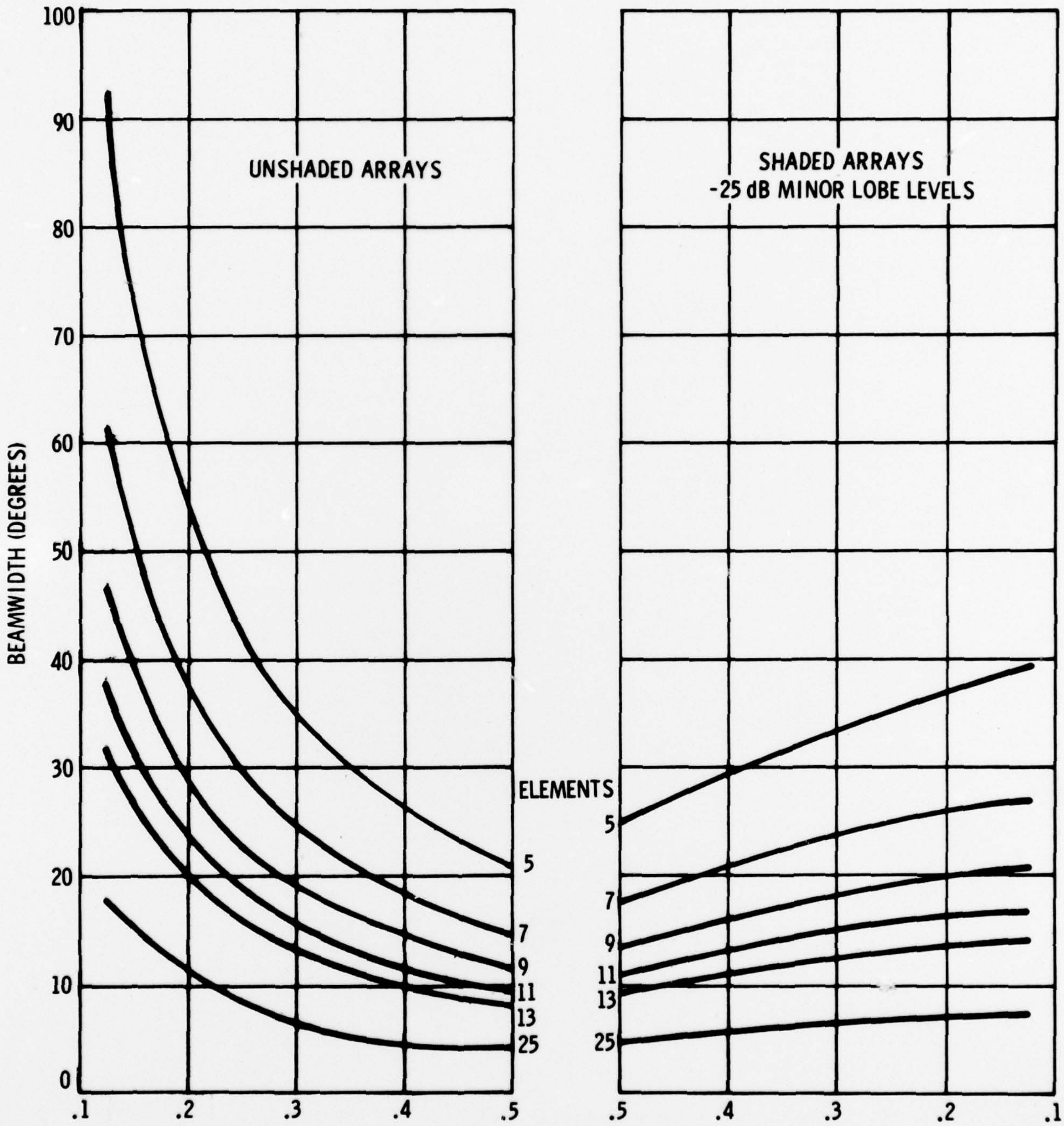

Figure 4. Beamwidth at -3 dB level.

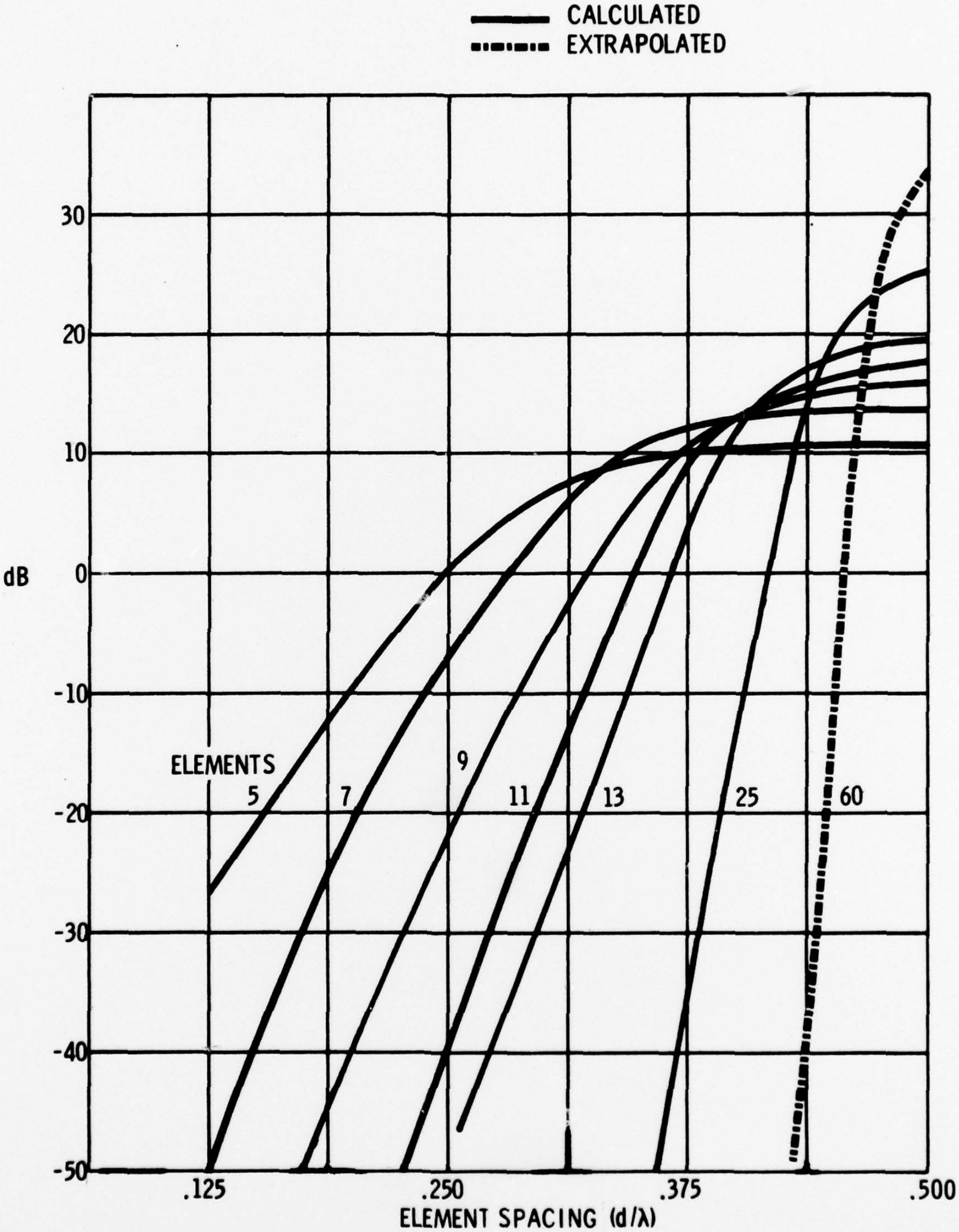

Figure 5. Signal gain (dB re single element) vs spacing. -25 dB minor lobe level.

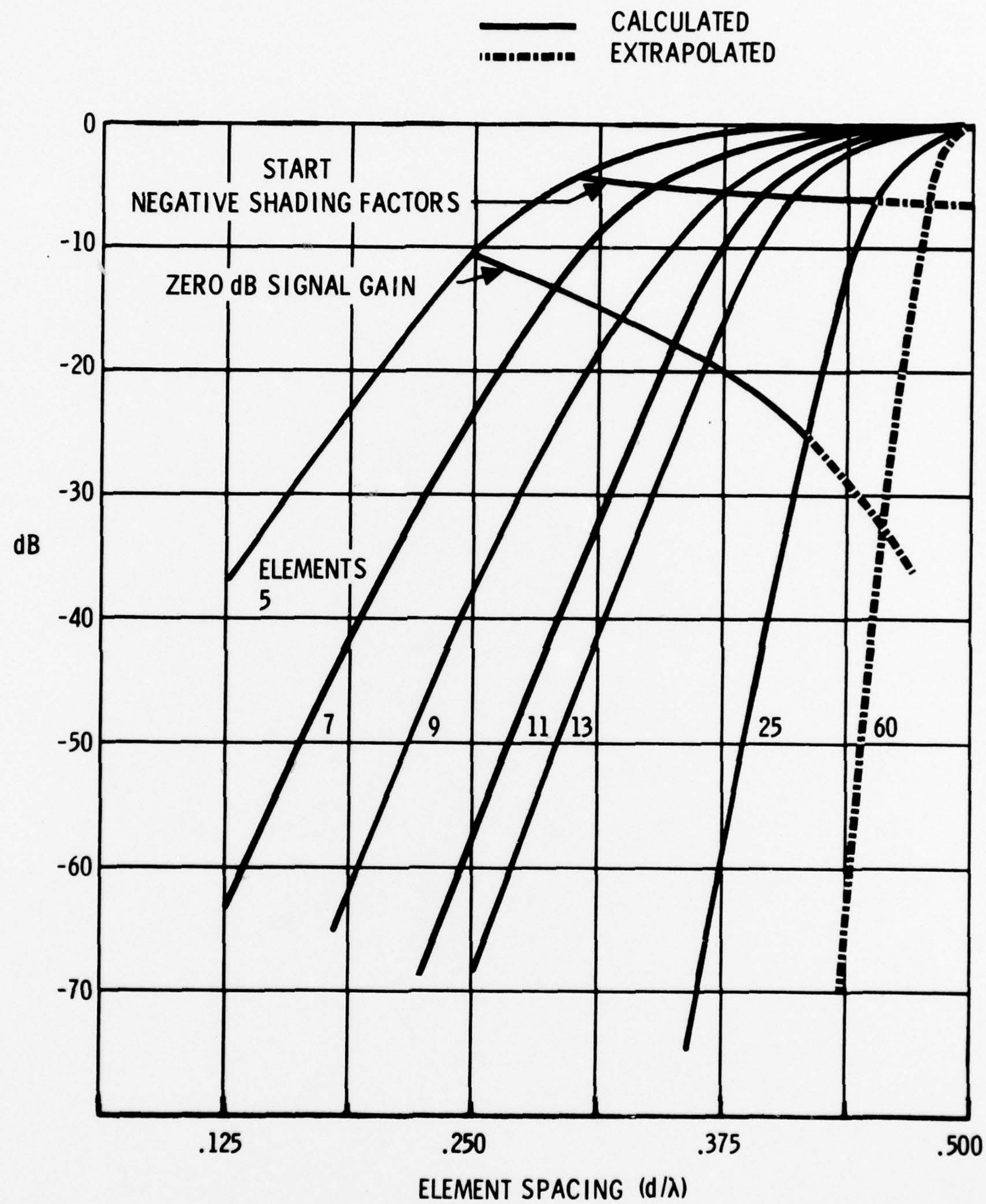

Figure 6. Loss of signal gain vs spacing. Normalized. -25 dB minor lobe level.

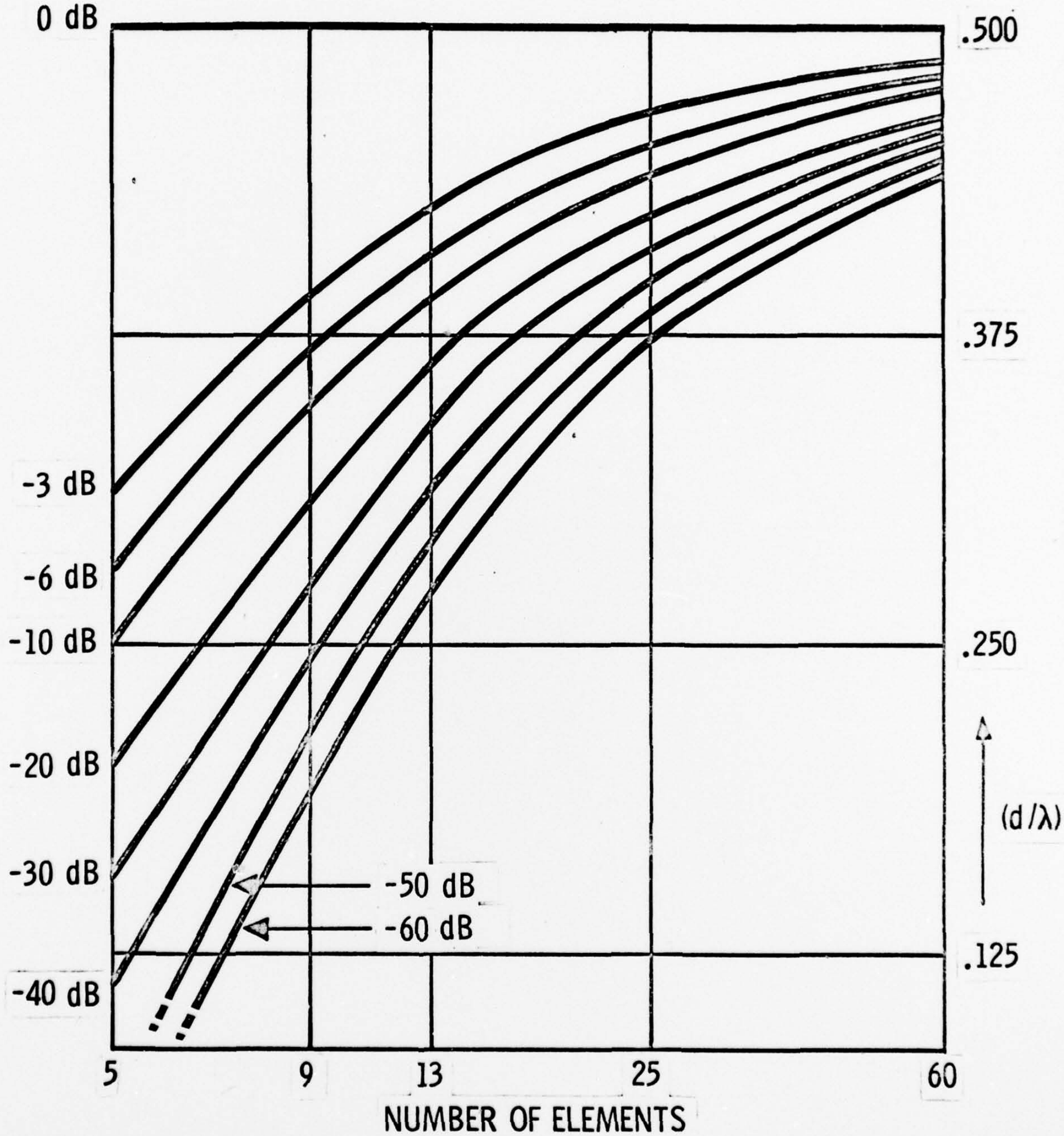

Figure 7. Decrease of signal gain vs spacing and number of elements. Normalized. -25 dB minor lobe level.

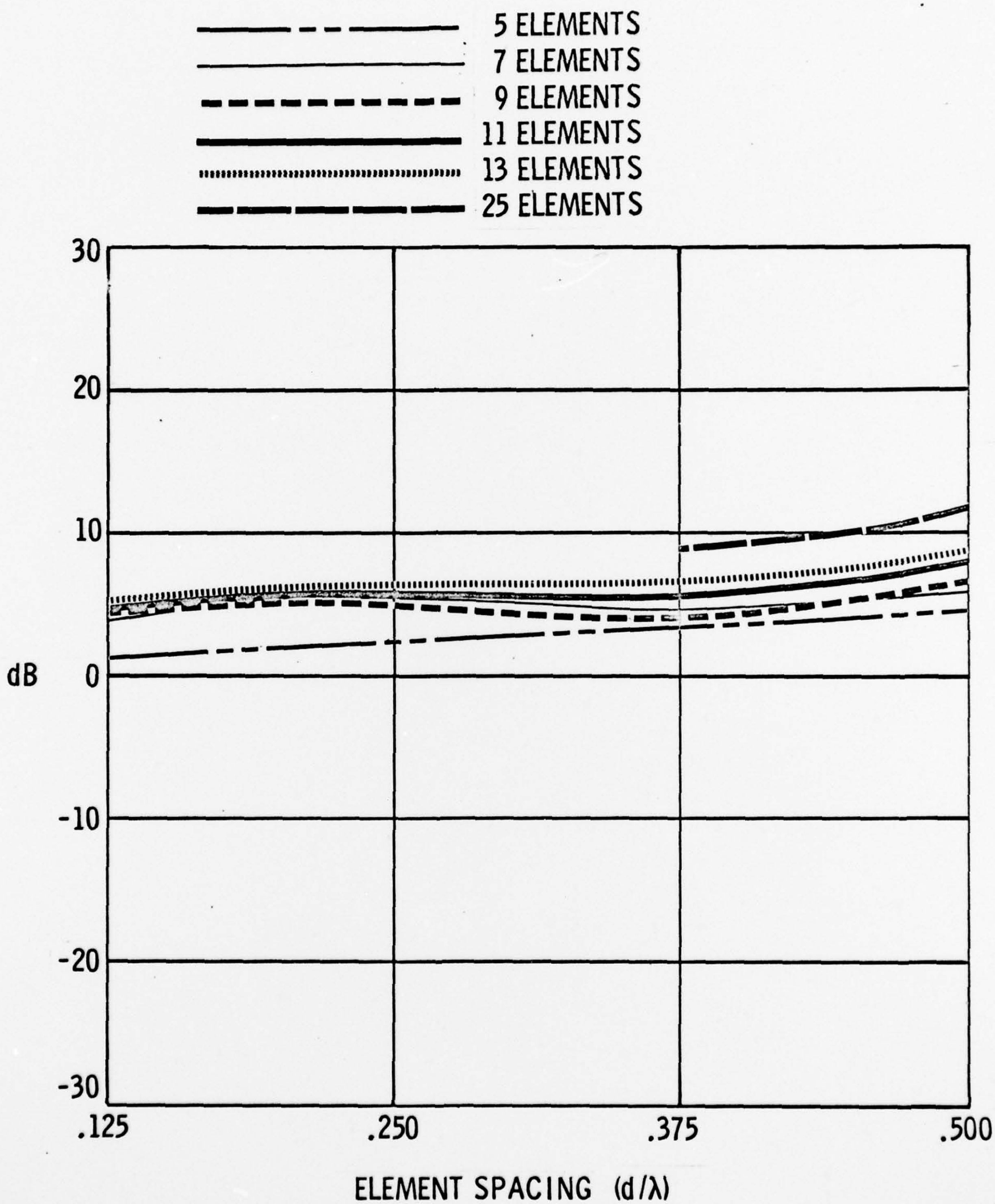

Figure 8. Noise gain (dB re single element) vs spacing. -25 dB minor lobe level.

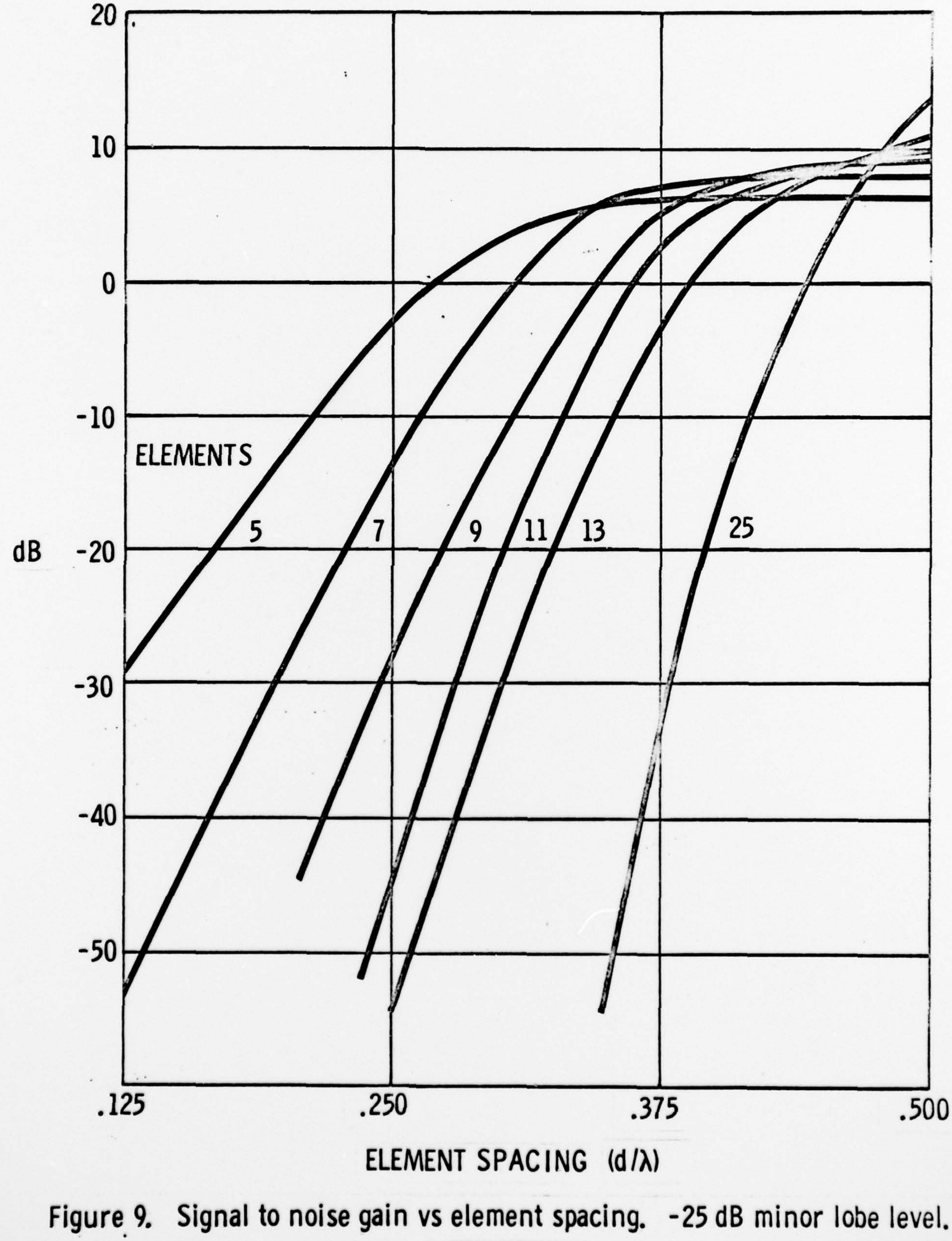

Figure 9. Signal to noise gain vs element spacing. -25 dB minor lobe level.

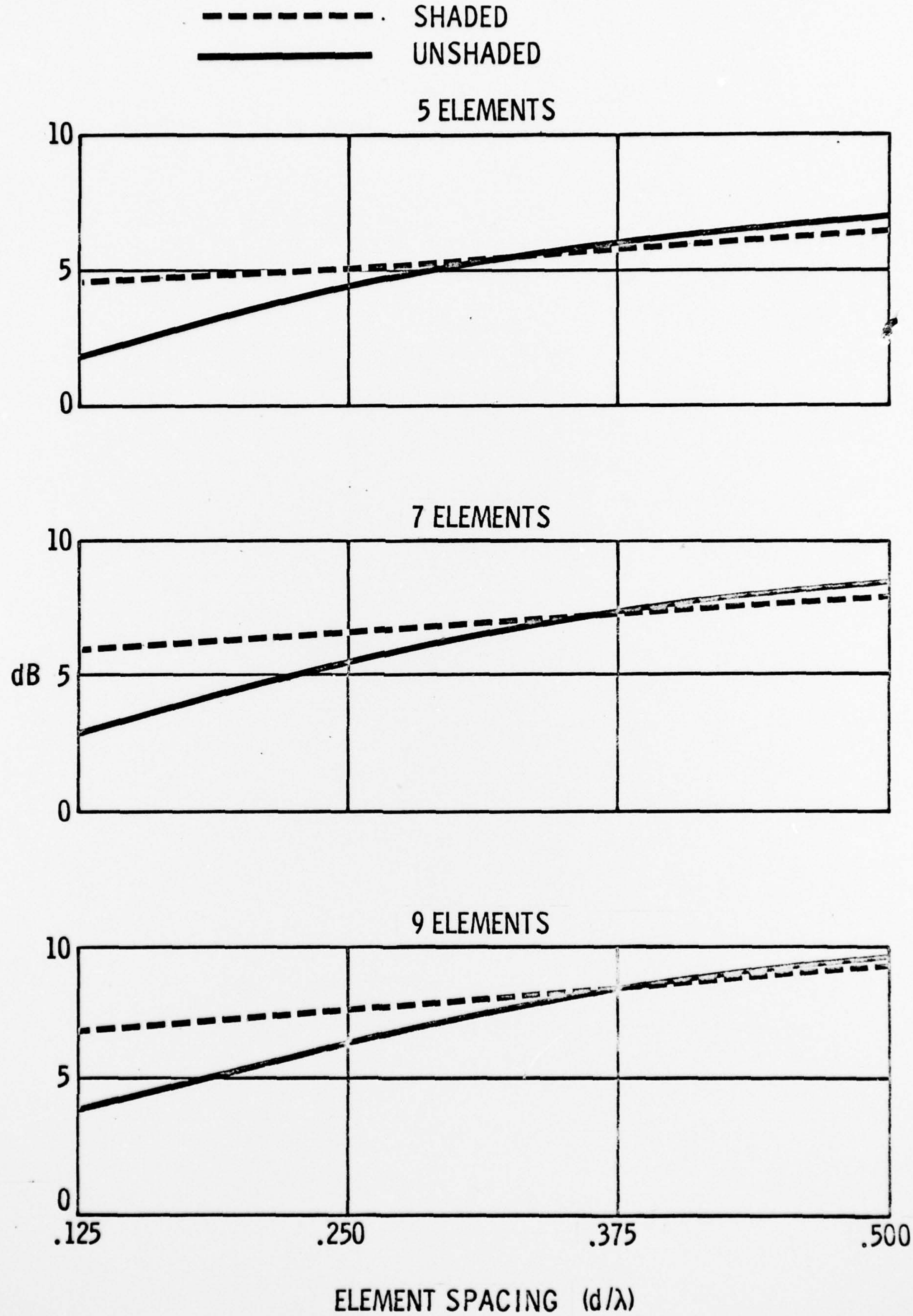

Figure 10. Directivity index vs spacing for unshaded and shaded arrays. -25 dB minor lobe level.

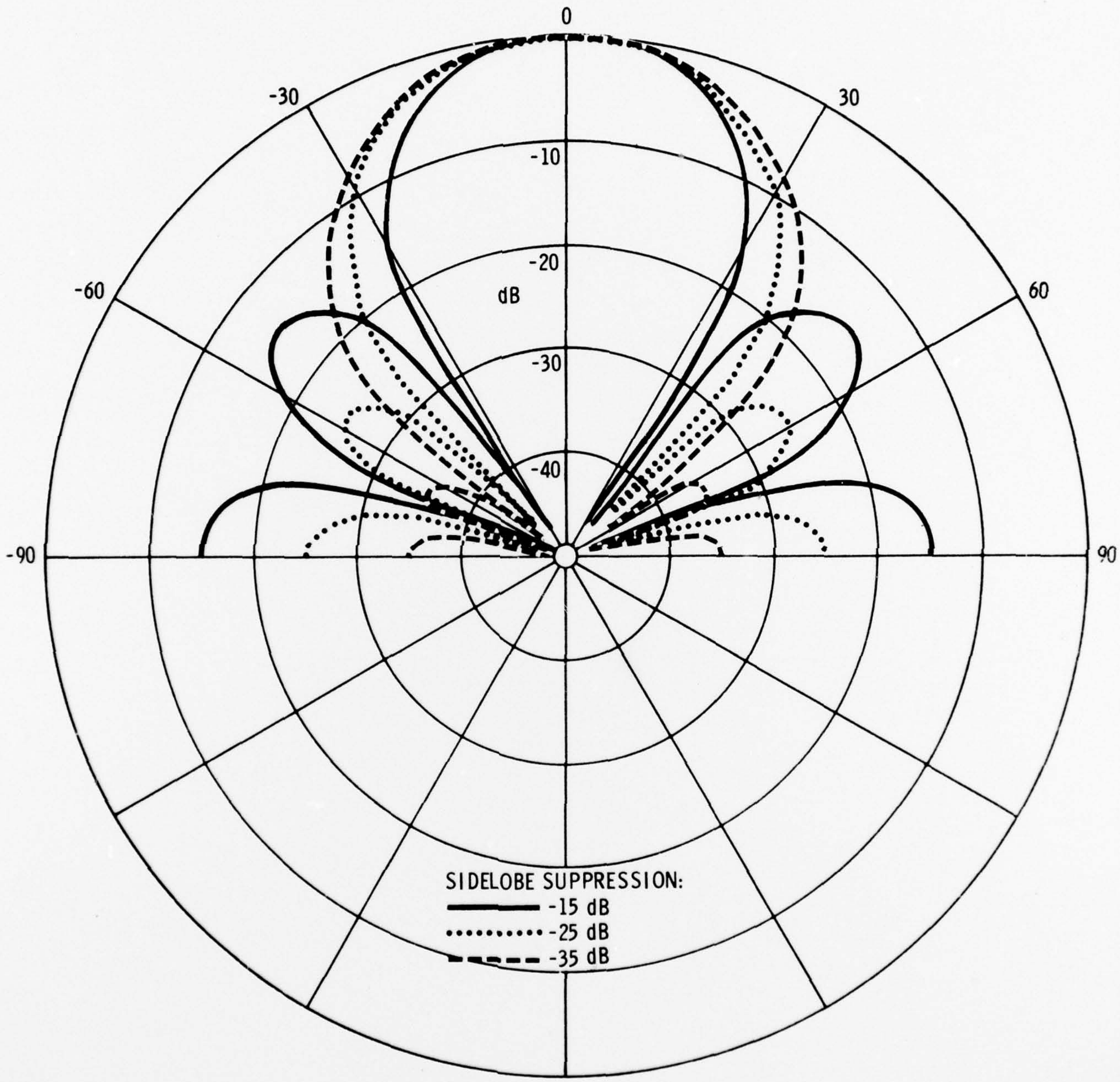

Figure 11. Directivity pattern of five element array, λ/4 spacing.

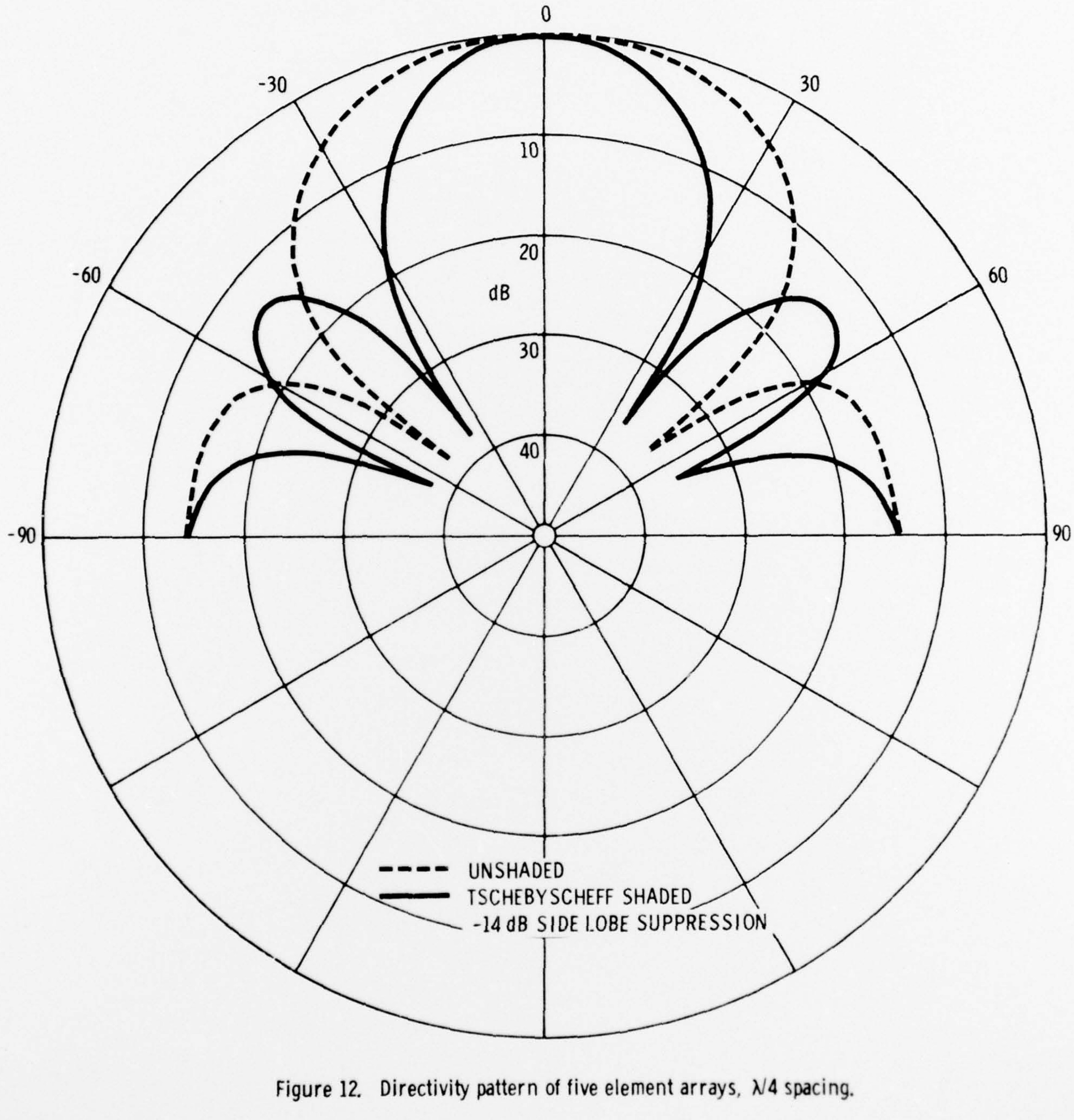

Figure 12. Directivity pattern of five element arrays, λ/4 spacing.

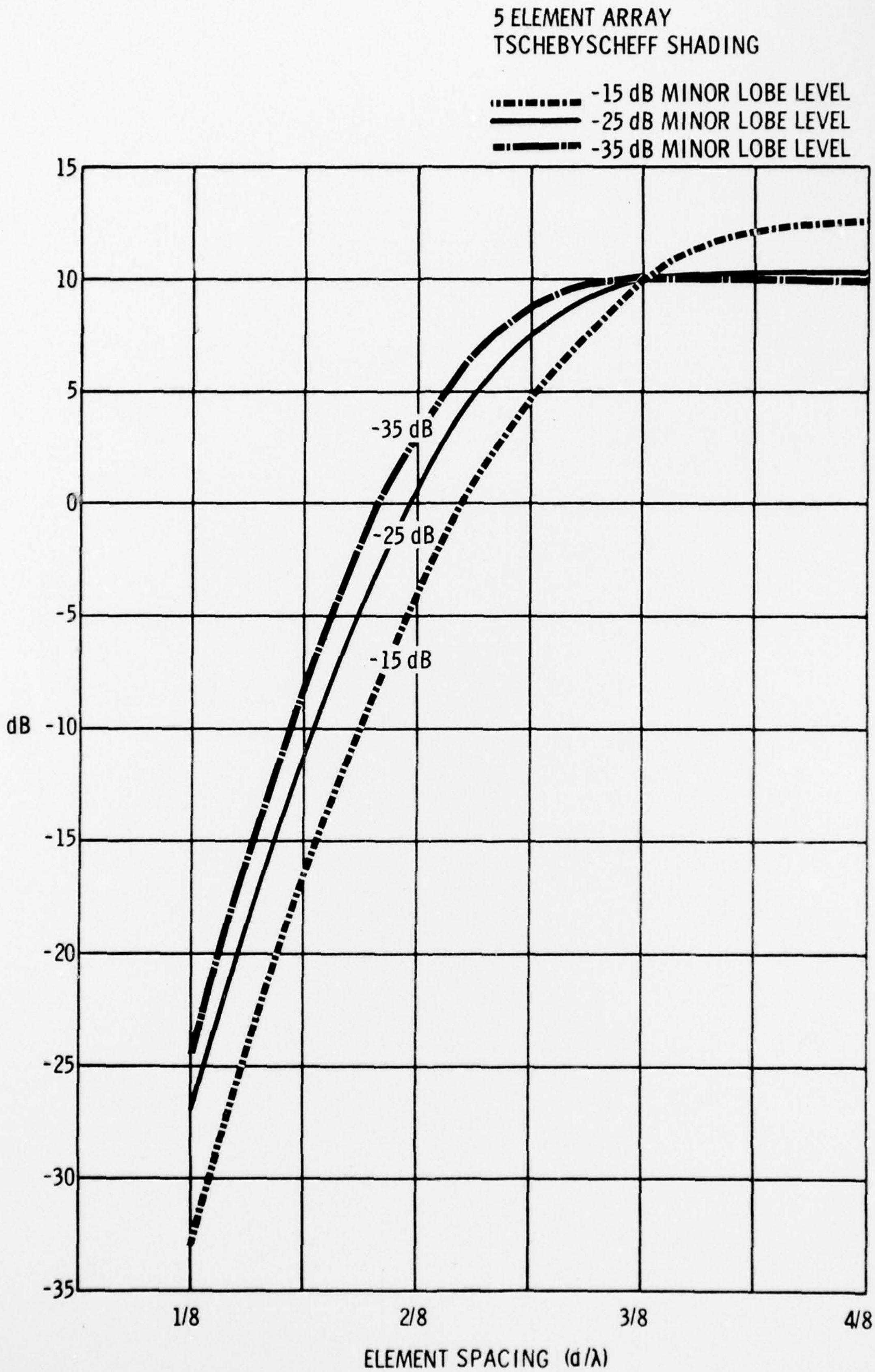

Figure 13. Signal gain (dB re single element) vs spacing and shading.