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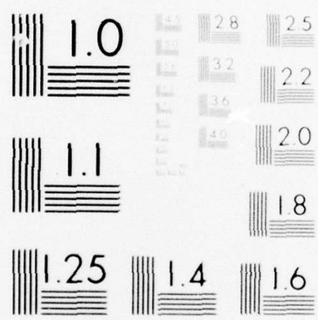

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

During this period the principal investigator was partially supported and performed research in the areas of shock models and wear processes, maintenance and replacement policies for devices subject to shocks or wear, and positive dependent of life distributions. This research activities resulted in six publications with the following titles: "Optimal Replacement Policies for Devices Subject to a Gamma Wear Process," "Optimality of the One Step Look-Ahead Stopping Times," "When to Stop a War," "Positive Dependence of the

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20. Abstract

Bivariate and Trivariate, Absolute Normal, t, Chi-Square, and F Distributions," "Discrete Version of Dynkin's Identity," and "Optimal Replacement of Damaged Devices." In addition to performing the research that resulted in the above papers, the principal investigator presented invited papers at three different conferences.

AFOSR-TR- 77-0941

Final Scientific Report
Grant AFOSR-76-2999
Prepared by

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1. Publication Resulting from Grant AFOSR 76-2999 to Date

Under the present grant we produced the following papers and reports:

(1) Abdel-Hameed, M. S. (1976) Optimal replacement policies for devices subject to a gamma wear process, AFOSR Technical Report No. 1, University of North Carolina at Charlotte. Appeared in The Theory and Applications of Reliability with Emphasis on Bayesian and Nonparametric Methods, ed. by C. P. Tsokos and I. N. Shimi (Invited Paper), Academic Press, 397-412.

(2) Abdel-Hameed, M. S. (1977) Optimality of the one step look-ahead stopping times, AFOSR Technical Report No. 2, University of North Carolina at Charlotte. Appeared in J. Appl. Probability 14, 162-169.

(3) Abdel-Hameed, M. S. (1976) When to stop a war, AFOSR Technical Report No. 4, University of North Carolina at Charlotte. Presented at the 1976 IEEE Conference on Decision and Control. (Invited Paper)

(4) Abdel-Hameed, M. S. and Sampson, A. (1976) Positive dependence of the bivariate and trivariate, absolute normal, t, x^2, and F distributions, AFOSR Technical Report No. 5, University of North Carolina at Charlotte. Submitted for publication in Ann. Statist.

(5) Abdel-Hameed, M. S. (1976) Discrete version of Dynkin's identity, AFOSR Technical Report No. 6, University of North Carolina at Charlotte.

(6) Abdel-Hameed, M. S. Optimal replacement of damaged devices, AFOSR Technical Report No. 7, University of North Carolina at Charlotte. To appear in the March issue of J. Appl. Probability.

In addition to performing the research from which the above papers and reports resulted, the Principle Investigator presented invited papers and otherwise participated in the following conferences:

1. Conference on "Theory and Applications of Reliability with Emphasis on Bayesian and Nonparametric Methods," held at the University of South Florida. (Invited)

2. The Twelfth Annual Conference in Statistics and Computer Sciences, held at The Institute of Statistical Studies and Research-Cairo University. (Invited)

3. The 1976 IEEE Conference on Decision and Control, Sponsored by SIAM and IBM. (Invited)

2. Research Progress in Probabilistic Aspects of Shock Models and Wear Processes

2.A. Abdel-Hameed and Proschan (1973 and 1975) studied life distributions of devices subject to a sequence of shocks occurring randomly in time as events in a nonhomogeneous Poisson process and pure birth process. Assume that $\{N(t),\ t \geq 0\}$ is a process describing the amount of shocks in the interval $[0, t]$. Let $\bar{P}_k$ be the probability that the device survives the first k shocks, $k = 0, 1, \ldots,$ then the continuous time survival probability (the probability that the device survives till a given time) $\bar{H}(t)$ can be represented as:

$$\text{(2.1)} \qquad \bar{H}(t) = E\bar{P}_{N(t)}\ ,\ t \geq 0.$$

Note that in the above model the probability of surviving the first k shocks, $\bar{P}_k$, is independent of the time at which shocks occur. Under the present grant we treated the more general model where we assume that these probabilities are time dependent. More explicitly, we let $\bar{P}_k(t)$ denote the probability of surviving the first k shocks conditional on the fact that these shocks occurred in the interval $[0, t]$. In this case we have that

$$\text{(2.2)} \qquad \bar{H}(t) = E\bar{P}_{N(t)}(t)\ ,\ t \geq 0.$$

A sample of the results obtained are: If $\{N(t),\ t \geq 0\}$ is a Poisson process, then

(a) If for each $t > 0$ $\{\bar{P}_k(t)\}_{k=0}^{\infty}$ is a discrete increasing failure rate (IFR) survival probability, and if for each k in $\{0, 1, \ldots\}$ $\{\bar{P}_k(t),\ t \geq 0\}$ is decreasing in $t \geq 0$, then $\bar{H}(t)$ is a continuous IFR survival probability.

(b) If for each $t > 0$ $\{\overline{P}_k(t)\}_{k=0}^{\infty}$ is a discrete increasing failure rate average (IFRA) survival probability, and if for each k in $\{0, 1, \ldots\}$ $\{\overline{P}_k(t), t \geq 0\}$ is decreasing in $t \geq 0$, then $\overline{H}(t)$ is a continuous IFRA survival probability.

(c) If for each $t > 0$, $\{\overline{P}_k(t)\}_{k=0}^{\infty}$ is a new better than used (NBU) survival probability, and for each k in $\{0, 1, \ldots\}$ $\{\overline{P}_k(t), t \geq 0\}$ is submultiplicative, then $\overline{H}(t)$ is a continuous NBU survival probability.

These results, along with a number of related results, will appear in an AFOSR Technical Report in the near future. These results include stochastic comparison between pairs of systems governed by the above shock processes having different shock survival probabilities $\{\overline{P}_{1,k}(t)\}_{k=0}^{\infty}$, and $\{\overline{P}_{2,k}(t)\}_{k=0}^{\infty}$.

2.B. Abdel-Hameed (1975) assumed that a device is subject to wear occurring randomly in time as a nonhomogeneous gamma process $\{X(t), t \geq 0\}$, with mean $\Lambda(t)$. If the device has probability $\overline{G}(x)$ of surviving x amount of wear, then the continuous time survival probability, $\overline{H}(t)$, can be written in the form

$$(2.3) \qquad \overline{H}(t) = E\overline{G}(X(t)).$$

He shows that life distribution properties of the wear survival $\overline{G}(x)$ are inherited as a corresponding property of the continuous time survival probability $\overline{H}(t)$.

Under the present grant we treat the more general case where the probability of surviving an x amount of wear depends on time of wear. In this case we replace $\overline{G}(x)$ by $\overline{G}(x, t)$ and Equation (2.3) is then replaced by

$$(2.3') \qquad \overline{H}(t) = E\overline{G}(X(t), t)$$

Results similar to those mentioned in 2A have been obtained under the present grant.

3. Research Progress on Maintenance and Replacement Policies for Devices Subject to Shocks or Wear.

3.A. One of the major accomplishments under the present grant is the research done by the principle investigator in this crucial and useful area of reliability theory. Taylor (1975) discusses optimal replacement policies for devices subject to a sequence of shocks occurring randomly in time as events in a homogeneous Poisson process. He assumed that shocks cause ~~damage~~ and damages accumulate additively. Replacement can occur before or at failure. Replacement at failure costs c dollars and replacement at failure costs c - k dollars, $0 < k < c$. He showed that the optimal replacement policy is the first time the accumulated damage exceeds a fixed threshold. Feldman (1976) treated the case where shocks cause damage and damages accumulate additively, but the accumulated damage process is semi-Markovian. He defines control limit policy to be the one in which there is a fixed real value α such that the system is replaced as soon as the accumulated damage is greater than α or when a failure occurs, whichever comes first; otherwise the system is left alone. He then finds the optimal control limit policy. In a recent paper, that will appear in J. App. Probability, the principle investigator shows that the optimal control limit policy is optimal not only among the class of control limit policies but among the class of all replacement policies that depends on the accumulated damage. However, this happened to be true when the shocks occur at a discrete time intervals and according to a Poisson process.

3.B. Abdel-Hameed (1975) treats a continuous version of wear processes. In particular, he assumes that a device is subject to wear and wear occurs randomly in time as a gamma process. If $\overline{G}(x)$ is the probability that the device survives x units of wear, $x \geq 0$, he relates life distribution properties of $\overline{G}$ to the corresponding property of the continuous time survival probability. Under the present grant we treat optimal replacement policies for such devices. The result of this research appeared in The Theory and Applications of Reliability with Emphasis on Bayesian and Nonparametric Methods, Vol. 1.

3.C. The research accomplished under the present grant in the area of maintenance and replacement policies for devices subject to shocks or wear led to the following result which provides a necessary condition for the optimality of the one step look-ahead stopping times. Let $\{X_n, n = 0, 1, \ldots\}$, $X_0 = x$ be a discrete-time homogeneous Markov process with state space $(E, \mathcal{B})$, g and c are two $\mathcal{B}$ measurable functions, and α is in $(0, 1]$. Assume that $A_\alpha g(x)$ is the expected conditional change derived by continuing one more stage then stopping now, i.e. $A_\alpha g(x) = \alpha E_x g(x_1) - g(x)$. Consider the set $B \in \mathcal{B}$, defined by the relation $B = \{x : A_\alpha g(x) \leq c(x)\}$. Let $F = \{\tau(\text{stopping time}) : E_x[\alpha^N g(x_N) I(\tau > N)]$ and $E_x[\sum_{n=0}^{N-1} \alpha^n A_\alpha g(x_n) I(\tau > N)] \to 0$ as $n \to \infty\}$. Then the Markov time $\tau^* = \inf\{n : x_n \in B\}$ is optimal in the sense that

$$E_x[\alpha^{\tau^*} g(x_{\tau^*}) - \sum_{n=0}^{\tau^*-1} \alpha^n c(x_n)] = \sup_{\tau \in F} [\alpha^\tau g(x_\tau) - \sum_{n=0}^{\tau-1} \alpha^n c(x_n)].$$

Applications to reliability and probability is considered. These results appeared in J. App. Probability (1977), 14, 162-169

3.D. From the experiences gained from studying the above replacement policies, it became clear that the following general result in Markov processes is present: Let $(X_n, \mathcal{F}_n, P_x)$, $n = 0,1,\ldots;$ $X_0 = x$ be a discrete time homogeneous strong Markov process. The state space $(E, \mathcal{B})$ is assumed to be a measurable space, where $\mathcal{B}$ is a sigma-algebra containing all subsets of E consisting of one point. Assume that f is a $\mathcal{B}$ measurable real valued function, and g is a function with support $\{0, 1,\ldots\}$. Define the infinitesimal generator of f by the relationship

$$Af(x) = E_x f(X_1) - f(x),$$

and let

$$Ag(n) = g(n + 1) - g(n),$$

then for any Markov time τ satisfying the conditions:

$$\overline{\lim_{N \to \infty}} \; E_x[|g(N)f(X_N)|I(\tau > N)] = 0,$$

$$\overline{\lim_{N \to \infty}} \; E_x[\sum_{n=0}^{N-1} |g(n + 1)Af(X_n)|I(\tau > N)] = 0,$$

and

$$\overline{\lim_{N \to \infty}} \; E_x[\sum_{n=0}^{N-1} |Ag(n)f(X_n)|I(\tau > N)] = 0,$$

we have the following identity:

$$E(g(\tau)f(x_\tau) - g(0)f(x) = E_x[\sum_{n=0}^{\tau-1} g(n + 1)Af(X_n)] + E_x[\sum_{n=0}^{\tau-1} f(X_n)Ag(n)].$$

This result provides a generalization of Dynkin's identity which is given in pp. 131-134 of Vol. I of his book on Markov processes.

In reliability context if $\{X_n, n = 0, 1,\ldots\}$ is a process describing the total damage a device is subject to, and if $f(x)$ is a function describing the cost of replacing the device when it is subjected to an x amount of damage, τ is a replacement policy, and $g(n)$ is the discounting factor at time n, then the left-hand side of the identity, appropriately modified by subtracting a constant, describes the expected cost using the policy τ. This identity provides means for expressing the expected cost in terms of the generators of the cost function and the discounting factor g. It played a major role in the paper "Optimal replacement policy for devices subject to a gamma wear process," and "optimal replacement of damaged devices" discussed in the first page of the report.

4. Research Progress in Positive Dependence of Life Distributions

Multivariate life distributions i.e., multivariate distributions of nonnegative random variables, have so far received a limited amount of attention in the literature. The methods of generating them in reliability context have been motivated by shock models. Examples of such distributions are the multivariate exponential distribution defined by Marshall and Olkin (1967), and the multivariate weibull distribution, defined by Johnson and Kotz (1975). Marshall (1975) defines the concept of hazard gradient and relates it to multivariate life survival probability.

The shock model basis for such multivariate life distributions guarantees the existence of some kind of positive dependence between the components of such distributions. There are many situations, however, in which the positive dependence of multivariate life distributions cannot be adequately modeled by shock processes. This leads to the following question: Can one develop a technique for generating associated multivariate life distributions?

Under the present grant the Principle Investigator and Allan Sampson provided a partial answer to the above raised question. They provided necessary (and in some cases sufficient) conditions for some of the well-known multivariate life distributions to be positively dependent. In particular they show that the bivariate and the trivariate absolute t-distributions are associated. These results are then used to show that a generalized bivariate and trivariate x^2, and F distributions are associated.