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L. G. Taff
J. M. Sorvari

Artificial Satellite Search Strategies

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FOR THE COMMANDER

Raymond L. Loiselle

Raymond L. Loiselle, Lt. Col., USAF
Chief, ESD Lincoln Laboratory Project Office

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

ARTIFICIAL SATELLITE SEARCH STRATEGIES

L. G. TAFF
J. M. SORVARI
Group 94

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ABSTRACT

Satellite search in four dimensions - right ascension, declination, magnitude, and parallax - is examined. The important sources of interference with search schemes are enumerated and their significance in relation to four specific search strategies is discussed.

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I. INTRODUCTION

The search for artificial satellites via reflected light is usually considered to be carried out in two dimensions - the two angular coordinates necessary (and sufficient) to specify position on the celestial sphere. A consideration of actual search schemes, however, shows that at least a third dimension, brightness, and perhaps a fourth dimension, distance, must be added. This should cause no conceptual difficulties since many readers are already familiar with the four-dimensional search (location plus radial velocity) of radar. The purpose of this Note is to discuss the implications of this higher dimensionality in the generalized search context.

The fundamental search technique consists of assigning coordinates (X) in the search space (S) to all of the objects detected and then defining a limited region of this space (s) which contains all of the objects of interest. Ideally, this limited region is chosen such that it contain only artifical satellites and no artificial satellites are in the domain S-s. In particular, if S consists of the product of the spaces (1) topocentric right ascension, (α) and declination (δ), (2) apparent magnitude (m), and (3) parallax (Π) then the definition of s is simple. This is because the vast majority of the objects detected (i.e., stars) have fixed values of α, δ, and m and have values of $\Pi < 1'' \times$ (mean equatorial radius of the earth/1 A.U.) $\leq 10^{-4}$". On the other hand, artificial satellites have variable values of α, δ and possibly varying values of m, together with Π values $\gtrsim 3422''.61$ (the mean equatorial horizontal parallax of the Moon).

Although the fundamental search technique is simple, trade-offs still exist. For example, to conduct a search rapidly a large field of view (for the telescope) is required. Concommitant with the large field of view, there is a brighter limiting magnitude and a poorer ability (in fixed time) to detect variations in right ascension or declination. The optimal solution of the trade-off problems, including the technical risks involved in the auxiliary equipment, is the most difficult part of the design of the best search scheme. We do not solve this problem here.

A small percentage of all of the detected objects may share one or more of the properties (in s) of artificial satellites. Hence, no matter what s we choose, we will be forced to consider multiple spurious detections (per field of view) and the non-negligible probability of failing to correctly identify a satellite. The objects which fill the satellite's s occupy a wide region in s. They range from meteorites to extragalactic supernovae. Section II considers the distribution of all such objects in S.

In addition to these naturally occurring extraneous objects, there are certain physical conditions beyond our control which may interfere with the logic of some detection schemes (i.e., the choice of s). These effects blur the boundaries between s and S-s and can make stars appear to have some properties of satellites and vice-versa. When present, these effects will give rise to incorrect discriminations. Section III considers these effects.

Next, Section IV discusses four types of search schemes and the ability of these schemes to fix s optimally. The four schemes involve the use of a permanent catalog, a temporary catalog, variations in α and δ, and parallax. The trade-off problems that arise will be sketched but not solved here. Nor will inclusion of even higher dimensionality spaces (using polarization, multicolor photometry, radial velocity, etc.) be considered here. Finally, in Section V, the possibility of composite schemes is discussed.

II. THE REST OF THE UNIVERSE

Many naturally occurring objects have coordinates in S which lie in one of the factor spaces of s. These objects thus mimic artificial satellites and may give rise to false alarms in various detection schemes. Starting at the earth and moving out, we need to consider meteors, asteroids, comets, other members of the solar system, variable stars within the galaxy, variable stars in other galaxies, and objects such as quasars and BL Lac objects.

A. Meteors

A meteor is the visible effect of a meteorite passing through the Earth's atmosphere. The duration of the phenomenon is on the order of seconds and occurs at a height 50-100km above the Earth's surface. Meteorites enter the Earth's atmosphere with a mean geocentric speed of $\simeq$ 40 km/sec. The geometry of the Earth-meteorite collision is such to produce more meteors after local midnight than before it. The V magnitudes of meteors range upward from -2^{m} with a mean color of B-V $\simeq$ $-.\,16\overset{m}{.}0$ The rate of sporadic meteors visible to the naked eye is 15/hr while for a telescope with a limiting magnitude near $16\overset{m}{.}0$, the rate increases to 1500/hr. On occasion, a meteor shower will occur when a large number (naked-eye-visible rates $\sim$ 100/hr) of meteors is visible for a short period of time. All of these meteors appear to come from a single point on the celestial sphere (the radiant) and the occurrence of the shower is usually predictable, as is the radiant.

B. Comets

A comet is a conglomeration of volatiles such as H_2O and CH_4 containing embedded grains of refractory materials, which becomes visible when the object nears the sun. The heliocentric orbits of comets are nearly parabolic except for the well cataloged short period ($\lesssim$ 50 yr) comets. The distribution of the direction of the first detection of comets ($B \simeq 12^m$) is isotropic on the celestial sphere. Comets rarely pass the Earth within the Moon's orbit. Since 1892, approximately 550 comets have been observed. A rough estimate for their apparent magnitude can be obtained from

$$V = 6 + 5\log\Delta + 10\ \log r$$

where Δ is the comet's geocentric distance in A.U. and r is its heliocentric distance also in A.U.

C. Asteroids

The bulk of the minor planets are confined to a plane inclined by $7\overset{\circ}{.}9$ to the ecliptic. The standard deviation of asteroid inclination is $5\overset{\circ}{.}7$. Almost all (99.8%) of the asteroids are confined to heliocentric distances between 1.524 A.U. and 5.203 A.U. (i.e., between the orbits of Mars and Jupiter). Asteroids have a mean color of $B-V = 0\overset{m}{.}86$. The number of asteroids with mean opposition magnitude within dm_o of m_o is

$$N(m_o)dm_o = 2\mathrm{dex}\ (-3.347 + 0.388m_o)\ dm_o$$

for m_o between $9\overset{m}{.}1$ and $20\overset{m}{.}6$.

Ignoring the opposition effect, at the average heliocentric distance of 2.7 A.U., an asteroid is $0\overset{m}{.}9$ fainter at quadrature than opposition. The number of asteroids brighter than $m_o = 16^m$ is approxiamtely 1600. Most of these have known orbits.

D. Other Members of the Solar System

For all of the other natural members of the Solar System, one can predict topocentric right ascension, declination, apparent magnitude and topocentric distance. Hence, these can provide no possible intrusion into s. The probability of discovering a new moon or planet brighter than $V = 16\overset{m}{.}0$ is negligible.

E. Variable Stars Within the Galaxy

Although there are not as many types of variables as there are variable stars, there is a sufficient number to preclude even a rudimentary exposition here. However, whether they be novae, Algol type variables, RR Lyr type variables, δ Cep type variables, β Ser type variables, U Gem stars, UV Cet stars, W Vir stars, β CMa stars, supernovae of Type I or Type II, β Lyr type variables, RV Tau type variables, or Z Cam type variables, we can be sure of the following: the GEODSS System, if it tried, would find thousands of them per year, every year. Herein lies a useful astronomical application of the GEODSS technology.

Variable stars have periods from fractions of a day (for β Lyr type) to hundreds of years (for recurrent novae). Moreover, because of selection effects, very few variables with periods in excess of 1 year are known. Variable stars have magnitude variations ranging from less than $0^{m}.1$ (e.g., β CMa type) to more than 14^{m} (supernovae). Stars that vary periodically may do so on a time scale of seconds with an amplitude of 10^{m} (UV Ceti type). Brighter variable stars appear to be isotropically distributed on the celestial sphere but the fainter ones follow the distribution of the brighter portions of the Milky Way.

F. Extragalactic Variable Stars

Aside from δ Cep variables in the nearer galaxies (LMC, SMC, M31, etc.) supernovae provide the principal detectable contribution from variable stars in other galaxies. Supernovae are generally divided in Type I and Type II. Type I generally reach $M_{V} = -18^{m}.9$ declining $2\text{-}3^{m}$ $20\text{-}30^{d}$ after maximum. Thereafter, their decline is expontential with an e-folding time $\simeq$ 0.5yr. Type II supernovae reach $M_{V} \simeq -17^{m}.5$ with a broader maximum, the initial drop being $\simeq 1^{m}.5$, and a decline faster than Type I thereafter. Since the absolute magnitude of a supernova is about 2^{m} fainter than the galaxy it occurs in, and the number of galaxies per square degree brighter that m_{V} is dex $[0.5(m_{V}-14.4)]$, and supernovae occur with a frequency of $\simeq$ 1/300 yr/galaxy, there will be $\simeq$ 90 supernovae brighter than $m_{V} = 16^{m}$ per year. Hence, one every $\simeq$ 4 days.

G. Extragalactic Variables

Aside from knowing very little about the numbers, distribution on the celestial sphere, magnitude distribution, amplitude distribution, etc., for quasars, BL Lac objects, Seyfert galaxies, N galaxies, etc., we do know that they are all fairly faint. For instance, the brightest quasar (3C273), has $m_v = +\, 12\overset{m}{.}8$, but the second brightest (3C351) is $15\overset{m}{.}3$ and there are only 4 (out of hundreds known) with $m_v \leq 16^m$.

III. PHYSICAL LIMITATIONS

The physical limitations which directly affect any search scheme may be naturally divided into three areas. The first area is limitations imposed by the Earth's atmosphere. Light from all extra-atmospheric objects suffers extinction when passing through the atmosphere. The magnitude of this extinction is variable at all temporal and spatial frequencies. In addition, as the light rays traverse the atmosphere their paths deviate from a straight line due to refraction. The amount of refraction is also a variable. The second area is limitations due to the finite resolving power of the telescope-camera system. Since the resolving power of the system interacts with other system parameters, the treatment of its effects gives rise to a variety of trade-off problems. The third area is limitations due to noise. The noise may be natural (e.g., shot noise in the photoelectron flux) or less natural but nonetheless unavoidable.

The net result of all of these limitations has the same general effect on search schemes: they make clear-cut discrimination (i.e., the choice of s) of artificial satellites from all other objects, based on one parameter, impossible. In extreme cases this may force the reduction of the dimensionality of S.

A. Atmosphere

When light traverses a portion of the Earth's atmosphere, it suffers both absorption and scattering. The net result is an apparent diminution of the brightness of an object. This extinction is usually expressed as εX where ε is the extinction/unit air mass and X ($\simeq \sec z$, z = zenith distance) is the number of air masses traversed by the light rays. Under the best conditions at the best astronomical sites $\varepsilon \simeq 0\overset{m}{.}1$-$0\overset{m}{.}2$/air mass and, of course, it may exceed 30^{m}/air mass. Moreover, ε may vary appreciably on a short time scale.

We have obtained data on this variation at the ETS on the GEODSS system under "good" observing conditions. That is, there was no visible haze nor weather fronts. Under these conditions, $d(\varepsilon X)/dt \simeq 0.05/\mathrm{hr}$. Measurements of 5^{s} duration taken 20^{s} apart indicate that εX may vary as much as $0\overset{m}{.}02$ on this time scale. On even shorter time scales, the phenomenon of scintillation becomes more important than variations in the extinction under most observing conditions. Scintillation is discussed in § III C. The extinction variations mentioned above are, of course, much smaller than the results for an average night at the ETS. The arrival or departure of a light haze can easily give rise to $d(\varepsilon X)/dt = 2^{m}/\mathrm{hr}$.

The mean effect of atmospheric refraction is to displace extra-atmospheric objects towards the observer's zenith. This is well understood and can be corrected. The variation from mean refraction gives rise to a different sort of effect. When the telescope aperture is small, the collected light has traversed a number of randomly oriented prisms which travel with the winds. As long as the size of the average prism (coherent mass of air) is greater than the telescope aperture, there is no dispersal of the light rays. As the aperture increases, this inequality is less likely so that the telescope focuses a number of separate images of the source. In either case, the size of the seeing disk is determined by the atmosphere and not the telescope (i.e., seeing disk > diffraction disk). For the ETS the typical seeing disk $\simeq$ 2". The smaller the site's height above mean sea level the larger the seeing disk will be. The seeing disk will rarely exceed its typical size by a factor of 2. However, if a weather front passes through the field of view momentary displacements of the image $\simeq$ 10" can occur, but these will be uniform across the front line.

B. Resolving Power

The large majority of the time (for the GEODSS system) the diffraction disk will be smaller that the seeing disk which will be smaller that the camera resolution element (resel). Hence, in the atmospheric/telescope/camera imaging process, it is the camera that limits the resolving power. A useful simplified model relates the diameter of a resel (α) to the telescope's focal length (f in m), the minification from the focal plane to the camera target (m), and the size of the resel on the camera target (s in μ) producing α. The result is:

$$\alpha = 0\overset{''}{.}206\ sm/f.$$

The number of resels per square degree n, is given by

$$n = 3.05 \times 10^{8} \, (f/ms)^{2}.$$

The field of view, ϕ, in degrees for a camera target d (mm) is

$$\phi = 0\overset{\circ}{.}0573 \, dm/f.$$

For the system at the ETS, d = 25, s = 50 and a variety of values of m and f is available.

C. Noise

The two primary sources of noise are shot noise and atmospheric scintillation. By incorporating a relatively noise free high gain stage in front of the camera, the noise associated with the electron scanning beam and other electronics can be made negligible. Two additional sources of noise, ion scintillation and small scale variations in the camera tube sensitivity, give rise to less important effects.

Shot noise is the statistical noise in the random emmission of photoelectrons from the first photo-sensitive surface. There are two contributions to shot noise: fluctuations in the photoelectron flux from the object (star) being observed and fluctuations in the photoelectron flux due to the sky background. The importance of the background noise depends on both the resel size and the brightness of the sky. The effect of the background noise on the star's signal depends on the amplitude of that signal. The effects of fluctuations in the signal due to the star are easier to evaluate. Table I shows the average noise in the stellar signal for typical ETS parameters on the 31" telescope. For the 14" telescope $\simeq 2^m$ should be subtracted.

TABLE I

FLUCTUATIONS IN TARGET SIGNAL EXPRESSED IN MAGNITUDES FOR VARIOUS TARGET MAGNITUDES AND INTEGRATION TIMES.

m/τ	10s	1s	1/30s
13	$0^m.01$	.02	.09
14	.01	.03	.14
15	.01	.04	.21
16	.02	.07	.32
17	.03	.10	.47

The same atmospheric turbulence which causes a seeing disk also causes the phenomenon of atmospheric scintillation. This is a rather complicated phenomenon and has been well treated by Young.* Combining his analysis with scintillation data from measurements made with the 6" photometer at the ETS implies that the scintillation noise in the 31" telescope/camera system will be

$$\sigma \simeq 0\overset{m}{.}02/\sqrt{\tau} ,$$

where τ is the integration time in seconds.

In most instances, the contribution from scintillation is negligible for objects at the threshold of detection. If the fluctuations are Poissonian for both star and background noise then the total noise in a threshold signal will be

$$\sigma^2 = \sigma_t^2 \; (1 + 2B/T),$$

where σ_t is taken from Table I and B/T is the ratio of the background to stellar signal in a resel on the camera's target, given by

$$B/T = \alpha^2 \text{ dex } [-0.4(m_{sky}-m_{star})] .$$

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* YOUNG, A.T., Methods of Experimental Physics (Academic Press, N.Y., 1974) Vol 12A, ch.2.

IV. SEARCH SCHEMES ANALYSIS

All of the search schemes to be analyzed here share a single basic principle of operation: In each case one compares two or more "photographs" of the same area of the celestial sphere. Hence, they extend the classical asteroid/comet/planet/variable star search techniques. Three of the search schemes analyzed here separate the photographs in time. For the permanent catalog method the time interval would be measured in months or years. For the temporary catalog method the time interval is measured in minutes while for proper motion method the duration is seconds. Discrimination and detection in the parallax technique are based on the comparison of two simultaneous photographs but the two cameras are spatially separated.

Each of these search schemes suffers from various problems. These are reflected in the detection capability, the discrimination capability, or scan rates. In addition, the phenomena discussed in Sections II and III also cause unwanted problems, with differing degrees of complication. These factors are discussed for each of the search schemes in turn. The discussion is both qualitative and quantitative. Since no scheme is problem free, Section V considers composite schemes based on two at a time combinations of the four basic schemes.

A. Permanent Catalog

Over a period of time (months to years) a set of photographs of that portion of the celestial sphere visible to each site is iteratively "cleaned". Cleaning refers to the elimination of artificial satellites, meteors, asteroids, comets, planets, and the replacement of variable stars at less than maximum brightness values. It is important to note however, that the largest component of unwanted objects can be eliminated in one month and that the catalog is site independent (given the current GEODSS site distribution). Once the cleaned catalog is created, the comparison of the real time image of a portion of the celestial sphere with its catalog image automatically indicates the presence of all artificial satellites, meteors, comets, asteroids, planets, and very long period variables near maximum brightness. However, while the detection capability of the permanent catalog is excellent, its discrimination capability is poor.

For objects brighter than the limiting magnitude of the search, detection is immediate as long as one image (that of an artificial satellite) is not superimposed over another image (that of a star). This effect is known as crowding. Its importance can be calculated by simply computing the ratio of the number of resels occupied by stars (including the effects of blooming), N_{star}, to the total number of resels in a field of view, N_{resel}. As long as N_{star}/N_{resel} is very much less than unity crowding will be unimportant.

Even with crowding, the detection probability can still exceed $1 - N_{star}/N_{resel}$ if S includes magnitudes. If an artificial satellite and a star occupy the same resel in the real time photograph, then this resel will have a magnitude less than its value in the library photograph. The usefulness of this detection method is limited by noise. If m is a part of S then to reduce the false alarm rate one must ignore differences in resel magnitudes of amount Δm. If the mechanism causing the magnitude fluctuations is Gaussian then Δm must be at least 3.290 standard deviations for a false alarm rate less than 0.1%. The value of the standard deviation, σ, depends on the resel brightness, the integration time, the brightness of the sky background, the telescope's focal length, the limiting magnitude of the search, and sundry other factors. Given Δm, the satellite's magnitude must be brighter than the star's apparent magnitude minus $2.5 \log [\mathrm{dex}(0.4\ \Delta m) - 1]$ for the inclusion of m in S to affect the detection probability. If $\Delta m = 0\overset{m}{.}5$ then the magnitude difference for a successful detection must be at least $0\overset{m}{.}6$.

The usefulness of including m in S now depends on the apparent magnitude of the satellites we wish to find, the effects of crowding, and the number of stars within $0\overset{m}{.}6$ (for $\Delta m = 0\overset{m}{.}5$) of the artificial satellite's apparent magnitude. For the fainter satellites little , if any, advantage accrues because the number of stars with magnitudes brighter than m is an expontentially increasing function of m. That $0\overset{m}{.}5$ is a reasonable estimate for

Δm may be confirmed from the current ETS configuration. In the zoomed state of the 31" telescope, a limiting magnitude of 17^m, and a sky background of $20^m/\text{sec}^2$, the value of σ is $0\overset{m}{.}18$, whence $\Delta m = 0\overset{m}{.}59$. For the 14" under the same circumstances, $\sigma = 0\overset{m}{.}33$ so $\Delta m = 1\overset{m}{.}09$.

The inclusion of m in S as a discriminatory parameter has other problems due to the naturally occurring variables, the presence of an enhanced background due to moonlight, and fluctuations due to atmospheric effects. Either the false alarm rate will be too high or the limiting magnitude for discrimination so low as to render this discrimination technique unfeasible.

The last problem associated with detection is the one involved with the precise matching of the two photographs. If this can not be accurately and rapidly performed, then the discriminatory capability of this technique would be severely degraded. However, this is a software problem already successfully solved by the SAO.

Finally, we discuss the scanning rate of this search technique. Since the permanent catalog method has a very poor discriminatory capability, it will have to be supplemented by some other method. The scanning rate for detection is limited by the speed with which the telescope can be moved. Hence, the total search rate is determined by the discrimination technique. Independent of this consideration, if one wants to parameterize the search scheme by a fixed signal to noise ratio then this also fixes the scan rate. To see this, remember that for objects near the limiting magnitude of the search the ratio of the background signal to object signal is large. From Section III C, it follows that $\sigma \propto 1/(f\tau^{\frac{1}{2}})$ and since the scan rate is proportional to area/time, which in turn is proportional to $1/(f^2\tau)$, the scan rate itself depends on σ^2. The remaining variables, telescope aperture, target diameter, and resel size, offer the only area for improving the system performance.

B. Temporary Catalog

The temporary catalog search scheme differs from the permanent catalog search scheme only in the time interval between the two photographs. This gives the temporary catalog a much better discriminatory capability than the permanent catalog method and a slightly worse detection capability. Both detection and discrimination rest on the movement of the object in the time interval between photographs. Below some minimum value of angular speed the object is a star, asteroid, or comet. Above some maximum value of the angular speed the object is a meteor, a satellite undergoing destructive reentry, or an airplane.

The problem of crowding and precision plate alignment is the same as for the permanent catalog scheme but the noise problem is somewhat worse. It is worse by exactly the square root of two, for both photographs contain errors whereas the library image in the permanent catalog system was error free. The net effect is to make the temporary catalog scheme have a brighter limiting magnitude and slower scan rate than the permanent catalog scheme.

The temporary catalog scheme has one other serious problem associated with it. This due to the possibility of a rapidly changing extinction. If the sky transparency improves in the time interval between the photographs then the false alarm rate will increase expontentially with the decreasing extinction and value of Δm. In particular, if $\Delta\varepsilon$ is the decrease in extinction measured in magnitudes, then all of the stars within $\left[(\Delta m)^2 + (\Delta\varepsilon)^2\right]^{1/2}$ magnitudes of the limiting magnitudes of the first photographs will yield false alarms. Decreasing transparency yields no benefits.

The scan rate for this scheme is limited by the rapidity with which the telescope can be moved.

C. Proper Motion

This search scheme relies only on proper motion for detection and discrimination. As such it bears a resemblance to the temporary catalog scheme, the principal difference being in the ratio of the maximum angular speed, Ω_{max}, to the minimum angular speed Ω_{min}. Reducing Ω_{min} implies that one has increased the telescope's focal length, decreased the target resel size, decreased the minification, or increased the exposure time of the photograph. The importance of crowding is the same as above but the noise problem is much less severe. This follows from the geometry of the artificial satellite's motion and the fact that at least three photographs must be compared to yield an acceptably low value for the false alarm rate. The alignment problem referred to above is also minimal as long as the sidereal drive of the telescope is sufficiently precise. Interference from naturally occurring objects will be minimal.

The scan rate is proportional to the product of the square of the field of view and the time for the object to cross a resel. Using the relationships in Section III B, we can write scan rate $\propto \phi^2\omega/\alpha = \omega d^2 m/(sf)$. Therefore, the most efficient way to improve the scan rate is to increase the size of the camera's target.

As mentioned above, an important parameter of the search scheme is the ratio of Ω_{min} to Ω_{max}. Consider the problem of maintaining a leakproof fence of length L (L in radians) against a maximum angular speed given by Ω_{max}. The time to sweep the fence must be less than the quotient of the field of view and Ω_{max}. The actual time required to sweep the is the time spent in each field of view. The time spent in each field of view is at least as large as the time required to traverse one resel with an angular speed of Ω_{min}. Hence,

$$\text{Tsweep} < \phi/\Omega_{max},$$

$$\text{Tsweep} = (L/\phi)(\alpha/\Omega_{min}),$$

$$\Omega_{max}/\Omega_{min} < \phi^2/(\alpha L) = md^2/(Lsf)$$

For the 31" telescope in full field, $d = 25\text{mm}$, $m = 3$, $f = 3.94\text{m}$, and $s = 50\mu$ so

$$\Omega_{max}/\Omega_{min} < 9.52/L.$$

For any reasonable length of the fence this is an unsatisfactory situation. For the 14" telescope the ratio can be improved by a factor of 10 but the limiting magnitude of the search is lowered by 2^m. Because of the neglect of field overlap, telescope movement between fields, etc., this is clearly an upper limit.

D. Parallax

The parallax search scheme is the best of all of the schemes considered here in terms of discriminatory power. It also shares the fastest scan rate since this is limited only by the rapidity with which the telescope can be moved. Its discriminatory power derives from the fact that the space between the Earth and the Moon is empty except for meteorites and artificial satellites. Meteorites are too faint to be detected. It does have problems in that one requires two telescopes at different geophysical locations to intimately couple. Not only must they both be synchronized but a large amount of information must flow between them. A possible solution to the data handling problem is an FM conversion of the video signal and intrasite communication via a microwave link. FM conversion of the video signal would have additional side benefits in other aspects of the signal processing (e.g., digitizing the video, controlling drifts, etc.).

The separation between the telescopes is determined by the amount of displacement desired when the two images are combined. For an object with a relative parallax of Π to yield a net displacement of N resels,

$$N = \Pi/\alpha = \ell f/(rsm),$$

where ℓ is the separation of the telescopes and r is the satellite's topocentric distance from a point midway between the telescopes. For N = 2 and the 31" telescope of the ETS in the zoom configuration, a 1.4km separation is required at synchronous distance. On the other hand, a 24" f/13 telescope which would have the same limiting magnitude, requires half this separation. The separation can be further reduced by increasing the focal length. This, however, reduces the field of view and thus greatly reduces the scan rate.

V. COMPOSITE SYSTEMS

The discussion of Section IV makes it clear that none of the schemes is ideal, each having its own strengths and weaknesses. To summarize: The permanent catalog is an "instantaneous" detector which can work to the telescope threshold of detection, but which has no discriminatory power. The temporary catalog gains some discriminatory power at the expense of limiting magnitude and has a potential for serious false alarm problems. Proper motion has good discrimination but is slower and in particular has a poor Ω_{max} to Ω_{min} ratio. Parallax has excellent discrimination, but is either very slow or requires large separation of two telescopes.

At the next level of complexity are two-at-a-time combinations of the four schemes discussed. Of the six possible combinations, two - permanent catalog combined with temporary catalog and proper motion combined with parallax - can be dismissed out of hand. This is because each pairing duplicates strengths and weaknesses and thus produces no gain. Two additional combinations - permanent catalog combined with proper motion and temporary catalog combined with parallax are similar but somewhat inferior to the remaining two combinations, discussed below.

If search is based on a combination of temporary catalog and proper motion, it is possible to gain the good discrimination of proper motion while retaining most of the speed of the temporary catalog. During an individual pass over a given area in the sky rapidly moving satellites are detected via the proper motion. Since no attempt is made to find the slow satellites in a single pass, it is not necessary to sit on a field of view for an extended time and so scan rates may be kept high. During successive passes, the temporary catalog technique is used to identify the slowly moving satellites. The parameters of the search can be adjusted so that any satellite moving fast enough to escape during coverage of a fence will be detected due to its proper motion. Unfortunately, this combination is unable to solve the potential false alarm problem of the temporary catalog. Use of a permanent catalog does solve this, but introduces the detection of asteroids.

Combination of permanent catalog with parallax adds the excellent discrimination of parallax to the scan speed and faint limiting magnitude of the permanent catalog. In this scheme detections are made via the permanent catalog and all suspects are handed off to another telescope or telescope pair for discrimination. Since each of the two major tasks is done by a different telescope, it is possible to optimize each telescope for its task. In particular, since a large field of view is not useful in discrimination, a pair of long focal length telescopes could be used, greatly reducing the size of the baseline needed. The remaining problem with this combination is the possible need to hand off large numbers of objects such as asteroids thus slowing the scan. This can be solved by the creation of a temporary catalog of non-interesting suspects.

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Satellite search in four dimensions – right ascension, declination, magnitude, and parallax – is examined. The important sources of interference with search schemes are enumerated and their significance in relation to four specific search strategies is discussed.