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**LANDING AND TAKEOFF ROLL-OUT AUGMENTATION**

**Vincent Muehter**

**The Bendix Corporation  
Flight Systems Division  
Teterboro, New Jersey**

**DECEMBER 1974**

**FINAL REPORT A007**

**Prepared for**

**Air Force Flight Dynamics Laboratory/FGT  
United States Air Force  
Wright-Patterson A.F.B., Ohio 45433**

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19 REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
18 1. REPORT NUMBER AFFDL-TR-77-69 ✓	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
6 4. TITLE (and Subtitle) LANDING AND TAKEOFF ROLL-OUT AUGMENTATION.	9	5. TYPE OF REPORT & PERIOD COVERED FINAL rept. Jan 1972 - Dec 1974
7. AUTHOR(s) VINCENT/MUEHTER	10	8. CONTRACT OR GRANT NUMBER(s) F33615-72-C-1753 new
9. PERFORMING ORGANIZATION NAME AND ADDRESS THE BENDIX CORPORATION FLIGHT SYSTEMS DIVISION TETERBORO, NEW JERSEY 07608	15	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62201F 17 P1 82260121
11. CONTROLLING OFFICE NAME AND ADDRESS AIR FORCE FLIGHT DYNAMICS LABORATORY/FGT WRIGHT-PATTERSON AFB, OH 45433	11	12. REPORT DATE DECEMBER 1974
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12 32p.		13. NUMBER OF PAGES 29
15. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Automatic; Aircraft Landings		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The results of a computer study on four roll-out guidance control laws and a nosewheel steering configuration is included herein. Beam convergence, ground speed, and crosswind effects were included in the simulation.		

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## FOREWORD

This report was prepared for the Air Force Flight Dynamics Laboratory, Flight Research and Test Branch, by the Flight Systems Division, The Bendix Corporation, Teterboro, New Jersey under Air Force Contract No. F33615-72-C-1753, data document item A007, Landing and Takeoff Roll-Out Augmentation. This work was performed in conjunction with other program tasks during the time period January, 1972, through December, 1974. The Air Force Program Manager was Capt. T. Imrich with Project Engineers Lt. R. P. Denaro and Lt. B. Kunciw.

The Flight Systems Division effort was under the direction of Mr. F. G. Adams, principal investigator. The simulation and analysis task was conducted by Mr. V. Muehter with contributions by Mr. K. Moses, Assistant Chief Engineer. Supporting data and configuration requirements were supplied by Messrs. J. Woloshen, M. Sforza and S. Skaritka.

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## SECTION 1

### INTRODUCTION

This study, in response to contract item A007, "Landing and Takeoff Roll-Out Augmentation," was performed to review the possibility of increasing the accuracy and performance of roll-out guidance systems. Parameters such as groundspeed ( $V$ ) and desensitized beam ( $\frac{R\eta}{R_0}$ ) were to be considered available to help provide desirable closure performance on the beam centerline considering range ( $R$ ) and groundspeed conditions during landing and takeoff at groundspeeds of over 80 knots.

The feasibility of automatic ground roll control at lower speeds where the rudder is ineffective, through the use of nosewheel steering, was also to be considered.

The system control laws evaluated for the rudder control portion are tabulated below:

- 1) Present System:  $\hat{\delta}_r = K_\eta \eta + K_\psi \psi$
- 2) Mod 1:  $\hat{\delta}_r = K_\eta \left( \frac{V_0}{V} \right) \left| \frac{R\eta}{R_0} \right| + K_\psi \psi$
- 3) Mod 2:  $\hat{\delta}_r = K_\eta \left( \frac{V_0}{V} \right)^2 \left| \frac{R\eta}{R_0} \right| + K_\psi \left( \frac{V_0}{V} \right) \psi$
- 4) Mod 3:  $\hat{\delta}_r = \left[ K_\eta \eta + K_\psi \psi \right] \left( \frac{V_0}{V} \right)$

## SECTION 2

### DISCUSSION

#### 1. Roll-Out Guidance Prior to Nosewheel Engagement

##### a) Present Roll-Out Configuration

The present roll-out system, Figure 1, uses heading and non-desensitized beam error and therefore its performance is a function of groundspeed and range. Its control law can be considered to be of the form,

$$\hat{\delta}_r = K_\eta \eta + K_\psi \psi = K_\eta \frac{Y}{R} + K_\psi \frac{\dot{Y}}{V},$$

where the parameter symbols are defined in Figure 12. Thus, the use of pure heading results in an increase in effective Y gain with decreasing groundspeed and the use of non-desensitized beam error results in an increase in effective Y gain due to a reduction in range (beam convergence).

In the case of a landing, these effective gain increases tend to compensate for a decrease in rudder effectiveness.

In the case of takeoff or touch and go, the landing beam error gain would be excessive since the increased airspeed renders the rudder surface more effective. For this reason provisions have been included to reduce the gain whenever the throttle levers are off the aft limit. However, this approach is not ideal since this gain adjustment is not continuous but is, rather, a step change.

Figure 2 shows the responses of this system during landings from touchdown to 80 knots, the point at which nosewheel steering would take over. These are responses to an initial offset in heading and crosstrack displacement and also to a constant 20 knot crosswind. Because the aircraft exhibits a tendency to weathercock, or turn upwind, some steady-state downwind rudder deflection is required to maintain the runway centerline. At the touchdown airspeed, the magnitude of this deflection is approximately equal to that provided at touchdown by the rudder channel integrator in the runway alignment mode. Since steady-state integration of the beam or heading term is not feasible in this mode, the rudder surface required to counteract weathercocking can only be supplied at the expense of steady-state tracking errors. If, however, the approximate rudder requirement is satisfied a priori, thus providing an operating point about which the tracking errors may be minimized.

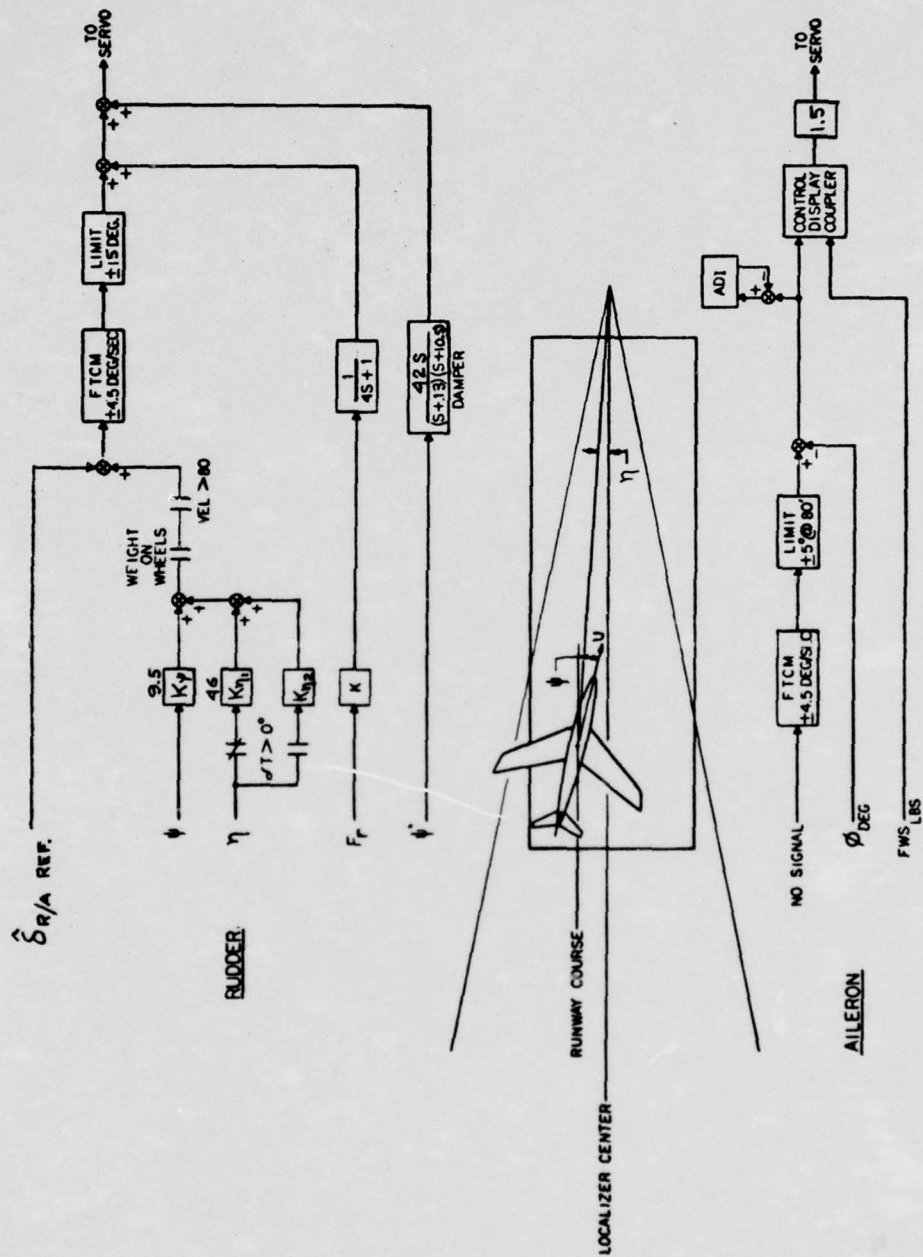


FIGURE 1. PRESENT ROLL-OUT GUIDANCE PROFILE



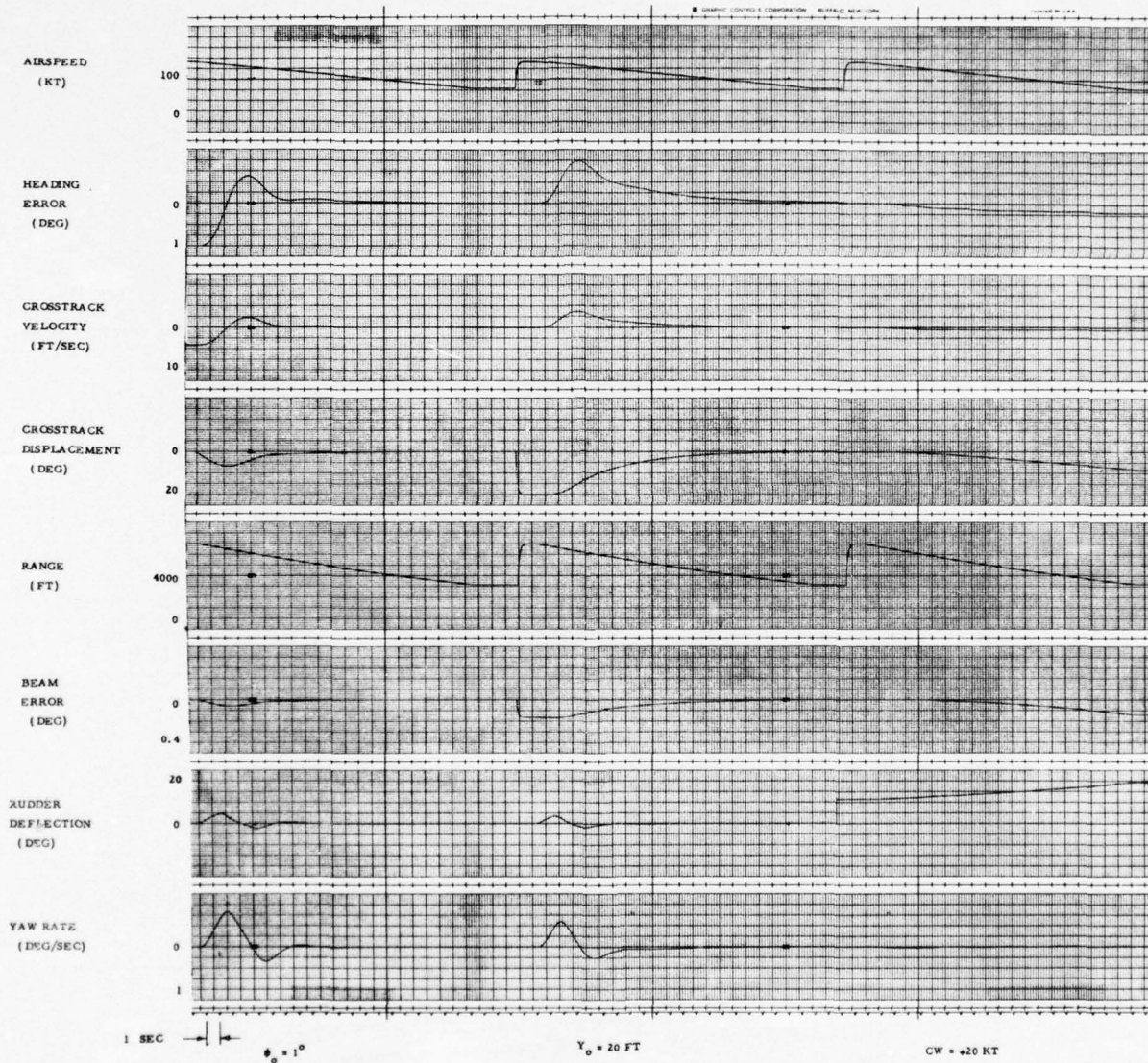


FIGURE 2. NOMINAL ROLL-OUT RESPONSE

As the airspeed decreases on the ground roll, the loss in rudder effectiveness requires a larger surface deflection until nosewheel frictional forces become appreciable. This requires a larger error from beam center, during roll-out, to command the increasing surface deflection, and is an area for improvement in the present Roll-Out system.

b) Modification #1

The Modification #1 configuration considered herein, as one alternative to the present system, utilizes desensitized beam, adjusted as a function of groundspeed. The control law is,

$$\hat{\delta}_r = K_\eta \left( \frac{V_0}{V} \right) \left( \frac{R}{R_0} \eta \right) + K_\psi \psi = \left[ K_\eta \left( \frac{V_0}{R_0} \right) Y + K_\psi \dot{Y} \right] \left( \frac{1}{V} \right)$$

Its responses to heading and crosstrack offsets and crosswind are shown in Figure 3. The similarity to the nowind case is quite evident, which indicates how closely the effects of range desensitization and groundspeed scheduling cancel each other. In both cases, one can observe a considerable departure from beam center due to a crosswind. In addition, as groundspeed decreases, the overall response to disturbances along the runway slows down in both cases.

The Y response to rudder ( $\delta_r$ ) is actually a function of (velocity)<sup>3</sup> since yaw acceleration ( $\ddot{\psi}$ ) is proportional to  $\delta_r V^2$  and  $\dot{Y} = \psi V$ . Therefore, both the nominal and Mod 1 control laws,

$$\hat{\delta}_r = K_\eta \frac{Y}{R} + K_\psi \frac{\dot{Y}}{V} \quad \text{and} \quad \hat{\delta}_r = K_\eta \left( \frac{V_0}{V} \right) \left( \frac{R}{R_0} \right) \frac{Y}{R} + K_\psi \frac{\dot{Y}}{V}$$

tend to cancel only one power of velocity in the numerator of the overall displacement loop gain. This accounts for the apparent system looseness as velocity decreases.

c) Modification #2

The next control law considered was,

$$\hat{\delta}_r = \left[ K_\eta \left( \frac{V_0}{R_0} \right) Y + K_\psi \dot{Y} \right] \left( \frac{V_0}{V^2} \right)$$

which is simply the previous control law multiplied by a  $(V_0/V)$  factor in an attempt to more accurately compensate the overall Y response for decreasing V. The responses of this system are depicted in Figure 4. They do appear to be somewhat faster than those of Figures 1 and 2 although the  $K_\eta$  and  $K_\psi$  gains may have to be readjusted to improve the damping somewhat. As  $\eta$  could be expected, there has been further improvement in the crosstrack excursion due to a constant crosswind.

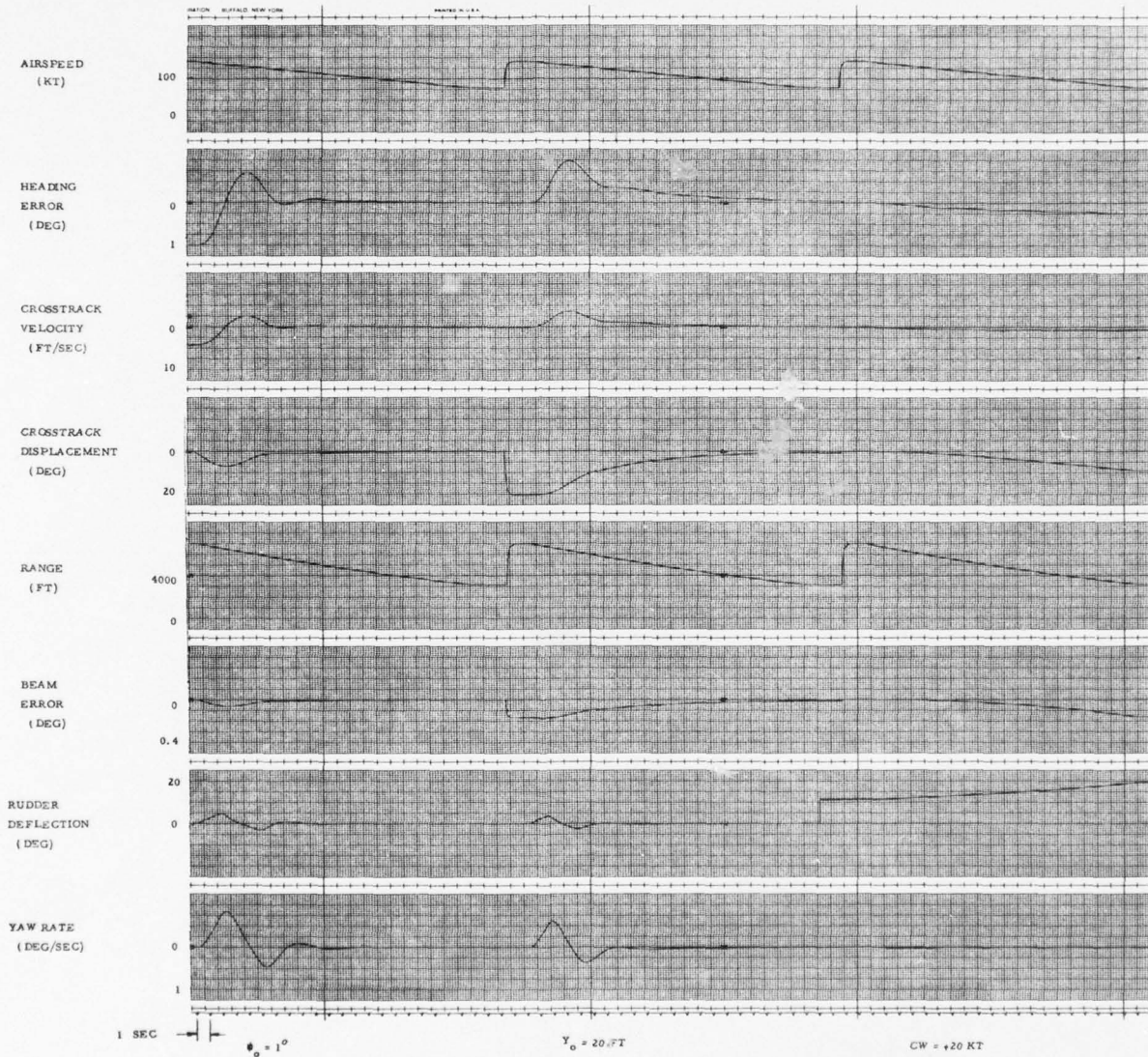


FIGURE 3. MOD 1 ROLL-OUT RESPONSE



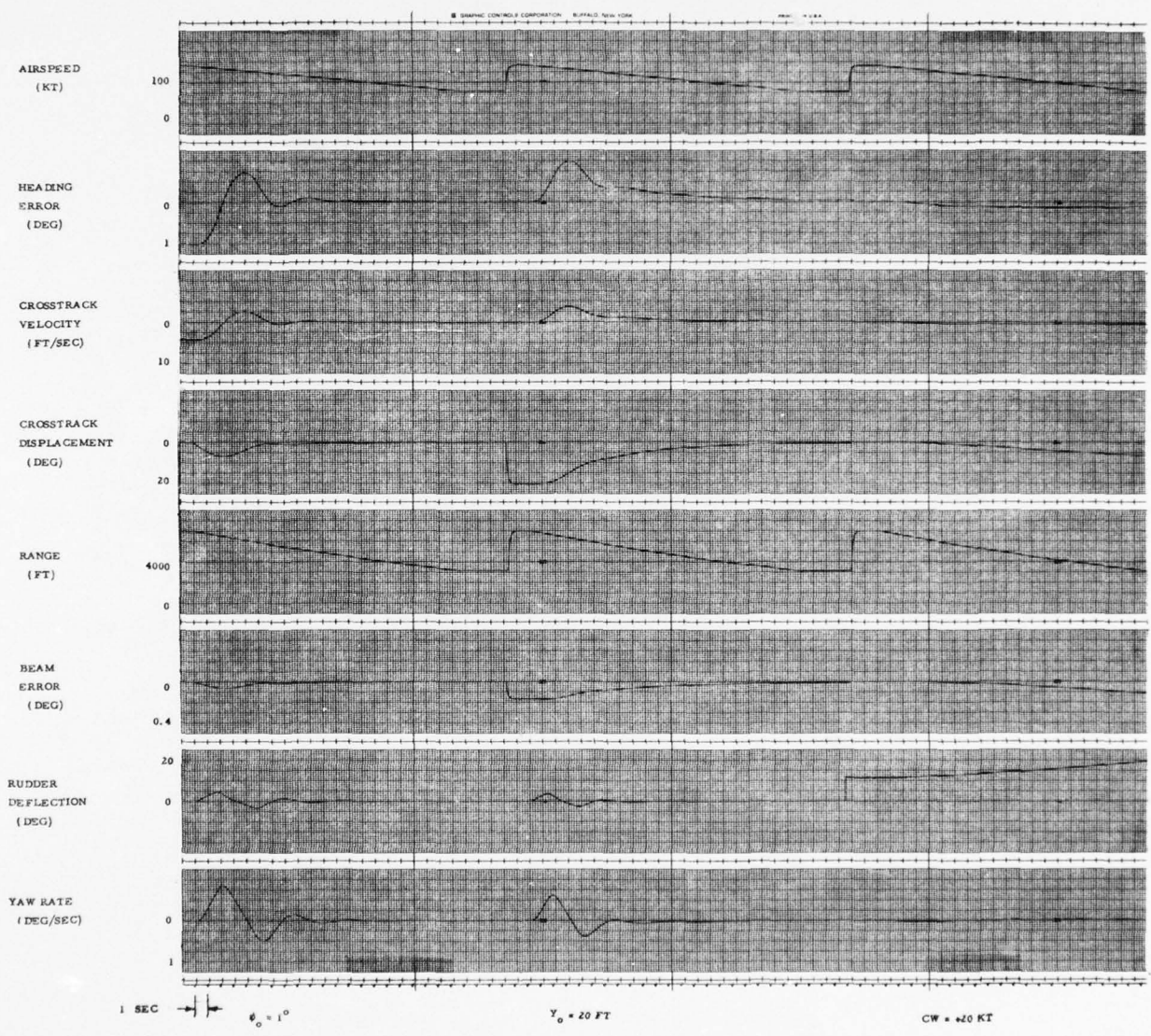


FIGURE 4. MOD 2 ROLL-OUT RESPONSE

d) Recommended Configuration

As an approximation to the Mod 2 configuration, the responses to the control law

$$\hat{\delta}_r + \left[ K_\eta \frac{Y}{R} + K_\psi \frac{\dot{Y}}{V} \right] \left( \frac{V_0}{V} \right) = \left[ K_\eta \eta + K_\psi \psi \right] \left( \frac{V_0}{V} \right)$$

are shown in Figure 5. These responses compare favorably to those of Figure 4 in that they retain system tightness during roll-out with some improvement in damping. This system has an advantage over the previous configuration in its ease of implementation from the present system. It is the result of multiplying the present control law by a  $\frac{V_0}{V}$  factor.

During takeoffs, the problem of increasing beam sensitivity with increasing rudder effectiveness is reduced by the increasing V term in the denominator of the equation.

Figures 6 and 7 illustrate the comparison of the present system with and without the additional groundspeed scheduling discussed above. These traces represent responses to disturbances occurring during roll-out, sometime after touch down. Considerable improvement due to groundspeed scheduling is evident in Figure 7. Specifically, Figure 7 demonstrates a shorter capture time and less sensitivity to crosswind during roll-out. Figure 7B is the better damped, since the  $K_\psi$  gain has been reduced to optimize this configuration.

2. Roll-Out Guidance Subsequent to Nosewheel Engagement

An Automatic Nosewheel Steering configuration is shown in Figure 8. It is applicable to airspeeds below 80 knots where the rudder is essentially ineffective. This configuration functionally corresponds to feeding the present roll-out rudder command signal to the nosewheel actuator with various gain adjustments. This mode would remain in effect down to a taxi groundspeed of roughly 50 knots.

The lateral displacement (Y,  $\dot{Y}$ ) response to nosewheel ( $\delta_n$ ) is a function of the square of velocity (V) since  $\dot{Y} = \psi V$  and  $\psi$  is proportional to  $\delta_n V$ . The control law, using a non-desensitized beam, is,

$$\hat{\delta}_r = K_1 \eta + K_2 \psi = K_1 \frac{Y}{R} + K_2 \frac{\dot{Y}}{V}$$

Therefore, the Y and  $\dot{Y}$  loop gains are proportional to  $\frac{V^2}{R}$  and  $\frac{V^2}{V}$ . Since V and R track each other closely, during Ground Roll between 80 and 50 knots, both terms can be taken as direct functions of V. This function of V yields less than a 40% change in the Y and  $\dot{Y}$  loop gains. This provides a sensitivity to groundspeed that is low enough to provide good nosewheel steering response during landings. Takeoffs, however, would result in an increasing Y loop gain due to an increasing velocity and a decreasing range. This tendency could be partly compensated by a reduction in Y gain whenever the throttle levers



FIGURE 5. MOD 3 (MODIFIED NOMINAL) ROLL-OUT RESPONSE



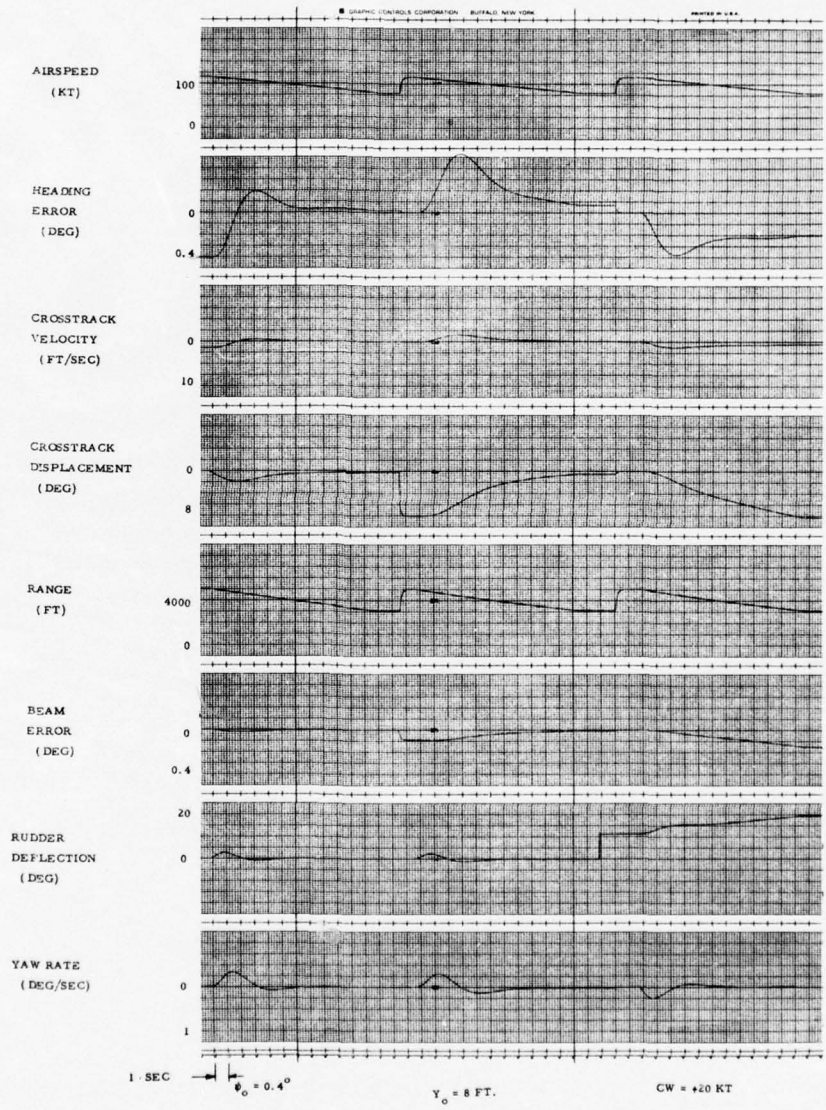


FIGURE 6. NOMINAL CONFIGURATION - CLOSE-IN RESPONSE

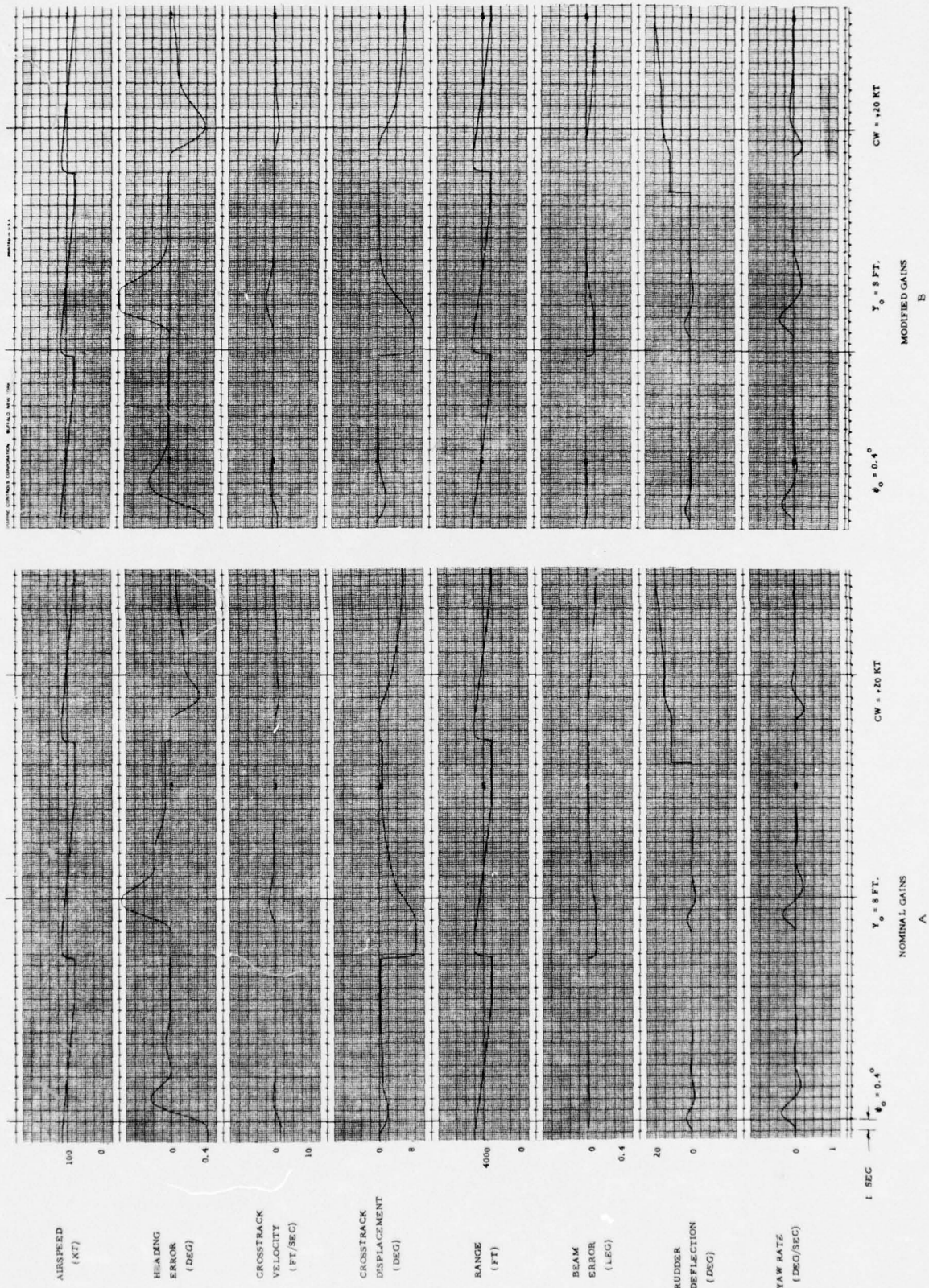


FIGURE 7. MOD 3 (MODIFIED NOMINAL) CONFIGURATION - CLOSE-IN RESPONSE

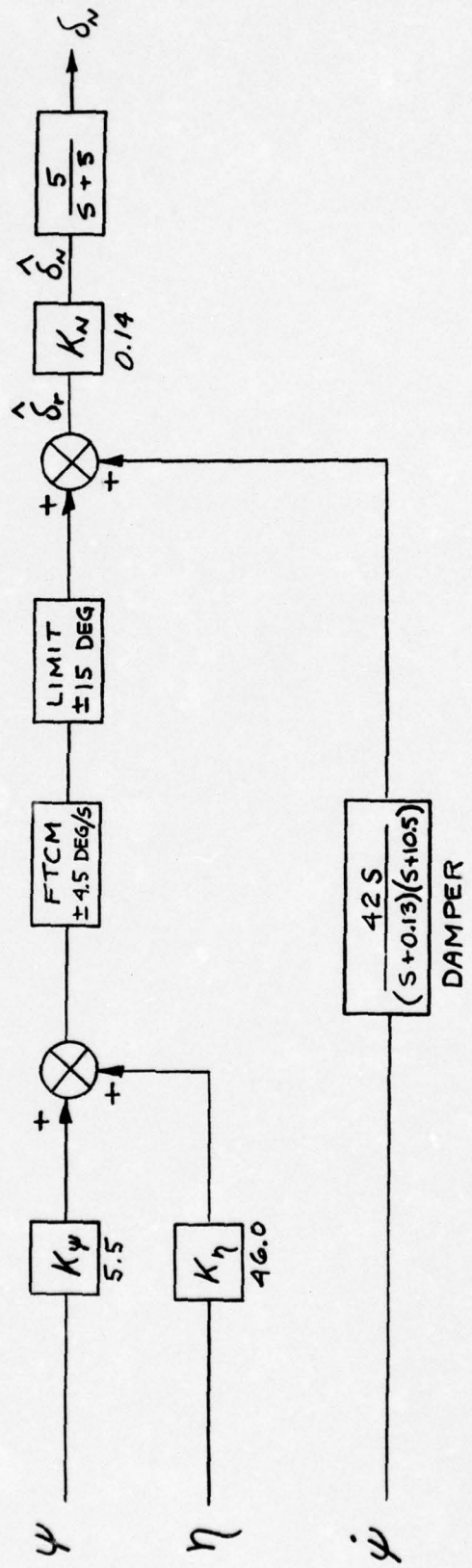


FIGURE 8. AUTOMATIC NOSEWHEEL STEERING CONFIGURATION



are off the aft limit. This scheme, however, would be a compromise approach and could not yield optimum response characteristics over the entire duration of the nosewheel steering mode. A more desirable approach would incorporate range information, if available, to schedule gains more accurately.

Figure 9 depicts responses to an initial  $1^\circ$  heading error and 8 ft. lateral offset occurring at 80 knots. As indicated by the responses, the system has recovered from both disturbances within 800 feet of runway.

The simulation, for the roll-out portion above 80 knots, was based upon a typical touch down range of 7,000 feet at 140 knots, linearly decelerated at 3 knots per second, although performance would be similar for other velocity profiles. The assumptions inherent in the equations are no lateral tire skid, and nose wheel off the runway. Since the castered nose gear does provide stabilizing frictional forces after it is lowered, the analog traces show a worst case, possibly icy runway conditions. Although the effect of nose gear on the runway would alter system response slightly from that illustrated, it would not seriously affect the choice of system gains.

The simulation of the automatic nosewheel steering mode was based upon a continuation of the roll-out conditions set up above. This corresponded to a range of 3,300 feet at 80 knots decelerating to 50 knots at 2,200 feet. Also, no slippage was assumed between nosewheel and runway.

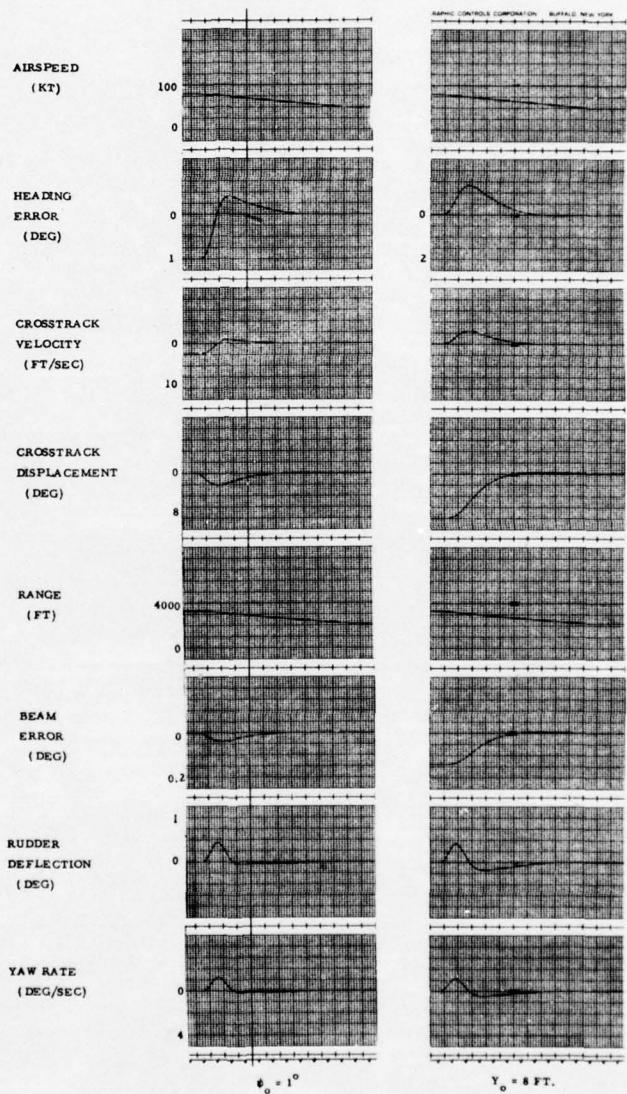


FIGURE 9. AUTOMATIC NOSEWHEEL STEERING RESPONSE

## SECTION 3

### CONCLUSIONS

The roll-out guidance configuration during takeoff, touch and go, and landing provides closed loop control on the runway at airspeeds down to taxi velocities. During crosswind landing the steady-state downwind rudder deflection integrated in during the RA mode is maintained on a short term basis to provide an "operating point" about which heading and beam error signals may be minimized. The wings level command to aileron and runway centerline command to rudder are maintained until disengaged by the pilot or rendered inoperative through force wheel/force link steering or nosewheel steering. The time interval of effective rudder control may vary from 10 to 20 seconds depending on initial airspeed, deceleration profile, pavement conditions, crosswind component, etc. At airspeeds below 80 knots where rudder is no longer effective, control is maintained through automatic nosewheel steering in the recommended configuration.

During takeoff operation, the runway centerline is maintained automatically through nosewheel steering until the indicated airspeed exceeds 80 knots. Engagement of the mode continues until rotate logic is provided by the R/GA computer. At aircraft rotation, the wings level mode is latched until 200 feet altitude while the rudder channel reverts to a yaw damper only mode.

In touch and go operation, the ROG mode is in effect from touchdown through rotation unless disengaged or overridden by the pilot. The wings level latch and synchronization of rudder commands at aircraft rotation is identical to the proposed takeoff mode.

It has been shown that improvements can be realized in the nominal Roll-Out Guidance System accuracy, using groundspeed information. Scheduling gains as a function of groundspeed, to compensate for changes in rudder effectiveness, results in a faster response and tighter crosswind control.

The system with the best control characteristics as well as the simplest implementation is the nominal configuration with its gains adjusted as inverse functions of groundspeed, i. e.,  $K_1 = K_2 \frac{1}{V}$ .

Figure 10 summarizes the crosswind performance of the four systems investigated. The divergences listed are for a 20 knot crosswind which was chosen as a typical example to compare the relative system performance. The divergence is that which develops between the time the aircraft touches down at 140 knots and the time at which nosewheel steering takes over at 80 knots.



SYSTEM CONTROL LAW	DIVERGENCE AT V = 80 KT		
	DEG	μ AMP	FT
$\delta_r = K_\eta \eta + K_\psi \psi$	0.14	10.5	7.6
$\delta_r = K_\eta \left(\frac{U_o}{U}\right) \left(\frac{R}{R_o} \eta\right) + K_\psi \psi$	0.16	12.0	8.7
$\delta_r = K_\eta \left(\frac{U_o}{U}\right)^2 \left(\frac{R}{R_o} \eta\right) + K_\psi \left(\frac{V_o}{V}\right) \psi$	0.11	8.2	6.0
$\delta_r = \left(K_\eta \eta + K_\psi \psi\right) \left(\frac{V_o}{V}\right)$	0.10	7.5	5.5

CW = 20 KT

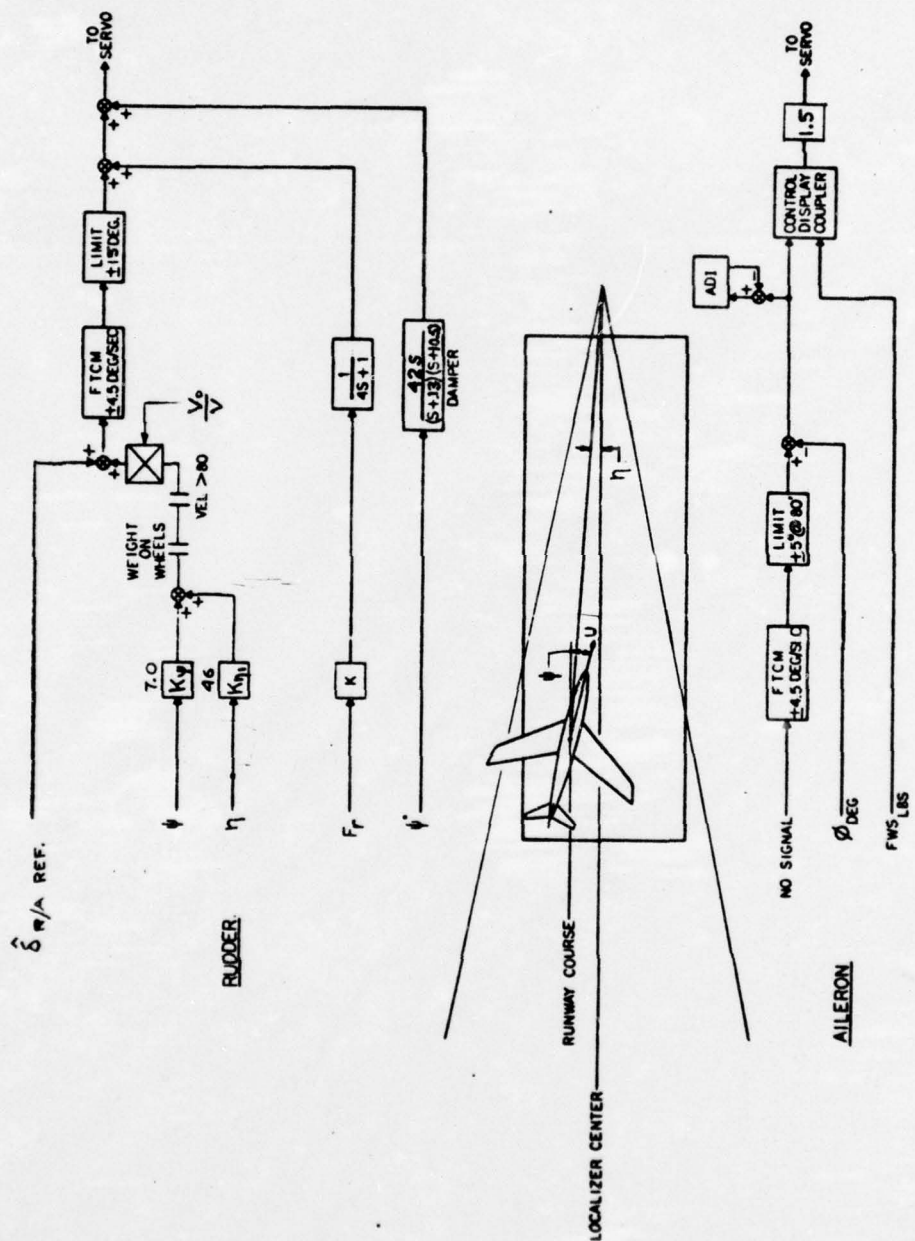
ROLL-OUT CROSSWIND DIVERGENCE  
FIGURE 10

Since no lateral tire slippage is considered to exist, any further divergence, due to crosswinds after nosewheel engagement, can be neglected.

The latter system, using,

$$\hat{\delta}_r = \left[ K_\eta \eta + K_\psi \psi \right] \frac{V_0}{V}$$

is recommended herein and it is illustrated in Figure 11.



PROPOSED ROLL OUT GUIDANCE PROFILE  
FIGURE 11



## APPENDIX I

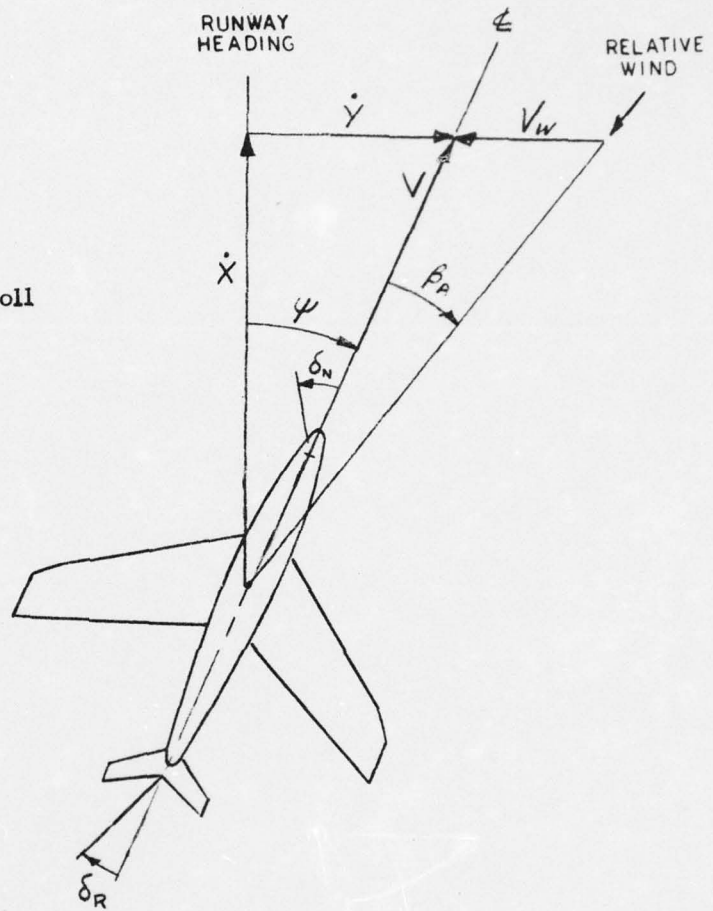
### DEVELOPMENT OF SIMULATION

#### 1. Roll-Out Guidance Prior to Nosewheel Engagement

The following steps were taken to adapt the 3 degrees of freedom perturbation stability axis equations to a valid form for simulation of the landing ground roll aerodynamics.

- a) Decouple and neglect the rolling degree of freedom.
- b) Introduce a frictional force,  $f$ , in the yawing moment and side force equations which represents the force exerted by the runway surface on the collective main gear, perpendicular to the aircraft reference line. Since the yawing moment equation is written about the aircraft c.g., the frictional term appears with a moment arm,  $d$ , equal to the distance between the nominal c.g. location and effective main gear center, approximately 5 feet for the aircraft in question. The effect of frictional forces on the nose gear has been neglected.
- c) Combine the yawing moment and side force equations to eliminate the frictional force.
- d) To allow for a variable velocity profile, convert the equation with dimensional (velocity dependent) coefficients to one containing non-dimensional (velocity independent) coefficients.
- e) Generate a velocity profile that is typical of the decelerating aircraft.

$\beta_I = 0$  on ground roll



<u>PARAMETER</u>	<u>SYMBOL</u>	<u>UNITS</u>	<u>DEFINED POSITIVE</u>
Nosewheel Deflection	$\delta_N$	Degrees	Turn Left
Rudder Deflection	$\delta_R$	Degrees	Left Rudder
Beam Error	$\eta$	Degrees	Plane Right of Beam
Heading Error	$\psi$	Degrees	Nose Right of Runway
Sideslip	$\beta_A$	Degrees	Relative Wind from Right
Lateral Displacement	$Y$	Feet	C.G. Right of Runway
Crosstrack Velocity	$\dot{Y}$	Ft./Sec.	C.G. Moving Right of Runway
Ground Speed	$V$	Ft./Sec.	Forward
Cross Wind Velocity	$V_w$	Ft./Sec.	Wind from Right of Runway

FIGURE 12. LATERAL AIRCRAFT GEOMETRY

The side force equation is:

$$(1) \quad \text{ft. - rad./sec.}^2 = V (\dot{\psi} + \dot{\beta}) + Y_r \dot{\psi} + Y_{\delta_R} \delta_R + Y_v v + f/m$$

where  $Y_r = \text{ft./sec.}$ ,  $Y_{\delta_R} = \text{ft./sec.}^2$ ,  $Y_v = \text{rad/sec.}$ ,  $f = \text{lb.}$   
 $m$  is aircraft mass in slugs,  $V$  is true airspeed in feet per second, and angles are expressed in radians.

Multiplying arguments by 57.3 and dividing through by  $V \text{ ft./sec.}$  gives:

$$(2) \quad \text{deg./sec.} = \dot{\psi} + \dot{\beta} = Y_r^* \dot{\psi} + Y_{\delta_R}^* \delta_R + Y_\beta \beta + 57.3 f/m V$$

where  $Y_r^* = \text{deg./sec.}$ ,  $Y_{\delta_R}^* = 1/\text{sec.}$ ,  $Y_\beta = Y_v = 1/\text{sec.}$ , and all angles are expressed in degrees.

Rearranging gives:

$$(3) \quad 57.3 f = mV \left[ \dot{\psi} + \dot{\beta} - Y_r^* \dot{\psi} - Y_\beta \beta - Y_{\beta_R}^* \delta_R \right]$$

The yawing moment equation is:

$$(4) \quad \text{rad./sec.}^2 = \ddot{\psi} = N_r \dot{\psi} + N_{\delta_R} \delta_R + N_\beta \beta - \frac{df}{I_z}$$

where  $N_r = 1/\text{sec.}$ ,  $N_{\delta_R} = 1/\text{sec.}^2$ ,  $N_\beta = 1/\text{sec.}^2$ ,  
 $I_z = \text{slug-ft.}^2$ ,  $d = \text{ft.}$ , and angles are expressed in radians.

Multiplying arguments by 57.3 results in  $\text{deg./sec.}^2 =$

$$\ddot{\psi} = N_r \dot{\psi} + N_{\delta_R} \delta_R + N_\beta \beta - 57.3 \frac{df}{I_z}$$

where angles are expressed in degrees.

Or, rearranging,

$$(5) \quad 57.3 f = \frac{I_z}{d} \left[ -\ddot{\psi} - N_r \dot{\psi} + N_\beta \beta + N_{\delta_R} \delta_R \right]$$



Combining (3) and (5) and rearranging yields

$$(6) \quad \ddot{\psi} = \dot{\psi} \left[ N_r + \frac{mVd}{I_z} (Y_r^* - 1) \right] - \frac{mVd}{I_z} \dot{\beta} + \beta \left[ N_\beta + \frac{mVd}{I_z} Y_\beta \right] + \delta_R \left[ N_{\delta_R} + \frac{mVd}{I_z} Y_{\delta_R}^* \right]$$

Converting (6) to non-dimensional coefficients,

$$(7) \quad \ddot{\psi} = \dot{\psi} \left[ \frac{\rho s V b^2}{4 I_z} C_{n_r} + \frac{mVd}{I_z} \left( \frac{\rho s b}{4m} C_{y_r} - 1 \right) \right] - \frac{mVd}{I_z} \dot{\beta} + \beta \left[ \frac{\rho s V^2 b}{2 I_z} C_{n_\beta} + \frac{mVd}{I_z} \frac{\rho s}{2m} C_{y_\beta} \right] + \delta_R \left[ \frac{\rho s V^2 b}{2 I_z} C_{n_{\delta_R}} + \frac{mVd}{I_z} \frac{\rho s V}{2m} C_{y_{\delta_R}} \right]$$

where the parameters  $C_{n_r}$ ,  $C_{y_r}$ ,  $C_{n_\beta}$ ,  $C_{y_\beta}$ ,  $C_{n_{\delta_R}}$ ,  $C_{y_{\delta_R}}$  are all dimensionless. The following values were used for this flight condition

$$\begin{aligned} C_{n_r} &= -.1599 & C_{y_r} &= .4103 & C_{n_\beta} &= .1347 \\ C_{y_\beta} &= -.7449 & C_{n_{\delta_R}} &= -.0859 & C_{y_{\delta_R}} &= .2292 \\ m &= 4968 \text{ slugs} & b &= 130.8 \text{ ft.} & \rho s &= 5.786 \text{ slugs/ft.} \end{aligned}$$

Substituting these values into (7), neglecting the  $\dot{\beta}$  term and scaling for the analog computer results in,

$$(8) \quad 10 \ddot{\psi} = -6.186 \frac{V\dot{\psi}}{100} + .5017 \frac{VV_w}{100} - .6357 \frac{V^2 \delta_R}{10^4}$$

where  $V_w = V\beta$  is airflow at right angles to the aircraft reference line.

For the small crab angles encountered during ground roll,  $V_w$  can be introduced in the simulation as a constant valued wind across the runway, hence, the reason for neglecting the  $\beta$  term. The airspeed term,  $V$ , was simulated as a constant deceleration of 5 ft./sec.<sup>2</sup> from an initial value of 140 knots or 234 feet per second. No differentiation has been made in this simulation between airspeed and groundspeed since motion of the aircraft can be assumed to be orthogonal to crosswinds at all times.

Note that the weathercock derivative,  $C_{n\beta}$  is a measure of the aircraft's tendency to turn upwind if sufficient downwind rudder deflection is not supplied to maintain the runway centerline. To determine the steady-state weathercock rate with zero rudder deflection, set  $\ddot{\psi} = \dot{\psi} = 0$  in equation (8) to obtain  $\dot{\psi}/V_w = .081$ ; (i. e.) the aircraft will yaw at a rate of .14 degrees per second per knot of crosswind assuming no nose-wheel resistance or lateral tire skid.

To determine rudder required to maintain the runway centerline, set  $\ddot{\psi} = \dot{\psi} = 0$  to obtain  $\delta_R = 50.17 V_w / .6357 V$ , or for the initial 234 ft./sec. airspeed,  $\delta_R / V_w = .3346$ ; (i. e.) approximately 14 degrees of rudder is required for a 25 knot crosswind.

Because of the assumptions involving frictional forces and the omission of aileron and nosewheel effectiveness, the equations outlined above are not expected to define the exact short term aircraft dynamics on the ground roll. However, the simulation was accurate enough to aid in the determination of the course and beam error system gains required to hold runway center.

## 2. Roll-Out Subsequent to Nosewheel Engagement

After nosewheel engage, the aircraft motion can be described by the following equation:

$$\dot{\psi} = - \frac{V \tan \delta_n}{d_n} \cong - \frac{V \delta_n}{d_n}$$

where:  $d_n$  = distance from nosewheel to main landing gear

$V$  = groundspeed

$\delta_n$  = nosewheel deflection, positive left

## APPENDIX II

### LIST OF SYMBOLS, ABBREVIATIONS

$\delta_a$	Aileron deflection, degrees
$\delta_R$	Rudder deflection, degrees
$\delta T$	Throttle position, degrees
$\eta$	Localizer beam error
$\phi$	Roll angle, degrees
$\phi_c$	Roll command, degrees
$\psi$	Runway heading error (PSC), degrees
$\beta_A$	Sideslip angle, degrees
$\beta_I$	Drift angle, degrees
$\psi + \beta_I$	Cross track angle, degrees
$\omega$	Frequency, radians per second
$\zeta$	Damping factor
$\tau$	Time constant, seconds
$\mu a$	Microamperes
ADI	Attitude director indicator
AFCS	Automatic flight control system
ALEC	Advanced lateral experimental computer
AP, A/P	Autopilot, autopilot engaged
CA	Coupling armed
DA	Drift angle
DEG	Degrees
DISC	Disconnect
ENG	Engage



LIST OF SYMBOLS, ABBREVIATIONS

(Continued)

$F_R$	Rudder force, pounds
$F_W$	Roll wheel force, pounds
FD, F/D	Flight director
FTCM	Full time command modifier
GA	Go-around
GS	Glide Slope
h	Altitude, feet
HDG	Heading mode
ILS	Instrument landing system
INU	Inertial navigation unit
K	Gain or relay designation
Kt	Knot, nautical mile per hour
LOC	Localizer
PB-20D	Autopilot designation
PSC	Preset course
R/GA	Rotate/Go-around
RA	Runway alignment prior to touchdown
RFDC	Roll flight director coupled
ROG	Roll out guidance
ROT	Rotate
S	Laplace variable
ST	Strut logic
SYNC	Synchronize

LIST OF SYMBOLS AND ABBREVIATIONS

(Continued)

TG	Touch and go
TO	Takeoff
TRK	Track
U	Indicated airspeed, knots
$V_A$	Axial velocity of main gear
$V_G$	Total ground speed
$V_w$	Total wind velocity with respect to ground
Y	Distance from aircraft c.g. to runway centerline, feet
$\dot{Y}$	Cross track velocity, feet per second
WL	Wings level
W/O	Washout







