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DETERMINATION OF THE QUALITY OF NAVY RECRUITS

BY RICHARD S. TOIKKA

TECHNICAL REPORT

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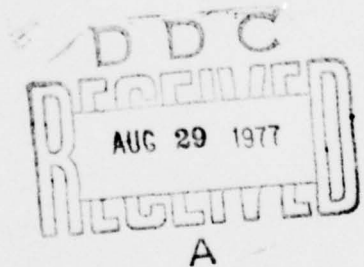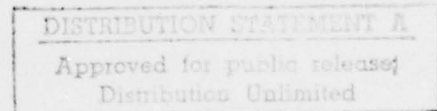
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In this technical report, a model of the Navy recruitment process is developed and preliminary tests are conducted using historical data on Navy recruits by mental test score. The model is based on a theory of selection by recruiters in which recruit quality is maximized subject to a quota filling constraint. Decision rules for recruiters are derived and then these rules are applied to see how recruitment standards would be expected to change in response to changes in the Navy's demand for accessions and the supply of Navy recruits. The model is empirically tested using data on civilian and military pay, unemployment draft pressure, and Navy enlistments by test score group. No statistically significant results emerge from the data analysis but some areas for further investigations are identified.

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I.   INTRODUCTION

The purpose of this paper is to analyze the determinants of the
quality of Navy recruits. The analysis considers the factors which affect
the supply of potential recruits to the Navy and also the process by which
recruitment standards are set. The outcome of the analysis is a behavioral
model which identifies those factors which are important in determining the
distribution of Navy recruits along a quality spectrum. This model is then
tested using time series data on the distribution of Navy recruits by score
on the Armed Forces Qualifying Test.[1]

The results of this research will be useful to the Navy in two ways.
First, to the extent that policy variables such as military compensation
and force level affect the quality of recruits, quantitative estimates of
those effects can be used to assess the consequences of various policy
strategies for recruit quality. Since these policy decisions interact with
the economic environment, particularly conditions in civilian labor markets,
the effects of policy in different economic environments can be explored.
Second, the results of this study may be used to forecast recruit quality
under various assumptions about force level, compensation, population, and
conditions in civilian labor markets. These forecasts can then be translated
into occupational assignment patterns for use in assessing the implications
of changing recruit quality for productivity and attrition in given military
assignments.

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1. This test has been recently eliminated in favor of the Armed
Services Vocational Aptitude Test.

II.   THE NAVY RECRUITMENT PROCESS

The Navy's recruitment system operates through the establishment of
quotas with both qualitative and quantitative dimensions. The quotas are
set by geographic area based on estimates of the numbers of qualified military
available (QMA) by area. The basic data input to this process is population
data from the 1970 Census. The Census data is then augmented with internal
Navy data on the proportion of the youth population which can meet Navy
recruitment standards.

The aggregate quotas are based on an assessment of the Navy's accession
requirements. New accessions may be required because of retirements and
other attrition as well as growth in total manpower requirements. Estimates
of total force requirements are made on the basis of staffing requirements
of Navy hardware. Then, attrition estimates are made using data on distribution
of current force level by length of service and pay grade. Based on the force
requirement and attrition estimates, an aggregate recruiting quota is derived.
This aggregate quota then is apportioned by area based on estimates of QMA
for each area.

With these recruitment quotas set and Navy compensation fixed in the
short-run, the recruiter's decision problem is to meet the assigned quota
subject to recruitment standards. Recruitment standards have historically
been defined in dimensions of test score and educational attainment. There
is evidence that military recruitment standards do respond to changes in
the supply of recruits. The report of the Military Manpower Commission
states:

"The Services change their respective enlistment standards to insure that only applicants of the highest caliber are accepted for enlistment. Enlistment standards are raised when there is an abundance of applicants and lowered when the supply falls short of expectations."[1]

Recruitment standards respond to changes in relative supply of applicants only when the supply of applicants exceeds the quota. If the quota is greater than the available supply then a shortfall is inevitable. When the draft was in effect, the presence of draft motivated enlistments allowed the services to be selective in picking among the supply of recruits. In the all volunteer force (AVF) period this selectivity has still been possible to some degree because of an increased recruiting effort by the services, an increase in military pay, and the lack of civilian job opportunities. A study of the quality of Air Force recruits (Cook and White, 1970) revealed that the average score on the Armed Forces Qualifying Test (AFQT) was sensitive to the relative supply of enlistees. This study presented quantitative evidence on the importance of short-run adjustments in recruitment standards on recruit quality. They estimate that a one percent change in the relative military-civilian pay ratio is associated with a .5 percent change in average AFQT score.

It is reasonable to believe that recruiting standards change in response to supply pressures. If recruiting quotas are viewed as requirements to be met in all but exceptional circumstances, recruiters have limited options to avoid a shortfall in circumstances of supply shortages. The following options are available and apparently in use. First, the Navy advertises

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1. Defense Manpower Commission, Report to the President and the Congress, April 1976, p. 200.

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and promotes its career opportunities to increase the supply of qualified recruits. Second, recruiters offer educational and assignment opportunities to prospective recruits as a means of enticing them to sign up. Both of these strategies increase the supply of applicants. However, there are budget and resource constraints on the utilization of these measures. Advertising budgets are fixed in the short-run and the availability of assignments and educational opportunities will be limited both by the Navy's total resources and the commitments already made to other recruits and Navy personnel. If quotas are not met through enhancing supply, the most available recourse is to relax recruitment standards.

The incentive to relax recruiting standards operates in reverse when quotas stand to be exceeded. Opportunities then exist for saving resources on advertising and promotion, and for increasing the quality of recruits by taking applicants with more education and higher test scores.

In the following section, a formal model of the recruiter's decision problem which reflects the considerations suggested here is developed. This model is then used to derive expressions for the number of applicants accepted in each AFQT score category.

III. A MODEL OF THE RECRUITMENT PROCESS

The recruiter's decision problem can be represented as maximizing the quality of recruits subject to a quota filling requirement. Quality is not directly observed but is derived as the outcome of an evaluative process in which the recruiter combines available information on test scores, education, background, personality, etc. Additional inputs into this process are the recruiter's own judgement and experience. The perceived quality of a potential recruit (q) may be functionally related to a series of determinants. Denote those determinants by a vector $\underline{x} = (x_1, x_2, \ldots, x_n)$. The quality relation for an applicant may be written as:

$$q = q(\underline{x}) \tag{1}$$

The distribution of quality in the recruit population is derived from the joint distribution of the quality determinants $(\underline{x})$. Given that joint distribution $q(x_1, \ldots, x_n)$, and the quality relation $q(\underline{x})$, the marginal distribution of quality in the recruit population may be derived. This quality distribution will be denoted by $f(q)$.

If recruiters maximize recruit quality subject to a quota filling constraint, the decision problem may be represented formally as:

$$\underset{q_o}{\text{Max}} \quad E(q|q > q_o)$$

subject to $[E(R) - \bar{R} = 0]$

where E is the expectations operator, R is the number of recruits, $\bar{R}$ is the quota, and q_o is the lowest acceptable level of recruit quality. The expected level of quality given an acceptance level q_o may be shown to be monotonically increasing in q_o (see the Appendix for a proof). The optimal strategy is to set q_o as high as possible subject to the quota filling constraint. The expected number of recruits E(R) is

$$E(R) = \left[\int_{q_o}^{\infty} f(q)\,dq \right] S \qquad (3)$$

Since $\dfrac{\partial E(R)}{\partial q_o} = -f(q_o)S < 0$, the constraint $[E(R)-\bar{R}]$ is always binding. Substituting (3) into the constraint and dividing through by S indicates that the optimal value of q_o is a function of the ratio $\bar{R}/S$:

$$\frac{\bar{R}}{S} - \int_{q_o}^{\infty} f(q)\,dq = 0 \qquad (4)$$

Solving (4) for q_o gives the optimal value q_o^* :

$$q_o^* = q_o^* (\bar{R}/S) \qquad (5)$$

and the optimal decision rule is

$$\left| \begin{array}{l} \text{accept i if } q_i \geq q_o^* \\[2mm] \text{reject i, otherwise} \end{array} \right. \qquad (6)$$

It can be seen from equation (5) that recruitment standards adjust in this model in response to changes in the ratio of the quota mandated accessions $\bar{R}$ to the total supply of potential recruits S. Differentiating (4) gives the partial derivative

$$\frac{\partial q_o}{\partial (R^*/S)} = - \frac{1}{f(q_o)} < 0 \qquad (7)$$

The negative sign of the partial derivative indicates that a statically optimal strategy is to set recruitment standards high when the quota is large relative to the available supply of recruits and to set it low when the quota is small relative to the supply.

The consequences of this behavior for recruit quality can be assessed directly by observing the effect of truncating the distribution of recruits by quality. The truncated distribution becomes

$$f(q|q>q_o) = \left| \begin{array}{l} \dfrac{f(q)}{\displaystyle\int_{q_o} f(q)\,dq} \text{ for } q>q_o \\[4mm] \qquad 0 \quad \text{otherwise} \end{array} \right. \qquad (8)$$

The moments of the distribution may be derived by integration. One result of particular interest is how changes in the recruiting quota and the supply of recruits affect the average quality of recruits. The expected value of the truncated quality distribution is given by

$$
E(q \mid q > q_o) = \int_{q_o(\bar{R}/S)}^{\infty} q \; f(q \mid q > q_o) dq = \frac{\int_{q_o(\bar{R}/S)}^{\infty} q \; f(q) dq}{\int_{q_o(\bar{R}/S)}^{\infty} f(q) dq} \tag{9}
$$

The partial derivative of expected quality with respect to the quota-supply ratio $\bar{R}/S$ is

$$
\frac{\partial E(q \mid q > q_o)}{\partial (\bar{R}/S)} = \frac{\partial E(q \mid q > q_o)}{\partial q_o} \; \frac{\partial q_o}{\partial (\bar{R}/S)} < 0 \tag{10}
$$

The effect of the quota-supply ratio on average quality is the product of two partial effects: first, the effect of the quota-supply ratio on the recruitment standard q_o which has been shown to be negative and the effect of the recruitment standard on average quality which is shown in the appendix to be positive. Thus, the quota-supply ratio is inversely related to average recruit quality under the quality maximization decision rule examined in this paper.

Quality in the sense defined by equation (1) is not likely to be observed since it is the result of a judgemental process. If the arguments of the quality function are known, however, tests of the validity of this theory may be carried out using data on those variables. Changes in recruitment standards will be reflected in the distribution of recruits by characteristics in the quality function. For example, if test score is

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positively correlated with perceived quality then a lowering of the recruiting standard would be expected to result in an increase in the relative number of recruits with low test scores and an increase in the relative number with high test scores.

The expected number of recruits with a particular set of characteristics represented by vector $\underline{x}^*$, where $\underline{x}^*$ contains a subset of the elements in vector $\underline{x}$, can be expressed by a conditional recruitment function defined as

$$E(R|\underline{x}^*) = s(\underline{x}^*) \int_{q_0}^{\infty} g(q|\underline{x}^*)dq \qquad (11)$$

where $s(\underline{x}^*)$ is the supply of recruits with characteristics $\underline{x}^*$ and $g(q|\underline{x}^*)$ is the conditional distribution of q in the population of recruits with characteristics $\underline{x}^*$. Total expected applicants (S) is given by

$$S = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} s(\underline{x}^*)f(\underline{x}^*)d\underline{x}^* \qquad (12)$$

By differentiating (11) the effects of the quota $\bar{R}$ and the supply of enlistees S on the conditional mean functions can be determined. The derivative with respect to the quota $\bar{R}$ is

$$\frac{dE(R|\underline{x}^*)}{d\bar{R}} = \frac{\partial E(R|\underline{x}^*)}{\partial q_0} \frac{\partial q_0}{\partial \bar{R}}$$

$$= \frac{f(q_0|\underline{x}^*)s(\underline{x}^*)}{f(q_0) S} > 0 \qquad (13)$$

The positive sign of the partial derivative indicates that the expected number of recruits is positively correlated with the size of the quota. This positive effect holds for all values of $\underline{x}^*$ (all quality levels). When the quota increases (decreases) recruitment standards fall (rise), thus causing the number of recruits in all quality categories to increase (decrease).

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The effect of recruitment supply on the expected number of recruits with a particular set of characteristics $\underline{x}^*$ can be seen in the derivative

$$\frac{dE(R|\underline{x}^*)}{dS} = \frac{\partial E(R|\underline{x}^*)}{\partial q_o} \frac{\partial q_o}{\partial S} + \frac{\partial E(R|\underline{x}^*)}{\partial S}$$

$$= - \frac{s(\underline{x}^*)}{S} \frac{f(q_o|\underline{x}^*)}{f(q_o)} \int_{q_o}^{\infty} f(q)dq \qquad (14)$$

$$\frac{\partial s(\underline{x}^*)}{\partial S} \int_{q_o}^{\infty} f(q|\underline{x}^*)dq$$

This derivative cannot be signed. The first term on the RHS is negative and the second term will generally be positive (if $\frac{\partial s(\underline{x}^*)}{\partial S} > 0$). A moment's reflection gives us the reason for the indeterminacy. As aggregate supply increases (decreases), recruitment standards rise (fall). The changes in recruitment standards have different impacts on the flow of recruits at different points on the quality spectrum. For example, lowering recruitment standards will tend to increase the number of low quality recruits and decrease the number of high quality recruits. With the quota fixed, the effect of changes in recruitment standards is to redistribute recruits along the quality spectrum. Thus, the sign of the expression on the RHS of (14) will be positive where the flow of relatively high quality recruits increases in response to raised recruitment standards, and negative where the flow of relatively low quality recruits decreases in response to raised recruitment standards.

Under certain conditions which are derived in the appendix, the marginal impact of supply of the expected number of recruits at any combination of characteristics $\underline{x}^*$ increases smoothly, being negative and large at values of $\underline{x}^*$ corresponding to very low quality and becoming positive and large at values of $\underline{x}^*$ corresponding to very high quality. These conditions are likely to be met if $\underline{x}^*$ is strongly correlated with the probability of acceptance into the service.

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IV. APPLICATION

The above analysis may be applied to data on the number of recruits by AFQT test score category. We may think of $\underline{x}^*$ as indexing test score category so that $x^* = 1, \ldots, 3$ for test score categories I-II, III and IV.[1]

The hypotheses suggested by the theory developed above are that the impacts of the aggregate number of applicants (S) and recruiting quota ($\bar{R}$) on the number of recruits differ by test score level. If the number of recruits in categories I-II,[2] III and IV are denoted by r_1, r_2, and r_3, respectively, the impact of changes in the aggregate quota $\bar{R}$ is expected to be positive at all quality levels. That is, we expect that

$$\frac{\partial r_i}{\partial \bar{R}} > 0 \qquad\qquad i = 1, \ldots, 3$$

The impact of aggregate recruit supply is more ambiguous, but if the conditions for the marginal impact increasing monotonically with test score are met (i.e., if test score is correlated strongly enough with the probability of acceptance into the Navy), then we expect

$$\frac{\partial r_1}{\partial S} < \frac{\partial r_2}{\partial S} < \frac{\partial r_3}{\partial S}$$

In the previous section of this paper it was established that the expected number of recruits at any test score level $i(r_i)$ is related to the supply of recruits at that level s_i and the probability of being accepted into the service according to:

$$r_i = s_i \int_{q_o (\bar{R}|S)}^{\infty} f(q|x^* = i)dq = s_i P_i \qquad (15)$$

where P_i is the probability of an applicant in test score category i being accepted into the Navy.

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1. There are five test score groups based on percentiles. These percentiles are given in Table 1.

2. These two groups are combined to increase the sample size.

TABLE 1. TEST SCORE CATEGORY AND PERCENTILE RANGE

AFQT Category	Percentile
I	93-100
II	65-92
III	31-64
IV	10-30
V	0-9

Data taken from Armed Forces Qualitative
Distribution Report of Male Enlistments,
Inductions, and Rejections, January 1973.

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The probability of acceptance may be approximated by an nth order polynomial in logarithms of the quota-aggregate supply ratio of the form:

$$P_i = e^{\sum_{j=1}^{n} \beta_{ij} \ln (\bar{R}/S)^j} \tag{16}$$

since when $\bar{R} = S$, $P_i = 1$ for all i. Further, if the distribution of the supply of recruits by test score level is constant, it follows that the supply at any level is a constant fraction of aggregate supply so that

$$\frac{s_i}{S} = e^{\lambda_i} \tag{17}$$

Substituting (16) and (17) into (15) and taking logarithms gives the resulting expression for recruits at the ith test score level

$$\ln r_i = \lambda_i + \ln S + \sum_{j=1}^{n} \beta_{ij} (\ln \bar{R} - \ln S)^j \tag{18}$$

$$i = 1, \ldots, 3$$

If time series data on total recruit supply (S), quotas ($\bar{R}$), and recruits by test score were available, the system of equations (18) could be estimated directly. Unfortunately, complete data are not available on persons rejected by the Navy, so that total recruit supply is not measured.[1] However, enough prior research has been done on the determinants of the supply of recruits so that a structural equation for total supply can be borrowed from the literature. In previous studies of enlistment supply hypothesized determinants have included military and civilian pay, unemployment, draft pressure, and other factors. For a summary of previous studies see Amey, Fechter, Grissmer, and Sica (1976), Chapter 2 and for a study of the sensitivity of estimates to model characteristics see Fechter (1976). Aggregate enlistment supply will be approximated as an exponential function of a vector of explanatory variables $(x_1, \ldots, x_m)$. That is

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1. Data are available from the AFEES Qualitative Distribution Reports although the rejection can be for both mental and physical reasons. Further analysis of these data is planned.

$$S = e^{\alpha_0 + \sum_{i=1}^{m} \alpha_i x_i} \qquad (19)$$

The logarithm of aggregate enlistment supply is then linear in the x variables.
That is

$$\ln S = \alpha_0 + \sum_{i=1}^{m} \alpha_i x_i \qquad (20)$$

The list of explanatory variables used in the analysis is given in Table 2.

Substitution of (20) into (18) gives a system of estimable recruit
functions. However, because of the higher order terms in S, the number of
explanatory variables increases rapidly with the number of terms in the poly-
nomial. The large number of terms makes direct estimation impractical. How-
ever, there is an iterative procedure which simplifies the estimation. The
estimation procedure will now be described.

An initial estimate of aggregate recruit supply is obtained from a
regression of $\ln r_1$ on the variables in the recruit supply equation. Under
the assumption that recruits in categories I and II are supply limited (i.e.,
all applicants are accepted) the coefficients in (16) are zero for that group.
Thus, the regression

$$\ln r_1 = \lambda_1 + \ln S$$

$$= \pi_{01} + \sum_{i=1}^{m} \alpha_i x_i \qquad (21)$$

where $\pi_{01} = \lambda_1 + \alpha_0$

may be used to estimate the slope coefficients α_i, i=1, ..., m directly.
This approach has been used by other researchers in identifying supply func-
tions for "high quality" recruits. Our approach differs from theirs in that
we assume that the coefficients in (16) are approximately zero for our highest
recruit qualtiy category and use this regression to obtain initial estimates

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TABLE 2. EXPLANATORY VARIABLES IN THE RECRUIT SUPPLY FUNCTION[*]

W^c	Civilian Pay (in logarithms)
W^m	Military Pay (in logarithms)
E	Employment Rate for Males, Aged 20-24 (in logarithms)
DP	Draft Pressure Variable (defined as the logarithm of $(1 - I/P)$, where I/P is the induction rate[**])
VN	Vietnam Dummy (equals 1, for 196503-197204; Zero, otherwise)
BERLIN	Berlin Crisis Dummy (equals 1, for 196104-196201; Zero, otherwise)
KMAR	Marriage Deferment Dummy (equals 1, for 196303-196503; Zero, otherwise)
S1 S2 S3	Seasonal Dummies (equaling 1, for the 1st, 2nd, and 3rd quarter, respectively; Zero, otherwise)

[*] The variables are described in more detail in the data appendix.

[**] Inductions divided by male population aged 17-20.

of the recruit supply function. In order to identify the constant term in the supply function (α_o) from the estimate of the constant (π_{o1}) in (21), a value for λ_1 is required. Since the rejection rate for categories I and II is expected to be lower than that for all recruits, we expect that

$$\frac{s_1}{S} < \frac{r_1}{R} \tag{22}$$

that is, category I-II's share of total recruit supply (applicants) will be less than its share of total recruits. Thus, an upper bound estimate for s_1/S is r_1/R, the category is share of total recruits. The average value for r_1/R over the period 1958-1972 was .34. To obtain an initial estimate of s_1/S, the value .32 was used. This implies $\lambda_1 = -1.12$. The first round regression estimates with the implied values for the parameters of the aggregate recruit supply function are given in Table 3.

The estimated parameters of the aggregate recruit supply function were then applied to the time series observations on the explanatory variables to create a time series of estimated aggregate recruit supply (in logarithms). Denote this variable by $\ln \hat{S}$. This synthetic variable was then substituted for $\ln S$ in (18) to obtain the system of estimable equations

$$\ln r_i = \pi_{oi} + \sum_{j=1}^{m} \alpha_j x_j + \sum_{j=1}^{n} \beta_{ij} (\ln R - \ln \hat{S})^j \tag{23}$$

where $\quad \pi_{oi} = \lambda_i + \alpha_o$

for i = 1, 2, 3

This system was estimated imposing the constraint that the α's were constant across equations. However, the constant terms π_{oi}, and the slope coefficients in the polynomial β_{ij} were allowed to vary across equations.

To identify the λ_i's and the constant α_o from the regression coefficients an additional constraint is required. This constraint is available since the

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TABLE 3. INITIAL HIGH QUALITY RECRUIT REGRESSION

Dependent Variable: Logarithm of the Enlistment Rate
for Category I-II Recruits*

Sample Period: 195802-197204 (Quarterly)

$R^2 = 0.32$; $\bar{R}^2 = 0.17$ (corrected for degrees of freedom)

Variable	Coefficient			t-statistic	Mean
Constant	π_{o1}	=	12.37	0.89	1.00
	λ_1	=	-1.13		
	α_o	=	13.51		
W^m	α_1	=	0.28	0.39	8.78
W^c	α_2	=	-2.33	1.41	9.47
E	α_3	=	-4.91	1.49	-0.08
DP	α_4	=	6.29	0.32	-0.01
VN	α_5	=	0.30	0.66	0.51
BERLIN	α_6	=	0.24	0.66	0.03
MAR	α_7	=	0.02	0.07	0.15
S1	α_8	=	0.11	0.68	0.24
S2	α_9	=	-0.10	-0.52	0.25
S3	α_{10}	=	-0.33	-1.94	0.25

* Total recruits in categories I and II divided by
male population aged 17-20.

recruit supplies in each category must sum to total recruit supply. Thus, the following expression must hold:

$$\sum_{i=1}^{3} \frac{s_i}{S} = \sum_{i=1}^{3} e^{\lambda_i} = 1 \tag{24}$$

When (24) is added to the system:

$$\pi_{oi} = \lambda_i + \alpha_o \tag{25}$$

$$i = 1, 2, 3$$

there is enough information to exactly identify α_o, λ_1, λ_2, and λ_3. The regression results from the constrained estimation are reported in Table 4.

Since the results in Table 4 are conditional on the aggregate recruit supply variable being a consistent estimate, another estimate of aggregate recruit supply was made using the parameters reported in Table 4. Equation (23) was then reestimated with the new supply estimate. These second round estimates are reported in Table 5.

This iterative procedure was repeated for several times to see if the parameter estimates converged to a single set of values. Convergence did not occur. Instead, the parameter values changed slightly from run to run (in some cases the signs of the coefficients changed). One of the reasons for the apparent instability in the estimates is the fact that the standard errors are large. One encouraging fact was that the signs of the coefficients on the military and civilian pay variables were those that were expected and remained so throughout the repeated estimation. Both pay variables are deflated by the Consumer Price Index. Real military pay has a positive impact and real civilian pay has a negative impact on the rate of enlistments. However, neither estimate is statistically significant from zero at the 5% level.

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TABLE 4. FIRST ROUND CONTRAINED REGRESSION

Dependent Variables: Logarithms of the Enlistment
Rates for Recruits in Categories I-II, III, and IV

Sample Period: 195802-197204 (Quarterly)

R^2 = .57; R^2 (corrected) = .52

Variable	Coefficient		t-statistic	Mean
Constants	$\pi_{o1}-\pi_{o3}$ =	1.11	7.66	
	$\pi_{o1}-\pi_{o3}$ =	1.53	10.75	
	π_{o3} =	-2.08	0.23	
	λ_1 =	-1.05		
	λ_2 =	-0.63		
	λ_3 =	-2.16		
	α_o =	0.08		
W^m	α_1 =	0.60	1.12	8.78
W^c	α_2 =	-1.16	0.96	-9.47
E	α_3 =	-0.47	0.18	-0.08
DP	α_4 =	8.57	0.60	-0.01
VN	α_5 =	0.02	0.08	0.50
BERLIN	α_6 =	0.50	1.81	0.03
MAR	α_7 =	0.05	0.33	0.15
S1	α_8 =	-0.02	0.16	0.25
S2	α_9 =	0.04	0.27	0.25
S3	α_{10} =	0.14	1.01	0.25
$\ln R-\ln\hat{S}$	β_{11} =	-0.06	0.21	0.00
$(\ln R-\ln\hat{S})^2$	β_{12} =	-0.57	0.85	0.03
$\ln R-\ln\hat{S}$	β_{21} =	0.08	0.28	0.00
$(\ln R-\ln\hat{S})^2$	β_{22} =	-0.48	0.72	0.03
$\ln R-\ln\hat{S}$	β_{31} =	0.63	2.31	0.00
$(\ln R-\ln\hat{S})^2$ *	β_{32} =	0.12	0.18	0.03

* The number of terms in the polynomial (n) was set at 2. This
will be increased in later research.

TABLE 5. SECOND ROUND CONSTRAINED REGRESSION

Dependent Variables: Logarithms of the Enlistment Rates for Recruits in Categories I-II, III, and IV

Sample Period: 195802-197204 (Quarterly)

$R^2 = .60$; R^2 (corrected) $= .56$

Variable	Coefficient		t-statistic	Mean
Constants	$\pi_{o1} - \pi_{o3}$ =	1.22	8.83	
	$\pi_{o1} - \pi_{o3}$ =	1.53	11.31	
	π_{o3} =	-2.52	-0.29	
	λ_1 =	-1.01		
	λ_2 =	-0.70		
	λ_3 =	-2.21		
	α_o =	-0.31		
W^m	α_1 =	0.59	1.14	8.78
W^c	α_2 =	-1.12	0.98	9.47
E	α_3 =	-0.49	0.21	-0.08
DP	α_4 =	13.77	1.0	-0.01
VN	α_5 =	0.18	0.70	0.50
BERLIN	α_6 =	0.61	2.33	0.03
MAR	α_7 =	0.08	0.54	0.15
S1	α_8 =	0.05	0.42	0.24
S2	α_9 =	0.07	0.49	0.25
S3	α_{10} =	0.11	0.86	0.25
$\ln R - \ln \hat{S}$	β_{11} =	0.68	1.74	-0.01
$(\ln R - \ln \hat{S})^2$	β_{12} =	-2.37	1.93	0.02
$\ln R - \ln \hat{S}$	β_{21} =	0.46	1.20	-0.01
$(\ln R - \ln \hat{S})^2$	β_{22} =	-0.46	0.38	0.02
$\ln R - \ln \hat{S}$	β_{31} =	0.78	2.00	-0.01
$(\ln R - \ln \hat{S})^2$	β_{32} =	1.03	0.86	0.02

The coefficient on the employment rate variable has the expected (i.e., negative) sign in the early iterations (see Tables 4 and 5), however, it does change sign in the later iterations. Throughout, the t-statistic is very small. The coefficient on the draft pressure variable is consistently positive, which is not consistent with its purpose in the equation (since it represents the probability of remaining a civilian and should be negatively related to the enlistment rate). In general, the dummy variables do not add much to the regression in terms of explanatory power. The one exception is the BERLIN dummy which has a positive coefficient (the reverse of what was expected based on a risk aversion theory).

Overall, the empirical results were disappointing. Several areas for further inquiry have been identified.

First, there was a good deal of multicolinearity in the data over the time period 195803-197204. It was our intention to estimate the model over the period in which a draft was in effect to get a better estimate of the impact of draft pressure. An extension of the time period seems indicated. There was a good deal of variation in the unemployment rate in 1974-76. These changes were largely independent of military demand so that it may be possible to get better estimates by extending the time period.

Second, two stage least squares (TSLS) should be used to estimate the constrained system of recruit functions. The total number of recruits appears on the RHS of the regressions. This variable is really endogenous since it is related to the dependent variable through a summation identity (i.e., recruits in the three groups sum to total recruits). TSLS estimation was attempted using four terms in the polynomial, but the estimation was not successful because of severe multicolinearity. Alternative estimation techniques are being considered.

Third, more use should be made of data on rejections by test score group. These data are available monthly in the Qualitative Distribution Reports and can be used to provide independent estimates of the acceptance probabilities.

In conclusion, the empirical analysis discussed in this section should be regarded as a first step toward an estimated empirical model of recruit quality. The theory has been well enough developed so that the major effort in the second year of the contract will focus on obtaining better empirical estimates of recruit functions by test score group. In addition, the analysis will be expanded to include education and possibly race.

APPENDIX

It was asserted without proof in the text that the expected value of recruit quality was positively related to the recruit acceptance standard q_o. The proof is as follows:

$$\text{Let} \quad P(q_o) = \int_{q_o}^{\infty} f(q)\,dq \tag{A1}$$

$$\text{then} \quad E(q|q>q_o) = \frac{\int_{q_o}^{\infty} q f(q)\,dq}{P(q_o)} \tag{A2}$$

$$\text{and} \quad \frac{dE(q|q>q_o)}{dq_o} = \frac{- P(q_o) q_o f(q_o) - \left[\int_{q_o}^{\infty} q f(q)\,dq\right] f(q_o)}{[P(q_o)]^2} \tag{A3}$$

$$= \frac{f(q_o)}{P(q_o)} \ [E(q|q>q_o) - q_o] \ > 0 \tag{A4}$$

It was also asserted that under certain conditions the marginal impact of aggregate supply on the number of recruits with given characteristics $(\underline{x}^*)$ was monotonic in the level of a particular characteristic (x_i) under these sufficient conditions:

1. The distribution of the aggregate supply of recruits across characteristics is constant. This assumption implies that

$$\frac{\partial s(\underline{x}^*)}{\partial S} = \frac{s(\underline{x}^*)}{S} \tag{A5}$$

The elasticity (ϵ) may be written as

$$\epsilon = \frac{S}{E(R|\underline{x}^*)} \ \frac{\partial E(R|\underline{x}^*)}{\partial S} = \frac{S}{P(q_o|\underline{x}^*)\ s(\underline{x}^*)} \ \frac{\partial E(R|\underline{x}^*)}{\partial S} \tag{A6}$$

where $P(q_o|\underline{x}^*) = \int_{q_o}^{\infty} f(q|\underline{x}^*)\,dq$

Substituting (14) from the text and (A5) into (A6) gives

$$\varepsilon = 1 - \frac{f(q_o|\underline{x}^*)}{P(q_o|\underline{x}^*)} \frac{P(q_o)}{f(q_o)} \tag{A7}$$

where $P(q_o) = \int_{q_o}^{\infty} f(q)dq$

The elasticity is montonic in x_i if the derivative $\frac{d\varepsilon}{dx_i}$ is strictly positive or negative. The derivative may be written as:

$$\frac{d\varepsilon}{dx_i} = - \frac{P(q_o)}{f(q_o)} \frac{\left[P(q_o|\underline{x}^*) \frac{\partial f(q_o|\underline{x}^*)}{\partial x_i} - f(q_o|\underline{x}^*)\frac{\partial P(q_o|\underline{x}^*)}{\partial x_i}\right]}{[P(q_o|\underline{x}^*)]^2} \tag{A8}$$

It follows that

$$\frac{d\varepsilon}{dx_i} \gtreqless 0 \text{ as } \frac{\partial P(q_o|\underline{x}^*)}{\partial x_i} \gtreqless \frac{P(q_o|\underline{x}^*) \frac{\partial f}{\partial x_i}(q_o|\underline{x}^*)}{f(q_o|\underline{x}^*)} \tag{A9}$$

Equation (A9) implies the second sufficient condition:

2. If $P(q_o|\underline{x}^*)$, the acceptance probability for an applicant with characteristics $\underline{x}^*$, is strongly monotonic in x_i, the elasticity ε will be monotonic in the same direction.

For example, if test score were strongly and positively correlated with quality, one would expect the (signed) elasticity of recruits with respect to aggregate supply to increase with test score level.

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DATA APPENDIX

The variables used in the regressions will be
described briefly here. Further information about
data and procedures is available upon request from
the author.

DATA APPENDIX

1.   Military Pay

Military pay was estimated by summing basic pay plus allowances, quarters and tax advantage over the four lowest years in service categories for pay grades E1-E6. This is equivalent to assuming a zero discount rate and a four year term of service. This pay variable was then divided by the Consumer Price Index to get a real military pay variable. The natural logarithm of real military pay was used in the regressions.

2.   Civilian Pay

Civilian pay was estimated by using median annual earnings of full year workers. The median earnings for those aged 16-19 and 20-24 were interpolated to get quarterly values and then each series was multiplied by two and the resulting series were summed. This procedure yields an estimate of the sum of four years civilian pay on the assumption that a typical recruit enlists for a four year term at age 18 and is consistent with the military pay variable. This civilian pay variable was then divided by the Consumer Price Index to get an estimate of real civilian pay. The natural logarithm of real civilian pay was used in the regressions.

3.   Employment Rate for Males Aged 20-24

This variable was taken from Employment and Earnings published by the Bureau of Labor Statistics. The employment rate is one minus the reported unemployment rate. The age group was selected to give a meaningful index of job opportunities for young men and yet to minimize the problem arising because the male youth unemployment rate is so sensitive to enlistment

behavior. The unemployment rate of males 16-19 would be expected to move inversely with enlistments since as more young men enter the service fewer are unemployed. Thus, if the employment rate for males 16-19 were used as a regressor, there would be a likely positive bias in the estimated coefficient. Using the rate for males aged 20-24 reduces this problem somewhat since there are fewer enlistees from the 20-24 age group.

4. Draft Pressure Variable

The draft pressure variable is the sum as that used by Fisher (1969) in his study of the cost of the draft. It is the natural logarithm of one minus the ratio of inductions to youth population aged 17-20. This variable represents the probability of remaining a civilian if one does not enlist and is expected to be negatively correlated with the enlistment rate.

5. International Tension and Deferment Dummy Variables

To account for possible influences of the likelihood of combat on enlistment behavior, variables were introduced to control for the "Berlin Crisis" in 1961-62 and the intense period of the Vietnam war. A variable was also introduced for the period in which married men were dropped to the lowest order of call in the draft.

6. Seasonal Dummies

Three quarterly dummies were introduced to account for predictable interyear variation in enlistment behavior.

7. Enlistment Rate

The enlistment rate was computed by taking the number of enlistees as reported in Monthly Qualitative Distribution Reports for various test score categories and dividing by male population aged 17-20. The natural logarithm of the enlistment rate was used in the regressions. The total enlistment rate was also computed and the natural logarithm of the total rate was used as the variable ℓnR in the polynomial part of the regressions.

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