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OFF-LINE ORTHOPRINTER SOFTWARE

DBA Systems Inc

DDC

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**ROME AIR DEVELOPMENT CENTER**
Air Force Systems Command
Griffiss Air Force Base, New York 13441

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This effort developed experimental software to expand the digital terrain data input options to the Off-Line Orthophoto Printer (OOP). The additional sources addressed were random terrain relief points, digitized contour data and multiple AS-11B-1 stereoplotter profile data sets. Profile sets compiled from earlier coverage can now be used with more current imagery.

The major task was to develop algorithms which could generate profiles compatible with the OOP operating software from the various terrain data sets.
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Profiles were generated using terrain modeling as well as interpolation schemes. Terrain modeling investigations used polynomial and multiquadratic functions. The multiquadratic function appeared to work with evenly spaced input such as AS-11B-1 profile data while the interpolation scheme was the best choice for digitized contour data.

The other major task was to transform the image orientation data and the digital terrain data into a common coordinate system compatible with the OOP operating system. The report covers the mathematics, subroutine descriptions, and test results.

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## EVALUATION

This effort provides an in-house capability to demonstrate advanced techniques in off-line orthophoto production. Exploitation of multiple digital terrain data sources has the potential to greatly expand the Defense Mapping Agency orthophoto production capability. Orthophotos are a viable form for photogrammetric targeting data bases which are currently being supported by TPO 2. Positional data bases in an orthophoto format require minimal field exploitation equipment complexity. Orthophotos, however, are relatively expensive to produce due to the high cost of digital terrain compilation. By taking advantage of terrain data already in existance, the need for new compilation with new image coverage is negated. While contractual resources for complete testing were not available, this effort has provided experimental software which will allow future evaluation of this potential.

ANTHONY JERIC
Project Engineer

1.    INTRODUCTION

This report, submitted in fulfillment of RADC Contract No.
F30602-74-C-0297, presents a description of the PDP-11/45 software
developed by DBA Systems, Inc. for the offline processing of digital
relief data on the Offline Orthophoto Printer (OOP). The primary
objectives of this contract consisted of the following items:

a)  to eliminate the dependence of the OOP on the AS-11B-1,

b)  to make use of available digital data base information
    on the OOP,

c)  to allow for more flexible use of data created by the
    AS-11B-1,

d)  to allow for creation of orthophotographs from limited
    relief information (discrete random points).

The Offline Orthophoto Printer Software (OOPS) contains a new
approach to offline processing of digital data.  The OOPS accepts in-
puts consisting of terrain data and photographic orientation, performs
a series of computations, and outputs two data files.  These two files
duplicate the format of those generated by the AS-11B-1 stereoplotter
and can be directly input into the Offline Orthophoto Printer (OOP).
The first file, the ShutDown Tape (SDT), contains the information
necessary for orientation of the photographic imagery on the OOP. The
second file, the OOPS Profile Tape, contains the derived terrain infor-
mation necessary to produce a rectified orthophotograph.

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A detailed description of the software written under the OOPS contract is presented in the next eight sections. Section two contains a system description. This includes an overview of the previous means of generating an orthophoto as well as a description of the new approach developed. Section three presents a detailed description of the methods implemented to process available digital topographic data. This description includes profile, contour and random point data processing along with the processing of multiple sets of data. Section four contains the mathematical concepts used to develop the OOPS. Section five contains the user's input description. Section six presents the system and subroutine flow diagrams. Section seven contains a short description of all subroutines used in the software development.

The input formats of the terrain data sources is addressed in Section eight. A description of the two output files is contained in Section nine. Results of various data runs are described in Section ten. A summary containing conclusions and recommendations is presented in Section eleven.

2.      SYSTEM DESCRIPTION

The OOPS system has substantially increased the production capability of the orthophoto printer at RADC.  Before the implementation of this software system, the OOP was totally dependent on the AS-11B-1 stereoplotter for the generation of all input data.  OOPS was designed to expand the capability of the OOP by accepting various types of digital data for the computation and output of profiles in AS-11B-1 format.  The software includes options to input orientation data in various coordinate systems, transform such data into the appropriate coordinate system for generating a punched shutdown tape for input to the OOP.

The OOPS system is written primarily in Fortran IV but includes some Assembly language routines for special I/O handling.  The system operates on RADC's PDP 11/45 in the 32K of available core using the DOS operating system.  Available peripherals include a card reader, paper tape reader/punch, two seven (7) track and one nine (9) track magnetic tape drives, and an 80 column/line printer.

2.1     System Overview

The capability of generating an orthophoto has significantly changed with the creation of the OOPS system.  It is no longer necessary for the input photograph to be one of the stereo-pair used to generate the digital data tape.  The terrain information can now be in the form of cartographic data, AS-11B-1 profile data, or discrete points, while the photographic orientation parameters can be derived from block or strip triangulations, resections or AS-11 absolute model data.

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Previously the OOP was entirely dependent on the AS-11B-1 stereo-plotter for the generation of orthophotos. Figure 2.1 shows the means by which an orthophoto was produced on the OOP before the implementation of the offline orthophoto software. This figure clearly demonstrates the dependence on the AS-11B-1 for all data input to the OOP. In addition to dependence on input data, the input photograph was required to be one of the stereo-pair used to generate these data tapes.

Figure 2.2 represents an overview of the OOPS system. Comparison of Figure 2.2 to Figure 2.1 shows the flexibility of input data resulting from the OOPS. The OOPS system will accept terrain data formatted as MMS-32 Cartographic data, AS-11B-1 profile data, or discrete points, such as, triangulation data and survey or geodetic control. Combinations and multiple sets of the above terrain data are also accepted. The source imagery accepted by OOPS includes both panoramic and frame photography. Position and orientation data generated for this source material can be input in nearly any form. Optional coordinate systems include USR, LSR, LV, geographics, and AS-11 absolute model parameters.

## 2.2    System Computational Flow

Figure 2.3 presents the four major sequential elements used in the computation of simulated AS-11B-1 data necessary for orthophoto generation. These four elements are input, extraction of data, computation

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FIGURE 2.1. Previous Means of Generating an Orthophoto Printer Product

Stereo model setup and profiling step

Orthophoto product generation step

*The term orthophoto product will be used to represent rectified photo, orthophoto and companion stereomates.

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FIGURE 2.2. Off-line Orthophoto Printer Software (OOPS) Input Data

FIGURE 2.3. OOPS (Off-line Orthophoto System)

of functional terrain representation and output of newly created pro-
files to a tape formatted similar to that created on the AS-11B-1.
The input element contains the logic to control the input data necessary
for the computation of orientation and selection of the correct terrain
data link.  An explanation of the many input options follows in Section
4.1.

The general concept for extracting data from the source tape is
the same in all modes.  Basically, the geographical area of interest is
determined by projecting the film format corners to the ground and then
transforming these ground coordinates to digital data coordinates.  This
computation defines the area from which data is to be extracted.  As the
terrain data tape is read, only data within the desired area is stored
on random disc, thus efficiently using the random storage routine.  When-
ever possible, data sorting is accomplished during this storage process
to eliminate unnecessary I/O.  The sorting algorithms for each data type
are explained in detail in Section 3.

Computation of the relief model and photographic orientation are
two of the most important areas in the OOPS program.  Although each data
type is handled differently due to considerations of program efficiency
and accuracy, the basic computational objective is the derivation of a
mathematical representation of the terrain surface.  A description of
these mathematical functions is presented in Section 3.  Other computa-
tions required include transformation of the input photo orientation

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and position parameters to the coordinate system of the OOP.  These transformations are explained in Section 4.1.

The last major area of OOPS deals with data output.  The shutdown tape contains the necessary information for photo orientation along with values for atmospheric refraction, earth curvature, focal length and various other values needed on the BX-272.  The shutdown tape is punched on paper tape in RCA 501 code.  Hard copy output of the shutdown tape information is tabulated in a convenient format.

The only other major output of the OOPS program is the simulated AS-11B-1 profile tape.  This output tape is formatted as "X,Y,Z" coordinates in the LSR system required by the OOP system.  The detailed format of this tape is documented in Section 8.

3.    METHODS OF PROCESSING VARIOUS DATA TYPES

The OOPS system will accept various types of digital relief information and derive profiles in the desired model coordinate system. The methodology selected for processing a specific data type resulted from considerations of program efficiency and resultant accuracy. The following sub-sections describe in detail the method of processing each data type.

An algorithm based upon the multiquadric function is used to model the terrain relief in the profile and random point modes, while a polynomial function is used in the contour mode. These algorithms will be presented at this level in order to clarify subsequent sections. The basic equation for the multiquadric function is of the form developed by Hardy (see Reference 1) and is given by:

$$Z_j = \sum_{i=1}^{n} a_i / ((X_i - X_j)^2 + (Y_i - Y_j)^2 + C)^{\frac{1}{2}}, \qquad (3.0.1)$$

where

$Z_j$  -  Z coordinate of jth data point in local space rectangular system

$a_i$  -  coefficient associated with the ith nodal point

$X_i, Y_i$  -  coordinates of ith nodal point

$X_j, Y_j$  -  coordinates of jth data point

$C$  -  an arbitrary constant term representing the area of influence of nodal points.

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It is the unique manner in which Hardy's formulation is applied that is important in the OOPS system. This application is described in Section 3.1 and 3.3.

The equation for the polynomial representation of relief in the contour mode is given by:

$$Z \ = \ a_1 + a_2 \hat{x} + a_3 \hat{y} + a_4 \hat{x}^2 + a_5 \hat{y}^2 + a_6 \hat{x}^3 + a_7 \hat{y}^3 \tag{3.0.2}$$

where

$$\hat{x} \ = \ x - \bar{x}$$

$$\hat{y} \ = \ y - \bar{y}$$

$$\bar{x} \ = \ \sum_{i=1}^{n} x_i$$

$$\bar{y} \ = \ \sum_{i=1}^{n} y_i$$

n = total number of data points used in a given raster fit

x,y = model coordinates.

The number of polynomial coefficients used in a specific raster fit is dependent upon the number of available data points.

3.1     Profile

Data input to the profile mode is currently derived from the AS-11 plotter system. This data is formatted in a very systematic manner. The data is very dense and is formatted as regularly spaced "X,Y" model coordinates with associated "Z" values in a local earth

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tangent coordinate system. Advantage is taken of the systematic format of this data in a very unique manner.

The partial derivative of equation 3.1 with respect to the unknown coefficients is

$$\partial Z_j/\partial a_i = 1.0/(X_i - X_j)^2 + (Y_i - Y_j)^2 + C)^{\frac{1}{2}}. \qquad (3.1.1)$$

Thus, the partial derivative is simply a function of the distance of the jth data point from the ith nodal point. With evenly grided data this is an important criteria, for if one selects subsets of the grided data and establishes an origin at the center of each subset, the data becomes perfectly symmetrical. This means that if a consistent nodel point grid is designated within every data subset the partial derivatives computed using equation (3.1.1) are identical for the jth data point in all subsets. This implies that the least squares normal equations for all subsets are identical and only the constant vector changes as a result of the changes in elevation within each subset. As a matter of fact, one can form and invert the normal equations and reuse the same inverted normal equations to solve for coefficients in all subsets which have equivalent spacing. The OOPS system currently employs this technique in the profile mode. The inverted normal equations are formed and stored for only one subarea of the total data set and reused for all subsequent subareas. In this regard, future versions of the program could be designed to accept as input one or more sets of inverted normal equations.

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Figure 3.1.1 shows the method of subarea division implemented. The origin of each subarea is designated by the large solid triangles. The wide dashed lines surround data areas that are stored as a unit. New profiles are to be derived over the area bounded by the dotted lines. Although the multiquad solution is derived over four of the squares outlined by the wide dashed lines this representation is only used over the inner subarea outlined by the solid black lines with the solid black triangles as the center. The excess area is included in the least squares solution to avoid discontinuity problems between adjacent areas. The total area is configured as a square to simplify the computational algorithms.

In the profile mode, the digital data is handled in four basic steps. First the software determines the limits of the required area; next this required data is stored in a systematic manner; the third step entails the derivation of the multiquadric coefficients; and the final step is the derivation of the new profiles using the derived coefficients.

Initially the program software derives the object space boundaries of the relief data required to produce the new orthophotograph (see Section 4 for coordinate system definition). These boundaries are determined by projecting the corners of the user designated photo area to the earth's ellipsoid. These projected corners are referred to as the footprint of the required orthophoto area.

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▲ Origin of subareas

▪▪▪▪▪▪▪▪ Data subsets stored as units

•••••••• Area new profiles are to be derived over

――― Area coefficients derived over

FIGURE 3.1.1. Storage and Coefficient Subareas for Profile Mode

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The digital terrain data, as required to represent the projected footprint, is read from magnetic tape. The required data is stored in a systematic manner on random disc. Figure 3.1.1 shows the subareas of digital terrain data which are stored as units.

The next step involved in the generation of profile data is the actual formation of the normal equations and solution for the multi-quadric coefficients describing the terrain. Nodes are computed over an area that is two times the size of the coefficient block in both the x and y direction. These nodes are evenly spaced about the center of this square area. The square area is four times the size of the storage block to allow for a fifty percent overlap on all sides.

The normal equations are then formed. The equation used is a function of the x and y coordinates (see equation 3.0.1). Recall, be-cause of the grided format of the profile data, the least squares normal equations are the same for all subareas. Therefore the inverse for the normal equations is stored on disc after the first solution and used throughout the digital data area. Only the constant vector is recomputed for subsequent blocks. Having recomputed the constant vector, the multi-quadric coefficients describing the terrain are derived (Section 4.3) and stored on a disc file for later use in computing the final output pro-files. In order to evaluate the adequacy of the derived terrain function, a RMS value between the derived "Z" coordinates and the input profile values is computed. This is computed by deriving "Z" values using the

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multiquadric function at all data points of every profile used for that particular solution. The difference between the original value and the derived value is used to compute the MEAN and RMS for each specific subarea. This statistic is output so the user can determine the quality of the fit over each subarea. Also output, are summary statistics which derive the MEAN and RMS values for the entire area.

With all the multiquadric coefficients computed and stored on a disc file, the last step in deriving the new profiles begins. The coefficients for a subarea are retrieved and used to derive the "Z" coordinates for "x,y" values within the bounds of that subarea. These derived profiles are reformatted to correspond to the AS-11 format in the output link (see Section 9).

## 3.2    Contour

One of the objectives of the OOPS system is to exploit data from the Automatic Cartographic System (ACS). The format of the digital data currently used in the ACS has been designated as MMS-32. The terrain relief data used in the ACS is stored as contour data. Thus, the MMS-32 contour data does not have the symmetrical "x,y" grid pattern of the profile data. It does, however, have an important characteristic. As terrain slope increases, a greater number of contours are used to represent an area. That is, relief information is denser in areas characterized by significant changes in ground elevation.

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The following general approach for deriving profiles from the MMS-32 digital contour data has been implemented. The final model system is established (see Section 4) and the segment of contour data near a desired profile is stored as "raw" information for that profile. The "raw" profile information is discrete points which are taken from the digital contour data. These extracted discrete points are those nearest the intersection of a contour and the line along which a profile is to be derived. Desired profiles are those which are to be created over a user specified photo area. These profiles have a constant "x" coordinate in the newly established model system. The "y" coordinates are increments of 250μm, while the "z" coordinate represents the terrain elevation above an established datum. When all the contour information has been stored on random disc in this manner, the "raw profiles" are processed in sets of four. Polynomial functions are fit over raster areas covering the width of four profiles, but vary in length depending on the density of the data. Thus the size of the area analytically represented is directly dependent on the terrain slope. Following is a detailed description of the processing sequence performed by the contour link.

The MMS-32 tape, which contains cultural, hydrographic, and topographic data, is searched until a data block containing contour data is found. Each data point within that block is then transformed to the desired profile system and checked to see if it lies within the

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desired model limits of the designated orthophoto area. All data points

which are within the required limits are stored along with their profile

identification. The available core storage is divided into blocks ac-

cording to the total desired number of output profiles. Thus, a parti-

cular set of profile data is stored in a block of core (see Figure 3.2.1).

For example profiles 1-5 may be stored in block 1 while profiles 6-10

would be stored in block 2, etc., through block 19. Block number 20

contains the profiles which are not evenly divisible into the first 19

blocks.



FIGURE 3.2.1. Data Storage (Contour)

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As each block fills, it is transferred to random disc. Each series of blocks are chained together on random disc by including the address of the previously written block in the subsequent block. Thus, all blocks containing a particular subset of profiles can be retrieved by knowing the address of the final block. Each subset of profiles is retrieved and the profiles are written on a sequential file in order of ascending "y" coordinates. If more than one point is found for the specified "y" profile interval, only the point closest to the desired profile increment is retained.

The profiles are retrieved in blocks of four and polynomial fits are computed over small raster areas of the four consecutive profiles. Using a set of four profiles the two center profiles are derived and output for each raster area. This concept is illustrated in Figure 3.2.2. For a normal raster, this is accomplished by deriving a least squares solution for the seven polynomial coefficients in equation 3.0.2. The data points used for this solution are the raw profile points as described earlier in this section, plus the two end points derived during the previous raster fit. A normal raster is defined as one not containing the end of a profile set. Thus, ten (10) new data points are available and constitute a raster area. For nonnormal rasters, less than ten (10) new data points are available, the number of coefficients used in the least squares fit is reduced. If four (4) through nine (9) new points are available five (5) coefficients are used, if one (1) through three (3) new points are available three (3) coefficients are used.

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After the initial fit the two end points from the profiles derived in the previous raster are retained and included as data in the subsequent raster fit. These data points are retained to assure that consecutive rasters fit together properly. Each of these retained data points are weighted as three points to assure raster compatibility. Consistency between profile sets is accomplished by saving two profiles from the previous set and utilizing only two new profiles.

These algorithms, implemented in the OOPS contour link, provide an efficient and accurate method of deriving profiles from the MMS-32 contour data. Using this set of four profiles two output profiles are derived along the center two. This concept is illustrated below.



FIGURE 3.2.2  Raster Area

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The above method for deriving profiles using contour data pro-
vided excellent results over areas of moderately sloping terrain. How-
ever, over areas of steep terrain, the polynomial function did not com-
pletely conform to the terrain. To more accurately model the steep
terrain, an interpolation routine was developed from which an elevation
could be computed based on the distance the desired data point was from
the known data point. The distance from the desired data point to the
known data point is computed from:

$$dxsj = (x_t - x_j)^2$$
$$dysj = (y_t - y_j)^2$$

where

$x_t, y_t, z_t$ = coordinates of desired data point ($t$)

$x_j, y_j, z_j$ = coordinates of known data point ($j$).

The desired elevation value is computed from

$$Z_t = \frac{\sum_{J=1}^{N} Z_j / XYZ_j}{\sum_{J=1}^{N} 1.0 / XYZ_j} ,$$

where

$XYZ_j$ = dxsj + dysj + C

$C$ = constant (presently set at .01).

The interpolation routine used the same algorithms for storage of raw
contour data and outputting of derived profiles. Only the method for
computing terrain elevations of data points was changed.

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## 3.3    Random Points

Unlike the contour or profile mode, the random point mode computes coefficients to represent an entire area of interest.  The objective of this link is to take rather sparse discrete point data and develop a terrain function which is adequate to produce an orthophotograph.  These discrete points can be acquired from a photogrammetric triangulation, available ground survey data, or points digitized from available map sources.  Coordinate systems allowed as input are: Universal Space Rectangular (USR), Geographics, or Universal Transverse Mercador (UTM).

A multiquadric function is used to represent the terrain relief over the entire area for which the orthophotograph is to be produced (see Section 4.3).  Initially, the object space area is then grided into theoretical rows and columns.  The number of rows and columns to be established is designated by the user.  The points of intersection of each row and column is defined as a nodal point.  Equation 3.0.1 defines the manner in which these nodal point coordinates are applied in the multiquadric solution.  The maximum number of designated nodal points cannot exceed 70.  Additional nodal points improve the accuracy of the terrain representation; however, if an excessive number of nodal points, relative to the available relief data, are introduced the solution can become ill-conditioned.  After the least squares solution for the multiquadric coefficients has been computed they are used to derive "z" coordinates for the new profiles.

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## 3.4 Combinations of Data Types

Combinations of multiple profile tapes (profile, contour, and random point data) can be accommodated using the OOPS software. With this option, profiles generated from several data sets, each of which covers only a portion of the total area, can be combined to generate one complete profile set. This combination is accomplished in the following manner. The user inputs the total number of data sets available. Each data set is then processed in the normal manner and profiles are output to a single sequential file in the OOPS model system. After all data sets have been processed the combined sequential file is searched for profiles with common coordinates. The "Z" coordinates of common points are averaged and an output tape formatted for the off-line orthophoto printer is output.

Random data points may be included directly into either the profile or contour mode. These included points are used to strengthen either the profile or contour solution in a specific area. The random points entered in this manner are input in either a UTM, USR or geographic coordinate frame.

The random points are transformed from UTM, USR, or Geographic coordinates to the coordinate system of the data type being processed. These additional points are then combined with the original data set and processed in the normal manner.

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4.      MATHEMATICAL CONCEPTS

The mathematics of the OOPS software system can be divided into three general areas: (1) establishment of coordinate systems and coordinate transformations, (2) computation of the photographic footprint and digital data limits, and (3) derivation of the functional coefficients which represent the terrain surface.

4.1      Coordinate Systems and Transformations

Orientation data can be input in various coordinate systems. This allows for complete flexibility in the choice of source imagery to be rectified on the OOP. Initially the available digital data source must be related to the photographic imagery from which the orthophotograph is to be produced. In addition, the parameters relating the photographic imagery to an absolute coordinate frame must be transformed into the model system of the orthophoto printer.

The photographic imagery from which the orthophotograph is to be produced can be related to any of several absolute coordinate frames. The photographic exposure station can be defined in Universal Space Rectangular (USR or geocentric), geographics, absolute AS-11 model system, or an earth tangent LSR system. The photographic attitude can be expressed as angles in the following systems: USR, AS-11 and Local Vertical (LV). If the user desires, an orientation matrix in either USR or LV may be input to designate the photo attitude. The photographic positional rates must be input in a local vertical system or

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the AS-11 system. The associated photographic attitude rates are input as local vertical angular rates, a local vertical rate matrix or AS-11 angular rates. In order to establish a unique orthophoto model system regardless of the input data source, the projected principle point of the photograph is established as the origin of an arbitrary earth tangent local model system. The "y" axis of this local model system is established as being the projected positive "y" axis of the output photo. The "x" axis is then defined to form a right handed local system. Figure 4.1.1 is an illustration of this earth tangent model system.

The detailed mathematics for each specific coordinate transformation is contained in the subroutine descriptions presented in Section 7. Only the general flow of the required transformations will be presented in this section. Each function is identified with the section containing its documentation so the reader can easily reference the detailed mathematics.

Following is a description of the transformations required depending on the type of orientation information with the photographic imagery. Initially all orientation data is taken to a local vertical system. <u>Note</u>: The notation $f_n$ refers to a function for which the mathematics is developed in the subroutine documentation (Section 7). The subscript "n" refers to the sub-section that specifically documents that function.

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FIGURE 4.1.1.  Model Coordinate System Definition

### 4.1.1 USR Orientation

$$\begin{bmatrix} \varphi \\ \lambda \\ H \end{bmatrix}_{GEO} = f_{7.97}(X,Y,Z)_{USR},$$

$$T_{LV} = R^T_{USRLV} \, T_{USR},$$

where

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{USR}$$ — are the USR coordinates of the photo exposure station

$$\begin{bmatrix} \varphi \\ \lambda \\ H \end{bmatrix}_{GEO}$$ — are the geographic coordinates of the exposure station

$T_{USR}$ — is the USR orientation matrix

$R^T_{USRLV}$ — is the rotation matrix relating the USR and local vertical systems

$T_{LV}$ — is the local vertical orientation matrix.

### 4.1.2 AS-11 Orientation

The AS-11 absolute model orientation is given as:

$$\lambda_M \begin{bmatrix} A_1 & A_2 & 0 \\ -A_2 & A_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_g - \bar{X}_g \\ Y_g - \bar{Y}_g \\ E_g - \bar{E}_g \end{bmatrix} = \begin{bmatrix} X_m - \bar{X}_m \\ Y_m - \bar{Y}_m \\ E_m - \bar{E}_m \end{bmatrix},$$

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where

$$\lambda_M \quad = \quad \text{the model to ground scale factor}$$

$$A_1 \quad = \quad \cos(AZ)$$

$$A_2 \quad = \quad \sin(AZ)$$

$$X_g, Y_g, E_g \quad = \quad \text{ground coordinates}$$

$$\overline{X}_g, \overline{Y}_g, \overline{E}_g \quad = \quad \text{point of tangency of the local model system}$$

$$X_m, Y_m, E_m \quad = \quad \text{AS-11 model coordinates}$$

$$\overline{X}_m, \overline{Y}_m, \overline{E}_m \quad = \quad \text{center of gravity of model system.}$$

Presently the OOPS software system assumes that the local model point of tangency is given in UTM coordinates. This can be easily expanded if additional user requirements are identified. This absolute model orientation is converted to local vertical in the following manner.

$$\begin{bmatrix} \varphi_0 \\ \lambda_0 \end{bmatrix} = f_{7.92} \, (\overline{X}_g, \overline{Y}_g, \overline{Z}_g)_{UTM}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{USR} = f_{7.59} \, (\varphi_0, \lambda_0)$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{LSR} = \lambda_M * \begin{bmatrix} BX \\ BY \\ BZ \end{bmatrix}_{AS-11} .$$

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The azimuth of the local system is defined as:

$$AZ = \text{arc tan } (A2, A1).$$

The rotation to a local space system is then defined in functional form as:

$$R_{USRLSR} = f_{7.91}(\varphi_0, \lambda_0, AZ).$$

The USR coordinates of the exposure station are computed as:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{USR} = R_{USRLSR} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{LSR} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{USROR}.$$

The USR exposure station coordinates are then converted to geographics

$$\begin{bmatrix} \varphi \\ \lambda \\ H \end{bmatrix} = f_{7.97}(X, Y, Z)_{USR}.$$

The AS-11 orientation matrix is formed, rotated to the USR coordinate system and then to local vertical

$$T_{USR} = R_{USR.LSR} \, T_{AS-11}$$

$$T_{LV} = R^T_{USR.LV} \cdot T_{USR} \,.$$

### 4.1.3  Local Vertical Orientation

If the orientation data is entered in local vertical no transformations are required.

### 4.1.4  Conversion of Orientation to Model System

After the relationship of the photograph to the local vertical coordinate frame has been established, the required orthophoto area is projected to the earth's surface (see Section 7.63).  This defines the footprint of the photograph on the earth's surface.  This information is stored for later determination of the orthophoto limits.  The photographic orientation as defined in the local vertical coordinate frame is converted to an AS-11 model system in order to output the orthophoto printer shutdown tape.  That relationship is computed in the following manner.

The projected principle point of the photograph is established as the AS-11 model origin.  The projected +Y photo axis is defined as +Y model axis.

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The local vertical orientation matrix is rotated to USR and
then to a local space rectangular system.

$$T_{LSR} = R_{USRLSR}^{T} \; R_{USRLV} \cdot T_{LV} \, ,$$

where

$T_{LV}$ = Local vertical orientation matrix

$R_{LVUSR}$ = Rotation from local vertical to USR

$R_{LSRUSR}^{T}$ = Rotation from USR to Local Space Rectangle.

The AS-11 angles are extracted from the $T_{LSR}$ matrix (see
Section 7.28).

The exposure station coordinates in the model system are computed
as:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{AS-11} = 1/\lambda_m \; R_{USRLSR}^{T} \begin{bmatrix} X_{USR} & X_{OR} \\ Y_{USR} & - & Y_{OR} \\ Z_{USR} & Z_{OR} \end{bmatrix} ,$$

where

$[XYZ]_{USR}$ = USR exposure station

$[XYZ]_{OR}$ = local model origin.

All the AS-11 values are output on paper tape for use in orthophoto
orientation of the photograph when generating an orthophoto on the OOP.

## 4.2    Computation of Photographic Footprints and Digital Data Limits

One of the required inputs to the OOPS system is the coordinate limits of the digital data tape in both the digital data coordinate system and a ground coordinate frame (USR, GEOGRAPHIC, UTM). Given this information, a translation and rotation representing the relationship from ground to digital data can be computed.

The ground coordinates are initially transformed into a USR coordinate system and then transformed to local coordinates.

The transformation from UTM coordinates to USR coordinates is computed by:

$$
\begin{bmatrix} \varphi \\ \lambda \\ H \end{bmatrix} = f_{7.92} \ (\text{UTM coordinates})
$$

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{USR} = f_{7.59} \ (\varphi, \lambda, H).
$$

If the ground coordinates are input in geographics, the transformation to USR is simply

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{USR} = f_{7.59} \ (\varphi, \lambda, H).
$$

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The transformation to LSR coordinates is then computed by:

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{LSR} = R^{T}_{USRLV} \cdot \begin{bmatrix} X & X_o \\ Y & -Y_o \\ Z & Z_o \end{bmatrix}_{\substack{USR \\ ORIGIN}} .
$$

Where the USR origin is the projected principal point of the photograph as described in Section 4.1.

A transformation is then computed relating the digital data corners to the derived local ground coordinates. The equation used for this transformation is:

$$
\begin{bmatrix} X \\ Y \end{bmatrix}_{LSR} = S \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}_{DD} - \begin{bmatrix} X \\ Y \end{bmatrix}_{OFF} .
$$

The least squares solution for this transformation is then derived in the following manner:

$$
\Delta = N^{-1} C
$$

where

$$
N = B^T B
$$

and

$$
C = B^T \epsilon .
$$

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The B matrix is computed as:

$$[B]_{2 \times 4} = \begin{bmatrix} \partial f_x/\partial \alpha & \partial f_x/\partial x_{O_{ff}} & \partial f_x/\partial y_{O_{ff}} & \partial f_x/\partial s \\ \partial f_y/\partial \alpha & \partial f_y/\partial x_{O_{ff}} & \partial f_y/\partial y_{O_{ff}} & \partial f_y/\partial s \end{bmatrix},$$

where

$$f_x = (\cos \alpha \cdot x_{DD} \cdot s + \sin \alpha \cdot y_{DD} \cdot s - X_{O_{ff}}) - X_{LSR}$$

$$f_y = (-\sin \alpha \cdot x_{DD} \cdot s + \cos \alpha \cdot y_{DD} \cdot s - Y_{O_{ff}}) - X_{LSR}$$

The constant vector is computed as:

$$[E] = \begin{bmatrix} X_{LSR} - \cos \alpha \cdot x \cdot s - \sin \alpha \cdot y \cdot s + X_{O_{ff}} \\ Y_{LSR} + \sin \alpha \cdot x \cdot s - \cos \alpha \cdot y \cdot s + Y_{O_{ff}} \end{bmatrix}.$$

Using the coordinates of the corners of the film format, USR coordinates are projected on the earth (see Section 7.63). Knowing these USR coordinates and the rotation and translation computed above, the model format is transformed into the digital data system (see Section 7.6). These model format coordinates in the digital data system are compared with the limits of the digital data tape. This comparison yields the location of the desired area. These computed boundary values are used in deriving the final output profiles.

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4.3    Multiquadric Representation

The basic equation used in the OOPS multiquadric mathematics is

$$Z_j = \sum_{i=1}^{N} a_i \left( (X_j - X_i)^2 + (Y_j - Y_i)^2 + C \right)^{-\frac{1}{2}},$$

where

$X_j, Y_j, Z_j$ = coordinates of the jth data point

$a_i$ = coefficient associated with the ith nodal point

$X_i, Y_i$ = coordinates of the ith nodal point

$C$ = an arbitrary constant term representing the area of influence of nodal points

The least squares approach is used in computing the value of $a_i$ in the above equation. The least squares solution involves solving the equation

$$\underset{(N,1)}{a} = \underset{(N,N)}{N^{-1}} \underset{(N,1)}{C}$$

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$$N \atop (N,N) \quad = \quad \sum_{j=1}^{M} \quad B_j^T \quad B_j \atop (N,1)(1,N)$$

$$C \atop (N,1) \quad = \quad \sum_{j=1}^{M} \quad B_j^T \quad \epsilon_j \atop (N,1)(1,1)$$

$$B_j \quad = \quad [B_{1j} \quad B_{2j} \quad B_{3j} \quad \dots B_{Nj}]$$

$$\epsilon \atop (1,1) \quad = \quad Z_j$$

$$B_{1j} \atop (1,1) \quad = \quad 1.0/(X_j - X_1)^2 + (Y_j - Y_1)^2 + C)^{\frac{1}{2}} \quad .$$

The least squares solution is applied differently to each data type due to the different amount and distribution of data and different storage techniques used. The mathematics as explained above are the same for all data types, the method of formation is the only difference (see Section 3).

Profile generation is accomplished by evaluating the multi-quadric equation for height z given various x and y values. These values are then output to a magnetic tape for conversion to a form acceptable on the OOP. Section 3 explains profile generation for each data type.

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5.    USER'S INPUT DESCRIPTION

The OOPS system requires that some data be input as card images. These card images can be input either through a card deck or a disc file. Contained on these images is information describing the photograph, input terrain data and final model.

Data required to describe the photograph includes the exposure station and orientation. The exposure station coordinates can be input in geographic, LSR, USR or AS-11 coordinates. The orientation coordinate system can be in LSR, USR, Local Vertical or AS-11. Other information needed to describe the photograph is photo type and camera parameters. Coordinates of the input terrain data corners and the associated ground coordinates of those points are input to allow the computation of a relationship between the input terrain data and the final model system. Other information required for terrain data processing includes file storage sizes and nodal point patterns.

The model corners and profile increments must be input and used for describing the final model. This information along with data describing the photograph will be used to compute the information needed to set-up the OOP model.

A detailed description of the setup of the card images used to input all necessary data is contained in the Off-line Orthoprinter Software (OOPS) Computer Program Documentation.

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6.        SYSTEM AND SUBROUTINE FLOW DIAGRAMS

A diagram illustrating the complete OOPS system is presented
in this section.  This system diagram (Figure 6.1) illustrates the
tree structure used in overlaying the OOPS software.  Presented on
this diagram is the object module name along with the name of the
individual subroutines which make-up each object module.

The tree structure is arranged such that subroutines which are
repeatedly called by different object modules are contained in the ob-
ject module called MAIN.  During the execution of the OOPS system, the
first object module to be called from the main program is the INPUT
module.  By exercising various options, the three modules on the lower
portion of the tree, below INPUT, will be utilized.  After completing
the computations in the INPUT module, program flow will proceed to
either EXTRA, PROFIL or CONTR, depending on the type of data to be pro-
cessed.  Following the processing of the data type, the MERGE module
will create a final output tape.

Subroutine flow charts of the major subroutines are presented
in the Off-line Orthoprinter Software (OOPS) Computer Program Documen-
tation.  These flow charts give overall logical flow and not the de-
tailed program structure.

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FIGURE 6.1 OOPS System

7.     SUBROUTINE DESCRIPTIONS

Each subroutine developed for the OOPS software system is explained in this section.  The purpose, entry points, calling elements, common area used, subroutines called and link containing the subroutine is all presented on the following pages.  A mathematical description is included for subroutines containing major mathematical techniques.  Subroutines involving storage or indexing, or those considered standard contain no mathematical technique section.

**7.1** Subroutine ADD (IAP)

Purpose:          Determine location of coefficient in random file.

Entry Points:     None

Elements:

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| 0 | I | IAP | - | Location on disc |

Common Areas:     /FPINT/,/STORE/,/CONST/,/FPDLPR/,

Subroutines Used:    None

Link:             WORK


**7.2** Subroutine ADDATA

Purpose:          To transform survey or terrain data from USR coordinates
to digital data coordinates.

Entry Points:     None

Elements:        None

Common Areas:     /UNITS/,/LVTRAN/,/PHOTO/

Subroutines Used:    MATMPY,MOVE,PLHXYZ

Link:             COR

Mathematics:

Given the origin in Geographic coordinates, the USR coordinates
are computed by:

$$
\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}_{USR} = f_{7.60} \begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}_{GEO} .
\tag{7.2.1}
$$

Knowing the USR coordinates of the origin and the USR
coordinates of the survey or terrain data points. The digital data coordinates are
computed by computing the local USR coordinates from:

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\substack{LOCAL \\ USR}} = \begin{bmatrix} X_A - X_0 \\ Y_A - Y_0 \\ Z_A - Z_0 \end{bmatrix} ,
\tag{7.2.2}
$$

where

$X_A, Y_A, Z_A$ = USR coordinates of the additional survey or
terrain points.

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Knowing the local USR coordinates, the digital data coordinates are simply a rotation and scale application:

$$
\begin{bmatrix} x_{DD} \\ y_{DD} \\ z_{DD} \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\substack{LOCAL \\ USR}} \cdot \lambda_{DD/G}
$$

(7.2.3)

where

$\alpha$ = azimuth ground to model axis

$\lambda_{DD/G}$ = scale (digital data to ground)

Equation (7.2.2) and (7.2.3) are repeated for all added points.

7.3    Subroutine ADDRES (IAP)

| | |
|---|---|
| Purpose: | To determine location of coefficient in random file. |
| Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| 0 | I | IAP | - | Location on disc |

| | |
|---|---|
| Common Areas: | /FPINT/,/STORE/,/CONST/,/FPDLPR/ |
| Subroutines Used: | None |
| Link: | CREATE |

**7.4**  <u>Subroutine AMXMN (XMMN, YX, YM)</u>

| Purpose: | To locate the minimum and maximum X and Y values of the tape. |
|---|---|
| Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | XMMN | (2,4) | Corner coordinates |
| 0 | R | YX | (2) | Maximum X, Maximum Y |
| 0 | R | YM | (2) | Minimum X, Minimum Y |

| Common Areas: | None |
|---|---|
| Subroutines Used: | None |
| Link: | COR |

**7.5**  <u>Subroutine ARATES (XC, ANGLV, ANGR)</u>

| Purpose: | Compute angular rates for AS11 shutdown tape. |
|---|---|
| Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | XC | (7) | Exposure station coordinates and angles |
| I | R | ANGLV | (6) | L.V. angle and rate |
| 0 | R | ANGR | (3) | AS11 rate angle |

| Common Areas: | None |
|---|---|
| Subroutines Used: | AS11AN, CLEAR, GEOPM, MATMPY, MINV, OMATLV, USRLVR, XYZPLH |
| Link: | SDT |

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**Mathematics:**

Given the Local vertical angles roll ($\omega$), pitch ($\varphi$), and yaw ($\alpha$) and their associated local vertical angular rates ($d\omega/dt$), ($d\varphi/dt$), and ($d\alpha/dt$) this subroutine transforms the known vertical rates to the AS-11 system.

The AS-11 orientation matrix is defined as:

$$T_{AS-11} = T_{LV} (R_{USRLV})^{\intercal} (R_{USR\,AS-11}) \qquad (7.5.1)$$

where

| | | |
|---|---|---|
| $T$ | = | local vertical orientation matrix |
| $R_{USRLV}$ | = | rotation matrix from USR to local vertical |
| $R_{USR\,AS-11}$ | = | rotation matrix relating USR to the AS-11 local space rectangular system |

let

$$R = (R_{USRLV})^{\intercal} (R_{USR\,AS-11}) \qquad (7.5.2)$$

then

$$T_{AS-11} = T_{LV} R$$

$$R = T_{LV}^{\intercal} \cdot T_{AS-11}$$

$$dT_{AS-11} = T_{LV} dR + dT_{LV} R$$

or

$$dT_{AS-11} = T_{LV} dR + dT_{LV} T_{LV}^{\intercal} T_{AS-11} \qquad (7.5.3)$$

where

$A = dT_{LV} T_{LV}^{\intercal}$ from equation (7.5.3) can be derived as:

$$A(1,1) =$$

$$\begin{bmatrix} 0 & -\cos\varphi\cos\omega \, d\alpha/dt + \sin\omega \, d\varphi/dt & \cos\varphi\sin\omega \, d\alpha/dt + \cos\omega \, d\varphi/dt \\ \cos\varphi\cos\omega \, d\alpha/dt - \sin\omega \, d\varphi/dt & 0 & -\sin\varphi \, d\alpha/dt + d\omega/dt + d\omega/dt \\ -\sin\omega\cos\varphi \, d\alpha/dt - \cos\omega \, d\varphi/dt & \sin\varphi \, d\alpha/dt - d\omega/dt & 0 \end{bmatrix} \qquad (7.5.4)$$

$$dR = (R_{USRLV})^{\intercal} dR_{USR\,AS-11} + dR_{USRLV}^{\intercal} \cdot R_{USR\,AS-11}$$

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The derivative of rotation matrix from USR to the local space system of the AS-11 plotter is zero and this equation becomes:

Since

$$dR_{USR\,AS-11} = 0$$

$$dR = dR_{USR\,LV}^{T} \cdot R_{USR\,AS-11} \qquad (7.5.5)$$

this equation can be rewritten as

$$(7.5.6)$$

$$dR = [dT_{\gamma^c}^{T} \cdot T_{\varphi^c}^{T} \cdot T_{\lambda^c}^{T} + T_{\gamma^c}^{T} \cdot dT_{\varphi^c}^{T} \cdot T_{\lambda^c}^{T} + T_{\gamma^c}^{T} \cdot T_{\varphi^c}^{T} \cdot dT_{\lambda^c}^{T}] \cdot R_{USR\,AS-11}$$

where

$$T_{\gamma^c}^{T} = \begin{bmatrix} \cos\gamma^c & -\sin\gamma^c & 0 \\ \sin\gamma^c & \cos\gamma^c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{\varphi^c}^{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\varphi^c & \cos\varphi^c \\ 0 & -\cos\varphi^c & \sin\varphi^c \end{bmatrix}$$

$$T_{\lambda^c}^{T} = \begin{bmatrix} -\sin\lambda^c & \cos\lambda^c & 0 \\ -\cos\lambda^c & -\sin\lambda^c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$dT_{\gamma^c}^{T}$, $dT_{\varphi^c}^{T}$, and $dT_{\lambda^c}^{T}$ are derivatives of these matrices with respect to time.

The AS-11 rates can then be extracted from the rate matrix derived using equation (7.5.3).

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$$d\varphi/dt \ AS\text{-}11 = dT_{AS\text{-}11}(1,3)/\cos\varphi$$

$$d\omega/dt \ AS\text{-}11 = (T(2,3) + \sin\omega\sin\varphi d\varphi)/(\cos\omega\cos\varphi)$$

$$dK/dt \ AS\text{-}11 = (T(1,2) + \sin\varphi\cos\varkappa \ d\varphi)/(-\cos\varphi\sin\varkappa)$$

**where**

$\varphi, \omega, \varkappa$ are the AS-11 orientation angles

$$T(1,3) = \cos\varphi d\varphi$$
$$T(2,3) = \cos\omega\cos\varphi d\omega - \sin\omega\sin\varphi d\varphi$$
$$T(1,2) = -\sin\varphi\cos\varkappa d\varphi - \cos\varphi\sin\varkappa d\varkappa$$

These derived rates are a function of time and must be converted to be a function of the scan angle for use in the AS-11 system. Thus:

$$d\varphi/d\overline{\alpha} = d\varphi/dt \cdot dt/d\overline{\alpha}$$
$$d\omega/d\overline{\alpha} = d\omega/dt \cdot dt/d\overline{\alpha}$$
$$d\varkappa/d\overline{\alpha} = d\varkappa/dt \cdot dt/d\overline{\alpha}.$$

## 7.6    Subroutine ARXY (XY)

| | |
|---|---|
| **Purpose:** | To find maximum and minimum corner values needed for stripping tape. |
| **Entry Points:** | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | XY | (2,4) | Image corner measurement |

**Common Areas:**    /FPDLPR/,/UNITS/,/GROUND/,/OPTION/,/PHOTO/, /MINMAX/,/MODELP/,/LVTRAN/

**Subroutines Used:**    AMXMN,MATMPY,PLHXYZ,MOVE

**Link:**    COR

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**Mathematics:**

Given the origin in geographic coordinates, the USR coordinates are computed by:

$$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = f_{7.80} \begin{bmatrix} \varphi_0 \\ \lambda_0 \\ h_0 \end{bmatrix}. \tag{7.6.1}$$

Knowing the USR coordinates, the rotation matrix relating USR to LV is

$$R_{USRLV} = f_{7.83} \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}. \tag{7.6.2}$$

Given the projected film corners in geographic coordinates, the USR coordinates are computed using Equation (7.6.1). The Local Vertical coordinates are then computed by:

$$\begin{bmatrix} X_{LV} \\ Y_{LV} \\ Z_{LV} \end{bmatrix} = R_{USRLV} \begin{bmatrix} X_i - X_0 \\ Y_i - Y_0 \\ Z_i - Z_0 \end{bmatrix}, \tag{7.6.3}$$

where

$X_i, Y_i, Z_i$ = USR coordinates of corner i

$X_{LV}, Y_{LV}, Z_{LV}$ = Local Vertical coordinates of corner i

The digital data coordinates of the projected film corners are then computed by:

$$\begin{bmatrix} x_{00} \\ y_{00} \\ z_{00} \end{bmatrix} = R \begin{bmatrix} X_{LV} - X_{OFF} \\ Y_{LV} - Y_{OFF} \\ Z_{LV} - Z_{OFF} \end{bmatrix} \cdot 1/S_{M/a} \tag{7.6.4}$$

where

$R$ = rotation matrix (Section 7.88)

$X_{OFF}, Y_{OFF}, Z_{OFF}$ = translation elements (Section 7.88)

$S_{M/a}$ = scale relating model to ground.

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Comparisons are then made between the digital data coordinates of the projected film format and the input limits of the digital data tape. The minimum and maximum values of these comparisons are used for extraction of data and final computations. The area covered by this final output tape, in model scale, can be computed by transforming the minimum and maximum values from digital data to ground and then ground to model. These transformations are performed as follows:

$$\begin{bmatrix} X_D \\ Y_D \end{bmatrix} = \left( R_{M/G} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \cdot \lambda \right) - \begin{bmatrix} X_{OFF} \\ Y_{OFF} \end{bmatrix}, \tag{7.6.5}$$

where

$R_{M/G}$ = rotation matrix $\begin{bmatrix} \cos(AZ) & \sin(AZ) \\ -\sin(AZ) & \cos(AZ) \end{bmatrix}$

$AZ$ = azimuth relation model to ground

$x, y$ = digital data coordinates

$X_D, Y_D$ = digital data corner in local vertical ground coordinate system

$\lambda$ = scale digital data to ground

$X_{OFF}, Y_{OFF}$ = ground translation values.

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \cos(AZIM) & -\sin(AZIM) \\ \sin(AZIM) & \cos(AZIM) \end{bmatrix} \cdot \begin{bmatrix} X_D \\ Y_D \end{bmatrix} \cdot 1/S_{G/M}, \tag{7.6.6}$$

where

$AZIM$ = azimuth relating model to ground

$S_{G/M}$ = scale relating ground to model

$x_m, y_m$ = model coordinate of digital data corners.

## 7.7  Subroutine ASTRP

| | |
|---|---|
| **Purpose:** | To compute the delta X between profiles. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /FPDLPR/,/UNITS/,/CONST/ |
| **Subroutines Used:** | None |
| **Link:** | STORE |

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## 7.8   Subroutine AS11AN (TLSR,ANAS11)

| | Purpose: | To compute the AS11 angles from an LSR matrix. |
|---|---|---|
| | Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | TLSR | (3,3) | LSR matrix |
| 0 | R | ANAS11 | (3) | AS11 angles |

| | |
|---|---|
| Common Areas: | None |
| Subroutines Used: | None |
| Link: | SDT |

## 7.9   Subroutine AS11M (ANG,OMAT)

| | Purpose: | To compute photo to ground matrix for AS11. |
|---|---|---|
| | Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | ANG | (3) | AS11 angles $(\chi,\omega,\varphi)$ |
| 0 | R | OMAT | (3,3) | AS11 matrix |

| | |
|---|---|
| Common Areas: | None |
| Subroutines Used: | None |
| Link: | INPUT |

Mathematics:

Given the AS-11 angles the matrix relating model coordinates to photograph coordinates is defined as:

$$(7.9.1)$$

$$
\begin{bmatrix}
\cos\chi\cos\varphi - \sin\chi\sin\omega\sin\varphi & \sin\chi\cos\varphi + \cos\chi\sin\omega\sin\varphi & -\cos\omega\sin\varphi \\
-\sin\chi\cos\omega & \cos\chi\cos\omega & \sin\omega \\
\cos\chi\sin\varphi + \sin\chi\sin\omega\cos\varphi & \sin\chi\sin\varphi - \cos\chi\sin\omega\cos\varphi & \cos\omega\cos\varphi
\end{bmatrix} .
$$

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**7.10**   Block Data

| | |
|---|---|
| **Purpose:** | To set up common blocks with associated fixed no. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /UNITS/,/IDXBL/,/INDEX/,/MODELP/ |
| **Subroutines Used:** | None |
| **Link:** | MAIN |

**7.11**   Subroutine BMAT (X,Y,B)

**Purpose:**  Compute the B matrix.

**Entry Points:**  None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | X | – | X value |
| I | R | Y | – | Y value |
| 0 | R | B | (70) | B matrix |

**Common Areas:**  /NODES/,/CONST/

**Subroutines Used:**  None

**Link:**  WORK

**Mathematics:**

Computes the matrix of the partial derivatives over all nodes.

This matrix is defined as:

$$B(I) = 1.0/(\Delta X^2 + \Delta Y^2 + C)^{\frac{1}{2}} \tag{7.11.1}$$

where

$$\Delta X = X - XNODE(I)$$

$$\Delta Y = Y - YNODE(I)$$

$C$ = constant value related to data amount

$X, Y$ = digital data coordinates

$XNODE(I), YNODE(I)$ = coordinates of node I.

$I$ = 1, number of nodes

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**7.12**   Subroutine BMATRX (X, Y, B)

Purpose:          To compute the B matrix.

Entry Points:     None

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | X | – | X value |
| I | R | Y | – | Y value |
| 0 | R | B | (70) | B matrix |

Common Areas:     /NODES/,/CONST/

Subroutined Used: None

Link:             CREATE

Mathematics:      See BMAT (Section 7.11).

**7.13**   Subroutine BMATX (X, Y, B)

Purpose:          Form the B matrix.

Entry Points:     None

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | X | – | X model value |
| I | R | Y | – | Y model value |
| 0 | R | B | (70) | B matrix computed by the contribution of input X&Y |

Common Areas:     /NODES/,/CONST/

Subroutines Used: None

Link:             EXTRA

Mathematics:      See BMAT (Section 7.11).

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**7.14**   Subroutine BOUND

| | |
|---|---|
| **Purpose:** | To read input tape, select necessary data and store to random disk file. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /STORE/,/FPDLPR/,/DATA/,/FPINT/,/UNITS/,/CONST/ |
| **Subroutines Used:** | BUFFIN,DISC,MXMN,RWRITE,XFILL,YFILL,MOVE,PK |
| **Link:** | STORE |

**7.15**   Subroutine BUFFIN (IFLAG,NWD)

**Purpose:**   To read in data from tapes created on 18-bit machines.

**Entry Points:**   None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | I | IFLAG | – | Flag for contour or profile |
| I | I | NWD | – | Number of 16-bit words |

| | |
|---|---|
| **Common Areas:** | /DATA/ |
| **Subroutines Used:** | MTREAD |
| **Link:** | STORE,CONTR |

**7.16**   Subroutine BUFFOT

| | |
|---|---|
| **Purpose:** | To bring in 2 16-bit words and create 1 18-bit word. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /DATA/,/BUFFER/ |
| **Subroutines Used:** | MTREAD |
| **Link:** | MERGE |

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**7.17**   Subroutine CLEAR (A, N)

**Purpose:**        To set N elements of array A to zero

**Entry Points:**   None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I/O | R | A | (1) | Array to be set to zero |
| I | I | N |  | Number of words in A to be set to zero |

**Common Areas:**       None

**Subroutines Used:**   None

**Link:**               MAIN


**7.18**   Subroutine CONTF (Z)

**Purpose:**        Will compute a Z value given an X and Y coordinate.

**Entry Points:**   None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| O | R | Z | – | Computed Z value |

**Common Areas:**       /STORE/, /CONST/, /DATA/, /FPDLPR/, /UNITS/, /PARMS/

**Subroutines Used:**   ADDRES, BMATRX, UNPRPK

**Link:**               CREATE

**Mathematics:**

Given the x, y coordinates, the storage address for the coefficients can be computed (Section 7.3). The center of this coefficient area is subtracted from the coordinates to put them in a local system.

$$X_L = X - X_{OFF} \tag{7.18.1}$$

$$X_L = Y - Y_{OFF} \tag{7.18.2}$$

where

$X, Y$ = input digital data coordinates

$X_{OFF}, Y_{OFF}$ = coordinates of center of coefficient area.

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Form B matrix where

$$[B] = f_{7.12}(X_L, Y_L).$$ (7.18.3)

Compute the Z coordinate from:

$$Z = [B] \cdot [P]$$ (7.18.4)

where

$[P] =$ computed coefficients representing terrain (Section 7.49).

## 7.19 Subroutine CONTR

| | |
|---|---|
| Purpose: | To locate contour header from digital data tape and begin collecting data. |
| Entry Points: | None |
| Elements: | None |
| Common Areas: | /INDEX/,/MODELP/,/ADDRES/,/UNITS/,/DATA/,/PHOTO/, /PARAM/ |
| Subroutines Used: | IMVSTR,IRANIO,ISTOR,LSTOR,MVSTR,SORTBL,BUFFIN,SORTPR |
| Link: | CONTR |

## 7.20 Subroutine CORNER

| | |
|---|---|
| Purpose: | Determine minimum and maximum values of model system. |
| Entry Points: | None |
| Elements: | None |
| Common Areas: | /FPDLPR/,/MINMAX/,/PHOTO/ |
| Subroutines Used: | MOVE |
| Link: | EXTRA |

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## 7.21    Subroutine CORUSR (ICARD, ITYPE)

| Purpose: | Reads digital data corner information in image coordinates and object coordinates. A matrix is then formed to relate these two coordinate systems. |
|---|---|

**Entry Points:**    None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | I | ICARD | – | Option: data input (CARDS, TAPE) |
| I | I | ITYPE | – | Option: data input (USR, GEO, UTM) |

**Common Areas:**    /UNITS/, /DIGIT/, /OPTION/

**Subroutines Used:**    DMSRAD, PLHXYZ, TRANS, ADDATA, ARXY

**Link:**    INPUT

**Mathematics:**

Given the UTM coordinates of a point, the USR coordinates are computed by first converting to geographic coordinates. This computation is computed by:

$$\begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix} = f_{7.94} \begin{bmatrix} X_{UTM} \\ Y_{UTM} \end{bmatrix}, \tag{7.21.1}$$

where

$X_{UTM}, Y_{UTM}$ = UTM coordinates of the corner.

Knowing the geographic coordinates, the conversion to USR is:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{USR} = f_{7.60} \begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}. \tag{7.21.2}$$

7.22    Subroutine DDTOMD(XT,YT,XPNT,YPNT)

Purpose:            To convert digital data to model coordinates.

Entry Points:       None

Elements:

| I/O | Type | Variable | Dimension | | Description |
|-----|------|----------|-----------|---|-------------|
| I | R | XT | – | X | Input digital data |
| I | R | YT | ·· | Y | coordinates |
| O | R | XPNT | – | X | Model coordinates |
| O | R | YPNT | – | Y | |

Common Areas:       /LVTRAN/,/PHOTO/,/PARAM/

Subroutines Used:   None

Link:               CREATE

Mathematics:

Given the digital data coordinates of a point, the model coordinates are computed by converting the digital data to Universal Space Rectangular (USR) coordinates and then transforming these USR coordinates to model coordinates.

The transformation of digital data to USR is accomplished by:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \cdot \lambda_{G/DD} - \begin{bmatrix} X_D \\ Y_D \end{bmatrix}, \quad (7.22.1)$$

where

$\alpha$ = azimuth relating digital data to ground

$x, y$ = digital data coordinates

$X_D, Y_D$ = origin of digital data in USR coordinates

$\lambda_{G/DD}$ = scale (ground to digital data).

The conversion from ground (USR) to model is computed by the applying of scale and a rotation matrix:

$$
\begin{bmatrix} X_M \\ Y_M \end{bmatrix} = \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \cdot 1/\lambda_{G/M}, \tag{7.22.2}
$$

where

$X_M , Y_M$ = model coordinates

$\gamma$ = azimuth relating ground to model

$\lambda$ = scale relating ground to model

$X, Y$ = USR coordinates.

## 7.23 Function DISC (L,X,YLM,YLIM)

| | |
|---|---|
| Purpose: | To compute location stored in random file. |
| Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | I | L | - | Profile number within 1 column |
| I | R | X | -. | X value of profile |
| I | R | XLM | - | Upper limit of block |
| I | R | YLIM | - | Lower limit of block |

| | |
|---|---|
| Common Areas: | /FPDLPR/,/FPINT/,/UNITS/ |
| Subroutines Used: | None |
| Link: | PROFIL |
| Mathematics: | |

Compute the disc storage location where

$$ DISC = IBASE + N \tag{7.23.1} $$

$$ IBASE = 10 * ((IX - IXMIN)/IBLOCK) * ((IYMAX - IYMIN)/IBLOCK) \tag{7.23.2} $$

$$ N = (L - 1) * ((IYMAX - IYMIN)/IBLOCK) + IPROF + 1 \tag{7.23.3} $$

where

$$IX = x \text{ value of profile}$$
$$IXMIN = \text{minimum } x \text{ value}$$
$$IBLOCK = \text{block size}$$
$$IYMAX = \text{maximum } y \text{ value}$$
$$IYMIN = \text{minimum } y \text{ value}$$
$$L = \text{profile number}$$
$$IPROF = \text{blocks previously stored from profile } L.$$

## 7.24   Subroutine DMSRAD (L,M,R,B)

| | | | | |
|---|---|---|---|---|
| **Purpose:** | To convert deg., min., and sec. to radians. | | | |
| **Entry Points:** | None | | | |

| **Elements:** | I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|---|
| | I | I | L | – | Integer degrees |
| | I | I | M | – | Integer minutes |
| | I | R | R | – | Seconds (real) |
| | 0 | R | B | – | Radians |

| | |
|---|---|
| **Common Areas:** | None |
| **Subroutines Used:** | None |
| **Link:** | MAIN |

## 7.25   Subroutine DOMLV (ANG,DOM)

| | |
|---|---|
| **Purpose:** | To compute the local vertical rate matrix |
| **Entry Points:** | None |

| **Elements:** | I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|---|
| | I | R | ANG | (6) | L.V. angles and rates |
| | 0 | R | DOM | (3,3) | L.V. rate matrix |

| | |
|---|---|
| **Common Areas:** | None |
| **Subroutines Used:** | None |
| **Link:** | SDT |

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Mathematics:

$A(1,1) = \cos\varphi \cos\alpha \, d\alpha - \sin\alpha \sin\varphi \, d\varphi$

$A(1,2) = -\cos\varphi \sin\alpha \, d\alpha - \cos\alpha \sin\varphi \, d\varphi$

$A(1,3) = \cos\varphi \, d\varphi$

$A(2,1) = -\cos\omega \, d\omega \sin\varphi \sin\alpha - \sin\omega \cos\varphi \cos\alpha \, d\varphi + \sin\omega \sin\varphi \sin\alpha \, d\alpha$
$\qquad\qquad + \sin\omega \cos\alpha \, d\omega + \cos\omega \sin\alpha \, d\alpha$

$A(2,2) = -\cos\omega \sin\varphi \cos\alpha \, d\omega - \sin\omega \cos\varphi \cos\alpha \, d\varphi - \sin\omega \sin\varphi \sin\alpha \, d\alpha$
$\qquad\qquad - \sin\omega \sin\alpha \, d\omega + \cos\omega \cos\alpha \, d\alpha$

$A(2,3) = \cos\omega \cos\varphi \, d\omega - \sin\omega \sin\varphi \, d\varphi$

$A(3,1) = \sin\omega \sin\varphi \sin\alpha \, d\omega - \cos\omega \cos\varphi \sin\alpha \, d\varphi - \cos\omega \sin\varphi \cos\alpha \, d\alpha$
$\qquad\qquad + \cos\omega \cos\alpha \, d\omega - \sin\omega \sin\alpha \, d\alpha$

$A(3,2) = \sin\omega \sin\varphi \cos\alpha \, d\omega - \cos\omega \cos\varphi \cos\alpha \, d\varphi + \cos\omega \sin\varphi \sin\alpha \, d\alpha$
$\qquad\qquad - \cos\omega \sin\alpha \, d\omega - \sin\omega \cos\alpha \, d\alpha$

$A(3,3) = -\sin\omega \cos\varphi \, d\omega - \cos\omega \sin\varphi \, d\varphi$

## 7.26 Subroutine EFIT

| | |
|---|---|
| Purpose: | Evaluate the fit of the coefficients using known coordinates. |
| Entry Points: | None |
| Elements: | None |
| Common Areas: | /UNITS/,/PARMS/,/CONST/,/NODES/,/FPINT/ |
| Subroutines Used: | BMATX,MATMPY |
| Link: | EXTRA |

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Mathematics:

Using the x,y image coordinates, the matrix of partial derivatives can be computed from

$$[B] = f_{7.13}(x,y).$$ (7.26.1)

Having computed the B matrix, the associated Z value can be computed from

$$Z_c = [B] \cdot [P]$$ (7.26.2)

where

$[P]$ = coefficients derived from normal solution of multiquadratic.

Given the "Z" value read from the input digital data tape and the newly computed "Z" value, a difference can be computed

$$\Delta Z = Z_R - Z_c$$ (7.26.3)

where

$Z_R$ = "Z" read from tape

$Z_c$ = computed Z.

The sum of the squares of $\Delta Z$ is used to compute the mean error in Z.

$$Z_{ME} = \sum_{i=1}^{N} (Z_{D_i})^2 / N$$ (7.26.4)

where

$N$ = number of points evaluated.

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## 7.27   Subroutine EVAL (ZME)

| | | | | |
|---|---|---|---|---|
| Purpose: | | To evaluate fit of derived coefficients. | | |
| Entry Points: | | None | | |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| 0 | R | ZME | – | Mean error in Z |

Common Areas:   /CONST/,/DATA/,/STORE/,/FPINT/,/UNITS/,/FPDLPR/, /PARMS/

Subroutines Used:   RREAD,VAL

Link:   WORK

Mathematics:

Using the $x, y$ image coordinates, the matrix of partial derivatives can be computed from:

$$[B] = f_{7.11}(x, y). \qquad (7.27.1)$$

Having computed the $[B]$ matrix, the associated Z value can be computed from

$$Z_c = [B][P], \qquad (7.27.2)$$

where

$[P]$ = coefficients derived from normal solution of multiquadratic.

Given the "Z" value read from the input digital data tape and the newly computed "Z" value, a difference can be computed.

$$\Delta Z = Z_R - Z_c \qquad (7.27.3)$$

where

$Z_R$ = "Z" read from tape

$Z_c$ = computed "Z".

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The sum of the squares of $\Delta Z$ is used to compute the mean

error in Z.

$$Z_{ME} = \sum_{i=1}^{N} (\Delta Z_i)^3 / N \qquad\qquad (7.27.4)$$

where

$N$ = number of points evaluated.

## 7.28   Subroutine EXLVAN (ANGLS)

| Purpose: | To extract local vertical angles and rates |
| --- | --- |

**Entry Points:**   None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
| --- | --- | --- | --- | --- |
| 0 | R | ANGLS | (6) | L.V. angles and rates |

| Common Areas: | Blank |
| --- | --- |
| Subroutines Used: | None |
| Link: | SDT |

**Mathematics:**

Given the local vertical orientation matrix and local vertical
rate matrix, the local vertical angles and rates can be extracted.

$$\varphi = \text{arc tan} \left[ \frac{OM(1,3)}{OM(3,3)/\cos \omega} \right] \qquad\qquad (7.28.1)$$

$$\omega = \text{arc tan} \left[ \frac{OM(2,3)}{OM(3,3)} \right] \qquad\qquad (7.28.2)$$

$$\alpha = \text{arc tan} \left[ \frac{OM(1,1)}{OM(1,2)} \right] \qquad\qquad (7.28.3)$$

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$$d\varphi = DOM(1,3)/\cos\varphi \tag{7.28.4}$$

$$d\omega = [DOM(2,3) + \sin\omega \sin\varphi \, d\varphi]/(\cos\omega \cos\varphi) \tag{7.28.5}$$

$$d\alpha = [DOM(1,2) + \cos\alpha \sin\varphi \, d\varphi]/(-\cos\varphi \sin\alpha) \tag{7.28.6}$$

where

$$OM = \begin{bmatrix} \cos\varphi \sin\alpha & \cos\varphi \cos\alpha & \sin\varphi \\ \begin{aligned} -\sin\omega \sin\varphi \sin\alpha \\ -\cos\omega \cos\alpha \end{aligned} & \begin{aligned} -\sin\omega \sin\varphi \cos\alpha \\ +\cos\omega \sin\alpha \end{aligned} & \sin\omega \cos\varphi \\ \begin{aligned} -\cos\omega \sin\varphi \sin\alpha \\ +\sin\omega \cos\alpha \end{aligned} & \begin{aligned} -\cos\omega \sin\varphi \cos\alpha \\ -\sin\omega \sin\alpha \end{aligned} & \cos\omega \cos\varphi \end{bmatrix}, \tag{7.28.7}$$

$$DOM(1,2) = -\sin\varphi \cos\alpha \, d\varphi - \sin\alpha \cos\varphi \, d\alpha \tag{7.28.8}$$

$$DOM(1,3) = \cos\varphi \, d\varphi$$

$$DOM(2,3) = \cos\omega \cos\varphi \, d\omega - \sin\varphi \sin\omega \, d\varphi.$$

## 7.29  Subroutine FPBOND

**Purpose:** Arranges footprint corners so first point has minimum X coordinates. Also limits of boundaries are determined.

**Entry Points:** None

**Elements:** None

**Common Areas:** /FPDLPR/,/MINMAX/

**Subroutines Used:** MOVE

**Link:** STORE

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**7.30** Subroutine GENODE

| | |
|---|---|
| **Purpose:** | To generate the coordinates of the nodes. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /FPDLPR/,/NODES/,/CONST/,/UNITS/ |
| **Subroutines Used:** | None |
| **Link:** | STORE |
| **Mathematics:** | |

The coordinates of the nodes are computed from the equations:

$$NODX(I) = DELX*(J-1) - XSIZE \qquad (7.30.1)$$
$$NODY(I) = DELY*(K-1) - YSIZE$$

where

$NODX(I), NODY(I)$ = coordinates of node I

$XSIZE$ = block size in X

$YSIZE$ = block size in Y

$DELX$ = (2.*XSIZE)/NODEX

$NODEX$ = total number of nodes in X

$DELY$ = (2.*YSIZE)/NODEY

$NODEY$ = total number of nodes in Y

$J$ = variable from 1 → NODEX

$K$ = variable from 1 → NODEY

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**7.31**   Subroutine GEOPM (XYZ,PLH,XYZPLH)

| | |
|---|---|
| **Purpose:** | To provide the matrix of partial derivatives of geocentric X,Y,Z with respect to geographic $\varphi,\lambda,H$. |
| **Entry Points:** | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | XYZ | (3) | Geocentric coordinates X,Y,Z |
| I | R | PLH | (3) | Geographic coordinates $\varphi,\lambda,H$ |
| 0 | R | XYZPLH | (3,3) | Matrix of partial derivatives of X,Y,Z with respect to $\varphi,\lambda,H$ |

| | |
|---|---|
| **Common Areas:** | /GEODET/ |
| **Subroutines Used:** | None |
| **Link:** | SDT |

**Mathematics:**

Given the geocentric coordinates and geographic coordinates, a matrix of partial derivatives of geocentric X,Y,Z with respect to geographic $\varphi,\lambda,h$. The matrix is composed of the following elements:

$$(1,1) = -[A/(1 - e^2 \sin^2 \varphi \cos\lambda + [[e^2 (A/(1 - e^2 \sin^2 \varphi)^{\frac{1}{2}}) \sin\varphi \cos\varphi]$$
$$/(1 - e^2 \sin^2 \varphi)] \cos\varphi \cos\lambda$$

$$(1,2) = -Y$$

$$(1,3) = \cos\varphi \cos\lambda$$

$$(2,1) = -[(A/(1 - e^2 \sin^2 \varphi)^{\frac{1}{2}}) + h] \sin\varphi \sin\lambda + [[e^2 (A/(1 - e^2 \sin^2 \varphi)^{\frac{1}{2}}) \sin\varphi \cos\varphi]$$
$$/(1 - e^2 \sin^2 \varphi)^{\frac{1}{2}}] \cos\varphi \sin\lambda$$

$$(2,2) = X$$

$$(2,3) = \cos\varphi \sin\lambda$$

$$(3,1) = \cos\varphi[(A/(1 - e^2 \sin^2 \varphi)^{\frac{1}{2}})(1 - e^2) + h] + [[e^2 [A/(1 - e^2 \sin^2 \varphi)^{\frac{1}{2}}] \sin\varphi \cos\varphi]$$
$$/(1 - e^2 \sin^2 \varphi)] (1 - e^2) \sin\varphi$$

$$(3,2) = 0$$

$$(3,3) = \sin\varphi \qquad\qquad\qquad (7.31.1)$$

**where**

$$A = \text{semi-major axis of the earth}$$

$$e^2 = \text{eccentricity squared.}$$

## 7.32 Subroutine GEUTM(OPHI,OLAM,PHI1,ALAM,OK,X,Y)

| | | | | |
|---|---|---|---|---|
| **Purpose:** | To transform geographic coordinates to UTM coordinates. | | | |
| **Entry Points:** | None | | | |
| **Elements:** | | | | |

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | R | OPHI | – | UTM origin in $\varphi$ |
| I | R | OLAM | – | UTM origin in $\lambda$ |
| I | R | PHI1 | – | Input $\varphi$ |
| I | R | ALAM | – | Input $\lambda$ |
| I | R | OK | – | SCALE |
| 0 | R | X | – | Computed UTM value |
| 0 | R | Y | – | Computed UTM value |

| | |
|---|---|
| **Common Areas:** | /GEODET/ |
| **Subroutines Used:** | None |
| **Link:** | CORD |
| **Mathematics:** | |

Given the geographic coordinates of a point, the UTM

coordinates can be computed from

$$X = OK\ (XPR) + BX$$
$$Y = OK\ (YPR) \tag{7.32.1}$$

**where**

$OK = \text{scale}$

$BX = \text{air base component in X}$

$XPR = EN + (DLC + DLC^3\ (1 - (\tan\varphi_1)^2 + ET)/6$
$\qquad\qquad + DLC^5\ ((5 - 18(\tan\varphi_1)^2 + (\tan\varphi_1)^4)/120$

$EN = A/(1 - e^2\ (\sin\varphi_1)^2)^{\frac{1}{2}}$

$DLC = \Delta\lambda\ \cos\varphi_1$

$ET = [e^2/(1 - e^2)](\cos\varphi_1)^2$

$$YPR = S + EN((DLC - \sin \varphi_1 \Delta\lambda)/2 + DLC^3 \sin \varphi_1 \Delta\lambda$$
$$(5 - (\tan \varphi_1)^2 /24)$$

$$S = A(1-e^2)[A_1(\varphi_1 - \varphi_0) - (B/2)(\sin(2\varphi_1) - \sin(2\varphi_0))$$
$$+ C/4 (\sin(4\varphi_1) - \sin(4\varphi_0)) - (D/6)(\sin(6\varphi_1)$$
$$- \sin(6\varphi_0))]$$

$$\Delta\lambda = \lambda_0 - \lambda_1$$

$A$ = semi-major axis

$e^2$ = eccentricity

$\varphi_0, \lambda_0$ = geographic coordinates of origin

$\varphi_1, \lambda_1$ = geographic coordinates of point

$$A1 = 1 + 3/4 \, e^2 + 45/64 \, e^4 + 175/256 \, e^6$$
$$B = 1/2[3/4 \, e^2 + 15/16 \, e^4 + 525/512 \, e^6]$$
$$C = 1/4[15/64 \, e^4 + 105/256 \, e^6]$$
$$D = 1/6[35/512 \, e^6]$$

## 7.33 Subroutine GNODES (NODX,NODY)

| | |
|---|---|
| **Purpose:** | Compute nodal points for multi-quad solution. |
| **Entry Points:** | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | I | NODX | – | Number of nodes in X |
| I | I | NODY | – | Number of nodes in Y |

| | |
|---|---|
| **Common Areas:** | /FPDLPR/,/NODES/,/CONST/,/UNITS/,/FPINT/ |
| **Subroutine Used:** | None |
| **Link:** | EXTRA |

The equation for the computation of the nodel point coordinates are:

$$NODX(I) = XMIN + DELX * (J-1)$$
$$NODY(I) = YMIN + DELY * (K-1)$$

(7.33.1)

where

$NODX(I), NODY(I)$ = coordinates of node I

XMIN = minimum x value for computation area

YMIN = minimum y value for computation area

DELX = (XMAX - XMIN)/(NODEX-1)

    XMAX = maximum x value

    NODEX= number of nodes in x

DELY = (YMAX - YMIN)/(NODEY - 1)

    YMAX = maximum y value

    NODEY= number of nodes in Y

J    = variable from 1 →NODEX

K    = variable from 1 →NODEY

## 7.34  Subroutine IMOVE (IA,IB,N)

| | | | | |
|---|---|---|---|---|
| Purpose: | Integer move of data in labeled common block /DATA/ | | | |
| Entry Points: | None | | | |

| Elements: | I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|---|
| | I | I | IA | – | First address of array to move |
| | I | I | IB | – | First address of array moving to |
| | I | I | N | – | Number of words to be relocated |

| | |
|---|---|
| Common Areas: | /DATA/ |
| Subroutines Used: | None |
| Link: | MERGE,CONTR |

## 7.35    Subroutine IMVSTR

**Purpose:**            To transform boundary limits to new coordinate system.

**Entry Points:**       None

**Elements:**           None

**Common Areas:**       /MODELP/,/INDEX/,/UNITS/,/PARAM/,/DATA/,/LVTRAN/, /FPDLPR/,/PHOTO/

**Subroutines Used:**   None

**Link:**               SORTPR

**Mathematics:**

Given the digital data corners in the digital data coordinate system, the conversion to USR coordinates is computed as:

$$\begin{bmatrix} X_i \\ Y_i \end{bmatrix} = \begin{bmatrix} \cos(AZ) & \sin(AZ) \\ -\sin(AZ) & \cos(AZ) \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} \cdot \lambda_{G/DD} - \begin{bmatrix} X_{ORG} \\ Y_{ORG} \end{bmatrix}, \quad (7.35.1)$$

where

$$AZ \quad = \text{azimuth relating ground to digital data}$$
$$x_i, y_i \quad = \text{image coordinates of point i}$$
$$\lambda_{G/DD} \quad = \text{scale relating ground to digital data}$$
$$X_{ORG}, Y_{ORG} = \text{USR coordinates of digital data origin}$$

With these newly computed USR coordinates, the model coordinates are derived from:

$$\begin{bmatrix} x_M \\ y_M \end{bmatrix}_i = \begin{bmatrix} \cos(AZIM) & -\sin(AZIM) \\ \sin(AZIM) & \cos(AZIM) \end{bmatrix} \cdot \begin{bmatrix} X_i \\ Y_i \end{bmatrix} \cdot 1.000/\lambda_{G/M}, \quad (7.35.2)$$

where

$$AZIM = \text{azimuth relating ground to model}$$
$$X_i, Y_i = \text{USR coordinates of point i}$$
$$\lambda_{G/M} = \text{scale relating ground to model.}$$

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A comparison between the four digital data corners and the coordinates of the model corners will define the area over which profiles are to be created and the number of profiles to be generated.

## 7.36 Subroutine INCLUD (IC)

| | | | | |
|---|---|---|---|---|
| Purpose: | To include additional data points. | | | |
| Entry Points: | None | | | |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | I | IC | - | Number of profiles currently being processed |

| | |
|---|---|
| Common Areas: | /DATA/,/MODELP/,/LASTPT/,/UNITS/ |
| Subroutines Used: | None |
| Link: | DERIVE |

## 7.37 Subroutine INPHDT

| | |
|---|---|
| Purpose: | To input data necessary for computing model and photo variables. |
| Entry Points: | None |
| Elements: | None |
| Common Areas: | /UNITS/,/PARAM/,/GEODET/,/ADDPT/,/OFF/,/OPTION/, /UNPAR/,/GROUND/,/PHOTO/ |
| Subroutines Used: | CLEAR,DMSRAD,EXLVAN,MATMPY,MOVE,PLHXYZ,PROJ, PTAPE,RGEO,RUSR,USRLVR,XYZPLH,OMATLV,DOMLV, RAS11,RUTM |
| Link: | INPUT |

Mathematics:

If the exposure station coordinates are input in USR or LSR coordinates, a transformation from USR to $\varphi,\lambda,h$ must be performed

$$\begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix} = f_{7.99} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\substack{USR \\ LSR}} . \tag{7.37.1}$$

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Knowing the geographic coordinates of the exposure station, the Local Vertical Orientation Matrix can be computed from

$$R_{USR} = f_{7.93} \begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}, \tag{7.37.2}$$

$$O_{LV} = R_{USRLV} \; O_{USR}, \tag{7.37.3}$$

where

$O_{USR}$ = USR orientation matrix.

From the local vertical orientation matrix, the Local vertical angles can be extracted

$$\begin{bmatrix} \varphi_{LV} \\ \lambda_{LV} \\ \gamma_{LV} \end{bmatrix} = f_{7.28} \; (O_{LV}). \tag{7.37.4}$$

Given the film format coordinates, the ground coordinates of the film corners can be computed by:

$$\begin{bmatrix} \varphi_{ci} \\ \lambda_{ci} \\ h_{ci} \end{bmatrix} = f_{7.64} \begin{bmatrix} x_i \\ y_i \end{bmatrix}, \tag{7.37.5}$$

where

$x_i, y_i$ = coordinates of corner i as i = 1-4.

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**7.38** Subroutine IRANIO (ICUR,ITYPE,IPRES)

| | | | | | |
|---|---|---|---|---|---|
| **Purpose:** | Initialize random storage routine. | | | | |
| **Entry Points:** | None | | | | |
| **Elements:** | I/O | Type | Variable | Dimension | Description |
| | I | I | ICUR | - | Initialize storage area |
| | I | I | ITYPE | - | Not used |
| | I | I | IPRES | - | Precision of word |
| **Common Areas:** | /UNITS/,/DATA/,/RANDBL/,/PDPIND/ | | | | |
| **Subroutines Used:** | None | | | | |
| **Link:** | PROFIL,EXTRA,CONTR | | | | |

**7.39** Subroutine ISTOR

| | |
|---|---|
| **Purpose:** | Set up common areas and clear arrays to be used in STOR routine. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /MODELP/,/UNITS/,/IDXBL/,/ADDRES/,/DATA/ |
| **Subroutines Used:** | KLEAR |
| **Link:** | SORTPR |

**7.40** Subroutine KLEAR (IDATA,N)

| | | | | | |
|---|---|---|---|---|---|
| **Purpose:** | Clear integer array. | | | | |
| **Entry Points:** | None | | | | |
| **Elements:** | I/O | Type | Variable | Dimension | Description |
| | I/O | I | IDATA | (N) | Array to clear |
| | I | I | N | - | Number of members of array to clear |
| **Common Areas:** | None | | | | |
| **Subroutines Used:** | None | | | | |
| **Link:** | CONTR | | | | |

**7.41** Subroutine LIMIT (X,YINTR,RMXY)

| | | | | | |
|---|---|---|---|---|---|
| **Purpose:** | | To compute maximum and minimum Y value given an X value. Also gives X and Y limits of the sheet. | | | |
| **Entry Points:** | | None | | | |
| **Elements:** | I/O | Type | Variable | Dimension | Description |
| | I | R | X | – | X value |
| | 0 | R | YINTR | (2) | Y intersections given Y |
| | 0 | R | RMXY | (4) | Max & min X&Y value of sheet |
| **Common Areas:** | | /PHOTO/,/FPDLPR/ | | | |
| **Subroutines Used:** | | MOVE | | | |
| **Link:** | | DERIVE | | | |


**7.42** Subroutine LSTOR

| | |
|---|---|
| **Purpose:** | To store the last selected points to the random file and write a flag to indicate that this is the last profile. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /MODELP/,/UNITS/,/IDXBL/,/ADDRES/,/DATA/ |
| **Subroutines Used:** | RWRITE |
| **Link:** | SORTPR |


**7.43** Subroutine MAIN

| | |
|---|---|
| **Purpose:** | Main program – controls program flow |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /GEODET/,/OPTION/,/UNITS/,/PHOTO/,/PDPIND/, |
| **Subroutines Used:** | INPHDT,CORUSR,CONTR,PROFIL,SCATER,MRGE |
| **Link:** | MAIN |

**7.44** Subroutine MATMPY (LAA,NRA,NCA,LAB,NRB,NCB,LAC,ICT,ICC)

| | | |
|---|---|---|
| **Purpose:** | | To compute the matrix product of arrays LAA and LAB. The result of this multiplication is stored in LAC. |
| **Entry Points:** | | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | R | LAA | (1) | Array LAA |
| I | I | NRA | – | Number rows in LAA |
| I | I | NCA | – | Number columns in LAA |
| I | R | LAB | (1) | Array LAB |
| I | I | NRB | – | Number rows in LAB |
| I | I | NCB | – | Number columns in LAB |
| I/O | R | LAC | (1) | Array LAC (product) |
| I | I | ICT | – | Flag to designate which matrix is to be transposed |

$$0 \rightarrow R = AB$$
$$1 \rightarrow R = A^T B$$
$$2 \rightarrow R = AB^T$$
$$3 \rightarrow R = A^T B^T$$

| I | I | ICC | – | Flag to designate summation or subtraction |
|---|---|-----|---|---------|

$0 \rightarrow R$ is stored in LAC
$1 \rightarrow LAC + R$ is stored in LAC
$2 \rightarrow LAC - R$ is stored in LAC
$3 \rightarrow \quad - R$ is stored in LAC

| | |
|---|---|
| **Common Areas:** | None |
| **Subroutines Used:** | None |
| **Link:** | MAIN |

**7.45** Subroutine MAXM(X,Y1,Y2)

| | | | |
|---|---|---|---|
| Purpose: | To test for slope of line and return maximum and minimum Y values. | | |
| Entry Points: | None | | |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | X | – | X value |
| 0 | R | Y1 | – | Maximum Y value |
| 0 | R | Y2 | – | Minimum Y value |

| | |
|---|---|
| Common Areas: | /CONST/,/FPDLPR/ |
| Subroutines Used: | XMN |
| Link: | WORK |

**7.46** Subroutine MDTODD (X,Y,X1,Y1)

| | |
|---|---|
| Purpose: | To convert model coordinates to input digital data coordinates. |
| Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | |
|---|---|---|---|---|
| I | R | X | – | X ⎫ Model coordinates |
| I | R | Y | – | Y ⎭ |
| 0 | R | X1 | – | X ⎫ Input digital data |
| 0 | R | Y1 | – | Y ⎭ system coordinates |

| | |
|---|---|
| Common Areas: | /LVTRAN/,/PARAM/,/PHOTO/ |
| Subroutines Used: | None |
| Link: | CREATE |

Mathematics:

Given the model coordinates of a point, the computation of the digital data corners involve transformation from model to ground and then ground to model.

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The transformation from model to ground is accomplished by:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} x_M \\ y_M \end{bmatrix} \lambda_{g/M} + \begin{bmatrix} X_D \\ Y_D \end{bmatrix},$$ (7.46.1)

where

$\gamma$  = azimuth relating model to ground

$x_M , y_M$  = model coordinates

$\lambda_{g\,M}$  = scale relating ground to model

$X_D , Y_D$  = origin of digital data in USR coordinates

$X, Y$  = USR coordinates of point $(x_M , y_M)$.

The digital data coordinates can now be computed from these

computed ground coordinates where

$$\begin{bmatrix} x_{DD} \\ y_{DD} \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \cdot 1.0/\lambda_{g/DD},$$ (7.46.2)

where

$\alpha$  = azimuth relating digital data to ground

$\lambda_{g/DD}$  = scale relating ground to digital data

$x_{DD} , y_{DD}$  = digital data coordinates of given model point.

## 7.47  Subroutine MINV (B,N)

| | | | | |
|---|---|---|---|---|
| **Purpose:** | To compute matrix inversion. | | | |
| **Entry Points:** | None | | | |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I/O | R | B | (N,N) | Matrix to be inverted |
| I | I | N | – | Number of rows and columns in B |

| | |
|---|---|
| **Common Areas:** | None |
| **Subroutines Used:** | None |
| **Link:** | DERIVE,INPUT |

**7.48**   Subroutine MOVE (A, B, N)

| | Purpose: | Move data from array A to array B (does not destroy data in array A) |
|---|---|---|

| | Entry Points: | None |
|---|---|---|

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I/O | R | A | (1) | Array data to be moved from |
| 0 | R | B | (1) | Array data to be moved to |
| I | I | N | - | Number of words moved |

| | |
|---|---|
| Common Areas: | None |
| Subroutines Used: | None |
| Link: | MAIN |

**7.49**   Subroutine MQFIT (KFLAG)

| | |
|---|---|
| Purpose: | To compute multi-quad. |
| Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I/O | I | KFLAG | - | Flag for forming normals |

| | |
|---|---|
| Common Areas: | /DATA/,/OPTION/,/STORE/,/CONST/,/PARMS/,/FPDLPR/, /FPINT/,/UNITS/ |
| Subroutines Used: | ADD,CLEAR,DISC,EVAL,MATMPY,MRTPLY,RREAD,RWRITE, SURV,TRIPLY,TRIVRT,MAXM,UNPRPK,BMAT,PROPK |
| Link: | WORK |

Mathematics:

The solution for the coefficients representing the terrain surfaces involves the solution of the matrix of partial derivatives where:

$$[B] = f_{7.11}(x,y). \tag{7.49.1}$$

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Computing this matrix for all $(x,y)$ in a block, the normal solution and constant vector can be computed from

$$N = \sum_{j=1}^{M} [B][B^T],\qquad(7.49.2)$$

and

$$C = \sum_{j=1}^{M} [B] Z_j,\qquad(7.49.3)$$

where

$j = 1 \rightarrow$ number of points in block

$Z_j$ = height of the jth point.

The matrix $[N]$ is formed and inverted for one block of data only. The solution of the coefficients involves the equation

$$[P] = N^{-1} C.\qquad(7.49.4)$$

This $[P]$ is computed for all data blocks and stored to the random file.

7.50    Subroutine MRGE

| | |
|---|---|
| Purpose: | To combine multiple data sets for the final profile tape. |
| Entry Points: | None |
| Elements: | None |
| Common Areas: | /UNITS/,/DATA/,/PHOTO/,/MINMAX/,/RANDBL/,/BUFFER/ |
| Subroutines Used: | MOVE,BUFFOT,RWRITE,IMOVE,RREAD |
| Link: | MERGE |

**7.51**   Subroutine MTREAD (IWD, KBUF, ISTAT)

| | | |
|---|---|---|
| **Purpose:** | | Assembly language subroutine for buffering in or out a physical record from magnetic tape. |
| **Entry Points:** | | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | I | IWD | – | Number of words to buffer in or out |
| I | I | KBUF | (768) | Array containing data |
| 0 | I | ISTAT | – | Status of operation |

| | |
|---|---|
| **Common Areas:** | None |
| **Subroutines Used:** | None |
| **Link:** | STORE, CONTR, MERGE |

**7.52**   Subroutine MRTPLY (R, NRR, NCR, T, NRT, NCT, F, IRS)

| | |
|---|---|
| **Purpose:** | To multiply a rectangular matrix and a triangular matrix and output rectangular matrix. |
| **Entry Points:** | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | R | R | (1) | Rectangular matrix |
| I | I | NRR | – | Number rows in R |
| I | I | NCR | – | Number columns in R |
| I | R | T | (1) | Triangular matrix |
| I | I | NRT | – | Number rows in T |
| I | I | NCT | – | Number columns in T |
| 0 | R | F | (1) | Resulting matrix |
| I | I | IRS | – | Option: |
| | | | | $-1$  F = – R*T |
| | | | | 0  F = F + R*T |
| | | | | 1  F =   R*T |

| | |
|---|---|
| **Common Areas:** | None |
| **Subroutines Used:** | None |
| **Link:** | WORK |

**7.53**    Subroutine MULTIQ (KNOD)

| | Purpose: | To compute the multiquadratic fit and derive final profile data. |
|---|---|---|

**Entry Points:**    None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | I | KNOD | – | – |

**Common Areas:**    /NOFIT/,/UNITS/,/MODELP/,/ADDRES/,/PHOTO/, /LASTPT/,/DATA/,/RANDBL/

**Subroutines Used:**    CLEAR,MATMPY,MINV,LIMIT,KLEAR,MOVE,RWRITE,RREAD

**Link:**    DERIVE

Mathematics:

This subroutine will compute the output profiles representing the terrain using data from the MMS-32 cartographic data tape. On option, this terrain will be represented by a three to seven term polynomial.

The coefficients for the polynomial solution involves the equations:

$$N = \sum_{i=1}^{n} B^T B$$

$$\tag{7.53.1}$$

$$C = \sum_{i=1}^{n} B^T Z$$

where

$$B = \text{partial derivative of } f_x$$
$$f_x = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2$$
$$x = x_i - x_0$$
$$y = y_i - y_0$$
$$x_i, y_i = \text{coordinates of data point i}$$
$$x_0, y_0 = \text{coordinates of local origin}$$
$$Z = Z_i - f_{x_i}$$
$$\Delta = N^{-1} C. \tag{7.53.2}$$

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The number of coefficients used to represent a specific raster area is dependent upon the number of data points available. Thus, the number of polynomial terms varies from a minimum of three (3) to a maximum of five (5). See Section 3.2 for additional detail.

The terrain representation is computed as:

$$Z = B \cdot A \qquad\qquad (7.53.3)$$

where

$B$ = partial derivatives

$A$ = computed coefficients.

Equation 7.53.3 is evaluated for all $(x,y)$ values within the area the coefficients were computed over.

## 7.54 Subroutine MVSTR (IL, NBLTP, FELEV)

| | | | | |
|---|---|---|---|---|
| Purpose: | To locate and store data with respect to specific profiles. | | | |
| Entry Points: | None | | | |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | I | IL | – | Data index within physical block |
| I | I | NBLTP | – | Physical data block count |
| I | R | FELEV | – | Elevation associated with current data |

| | |
|---|---|
| Common Areas: | /MODELP/,/INDEX/,/UNITS/,/PHOTO/,/DATA/,/LVTRAN/ |
| Subroutines Used: | STOR |
| Link: | SORTPR |

**Mathematics:**

Having read a block of data from the MMS-32 contour tape, all coordinates must be converted to the model coordinate system for comparison to desired profile coordinates. If these coordinates fall within a boundary surrounding a profile, the values are stored.

The transformation from MMS-32 coordinates to model coordinates involve:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos(AZ) & \sin(AZ) \\ -\sin(AZ) & \cos(AZ) \end{bmatrix} \begin{bmatrix} X_{DD} \\ Y_{DD} \end{bmatrix} \cdot \lambda_{g/DD} - \begin{bmatrix} X_{ORG} \\ Y_{ORG} \end{bmatrix}, \qquad (7.54.1)$$

where

$$AZ \qquad = \text{azimuth relating digital data to ground}$$
$$X_{DD}, Y_{DD} \quad = \text{digital data coordinates of point}$$
$$\lambda_{g/DD} \qquad = \text{scale relating ground t digital data}$$
$$X_{ORG}, Y_{ORG} = \text{USR coordinates of digital data origin.}$$

$$\begin{bmatrix} x_M \\ y_M \end{bmatrix} = \begin{bmatrix} \cos(AZIM) & -\sin(AZIM) \\ \sin(AZIM) & \cos(AZIM) \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \cdot 1000/\lambda_{G/M},$$

where

$$AZIM = \text{azimuth relating ground to model}$$
$$X, Y = \text{USR coordinates of point}$$
$$\lambda_{g/M} = \text{scale relating ground to model.}$$

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**7.55**   Subroutine MXMN (X,YMX,YMN)

| | | | | |
|---|---|---|---|---|
| **Purpose:** | To compute minimum and maximum Y for each block. | | | |
| **Entry Points:** | None | | | |
| **Elements:** | | | | |

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | X | – | X input |
| 0 | R | YMX | – | Maximum Y |
| 0 | R | YMN | – | Minimum Y |

| | |
|---|---|
| **Common Areas:** | /FPDLPR/,/UNITS/ |
| **Subroutines Used:** | XMN |
| **Link:** | PROFIL |

**7.56**   Subroutine NEW

| | |
|---|---|
| **Purpose:** | To compute the new profiles. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /PARMS/,/MINMAX/,/FPDLPR/,/PHOTO/,/DATA/, /UNITS/,/CONST |
| **Subroutines Used:** | DMATX,MATMPY,PPACK,CLEAR,COFF |
| **Link:** | EXTRA |
| **Mathematics:** | |

The computed height of the terrain at point $(x,y)$ is computed by:

$$[B] = f_{7.13}(x,y) \qquad (7.56.1)$$

$$Z = [B] \cdot [P] \qquad (7.56.2)$$

where

$[B]$ = matrix of partial derivatives evaluated for all nodes

$[P]$ = computed coefficients representing terrain.

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**7.57** Subroutine OMATLV (ANG,OMATRX)

| | | | | |
|---|---|---|---|---|
| **Purpose:** | To compute L.V. orientation matrix given L.V. angles. | | | |
| **Entry Points:** | None | | | |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | ANG | (3) | L.V. angles $\varphi, \omega, \alpha$ |
| 0 | R | OMATRX | (9) | Array containing computed orientation matrix |

| | |
|---|---|
| **Common Areas:** | None |
| **Subroutines Used:** | None |
| **Link:** | SDT |
| **Mathematics:** | |

The ground to photograph local vertical orientation matrix is

defined as:

$$T_{LV}$$

where

$$T_{LV}(1,1) = \sin \alpha \cos \varphi$$
$$T_{LV}(1,2) = \cos \alpha \sin \varphi$$
$$T_{LV}(1,3) = \sin \varphi$$

$$T_{LV}(2,1) = \cos \alpha \cos \omega - \sin \alpha \sin \varphi \sin \omega$$
$$T_{LV}(2,2) = -\sin \alpha \cos \omega - \cos \alpha \sin \varphi \sin \omega \qquad (7.57.1)$$
$$T_{LV}(2,3) = \cos \varphi \sin \omega$$

$$T_{LV}(3,1) = -\cos \alpha \sin \omega - \sin \alpha \sin \varphi \cos \omega$$
$$T_{LV}(3,2) = \sin \alpha \sin \omega - \cos \alpha \sin \varphi \cos \omega$$
$$T_{LV}(3,3) = \cos \varphi \cos \omega$$

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**7.58** Subroutine OUTPRO (K,NOY)

| | | | | |
|---|---|---|---|---|
| Purpose: | Outputs profile data to file called IOUT. | | | |
| Entry Points: | None | | | |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | I | K | - | Profile number |
| I | I | NOY | - | Total number of Y coordinate values in profile |

| | |
|---|---|
| Common Areas: | /IDXBL/,/MODELP/,/UNITS/,/DATA/ |
| Subroutines Used: | IMOVE |
| Link: | SORTPR |

**7.59** Subroutine PACK (K,FX,FY,FZ,IDX)

| | |
|---|---|
| Purpose: | To move a 32-bit real word into two 16-bit integer words. |
| Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | I | K | - | Profile ID |
| I | R | FX | - | X value |
| I | R | FY | - | Y value |
| I | R | FZ | - | Z value |
| I | R | IDX | - | First address of point in common block |

| | |
|---|---|
| Common Areas: | /DATA/ |
| Subroutines Used: | None |
| Link: | CONTR |

**7.60    Subroutine PK (VALE,IFX)**

| | | | | | |
|---|---|---|---|---|---|
| Purpose: | | To pack one real word into two integer words. | | | |
| Entry Points: | | None | | | |
| Elements: | I/O | Type | Variable | Dimension | Description |
| | I | R | VALE | – | Real word input |
| | 0 | I | IFX | (2) | Integer word array |
| Common Areas: | | None | | | |
| Subroutines Used: | | None | | | |
| Link: | | STORE | | | |

**7.61    Subroutine PLHXYZ (PLH,XYZ)**

| | | | | | |
|---|---|---|---|---|---|
| Purpose: | | To convert geographic coordinates of geocentric coordinates. | | | |
| Entry Points: | | None | | | |
| Elements: | I/O | Type | Variable | Dimension | Description |
| | I | R | PLH | (3) | Geographic coordinates |
| | 0 | R | XYZ | (3) | Geocentric coordinates |
| Common Areas: | | /GEODET/ | | | |
| Subroutines Used: | | None | | | |
| Link: | | INPUT | | | |

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**7.62**   Subroutine PPACK (I,X,Y,Z,K)

Purpose:              To pack real words in two interger words.

Entry Points:         None

Elements:

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | I | I | - | First location of real words in integer array |
| I | R | X | - | X value to be stored |
| I | R | Y | - | Y value to be stored |
| I | R | Z | - | Z value to be stored |
| I | I | K | - | K offset from first location in integer array |

Common Areas:         /DATA/

Subroutines Used:     None

Link:                 EXTRA


**7.63**   Subroutine PROFIL

Purpose:              Main program for processing profile data.

Entry Points:         None

Elements:             None

Common Areas:         /OPTION/,/CONST/,/PARMS/,/UNITS/,/FPDLPR/,
                      /STORE/,/FPINT/,/MINMAX/,/NODES/,/PHOTO/,
                      /PARAM/,/LVTRAN/

Subroutines Used:     BOUND,FPBOND,GENODE,IRANIO,MQFIT,PROFLE,ASTRP

Link:                 PROFIL


**7.64**   Subroutine PROFLE

Purpose:              To compute final output profiles.

Entry Points:         None

Elements:             None

Common Areas:         /UNITS/,/DATA/,/MINMAX/,/FPDLPR/,/PHCTO/,
                      /CONST/,/PARAM/,/LVTRAN/

Subroutines Used:     CONTF,PROPAK,XYINT

Link:                 CREATE

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## 7.65 Subroutine PROJ (XY,PHLAHT)

**Purpose:** To calculate universal space rectangular coordinates of the point of intersection of a single ray from a spatial photograph with a reference ellipsoid.

**Entry Points:** None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | R | XY | (2) | Image coordinate of point |
| O | R | PHLAHT | (3) | $\varphi, \lambda, H$ of image input |

**Common Areas:** /GEODET/,/OPTION/,/PARAM/,/UNPAR/

**Subroutines Used:** MATMPY,MOVE,OMATLV,XYZPLH,USRLVR

**Link:** SDT

**Mathematics:**

Given the image coordinates and focal length of the camera, the direction cosines can be computed by:

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = T_{USR} \begin{bmatrix} x \\ -y \\ -f \end{bmatrix}, \qquad (7.65.1)$$

where

$x, -y$ = image coordinates of point

$-f$ = focal length of camera

$T_{USR}$ = USR orientation matrix.

Knowing the direction cosines, the USR coordinates can be computed by:

$$\begin{aligned}
Z(1) &= (-B+D)/2A \\
Z(2) &= (-B-D)/2A \\
X(1) &= U/W\,(Z1) + (X + \dot{X}(t)) - U/W(t) \\
X(2) &= U/W\,(Z2) + (X + \dot{X}(t)) - U/W(t) \qquad (7.65.2) \\
Y(1) &= V/W\,(Z1) + (Y + \dot{Y}(t)) - V/W(t) \\
Y(2) &= V/W\,(Z2) + (Y + \dot{Y}(t)) - V/W(t)
\end{aligned}$$

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where

$$A = (U/W)^2 + (V/W)^2 + 1/(1 - e^2)$$

$$B = 2[U/W((X + \dot{X}(t)) - U/W(Z + \dot{Z}(t))) + (V/W)((Y + \dot{Y}(t)) - V/W(Z - \dot{Z}(t)))]$$

$$C = [X + \dot{X}(t) - U/W(Z - \dot{Z}(t))]^2 + [Y + \dot{Y}(t) - V/W(Z - \dot{Z}(t))]^2 - a^2$$

$$D = [B^2 - 4AC]^{\frac{1}{2}}$$

$$e^2 = \text{eccentricity}$$

$$t = \text{time from center of scan (for pan)}$$

$$X, Y, Z = \text{exposure station coordinates}$$

$$\dot{X}, \dot{Y}, \dot{Z} = \text{linear rates}$$

$$a = \text{semi-major axis.}$$

From the above computed $X, Y, Z$ values, the distance from the origin can be computed by:

$$R(1) = [(X(1) - X_0)^2 + (Y(1) - Y_0)^2 + (Z(1) - Z_0)^2]^{\frac{1}{2}}$$

$$R(2) = [(X(2) - X_0)^2 + (Y(2) - Y_0)^2 + (Z(2) - Z_0)^2]^{\frac{1}{2}}$$

(7.65.3)

where

$$X_0, Y_0, Z_0 = \text{coordinates of the exposure station.}$$

The comparison of the R values determine desired point of intersection. The USR coordinates will be $X(1), Y(1), Z(1)$ unless $R(1) > R(2)$, then the USR coordinates $X(2), Y(2), Z(2)$ are used.

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## 7.66   Subroutine PROPAK (K,X,Y,Z)

| | |
|---|---|
| **Purpose:** | To pack a real word into common area occupied by two integer words. |
| **Entry Points:** | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | I | K | – | Point location in array |
| I | R | X | – | X value |
| I | R | Y | – | Y value |
| I | R | Z | – | Z value |

| | |
|---|---|
| **Common Areas:** | /DATA/ |
| **Subroutines Used:** | None |
| **Link:** | CREATE |

## 7.67   Subroutine PROPK (I,X,Y,Z,K)

| | |
|---|---|
| **Purpose:** | To pack a real word into area occupied by two integer words. |
| **Entry Points:** | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | I | I | – | Indicates point location in array |
| I | R | X | – | X value to store |
| I | R | Y | – | Y value to store |
| I | R | Z | – | Z value to store |
| I | I | K | – | Offset of storage location from first location in array |

| | |
|---|---|
| **Common Areas:** | /DATA/ |
| **Subroutines Used:** | None |
| **Link:** | PROFIL |

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**7.68** Subroutine PTAPE

| | |
|---|---|
| **Purpose:** | Prepare data for punching AS11 shutdown tape. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /UNITS/,/OPTION/,/UNPAR/,/GEODET/,/GROUND/, /PARAM/,/PHOTO/,/OFF/ |
| **Subroutines Used:** | ARATES,AS11AN,CLEAR,MATMPY,MOVE,OMATLV, PLHXYZ,PUNDEC,USRLVR,PUNTAP |
| **Link:** | SDT |
| **Mathematics:** | |

Compute Y axis in LV system

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = R_{USRLV} \begin{bmatrix} X_1 - X_0 \\ Y_1 - Y_0 \\ Z_1 - Z_0 \end{bmatrix}, \qquad (7.68.1)$$

where

$X_1, Y_1, Z_1$    are USR values of projected Y axis

$X_0, Y_0, Z_0$    USR values of origin

$X_2, Y_2, Z_2$    LV values of projected Y axis

$R_{USRLV}$    is $f_{7.94}$.

Compute azimuth of model system

$$AZIM = \arctan [X_2/Y_2]. \qquad (7.68.2)$$

Compute $B_x, B_Y, B_Z$

$$\begin{bmatrix} B_x \\ B_Y \\ B_Z \end{bmatrix}_{LSR} = R_{USRLSR} \begin{bmatrix} X_\ell - X_0 \\ Y_\ell - Y_0 \\ Z_\ell - Z_0 \end{bmatrix}_{USR}, \qquad (7.68.3)$$

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**where**

$$X_\xi, Y_\xi, Z_\xi = \text{coordinates of exposure station}$$
$$B_x, B_Y, B_z = \text{air base components.}$$

The atmospheric refraction coefficients are computed using a polynomial derived to fit the graphic table shown in Figure 6-1 and Figure 6-2 of the volume I of the AS-11B-1 manuals. If the flying height is above 50 kilometers a maximum set of values are used. For under 50 kilometers the following equation is used to compute the AS-11 atmospheric coefficients (CE and CH).

$$CH = (a_0 + a_1 H + a_2 H^2 + a_3 H^3 + a_4 H^4 + a_5 H^5)(1/MS)(C)$$
$$CE = (b_0 + b_1 H + b_2 H^2 + b_3 H^3 + b_4 H^4 + b_5 H^5)(MS/1000)$$

(7.68.4)

**where**

MS = Model Scale

H = flying Height of vehicle (in kilometers)

C = 1.0E + 06

| | | | |
|---|---|---|---|
| $a_0$ = -243.001 | | $b_0$ = 1.698E-03 | |
| $a_1$ = 115.042 | | $b_1$ = -3.943E-04 | |
| $a_2$ = - 7.168 | | $b_2$ = 4.030E-05 | |
| $a_3$ = 18.5999 | | $b_3$ = -1.911E-06 | |
| $a_4$ = - .8107 | | $b_4$ = 4.067 | |
| $a_5$ = .00928 | | $b_5$ = -3.133 | |

**7.69** Subroutine PUNCH (ICH)

| Purpose: | Assembly language routine for punching paper tapes. | | | | |
|---|---|---|---|---|---|
| Entry Points: | None | | | | |
| Elements: | I/O | Type | Variable | Dimension | Description |
| | I | I | ICH | - | Character to be punched |
| Common Areas: | None | | | | |
| Subroutines Used: | None | | | | |
| Link: | SDT | | | | |

**7.70** Subroutine PUNDEC (DNUM,NODEC)

| | | | | | |
|---|---|---|---|---|---|
| Purpose: | To punch data on paper tape. | | | | |
| Entry Points: | None | | | | |
| Elements: | I/O | Type | Variable | Dimension | Description |
| | I | R | DNUM | – | Decimal number to be punched |
| | I | I | NODEC | – | Number of decimal places |
| Common Areas: | /UNITS/ | | | | |
| Subroutines Used: | CLEAR | | | | |
| Link: | SDT | | | | |

**7.71** Subroutine PUNTAP

| | |
|---|---|
| Purpose: | Paper tape punch routine. |
| Entry Points: | None |
| Elements: | None |
| Common Areas: | /UNITS/ |
| Subroutines Used: | PUNCH |
| Link: | SDT |

**7.72** Subroutines PXYZ (PLH,XYZ)

| | | | | | |
|---|---|---|---|---|---|
| Purpose: | Transform geographic coordinates to geocentric coordinates. | | | | |
| Entry Points: | None | | | | |
| Elements: | I/O | Type | Variable | Dimension | Description |
| | I | R | PLH | (3) | Geographic latitude, longitude and height |
| | O | R | XYZ | (3) | USR coordinates |
| Common Areas: | /GEODET/ | | | | |
| Subroutines Used: | None | | | | |
| Link: | EXTRA | | | | |

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**7.73** <u>Subroutine RAS11</u>

| | |
|---|---|
| **Purpose:** | Input AS11 shutdown tape data and compute rotation matrix. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /PARAM/,/UNITS/,/GEODET/ |
| **Subroutines Used:** | AS11M,UTMPRP,UTMGE,USRLVR,PLHXYZ,MATMPY,XYZPLH, MOVE |
| **Link:** | CORD |

**7.74** <u>Subroutine RGEO</u>

| | |
|---|---|
| **Purpose:** | To create additional point file.  Data read in geographics. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /UNITS/ |
| **Subroutine Used:** | DMSRAD,PLHXYZ |
| **Link:** | SDT |

**7.75** <u>Subroutine RREAD (IUN,IFWA,NWDS,IDIS)</u>

| | |
|---|---|
| **Purpose:** | To read from the random file. |
| **Entry Points:** | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I/O | I | IUN | – | Unit reading from |
| I | I | IFWA | – | First location to read |
| I | I | NWDS | – | Number of words |
| I | I | IDIS | – | Storage location on random file |

| | |
|---|---|
| **Common Areas:** | /RANDBL/,/UNITS/,/DATA/ |
| **Subroutines Used:** | None |
| **Link:** | PROFIL,EXTRA,MERGE,CONTR |

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**7.76** Subroutine RTPLY (R,NRR,NCR,T,NRT,NCT,F,IRS)

| | Purpose: | To multiply a rectangular matrix and a triangular matrix. Output is a rectangular matrix. |
|---|---|---|

**Entry Points:** None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | R | (1) | Rectangular matrix |
| I | I | NRR | – | Number rows in R |
| I | I | NCR | – | Number columns in R |
| I | R | T | (1) | Triangular matrix |
| I | I | NRT | – | Number rows in T |
| I | I | NCT | – | Number columns in T |
| O | R | F | (1) | Resulting matrix |
| I | I | IRS | – | Options: |

|  |  |
|---|---|
| -1 | -R*T |
| 0 | C+R*T |
| 1 | R*T |

**Common Areas:** None

**Subroutines Used:** None

**Link:** EXTRA

**7.77** Subroutine RUSR

**Purpose:** To create additional point file. Data read in USR coordinates.

**Entry Points:** None

**Elements:** None

**Common Areas:** /UNITS/

**Subroutines Used:** None

**Link:** SDT

7.78    Subroutine RUTM (OPHI,OLAM)

Purpose:            To read in additional points in UTM coordinates and output
                    on a file their USR coordinates.

Entry Points:       None

Elements:

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | R | OPHI | – | Origin of the UTM values |
| I | R | OLAM | – | |

Common Areas:       /UNITS/,/PHOTO/,/GEODET/,/OPTION/

Subroutines Used:   UTMPRP,UTMGE,PLHXYZ

Link:               CORD


7.79    Subroutine RWRITE (IUN,IFWA,NWDS,IDIS)

Purpose:            To write to the random file

Entry Points:       None

Elements:

| I/O | Type. | Variable | Dimension | Description |
|-----|-------|----------|-----------|-------------|
| I/O | I | IUN | – | Unit to write on |
| I | I | IFWA | – | First location in common |
| I | I | NWDS | – | Number of words |
| 0 | I | IDIS | – | Storage location on random file |

Common Areas:       /UNITS/,/RANDBL/,/DATA/

Subroutines Used:   None

Link:               PROFIL,EXTRA,MERGE,CORTR


7.80    Subroutine SCATER

Purpose:            Control program flow when using scattered points for making
                    an orthophoto.

Entry Points:       None

Elements:           None

Common Areas:       /UNITS/,/GEODET/,/DATA/,/FPDLPR/,/NODES/,/CONST/,
                    /PHOTO/,/PARAM/,/MINMAX/,/PARMS/,/FPINT/

Link:               EXTRA

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7.81  Subroutine SOLVE

| | |
|---|---|
| **Purpose:** | Form and solve the normal equations and compute the coefficients. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /NODES/,/DATA/,/CONST/,/PARMS/,/FPDLPR/,/UNITS/, /FPINT/ |
| **Subroutines Used:** | BMATX,TRIPLY,MATMPY,TIVRT,RTPLY,CLEAR |
| **Link:** | EXTRA |

**Mathematics:**

The solution for the coefficients representing the terrain surface involves the solution of the matrix of partial derivatives were

$$[B] = f_{7.13} (x,y). \tag{7.81.1}$$

Computing this matrix for all $(x,y)$, the normal solution and constant vector can be computed from

$$N = \sum_{J=1}^{M} [B][B^T] \tag{7.81.2}$$

and

$$C = \sum_{J=1}^{M} [B] Z_J \tag{7.81.3}$$

where

$J$ = 1 → number of points

$Z_J$ = height of the $J$ th point.

The matrix $[N]$ is found and inverted for one block of data only. The solution for the coefficients involves the equation

$$[P] = N^{-1} C. \tag{7.81.4}$$

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7.82   Subroutine SORTBL

| | |
|---|---|
| **Purpose:** | To read data from random file and sort as to profiles. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /IDXBL/,/UNITS/,/ADDRES/,/MODELP/,/ADIDBL/,/DATA/ |
| **Subroutines Used:** | KLEAR,OUTPRO,PACK,RREAD,RWRITE,UNPACK |
| **Link:** | SORTPR |

7.83   Subroutine SORTPR

| | |
|---|---|
| **Purpose:** | To read in sets of four profiles. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /UNITS/,/LASTPT/,/NOFIT/,/MODELP/,/DATA/,/OPTION/, /RANDBL/ |
| **Subroutines Used:** | KLEAR,INCLUD,SRFILL |
| **Link:** | DERIVE |

7.84   Subroutine SRFIL

| | |
|---|---|
| **Purpose:** | To sort profile sets into raster data. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /LASTPT/,/NOFIT/,/UNITS/,/MODELP/,/DATA/ |
| **Subroutines Used:** | MULTIQ |
| **Link:** | DERIVE |

**7.85** <u>Subroutine STOR (K,FX,FY,FZ)</u>

| | | | | |
|---|---|---|---|---|
| **Purpose:** | To store selected points of contour data to a random file. | | | |
| **Entry Points:** | None | | | |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | I | K | – | Profile ID |
| I | R | FX | – | Profile X value |
| I | R | FY | – | Profile Y value |
| I | R | FZ | – | Profile Z value |

| | |
|---|---|
| **Common Areas:** | /MODELP/,/UNITS/,/IDXBL/,/ADDRES/,/DATA/ |
| **Subroutines Used:** | PACK,RWRITE |
| **Link:** | SORTPR |

**7.86** <u>Subroutine SURV (C)</u>

| | |
|---|---|
| **Purpose:** | To include random point contribution in normal equation. |
| **Entry Points:** | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I/O | R | C | (70) | Constant vector |

| | |
|---|---|
| **Common Areas:** | /CONST/,/FPDLPR/,/DATA/,/UNITS/ |
| **Subroutines Used:** | BMAT,MATMPY,TRIPLY |
| **Link:** | WORK |

**Mathematics:**

Given image coordinates of the randomly distributed survey or terrain points, the partial derivative matrix is computed as:

$$[B] = f_{7.11}(x,y). \tag{7.86.1}$$

This [B] matrix contribution is summed into the normal equation and constant vector formed in the profile block (Section 7.50).

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$$N = \sum_{J=1}^{N} [B][B^T] \qquad\qquad (7.86.2)$$

$$C = \sum_{J=1}^{N} [B] Z_J \qquad\qquad (7.86.3)$$

where

$J = 1 \rightarrow$ total number of points.

These matrices are used in the subroutine described in Section 7.49. They are entered as the solution to equations 7.49.2 and 7.49.3.

7.87 <u>Subroutine TIVRT (TMT,SVC,MAR,MFI)</u>

| Purpose: | Inversion and/or solution for a triangular matrix. |
| --- | --- |
| Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
| --- | --- | --- | --- | --- |
| I/O | R | TMT | (1) | Matrix stored upper triangular |
| I/O | R | SVC | (1) | Solution vector |
| I | I | MAR | – | Matrix rank |
| I | I | MFI | – | Options: |

<div style="text-align:right">

-1 inverse only, no solution
0 solution only, no inverse
1 both inverse and solution

</div>

| Common Areas: | None |
| --- | --- |
| Subroutines Used: | None |
| Link: | EXTRA |

**7.88** <u>Subroutine TR (X,Y,Z,Z1,XX,YY,ZZ)</u>

| Purpose: | To transform points from USR coordinates to model scale. |

| Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | R | X | - | X USR coordinate |
| I | R | Y | - | Y USR coordinate |
| I | R | Z | - | Z USR coordinate |
| I | R | Z1 | - | Z UTM value |
| O | R | XX | - | x model coordinate |
| O | R | YY | - | y model coordinate |
| O | R | ZZ | - | z model coordinate |

| Common Areas: | /PHOTO/,/PARAM/ |
| Subroutines Used: | PXYZ,MATMPY |
| Link: | EXTRA |

Mathematics:

Given the geographic coordinates of the origin, the USR coordinates can be computed from:

$$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = f_{7.71} \begin{bmatrix} \varphi_0 \\ \lambda_0 \\ h_0 \end{bmatrix} \cdot \tag{7.88.1}$$

Knowing the USR coordinates of the origin and the USR coordinates of a point, the model coordinates can be computed by:

$$\begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = R^T_{USR\,LSR} \begin{bmatrix} X_U - X_0 \\ Y_U - Y_0 \\ Z_U - Z_0 \end{bmatrix}, \tag{7.88.2}$$

where

$X_U, Y_U, Z_U$ = USR coordinates of point

$R_{USR\,LSR}$ = rotation from USR to LSR system.

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(AZ) & \sin(AZ) & 0 \\ -\sin(AZ) & \cos(AZ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} \cdot \lambda_{G/M} \qquad (7.88.3)$$

where

$AZ$ = azimuth relating ground to model

$\lambda_{G/M}$ = scale relating ground to model.

## 7.89    Subroutine TRANFR

| | |
|---|---|
| **Purpose:** | To transform data from USR coordinates to model coordinates. |
| **Entry Points:** | None |
| **Elements:** | None |
| **Common Areas:** | /PHOTO/,/UNITS/,/CONST/,/PARAM/ |
| **Subroutines Used:** | MOVE,PXYZ,MATMPY,TR |
| **Link:** | EXTRA |

## 7.90    Subroutine TRANS (XY,XYZUSR)

**Purpose:**    Compute a two-dimensional transformation.

**Entry Points:**    None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | XY | (2,4) | Image coordinate of D.D. corners |
| I | R | XYZUSR | (3,4) | Object coordinate of D.D. corners |

**Common Areas:**    /UNITS/,/GEODET/,/PHOTO/,/LUTRAN/

**Subroutines Used:**    CLEAR,MATMPY,MINV,MOVE,PLHXYZ,USRLVR

**Link:**    COR

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Mathematics:

Given the equations

$$f_x = (x \cos \alpha + y \sin \alpha)S - X_D \qquad (7.90.1)$$

$$f_y = (-x \sin \alpha + y \cos \alpha)S - Y_D \qquad (7.90.2)$$

where

$$x, y = \text{image coordinates}$$

$$\alpha = \text{azimuth}$$

$$S = \text{scale}$$

$$X_D, Y_D = \text{LSR coordinates of corners}.$$

To solve equations 7.90.1 and 7.90.2 an iteration of the least square solution is performed.

The matrix of partial derivatives of $f_x$ and $f_y$ is computed. The elements comprising the partial derivative matrix is:

$$[B] = \begin{bmatrix} \partial f_x/\partial \alpha & \partial f_x/\partial X_D & \partial f_x/\partial Y_D & \partial f_x/\partial S \\ \partial f_y/\partial \alpha & \partial f_y/\partial X_D & \partial f_y/\partial Y_D & \partial f_y/\partial S \end{bmatrix}, \qquad (7.90.3)$$

The constant vector is comprised of

$$[C] = \begin{bmatrix} X - (x \cos \alpha + y \sin \alpha)S + X_D \\ Y - (-x \sin \alpha + y \cos \alpha)S + Y_D \end{bmatrix}. \qquad (7.90.4)$$

The least squares solution involves

$$N = \sum_{i=1}^{4} B^T B \qquad (7.90.5)$$

$$\bar{C} = \sum_{i=1}^{4} B^T C \qquad (7.90.6)$$

$$\Delta = N^{-1} \bar{C} \qquad (7.90.7)$$

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the values contained in the $[\Delta]$ matrix are corrections for $\alpha, X_D, Y_D, S$ where

$$\alpha_N = \alpha_O + \Sigma\Delta\alpha$$
$$X_{DN} = X_{DO} + \Sigma\Delta X_D$$
$$Y_{DN} = Y_{DO} + \Sigma\Delta Y_D \qquad\qquad (7.90.8)$$
$$S_N = S_O + \Sigma\Delta S_D$$

where

$$\Delta\alpha, \Delta X_D, \Delta Y_D, \Delta S = \text{change in } \alpha, X_D, Y_D, S$$
$$\alpha_O, X_{DO}, Y_{DO}, S_O = \text{initial approximations}$$
$$\alpha_N, X_{DN}, Y_{DN}, S_N = \text{updated values.}$$

## 7.91   Subroutine TRIPLY (LAA, NRA, NCA, LAB, NRB, NCB, LAC, ICT, ICC)

| | |
|---|---|
| **Purpose:** | To compute the matrix product of arrays LAA and LAB storing the upper triangular portion only. The result of this multiplication is stored in LAC. |
| **Entry Points:** | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | LAA | (1) | Array LAA |
| I | I | NRA | – | Number rows in LAA |
| I | I | NCA | – | Number columns in LAA |
| I | R | LAB | (1) | Array LAB |
| I | I | NRB | – | Number rows in LAB |
| I | I | NCB | – | Number columns in LAB |
| I/O | R | LAC | (1) | Array LAC (product) |
| I | I | ICT | – | FLAG to designate which matrix are to be transposed: |

$$0 \rightarrow R = A\,B$$
$$1 \rightarrow R = A^T B$$
$$2 \rightarrow R = A\,B^T$$
$$3 \rightarrow R = A^T B^T$$

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | I | ICC | | FLAG to designate summation or subtraction |

$$0 \rightarrow R \text{ is stored in LAC}$$
$$1 \rightarrow \text{LAC} + R \text{ is stored in LAC}$$
$$2 \rightarrow \text{LAC} - R \text{ is stored in LAC}$$
$$3 \rightarrow \qquad - R \text{ is stored in LAC}$$

| | |
|---|---|
| **Common Areas:** | None |
| **Subroutines Used:** | None |
| **Link:** | MAIN |

## 7.92 Subroutine TRIVRT (TMT,SVC,MAR,MFI)

| | Purpose: | Inversion solution for a triangular matrix. |
|---|---|---|
| | Entry Points: | None |

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I/O | R | TMT | (1) | Matrix stored upper triangular |
| I/O | R | SVC | (1) | Solution vector |
| I | I | MAR | – | Matrix rank |
| I | I | MFI | – | FLAG to designate solution and/or inversion |

Elements:

-1 – inverse only, no solution
0 – solution only, no inverse
1 – both invert + solution

| | |
|---|---|
| Common Areas: | None |
| Subroutines Used: | None |
| Link: | WORK |

## 7.93 Subroutine UNPACK (K,FX,FY,FZ,IDX)

| | Purpose: | To move two 16-bit integer words to a 32-bit real word. |
|---|---|---|
| | Entry Points: | None |

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| 0 | I | K | – | Profile ID |
| 0 | R | FX | – | X value |
| 0 | R | FY | – | Y value |
| 0 | R | FZ | – | Z value |
| I | I | IDX | – | Address of point in common block |

Elements:

| | |
|---|---|
| Common Areas: | /DATA/ |
| Subroutines Used: | None |
| Link: | CONTR |

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**7.94**  **Subroutine UNPRPK (K,L,X,Y,Z)**

| Purpose: | To unpack real words which have been stored in two integer word locations. |
|---|---|

Entry Points:     None

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | I | K | – | Word location in common array |
| I | I | L | – | Number to indicate words per point |
| 0 | R | X | – | X value |
| 0 | R | Y | – | Y value |
| 0 | R | Z | – | Z value |

Common Areas:     /DATA/

Subroutines Used:   None

Link:            PROFIL


**7.95**  **Subroutine USRLVR (PLG,ROTMAT)**

| Purpose: | To compute transformation from geocentric to local space rectangular. |
|---|---|

Entry Points:     None

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | PLG | (3) | Geographic $\varphi,\lambda,\gamma$ of LSR system |
| 0 | R | ROTMAT | (3,3) | Transformation matrix |

Common Areas:     None

Subroutines Used:   None

Link:            INPUT

Mathematics:

Given the geographic coordinates for the origin of the LSR system and the azimuth of the "Y" axis from north the rotation matrix from USR to local space rectangular (LSR) is defined as:          (7.95.1)

$$
\begin{bmatrix}
-\cos\gamma\sin\lambda + \sin\gamma\sin\varphi\cos\lambda & -\sin\dot\gamma\sin\lambda - \cos\gamma\sin\varphi\cos\lambda & \cos\varphi\cos\lambda \\
\cos\gamma\cos\lambda + \sin\gamma\sin\varphi\sin\lambda & \sin\gamma\cos\lambda - \cos\gamma\sin\varphi\sin\lambda & \cos\varphi\sin\lambda \\
-\sin\gamma\cos\varphi & \cos\gamma\cos\varphi & \sin\varphi
\end{bmatrix} .
$$

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**7.96**   Subroutine UTMGE (OPHI,OLAM,XG,YG,OK,OUT,PHI,ALAM)

| Purpose: | Transform UTM to geodetic coordinates. |
| --- | --- |
| Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
| --- | --- | --- | --- | --- |
| I | R | OPHI | – | UTM origin on map sheet ($\omega$) |
| I | R | OLAM | – | UTM origin on map sheet ($\lambda$) |
| I | R | XG | – | UTM coordinates to be transformed |
| I | R | YG | – | UTM coordinates to be transformed |
| I | R | OK | – | Scale factor |
| I | R | OUT | (9) | Matrix necessary for UTM conversion |
| O | R | PHI | – | $\varphi$ of converted coordinate |
| O | R | ALAM | – | $\lambda$ of converted coordinate |

| Common Areas: | /GEODET/ |
| --- | --- |
| Subroutines Used: | None |
| Link: | CORD |

Mathematics:

Conversion process for converting Universal Transverse Mercator to Geographics

$$x = (X_G - B_x)/OK \tag{7.96.1}$$
$$y = Y_G/OK$$

where

$X_G, Y_G$ = UTM coordinates of point G

OK = scale factor

$B_x$ = air base component in X

$$R = a(1 - e^2)/(1 - e^2 . \sin^2 \varphi_0)^{3/2} \tag{7.96.2}$$

where

a = semi-major axis

$e^2$ = eccentricity

$\varphi_0$ = UTM origin in $\varphi$ of Geographics

$$\varphi' = \varphi_0 + y/R \tag{7.96.3}$$

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$$A = OUT(3)$$
$$B = OUT(4)$$
$$C = OUT(5) \tag{7.96.4}$$
$$D = CUT(6)$$

where

$OUT(9)$ = nine element matrix from
UTMPRP (Section 7.96)

$$S = a(1-e^2)[A(\varphi'-\varphi_o) - B(\sin 2\varphi' - \sin 2\varphi_o) + \tag{7.96.5}$$
$$C(\sin 4\varphi' - \sin 4\varphi_o) - D(\sin 6\varphi' - \sin 6\varphi_o)]$$

$$\varphi' = \varphi' - (s-y)/R \tag{7.96.6}$$

Return to equation 7.96.5 until s-y < .005 or until ten iterations are made.

$$N = a/(1-e^2 \sin^2 \varphi')^{\frac{1}{2}} \tag{7.96.7}$$

$$\eta^2 = \cos^2 \varphi' [e^2/1 - e^2] \tag{7.96.8}$$

$$R = a(1-e^2)/(1-e^2 \sin^2 \varphi')^{3/3} \tag{7.96.9}$$

$$\Delta\varphi = \tan\varphi' [-x^2/2R(X/N) + x^4/24RN^3(5+3\tan^2 \varphi')] \tag{7.96.10}$$

$$\varphi = \varphi' + \Delta\varphi \tag{7.96.11}$$

$$\Delta\lambda = 1/\cos\varphi'[X/N - 1/6(X/N)^3(1+2\tan^2 \varphi' + \eta^2) \tag{7.96.12}$$
$$+ 1/120(X/N)^5(5+28\tan^2 \varphi' + 24\tan^4 \varphi')]$$

$$\lambda = \lambda_0 - \Delta\lambda \ (\lambda \text{ positive west of Greenwich}). \tag{7.96.13}$$

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## 7.97   Subroutine UTMPRP (OPHI,OUT)

| Purpose: | Data preparation for computing USR coordinates from UTM coordinates. |
| --- | --- |
| Entry Points: | None |

**Elements:**

| I/O | Type | Variable | Dimension | Description |
| --- | --- | --- | --- | --- |
| I | R | OPHI | – | PHI of origin of UTM coordinates |
| 0 | R | OUT | (9) | Matrix necessary for UTM conversion |

| Common Areas: | /GEODET/ |
| --- | --- |
| Subroutines Used: | None |
| Link: | CORD |

**Mathematics:**

This routine forms a nine element array of constants needed for transforming UTM coordinates to geographics

$$OUT(1) = \sin \varphi$$

$$OUT(2) = \cos \varphi$$

$$OUT(3) = 1 + (3/4)\, e^2 + (45/64)\, e^4 + (175/256)\, e^6$$

$$OUT(4) = 1/2\left[(3/4)\, e^2 + (15/16)\, e^4 + (525/512)\, e^6\right] \qquad (7.97.1)$$

$$OUT(5) = 1/4\left[(15/64)\, e^4 + (105/256)\, e^6\right]$$

$$OUT(6) = 1/6\left[(35/512)\, e^6\right]$$

$$OUT(7) = \sin 2\varphi$$

$$OUT(8) = \sin 4\varphi$$

$$OUT(9) = \sin 6\varphi.$$

**7.98**   Subroutine XFILL (IPR,Z,X,J)

| | Purpose: | To fill block in X direction if EOF is reached first |
|---|---|---|
| | Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I/O | I | IPR | - | Profile number |
| I | R | Z | - | Z value of last point |
| I | R | X | - | X value of profile to store |
| I | I | J | - | Index of data storage location |

| | |
|---|---|
| Common Areas: | /DATA/,/FPINT/,/STORE/,/UNITS/,/CONST/,/FPDLPR/ |
| Subroutines Used: | DISC,RWRITE,PK |
| Link: | STORE |

**7.99**   Subroutine XMN (X,YMX,YMN)

| | Purpose: | To solve for the maximum and minimum Y value given an X value |
|---|---|---|
| | Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | X | - | X value input |
| O | R | YMX | - | Maximum Y value |
| O | R | YMN | - | Minimum Y value |

| | |
|---|---|
| Common Areas: | /FPDLPR/,/UNITS/ |
| Subroutines Used: | None |
| Link: | PROFIL |

**7.100**   Subroutine XYINT (X,YINTR,RMXY)

| | Purpose: | Computes maximum Y and minimum Y value given an X. Also computes the limits in X and Y. |
|---|---|---|
| | Entry Points: | None |

Elements:

| I/O | Type | Variable | Dimension | Description |
|---|---|---|---|---|
| I | R | X | - | X value |
| O | R | YINTR | 2 | Y intersections given X value |
| O | R | RMXY | 4 | Maximum and minimum X&Y values for particular X value |

| | |
|---|---|
| Common Areas: | /PHOTO/,/MINMAX/,/FPDLPR/ |
| Subroutines Used: | MOVE |
| Link: | CREATE |

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## 7.101 Subroutine XYZPLH(XYZ,PLH)

**Purpose:** To convert from geocentric coordinates X,Y,Z to geographic coordinates latitude, longitude (in radians) and height (in meters)

**Entry Points:** None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | R | XYZ | (3) | Geocentric XYZ in meters |
| O | R | PLH | (3) | Geographic latitude, longitude and height |

**Common Areas:** /GEODET/

**Subroutines Used:** None

**Link:** INPUT

## 7.102 Subroutine YFILL (Z,YI,IYX,I,IPR,X,YM,YLM)

**Purpose:** To fill block IF maximum Y value on tape is encountered before block is full

**Entry Points:** None

**Elements:**

| I/O | Type | Variable | Dimension | Description |
|-----|------|----------|-----------|-------------|
| I | R | Z | - | Z value of last recorded point |
| I | R | YI | - | Y value of last recorded point |
| I | I | IYX | - | Maximum Y value of block |
| I | I | I | - | Location of last stored point in common array |
| I/O | I | IPR | - | Profile number |
| I | R | X | - | X value of profile |
| I/O | R | YM | - | Minimum Y value to store |
| I/O | R | YLM | - | Maximum Y value to store |

**Common Areas:** /DATA/,/FPINT/,/STORE/,/UNITS/,/FPDLPR/,/CONST/

**Subroutines Used:** PROPK,DISC,RWRITE

**Link:** STORE

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8.      DESCRIPTION OF OOPS INPUT FILE

The OOPS system uses three terrain data sources as input for the generation of profiles representing the terrain of the rectified photograph. The contour mode uses a MMS-32 data tape as input. The profile mode uses a reformatted AS-11B-1 profile tape. Input for the random point mode is a magnetic tape containing point ID and X,Y,Z ground coordinates of discrete points.


8.1     Contour

MMS-32 data tapes contain cultural, hydrographic, and topographic data. These different data types are identified within a header block. The x and y coordinates of each point in the contour data tape are contained in 36 bit words. A complete description of the input information for the contour mode can be found in the Automatic Cartographic System Mod III by Pennsylvania Research Associates, Inc. which was developed under RADC Contract No. RADC-TR-71-50.


8.2     Profile

The profile mode uses as input, a data tape generated on the PDP 11/45. This tape is the result of processing an AS-11B-1 data tape through a program designed to take out any irregularities in the data. The magnetic tape is blocked in 256, 16 bit words. Generation of this tape is performed by running a program called GRIDIN. This program was written by RADC personnel.

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The magnetic tape generated from the GRIDIN program is blocked in 256 integer words or 128 real words. All x,y and z coordinates are floating point numbers. The format of each block is as follows: real word 1 and 2 is blank, words 3-44 contain x values, words 45-86 contain y values and words 87-128 are the z values associated with the x and y coordinates. The x coordinates are incremented starting at the minimum x value in 1 millimeter increments. The y value is incremented every 250 microns. All coordinates are given in millimeters.

## 8.3    Random Points

Input to the random point mode consists of a magnetic tape containing the identification number and ground coordinates of discrete points. This data is written with an unformatted write consisting of one integer word (ID) and three real words (X,Y,Z). Each point is written in this manner. The file is ended when a point is input with an identification number of 9999.

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9.      DESCRIPTION OF OOPS OUTPUT FILE

        The generation of a rectified orthophotograph on the offline
orthophoto printer is accomplished by using two input files.  The first
of these files is the decimal shutdown tape.  This paper tape contains
the information necessary for orientation of the photograph on the OOP.
The second file is a magnetic tape containing the data representing the
terrain of the rectified orthophotograph.  Both tapes duplicate the
format of data output from the AS-11B-1.


9.1     Decimal Shutdown Tape

        The decimal shutdown tape contains all the constants necssary
for the orientation of the photograph on the OOP along with an alpha-
numeric identifier that precedes each data constant.  Thus shutdown
tape, in the form of a paper tape, is punched in RCA 501 code.  All
information punched on this paper tape is also tabulated on hard copy.

        Information pertaining to orientation is in an AS-11 coordin-
ate frame.  This coordinate frame uses the projected principle point as
the origin and the +Y photo axis as the +Y model axis (see Section 4.1).
The data constants contained on the shutdown tape include values for
both fore and aft photographs.  This allows the user to mount either
photo for rectification.  Data constants include the offsets in x and
y, focal length, radius of the earth, point of tangency of the photo,
and constants for atmopsheric refraction.  Nadir point values, air base

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components and linear rates, and orientation and orientation rates are also included.  In addition photograph type (pan or frame) is available on this shutdown tape.

## 9.2    OOPS Profile Tape

The OOPS profile tape contains the derived terrain information. This tape is formatted in 18-bit integer words of "X,Y,Z" coordinates and an associated identification label.  Between any two variables, there is an 18-bit word with a value of zero.  These 18-bit integer words are derived from one PDP real word.  The conversion from 16 to 18-bit words is accomplished by computing the binary representation of the real PDP word.  This binary representation is then output to a seven track magnetic tape to be input on the BX-272.

The output tape is compatible with the input requirements of the off-line orthophotoprinter.  Following is a diagram of the output tape format:

| GE Words | 36 bits | | 36 bits | | 36 bits | | 36 bits | |
|---|---|---|---|---|---|---|---|---|
| | 18 bits | 18 bits | 18 bits | 18 bits | 18 bits | 18 bits | 18 bits | 18 bits |
| BX-272 wds Word Type | Integer | | Integer | | Interger | | Integer | |
| Word Content | Zero | ID | Zero | X | Zero | Y | Zero | Z |

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The tape is blocked in physical records of 256 BX-272 18-bit words. Thus, 32 data points are contained in each physical record. The initial profile is increasing in the "Y" coordinate while the second profile decreases in "Y". The data is compacted between profiles such that data from two profiles can be contained in one physical record.

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## 10.    RESULTS

Considerable testing using both real and simulated data was accomplished during the OOPS contract. However, time did not permit extensive compilation of statistics. Both real and simulated tests were performed for the three program modes (contour, profile and random points) and for the coordinate transformations needed to generate the shut-down tape. The basic approach was to compare plots of the real data inputs with plots of the generated OOPS output.

The simulated data sets were all easily recognizable geometric figures such that any deviation from that geometric shape was readily discernible. For the contour and profile modes, two hemispheres placed side by side were used as fictitious data. These hemispheres were of different heights, with one hemisphere ranging from a height of 1,000 feet to 2,500 feet and the second from 1,000 feet to 1,500 feet. For the random points mode, a surface was created from a sine wave function. Each profile of this surface was an elongated sine curve with each successive profile being translated parallel to the previous one.

The real data tests were all developed over the South Mountain area near Phoenix, Arizona. This well known test area is noted for its sharp contrast in terrain relief. Slope in this area varies from approximately $0^0$ to in excess of $60^0$, which presents a difficult problem of terrain modeling. The following paragraphs describe the results obtained from each of the three modes and the coordinate transformations.

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*Contour Mode*.  Figures 10.1-10.5 illustrate the simulated
data inputs, results from simulated data, actual data inputs, and re-
sults from real data inputs, respectively for the contour mode.  The
simulated hemispheres, as shown in Figure 10.1, were digitized in MMS-
32 format by RADC.  This digitized data was converted to profile infor-
mation and the results, as output from the OOPS software, are illustra-
ted in Figure 10.2.  Excellent results were obtained from the contour
mode over areas of moderately sloping relief using a polynomial func-
tion, however, an interpolation routine provided more successful results
over extremely rugged terrain.  Figure 10.4 illustrates results obtain-
ed from the real data input set in Figure 10.3 using a polynomial func-
tion to derive the profiles.  These results are less accurate due to
the sparsely digitized contours and extreme magnitude of the terrain
slope changes.  However, when compared with the actual AS-11B-1 profiles,
shown in Figure 10.7, these profiles do conform to the actual terrain
in most areas.  The same input data was used to generate profiles using
an interpolation routine.  As illustrated in Figure 10.5, this method
of generating profiles from rugged areas more closely resemble the actual
terrain.  Figures 10.4 and 10.5 were plotted at a different scale and
orientation than the actual AS-11B-1 profiles shown in Figure 10.7. Gen-
eral terrain characteristics can be compared, however, a direct compari-
son between the three figures cannot be made.

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*Profile Mode*.   The profile mode was tested in a very similar

manner.   The profiles, as derived from the hemispheres in the contour

mode, were used as AS-11B-1 input data to the profile mode.   This simu-

lated data was processed through the profile mode and the results are

illustrated in Figure 10.6.   This plot of a small portion of the sphere

illustrates an inherent problem in modeling smooth surfaces with the

multiquadric function.   Each of the small bumps on the spherical sur-

face are nodal points about which the multiquadric function was deri-

ved.   Recall, Figure 10.7 is a plot of the original profile data as

derived manually from the AS-11B-1.   The profiles derived by the OOPS

software were oriented to correspond to that of the input, so that a

comparison could be made, however, these output profiles can be oriented

to any desired azimuth angle.   The output from the profile mode is illus-

trated in Figure 10.8.   These results are felt to be adequate for ortho-

photo production.   Some features are generalized due to the multiquadric

functional representation.   This, however, is quite desirable for ortho-

photo compilation.

*Random Points*.   Two data sets were processed using the random

points mode.   The first was a simulated data case where scattered points

were generated from a three-dimensional sine wave function.   The second

data set was real data which consisted of discrete points measured from

a map sheet in a UTM coordinate frame.   The real data set again covered

the South Mountain area.   Plots are not available of the generated

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profiles for either the simulated or real data cases. However, these profiles were generated from the same mathematical expression as that used in the profile mode. The results from the sine wave function were consistent and seemed to approximate the original sine function quite well. The resultant RMS value for all points in the simulated area was 18 meters. Over the real data area 262 data points were available, with an RMS value for these points of 35 meters.

*Coordinate Transformations*. Coordinate transformations developed during this contract for the generation of the footprint boundaries and shut-down tape were validated using real data from previous DBA frame and panoramic reductions. Transformations were developed relating USR, LSR, LV, Geographics and UTM coordinates. Photograph orientations which were converted from one coordinate system to another were verified by projecting known image points, using the transformed orientation matrix, and comparing the resultant object space coordinates to their known values.

Routines were developed for computing footprint boundaries or ground coordinates, by projecting image coordinates to the earth. These routines were tested using both frame and panoramic image coordinates with known ground coordinates.

FIGURE 10.1.   Simulated Data Used for Contour Mode

FIGURE 10.2. Resultant Profile of Simulated Data from Contour Mode

FIGURE 10.3.   Contour Input, South Mountain Area

FIGURE 10.4.  Profiles from Contour Mode Using Raster Polynomial Method

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FIGURE 10.5. Profiles from Contour Mode Using Interpolation Routine

FIGURE 10.6.   Profiles Derived from Input Hemispheres

FIGURE 10.7. AS-11B-1 Original Profile Data, South Mountain Area

FIGURE 10.8. Resultant Profiles South Mountain Area

## 11. CONCLUSIONS AND RECOMMENDATIONS

Several methods of modeling terrain relief were investigated and implemented during the OOPS contract. These methods included; multiquadric, polynomial, interpolation and a combination of interpolation and multiquadric. For patterned input data, such as the profile data, the multiquadric function proved to be a satisfactory method of representing the terrain. For unpatterned data, three of the above mathematical approaches were developed. These were: polynomial, interpolation and a combination of interpolation and multiquadric. Random points are processed using a multiquadric function, which provided to be quite adequate. Contour data is processed using both a polynomial function and an interpolation routine. The results achieved using the polynomial function are excellent for areas of moderately sloping terrain, however, in areas of steep terrain, an interpolation routine provided the best results. These mathematical approaches are applied to data of various types to provide data in a format suitable for the off-line orthophoto printer. Orientation of the final model can be derived from nearly any input coordinate system, including USR, LSR, LV, Geographics and AS-11 absolute model parameters. Data used for the generation of output profiles can be computed from AS-11B-1 profiles, MMS-32 contour data and random points. Combinations of the above data types are also acceptable. This comprehensive software system provides a major contribution for expanding the capability of the off-line orthophoto system.

The OOPS system was designed and implemented primarily as a research and development program. Several modifications to this system could be made to improve flexibility and computation efficiency. Employing a combination of array algebra and the multiquadric function in the profile and random point modes would improve software efficiency. Considering the unpatterned data distribution in the random point mode, an interpolation routine would be required prior to beginning the array algebra solution. This interpolation routine would establish grid points having x and y coordinates and an interpolated height. The interpolated height of any grid point would be computed from a function which utilizes distances and heights of surrounding data points. Because the input profile data is already in a gridded format, no data reformatting is required for that mode. Array algebra could greatly reduce <u>both</u> the time and computer core required to compute the coefficients representing terrain relief. At the present time, the solution of an area 70mm x 110mm with a 6x6 nodal pattern requires approximately 3.5 hours. Modification of the software to model the multiquadric function using array algebra could easily reduce this computational time by a factor of ten. It is strongly recommended that the OOPS system be modified so that the solution for the multiquadric function be performed using array algebra.

A second improvement to OOPS would be to determine and use the "best" nodal pattern and constant value for each different type of terrain, whenever the multiquadric math representation is employed.

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The determination of these two items could be performed either manually or automatically. This improvement would result in time savings as well as increased accuracy. Areas with fairly level terrain require a sparce nodal pattern; the number of computations become less and the size of the matrices in core are smaller. When comparing a nodal point pattern of 3x3 with one of 7x7, a significant amount of computation time is saved. There are approximately one-fifth as many computations in a 3x3 nodal pattern as compared to a 7x7 nodal pattern. Thus, one could select the optimum nodal pattern to achieve maximum efficiency while maintaining adequate accuracy.

The modular design of the OOPS system could be expanded to accept new data types. One data type which presently exists and was not addressed during this contract is LIS. This data consists of vector rays between data points. Other data types which could be incorporated into the OOPS system are TERCOM and DRLMS. Such data is given in an evenly gridded format and therefore would be ideally suited to the multiquadric function solution using array algebra. The ability to create profiles using any of the above data types would greatly increase the capability of the OOP. Another improvement which could be incorporated in the OOPS is the ability to read an AS-11B-1 data tape created directly from the stereo plotter. Presently this raw data is input to a separate software system for editing prior to entry into OOPS. This editing function could be accomplished directly in OOPS without an intermediate software step.

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The major obstacle encountered during the OOPS development was the extremely large computer storage requirement. The core requirement resulted from both the large amount of input data to be processed and the software code itself. The available mass storage was also very limited since only one RK05 disc is available on the RADC PDP 11/45. This created several I/O problems and considerable time was lost in cleaning and compacting disc files. It is recommended that a minimum of one more RK05 disc be purchased. During this contract, files had to be limited and therefore data sizes were also limited. Much of the I/O currently done on magnetic tape could be accomplished more efficiently on disc. Further recommendations as to hardware would include the purchase of the RSX-11M operating system with Fortran IV Plus. The purchase of RSX-11M would allow the user to access the full 64K of core. Fortran IV plus would enable the programmer to use such features as subroutine entry points and double word integers.

During any potential upgrade of the OOPS, it is recommended that the individual processes be broken up into subtasks under control of a RSX monitor. Between these subtasks, checkpointing or operator interaction is recommended. Here the operator can analyze the output at such points and either rerun that part of the program or continue. This modification would result in a more efficient software system in terms of utilization of computer operation time. After debugging a complete mode, such as contour, profile or random points reduction,

the programs should be tested using several data cases to exercise each option. After all modes have been debugged in this manner, the complete system could be incorporated. Each system link of the OOPS software requires approximately 35 minutes of run time. Therefore, much time could be lost by not debugging programs as modules. However, although development and "debug" is easier when the software is in separate modules, eventual utilization is tedious unless the individual modules are combined under a single controller.

In summary, the development of the OOPS has enhanced the capability of the off-line orthophoto printer in that various types of data can now be used for the computation of profile tapes for the Offline Orthophoto Printer. The current software system demonstrates the feasibility of the multiquadric and polynomial functions to represent terrain surfaces with acceptable accuracies. Incorporation of the above recommendations into the OOPS would result in a substantial improvement in run time, as well as an increased product accuracy.

# REFERENCES

1.  Hardy, R.L., 1972. "The Analytical Geometry of Topographic Surfaces", ACSM 1972 Proceedings of the 32nd Annual Meeting: 163-181.

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