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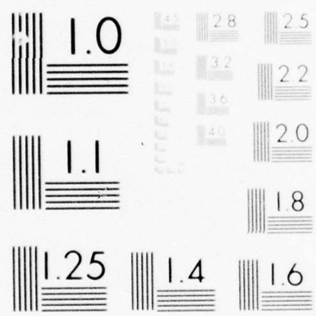

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Final Technical Report
August 1977

A GTD ANALYSIS OF THE CIRCULAR REFLECTOR ANTENNA INCLUDING FEED AND STRUT ~~ANTENNA~~ Scatter

The Ohio State University

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ROME AIR DEVELOPMENT CENTER
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Recently, the wide angle side lobes of a circular reflector antenna pattern were calculated by the Geometrical Theory of Diffraction and an overall pattern can be obtained from a combination of GTD and aperture integration methods. In this report, the analysis is extended to include the scattering from the feed supports. The feed support scattering is treated using the concept of equivalent current line source. The cylinder scattering model developed in this report gives the effect of strut scattering on the wide angle side lobes of the reflector. The scattering from the feed struts

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) always has its greatest effect near the scattering cones for each strut. These scattering cones usually give rise to a maximum aperture blockage effect in certain off-principal plane patterns. The analysis given here can be used to compute the complete pattern of any arbitrary plane cut for a practical circular reflector antenna system.

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EVALUATION

This contractual effort resulted in the development of a geometrical theory of diffraction (GTD) computer code for circular parabolic reflectors with feed and strut scattering. This computer code analytically predicts the far field pattern of the parabolic reflector including the scattering from reflector edges, feed supports, and the feed structure.

The results of this effort provide a means of simulating a parabolic reflector with obstructing feeds and supporting structures and predicting the far outside lobe responses. This computer code can more accurately simulate the parabolic reflector environment than previously accomplished in the past.

This fits into the RADC Technology Plan (TPO I-B) for analytically simulating parabolic reflector antenna system before fabrication, installation and test.

The computer code, OSU PATT, is presently operational on the RADC-HIS 6180 computer facility and will be used to analyze the performance characteristics of radar and communication parabolic reflector type antenna systems.

Carmen S. Malagisi

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Project Engineer

PART I - THEORY

CHAPTER I
INTRODUCTION

The classical analysis of reflector antennas was well developed in the late 1940's. Accordingly, the radiation pattern in the forward hemisphere of a reflector antenna can be calculated by either the aperture field method or the current distribution method [1]. However, these methods are rather slow for electrically large antennas, and in general do not predict the wide angle side lobes accurately. Recently, the wide angle side lobes were calculated [2] by the GTD [3,4,5] which offers an efficient way to obtain the radiation pattern except for the forward axis region. Thus an overall pattern results from a combination of GTD and aperture integration methods. Using this approach, the far field patterns of the circularly symmetric parabolic reflector antenna with the feed at the focus were calculated in [2]. The effect of a rapid field variation at the edge of the reflector and the coupling between two reflector antennas were also analyzed using the GTD approach [6].

In this report, the same approach is extended to the scattering from the feed supports. Also the analysis has been extended to include calculation of the off-principal plane patterns. The feed horn blockage is also treated in a similar manner to that in [2] by replacing the feed structure by an equivalent circular or rectangular flat plate model whose area approximates the cross section of the feed structure. This approximation is justified in the forward direction, where the feed horn blockage is generally most significant. The flat plate scattering is analyzed by the conventional physical optics approach [7].

On the other hand, the feed support scattering is treated in a different way in which the scattered field of each individual strut is obtained using the concept of equivalent current line source [8,9]. The GTD is used to determine the equivalent line sources. The total effect of the feed support scattering is the sum of the scattered fields from each individual strut. Then the scattering from the feed horn and the feed supports is simply added to the other radiation components of the antenna. Thus the total pattern of the reflector antenna is composed of the radiation field from the reflector, the direct feed pattern and the scattering from the feed structure.

A user-oriented computer program is described in Part II of this report. The input to the program consists of the E- and H-plane patterns of the feed, the frequency, the dimensions of the reflector, the positions and diameters of the feed supports, and the physical cross section of the feed.

In addition to the reflector antenna code described in the body of this report, a second computer code developed by W. D. Burnside and R. F. Marhefka has been made operational on the RADC Honeywell computer system. This code referred to as the flat plate program is used to compute the far-zone scattered fields for antennas radiating in the near zone of structures made of flat plates. In its present form this code simulates structures such as buildings, or ships by a set of finite flat plates forming a convex structure for which the scattering from one flat plate to another is negligible. Using the present code, one can treat the structure by a single flat plate, a rectangular box, a rectangular pyramid, etc. Also a separate ground plane can be introduced. This additional effort was supported by the Naval Ocean Systems Center, San Diego, California under Contract N00123-76-C-1371.

The present code is limited to one structure which can be simulated by as many as 14 plates. This is based on the array dimensions in the code and is not a limitation of the theory. Each plate can consist of 6 corners; however, each corner must lie in a plane or the computer code will abort. The definition of the plates is made by first setting up a fixed cartesian coordinate system relative to the structure under investigation. The plates are, then, defined by the location of the corners. The antenna location is, also, specified in the same coordinate frame. One should note that the fixed coordinate system should be chosen such that one can easily define the structure. The program has the flexibility to handle arbitrary pattern cuts relative to this coordinate system as is discussed later.

The antenna presently considered in the computer code is simulated by a set of electric or magnetic elemental radiators. There is a maximum of six such radiators which is limited by the computer code dimension and not the theory. Each electric or magnetic radiator has cosine distribution, arbitrary length, arbitrary magnitude and phase, and arbitrary orientation. This elemental antenna is considered initially but can be easily modified in that the code is modular in construction. In this case, the SOURCE subroutines can be easily exchanged with another antenna pattern subroutine.

The present form of the computer code is not large in terms of computer storage and executes a pattern in short order. The storage is, of course, dependent on the dimensions which might vary; however, the present code requires approximately 100 K bytes. It will run a pattern cut of 360 points for a flat plate structure with one antenna in approximately 10 seconds on a CDC-6600 computer.

The limitations associated with the computer code results from the basic nature of the analysis. The solution is derived using the Geometrical Theory of Diffraction (GTD) technique which is a high frequency approach. In terms of the scattering from a finite flat plate, this means that each plate should have edges at least a wavelength long. In addition, antenna elements should not get closer than about a wavelength to any edge. In some cases, the previous wavelength limit can be reduced to a quarter wavelength.

A part of the work carried out under this contract concerned the development of diffraction coefficients for perfectly-conducting cylindrical scatterers. These diffraction coefficients were used to calculate the scattering from the feed support of a reflector antenna, as described in the body of this report. However, they also were used to predict the degradation of the pattern of the LAMPS antenna caused by the presence of a nearby cylinder. The LAMPS antenna is a 34" parabolic reflector antenna with a nominal frequency of 4.6 GHz. A report [13] was prepared on this task, and since the results were of interest in the analysis of a shipboard antenna configuration, its publication was supported by NOSC through the Naval Regional Procurement Office, Long Beach, California under Contract N00123-76-C-1371.

CHAPTER II
RADIATION PATTERN

A. Primary Feed

Reflector antennas are classified according to the geometry of the reflector surface as well as the shape of the reflector rim. In this report, a focus-fed paraboloid with circular rim is considered. However, this approach can also be applied to other kinds of reflector antennas. Since the feed is located at the focus, it is convenient to introduce a spherical coordinate system to describe the field of the primary feed with origin at the focus (P_f) and the y axis as the polar axis as shown in Figure 1a. To describe the radiation from the feed at the reflector, another spherical coordinate system also with origin at the focus, but having the z axis as the polar axis is used (see Figure 1b).

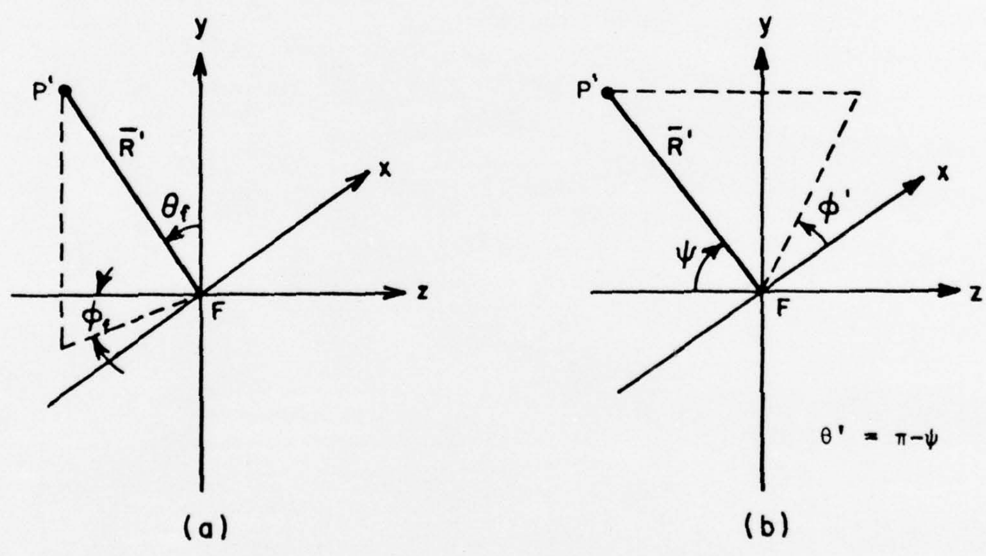


Figure 1. Coordinate systems for the primary feed.

The relations between the coordinate variables and unit vectors of these two systems are given by

$$\cos\psi = \cos\phi_f \sin\theta_f \tag{1}$$

$$-\cos\hat{\phi}' \sin\psi = \sin\phi_f \sin\theta_f \tag{2}$$

$$\hat{\psi} = -\theta_f \frac{\sin\phi' \cos\psi}{B} + \hat{\phi}_f \left(\frac{\cos\phi'}{B}\right) \tag{3}$$

$$\hat{\phi}' = -\hat{\theta}_f \frac{\cos\phi'}{B} - \hat{\phi}_f \frac{\sin\phi' \cos\psi}{B} \tag{4}$$

where

$$B = \sqrt{1 - \sin^2\psi \sin^2\phi'} \quad , \tag{5}$$

The coordinate systems for the reflector geometry are shown in Figure 2 where (R,θ,ϕ) are the spherical coordinates of the far field

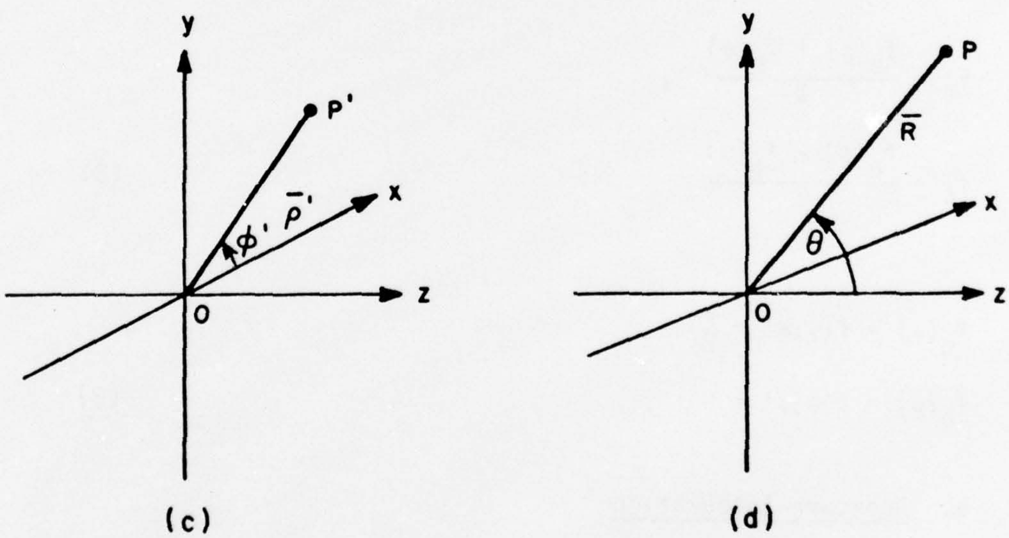


Figure 2. Coordinate systems for the reflector.

point. If we assume that the reflector is in the far zone region of the feed source, the field of the primary feed having the same polarization in the far zone as a dipole oriented in the y direction is given by

$$\vec{E}^f = -\hat{\theta}_f A\, g(\theta_f,\phi_f)\frac{e^{-jkR'}}{R'}$$

$$= -F\left(\hat{\theta}\,\frac{\cos\psi\,\sin\phi'}{B} - \hat{\phi}\,\frac{\cos\phi'}{B}\right) f(\psi,\phi')\,\frac{e^{-jkR'}}{R'} \qquad (6)$$

where $g(\theta f,\phi f) = f(\psi,\phi')$ is the primary feed pattern,

A is set equal to F, the focal length of the reflection, for convenience

and $R' = \frac{2F}{1+\cos\psi}$ is the distance from the feed to the reflector surface.

In the off-principal planes ($\phi \neq 0, \frac{\pi}{2}$), the feed patterns are approximated by interpolating between E- and H-plane patterns in the following way,

$$f(\psi,\phi') = f_T - f_\delta \cos 2\phi' \qquad (7)$$

where

$$f_T = \frac{f_e(\psi) + f_h(\psi)}{2} \quad ,$$

$$f_\delta = \frac{f_e(\psi) - f_h(\psi)}{2} \qquad (8)$$

and

$$f_e(\psi) = f(\psi,\phi' = \frac{\pi}{2})$$

$$f_h(\psi) = f(\psi,\phi' = 0) \quad . \qquad (9)$$

B. Aperture-Integration

In this and the next two sections we are considering the antenna pattern without aperture blockage or "to calculate most of the scattering from the reflector". The GTD together with the direct feed radiation provides a very efficient method to calculate most of the antenna pattern. The GTD fields can be calculated from only the feed illumination of the aperture edges. However, the main beam and the first few sidelobes depend on the fields over the entire aperture. Consequently, the classical technique of aperture integration is used to calculate the pattern in the forward axial region.

Using the concepts of geometrical optics and conservation of power, it can be shown [2] that the magnitude of the aperture field is related to that of the incident field by

$$|E^a| = |E^f(R')| \sqrt{R' \frac{d\psi}{d\rho}} \quad . \tag{10}$$

It follows from the above expression, adding the appropriate polarization and a phase factor, that the aperture field is given by

$$\overline{E}^a(\rho',\phi') = \hat{e}_r \; F \; f(\psi,\phi') \; \frac{e^{-jkR_o}}{R'} \tag{11}$$

where

$$\hat{e}_r = -\frac{1}{B} [\hat{\rho}' \cos\psi \sin\phi' + \hat{\phi}' \cos\phi']$$

$$= +\frac{1}{B} [\frac{1}{2} \sin 2\phi' (1 - \cos\psi) \hat{x}$$

$$- (\cos\psi \sin^2\phi' + \cos^2\phi') \hat{y}] \quad . \tag{12}$$

Ro is the distance from the feed to the rim of the reflector (see Figure 4), and

$$\rho' = R' \sin\psi \quad . \tag{13}$$

The coordinate system for describing the aperture field is shown in Figure 2a in which the origin is at the center of the reflector aperture.

In evaluating the radiation from the aperture, a spherical coordinate system, shown in Figure 2b, and also centered at 0, is introduced to describe the far zone field of the reflector antenna. By the equivalence principle with image theory, the equivalent magnetic current induced by the aperture field is

$$\overline{K} = 2 \, \overline{E}^a \times \hat{z} \quad .$$

The pattern function of this equivalent current source is given by

$$\overline{F} = -\frac{jk}{4\pi} \int_0^{2\pi} \int_0^a [\overline{K}(\rho',\phi') \times \hat{r}'] \, e^{jk\rho' \sin\theta \cos(\phi-\phi')} \rho' d\rho' d\phi' \tag{14}$$

where $\hat{r}'$ is the unit vector along r', the distance between the field point and the source, and a is the radius of the aperture.

In substituting $\overline{E}^a$ into Equation (14), we consider the x and y polarizations separately. As in [2], the aperture field E_y^a can be approximated by interpolating between the aperture distributions along the principal axes.

$$\overline{E}_e^a = E_y^a(\rho',\tfrac{\pi}{2}) = -\hat{y}\, F\, f_e(\psi)\, \frac{e^{-jkR_o}}{R'}$$

$$\overline{E}_h^e = E_y^a(\rho',0) = -\hat{y}\, F\, f_h(\psi)\, \frac{e^{-jkR_o}}{R'} \quad . \tag{15}$$

Thus the aperture field is given by

$$E_y^a(\rho',\phi') = T_f(\rho') - \delta_f(\rho') \cos 2\phi' \tag{16}$$

where

$$T_f(\rho') = \frac{E_e^a + E_h^a}{2} ,$$

and

$$\delta_f(\rho') = \frac{E_e^a - E_h^a}{2} \quad . \tag{17}$$

Hence for the y component of the aperture field in Equation (14), we have

$$\begin{aligned}
\overline{K}_y \times \hat{r}' &= 2(\hat{y}\, E_y^a \times \hat{z}) \times \hat{r}' \\
&\simeq 2\, \hat{x}\, E_y^a \times (\hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta) \\
&= 2(\hat{z} \sin\theta \sin\phi - \hat{y} \cos\theta)\, E_y^a
\end{aligned} \tag{18}$$

where we have assumed $\hat{r}' \simeq \hat{r}$ for far field calculation.

The pattern function of E_y^a is therefore given by

$$\overline{F}_y(\theta,\phi) = (\hat{y}\cos\theta - \hat{z}\sin\theta\sin\phi)\frac{jk}{2\pi} I_y \tag{19}$$

where

$$I_y = \int_0^{2\pi}\int_0^{a} E_y^a \, e^{jk\rho'\sin\theta\cos(\phi-\phi')} \rho' d\rho' d\phi' \quad .$$

The ϕ' integration can be analytically evaluated using the well-known properties of the Bessel function J_n; thus we get

$$I_y = 2\pi \int_0^{a} [T_f(\rho') \, J_0(k\rho'\sin\theta) + \delta_f(\rho') \cos 2\phi J_2(k\rho'\sin\theta)]\rho' d\rho' \ . \tag{20}$$

The above integration is numerically evaluated by Simpson's rule.

Next consider the x polarized aperture field. From Equations (11) and (12)

$$E_x^a = \frac{1}{2B}\sin 2\phi'(1-\cos\psi) \, F \, f(\psi,\phi') \frac{e^{-jkR_o}}{R'} \quad . \tag{21}$$

In the principal planes ($\phi = 0, \frac{\pi}{2}$), these x polarized fields vanish. Thus only the y-component of the aperture field contributes to the radiation patterns in the principal planes. In the off-principal planes, the x polarized component E_x^a may have significant contribution to the radiation pattern which reaches a maximum in the 45° plane. For the E_x^a component in Equation (14) we have

$$\overline{K}_x \times \hat{r}' \cong 2(-\hat{x}\cos\theta + \hat{z}\sin\theta\cos\phi) E_x^a \quad . \tag{22}$$

Thus the pattern function F_x becomes

$$\overline{F}_x(\theta,\phi) = (\hat{x}\cos\theta - \hat{z}\sin\theta\cos\phi)\frac{jk}{2\pi} I_x \tag{23}$$

where

$$I_x = \int_0^{2\pi}\int_0^{a} E_x^a \, e^{jk\rho'\sin\theta\cos(\phi-\phi')} \rho' d\rho' d\phi' \quad . \tag{24}$$

Using the interpolation for the primary feed pattern as given by Equation (8), the above integral becomes

$$I_x = \frac{F\, e^{-jkR_o}}{2} \int_0^{2\pi}\int_0^{a} \frac{1}{BR^\tau} [\sin 2\phi'(1-\cos\psi)][f_T - f_\delta \cos 2\phi']$$

$$\times\, e^{jk\rho'\sin\theta\cos(\phi-\phi')} \rho' d\rho' d\phi'$$

$$= \pi F\, e^{-jkR_o} \int_0^{a} \frac{1}{B}[\sin 2\phi(1-\cos\psi) \frac{f_T}{R^\tau} J_2(k\rho'\sin\theta)$$

$$+ \frac{1}{2}\sin 4\phi(1-\cos\psi) \frac{f_\delta}{R^\tau} J_4(k\rho'\sin\theta)]\rho' d\rho' \quad . \tag{25}$$

In carrying out the ϕ integration, we have made a stationary phase approximation that $\phi' \simeq \phi$ in the expression for B, i.e., $B = \sqrt{1 - \sin^2\psi \sin^2\phi}$. Again the above integration is evaluated by Simpson's rule. Hence the total far field as obtained from aperture integration is given by

$$\overline{E}(\overline{R}) = [\overline{F}_x(\theta,\phi) + \overline{F}_y(\theta,\phi)] \frac{e^{-jkR}}{R} \tag{26}$$

which is expressed in spherical components by

$$\overline{F}_y = -\,(\hat{\theta}\sin\phi + \hat{\phi}\cos\theta\cos\phi)\frac{jk}{2\pi} I_y$$

and

$$\overline{F}_x = (\hat{\theta}\cos\phi - \hat{\phi}\cos\theta\sin\phi)\frac{jk}{2\pi} I_x \quad .$$

C. Two Point Method Using GTD

The wide angle side lobes of a circular reflector have previously been analyzed by using the two point method based on GTD [2]. This method states that the diffracted field from the paraboloid is contributed mainly by two stationary points, Q_1 and Q_2 on the rim of the reflector (see Figure 3). The detailed formulations of the diffracted field in the E- and H-planes are given in [2]. The off-principal plane diffracted field pattern can be obtained by using a similar procedure. The far zone diffracted fields from Q_1 and Q_2 are given by

$$\bar{E}_1^d(P) = \bar{E}^f(Q_1) \cdot \bar{\bar{D}}(Q_1) \sqrt{\frac{a}{\sin\theta}} \; \frac{e^{-jk(R-a\sin\theta)}}{R} \tag{27}$$

and

$$\bar{E}_2^d(P) = \bar{E}^f(Q_2) \cdot \bar{\bar{D}}(Q_2) \sqrt{\frac{a}{\sin\theta}} \; \frac{e^{-jk(R+a\sin\theta) + j\pi/2}}{R} \tag{28}$$

respectively, where $\bar{\bar{D}}$ is the dyadic diffraction coefficient for a curved edge, $E^f(Q_{1,2})$ is the electric field of the feed at $Q_{1,2}$ and a is the radius of the reflector aperture. For rays normally incident on the edge, as is the case here, the dyadic diffraction coefficient [4] can be expressed as:

$$\bar{\bar{D}} = \hat{e}\,\hat{e}\,D_s + \hat{p}_d\,\hat{p}\,D_h \tag{29}$$

where D_s and D_h are the scalar diffraction coefficients for the soft and hard boundary conditions, respectively, and are given in [4], and

$\hat{e}$ is the unit vector tangent to the edge,

$\hat{p} = \hat{e} \times \hat{I}$; $\hat{I}$ being the unit vector in the direction of the incident ray,

$\hat{p}_d = \hat{e} \times \hat{d}$; $\hat{d}$ being the unit vector in the direction of the diffracted ray.

For the ray diffracted at Q_1

$\hat{e} = \hat{\phi}$,

$\hat{p}_d = \hat{e} \times \hat{d} = \hat{\theta}$,

$\hat{p} = \hat{e} \times \hat{I}_1 = \hat{\phi} \times \hat{I}_1$

$\psi = \alpha$ is the half angle spanned from the focus to the rim of the reflector.

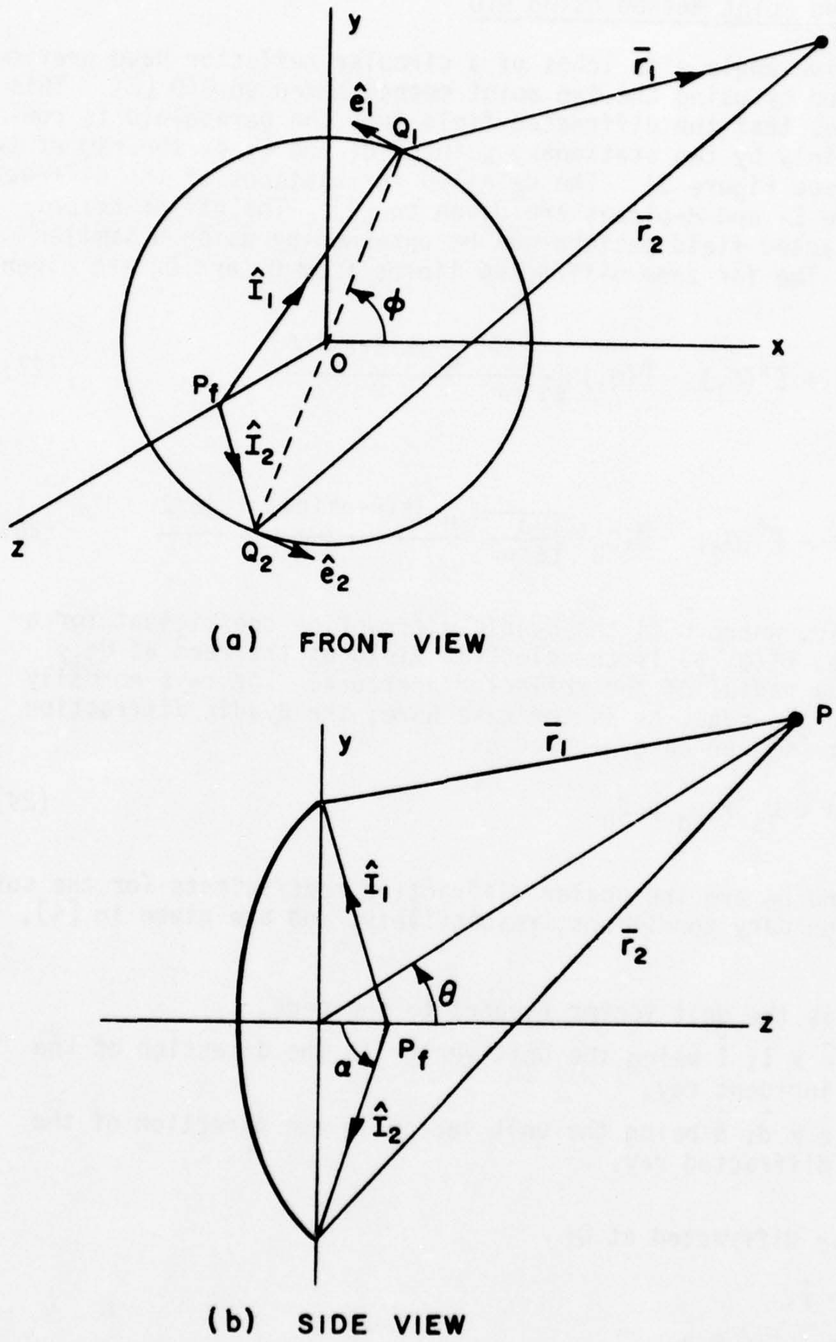


Figure 3. Geometry of the edge diffracted rays in an arbitrary ϕ plane.

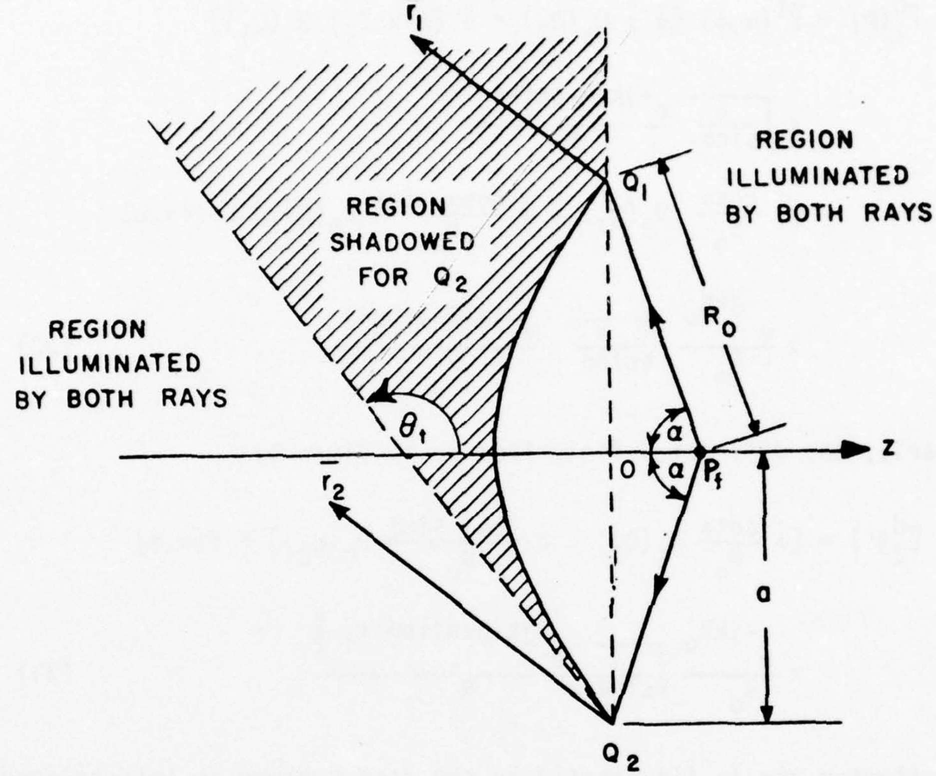


Figure 4. Different regions illuminated by the edge diffracted rays.

and

> $R' = R_0$ is the distance from the focus to the rim (see Figures 3 and 4).

Combining Equations (6), (27) and (29) with $\phi'=\phi$ and $\psi=\alpha$ (i.e., $B = B_0 = \sqrt{1 - \sin^2\alpha \sin^2\phi}$), the diffracted field from point Q_1 (see Figure 3) is given by

$$\overline{E}_1^d(P) = \overline{E}^f(\alpha,\phi)\,[\hat{\phi}\,\hat{\phi}\,D_s(Q_1) + \hat{\theta}\,(\hat{\phi}\times\hat{I}_1)\,D_h(Q_1)]$$

$$\times\sqrt{\frac{a}{\sin\theta}}\;\frac{e^{-jk(R-a\sin\theta)}}{R}$$

$$= [\hat{\phi}\,\frac{\cos\phi}{B_o}\,D_s(Q_1) - \hat{\theta}\,\frac{\cos\alpha\,\sin\phi}{B_o}\,D_h(Q_1)]\,F\,f(\alpha,\phi)$$

$$\times\frac{e^{-jkR_o}}{R_o}\;\sqrt{\frac{a}{\sin\theta}}\;\frac{e^{-jk(R-a\sin\theta)}}{R}\quad . \tag{30}$$

Similarly, the diffracted field from Q_2 is given by

$$\overline{E}_2^d(P) = [\hat{\phi}\,\frac{\cos\phi}{B_o}\,D_s(Q_2) - \hat{\theta}\,\frac{\cos\alpha\,\sin\phi}{B_o}\,D_h(Q_2)]\,F\,f(\alpha,\phi)$$

$$\times\frac{e^{-jkR_o}}{R_o}\;\sqrt{\frac{a}{\sin\theta}}\;\frac{e^{-jk(R+a\sin\theta)+j\frac{\pi}{2}}}{R}\quad . \tag{31}$$

The reflector rim is illuminated by the feed pattern as interpolated from the E- and H-planes. Thus

$$f(\alpha,\phi) = f_T(\alpha) - f_\delta(\alpha)\cos 2\phi \tag{32}$$

where f_T and f_δ are defined in Equations (3a) and (3b). The diffracted field in the different GTD regions is given by

$$\overline{E}^d = \begin{cases} \overline{E}_1^d + \overline{E}_2^d & \theta \leq \frac{\pi}{2} \\ \overline{E}_1^d & \frac{\pi}{2} < \theta < \theta_t \\ \overline{E}_1^d + \overline{E}_2^d & \theta_t \leq \theta \end{cases} \tag{33}$$

where θ_t is the angle of the shadow boundary for Q_2 (see Figure 4). Then the field of the primary feed is superimposed on the diffracted field in the region from $\theta = 0$ to $\theta = \pi-\alpha$ which is the shadow boundary of the field from the feed.

D. Ring Current Method

In the rear axis region, the ring current method is used to calculate the diffracted field, since the two point method is not valid there [2]. The E-plane and H-plane field patterns for θ close to π have been formulated in [2] as

$$\overline{E}_e = -\hat{\theta}\,\frac{\pi a}{\sqrt{\lambda}}\,\frac{e^{-j(kR-\frac{\pi}{4})}}{R}\,[A_1 J_o(x) + A_2 J_2(x) + A_3 J_4(x)], \tag{34}$$

where $x = ka\sin\theta$,

$$\begin{aligned} A_1 &= T_f(D_h - D_s\cos\theta) + \frac{\delta_f}{2}(D_h + D_s\cos\theta)\ , \\ A_2 &= -T_f(D_h + D_s\cos\theta) - \delta_f\,(D_h - D_s\cos\theta)\ , \\ A_3 &= \frac{\delta_f}{2}(D_h + D_s\cos\theta)\ , \end{aligned} \tag{35}$$

and

$$\overline{E}_h = \hat{y}\,\frac{\pi a}{\sqrt{\lambda}}\,\frac{e^{-j(kR-\frac{\pi}{4})}}{R}\,[B_1\,J_o(x) + B_2\,J_2(x) + B_3\,J_4(x)], \tag{36}$$

where

$$\begin{aligned} B_1 &= T_f(D_s - D_h\cos\theta) - \frac{\delta_f}{2}(D_s + D_h\cos\theta), \\ B_2 &= -T_f(D_s + D_h\cos\theta) + \delta_f(D_s - D_h\cos\theta), \\ B_3 &= -\frac{\delta_f}{2}(D_s + D_h\cos\theta). \end{aligned} \tag{37}$$

Note that the diffraction coefficients D_s and D_h are assumed to have their rear axis values, i.e., for $\theta = \pi$.

In the off-principal planes, the field in the rear axis region is obtained by interpolating between E_e and E_h as follows:

$$E_y = E_e\sin^2\phi + E_h\cos^2\phi. \tag{38}$$

This can be transformed to spherical components by

$$E_\theta = E_y \sin\phi \tag{39}$$

$$E_\phi = E_y \cos\phi \quad . \tag{40}$$

since $\pi-\theta$ is assumed to be small.

CHAPTER III
FEED STRUCTURE SCATTERING

In practice, the scattering from the feed horn and the feed supports increases the side lobe levels and reduces the gain of the antenna. Therefore, the blocking effect of the feed structure should be taken into account.

To analyze the scattering from the feed structure, two different models are used. An equivalent line source model is used for feed support scattering and a flat plate model is used for feed horn scattering.

A. Equivalent Line Source Model For feed Support Scattering

The effect of the feed support blocking is usually estimated by the projected shadow method in which the feed support or strut is replaced by its effective shadow on the reflector plane. The radiation field from the shadow area is then calculated and subtracted from the antenna pattern [10,11]. In this report, an alternative approach is developed in which the scattered fields from the struts are computed by the equivalent current approach [7]. Since struts often take the form of circular cylinders, only metallic circular cylinder struts are considered here. Also, the incident field on the struts is assumed to be the reflected wave from the reflector only. That is, the interaction of the direct feed radiation and the strut is not considered. The strut scattered field as reflected by the reflector is also neglected. Therefore, the feed support scattering problem is equivalent to a circular cylinder scattering with a plane wave incident at a specified incident angle; however the amplitude of the plane wave can be a slowly-varying function of position. The equivalent current approach is used to take into account the variations of the incident field along each strut, which includes the effect of the finite length of each strut. In this approach, the scattering from each element of a strut is assumed to be the same as that from an element on an infinite cylinder with the same incident field.

Let (χ,η,ξ), (ζ,γ,ξ) and (r,α,γ) denote the rectangular, cyindrical and spherical coordinates of the strut system, respectively. Consider a plane wave incident upon an infinite conducting cylinder of radius a, located at the origin, with an incident angle β, at $\gamma' = \pi$, as shown in Figure 5.

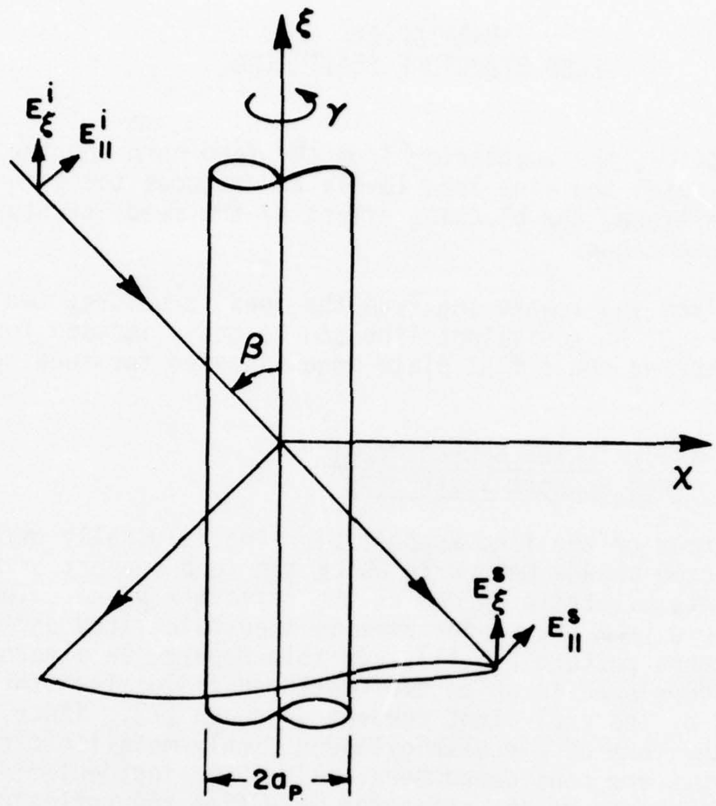


Figure 5. Geometry of the scattering cylinder with an incidence angle β.
(Note that E_ξ and $E_\|$ both lie in the plane of incidence, which contains the incident ray and the cylinder axis.)

The incident field can be expressed as

$$\overline{E}^i = (\hat{\chi}\cos\beta + \hat{\xi}\sin\beta)\, E_o\, e^{-jk(\chi \sin\beta + \xi \cos\beta)} \quad . \tag{41}$$

The field component parallel to the axis of the cylinder is given by

$$E^i_\xi = E_\| \sin\beta \;\; e^{-jk(\chi \sin\beta + \xi \cos\beta)} \quad . \tag{42}$$

Noting that

$$\chi = \zeta \cos\gamma \tag{43}$$

we have

$$E^i_\xi = E_\parallel \sin\beta \; e^{-jk\xi \cos\beta} \; e^{-jk\zeta \sin\beta \cos\gamma} \; . \tag{44}$$

For a wave incident at an angle β with respect to the cylinder axis, the analysis of the cylinder scattering closely follows that for $\beta = 90°$ [7]. Thus

$$E^i_\xi = E_\parallel \sin\beta \; e^{-jk\,\xi \cos\beta} \sum_{-\infty}^{\infty} j^{-n} J_n(k\zeta \sin\beta) \, e^{-jn\gamma} \; .$$

Then the scattered field component from the infinite circular cylinder is given by

$$E^s_\xi = E_\parallel \sin\beta \; e^{-jk\,\xi \cos\beta} \sum_{-\infty}^{\infty} j^{-n} a_n H^{(2)}_n(k\zeta\sin\beta) \times e^{jn\gamma} \; . \tag{45}$$

In the far field zone

$$E^s_\xi \simeq E_\parallel \sin\beta \; e^{-jk(\xi \cos\beta + \zeta \sin\beta)} \sqrt{\frac{2j}{\pi k\zeta\sin\beta}} \sum_{-\infty}^{\infty} a_n e^{jn\gamma}$$

$$= E_\parallel \sin\beta \; \frac{e^{-jks}}{\sqrt{s}} \; \frac{1}{\sin\beta} \sqrt{\frac{2j}{\pi k}} \sum_{n=0}^{\infty} \varepsilon_n a_n \cos n\gamma$$

where

$$a_n = - \frac{J_n(ka \sin\ )}{H^{(2)}_n(ka \sin\ )}$$

$$\varepsilon_n = \begin{cases} 1 & n = 0 \\ 2 & \text{otherwise} \end{cases}$$

and

$$s = \zeta/\sin\beta \; .$$

The scattered field for parallel polarization can be represented by

$$\overline{E}^s = \hat{\alpha} \frac{E^s_\xi}{\sin\beta} = \hat{\alpha} E_\parallel D_s(\gamma,\beta) \frac{e^{-jks}}{\sqrt{s}} \tag{46}$$

where the diffraction coefficient is given by

$$D_s(\gamma,\beta) = \frac{1}{\sin\beta}\sqrt{\frac{2j}{\pi k}}\sum_{n=0}^{\infty}\varepsilon_n\, a_n \cos n\gamma \quad . \tag{47}$$

Comparing $\overline{E}^s$ with the field of an infinite line source with current $I = I_o\, e^{-jk\xi\cos\beta}$, which is given by

$$\overline{E} = -\hat{\alpha}\,\frac{k^2 I_o}{4\omega\varepsilon}\sqrt{\frac{2j}{\pi k}}\,\frac{e^{-jks}}{\sqrt{s}} \tag{48}$$

we obtain an equivalent electric current line source for the conducting cylinder with a current equal to

$$I_o(\xi) = \frac{4\omega\varepsilon}{k^2}\,\frac{D_s(\gamma,\beta)}{\sqrt{\frac{2j}{\pi k}}}\,E_{\parallel}(\xi) \quad . \tag{49}$$

Then the scattering from a finite section of conducting cylinder can be obtained by calculating the radiation from the equivalent finite length line source. Thus the equivalent current I for the parallel polarized incident field becomes

$$I(\xi) = I_o(\xi)\, e^{jk\xi\cos\beta} \tag{50}$$

and the far zone radiation field is given by

$$E_\alpha = -\sqrt{\frac{jk}{2\pi}}\, D_s(\gamma,\beta)\,\frac{e^{-jkr}}{r}\,\sin\alpha\int_{\xi_1}^{\xi_2} E_{\parallel}(\xi)\, e^{jk\xi(\cos\alpha+\cos\beta)}\, d\xi \tag{51}$$

where (r,α,γ) specifies the spherical coordinates of the strut system. Similarly for the electric field component perpendicular to the axis of the cylinder, the scattered field can be obtained from $H_{\parallel}$ as

$$H_\alpha = \sqrt{\frac{jk}{2\pi}}\, D_h(\gamma,\beta)\,\frac{e^{jkr}}{r}\,\sin\alpha\int_{\xi_1}^{\xi_2} H_{\parallel}(\xi)\, e^{jk\xi(\cos\alpha+\cos\beta)}\, d\xi \tag{52}$$

where

$$D_h(\gamma,\beta) = \frac{1}{\sin\beta} \sqrt{\frac{2j}{\pi k}} \sum_{n=0}^{\infty} \varepsilon_n \, b_n \cos n\gamma$$

and

$$b_n = - \frac{J_n'(ka \sin\beta)}{H_n'^{(2)}(ka \sin\beta)} \quad .$$

The corresponding electric field for the perpendicularly polarized incident field is given by

$$E_\gamma = -\sqrt{\frac{jk}{2\pi}} \; D_h(\gamma,\beta) \frac{e^{-jkr}}{r} \sin\alpha$$

$$\times \int_{\xi_1}^{\xi_2} E_\perp(\xi) \, e^{jk\xi(\cos\alpha+\cos\beta)} \, d\beta \quad . \qquad (53)$$

B. Coordinate Transformation

In order to add the fields scattered from the feed struts to the radiation field from the reflector, each strut coordinate system needs to be transformed into the reflector coordinate system. As described in the previous section, the rectangular coordinates of the strut coordinate system are denoted by (χ,η,ξ); and (r,α,γ) denotes the spherical coordinates. The relationship between their respective unit vectors is given by

$$\begin{bmatrix} \hat{r} \\ \hat{\alpha} \\ \hat{\gamma} \end{bmatrix} = [A] \begin{bmatrix} \hat{\chi} \\ \hat{\eta} \\ \hat{\xi} \end{bmatrix} \qquad (54)$$

where

$$[A] = \begin{bmatrix} \sin\alpha\cos\gamma & \sin\alpha\sin\gamma & \cos\alpha \\ \cos\alpha\cos\gamma & \cos\alpha\sin\gamma & -\sin\alpha \\ -\sin\gamma & \cos\gamma & 0 \end{bmatrix} \qquad (55)$$

is the transformation matrix.

Similarly for the reflector coordinate system

$$\begin{bmatrix} \hat{R} \\ \hat{\theta} \\ \hat{\phi} \end{bmatrix} = [B] \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}$$

where

$$[B] = \begin{bmatrix} \sin\theta\ \cos\phi & \sin\theta\ \sin\phi & \cos\theta \\ \cos\theta\ \cos\phi & \cos\theta\ \sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} . \tag{57}$$

Now let us transform the (x,y,z) coordinates into the (χ,η,ξ) ones. The transformation consists of two steps: first rotate the x axis an angle ϕ_0 about the z axis, then the z axis is rotated an angle θ_0 about the y' (or η) axis, as shown in Figure 6. The resultant transformation can be expressed by the following representation:

$$\begin{bmatrix} \hat{\chi} \\ \hat{\eta} \\ \hat{\xi} \end{bmatrix} = \begin{bmatrix} \cos\theta_0\ \cos\phi_0 & \cos\theta_0\ \sin\phi_0 & -\sin\theta_0 \\ -\sin\phi_0 & \cos\phi_0 & 0 \\ \sin\theta_0\ \cos\phi_0 & \sin\theta_0\ \sin\phi_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} . \tag{58}$$

If we denote the angle between the strut along the ξ axis and the negative z-axis as β, and the angle between the x-axis and the strut projection on the xy plane as ϕ_s (see Figure 7), we have

$$\phi_0 = \phi_s \tag{59}$$

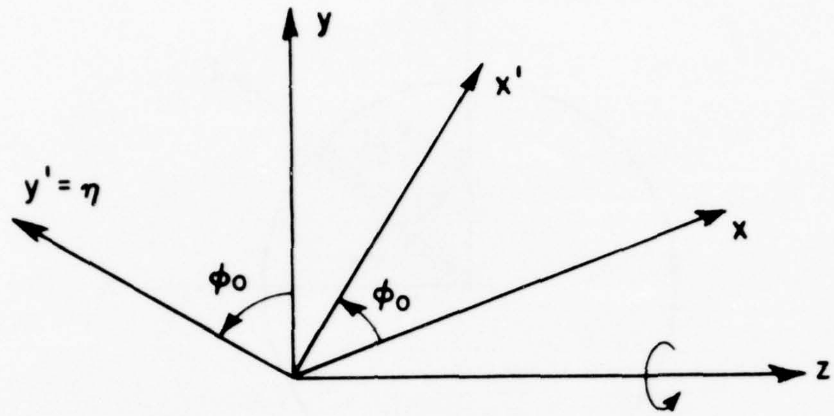
$$\theta_0 = \pi - \beta \tag{60}$$

and

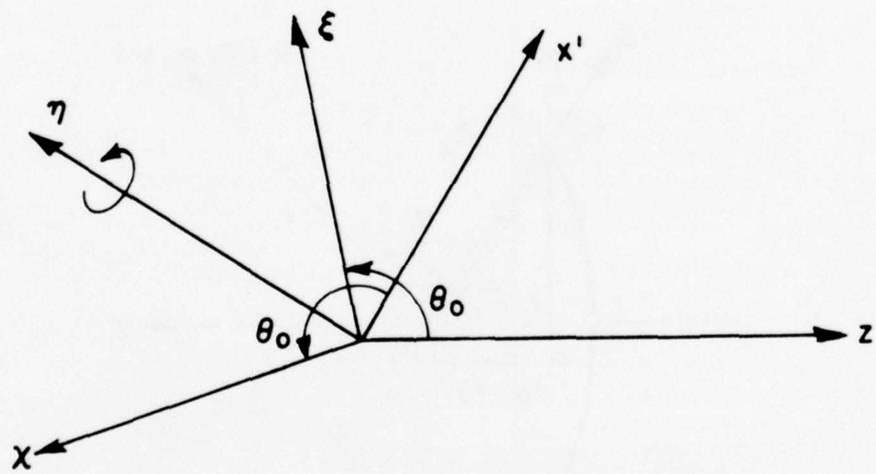
$$\begin{bmatrix} \hat{\chi} \\ \hat{\eta} \\ \hat{\xi} \end{bmatrix} = [T] \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \tag{61}$$

where

$$[T] = \begin{bmatrix} -\cos\beta\ \cos\phi_s & -\cos\beta\ \sin\phi_s & -\sin\beta \\ -\sin\phi_s & \cos\phi_s & 0 \\ \sin\beta\ \cos\phi_s & \sin\beta\ \sin\phi_s & -\cos\beta \end{bmatrix} . \tag{62}$$



(a) ROTATE ϕ_o ANGLE ABOUT z AXIS



(b) ROTATE θ_o ANGLE ABOUT η AXIS

Figure 6. Coordinate transformations between the strut and the reflector coordinate systems.

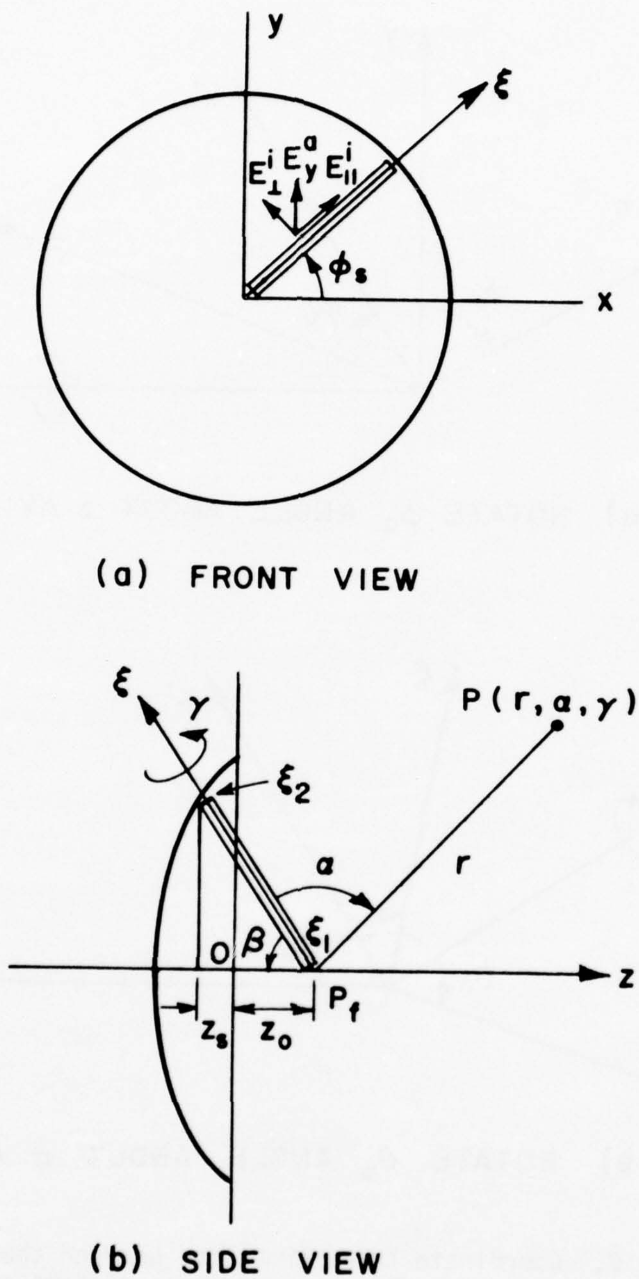


Figure 7. Geometry and coordinate system for an individual strut.

Then the relation between the two spherical coordinates (R,θ,ϕ) and (r,α,γ) is given by

$$\begin{bmatrix} \hat{r} \\ \hat{\alpha} \\ \hat{\gamma} \end{bmatrix} = [A][T][B]^{-1} \begin{bmatrix} \hat{R} \\ \hat{\theta} \\ \hat{\phi} \end{bmatrix} \tag{63}$$

where $[B]^{-1}$ is the inverse matrix of $[B]$. To express α and γ in terms of θ,ϕ,β, and ϕ_s, the components of χ,η,ξ are written in the (x,y,z) coordinate system as

$$\chi = -x\cos\beta\cos\phi_s - y\cos\beta\sin\phi_s - z\sin\beta \tag{64}$$

$$\eta = -x\sin\phi_s + y\cos\phi_s \tag{65}$$

$$\xi = x\sin\beta\cos\phi_s + y\sin\beta\sin\phi_s - z\cos\beta \quad . \tag{66}$$

But

$$\chi = R\sin\alpha\cos\gamma \tag{67a}$$

$$\eta = R\sin\alpha\sin\gamma \tag{67b}$$

$$\xi = R\cos\alpha \tag{67c}$$

and

$$x = R\sin\theta\cos\phi \tag{68a}$$

$$y = R\sin\theta\sin\phi \tag{68b}$$

$$z = R\cos\theta \tag{68c}$$

Putting Equations (67a) and (68) into Equation (64) and combining terms, we obtain

$$\sin\alpha\cos\gamma = -[\sin\theta\cos\beta\cos(\phi-\phi_s) + \cos\theta\sin\beta] \quad . \tag{69}$$

Similarly, putting Equations (67b) and (68) into Equation (65), we get

$$\sin\alpha\sin\gamma = \sin\theta\sin(\phi-\phi_s) \quad . \tag{70}$$

Dividing Equation (70) by Equation (69), we have

$$\tan\gamma = \frac{\sin\theta\sin(\phi-\phi_s)}{-[\sin\theta\cos\beta\cos(\phi-\phi_s) + \cos\theta\sin\beta]} \quad . \tag{71}$$

Also, substituting Equations (67c) and (68) into Equation (66), we get

$$\cos\alpha = \sin\theta \sin\beta \cos(\phi-\phi_s) - \cos\theta \cos\beta \quad . \tag{72}$$

For a given observation direction θ and ϕ, the angular coordinate α and γ for a given strut can be calculated from Equations (71) and (72), where β and ϕ_s specify the orientation of the strut. The field components in the strut coordinate system can be transformed to the reflector coordinate system by using the following dot products obtained from Equation (63) as

$$\begin{aligned}\hat{\alpha}\cdot\hat{\theta} = &- \cos\theta \cos(\phi-\phi_s)[\sin\alpha \sin\beta + \cos\alpha \cos\gamma \cos\beta] \\ &+ \cos\alpha \sin\gamma \sin(\phi-\phi_s) \cos\theta \\ &+ \sin\theta(\cos\alpha \cos\gamma \sin\beta - \sin\alpha \cos\beta)\end{aligned} \tag{73a}$$

$$\begin{aligned}\hat{\alpha}\cdot\hat{\phi} = &[\sin\alpha \sin\beta_i + \cos\alpha \cos\gamma \cos\beta] \sin(\phi-\phi_s) \\ &+ \cos\alpha \sin\gamma \cos(\phi-\phi_s)\end{aligned} \tag{73b}$$

$$\begin{aligned}\hat{\gamma}\cdot\hat{\theta} = &\sin\gamma[\cos\beta \cos\theta \cos(\phi-\phi_s) - \sin\beta \sin\theta] \\ &+ \cos\gamma \cos\theta \sin(\phi-\phi_s)\end{aligned} \tag{73c}$$

$$\hat{\gamma}\cdot\hat{\phi} = - \sin\gamma \cos\beta \sin(\phi-\phi_s) + \cos\gamma \cos(\phi-\phi_s) \tag{73d}$$

Thus the far field scattered from the strut can be expressed in the (R,θ,ϕ) coordinate system as

$$E_\theta = \hat{\theta}\cdot\hat{\alpha}\, E_\alpha + \hat{\theta}\cdot\hat{\gamma}\, E_\gamma \tag{74}$$

$$E_\phi = \hat{\phi}\cdot\hat{\alpha}\, E_\alpha + \hat{\phi}\cdot\hat{\gamma}\, E_\gamma \quad . \tag{75}$$

E_α and E_γ are given by Equation (51) and (53), respectively in which

$$E_\parallel = E_y^a \sin\phi_s \tag{76}$$

and

$$E_\perp = E_y^a \cos\phi_s \tag{77}$$

as shown in Figure 7.

In addition to the coordinate rotation described above, the transformation also consists of a linear displacement of the coordinate centers along the z-axis as shown in Figure 8. Thus

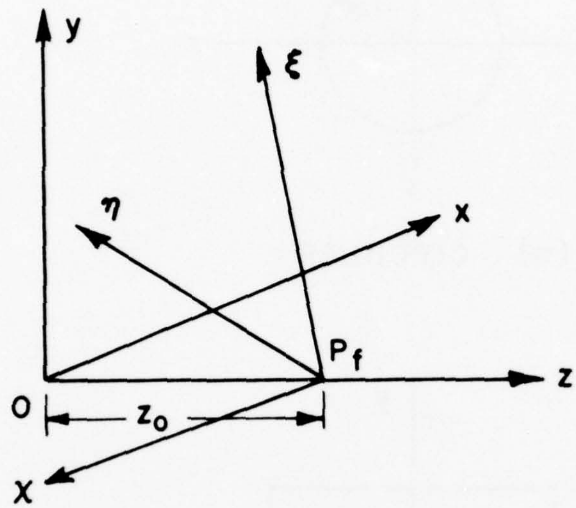


Figure 8. Linear axis placement of the strut and the reflector coordinate systems.

$r \approx R - z_o \cos\theta$ in the phase expression for the far field approximation in Equations (51) and (53), and $r \approx R$ in magnitude. The quantity z_o is the distance between the two coordinate centers, i.e., the distance from the center of the reflector aperture to the intersection of the strut on the z-axis.

C. Feed Horn Scattering Model

A commonly used method to analyze the feed horn blockage is to assume that the scattering from the feed aperture is the same as that from a conducting flat plate whose area is equal to the cross section of the feed. This approximation is justified in the forward region, where the feed blockage is generally most significant. Then the effect of the aperture radiation being blocked by the feed can be approximated by adding the pattern of the equivalent flat plate to that of the reflector. Since the feed aperture is relatively small compared with the antenna aperture and is located at the center, the aperture field can be assumed uniform over the flat plate. So

$$\overline{E}_c^a = -\hat{y}\, e^{-jkR_o} \tag{78}$$

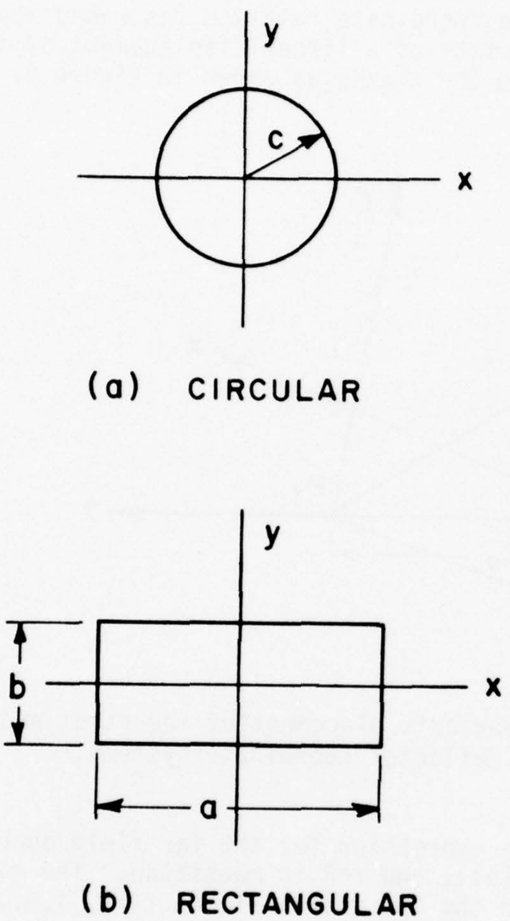


Figure 9. Flat plate models for the feed horn scattering.

for a feed with circular aperture of radius C (see Figure 9a), such as a circular feed horn. The far zone scattered field for small θ is given by

$$\overline{E}_c = - \hat{y} \frac{j}{\lambda} \frac{e^{-jkR}}{R} 2\pi c^2 \frac{J_1(k\ c \sin\theta)}{k\ c \sin\theta} e^{-jkR_o} \quad . \tag{79}$$

If the feed is a rectangular waveguide or horn, the scattered field is given by

$$\overline{E}_R = -\hat{y}\, j\, \frac{e^{-jkR}}{\lambda R}\; ab \left[\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\cos\phi\right)}{\frac{\pi a}{\lambda}\sin\theta\cos\phi)}\right] \times$$

$$\left[\frac{\sin\left(\frac{\pi b}{\lambda}\sin\theta\sin\phi\right)}{\frac{\pi b}{\lambda}\sin\theta\sin\phi}\right] \qquad (80)$$

where a and b are the width and height of the flat plate model, respectively, (see Figure 9b). The scattered field components can be obtained in spherical coordinates by using Equations (39) and (40).

CHAPTER IV
RESULTS AND DISCUSSION

Previous analyses [10,11,12] of feed support scattering have usually been limited to angles near the main beam region. However, the cylinder scattering approach developed in this report provides a model to treat the wide angle scattering of each strut. Therefore, in addition to the gain loss, this approach also gives the effect of strut scattering on the wide angle side lobes of the reflector.

According to the geometrical optics assumption, the forward scattering from a cylinder is identical to that from a strip with the same cross section provided that the diameter of the cylinder is large enough. Thus the field obtained from the two solutions should be nearly the same in the forward region. In Figure 10, the scattered pattern of a rectangular strip is compared with that of a vertical circular cylinder ($\beta = 90°$) with uniform incident field, whose E-field is parallel to the cylinder axis. As shown by the curves, the field near the forward axis in the H-plane (transverse to the cylinder axis) agrees within 1 dB for $ka_p = 7$, where a_p is the radius of the strut. For larger ka_p, the difference is even smaller since the geometrical optics approximation of the rectangular strip for the cylinder gets better. For wide angles ($\theta > 20°$), the agreement becomes poorer, because the shadow aperture model is not valid there. Another comparison is made between the Induced Field Ratio (IFR) coefficients in the paper by Rusch et al [12], and those calculated from the equations in this report. The IFR coefficient is defined to be the ratio of the field forward scattered by the cylinder to that of a flat strip with the same physical cross section as the cylinder. Thus the IFR can be obtained from the ratio of Equation (51) with $E_{\parallel} = 1$, $\xi_1 = 0$ and $\xi_2 = L$ to Equation (80) with $a = 2a_p$ and $b = L$. Values of the IFR obtained from this ratio are compared in Table 1 with those calculated by Rusch et al, where the IFR coefficients of [12] are taken from Figure 2 of that paper. The consistency of the two methods in the forward axis region is evident. Since the IFR as defined in [12] is restricted to forward scatter, the results of [12] cannot be used in this work, where the bistatic scattering from the cylindrical strut is required.

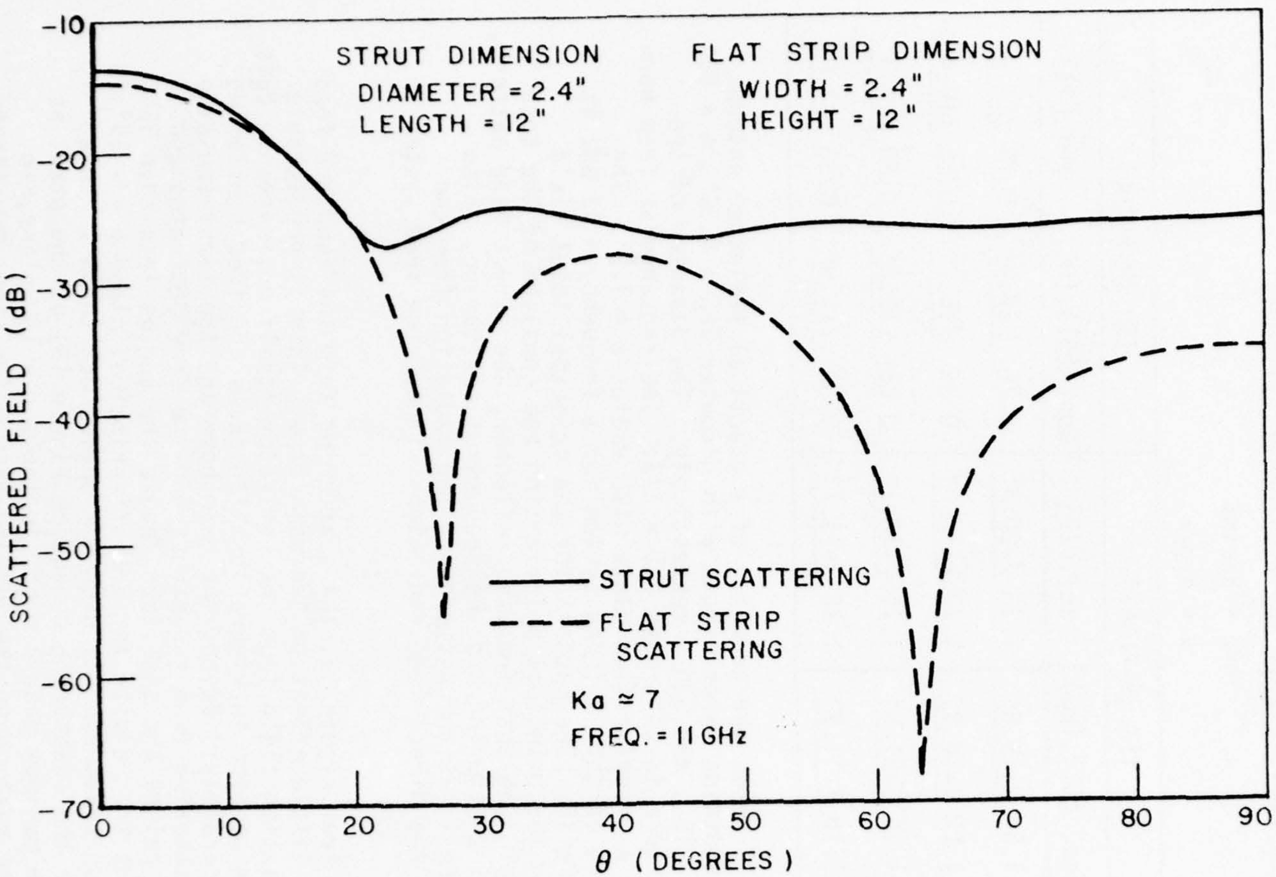


Figure 10. Comparison of the scattering from a circular cylinder (vertical strut) and a flat strip in the H-plane.

Table 1

IFR Coefficients for a Circular Cylinder

Arg = $ka\sin\beta$

$x = \text{Arg}/\pi$

		IFR (E-PLANE)		IFR (H-PLANE)	
Arg	x	Eqs. (51),(80)	Ref.[12]	Eqs. (51),(80)	Ref.[12]
2.54	0.81	1.34 $\underline{/-19.8°}$	1.33 $\underline{/-20.3°}$	0.78 $\underline{/22.9°}$	0.79 $\underline{/21.9°}$
4.62	1.47	1.22 $\underline{/-14.3°}$	1.22 $\underline{/-15.0°}$	0.85 $\underline{/15.7°}$	0.85 $\underline{/16.4°}$
6.74	2.15	1.16 $\underline{/-11.5°}$	1.16 $\underline{/-12.1°}$	0.88 $\underline{/12.4°}$	0.89 $\underline{/11.5°}$
7.02	2.24	1.16 $\underline{/-11.2°}$	1.15 $\underline{/-11.7°}$	0.89 $\underline{/11.5°}$	0.90 $\underline{/11.0°}$

Figure 11 shows the geometry of a practical reflector antenna system which contains three struts with diameter $2a_p$ = 0.37", β = 68° and ϕ_s = 90°, 210°, and 330°, respectively. The diameter of the reflector aperture is 24", with F/D = 1/3. The rectangular feed horn is mounted in a circular flat plate with radius c = 1.2". The H-plane pattern of this antenna system for a frequency = 11 GHz is shown in Figure 12. The pattern for the principal total field corresponds to the principal polarization and consists of the total radiation, including that from the reflector, the direct feed pattern, the feed model scattering and the feed strut scattering. Also shown in Figure 12 is the pattern for the radiation from the reflector and the direct feed radiation, without the feed system scattering.

As seen from Figure 12, the scattering from the feed and feed struts causes little effect on the main beam. This is predictable since the radiation field from the reflector itself dominates in this region, as can be seen in Figure 13 which shows the feed scattering components separately. Beyond the main beam and the first few side lobes, the scattering from the struts and the feed dominates and substantially raises the side lobe levels for angles less than 35°. Figure 14 shows the results for the off-principal plane ϕ = - 15° in which the strut scattering is high for angles out to 80°. This occurs because the scattering cone (see Figure 15) of the strut at ϕ_s = 90° has a maximum near θ = 65° in the ϕ = - 15° plane, in addition to the maximum on the forward axis. The other two struts make less contribution to the scattered field in this plane since their scattering cones are located in different directions. Therefore the feed support scattering effect in the ϕ = - 15° plane comes mainly from the strut at ϕ_s = 90°. The scattered field patterns of the

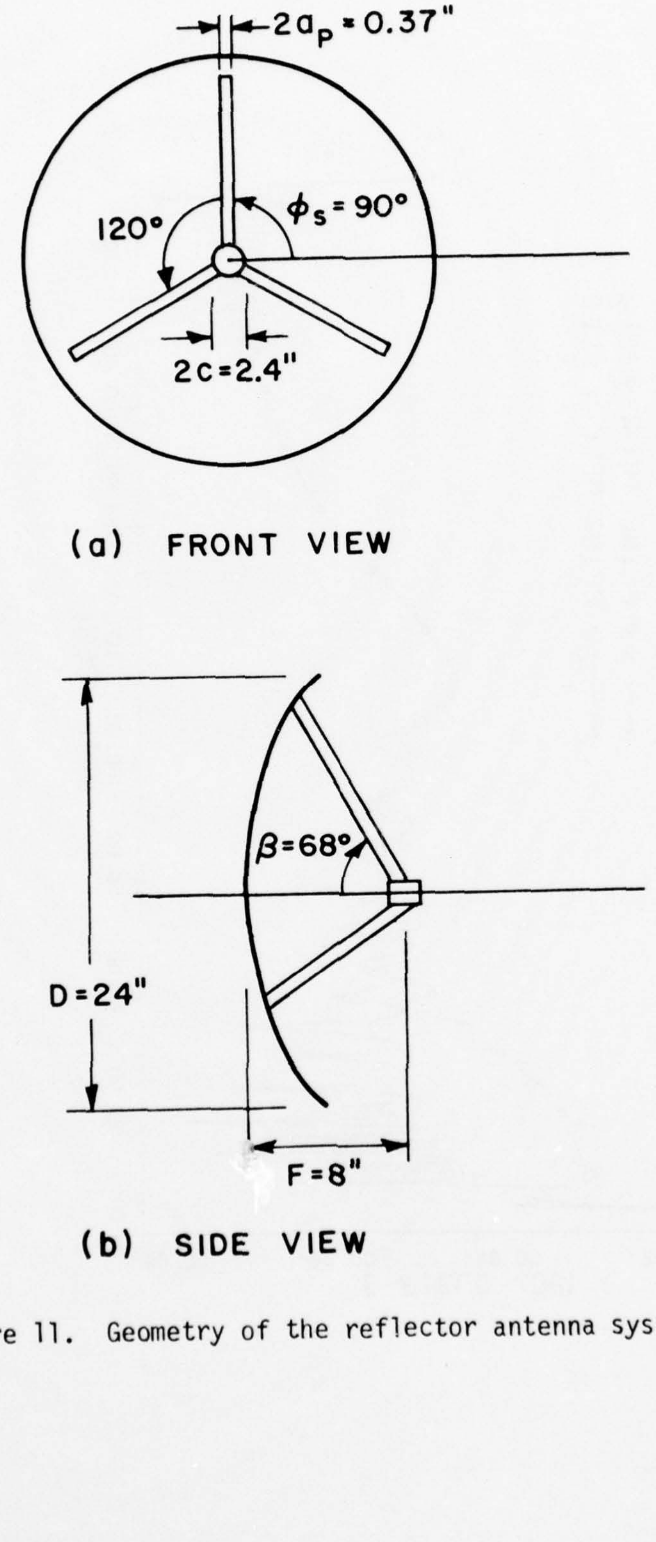


Figure 11. Geometry of the reflector antenna system.

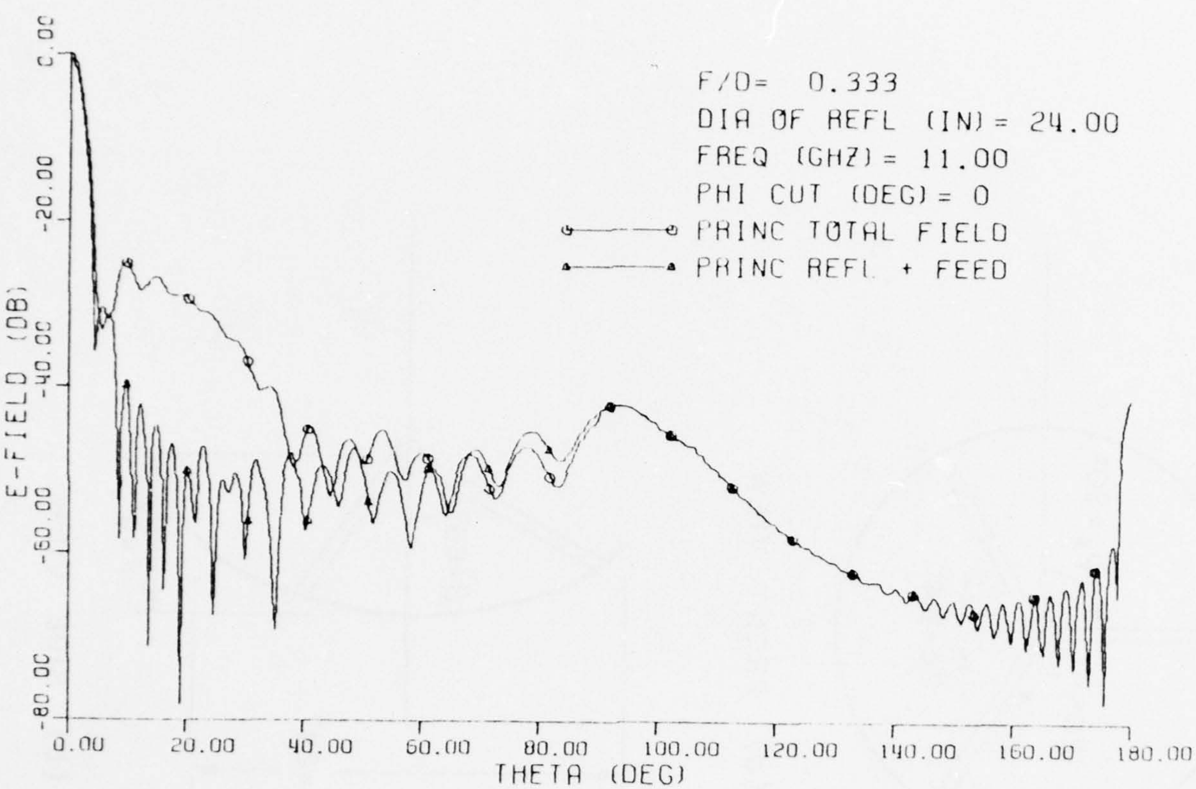


Figure 12. H-plane pattern (ϕ=0) of the reflector antenna.

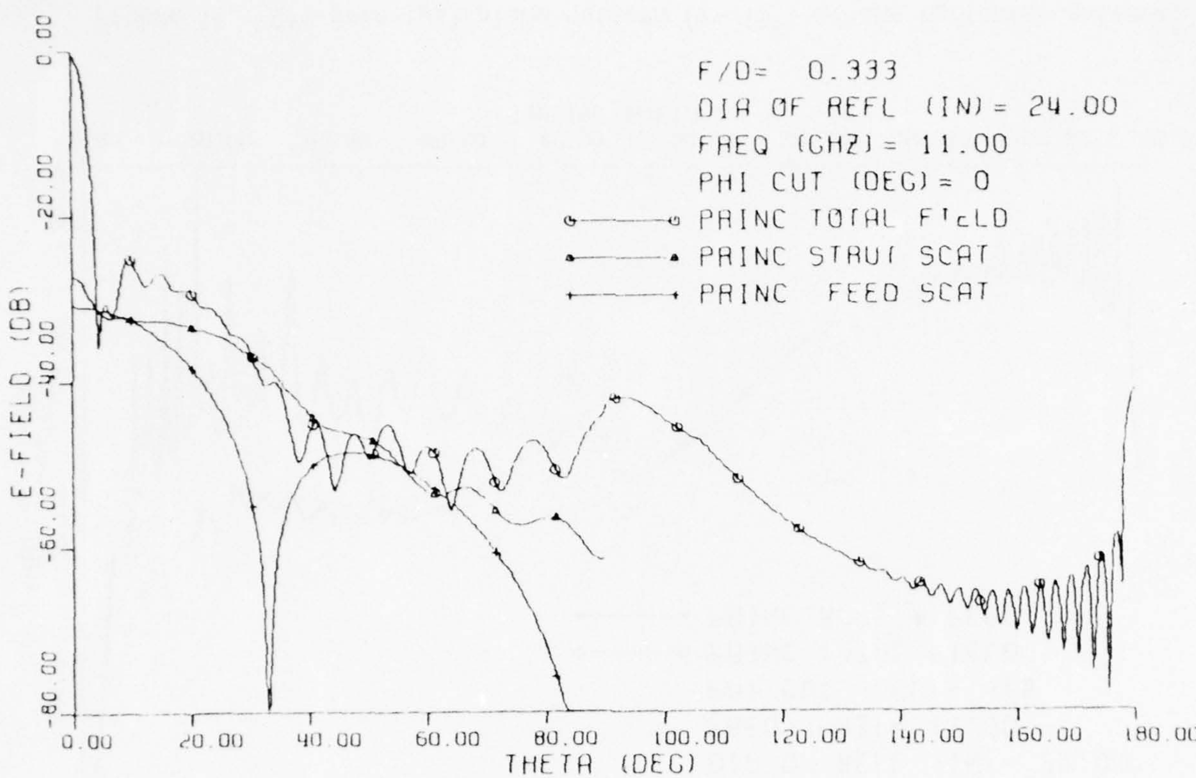


Figure 13. H-plane pattern (ϕ=0) of the feed strut scattering and feed horn scattering.

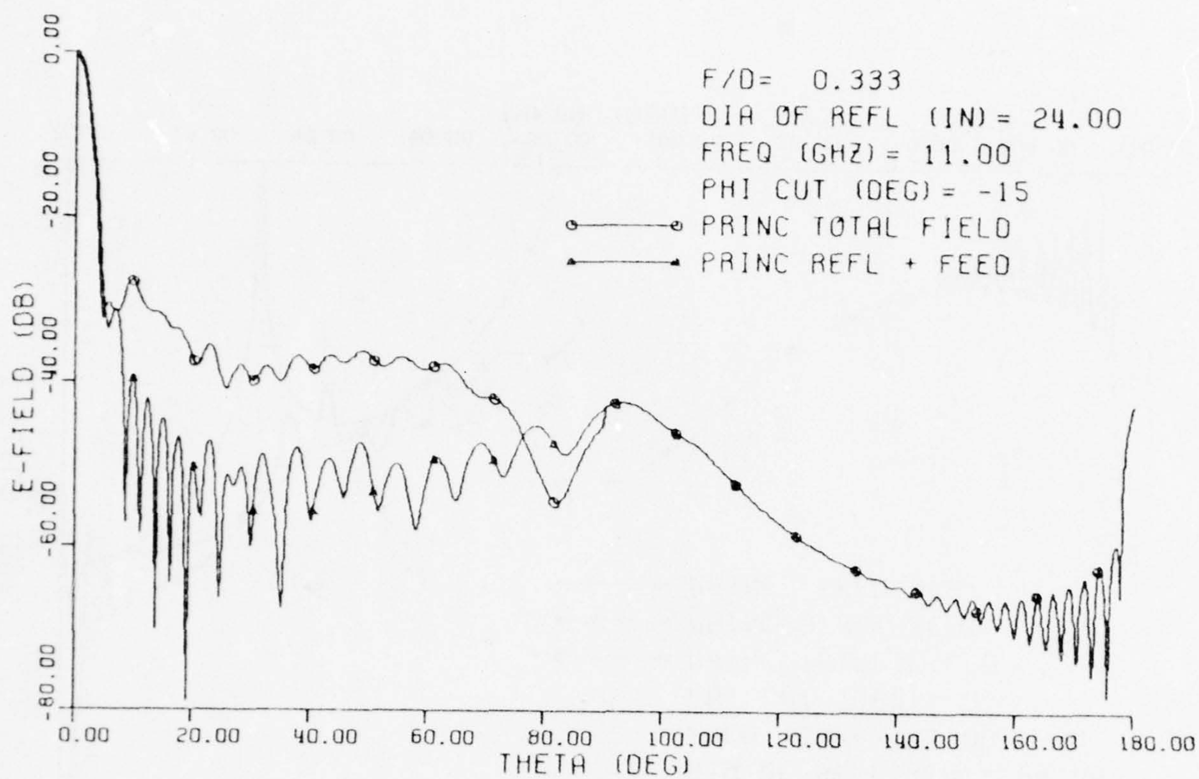


Figure 14. Off-principal plane pattern (ϕ=-15°) of the reflector antenna.

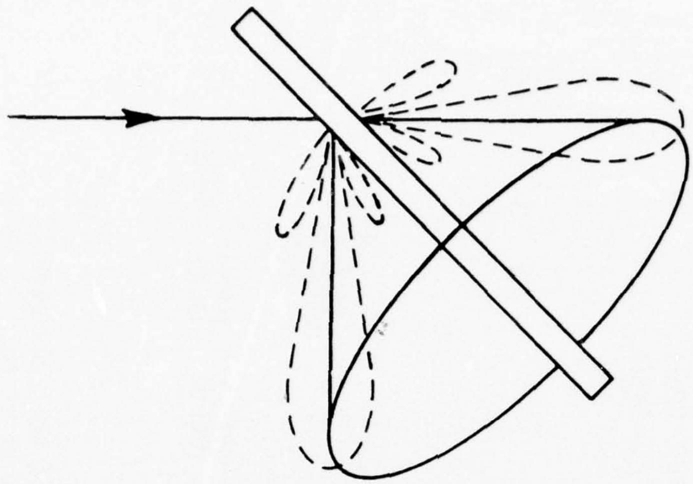


Figure 15. Scattering cone of a cylinder.

struts and the feed horn are shown in Figure 16. Note that the total pattern is no longer symmetric with respect to the z-axis for most off-principal plane cuts as can be seen by comparing Figure 14 and Figure 17 for the $\phi = -15°$ and $\phi = +165°$ plane pattern. Even though the feed pattern (and thus the aperture field) is assumed to be symmetric, the strut geometry is not symmetric in this plane.

The antenna patterns in these figures correspond to the principal polarization, defined in terms of a Huygen's source. Thus the θ and ϕ components can be transformed to principal and cross polarized components by

$$E_{princ} = \sin\phi \, E_\theta + \cos\phi \, E_\phi \tag{81}$$

$$E_{cross} = \cos\phi \, E_\theta - \sin\phi \, E_\phi \tag{82}$$

The fields scattered from the feed and struts are shadowed by the reflector surface for much of the rear hemisphere. However, the shadow boundary for the feed support scattering is difficult to define since part of the scattered field is blocked by the reflector as θ approaches 90°, and this blocking keeps on increasing as θ passes 90° until the scattered field is totally blocked out at $\theta = \pi - \alpha$, the shadow boundary for direct feed radiation. For simplicity, the shadow boundary for the feed support and feed horn scattering is set at $\theta = 90°$. This approximation should be reasonable since the scattered field from the struts is usually small at pattern angles $\theta > 90°$. However, a slight discontinuity still exists in the pattern at $\theta = 90°$ because of this truncation.

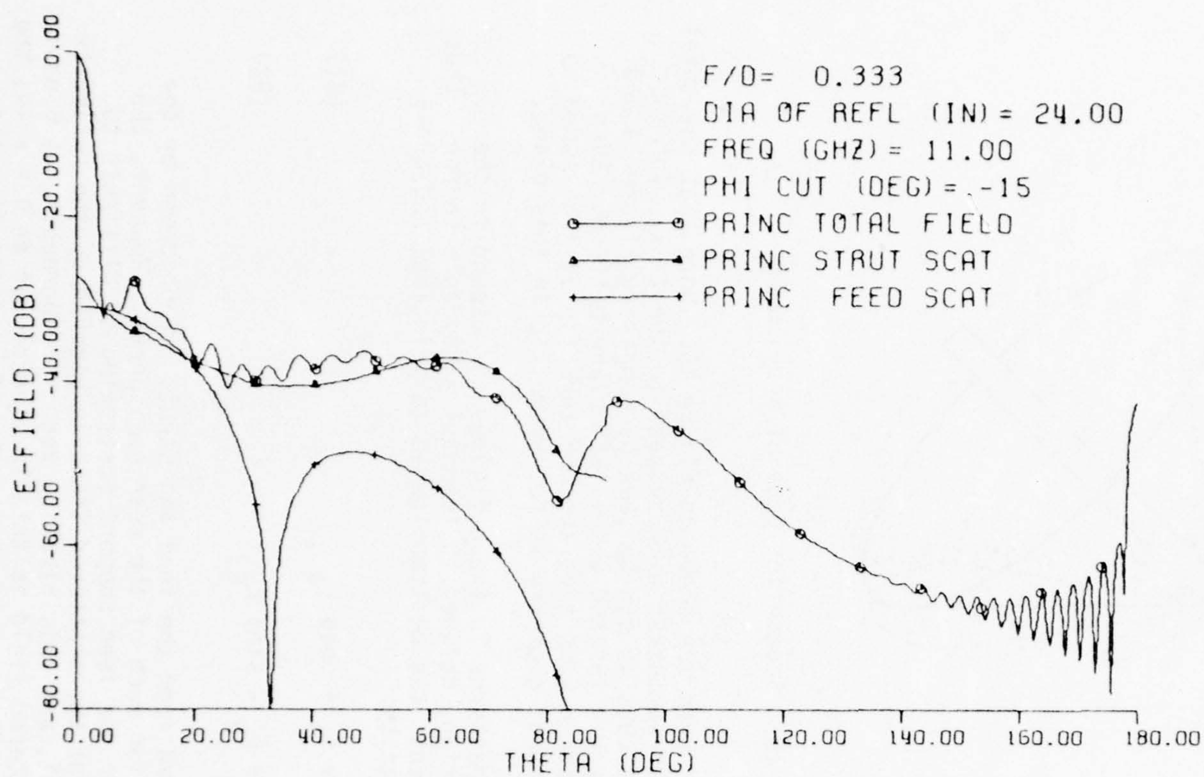


Figure 16. Off-principal plane (ϕ=-15°) of the feed strut scattering and the feed horn scattering.

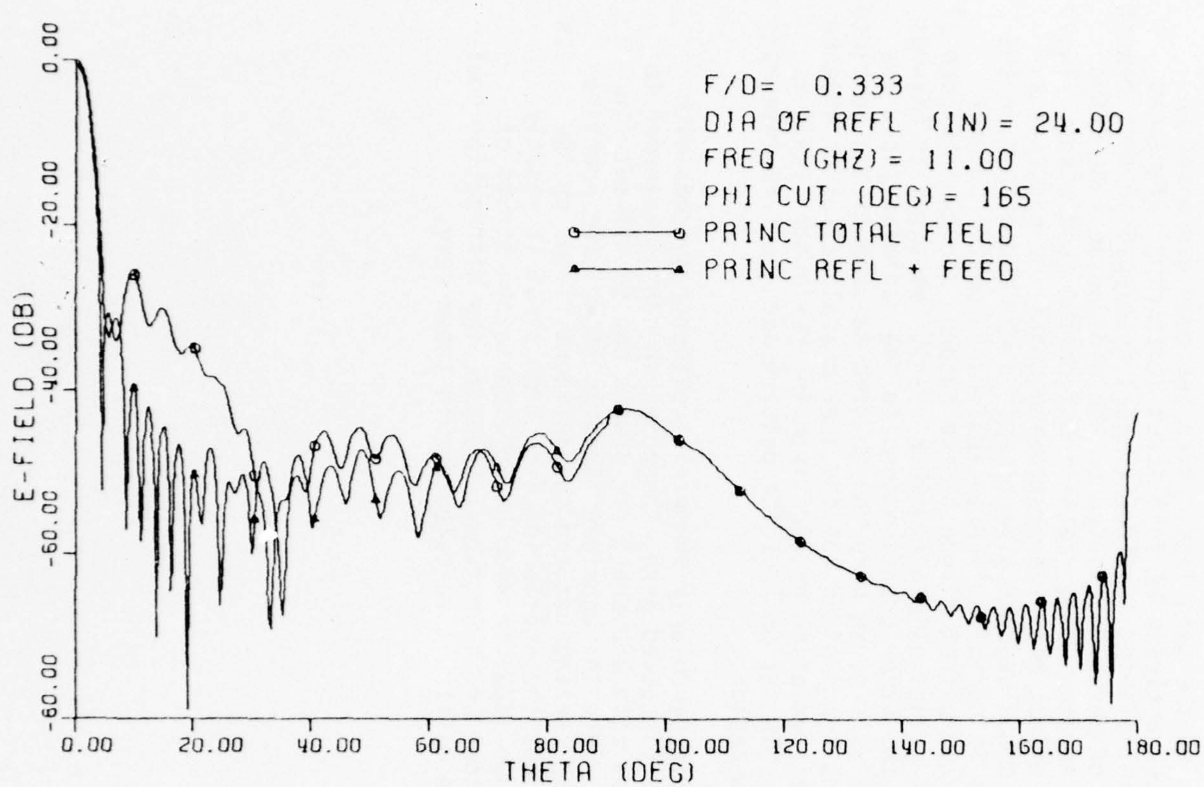


Figure 17. Off-principal plane pattern (ϕ=+165°) of the reflector antenna.

CHAPTER V
CONCLUSIONS

In addition to the gain loss, which has been discussed in previously reported analyses of feed strut scattering referenced in Chapter IV, the cylinder scattering model developed in this report gives the effect of strut scattering on the wide angle side lobes of the reflector antenna. The scattering from the feed struts always has its greatest effect near the scattering cones for each strut. These scattering cones usually give rise to a maximum effect on the antenna sidelobes in certain off-principal plane patterns. For example, the maximum effect of one of the antenna struts in Figure 11 occurs in the off-principal plane with $\phi = -15°$ as shown in Figures 14 and 16. When the other contributions to the radiated field are included, the analysis given here can be used to compute the complete pattern of any arbitrary plane cut for a practical reflector antenna system. The basic radiation pattern from the reflector itself is obtained by using GTD for most of the pattern and aperture integration for the main beam region.

Although only circularly symmetric reflector systems with focused feeds are discussed here, the approach can be extended to reflector antennas with arbitrary rim shapes and to off-set fed systems as well. Once the aperture field is known, the aperture integration can be carried out and the incident fields on the struts are available. Thus the scattering from each strut is readily obtained. The same idea can also be applied to the struts of rectangular or other cross sections; however, the diffraction coefficients are more difficult to calculate in these cases.

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PART II - USER'S MANUAL FOR COMPUTER PROGRAM
"CIRCULAR REFLECTOR ANTENNA WITH FEED AND STRUT SCATTER" (OSUPATT)

A. Description of Computer Program

OSUPATT is a computer program which calculates the total field radiated by a circularly symmetric parabolic reflector including effects of scattering by a number of circularly cylindrical struts and blockage by the feed horn. The user is free to specify the geometry of the reflector, the size of the feed horn cross-section, both the E- and H-planes of the feed pattern, and the size and orientation of the struts. This brief user's document will indicate the methods used and the limitations of the program, and required formats for input data; details of the theory are given in Part I of this report.

The basic geometry used in the program is shown in Figure 1.

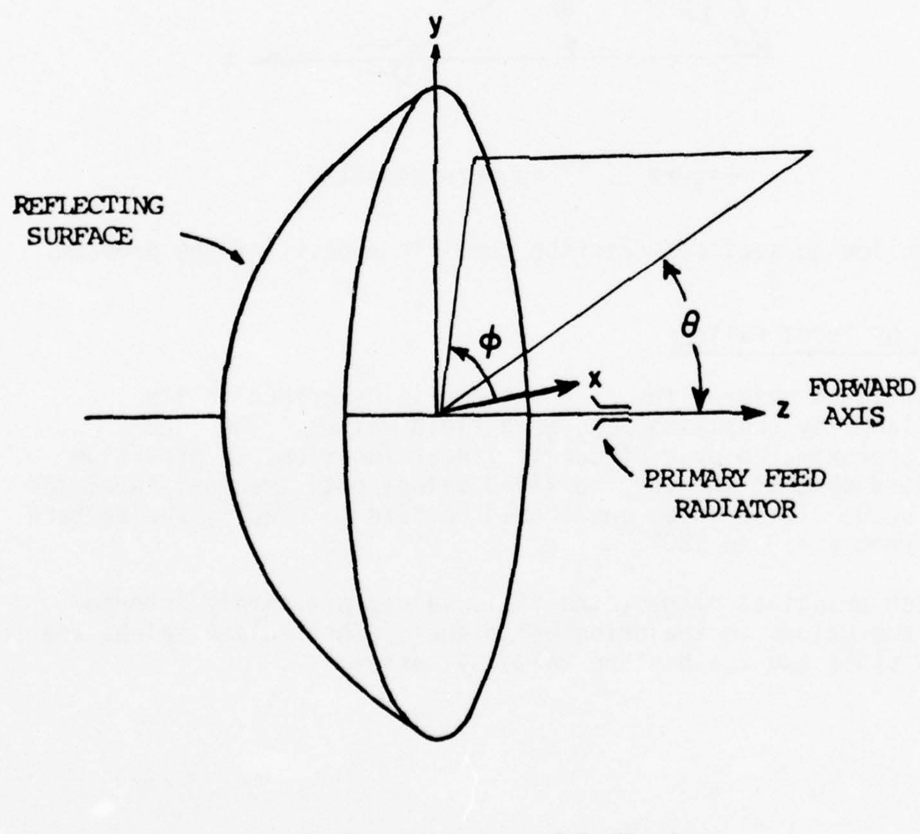


Figure 1. Geometry of Reflector.

The reflector points in the +z direction with the feed located at the focus of the parabola. The polarization of the feed is linear in the +y direction; thus E_ϕ is the principal polarization in the $\phi = 0$ plane and E_θ is the principal polarization in the $\phi = 90°$ plane. The E-plane of the feed horn and the reflector is thus the yz plane and the H-plane is the xz plane. The struts must be oriented so that each strut projects through the z axis. At this time the principal plane feed patterns must be symmetric. These patterns are specified by listing field values for discrete values of ψ, the angle away from the -z axis (see Figure 2).

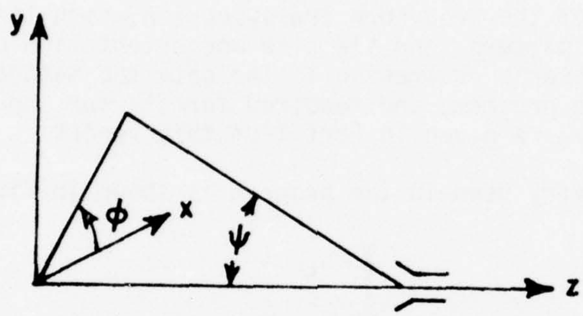


Figure 2. Feed horn geometry.

The following sections describe specific aspects of the problem.

DESCRIPTION OF INPUT PATTERN

As mentioned before, the feed pattern is described in its principal planes by supplying tabulated field values. The feed pattern is approximated by a piecewise linear function. A provision has been added so that the assumed field values past the last tabulated value will be 0. To be safe, one should be sure to specify the pattern explicitly from $\psi = 0$ to $180°$.

Between principal planes, the field values are simply interpolated from the values in the principal planes. The E-plane values are weighted by $\sin^2\phi$ and the H-plane value by $\cos^2\phi$.

REFLECTOR PATTERN CALCULATION

The reflector's secondary pattern is calculated by two different methods over two different ranges of polar angle θ. For small θ ($\frac{kD}{2} \sin \theta < 24$), the aperture fields are integrated as specified by physical optics. Due to the symmetry of this problem, the integration is still one dimensional and thus relatively large apertures can be used without prohibitive computer costs. For θ angles past the limit, the Geometrical Theory of Diffraction (GTD) is used. The fields calculated by the program are all normalized by the peak field of the reflector by itself. Any direct field from the feed is also included in the total field.

FEED BLOCKAGE

The effect of blockage by a feed cross-section is very simply modeled in the forward direction by Kirchkoff's method. When a rectangular feed is specified, the scattered pattern has the familiar $\sin(x)/x$ shaped pattern in each principal direction while for a circular feed, the pattern has $J_1(x)/x$ variation.

STRUT BLOCKAGE

Scattering by circular cylindrical struts is calculated by first finding the equivalent currents on segments of the struts and then integrating the reradiated fields which are produced by these currents. The equivalent currents are calculated from the diffraction coefficients of the cylindrical strut cross-section and the incident aperture field. The diffraction coefficients are a summation of eigenmodes for the circular cylinder.

LIMITATIONS

Due to limitations in theory or the arrangement of the computer program, the following limits should be observed.

STRUTS

STRUT must not extend past z axis, i.e., RL02 must not exceed $\frac{RS22}{\sin\beta}$. The case of a strut passing through the axis can be done by using two separate struts that meet at the axis.

STRUT diameter should not exceed four wavelengths. Maximum number of struts is 4. Beta should be in the range 0° to 90°.

For best results, strut angle BETA should be large enough so that sampling points are less than a 1/4 wavelength apart, i.e., $\sin\beta > \frac{2D/\lambda}{NAP}$.

FEED PATTERN

The maximum number of points to fit the pattern (N2) is 50.

REFLECTOR

Maximum diameter is 100 wavelengths.

INPUT FORMAT

GENERAL COMMENTS

All distance parameters required for input are in inches; all angles should be specified in degrees.

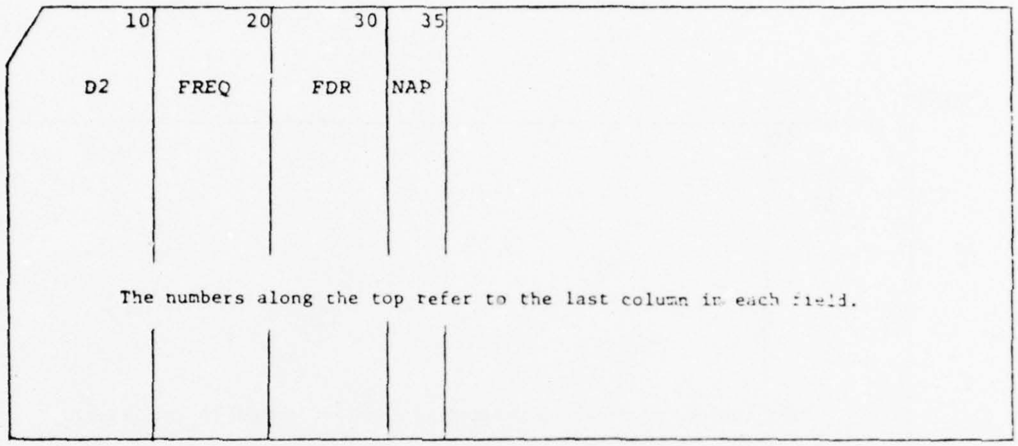
All decimal numbers may be anywhere within their specified field; all integers must be right adjusted. The decimal point must explicitly appear for all decimal numbers.

Comments may be typed on any card to the right of specified fields.

REFLECTOR GEOMETRY CARD

PURPOSE: To enter geometry of reflector

CARD:



PARAMETERS: (I integer, F decimal number)

D2 (F) -diameter of reflector

FREQ (F)- frequency in MHZ

FDR (F) - ratio of focal distance f to diameter D

NAP (I) - number of aperture integration points. If this parameter is less than 50, or if it is omitted, 50 will be used. If over 99 is used, 99 will be used.

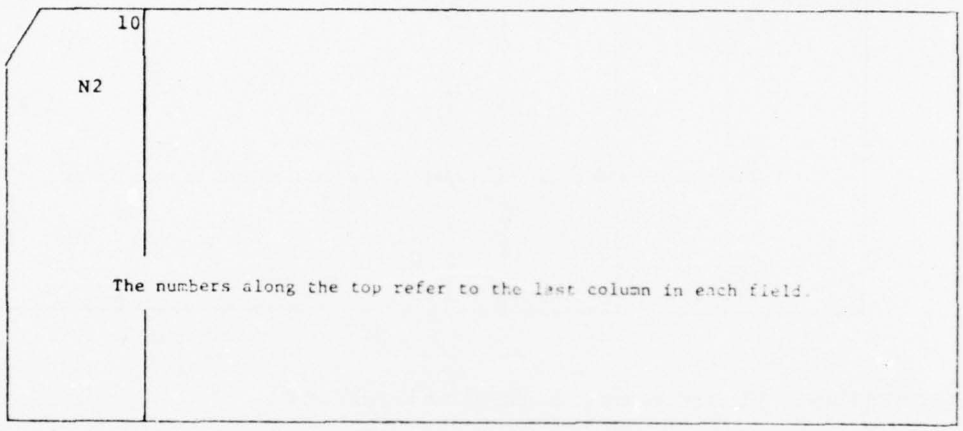
NOTES:

- The NAP parameter allows an increase of integration points over both the aperture and the strut for large sized reflectors, while allowing a smaller number of points and smaller computer costs for smaller problems. The value 99 may always be used if the computer cost is not objectionable.

FEED PATTERN CARD

PURPOSE: To provide data on number of points used to model primary feed pattern.

CARD:



PARAMETERS: (I integer, F decimal number)

N2 (I) - number of data points used to determine pattern in piecewise linear function.

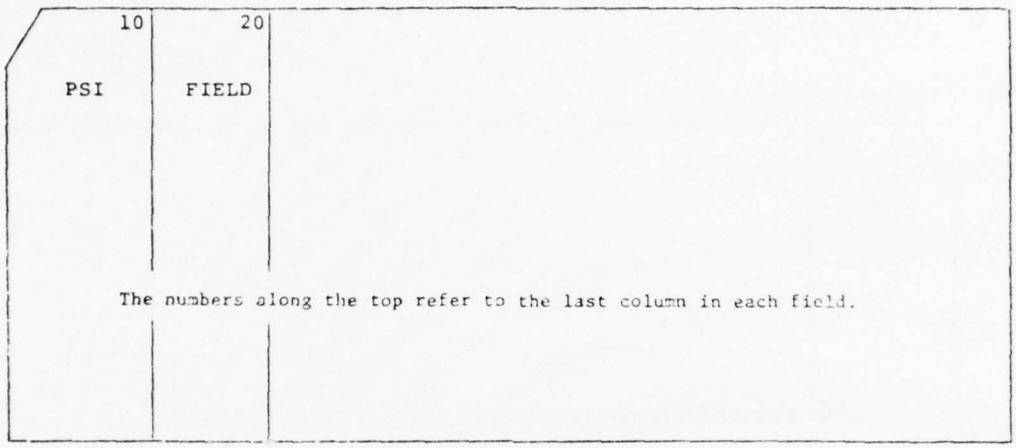
NOTES:

- This card determines how many cards of the next format will be read.

FEED PATTERN DATA CARDS

PURPOSE: To provide tabulated values of primary pattern.

CARD:



PARAMETERS: (I integer, F decimal number)

PSI (F) : Angle from axis of horn (-z axis of reflector, see figure 2) where tabulated value of field is to be supplied.

FIELD (F) : Field value of horn at that PSI angle.

NOTES:

- There should be 2 groups of cards of this format.

  N2 cards { Tabulated values to be interpolated by piece-wise linear function for E plane

  N2 cards { Tabulated values to be interpolated by piece-wise linear function for H plane

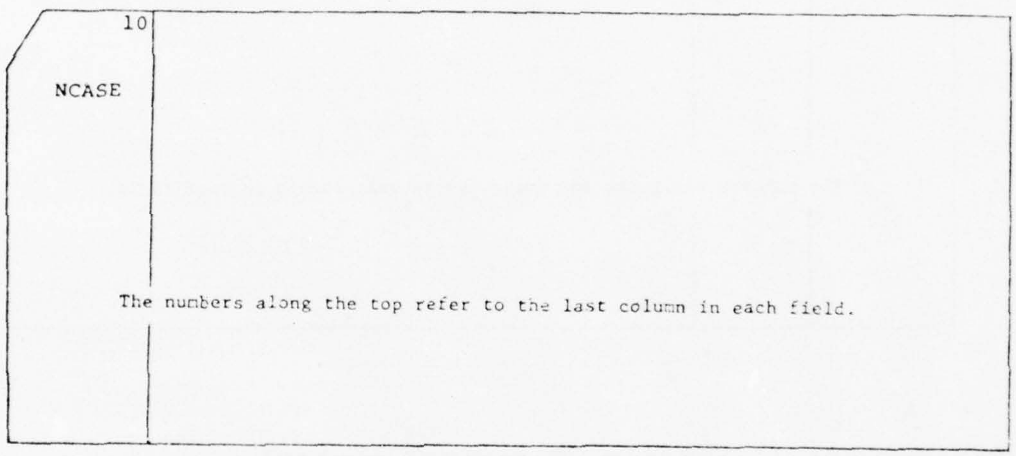
- For PSI larger than the last given value of PSI, a zero is assumed for the field value

- The maximum value of psi should subtend the edge of the reflector or erroneous results will occur.

NUMBER OF STRUT PROBLEMS CARD

PURPOSE: To indicate to the program how many problems will be done with the same reflector and feed pattern (above cards) but with different blockage parameters (below cards)

CARD:



PARAMETERS: (I integer, F decimal number)

NCASE (I) - number of different strut problems to be done with same reflector and feed pattern

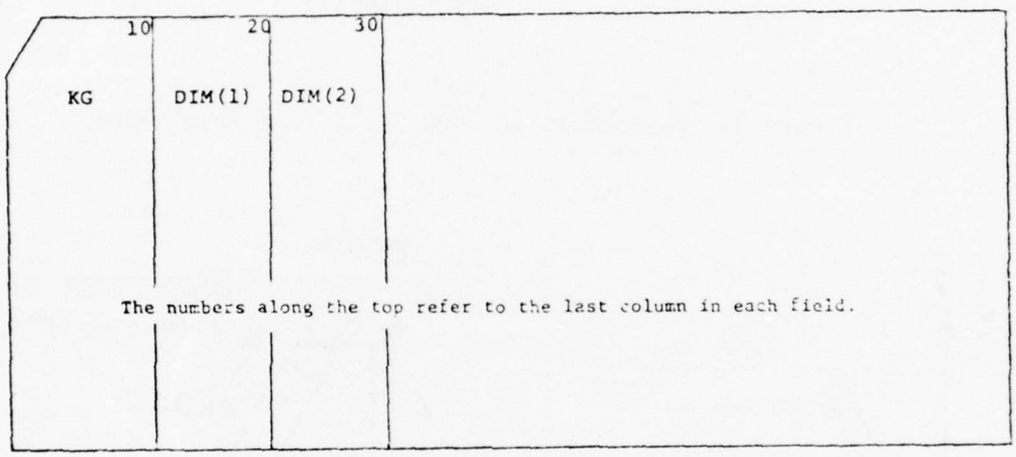
NOTES:

- If NCASE is not 1, the entire group of cards to be described below must be repeated NCASE times
- If the NCASE field is left blank, NCASE will be assigned 1. However, this card must always be present even if it is blank.

FEED SCATTERING MODEL CARD

PURPOSE: To describe size of feed aperture for blockage purposes.

CARD:



PARAMETERS: (I integer, F decimal number)

KG (I) - Code variable that indicates shape of feed aperture.
The values are: 1 - feed is circular
2 - feed is rectangular

DIM (1) (F) - These parameters are dimensions
DIM (2) (F) of feed (see figure 3)
If KG = 1, DIM (1) is radius
DIM (2) is ignored
If KG = 2, DIM (1) is length of feed in x direction
DIM (2) is length of feed in y direction

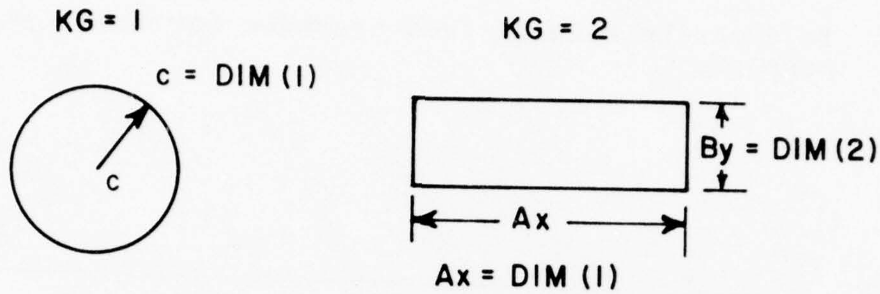


Figure 3. Parameters on Feed Scattering Model Card.

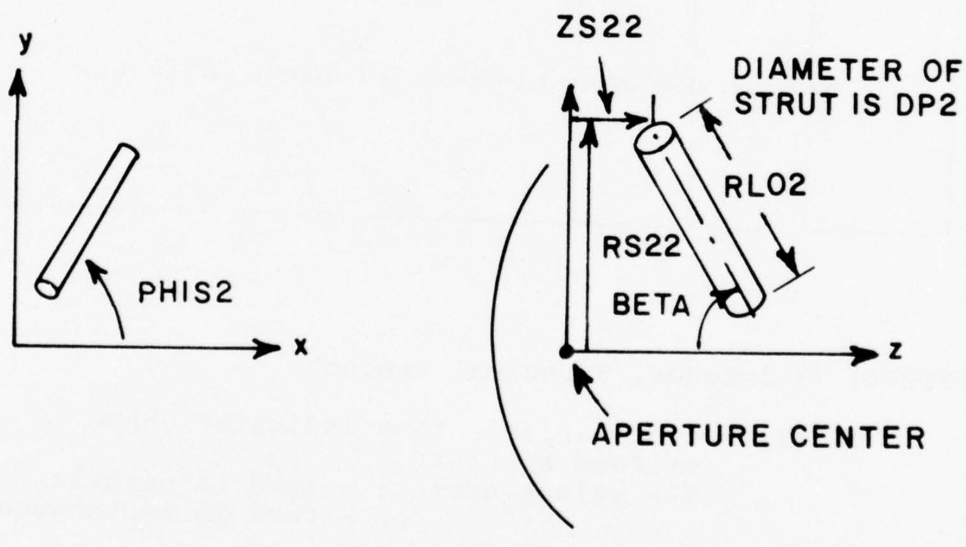
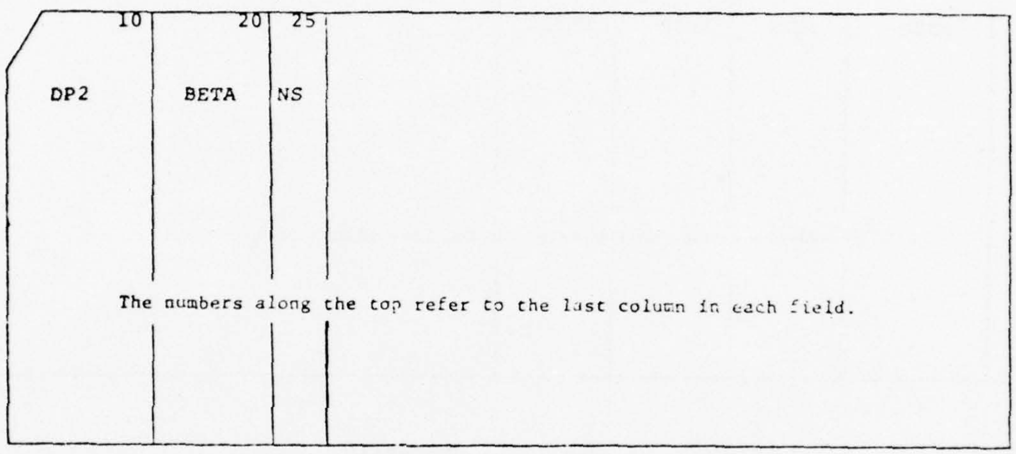


Figure 4. Strut Location Parameters.

COMMON STRUT DATA CARD

PURPOSE: To provide data that is common to all struts.

CARD:



PARAMETERS: (I integer, F decimal number)

DP2 (F) - diameter of struts

BETA (F) - angle of struts from -z axis (see figure 4)

NS (I) - number of struts (cannot be more than 4)

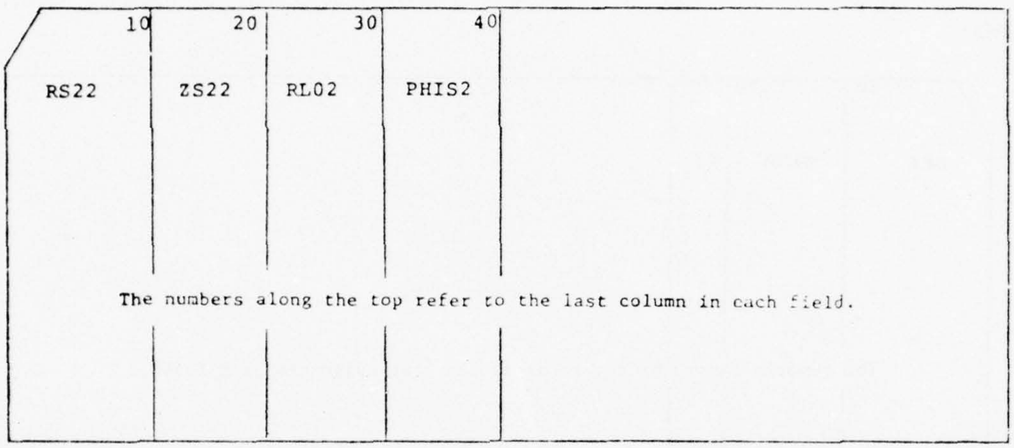
NOTES:

- This card causes NS cards of the next format to be read.

INDIVIDUAL STRUT DATA CARDS

PURPOSE: To describe data that may be different for each strut.

CARD:



PARAMETERS: (I integer, F decimal number)

RS22 (F) - Radial distance of one end of the strut from z axis (see figure 4)

ZS22 (F) - z coordinate of this same end

RL02 (F) - length of strut

PHIS2 (F) - ϕ(phi) angle of projection of strut in the xy plane

NOTES:

- For good accuracy there is a relationship between NAP, diameter of reflector in wave lengths, and angle BETA. For a given NAP, and diameter, the more nearly parallel the struts are to the z axis, the farther apart are integration points along the strut. This is the reason for the size restriction discussed under "limitations."
- Due to the analytical modeling done, any part of a strut whose projection falls outside the aperture will be ignored.
- There must be NS of these cards.

PHI CONTROL CARD

PURPOSE: To allow the user to specify which pattern cuts he wants.

CARD:

10	20	30	40	50	60	70	80
NP	ANP(1)	ANP(2)	ANP(3)	...	...	...	...

The numbers along the top refer to the last column in each field.

PARAMETERS: (I integer, F decimal number)

NP (I) - Absolute value is the number of phi cuts. If NP is positive, cuts will be evenly spaced. If NP is negative, cuts can be arbitrarily spaced.

ANP(1) (F)
ANP(2)
⋮
ANP(7) - Information on phi cuts: if evenly spaced case, ANP (1) is starting phi value and ANP (2) is increment between successive phi values. If arbitrary spaced case, the first NP values of ANP are the phi values.

NOTES:

- No comments are permitted on this one card since all possible fields are used.

- If arbitrary spaced phi's are used, limit is 7 different values. This limitation does not apply to evenly spaced cases.

THETA CONTROL CARD

PURPOSE: To set spacing and limits of field values within a phi cut.

CARD:

10	20	30	
NTHETA	THONE	DTHETA	

The numbers along the top refer to the last column in each field.

PARAMETERS: (I integer, F decimal number)

NTHETA (I) - number of theta values

THONE (F) - first theta value

DTHETA (F) - increment in theta values

NOTES:

- THONE should be assigned 0 to obtain proper normalization of secondary pattern.

PLOTTING CONTROL CARDS

A special group of cards, with a different mode of operation are used to control the plotting of data. This input, consisting of the AXIS, TITLE, CURVE, and NEXTPHI cards, can be rearranged to provide a wide variety of plotting options. Much of the data read in by these cards has an assumed default value. If the user does not provide some of this data, the program will assume a useful value. Hence, the user has the option of using standard size plots by entering only several numbers, or else, choosing his own plot format by entering additional data. Once the user has entered this data, it becomes the new assumed values for the rest of the program and this data does not need to be entered a second time unless it is desired to be changed.

AXIS CARD

PURPOSE: To cause the plotter to draw a new coordinate axis for the patterns to be subsequently plotted.

CARD:

10	20	30	40	50	60	
AXIS 1	XLENGTH	YLENGTH	VLOW	VHI	PHILAB	

The numbers along the top refer to the last column in each field.

PARAMETERS: (I integer, F decimal number)

AXIS 1 : The word AXIS in columns 1 thru 4 and the number 1 in column 10 signifies which type of plotting card has been read.

XLENGTH (F) : length of the horizontal axis (theta angle) in inches

YLENGTH (F) : length of the vertical axis (field pattern) in inches

VLOW (F) : value of the field pattern in db to labeled on lower point of vertical axis.

AXIS CARD (cont.)

VHI (F) : value of field pattern in db to be labeled on highest pcint of vertical axis.

PHILAB (F): decimal code that controls plotting of the phi values. The values are:
1 = phi value for each individual curve will be plotted. Use this option when plotting different phi cuts on the same axis.
0 = All the curves on a single axis are for the same phi cut and phi will be labeled only once for the graph.

NOTES:

- The default values for this card are:

  XLENGTH = 9. VLOW = -72.
  YLENGTH = 6. VHI = 0.
  PHILAB = 0.

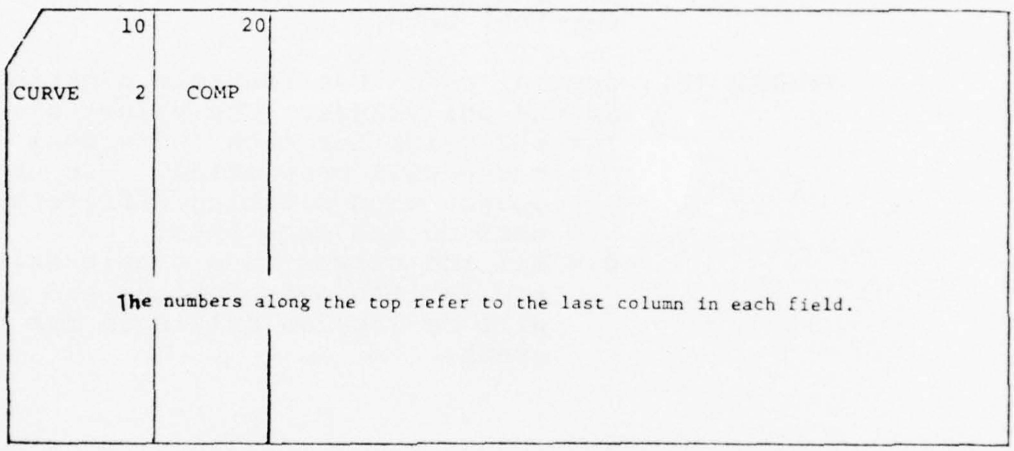
  Thus the standard default plot has an axis 6" x 9" with the 6" vertical scale ranging from 0 down to -72 db. Only one phi cut angle label is provided.

- Labels on graphs make complete graph larger than just dimension of coordinate system.

CURVE CARD

PURPOSE: To cause the computer to plot a selected component of the most recently computed phi cut and label it.

CARD:



PARAMETERS: (I integer, F decimal number)

CURVE 2 : The word CURVE in columns 1 thru 5 and the number 2 in column 10 signify which type of plotting card has been read.

COMP (I) : Component of field that is to be plotted. These component numbers are the same order as the magnitude in the output printout. The values are:

1 = for strut scattering, principal component
2 = for strut scattering, cross-polarized component
3 = for feed scattering, principal component
4 = for feed scattering, cross-polarized component
5 = for reflector + direct feed, principal component

CURVE CARD (cont.)

6 = for reflector + direct feed, cross-polarized component
7 = for total field, principal component
8 = for total field, cross-polarized component

NOTES:

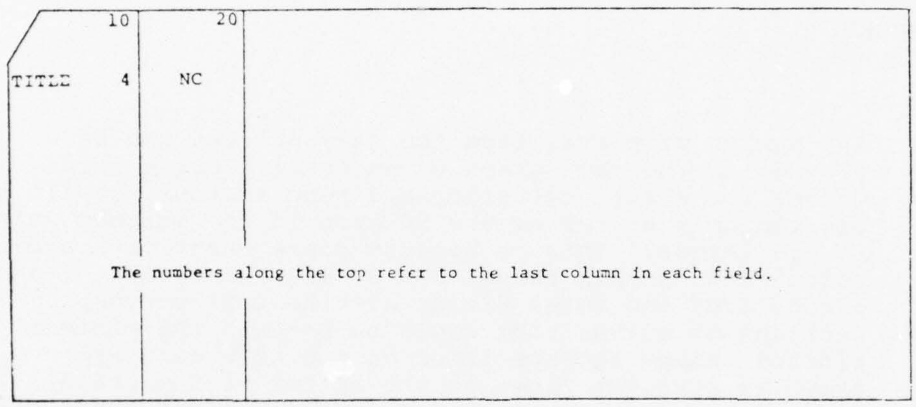
- Any number of curves from the same phi cut can be plotted in the same graph by successive curve cards.
- Curves for strut scattering and feed scattering will be plotted only as far as $\theta = 90^{\circ}$ even if the maximum value of θ is larger. This is because these quantities are only calculated in this range. For $\theta > 90^{\circ}$, the reflector and direct feed and total fields are the same curves.
- Sections of curves that would go beneath the minimum plotted values VLOWspecified on the AXIS card are drawn as straight lines at the bottom of the graph. This situation occurs for very deep nulls and for cross-polarized curves.

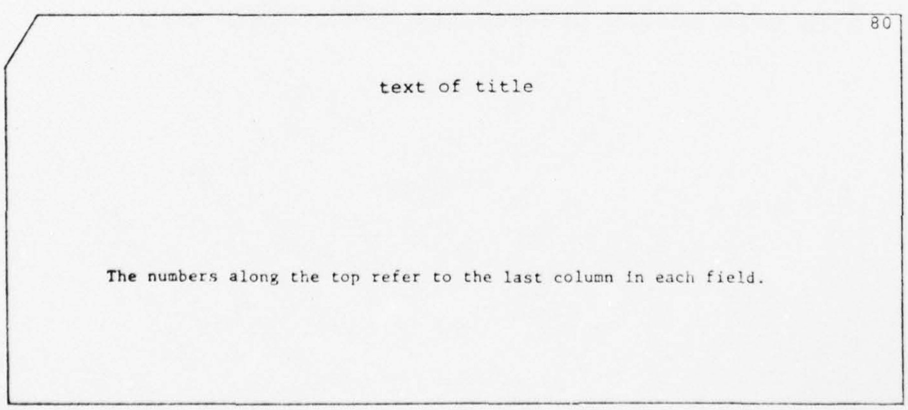
- AXIS card should precede the first CURVE card.

TITLE CARDS

PURPOSE: These two cards cause a user supplied title to be drawn beneath the coordinate axis.

CARD:





PARAMETERS: (I integer, F decimal number)

TITLE 4	:	The word TITLE in columns 1 thru 5 and the number 4 in column 10 signify that this and the next card provide title information.
NC (I)	:	The number of characters in the title which will be given on 2nd card
SECOND card	:	title: the first NC characters in this card will appear centered beneath the coordinate axis.

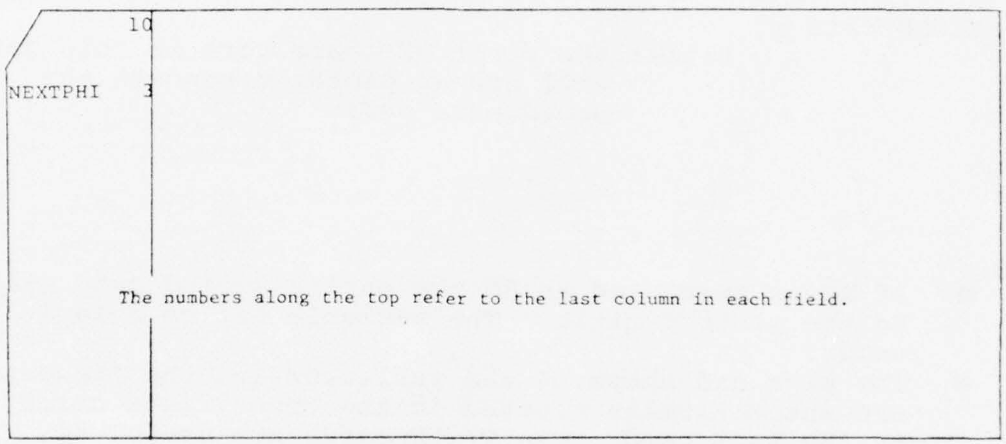
NOTES:

- If NC is specified as 80 the entire second card will be the plotted title. The variable has no default value.
- The size and shape of the reflector and the frequency are automatically plotted in the upper right corner by the AXIS card. The TITLE cards are useful to record the struts and feed details on the plot.

NEXTPHI CARD

PURPOSE: This card causes the next phi cut to be calculated after processing plot cards.

CARD:



PARAMETERS: (I integer, F decimal number)

NEXTPHI 3 : The word NEXTPHI in columns 1 thru 7 and the number 3 in column 10 signify what type of plot card this is. There are no other parameters on this card.

NOTES:

- This card is necessary to specify that the computer should calculate the next phi cut it has been previously given and not expect another plot command to plot a curve for the present phi cut.
- There should be as many NEXTPHI cards as there are phi cuts. Even if no plotting is desired the NEXTPHI card must appear. The program has been written so that if only NEXTPHI plot cards are used (no plotting is desired) then no calls to plot routines will be executed and no output plot file need be provided. The program is also written so that if plotting is never desired, only 1 card need be removed from the main program to eliminate reading NEXTPHI cards.

- After reading a NEXTPHI card, a new AXIS card needs to be read only if a new axis is desired. In this way, different phi cuts can be plotted on the same axis.

B. Example Computer Run

The input data for an example computer run are given in Table I. This example consists of three cases; the printed output data for the first case is given in Table II and the plotted output is given in Figures 5, 6 and 7. This example corresponds to a 24" diameter reflector with F/D = 1/3. For the first use there are three struts with diameter $2a_\rho$=0.37", β=68° and ϕ_S=90°, 210°, 330°, respectively. A circular flat plate with radius c=1.2" is used to model the feed. The pattern plane is ϕ=-15° and the frequency is 11 GHz.

The printout of the first case, given in Table II, first echoes the input data, and then prints an interpolated input pattern that has been deduced from the arbitrarily spaced primary feed pattern data points. The printout of the coefficients of the scattered wave was used in the development of the program and can usually be ignored by the user. The bulk of the printout is the different contributions to the secondary field of the reflector. In the sample a theta spacing of 1/4° was used so that the corresponding plot would have very smooth curves; a wider spacing can be used with increased roughness of plotted curves. The principal and cross-polarized components of each contribution are referenced to the Huygen polarization reference. After 90°, only the reflector and direct feed, and total fields are computed and listed. The printout for the rest of the three cases is similar and has been omitted.

Three examples of the computer plotting subroutines are shown in Figures 5, 6 and 7. The example in Figure 5 demonstrates the capability to plot the individual scattered field components: the basic contribution from the reflector and direct feed radiation, the strut scattering, the feed horn model scattering, and the total field which includes the various scattered field components. In this example only the principal polarization components are shown. The antenna geometry, pattern plane, and frequency correspond to the example computer run listed above. In this plot, the standard size and scale have been used.

The NEXTPHI 3 card causes more computations to be performed. Since only one value of phi has been specified above and since NCASE was read as 3 the next card is read as specifying a new strut and blockage problem with the same reflector geometry and primary feed. A new strut configuration with a single strut at phi=90° has been set. The second case, Figure 6 shows the capability to plot different pattern plane cuts: ϕ=-15°, 0°, 90° are shown. In this example only the principally polarized component of the scattered field from the struts are shown. Again the antenna geometry and frequency corresponds to the above computer run. Each NEXTPHI 3 card causes the phi values defined on the phi value card to be executed.

The capability to calculate cross-polarized components is shown in Figure 7 where only the total field is shown in both principal and cross polarizations. The reflector geometry, the direct feed pattern and the frequency are the same as above. However, instead of tripod struts, there is only one strut located at $\phi_S=20°$. The strut diameter is 2.0 inches (1.86λ) and $\beta=68°$. The strut length is the same as above. The strut angle $\phi_S=20°$ and the large strut diameter was chosen to demonstrate a large cross polarized component of the strut scatter. This cross polarization effect dominates for $\phi>45°$ in the $\phi=120°$ plane which is near the scattering cone of the strut.

TABLE I - Input Data Card Listing

10	20	30	
24.	11000.	.3333333	Reflector Geometry Card
14			Feed Pattern Card
0.	1.		Feed Pattern Data Cards (E-plane)
10.	0.966		
20.	0.8714		
30.	0.7375		
40.	0.59		
50.	0.4522		
60.	0.336		
70.	0.2456		
80.	0.1813		
90.	0.13778		
120.	0.0917		
132.	0.079		
170.	0.02114		
180.	0.02427		
0.	1.		Feed Pattern Data Cards (H-plane)
10.	0.9575		
20.	0.8419		
30.	0.684		
40.	0.52		
50.	0.3772		
60.	0.2664		
70.	0.1866		
80.	0.1358		
90.	0.10521		
120.	0.03588		
132.	0.03475		
160.	0.01884		
180.	0.0224		

TABLE I - Continued

```
         10        20        30        40        50
         3                                             Number of Strut Problems Card
         1  2.4                                        Feed Scattering Model Card
0.37        66.         5                              Common Strut Data Card
10.8       -0.8515      11.645    90.
10.8       -0.8515      11.645    210.                 Individual Strut Data Cards
10.8       -0.8515      11.645    330.
    -1  -15.                                           Phi Control Card
   361      0.          0.25                           Theta Control Card
AXIS     1
CURVE    2         7
CURVE    2         5
CURVE    2         1                                   Plotting Control Cards
CURVE    2         3
TITLE    4        60
FIELD COMPONENTS FOR TRIPOD STRUT SUPPORT 2.4 INCH FEED HORN
NEXTPHI  3

         1  2.4
0.37        68.         1                              Feed Scattering Model Card
10.8       -0.8515      11.64     90.                  Common Strut Data Card
        -3  -15.        0.0       90.                  Individual Strut Data Card
       361  0.          0.25                           Phi Control Card
AXIS     1                                   1         Theta Control Card
CURVE    2         1
NEXTPHI  3
CURVE    2         1                                   Plotting Control Cards
NEXTPHI  3
CURVE    2         1
NEXTPHI  3
```

TABLE I - Continued

10	20	30	40	50		
1	2.4					Feed Scattering Model Card
2.0	68.	1				Common Strut Data Card
10.8	-0.8515	11.64	20.			Individual Strut Data Card
-1	125.					Phi Control Card
721	0.	0.25				Theta Control Card
AXIS 1	9.	5.	-100.	0.	0.	
CURVE 2	7					Plotting Control Cards
CURVE 2	8					
NEXTPHI 3						

TABLE II
COMPUTER OUTPUT LISTING

CIRCULAR REFLECTOR ANTENNA FAR FIELD PATTERN WITH SCATTERING FROM FEED STRUTS AND FEED

ANTENNA GEOMETRY

APERTURE DIAMETER =	24.000	LAMDA =	1.074	INCHES
	0.610		0.027	METERS
	22.352			WAVELENGTH

FOCAL DISTANCE TO DIAMETER RATIO = 0.333

FREQUENCY = 11000.000 MHZ

NUMBER OF APERTURE POINTS = 50

NUMBER OF INPUT FEED VALUES = 14

PIECEWISE LINEAR FEED INPUT

PSI	F
0.0	1.00000
10.00	0.96600
20.00	0.87140
30.00	0.73750
40.00	0.59000
50.00	0.45220
60.00	0.33600
70.00	0.24560
80.00	0.18130
90.00	0.13778
120.00	0.09170
132.00	0.07900
170.00	0.02114
180.00	0.02427

PIECEWISE LINEAR FEED INPUT

PSI	F
0.0	1.00000
10.00	0.95750
20.00	0.84190
30.00	0.68400
40.00	0.52000
50.00	0.37720
60.00	0.26640
70.00	0.18660
80.00	0.13580
90.00	0.10521
120.00	0.03588
132.00	0.05475
160.00	0.01884
180.00	0.02240

FEED PATTERN

PSI	FE	FEDB	FH	FHDB
0.0	1.0000	0.0	1.0000	0.0
5.00	0.9930	-0.1489	0.9789	-0.1866
10.00	0.9660	-0.3065	0.9575	-0.3772
15.00	0.9187	-0.7365	0.8997	-0.9180
20.00	0.8714	-1.1956	0.8419	-1.4948
25.00	0.8045	-1.8900	0.7630	-2.3501
30.00	0.7375	-2.6448	0.6840	-3.2969
35.00	0.6637	-3.5599	0.6020	-4.4091
40.00	0.5900	-4.5830	0.5200	-5.6799
45.00	0.5211	-5.6616	0.4486	-6.9628
50.00	0.4522	-6.8934	0.3772	-8.4686
55.00	0.3941	-8.0879	0.3218	-9.8483
60.00	0.3360	-9.4732	0.2664	-11.4893
65.00	0.2909	-10.7281	0.2265	-12.8986
70.00	0.2456	-12.1954	0.1866	-14.5818
75.00	0.2134	-13.4141	0.1612	-15.8527
80.00	0.1813	-14.8320	0.1359	-17.3420
85.00	0.1595	-15.9426	0.1205	-18.3799
90.00	0.1378	-17.2163	0.1052	-19.5589
95.00	0.1301	-17.7144	0.0937	-20.5694
100.00	0.1224	-18.2429	0.0821	-21.7131
105.00	0.1147	-18.8057	0.0705	-23.0307
110.00	0.1071	-19.4074	0.0590	-24.5844
115.00	0.0994	-20.0540	0.0474	-26.4780
120.00	0.0917	-20.7526	0.0359	-28.9029
125.00	0.0864	-21.2689	0.0437	-27.1819
130.00	0.0811	-21.8178	0.0516	-25.7462
135.00	0.0744	-22.5648	0.0509	-25.8652
140.00	0.0668	-23.5020	0.0445	-27.0347
145.00	0.0592	-24.5527	0.0381	-28.3866
150.00	0.0516	-25.7482	0.0317	-29.9884
155.00	0.0440	-27.1350	0.0253	-31.9539
160.00	0.0364	-28.7860	0.0188	-34.4984
165.00	0.0288	-30.8263	0.0197	-34.0975
170.00	0.0211	-33.4979	0.0206	-33.7142
175.00	0.0227	-32.8775	0.0215	-33.3472
180.00	0.0243	-32.2986	0.0224	-32.9950

FEED SCATTERING MODEL

CIRCULAR FEED BLOCKAGE

DIAMETER = 2.400 INCHES
0.061 METERS
2.235 WAVELENGTH

FEED STRUTS

DIAMETER = 0.370 INCHES
0.009 METERS
0.345 WAVELENGTH

BETA = 68.0 DEGREES

NUMBER OF STRUTS = 3

STRUT NUMBER 1

PHIS = 90.0 DEGREES

RHOS	ZS	LENGTH	
10.800	-0.851	11.645	INCHES
0.274	-0.022	0.296	METERS
10.058	-0.793	10.845	WAVELENGTH

STRUT NUMBER 2

		RHOS	ZS	LENGTH	
PHIS =210.0 DEGREES					
		10.800	-0.851	11.645	INCHES
		0.274	-0.022	0.296	METERS
		10.058	-0.793	10.845	WAVELENGTH

STRUT NUMBER 3

		RHOS	ZS	LENGTH	
PHIS =330.0 DEGREES					
		10.800	-0.951	11.645	INCHES
		0.274	-0.022	0.296	METERS
		10.058	-0.793	10.845	WAVELENGTH

ARG= 1.0037

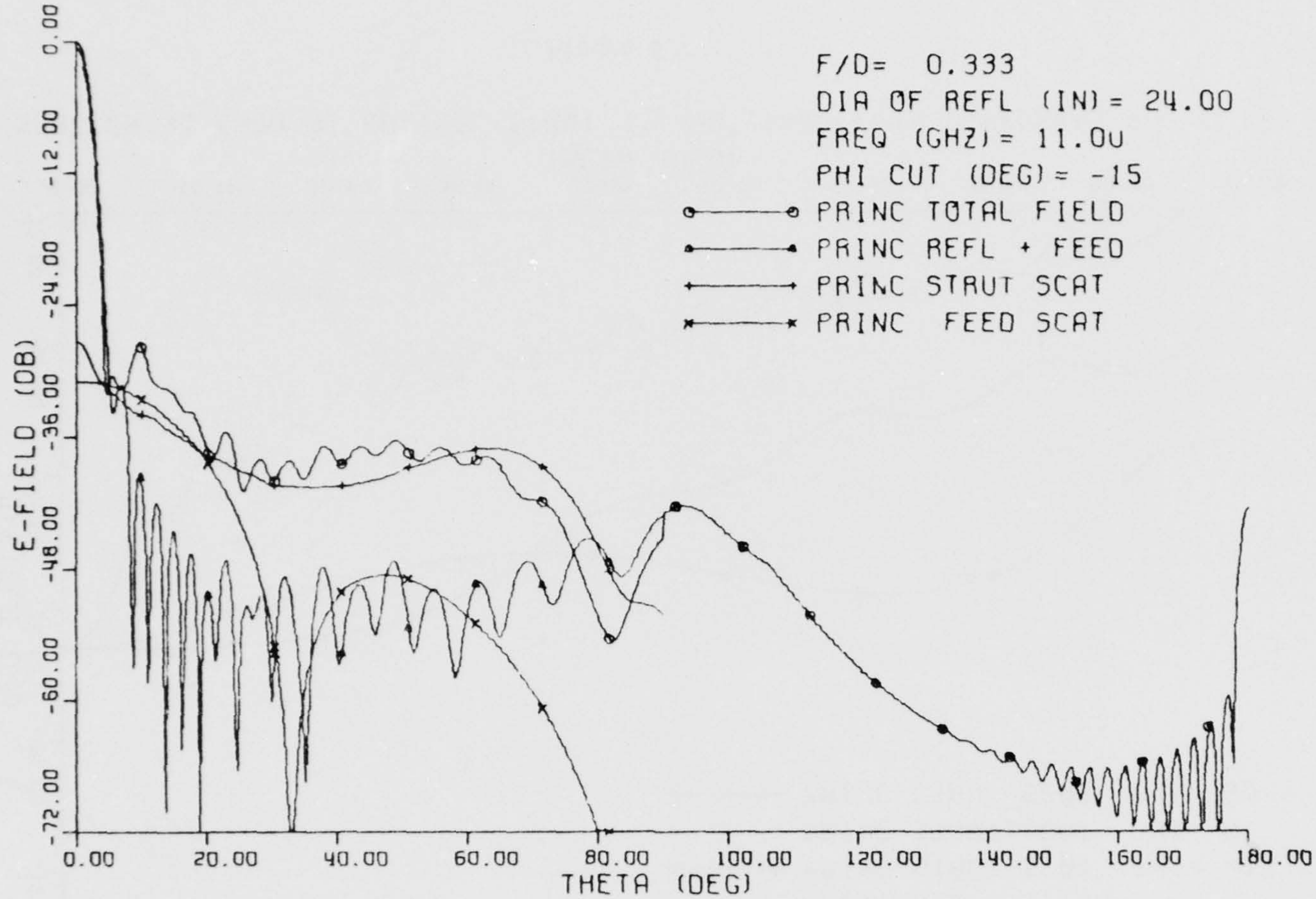
COEFFICIENTS OF THE SCATTERED WAVE FROM A CYLINDER

M	AM(M)		BM(M)	
1	-0.493E+00	-0.589E-01	-0.122E+00	0.215E+00
2	-0.243E+00	0.429E+00	-0.123E+00	-0.328E+00
3	-0.494E-02	0.701E-01	-0.710E-02	-0.840E-01
4	-0.118E-04	0.343E-02	-0.132E-04	-0.363E-02
5	-0.587E-08	0.766E-04	-0.610E-08	-0.781E-04
6	-0.990E-12	0.995E-06	-0.101E-11	-0.100E-05
7	-0.725E-16	0.851E-08	-0.732E-16	-0.856E-08
8	-0.268E-20	0.517E-10	-0.269E-20	-0.519E-10
9	-0.552E-25	0.235E-12	-0.554E-25	-0.235E-12
10	-0.685E-30	0.828E-15	-0.687E-30	-0.829E-15
11	-0.543E-35	0.233E-17	-0.544E-35	-0.233E-17
12	-0.287E-40	0.536E-20	-0.288E-40	-0.536E-20

PHI=-15.00 DEGREES

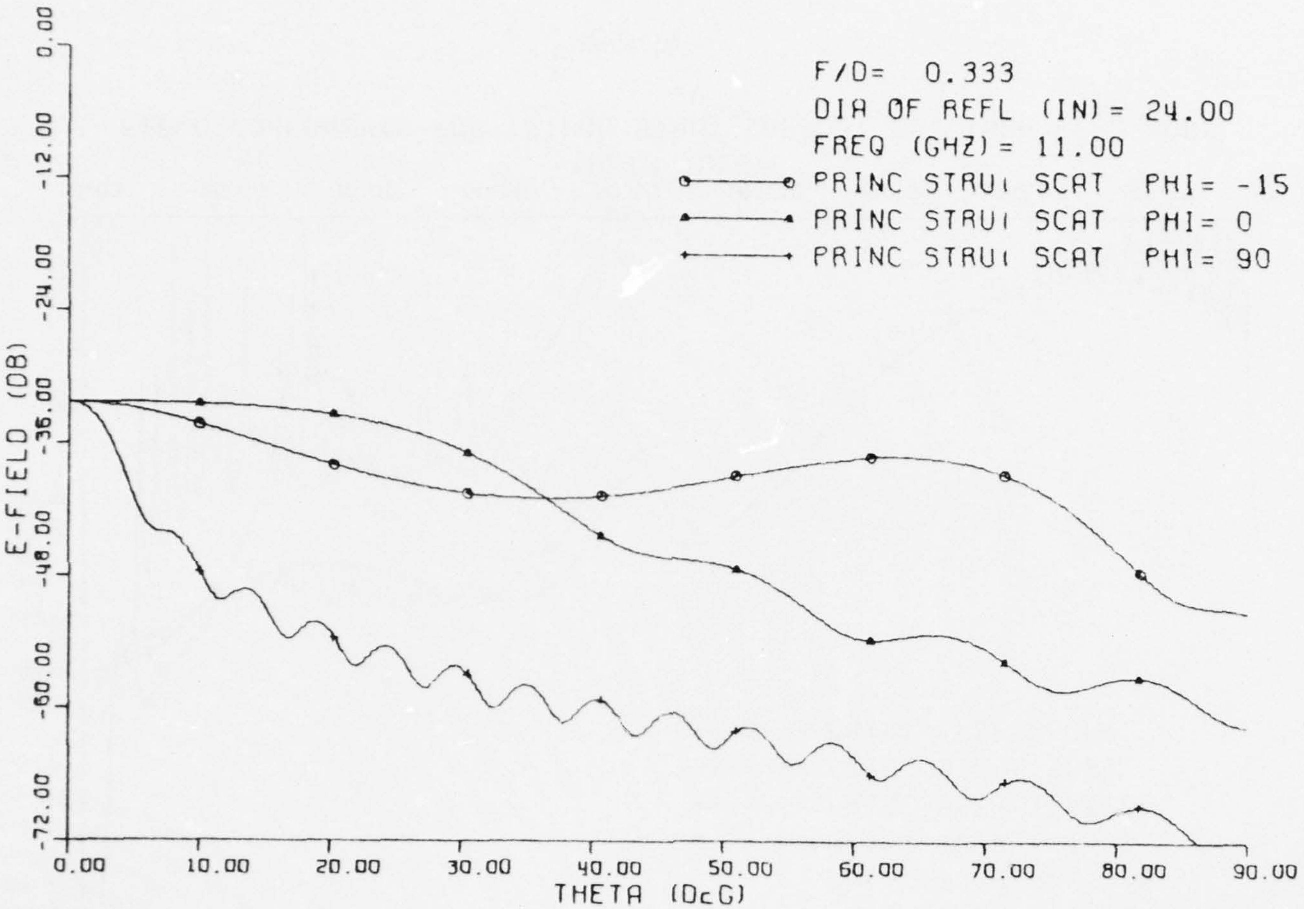
AXIAL FIELD = 42.84 DB

	STRUT SCAT PRINC DB	STRUT SCAT PRINC DEG	STRUT SCAT CROSS DB	STRUT SCAT CROSS DEG	FEED SCAT PRINC DB	FEED SCAT PRINC DEG	FEED SCAT CROSS DB	FEED SCAT CROSS DEG	REFL+DIRECT FEED PRINC DB	REFL+DIRECT FEED PRINC DEG	REFL+DIRECT FEED CROSS DB	REFL+DIRECT FEED CROSS DEG	TOTAL FIELD PRINC DB	TOTAL FIELD PRINC DEG	TOTAL FIELD CROSS DB	TOTAL FIELD CROSS DEG
0.0	-27.31	-154.7	-63.96	160.2	-30.96	-141.0	-175.02	-14.0	0.0	39.0	-139.16	90.0	-0.63	39.6	-63.96	165.2
0.25	-27.38	-155.4	-55.20	98.5	-30.96	-141.0	-187.32	0.0	-0.06	39.0	-66.95	-141.0	-0.70	39.7	-56.15	112.3
0.50	-27.50	-156.1	-49.71	89.1	-30.96	-141.0	-177.78	90.0	-0.26	39.0	-55.03	-141.0	-0.90	39.7	-51.94	171.6
0.75	-27.66	-157.0	-46.41	86.4	-30.96	-141.1	-180.33	26.6	-0.58	39.0	-48.21	-141.0	-1.24	39.8	-48.93	159.5
1.00	-27.86	-158.1	-44.13	85.4	-30.97	-141.2	-180.33	-26.6	-1.04	39.0	-43.51	-141.0	-1.72	39.8	-45.26	156.9
1.25	-28.10	-159.4	-42.45	85.2	-30.98	-141.3	-175.28	0.0	-1.64	39.0	-40.03	-141.0	-2.35	39.9	-42.84	170.0
1.50	-28.37	-160.9	-41.17	85.3	-30.99	-141.4	-177.32	71.6	-2.37	39.0	-37.35	-141.0	-3.14	40.1	-40.16	179.0
1.75	-28.67	-162.7	-40.19	85.5	-31.01	-141.5	-178.29	45.0	-3.26	39.0	-35.26	-141.0	-4.05	40.3	-37.93	-175.0
2.00	-28.99	-164.8	-39.43	85.8	-31.02	-141.7	-187.32	90.0	-4.29	39.0	-33.62	-141.0	-5.20	40.5	-36.12	-170.9
2.25	-29.33	-167.1	-38.87	86.2	-31.04	-141.9	-173.34	36.9	-5.49	39.0	-32.35	-141.0	-6.50	40.9	-34.70	-168.0
2.50	-29.66	-169.7	-38.48	86.5	-31.06	-142.1	-177.78	0.0	-6.86	39.0	-31.40	-141.0	-8.01	41.4	-33.63	-166.0
2.75	-30.00	-172.5	-38.22	86.8	-31.08	-142.3	-178.29	45.0	-8.41	39.0	-30.74	-141.0	-9.76	42.0	-32.98	-164.6
3.00	-30.31	-175.6	-38.08	87.1	-31.10	-142.6	-176.18	33.7	-10.17	39.0	-30.34	-141.0	-11.77	43.0	-32.42	-163.8
3.25	-30.61	-178.9	-38.06	87.2	-31.13	-142.9	-187.32	90.0	-12.13	39.0	-30.18	-141.0	-14.10	44.5	-32.23	-163.4
3.50	-30.87	177.8	-38.14	87.2	-31.16	-143.2	-184.31	45.0	-14.32	39.0	-30.27	-141.0	-16.81	46.9	-32.31	-163.4
3.75	-31.10	174.4	-38.31	87.1	-31.19	-143.5	-187.32	90.0	-16.76	39.0	-30.58	-141.0	-20.00	51.0	-32.66	-163.9
4.00	-31.30	171.0	-38.55	86.8	-31.22	-143.9	-180.33	26.6	-19.45	39.0	-31.14	-141.0	-23.30	58.5	-33.29	-164.8
4.25	-31.47	167.7	-38.87	86.3	-31.25	-144.2	-184.31	135.0	-22.37	39.0	-31.95	-141.0	-28.16	73.9	-34.22	-166.5
4.50	-31.61	164.6	-39.25	85.7	-31.29	-144.6	-175.28	0.0	-25.45	39.0	-33.02	-141.0	-31.87	103.8	-35.47	-169.1
4.75	-31.74	161.7	-39.68	84.8	-31.32	-145.0	-177.78	90.0	-28.51	39.0	-34.39	-141.0	-32.41	137.5	-37.05	-173.1
5.00	-31.85	159.0	-40.15	83.8	-31.37	-145.5	-500.00	0.0	-31.20	39.0	-36.10	-141.0	-31.33	155.7	-39.08	-179.5
5.25	-31.96	156.5	-40.64	82.5	-31.41	-145.9	-187.32	90.0	-33.06	39.0	-38.22	-141.0	-30.58	162.3	-41.45	169.9
5.50	-32.06	154.2	-41.14	81.1	-31.45	-146.4	-180.33	63.4	-33.79	39.0	-40.87	-141.0	-30.39	162.9	-43.36	152.3
5.75	-32.16	152.1	-41.63	79.7	-31.50	-146.9	-176.19	33.7	-33.57	39.0	-44.28	-141.0	-30.58	159.9	-45.35	127.2
6.00	-32.28	150.2	-42.12	78.1	-31.55	-147.4	-176.18	33.7	-32.85	39.0	-48.92	-141.0	-30.92	154.6	-45.13	102.2
6.25	-32.41	148.4	-42.57	76.6	-31.60	-148.0	-181.30	0.0	-32.06	39.0	-56.32	-141.0	-31.25	148.5	-44.02	85.1
6.50	-32.54	146.7	-42.99	75.3	-31.65	-148.5	-176.18	56.3	-31.47	39.0	-95.82	39.0	-31.47	142.8	-42.98	75.2
6.75	-32.68	145.0	-43.38	74.1	-31.71	-149.1	-181.30	0.0	-31.20	39.0	-59.61	39.0	-31.57	139.0	-42.32	69.6
7.00	-32.83	143.4	-43.72	73.3	-31.76	-149.7	-175.28	0.0	-31.32	39.0	-55.98	39.0	-31.58	137.8	-42.07	66.7
7.25	-32.98	141.7	-44.03	72.7	-31.82	-150.4	-176.18	33.7	-31.89	39.0	-55.37	39.0	-31.53	139.5	-42.20	65.7
7.50	-33.11	140.1	-44.30	72.6	-31.88	-151.0	-175.02	14.0	-32.99	39.0	-56.68	39.0	-31.38	143.8	-42.66	66.2
7.75	-33.27			72.7	-31.95	-151.7	-184.31	45.0	-34.72	39.0	-60.31	39.0	-31.07			68.2
8.00	-33			[illegible]	-32.01	-152.4	-176.18	56.3	-37.35	39.0	-70.47	39.0	-30			71.6
8.25					-32.08	-153.1	-180.33	26.6	-41.51	39.0	-67.25	-141.0				
8.5					-32.15	-153.9	-181.30	90.0	-49.64	39.0	-59.80	-141				
					22	-154.7	-177.78	90.0	-57.11	-141.0	-55.05					
						-155.4	-184.31	135.0	-45.45	-141.0	-53.0					
						56.3	-180.33	116.6	-41.61	-141.0						
							-181.30	0.0	-39.88	-141.0						
							94.31	45.0	-39.33	-14						
								45.0	-39.7							
														157.4	-7[illegible]	
													-68.27	143.5	-70.64	13
													-63.85	136.9	-68.11	133.
..0													-62.50	132.7	-66.73	131.1
.25													-61.99	129.4	-66.22	129.[illegible]
174.50													-62.27	126.3	-66.51	127.[illegible]
174.75													-63.42	122.9	-67.70	125.5
175.00													-65.78	117.4	-70.17	122.6
175.25													-70.31	106.3	-75.09	115.5
175.50													-78.02	48.7	-87.02	52.8
175.75													-71.00	-26.2	-75.6[illegible]	-36.[illegible]
176.00													-65.44	-39.9	-69.69	-44.7
176.25													-62.32	-45.1	-66.53	-47.6
176.50													-60.51	-48.0	-64.70	-49.3
176.75													-59.61	-50.0	-63.91	-50.4
177.00													-59.50	-51.7	-63.71	-51.4
177.25													-60.24	-53.4	-64.46	-52.3
177.50													-62.08	-55.5	-66.33	-53.5
177.75													-65.88	-59.5	-70.21	-55.4
178.00													-54.53	129.0	-59.30	129.0
178.25													-50.87	129.0	-55.64	129.0
178.50													-48.25	129.0	-53.02	129.0
178.75													-46.29	129.0	-51.06	129.0
179.00													-44.92	129.0	-49.59	129.0
179.25													-43.75	129.0	-48.52	129.0
179.50													-43.01	129.0	-47.71	129.0
179.75													-42.58	129.0	-47.25	129.0
180.00													-42.44	129.0	-47.21	129.0



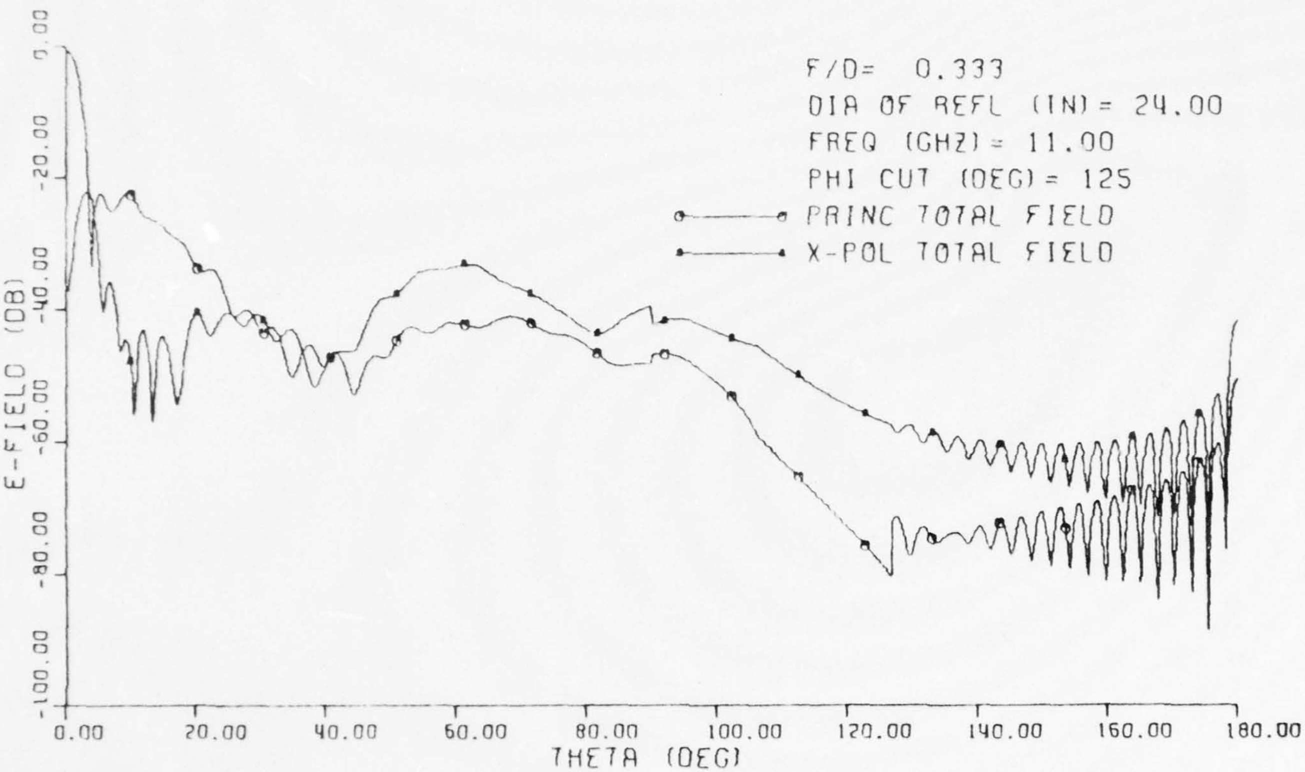
FIELD COMPONENTS FOR TRIPOD STRUT SUPPORT 2.4 INCH FEED HORN

Figure 5.



SCATTERING FROM A VERTICAL STRUT (ϕ_s=90°, β=68°) IN DIFFERENT PHI CUTS

Figure 6.



TOTAL FIELD COMPONENTS FOR SINGLE STRUT SUPPORT 2.4 INCH FEED HORN

Figure 7.

C. Appendix - Computer Program Listing

The listing for the computer program described in this report is given below:

"Circular Reflector Antenna with Feed and Strut Scatter" (OSUPATT)

```
C
C
C
C     *****N2  NUMBER OF LINEAR FEED COEFFICIENTS
C
C
C     DS       SOFT DIFFRACTION COEFFICIENT
C     DH       HARD DIFFRACTION COEFFICIENT
C
C     RL,WA,ILOP : REFER TO SUBROUTINE GTD
C
C
      DIMENSION G(100),H(100),ZY(100),ZX(100),DLE(50),DLH(50),BET(100),
     $TF(100),DF(100),TP(100),TM(100),PXE(50),PXH(50),FFE(50),FFH(50),
     $ BJ(25),ANP(10),DIM(2),RS2(4),ZS2(4),RLO(4),PHIS(4),PLOTST(8,721)
      COMPLEX J,CP,CP1,CP2,DS1,DS2,DH1,DH2,FP1,FP2,FT1,FT2,EFT,EFP,EF,
     $CP4,DS,DH,DSC,DHC,A1,A2,A3,B1,B2,B3,FY,ET,EP,CP3,AM(25),HNK(25),
     $HKP,EX,EZ,ESY,EST,ESP,ETHE,EPHI,ETT,ETP,CPP,BM(25),EAT,EAP,CPO,
     $EFBY,EFBT,EFBP,ESTT,ESTP,EPL,HPL
      REAL MAX
      COMMON /STR/J,CPO,AM,BM,PI,RK,MX,DEL,A,TF,DF,BETN
1     FORMAT (3F10.4,I5)
2     FORMAT ( //'  CIRCULAR REFLECTOR ANTENNA FAR FIELD PATTERN  WITH
     $SCATTERING FROM FEED STRUTS AND FEED',//T5,'ANTENNA GEOMETRY',//
     $T10,'APERTURE DIAMETER =' ,F8.3,T50,'LAMDA =' ,F8.3,T75,'INCHES',/
     $T29,F8.3,T57,F8.3,T75,'METERS',/T29,F8.3,T75,'WAVELENGTH',//T10,
     $'FOCAL DISTANCE TO DIAMETER RATIO =',F8.3,//T10,'FREQUENCY =',
     $F10.3,' MHZ',//T10,'NUMBER OF APERTURE POINTS =',I5,///)
    4 FORMAT(T23,'STRUT SCAT',T50,'FEED SCAT',T72,'     REFL+DIRECT FEE
     XD ',T110,'TOTAL FIELD'/  9X,4(6X,'PRINC',11X,'CROSS',2X)/
     X  9X,8(4X,'DB',5X,'DEG ')      )
C
5     READ (5,1,END=101) D2,FREQ,FDR,NAP
C
      IF (NAP.LT.50) NAP=50
      IF (NAP.GT.99) NAP=99
      PI=3.141592653
      RK=2.*PI
      J=(0.,1.)
      MAX=0.
      RLAM=300./FREQ
      FM=0.0254
      RLAM1=RLAM/FM
      D1=D2*FM
      D=D1/RLAM
      WRITE (6,2) D2,RLAM1,D1,RLAM,D,FDR,FREQ,NAP
      A=D/2.
      F=FDR*D
      MI=NAP+1
      DEL=A/NAP
      B=F-A*A/(4.*F)
      RO=SQRT(A*A+B*B)
      CPO=CEXP(-J*RK*RO)
      CP=J*CPO
      CP4=PI*A*CEXP(J*(PI/4.-RK*RO))
      ALP=ATAN2(A,B)
      ALPHA=ALP*180./PI
      SINA=SIN(ALP)
      COSA=COS(ALP)
C
C
C                       ******************************
C                       * LINEAR FEED COEFFICIENTS *
```

```
C                    ******************************
C
      READ (5,10) N2
10    FORMAT (I10)
      WRITE (6,11) N2
11    FORMAT (T10,'NUMBER OF INPUT FEED VALUES =',I3,//)
      CALL LNFD(DLE,PXE,FFE,N2)
      CALL LNFD(DLH,PXH,FFH,N2)
      N3=N2-1
C
C                              ****************
C                              * FEED PATTERN *
C                              ****************
C
      WRITE (6,13)
13    FORMAT (//T10,'FEED PATTERN',//T23,'PSI',T37,'FE',T51,'FEDB',T66,
     $'FH',T80,'FHDB',//)
      PSI=0.
      DO 17 L=1,37
      CALL LSUM(PSI,PXE,FFE,DLE,N3,FE)
      CALL LSUM(PSI,PXH,FFH,DLH,N3,FH)
      IF (FE.EQ.0.) FEDB=-500.
      IF (FH.EQ.0.) FHDB=-500.
      IF (FE.NE.0.) FEDB=20.*ALOG10(ABS(FE))
      IF (FH.NE.0.) FHDB=20.*ALOG10(ABS(FH))
      WRITE (6,16) PSI,FE,FEDB,FH,FHDB
16    FORMAT (T18,F9.2,4F15.4)
      PSI=PSI+5.
17    CONTINUE
C
C             *************************************************
C             * DIFFRACTION COEFFICIENTS FOR REAR AXIS FIELD *
C             *************************************************
C
      GAM=ATAN2(2.*F,A)
      GAMMA=GAM*180./PI
      PHIP=180.-GAMMA-ALPHA
      S=100.*RO**2
      SL=RO
      ILOP=4
      WA=0.0
      PH180=360.-GAMMA
      CALL GTD(PH180,PHIP,90.,S,RO,WA,SL,AS,ILOP,DS,DH)
      AN1=180.-ALPHA
      AN2=180.-GAMMA
C
C                          **************************
C                          * FEED SCATTERING MODEL *
C                          **************************
C
      READ (5,21) NCASE
      IF (NCASE .LT. 1) NCASE=1
      DO 100 NC=1,NCASE
      WRITE (6,20)
20    FORMAT (//T10,'FEED SCATTERING MODEL',//)
      READ(5,21) KG,(DIM(I),I=1,KG)
21    FORMAT (I10,2F10.5)
      DC1=DIM(1)*FM
      DC=DC1/RLAM
      C=DC/2.
      IF (KG.EQ.1) GO TO 23
      AX=DC
      BY1=DIM(2)*FM
      BY=BY1/RLAM
      CPP=AX*BY*CP*CEXP(-J*RK*B)
      WRITE (6,22) DIM(1),DIM(2),DC1,BY1,AX,BY
22    FORMAT (T15,'RECTANGULAR FEED BLOCKAGE',//T30,'WIDTH =',F8.3,T55,
     1'HEIGHT =',F8.3,T60,' INCHES',/T37,F8.3,T63,F8.3,T80,' METERS',/
```

```
     $T37,F8.3,T63,F8.3,T80,' WAVELENGTH',//)
      GO TO 25
23    WRITE (6,24) DIM(I),DCI,DC
24    FORMAT (T15,'CIRCULAR FEED BLOCKAGE',//T26,'DIAMETER =',F8.3,T55,
     $' INCHES',/T36,F8.3,T55,' METERS',/T36,F8.3,T55,' WAVELENGTH',//)
      CPP=2.*PI*C**2*CEXP(-J*RK*B)*CP
25    CONTINUE
C
C                        **************************
C                        * FEED STRUT LOCATIONS *
C                        **************************
C
      READ (5,31) DP2,BETA,NS
31    FORMAT (2F10.5,I5)
      DP1=DP2*FM
      DP=DP1/RLAM
      AP=DP/2.
      WRITE (6,32) DP2,BETA,NS,DP1,DP
32    FORMAT (T10,'FEED STRUTS',//T24,'DIAMETER =',F8.3,' INCHES',T60,
     $'BETA = ',F6.1,' DEGREES',T90,'NUMBER OF STRUTS = ',I2,/T34,F8.3,
     $' METERS',/T34,F8.3,' WAVELENGTH',//)
      DO 35 I=1,NS
      READ (5,33)  RS22,ZS22,RLO2,PHIS2
33    FORMAT (4F10.5)
      RS21=RS22*FM
      ZS21=ZS22*FM
      RLO1=RLO2*FM
      RS2(I)=RS21/RLAM
      ZS2(I)=ZS21/RLAM
      RLO(I)=RLO1/RLAM
      WRITE (6,34) I,PHIS2,RS22,ZS22,RLO2,RS21,ZS21,RLO1,RS2(I),ZS2(I),
     $RLO(I)
34    FORMAT (T25,'STRUT NUMBER',I2,//T50,'PHIS =',F5.1,' DEGREES',T80,
     $'RHOS',T93,'ZS',T103,'LENGTH',//T73,3F12.3,T115,'INCHES',/T73,
     $3F12.3,T115,'METERS',/T73,3F12.3,T115,'WAVELENGTH',//)
      PHIS(I)=PHIS2*PI/180.
35    CONTINUE
C
C                   ****************************************
C                   * COEFFICIENTS OF CYLINDER SCATTERING *
C                   ****************************************
C
      BETN=BETA*PI/180.
      SINB=SIN(BETN)
      ARG=RK*AP*SINB
      WRITE (6,41)  ARG
41    FORMAT (//T10,'ARG=',F8.4,//)
      MX=ARG+11
      DO 43 M=1,MX
      L=M-1
      CALL BESJ(ARG,L,BSJ,0.001,IER)
      CALL BESY(ARG,L,BN,IER)
      BJ(M)=BSJ
      HNK(M)=BSJ-J*BN
      AM(M)=-BJ(M)/HNK(M)
      IF (M.GT.1) GO TO 42
      AM(1)=0.5*AM(1)
      GO TO 43
42    BJP=BJ(L)-(L/ARG)*BJ(M)
      HKP=HNK(L)-(L/ARG)*HNK(M)
      BM(M)=-BJP/HKP
      IF (ARG.GT.1.) GO TO 43
      RRA=REAL(AM(M))/REAL(AM(1))
      IF (RRA.GT.1.0E-8) GO TO 43
      RIA=AIMAG(AM(M))/AIMAG(AM(1))
      IF (RIA.GT.1.0E-8) GO TO 43
      GO TO 44
43    CONTINUE
```

```
44    MX=M
      BM(1)=-0.5*BJ(2)/HNK(2)
      WRITE (6,45)
45    FORMAT(/T8,'COEFFICIENTS OF THE SCATTERED WAVE FROM A CYLINDER',
     $//T20,'M',T35,'AM(M)',T65,'BM(M)',//)
      WRITE (6,46)  (M,AM(M),BM(M),M=1,MX)
46    FORMAT (T16,I5,4E15.3)
C
C
      READ(5,53) NP,(ANP(K),K=1,7 )
53    FORMAT (I10,7F10.5)
      READ(5,53) NTHETA,THONE,DTHETA
      IEVEN=1
      IF (NP .LT. 0 ) IEVEN=0
      NP=IABS(NP)
      DO 95 K=1,NP
      PHI=ANP(K)
      IF (IEVEN .EQ. 1) PHI=ANP(1)+(K-1)*ANP(2)
      WRITE (6,55) PHI
55    FORMAT (//,T10,'PHI=',F6.2,' DEGREES',//)
      PHI=PHI*PI/180.
      SINP=SIN(PHI)
      COSP=COS(PHI)
      COS2P=COS(2.*PHI)
      SIN2P=SIN(2.*PHI)
      SIN4P=SIN(4.*PHI)
      COS4P=COS(4.*PHI)
      FAL=SQRT(1.-(SINA*SINP)**2)
      FTA=COSA*SINP/FAL
      FPA=COSP/FAL
C
C                               **********************
C                               * APERTURE FIELDS *
C                               **********************
C
      RHO=0.
      DO 61 I=1,MI
      BB=F-RHO*RHO/(4.*F)
      RP=SQRT(RHO*RHO+BB*BB)
      SI=ATAN2(RHO,BB)
      PSI=SI*180./PI
      COSS=COS(SI)
      SINS=SIN(SI)
      BET(I)=SQRT((1.-(SINS*SINP)**2))
      TP(I)=1.+COSS
      TM(I)=1.-COSS
      CALL LSUM(PSI,PXH,FFH,DLH,N3,FH)
      CALL LSUM(PSI,PXE,FFE,DLE,N3,FE)
      CALL LSUM(PSI,PXH,FFH,DLH,N3,FH)
      TF(I)=(FE+FH)*F/(2.*RP)
      DF(I)=(FE-FH)*F/(2.*RP)
      RHO=RHO+DEL
61    CONTINUE
      TS=TF(MI)
      DL=DF(MI)
      BP=TS-DL*COS2P
      WRITE (6,4)
C
C                             ***************************
C                             * PATTERN CALCULATION *
C                             ***************************
C
      DO 90 N=1,NTHETA
      THETA=THONE+(N-1)*DTHETA
      THE=THETA*PI/180.
      SINT=SIN(THE)
      COST=COS(THE)
      CP3=CEXP(J*RK*B*COST)
```

```
      X=RK*A*SINT
      IF (THETA.LT.90.) GO TO 62
      IF (X.LT.2.5) GO TO 76
      GO TO 70
62    IF (X.GT.24.) GO TO 70
C
C                        **************************
C                        * APERTURE INTEGRATION *
C                        **************************
C
      RHO=0.
      DO 65 I=1,MI
      XP=RK*RHO*SINT
      IF (XP.GT.4.0E-2) GO TO 63
      Y=(XP*XP)/4.
      BJ0=1.-Y+Y*Y/4.
      BJ2=Y/2.
      BJ4=0.
      GO TO 64
63    CALL BESJ(XP,0,BJ0,0.001,IER1)
      CALL BESJ(XP,2,BJ2,0.001,IER1)
      CALL BESJ(XP,4,BJ4,0.001,IER1)
64    H(I)=-(SIN2P*TE(I)*BJ2+0.5*SIN4P*DE(I)*BJ4)*RHO*TM(I)/(2.*BET(I))
      G(I)=-(TE(I)*BJ0+DE(I)*BJ2*COS2P)*RHO
      RHO=RHO+DEL
65    CONTINUE
      CALL QSF(DEL,H,ZX,MI)
      CALL QSF(DEL,G,ZY,MI)
      EX=CP*RK*COST*ZX(MI)
      EY=CP*RK*COST*ZY(MI)
      EZ=-CP*RK*SINT*(COSP*ZX(MI)+SINP*ZY(MI))
      EP=-EX*SINP+EY*COSP
      ET=EX*COST*COSP+EY*COST*SINP-EZ*SINT
      IF (THETA.NE.0.0) GO TO 77
      MAX=20.*ALOG10(CABS(EY))
      WRITE (6,66) MAX
66    FORMAT (//T15,'AXIAL FIELD =',F8.2,' DB',//)
      GO TO 77
C
C
C                        **********************
C                        * TWO POINT METHOD *
C                        **********************
C
70    T2=SQRT(A/SINT)
      PSI=180.-THETA
      SI=PI-THE
      SINS=SIN(SI)
      COSS=COS(SI)
      FAC=SQRT(1.-(SINS*SINP)**2)
      IF (FAC.EQ.0.) GO TO 71
      FT=COSS*SINP/FAC
      FP=COSP/FAC
      GO TO 72
71    FT=1.
      FP=0.
72    CP1=CEXP(J*RK*(A*SINT-RO))
      CP2=CEXP(-J*RK*(A*SINT+RO))
      PHI1=180.-GAMMA+THETA
      PHI2=180.-GAMMA-THETA
      IF (PHI2.LE.0.0) PHI2=360.+PHI2
      CALL GTD(PHI1,PHIP,90.,S,RO,WA,SL,AS1,ILOP,DS1,DH1)
      CALL GTD(PHI2,PHIP,90.,S,RO,WA,SL,AS1,ILOP,DS2,DH2)
      ET1=-T2*CP1*DH1*BP*FTA
      ET2=-J*T2*CP2*DH2*BP*FTA
      EP1=T2*CP1*DS1*BP*FPA
      EP2=J*T2*CP2*DS2*BP*FPA
C
```

```
C
C                              ***************
C                              * DIRECT FEED *
C                              ***************
C
      EF=0.
      IF (THETA.GT.AN1) GO TO 74
      CALL LSUM(PSI,PXE,FFE,DLE,N3,FE)
      CALL LSUM(PSI,PXH,FFH,DLH,N3,FH)
      EF=((FE+FH)/2.-COS2P*(FE-FH)/2.)*F*CP3
74    EFT=-EF*FT
      EFP=EF*FP
      IF (THETA.LE.90..OR.THETA.GT.AN2) GO TO 75
      EP2=0.
      ET2=0.
75    EP=EP1+EP2+EFP
      ET=ET1+ET2+EFT
      GO TO 77
C
C                          **********************
C                          * RING CURRENT METHOD *
C                          **********************
C
76    CONTINUE
      CALL BESJ(X,0,BJ0,0.002,IER)
      CALL BESJ(X,2,BJ2,0.001,IER)
      CALL BESJ(X,4,BJ4,0.001,IER)
      DSC=DS*COST
      A1=TS*(DH-DSC)+DL*(DH+DSC)/2.
      A2=-TS*(DH+DSC)-DL*(DH-DSC)
      A3=DL*(DH+DSC)/2.
      DHC=DH*COST
      B1=TS*(DS-DHC)-DL*(DS+DHC)/2.
      B2=-TS*(DS+DHC)+DL*(DS-DHC)
      B3=-DL*(DS+DHC)/2.
      EPL=-CP4*(A1*BJ0+A2*BJ2+A3*BJ4)
      HPL=CP4*(B1*BJ0+B2*BJ2+B3*BJ4)
      EY=-EPL*SINP**2+HPL*COSP**2
      ET=-EY*SINP
      EP=EY*COSP
77    CONTINUE
C
C                     ****************************
C                     * FEED BLOCKAGE SCATTERING *
C                     ****************************
C
      IF (THETA.LE.90.) GO TO 80
      EFBY=0.
      ET1=ET
      ETP=EP
      GO TO 85
80    IF (KG.EQ.1) GO TO 81
      XA=RK*AX*SINT*COSP/2.
      XB=RK*BY*SINT*SINP/2.
      IF (XA.EQ.0.0) GA=1.
      IF (XA.GT.0.0) GA=SIN(XA)/XA
      IF (XB.EQ.0.0) GB=1.
      IF (XB.GT.0.0) GB=SIN(XB)/XB
      EFBY=CPP*CP3*GA*GB
      GO TO 82
81    CONTINUE
      XC=RK*C*SINT
      CALL BESJ(XC,1,BJ1,0.001,IER1)
      IF (XC.EQ.0.) EFBY=CPP*CP3/2.
      IF (XC.GT.0.) EFBY=CPP*CP3*BJ1/XC
82    CONTINUE
      EFBP=EFBY*COSP
      EFBT=EFBY*SINP
```

```
C
C                              ********************
C                              * STRUT SCATTERING *
C                              ********************
C
      ESTT=(0.,0.)
      ESTP=(0.,0.)
      DO 84 I=1,NS
      PHS=PHIS(I)
      ZN2=ZS2(I)
      RN2=RS2(I)
      RNO=RLO(I)
      CALL STRUT(PHI,THETA,PHS,ZN2,RN2,RNO,EST,ESP)
      ESTT=ESTT+EST
      ESTP=ESTP+ESP
84    CONTINUE
      ETT=ET+ESTT+EFBT
      ETP=EP+ESTP+EFBP
      CALL HUYSRC(EFBT,EFBP,COSP,SINP)
      CALL DBPHS(EFBP,MAX)
      CALL DBPHS(EFBT,MAX)
      CALL HUYSRC(ESTT,ESTP,COSP,SINP)
      CALL DBPHS(ESTP,MAX)
      CALL DBPHS(ESTT,MAX)
      PLOTST(1,N)=REAL(ESTT)
      PLOTST(2,N)=REAL(ESTP)
      PLOTST(3,N)=REAL(EFBT)
      PLOTST(4,N)=REAL(EFBP)
85    CONTINUE
      CALL HUYSRC(ET,EP,COSP,SINP)
      CALL DBPHS(ET,MAX)
      CALL DBPHS(EP,MAX)
      CALL HUYSRC(ETT,ETP,COSP,SINP)
      CALL DBPHS(ETT,MAX)
      CALL DBPHS(ETP,MAX)
      PLOTST(5,N)=REAL(ET)
      PLOTST(6,N)=REAL(EP)
      PLOTST(7,N)=REAL(ETT)
      PLOTST(8,N)=REAL(ETP)
      IF (THETA.GT.90.) WRITE (6,88) THETA,ETT,ETP
88    FORMAT (F9.2,T100,2(F8.2,F7.1))
      IF (THETA.LE.90.) WRITE (6,89) THETA,ESTT,ESTP,EFBT,EFBP,ET,EP,ETT
     $,ETP
89    FORMAT (F9.2,8(F8.2,F7.1))
90    CONTINUE
      TMAX=THONE+(NTHETA-1)*DTHETA
      CALL PLTCK(TMAX,FDR,D2,PHI*180./PI,FREQ/1000.,NTHETA,PLOTST,DTHETA
     $)
95    CONTINUE
100   CONTINUE
      GO TO 5
101   STOP
      END
      SUBROUTINE PLTCK(TMAX,FOD,DIA,PHI,FREQ,NTHETA,PLOTST,DT)
      DIMENSION DATA(5),ITITLE(20),PLOTST(8,180)
      DATA IFIRST/1/,NCOM/7/
      DATA XAXIS/9./,   YAXIS/6./,   VL/-72./,   VT/0./, PC/0./,HT/.14/
      CW=1.* HT
      VS=2.* HT
      IX=10./DT+1
    5 READ(5,1) NTYPE,DATA
    1 FORMAT(8X,I2,5F10.0)
C     IF GO CASE RETURN
      IF(NTYPE .EQ. 3) RETURN
      IF (IFIRST .EQ. 0) GO TO 2
C     FIRST PLOTTING CALL ---- INITIALIZE
      CALL PLOT(2.,1.,-3)
    2 IF ( NTYPE .NE. 1) GO TO 3
```

```
C     AXIS CASE
        IF (IFIRST .EQ. 0.) CALL PLOT ( XAXIS+3.,0.,-3)
          IF ( DATA(1) .NE. 0.) XAXIS=DATA(1)
          IF ( DATA(2) .NE. 0.) YAXIS=DATA(2)
          IF ( DATA(3) .NE. 0.) VL=DATA(3)
          IF ( DATA(4) .NE. 0.) VT=DATA(4)
          IF ( DATA(5) .NE. 0.) PC=1.
          IF ( DATA(5) .EQ. 0.) PC=0
C      PC IS REAL CODE VARIABLE    1. IS INDIVIDUAL PHI LABELS
        DX=TMAX/XAXIS
        DY=(VT-VL)/YAXIS
        ICRV=0
        CALL AXIS(0.,0.,'THETA (DEG)',-11,XAXIS,0.,0.,DX,1.)
        CALL AXIS(0.,0.,'E-FIELD (DB)',12,YAXIS,90.,VL,DY,1.)
C       DRAW F/D, DIA AND FREQ LABELS
        XCOL=XAXIS-4.
        YCOL=YAXIS-.25
        XC=XCOL+5.*CW
        CALL SYMBOL(XC,YCOL,HT,'F/D=',0.,4)
        CALL NUMBER(XC+ 6.*CW,YCOL,HT,FOD,0.,3)
        YCOL=YCOL-VS
        CALL SYMBOL(XC,YCOL,HT,'DIA OF REFL (IN)=',0.,17)
        CALL NUMBER(XC+18.*CW,YCOL,HT,DIA,0.,2)
        YCOL=YCOL-VS
        CALL SYMBOL(XC,YCOL,HT,'FREQ (GHZ)=',0.,11)
        CALL NUMBER(XC+12.*CW,YCOL,HT,FREQ,0.,2)
        YCOL=YCOL-VS
        IF ( PC .EQ.1.) GO TO 4
C      DRAW COMMON PHI LABEL
        CALL SYMBOL(XC,YCOL,HT,'PHI CUT (DEG)=',0.,14)
        CALL NUMBER(XC+15.*CW,YCOL,HT,PHI,0.,-1)
        YCOL=YCOL-VS
        GO TO 4
      3 IF ( NTYPE .NE. 4) GO TO 6
C     PLOT TITLE
        NC=DATA(1)
        READ(5,7) ITITLE
      7 FORMAT(20A4)
        XT=(XAXIS-NC*CW)/2.
        CALL SYMBOL(XT,-.75,HT,ITITLE(1),0.,NC)
        GO TO  4
      6 IF (NTYPE .NE. 2) GO TO 5
C     PLOT CURVE
        IF ( DATA(1) .NE. 0.) NCOM=DATA(1)
        ICRV=ICRV+1
        NP=NTHETA
        IF ( NCOM.LT. 5 .AND. TMAX .GT. 90.) NP=90./DT+1
        DDX=XAXIS/(NTHETA-1)
        IC=3
        DO 8 I=1,NP
        X=(I-1)*DDX
        Y=(PLOTST(NCOM,I)-VL)/DY
        IF ( Y .LT. 0.) Y=0.
        IF ( Y .GT. YAXIS) Y=YAXIS
        CALL PLOT(X,Y,IC)
        IF ((I/IX)*IX.EQ.I) CALL SYMBOL(X,Y,.07,ICRV,0.,-1)
      8 IC=2
C      LABEL CURVE
        CALL SYMBOL(XC-1.,YCOL+HT/2.,.07,ICRV,0.,-1)
        CALL PLOT(XC-.2,YCOL+HT/2.,2)
        CALL SYMBOL(XC-.2,YCOL+HT/2.,.07,ICRV,0.,-1)
        CALL LABEL(XC,YCOL,HT,NCOM,'PRINC STRUT SCAT ',
       1 'X-POL STRUT SCAT ','PRINC  FEED SCAT ','X-POL  FEED SCAT ',
       2 'PRINC REFL + FEED',  'X-POL REFL + FEED','PRINC TOTAL FIELD',
       6 'X-POL TOTAL FIELD')
        IF ( PC .EQ. 0.) GO TO 9
C   LABEL INDIVIDUAL PHI CUT
        CALL SYMBOL(XC+18.*CW,YCOL,HT,'PHI=',0.,4)
```

```
      CALL NUMBER(XC+23.*CW,YCOL,HT,PHI,0.,-1)
    9 YCOL=YCOL-VS
      CALL PLOTE2
    4 IFIRST=0
      GO TO 5
      END
      SUBROUTINE LABEL(X,Y,H,N,C1,C2,C3,C4,C5,C6,C7,C8)
      DIMENSION C1(1),C2(1),C3(1),C4(1),C5(1),C6(1),C7(1),C8(1)
      GO TO (1,2,3,4,5,6,7,8),N
    1 CALL SYMBOL(X,Y,H,C1(1),0.,17)
      GO TO 9
    2 CALL SYMBOL(X,Y,H,C2(1),0.,17)
      GO TO 9
    3 CALL SYMBOL(X,Y,H,C3(1),0.,17)
      GO TO 9
    4 CALL SYMBOL(X,Y,H,C4(1),0.,17)
      GO TO 9
    5 CALL SYMBOL(X,Y,H,C5(1),0.,17)
      GO TO 9
    6 CALL SYMBOL(X,Y,H,C6(1),0.,17)
      GO TO 9
    7 CALL SYMBOL(X,Y,H,C7(1),0.,17)
      GO TO 9
    8 CALL SYMBOL(X,Y,H,C8(1),0.,17)
    9 RETURN
      END
      SUBROUTINE HUYSRC(ETHETA,EPHI,COSP,SINP)
C
C        ROUTINE CONVERT ETHETA,EPHI TO E HUYGEN PRINCIPAL AND CROSS
C        POLARIZAED COMPONENTS
C
      COMPLEX ETHETA,EPHI,STOR
      STOR=COSP*ETHETA-SINP*EPHI
      ETHETA=SINP*ETHETA+COSP*EPHI
      EPHI=STOR
      RETURN
      END
      SUBROUTINE STRUT(PHI,THETA,PHIS,Z2,RS2,RLO,EST,ESP)
C
C
      DIMENSION RSY(100),ASY(100),RZY(100),AZY(100),TF(100),DF(100)
      COMPLEX J,CO,CQ1,CQ2,CP5,DS,DH,TEMP,TERMS,TERMH,AM(25),BM(25),QY,
     $SUMY,EAT,EAP,EGT,EGP,EALP,EGAM,CPO,EST,ESP,OYD
      COMMON /STR/J,CPO,AM,BM,PI,RK,MX,DEL,A,TF,DF,BET
      THE=THETA*PI/180.
      COST=COS(THE)
      SINT=SIN(THE)
      SINPS=SIN(PHIS)
      COSPS=COS(PHIS)
      SINB=SIN(BET)
      COSB=COS(BET)
      COTB=COSB/SINB
      ZO=Z2+RS2*COTB
      RS1=RS2-RLO*SINB
      IF (RS2.GT.A) RS2=A
      R1=RS1/SINB
      R2=RS2/SINB
      L1=RS1/DEL+2
      L2=RS2/DEL+1
      N=L2-L1
      N1=N+1
      DELS=DEL/SINB
      RL1=DELS*(L1-1)
      RL2=DELS*(L2-1)
      CP5=CEXP(J*(PI/4.+RK*ZO*(COST-1.)))
      TEMP=CEXP(J*PI/4.)/(PI*SINB)
      PHIO=PHI-PHIS
      COSO=COS(PHIO)
```

```
      SIND=SIN(PHID)
      COSA=SINT*SINB*COSD-COST*COSB
      ALP=ARCOS(COSA)
      SINA=SIN(ALP)
      SSG=SINT*SIND
      SCG=-SINT*COSB*COSD-COST*SINB
      GAM=ATAN2(SSG,SCG)
      SING=SIN(GAM)
      COSG=COS(GAM)
      ALPHA=ALP*180./PI
      GAMMA=GAM*180./PI
      DS=(0.,0.)
      DH=(0.,0.)
      DO 10 M=1,MX
      COSM=COS((M-1)*(PI-GAM))
      TERMS=2.*AM(M)*COSM
      TERMH=2.*BM(M)*COSM
      DS=DS+TERMS
      DH=DH+TERMH
10    CONTINUE
      DS=TEMP*DS
      DH=TEMP*DH
      CAB=COSA+COSB
      RL=RL1
      DO 12 I=1,N1
      CQ=CEXP(J*RK*RL*CAB)
      K=I+L1-1
      EI=-(TF(K)-DF(K)*COSPS)
      SUMY=EI*CQ
      RSY(I)=REAL(SUMY)
      ASY(I)=AIMAG(SUMY)
      RL=RL+DELS
12    CONTINUE
      CALL QSF(DELS,RSY,RZY,N1)
      CALL QSF(DELS,ASY,AZY,N1)
      QYO=RZY(N1)+J*AZY(N1)
      CQ1=CEXP(J*RK*(R1+RL1)*CAB/2.)
      CQ2=CEXP(J*RK*(R2+RL2)*CAB/2.)
      EN1=-(RL1-R1)*(TF(L1)+TF(L1-1)-(DF(L1)+DF(L1-1)))/2.
      EN2=-(R2-RL2)*(TF(L2)-DF(L2))/2.
      QY=QYO+EN1*CQ1+EN2*CQ2
18    EALP=-CP5*DS*SINA*QY*SINPS*CPO
      EGAM=CP5*DH*SINA*QY*(-COSPS)*CPO
      FAT=-(COSA*COSG*COSB+SINA*SINB)*COST*COSD+COSA*SING*COST*SIND+SINT
     $*(COSA*COSG*SINB-SINA*COSB)
      FAP=(SINA*SINB+COSA*COSG*COSB)*SIND+COSA*SING*COSD
      FGT=SING*(COSB*COST*COSD-SINB*SINT)+COSG*COST*SIND
      FGP=COSG*COSD-SING*COSB*SIND
      EAT=EALP*FAT
      EAP=EALP*FAP
      EGP=EGAM*FGP
      EGT=EGAM*FGT
      EST=EAT+EGT
      ESP=EAP+EGP
      RETURN
      END
      SUBROUTINE LNFD(DSI,PX,FF,N)
      DIMENSION DSI(50),PX(50),FF(50)
      WRITE (6,2)
2     FORMAT (//T10,'PIECEWISE LINEAR FEED INPUT',//T18,'PSI',T31,'F',/)
      DO 20 I=1,N
      READ(5,10) PX(I),FF(I)
10    FORMAT (F10.2,F10.5)
15    FORMAT (T10,F10.2,F15.5)
      WRITE (6,15) PX(I),FF(I)
      IF (I.EQ.1) GO TO 20
      DSI(I-1)=(FF(I)-FF(I-1))/(PX(I)-PX(I-1))
20    CONTINUE
```

```
      RETURN
      END
      SUBROUTINE LSUM(PSI,PX,FF,DSI,N3,FE)
      DIMENSION DSI(50),PX(50),FF(50)
      DO 10 I=1,N3
      IF (PSI.GT.PX(I+1)) GO TO 10
      FE=FF(I)+DSI(I)*(PSI-PX(I))
      RETURN
10    CONTINUE
      FE=0.
      RETURN
      END
      SUBROUTINE DBPHS(E,MAX)
      COMPLEX E
      REAL DB,FASE,MAX
      PI=3.141592653
      AE=CABS(E)
      IF (AE.GT.0.) GO TO 10
      DB=-500.
      FASE=0.
      GO TO 20
10    DB=20.*ALOG10(AE)-MAX
      FASE=ATAN2(AIMAG(E),REAL(E))*180./PI
20    E=CMPLX(DB,FASE)
      RETURN
      END
      SUBROUTINE GTD(PHI,PHIP,BETAO,S ,SP,WA,SL,AS,ILOP,DS,DH)
C
C
C     PURPOSE - TO COMPUTE DIFFRACTION COEFFICIENTS DS,DH
C
C     PARAMETERS
C
C         ***INPUT***
C
C         PHI   = DIFFRACTED ANGLE
C         PHIP  = INCIDENT ANGLE AT THE EDGE
C         BETAO= ANGLE BETWEEN THE INCIDENT RAY THE TANGENT TO THE EDGE
C         S     = DISTANCE FROM FIELD POINT TO THE EDGE ALONG THE
C                                   DIFFRACTED RAY PATH
C         SP    = DISTANCE FROM THE SOURCE POINT TO THE EDGE ALONG THE
C                                   INCIDENT RAY
C         WA    = WEDGE ANGLE
C         SL    = DISTANCE PARAMETER WHEN EXTERNALLY PROVIDED (WHEN ILCP=4
C         ILOP = CONTROL NUMBER :
C                ILOP=1 : PLANE-WAVE INCIDENCE
C                ILOP=2 : CYLINDRICAL WAVE INCIDENCE
C                ILOP=3 : SPHERICAL OR CONICAL-WAVE INCIDENCE
C                ILOP=4 THE DISTANCE PARAMETER L=SL IS PROVIDED EXTERNALLY
C
C         ***OUTPUT***
C
C         AS    = AMPLITUDE OF THE FIELD ALONG DIFFRACTED RAY (NOT APPLICA
C                                   WHEN ILOP=4)
C         DS    = SOFT DIFFRACTION COEFFICIENT
C         DH    = HARD DIFFRACTION COEFFICIENT
C
C         ***UNITS***
C
C         ANGLE    : DEGREE
C         LENGTH   : WAVELENGTH
C     REQUIRED FUNCTIONS : TRANSITION FUNCTION F(X) AND SIGN FUNCTION SG
C
C     REFERENCE: A UNIFORM GEOMETRICAL THEORY OF DIFFRACTION FOR AN EDGE
C     IN A PERFECTLY CONDUCTING SURFACE , BY R.G.KOUYOUMJIAN AND P.H.
C     PATHAK , PROC. OF IEEE , VOL 62 NOV. 1974
C
C
```

```
      COMPLEX F,AJ,C0,C1,C2,C3,C4,DS,DH,FASE,Q
      REAL L,K1,K2
      A(T,WN,N)=2.0*COS(3.14159265*WN*N-T/2.0)**2
C
C *** COMPUTE NECESSARY CONSTANTS
C
      PI=3.14159265
      PII=2.0*PI
      PI4=PI/4.0
      PI180=PI/180.
      AJ=CMPLX(0.,1.)
      EPSI=.0001
      W=50.
      GS=1.
      GH=1.
      WG=360.-WA
C
      IF (PHIP.EQ.0..OR.PHIP.EQ.WG) GS=0.
      IF (PHIP.EQ.0..OR.PHIP.EQ.WG) GH=.5
C
C *** COMPUTE ANGLES BETA-ZERO,BETA-POSITIVE,BETA-NEGATIVE
C
      WN=WG/180.
      BETAP=(PHI+PHIP)*PI180
      BETAN=(PHI-PHIP)*PI180
C
C *** CHOOSE DISTANCE PARAMETER FOR TYPE OF EGDE ILLUMINATION
C
      IF(ILOP.EQ.1) L=S*SIN(BETAO*PI180)**2
      IF(ILOP.EQ.2) L=S*SP/(S+SP)
      IF(ILOP.EQ.3) L=S*SP/(S+SP)*SIN(BETAO*PI180)**2
      IF(ILOP.EQ.4) L=SL
C
C *** COMPUTE A(S,SP) THE AMPLITUDE OF THE FIELD ALONG THE DIFFRACTED RA
C
      IF(ILOP.EQ.1) AS=1.0/SQRT(S)
      IF(ILOP.EQ.2) AS=1.0/SQRT(S*SIN(BETAO*PI180))
      IF(ILOP.EQ.3) AS=SQRT(SP/(S*(S+SP)))
      IF(ILOP.EQ.4) AS=1.0
C
C *** COMPUTE ANGULAR ARGUMENTS FOR COTANGENT FUNCTION
C
      ANG1=(PI+BETAN)/(2.0*WN)
      ANG2=(PI-BETAN)/(2.0*WN)
      ANG3=(PI+BETAP)/(2.0*WN)
      ANG4=(PI-BETAP)/(2.0*WN)
C
C *** CALCULATE N+ AND N-
C
      NPN=IFIX((PI+BETAN)/(PII*WN)+0.5)
      NNN=IFIX((-PI+BETAN)/(PII*WN)+0.5)
      NPP=IFIX((PI+BETAP)/(PII*WN)+0.5)
C
C *** COMPUTE ARGUMENTS OF TRANSITION FUNCTION F(X) X=KLA
      ARG1=PII*L*A(BETAN,WN,NPN)
      ARG2=PII*L*A(BETAN,WN,NNN)
      ARG3=PII*L*A(BETAP,WN,NPP)
      ARG4=PII*L*A(BETAP,WN,NNP)
C
C *** DS AND DH CALCULATED BY KELLER'S FORM WHEN FIELD POINT IS NOT IN
      NNP=IFIX((-PI+BETAP)/(PII*WN)+0.5)
C                                            TRANSITION REGIONS
C
      IF (ARG1.LE.W) GO TO 50
      IF (ARG2.LE.W) GO TO 50
      IF (ARG3.LE.W) GO TO 50
      IF (ARG4.LE.W) GO TO 50
      C0=SIN(PI/WN)*CEXP(-AJ*PI4)/(WN*PII*SIN(BETAO*PI180))
```

AD-A043 400   OHIO STATE UNIV COLUMBUS DEPT OF ELECTRICAL ENGINEERING   F/G 9/1
A GTD ANALYSIS OF THE CIRCULAR REFLECTOR ANTENNA INCLUDING FEED--ETC(U)
AUG 77 S H LEE, R C RUDDUCK, C A KLEIN   F30602-76-C-0224
UNCLASSIFIED   4381-1   RADC-TR-77-259   NL

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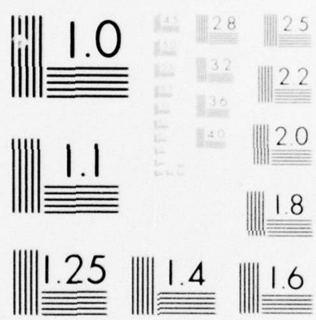

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
      K1=1.0/(COS(PI/WN)-COS(BETAN/WN))
      K2=1.0/(COS(PI/WN)-COS(BETAP/WN))
      DS=GS*CO*(K1-K2)
      DH=GH*CO*(K1+K2)
C
      RETURN
C
C *** CALCULATE THE FOUR TERMS OF THE DIFFRACTION COEFFICIET
C
   50 CO=-CEXP(-AJ*PI4)/(2.0*WN*PI1*SIN(BETAO*PI180))
      IF (ABS(ANG1-PI*NPN).GT.EPSI) GO TO 100
      EPS=(ANG1-PI*NPN)*2.*WN
      C1=Q(EPS,WN,L,PI)
      GO TO 150
C
  100 C1=COTAN(ANG1)*F(ARG1)
  150 IF (ABS(ANG2+PI*NNN).GT.EPSI) GO TO 200
      EPS=(ANG2+PI*NNN)*2.*WN
      C2=Q(EPS,WN,L,PI)
      GO TO 250
C
  200 C2=COTAN(ANG2)*F(ARG2)
  250 IF (ABS(ANG3-PI*NPP).GT.EPSI) GO TO 300
      EPS=(ANG3-PI*NPP)*2.*WN
      C3=Q(EPS,WN,L,PI)
      GO TO 350
C
  300 C3=COTAN(ANG3)*F(ARG3)
  350 IF (ABS(ANG4+PI*NNP).GT.EPSI) GO TO 400
      EPS=(ANG4+PI*NNP)*2.*WN
      C4=Q(EPS,WN,L,PI)
      GO TO 450
C
  400 C4=COTAN(ANG4)*F(ARG4)
  450 CONTINUE
C
C *** COMPUTE DS AND DH
C
      DS=GS*CO*(C1+C2-C3-C4)
      DH=GH*CO*(C1+C2+C3+C4)
      RETURN
      END
      FUNCTION SGN(X)
C     PURPOSE : TO GIVE MINUS SIGN IF X IS NEGATIVE OR VICE VERSA
      IF (X) 1,2,3
    1 SGN=-1.
      RETURN
    2 SGN=0.
      RETURN
    3 SGN=1.
      RETURN
      END
      COMPLEX FUNCTION F(X)
C         TRANSITION FUNCTION  F(X) FOR DIFFRACTION COEFFICIENT
C
C         F(X) IS COMPUTED BY FRESNEL INTEGRAL
C         FOR X>8.,OR X).2,  F(X) IS EVALUATED BY APPROXIMATION
C         ERROR BY APPROXIMATION IS WITHIN 1(
C         SUBROUTINE REQUIRED CSX(C,S,X)
C
C
      COMPLEX ZX,Z
      REAL PI/3.1415926535/,PI4/.78539816/, PQ / 2.5066283/
      COMPLEX CPI4/(.70710678,.70710678)/
      IF (X-8.) 20,20,10
   10 X2=X*X
      X3=X2*X
      X4=X2*X2
```

```
      F =CMPLX(1.-.75/X2+4.6875/X4,.5/X -1.875/X3)
      RETURN
20    IF (X-.2) 40,30,30
30    ZX=CMPLX(0.,X)
      CALL CSX(C,S,X)
      C1=.5-C
      S1=.5-S
      F=SQRT(X)*CEXP(ZX)*PQ*CMPLX(S1,C1)
      RETURN
40    Z=SQRT(PI*X)-2.*X*CPI4-2.*X*X/CPI4/3.
      F =Z*CEXP(CMPLX(0.,PI4+X))
      RETURN
      END
      SUBROUTINE CSX(C,S,X)
C
C        PURPOSE
C              COMPUTES THE FRESNEL INTEGRALS.
C
C        DESCRIPTION OF PARAMETERS
C              C  -  THE RESULTANT OUTPUT OF C(X)
C              S  -  THE RESULTANT OUTPUT OF S(X)
C              X  -  ARGUMENT
C
C        FROM SUBROUTINE CS(C,S,X) OF SSP
      Z=ABS(X)
    2 IF (Z-4.) 3,3,4
C
C        IF (Z.LE.4.)
C
    3 C=SQRT(Z)
      S=Z*C
      Z=(4.-Z)*(4.+Z)
      C=C*((((((5.100785E-11*Z+5.244297E-9)*Z+5.451182E-7)*Z
     1+3.273308E-5)*Z+1.020418E-3)*Z+1.102544E-2)*Z+1.840965E-1)
      S=S*(((((6.677681E-10*Z+5.883158E-8)*Z+5.051141E-6)*Z
     1+2.441816E-4)*Z+6.121320E-3)*Z+8.026490E-2)
      RETURN
C
C        IF (Z.GT.4.)
C
    4 D=COS(Z)
      S=SIN(Z)
      Z=4./Z
      A=(((((((8.768258E-4*Z-4.169289E-3)*Z+7.970943E-3)*Z-6.792801E-3)
     1*Z-3.095341E-4)*Z+5.972151E-3)*Z-1.606428E-5)*Z-2.493322E-2)*Z
     2-4.444091E-9
      B=((((((-6.633926E-4*Z+3.401409E-3)*Z-7.271690E-3)*Z+7.428246E-3)
     1*Z-4.027145E-4)*Z-9.314910E-3)*Z-1.207998E-6)*Z+1.994711E-1
      Z=SQRT(Z)
      C=.5+Z*(D*A+S*B)
      S=.5+Z*(S*A-D*B)
      RETURN
      END
      COMPLEX FUNCTION Q(EPS,WN,RL,PI)
      REAL L
      L=RL
      Q=WN*CEXP(CMPLX(0.,PI/4.))*(2.*PI*SQRT(L)*SGN(EPS)-4.*PI*L*EPS
     $*CEXP(CMPLX(0.,PI/4.)))
      RETURN
      END
      FUNCTION COTAN(X)
      S=SIN(X)
      IF (S .NE. 0.) GO TO 1
      COTAN=1.E50
      RETURN
    1 COTAN=COS(X)/S
      RETURN
      END
```

```
C        SUBROUTINE BESJ
C
C     ..................................................................
C
C
C        PURPOSE
C           COMPUTE THE J BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER
C
C        USAGE
C           CALL BESJ(X,N,BJ,D,IER)
C
C        DESCRIPTION OF PARAMETERS
C           X  -THE ARGUMENT OF THE J BESSEL FUNCTION DESIRED
C           N  -THE ORDER OF THE J BESSEL FUNCTION DESIRED
C           BJ -THE RESULTANT J BESSEL FUNCTION
C           D  -REQUIRED ACCURACY
C           IER-RESULTANT ERROR CODE WHERE
C              IER=0  NO ERROR
C              IER=1  N IS NEGATIVE
C              IER=2  X IS NEGATIVE OR ZERO
C              IER=3  REQUIRED ACCURACY NOT OBTAINED
C              IER=4  RANGE OF N COMPARED TO X NOT CORRECT (SEE REMARKS)
C
C        REMARKS
C           N MUST BE GREATER THAN OR EQUAL TO ZERO, BUT IT MUST BE
C           LESS THAN
C              20+10*X-X** 2/3   FOR X LESS THAN OR EQUAL TO 15
C              90+X/2            FOR X GREATER THAN 15
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           RECURRENCE RELATION TECHNIQUE DESCRIBED BY H. GOLDSTEIN AND
C           R.M. THALER,'RECURRENCE TECHNIQUES FOR THE CALCULATION OF
C           BESSEL FUNCTIONS',M.T.A.C.,V.13,PP.102-108 AND I.A. STEGUN
C           AND M. ABRAMOWITZ,'GENERATION OF BESSEL FUNCTIONS ON HIGH
C           SPEED COMPUTERS',M.T.A.C.,V.11,1957,PP.255-257
C
C     ..................................................................
C
      SUBROUTINE BESJ(X,N,BJ,D,IER)
C
      BJ=.0
      IF(N)10,20,20
   10 IER=1
      RETURN
   20 IF(X)30,30,31
   30 IER=2
      RETURN
   31 IF(X-15.)32,32,34
   32 NTEST=20.+10.*X-X** 2/3
      GO TO 36
   34 NTEST=90.+X/2.
   36 IF(N-NTEST)40,38,38
   38 IER=4
      RETURN
   40 IER=0
      N1=N+1
      BPREV=.0
C
C     COMPUTE STARTING VALUE OF M
C
      IF(X-5.)50,60,60
   50 MA=X+6.
      GO TO 70
   60 MA=1.4*X+60./X
   70 MB=N+IFIX(X)/4+2
```

```
      MZERO=MAXO(MA,MB)
C
C     SET UPPER LIMIT OF M
C
      MMAX=NTEST
  100 DO 190 M=MZERO,MMAX,3
C
C     SET F(M),F(M-1)
C
      FM1=1.0E-28
      FM=.0
      ALPHA=.0
      IF(M-(M/2)*2)120,110,120
  110 JT=-1
      GO TO 130
  120 JT=1
  130 M2=M-2
      DO 160 K=1,M2
      MK=M-K
      BMK=2.*FLOAT(MK)*FM1/X-FM
      FM=FM1
      FM1=BMK
      IF(MK-N-1)150,140,150
  140 BJ=BMK
  150 JT=-JT
      S=1+JT
  160 ALPHA=ALPHA+BMK*S
      BMK=2.*FM1/X-FM
      IF(N)180,170,180
  170 BJ=BMK
  180 ALPHA=ALPHA+BMK
      BJ=BJ/ALPHA
      IF(ABS(BJ-BPREV)-ABS(D*BJ))200,200,190
  190 BPREV=BJ
      IER=3
  200 RETURN
      END
C
C     ...............................................................
C
C        SUBROUTINE BESY
C
C        PURPOSE
C           COMPUTE THE Y BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER
C
C        USAGE
C           CALL BESY(X,N,BY,IER)
C
C        DESCRIPTION OF PARAMETERS
C           X   -THE ARGUMENT OF THE Y BESSEL FUNCTION DESIRED
C           N   -THE ORDER OF THE Y BESSEL FUNCTION DESIRED
C           BY  -THE RESULTANT Y BESSEL FUNCTION
C           IER-RESULTANT ERROR CODE WHERE
C               IER=0  NO ERROR
C               IER=1  N IS NEGATIVE
C               IER=2  X IS NEGATIVE OR ZERO
C               IER=3  BY HAS EXCEEDED MAGNITUDE OF 10**70
C
C        REMARKS
C           VERY SMALL VALUES OF X MAY CAUSE THE RANGE OF THE LIBRARY
C           FUNCTION ALOG TO BE EXCEEDED
C           X MUST BE GREATER THAN ZERO
C           N MUST BE GREATER THAN OR EQUAL TO ZERO
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
```

```
C          RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE
C          AS DESCRIBED BY A.J.M.HITCHCOCK,'POLYNOMIAL APPROXIMATIONS
C          TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TO RELATED
C          FUNCTIONS', M.T.A.C., V.11,1957,PP.86-88, AND G.N. WATSON,
C          'A TREATISE ON THE THEORY OF BESSEL FUNCTIONS', CAMBRIDGE
C          UNIVERSITY PRESS, 1958, P. 62
C
C     ..................................................................
C
      SUBROUTINE BESY(X,N,BY,IER)
C
C     CHECK FOR ERRORS IN N AND X
C
      IF(N)180,10,10
   10 IER=0
      IF(X)190,190,20
C
C     BRANCH IF X LESS THAN OR EQUAL 4
C
   20 IF(X-4.0)40,40,30
C
C        COMPUTE YO AND Y1 FOR X GREATER THAN 4
C
   30 T1=4.0/X
      T2=T1*T1
      P0=((((-.0000037043*T2+.0000173565)*T2-.0000487613)*T2
     1   +.00017343)*T2-.001753062)*T2+.3989423
      Q0=((((.0000032312*T2-.0000142078)*T2+.0000342468)*T2
     1   -.0000869791)*T2+.0004564324)*T2-.01246694
      P1=((((.0000042414*T2-.0000200920)*T2+.0000580759)*T2
     1   -.000223203)*T2+.002921826)*T2+.3989423
      Q1=((((-.0000036594*T2+.00001622)*T2-.0000398708)*T2
     1   +.0001064741)*T2-.0006390400)*T2+.03740084
      A=2.0/SQRT(X)
      B=A*T1
      C=X-.7853982
      Y0=A*P0*SIN(C)+B*Q0*COS(C)
      Y1=-A*P1*COS(C)+B*Q1*SIN(C)
      GO TO 90
C
C        COMPUTE YO AND Y1 FOR X LESS THAN OR EQUAL TO 4
C
   40 XX=X/2.
      X2=XX*XX
      T=ALOG(XX)+.5772157
      SUM=0.
      TERM=T
      Y0=T
      DO 70 L=1,15
      IF(L-1)50,60,50
   50 SUM=SUM+1./FLOAT(L-1)
   60 FL=L
      TS=T-SUM
      TERM=(TERM*(-X2)/FL**2)*(1.-1./(FL*TS))
   70 Y0=Y0+TERM
      TERM = XX*(T-.5)
      SUM=0.
      Y1=TERM
      DO 80 L=2,16
      SUM=SUM+1./FLOAT(L-1)
      FL=L
      FL1=FL-1.
      TS=T-SUM
      TERM=(TERM*(-X2)/(FL1*FL))*((TS-.5/FL)/(TS+.5/FL1))
   80 Y1=Y1+TERM
      PI2=.6366198
      Y0=PI2*Y0
      Y1=-PI2/X+PI2*Y1
```

```
C
C     CHECK IF ONLY Y0 OR Y1 IS DESIRED
C
   90 IF(N-1)100,100,130
C
C     RETURN EITHER Y0 OR Y1 AS REQUIRED
C
  100 IF(N)110,120,110
  110 BY=Y1
      GO TO 170
  120 BY=Y0
      GO TO 170
C
C    PERFORM RECURRENCE OPERATIONS TO FIND YN(X)
C
  130 YA=Y0
      YB=Y1
      K=1
  140 T=FLOAT(2*K)/X
      YC=T*YB-YA
      IF(ABS(YC)-1.0E70)145,145,141
  141 IER=3
      RETURN
  145 K=K+1
      IF(K-N)150,160,150
  150 YA=YB
      YB=YC
      GO TO 140
  160 BY=YC
  170 RETURN
  180 IER=1
      RETURN
  190 IER=2
      RETURN
      END
C
C     ..................................................................
C
C        SUBROUTINE QSF
C
C        PURPOSE
C           TO COMPUTE THE VECTOR OF INTEGRAL VALUES FOR A GIVEN
C           EQUIDISTANT TABLE OF FUNCTION VALUES.
C
C        USAGE
C           CALL QSF (H,Y,Z,NDIM)
C
C        DESCRIPTION OF PARAMETERS
C           H      - THE INCREMENT OF ARGUMENT VALUES.
C           Y      - THE INPUT VECTOR OF FUNCTION VALUES.
C           Z      - THE RESULTING VECTOR OF INTEGRAL VALUES. Z MAY BE
C                    IDENTICAL WITH Y.
C           NDIM   - THE DIMENSION OF VECTORS Y AND Z.
C
C        REMARKS
C           NO ACTION IN CASE NDIM LESS THAN 3.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           BEGINNING WITH Z(1)=0, EVALUATION OF VECTOR Z IS DONE BY
C           MEANS OF SIMPSONS RULE TOGETHER WITH NEWTONS 3/8 RULE OR A
C           COMBINATION OF THESE TWO RULES. TRUNCATION ERROR IS OF
C           ORDER H**5 (I.E. FOURTH ORDER METHOD). ONLY IN CASE NDIM=3
C           TRUNCATION ERROR OF Z(2) IS OF ORDER H**4.
C           FOR REFERENCE, SEE
C           (1) F.B.HILDEBRAND, INTRODUCTION TO NUMERICAL ANALYSIS,
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C                MCGRAW-HILL, NEW YORK/TORONTO/LONDON, 1956, PP.71-76.
C            (2) R.ZURMUEHL, PRAKTISCHE MATHEMATIK FUER INGENIEURE UND
C                PHYSIKER, SPRINGER, BERLIN/GOETTINGEN/HEIDELBERG, 1963,
C                PP.214-221.
C
C     ..................................................................
C
      SUBROUTINE QSF(H,Y,Z,NDIM)
C
C
      DIMENSION Y(1),Z(1)
C
      HT=.3333333*H
      IF(NDIM-5)7,8,1
C
C     NDIM IS GREATER THAN 5. PREPARATIONS OF INTEGRATION LOOP
    1 SUM1=Y(2)+Y(2)
      SUM1=SUM1+SUM1
      SUM1=HT*(Y(1)+SUM1+Y(3))
      AUX1=Y(4)+Y(4)
      AUX1=AUX1+AUX1
      AUX1=SUM1+HT*(Y(3)+AUX1+Y(5))
      AUX2=HT*(Y(1)+3.875*(Y(2)+Y(5))+2.625*(Y(3)+Y(4))+Y(6))
      SUM2=Y(5)+Y(5)
      SUM2=SUM2+SUM2
      SUM2=AUX2-HT*(Y(4)+SUM2+Y(6))
      Z(1)=0.
      AUX=Y(3)+Y(3)
      AUX=AUX+AUX
      Z(2)=SUM2-HT*(Y(2)+AUX+Y(4))
      Z(3)=SUM1
      Z(4)=SUM2
      IF(NDIM-6)5,5,2
C
C     INTEGRATION LOOP
    2 DO 4 I=7,NDIM,2
      SUM1=AUX1
      SUM2=AUX2
      AUX1=Y(I-1)+Y(I-1)
      AUX1=AUX1+AUX1
      AUX1=SUM1+HT*(Y(I-2)+AUX1+Y(I))
      Z(I-2)=SUM1
      IF(I-NDIM)3,6,6
    3 AUX2=Y(I)+Y(I)
      AUX2=AUX2+AUX2
      AUX2=SUM2+HT*(Y(I-1)+AUX2+Y(I+1))
    4 Z(I-1)=SUM2
    5 Z(NDIM-1)=AUX1
      Z(NDIM)=AUX2
      RETURN
    6 Z(NDIM-1)=SUM2
      Z(NDIM)=AUX1
      RETURN
C     END OF INTEGRATION LOOP
C
    7 IF(NDIM-3)12,11,8
C
C     NDIM IS EQUAL TO 4 OR 5
    8 SUM2=1.125*HT*(Y(1)+Y(2)+Y(2)+Y(2)+Y(3)+Y(3)+Y(3)+Y(4))
      SUM1=Y(2)+Y(2)
      SUM1=SUM1+SUM1
      SUM1=HT*(Y(1)+SUM1+Y(3))
      Z(1)=0.
      AUX1=Y(3)+Y(3)
      AUX1=AUX1+AUX1
      Z(2)=SUM2-HT*(Y(2)+AUX1+Y(4))
      IF(NDIM-5)10,9,9
    9 AUX1=Y(4)+Y(4)
```

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