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HYPOTHESIS TESTING AND STATE ESTIMATION FOR DISCRETE SYSTEMS WITH FINITE-VALUED SWITCHING PARAMETERS*

Interim rept.

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In this paper, we consider the problem of state estimation for discrete systems with parameters which may be switching within a finite set of values. In the general case it is shown that the optimal estimator requires a bank of elemental estimators with its number growing exponentially with time. For the Markov parameter case, it is found that the optimal estimator requires only N^2 elemental estimators where N is the number of possible parameter values.

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ABSTRACT

In this paper, we consider the problem of state estimation for discrete systems with parameters which may be switching within a finite set of values. In the general case it is shown that the optimal estimator requires a bank of elemental estimators with its number growing exponentially with time. For the Markov parameter case, it is found that the optimal estimator requires only N^2 elemental estimators where N is the number of possible parameter values.

1. Introduction

In most practical applications of recursive estimation theory, there are difficulties in obtaining an exact mathematical model of the physical dynamic process. The uncertain parts of the system are sometime represented by an unknown parameter vector. When the state estimation for this type of system has to be carried out, the variations of these parameters and their identification play a critical role.

Many approaches have been proposed in attempting to perform state estimation together with parameter identification. There is one class of adaptive estimation scheme which calls for the construction of a bank of elemental estimators with each matched to a possible parameter value [1] - [7]. The optimal state estimate is obtained via a weighted sum over the elemental estimates with the a posteriori hypothesis probabilities as weighting factors. Based upon this idea, algorithms for both adaptive estimation and control have been designed. These algorithms are optimum if 1) one of the assumed model matches with the physical process and 2) the unknown parameter is a constant vector. Various arbitrary modifications have been suggested to alleviate the above restrictions.

The purpose of this paper is to present the extension of the above adaptive estimation scheme to the case when the parameter vector is assumed to vary within a finite number of values. For a general parameter process when the present parameter value can depend on the past history of the parameter values, the optimum state estimate is found to be the weighted sum of elemental estimates obtained by estimators matched to all possible parameter histories. The weighting factor is the a posteriori hypothesis probability determined by 1) residual processes of all elemental estimators and 2) the conditional probability characterizing properties of the parameter process. Since each elemental estimator has to match to a given parameter history and the number of possible histories grows with time, the optimal estimator requires exponentially growing memory. When the parameter follows a Markov process, i.e., the present parameter value depends only on the previous parameter value, this great computational and memory requirement is relaxed. It is shown that the required number of elemental estimator stays at N^2 for the Markov parameter case where N is the number of possible parameter values. Since the estimator for the Markov parameter allows the parameter to change with time and its computational requirement is quite modest (when compared with the exponentially growing memory requirement), it may be used as a feasible suboptimal estimator design for the general time varying parameter estimation problem.

There are a number of potential application areas of the above proposed scheme. For example, the constant parameter multiple elemental estimator algorithm was applied in the F-8C aircraft real-time control problem [6]. In that application, the aerodynamic constants were discretized into several sets of values. A bank of Kalman filters were constructed to match each set of parameter values. The switching parameter algorithm discussed here will allow a more rapid switching among different sets of parameters when the true aerodynamic constants have indeed changed. Another potential application of this scheme is in the Re-entry Vehicle (RV) tracking area [8]-[10]. A RV reenters the atmosphere following a ballistic trajectory. It may perform aerodynamic maneuvers at a medium altitude region and return to ballistic at low altitude region. A detection scheme was suggested in [8] and [9] allowing the detection of a RV maneuver. This scheme suffers from detection delay and filter transients in the switch-over region. In addition, it cannot identify the change to a trajectory from maneuver to ballistic. Another solution is to construct two filters, a ballistic filter and a maneuvering filter. If one chooses the constant parameter algorithm, then a large bias error will develop in the ballistic filter when the RV initiates a maneuver. It therefore cannot identify the maneuver quench. The proposed algorithm allowing transition probabilities to exist between ballistic and maneuvering filters thus allow the rapid identification of a maneuvering as well as a ballistic trajectory.

The optimal estimator for continuous systems with continuously time-varying parameters was considered before, e.g. see References [2]-[4]. A representation theorem was given in [2] and [3] and the differential equation for the a posteriori hypothesis probability density function was given in [4]. It is, however, very difficult to derive the algorithm for the discrete case by directly discretizing the results of [2]-[4]. The derivation used in this paper is that of Bayesian approach [11]. Similar derivations were used in [5] and [7] where the adaptive control problem was discussed in [5] and a tutorial treatment of the multiple elemental estimator approach to adaptive estimation was presented in [7].

This paper is organised as follows. The state estimation problem considered in this paper is stated in Section 2. The optimal state estimators for 1) a general parameter process and 2) a Markov parameter process are presented in Section 3. The time evolution of the parameter process is characterized by a set of hypothesis processes which is also discussed in the Section 3. The implementation of the optimal state estimator depends upon the elemental estimators. This subject is briefly discussed in Section 4. Derivations of the estimation algorithms are given in the Appendix.

2. Problem Statement

Consider the following discrete system and measurement equations.

$$\underline{x}(t+1) = \underline{f}(\underline{x}(t), \underline{\gamma}(t), \underline{u}(t), \underline{\xi}(t)) \tag{2.1}$$

$$\underline{z}(t+1) = \underline{h}(\underline{x}(t+1), \underline{\theta}(t+1)) \tag{2.2}$$

where $\underline{x}(t)$ is the state vector

$\underline{u}(t)$ is the known control vector

$\underline{z}(t)$ is the measurement vector

$\underline{\xi}(t)$ is the system noise vector

$\underline{\theta}(t)$ is the measurement noise vector

$\underline{\gamma}(t)$ is the random, unknown, and time-varying parameter vector

t is the discrete time index

The parameter vector is restricted to take values from a finite set. Define a finite-dimensional parameter vector set R_q containing N distinct parameter vectors, i.e.,

$$R_q = \{\underline{\gamma} : \underline{\gamma} = \underline{\gamma}_i ; \quad i = 1, \ldots, N\} \tag{2.3}$$

The parameter may switch to any parameters in R_q at the next discrete time, i.e.,

$$\underline{\gamma} = \underline{\gamma}_i \quad \text{at} \quad t \quad \text{for} \quad \underline{\gamma}_i \in R_q$$

and

$$\underline{\gamma} = \underline{\gamma}_j \quad \text{at} \quad \tau \quad \text{for} \quad \underline{\gamma}_j \in R_q$$

when $t \neq \tau$, i may or may not be equal to j. Due to this property, $\underline{\gamma}$ is called a stochastic jump parameter. A similar parameter process was considered earlier in a control design problem [12].

Given the problem defined above, we would like to find the minimum variance estimator (true conditional mean) of the state

$$\hat{\underline{x}}(t/t) = E[\underline{x}(t)/Z(t)] \tag{2.4}$$

and the conditional covariance matrix

$$\underline{\Sigma}(t/t) = \text{cov}[\underline{x}(t) ; \underline{x}(t)/Z(t)] \tag{2.5}$$

where Z(t) is the set of all past measurements and controls, i.e.,

$$Z(t) = \{\underline{u}(0), \underline{u}(1), \ldots, \underline{u}(t-1), \underline{z}(1), \ldots, \underline{z}(t)\} \tag{2.6}$$

This is clearly a discrete nonlinear estimation problem. With the parameter vector defined above, it can be shown that the optimal estimator has a fixed structure. That is, the optimal estimate is a weighted sum of elemental estimates with each matched to a possible parameter history. Furthermore, when the system becomes linear even though the parameter process is still related to the state vector nonlinearly, the optimal estimates can be computed for a finite discrete time t.

3. Problem Solution

In this section, we present solutions to the described estimation problem for 1) a general parameter process, and 2) a Markov parameter process. In order to mathematically characterize the time evolution of the parameter process, we first define a set of hypothesis processes in the following subsection.

3.1 Hypothesis Processes

The parameter vector may switch to any vectors in R_q at the next discrete time, the number of parameter permutation therefore grows exponentially. A set of indices is defined to represent the time evoluation of the parameter permutation.

1) I is an index set denoting

$$I = \{ i : i = 1, \ldots, N \} \tag{3.1}$$

2) I_t is a t-tuple index set, i.e.,

$$I_t = \{ i : i = i_t \times i_{t-1} \times \ldots \times i_1 ; i_t, \ldots, i_1 \in I \} \tag{3.2}$$

3) $I_{t+1} = i_{t+1} \times I_t$ for $i_{t+1} \in I$

Notice that I is the index for parameter vectors in R_q and I_t is the index for the permutation of the parameter vector from the initial time to time t. In the following, two hypothesis processes representing the parameter time history are defined.

1) $H_i(t)$, $i \in I$: the hypothesis that $\underline{\gamma} = \underline{\gamma}_i$ at t. It is a "local" hypothesis since it only concerns the status of $\underline{\gamma}$ at t.

2) $\overline{H_i}(t)$, $i \in I_t$: the hypothesis that a given history of $\underline{\gamma}$ for up to time t is true. This history is indexed by a $i \in I_t$. It is a "global" hypothesis since it concerns the time history of $\underline{\gamma}$. For all $i \in I_t$, it defines all possible $\underline{\gamma}$ sequences for up to time t.

Clearly, the global hypothesis describes the time evolution of the parameter process. It is specified by local hypotheses at each discrete time instance, i.e.,

$$\overline{H_i}(t) = (H_{i_t}(t) , H_{i_{t-1}}(t-1) , \ldots , H_{i_1}(1)) \qquad (3.3)$$

where $i \in I_t$

$i_t , \ldots , i_1 \in I$

i is determined by a given sequence of $i_t , \ldots , i_1$

In the discussions to follow, we will frequently refer to the <u>a posteriori hypothesis probability</u>. It is the probability of a given hypothesis being true conditioned upon the past measurements, i.e.,

$$P_i(t) = \text{Prob (the i-th hypothesis is true / Z(t))} \qquad (3.4)$$

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1) For state estimate and covariance

$$\hat{\underline{x}}(t/t) = \sum_{i \in I_t} P_i(t) \, \hat{\underline{x}}_i(t/t) \tag{3.7}$$

$$\underline{\Sigma}(t/t) = \sum_{i \in I_t} P_i(t) \left[\underline{\Sigma}_i(t/t) + (\hat{\underline{x}}_i(t/t) - \hat{\underline{x}}(t/t)) \, (\hat{\underline{x}}_i(t/t) - \hat{\underline{x}}(t/t))^T \right] \tag{3.8}$$

where

$$\hat{\underline{x}}_i(t/t) = E\left[\underline{x}(t) \, / \, Z(t), \overline{H}_i(t) \right]$$

$$\underline{\Sigma}_i(t/t) = \text{cov}\left[\underline{x}(t); \underline{x}(t) \, / \, Z(t), \overline{H}_i(t) \right]$$

2) For the a posteriori hypothesis probability

$$P_j(t+1) = \frac{p(\underline{z}(t+1) \, / \, H_i(t+1), \overline{H}_k(t), Z(t))}{p(\underline{z}(t+1) \, / \, Z(t))} \cdot P(H_i(t+1) \, / \, \overline{H}_k(t), Z(t)) \, P_k(t) \tag{3.9}$$

where

1) $i \in I$, $k \in I_t$, and $j = i \times k$

2) $P(H_i(t+1) \, / \, \overline{H}_k(t), Z(t)) = \text{Prob}\,(\underline{Y}(t+1) = \underline{Y}_i \, / \, \overline{H}_k(t), Z(t))$

We make the following remarks:

1) The elemental estimate, $\hat{\underline{x}}_i(t/t)$, is obtained by constructing a minimum variance estimator based upon systems and measurement equations defined by (2.1) - (2.2) with the parameter process matched to the i-th global hypothesis, $\overline{H}_i(t)$.

2) The elemental estimates, $\hat{\underline{x}}_i(t/t)$, and covariances, $\underline{\Sigma}_i(t/t)$, are combined to obtain the optimal estimate $\hat{\underline{x}}(t/t)$ and covariance $\underline{\Sigma}(t/t)$ by using a weighted sum. The i-th weighting factor, $P_i(t)$, is the a posteriori hypothesis probability of the i-th global hypothesis being true conditioned upon all the past measurements and controls.

3) The above equations are independent of whether the system is linear or nonlinear. When the system is nonlinear, the elemental estimates can, of course, only be obtained approximately. The problem of realization will be discussed in the next section.

4) The probability density functions of (3.9) are computed by using residual processes of elemental estimators. That is

a) $p(\underline{z}(t+1) / H_i(t+1), \overline{H}_k(t), Z(t))$

= the density of the residual process of the elemental estimator matching to $\overline{H}_k(t)$ and $H_i(t+1)$.

b) $p(\underline{z}(t+1) / Z(t))$

$$= \sum_{m \in I_t} p(\underline{z}(t+1) / \overline{H}_m(t), Z(t)) P_m(t)$$

$$= \sum_{m \in I_t} \left[\sum_{n \in I} p(\underline{z}(t+1) / \overline{H}_m(t), H_n(t+1), Z(t)) \cdot P(H_n(t+1) / \overline{H}_m(t), Z(t)) \right] P_m(t) \qquad (3.10)$$

5) The conditional hypothesis probability, $P(H_i(t+1)/\overline{H}_k(t), Z(t))$, represents the property of the hypothesis process. It characterizes the evolution of $\underline{\gamma}$ in R_q. Notice that we do not restrict the behavior of $\underline{\gamma}$ variations. In the most uncertain situation such that the parameter may change to any parameters in R_q with equal probability one obtains

$$P(H_i(t+1) / \overline{H}_k(t), Z(t)) = \frac{1}{N} \quad \text{for all} \quad i \in I \tag{3.11}$$

If the parameter is a Markov process, it becomes the transition probability

$$P(H_i(t+1) / \overline{H}_k(t), Z(t))$$

$$= P(H_i(t+1) / H_k(t)) \qquad \text{for} \quad i, k \in I \tag{3.12}$$

6) Clearly, the difficulty of implementing this estimator lies in the fact that the number of elemental estimators grows exponentially with time. The time evolution of elemental estimators for $N = 2$ is shown in Fig. 1. Notice that at time t, the number of elemental estimators grows to 2^t.

This difficulty is however, alleviated when the parameter follows a Markov process. It can be shown that for a Markov parameter process, the number of elemental estimator is limited to N^2. This case is discussed in the next section.

3.3 The Markov Parameter Case

In presenting the algorithm for the Markov parameter case, we use the following simplified notations:

$$\hat{\underline{x}}_i(t/t) = E[\underline{x}(t) / H_i(t), Z(t)]$$

$$\underline{\Sigma}_i(t/t) = \text{cov}[\underline{x}(t); \underline{x}(t) / H_i(t), Z(t)]$$

$\hat{\underline{x}}_{ij}(t/t-1) = E[\,\underline{x}(t)/H_i(t),\ H_j(t-1),\ Z(t-1)\,]$ = the predicted state

$\hat{\underline{x}}_{ij}(t/t) = E[\,\underline{x}(t)/H_i(t),\ H_j(t-1),\ Z(t)\,]$ = the updated state

$P_i(t)$ = Prob ($H_i(t)$ is true / $Z(t)$)

P_{ij} = Prob ($H_i(t)$ is true / $H_j(t-1)$)

= the transition probability

$p_{ij}(t) = p(\underline{z}(t)\ /\ H_i(t),\ H_j(t-1),\ Z(t-1)\ )$

= the residual density of the (i, j)-th elemental estimator

We now proceed to state the estimator. The derivation is included in the Appendix.

1) For state estimate and covariance calculations:

1.1) At time t-1, we have

$\hat{\underline{x}}_i(t-1\ /\ t-1)$, $\underline{\Sigma}_i(t-1\ /\ t-1)$, $P_i(t-1)$; for i = 1, ..., N

1.2) Compute

$\hat{\underline{x}}_{ij}(t/t-1)$, $\underline{\Sigma}_{ij}(t/t-1)$; for i, j = 1, ..., N

using N^2 state and covariance predictors.

1.3) Compute

$\hat{\underline{x}}_{ij}(t/t)$, $\underline{\Sigma}_{ij}(t/t)$; for i, j = 1, ..., N

using N^2 state and covariance updators and the new measurement $\underline{z}(t)$.

1.4) Compute $\hat{\underline{x}}_i(t/t)$ and $\underline{\Sigma}_i(t/t)$ for i=1, ..., N using

$$\hat{\underline{x}}_i(t/t) = \sum_{j=1}^{N} \hat{\underline{x}}_{ij}(t/t)\, P\,(H_j(t-1)/H_i(t),\ Z(t)\,) \qquad (3.13)$$

$$\underline{\Sigma}_i(t/t) = \sum_{j=1}^{N} P\,(H_j(t-1)\,/\,H_i(t),\ Z(t)\,) \qquad (3.14)$$

$$\cdot\ [\,\underline{\Sigma}_{ij}(t/t) + (\hat{\underline{x}}_{ij}(t/t) - \hat{\underline{x}}_i(t/t)\,)\,(\hat{\underline{x}}_{ij}(t/t) - \hat{\underline{x}}_i(t/t)\,)^T\,]$$

where

$$P(H_j(t-1)/H_i(t),\ Z(t)\,) = \frac{p_{ij}(t)\,P_{ij}\,P_j(t-1)}{\sum_{j=1}^{N} p_{ij}(t)\,P_{ij}\,P_j(t-1)} \qquad (3.15)$$

1.5) The minimum variance (true conditional mean) state estimate $\hat{\underline{x}}\,(t/t)$ and covariance $\Sigma\,(t/t)$ are obtained by

$$\hat{\underline{x}}\,(t/t) = \sum_{i=1}^{N} P_i(t)\,\hat{\underline{x}}_i(t/t) \qquad (3.16)$$

$$\underline{\Sigma}\,(t/t) = \sum_{i=1}^{N} P_i(t)\,[\,\underline{\Sigma}_i(t/t) + (\,\hat{\underline{x}}_i(t/t) - \hat{\underline{x}}\,(t/t)\,)\,(\hat{\underline{x}}_i(t/t) - \hat{\underline{x}}\,(t/t)\,)^T\,] \qquad (3.17)$$

The above completes one cycle of the state estimate and covariance computation.

2) For the a posteriori hypothesis probability calculations

$$P_i(t) = \frac{\sum_{j=1}^{N} p_{ij}(t)\, P_{ij}\, P_j(t-1)}{\sum_{i=1}^{N} \sum_{j=1}^{N} p_{ij}(t)\, P_{ij}\, P_j(t-1)} \qquad (3.18)$$

We make the following remarks:

1) The above algorithm requires implementing N^2 elemental estimators. If the elemental estimator is a minimum variance estimator, then the state estimate $\hat{\underline{x}}(t/t)$ is optimum in the minimum variance sense.

2) Equation (3.18) may be re-written in a matrix form, let

$\underline{q}(t)$ be an Nx1 column vector consisting of all a posteriori local hypothesis probabilities at time t

$$\underline{q}(t) = \begin{bmatrix} p(H_1(t) / Z(t)) \\ \vdots \\ p(H_N(t) / Z(t)) \end{bmatrix}$$

and let

S(t+1/t) be an (NxN) matrix with the (i, j)-th element s_{ij} being equal to the product of the residual density of the elemental estimator matched to $H_i(t+1)$ and $H_j(t)$ and the probability of $H_i(t+1)$ being true conditional upon $H_j(t)$.

$$s_{ij}(t) = [\, p\,(\underline{z}(t+1) / H_i(t+1),\ H_j(t),\ Z(t))\ P(H_i(t+1) / H_j(t))\,]$$

Then

$$\underline{q}(t+1) = \frac{1}{p(\underline{z}(t+1) \;/\; Z(t))}\ S\,(t+1 \;/\; t)\ \underline{q}(t) \tag{3.19}$$

The time evolution of the hypothesis process for a Markov parameter process is illustrated in Fig. 2 for N = 2. A detailed flow chart illustrating the state estimation process for N = 2 is shown in Fig. 3.

A special case of the above is when the parameter is known to be time invariant. Under this condition, the above estimator is reduced to the estimator derived by Magill [1]. This can be obtained by substituting

$$P\,(H_i(t) / H_j(t-1)) = \begin{cases} 1 & \text{when } i = j \\ 0 & \text{otherwise} \end{cases} \tag{3.20}$$

into the estimator equations.

4. Elemental Estimator Implementation

The significance of the estimator presented in the previous section is that the estimator structure is the same for both linear and nonlinear systems. This conclusion was also found in the continuous case [2] - [4]. The estimator realization therefore depends on: 1) the realization of the elemental estimator and 2) the practicality of constructing a large number of elemental estimators. Three cases can be considered.

a) General Case

For the system and measurement equations considered in equations (2.1) - (2.2), $\hat{\underline{x}}_i(t/t)$ and $\Sigma_i(t/t)$ cannot be computed exactly. Suboptimal nonlinear filters such as the extended Kalman filter [13] or the second order filter [14] must be used. These filters have found many practical applications. They can be applied to approximate the optimal estimators.

b) Linear System Case

When the system is linear and noise vectors are additive, the optimal estimate can be computed exactly. In this case, the elemental estimator is constructed by using the discrete Kalman filter. In the general case, the number of Kalman filters grows exponentially with time. For the Markov parameter case, the number of Kalman filters is N^2.

c) Linear System and Constant Parameter Case

This is the case considered by Magill [1]. The estimator presented in this paper can be reduced to the Magill's estimator by properly setting the conditional probabilities.

5. Discussion and Conclusions

In this paper, we have presented the optimal estimator for discrete systems with finite-valued switching parameters. For a general parameter process in which the present parameter value can depend on the past history of the parameter values, the memory requirement of the estimator grows exponentially with time. When the present parameter value depends only on the previous parameter value and this dependence can be characterized by the transition probability, the required number of elemental estimator is found to be equal to N^2 where N is the number of possible parameter vectors. With this modest memory requirement, this algorithm is felt to be feasible for suboptimal estimator design for the general time-varying parameter estimation problem.

APPENDIX DERIVATIONS

A.1 Derivation of Equations (3.7) and (3.8)

This derivation involves a straightforward application of the definitions of $\hat{\underline{x}}(t/t)$ and $\underline{\Sigma}(t/t)$ and

$$p(\underline{x}(t) / Z(t)) = \sum_{i \in I_t} P_i(t)\, p(\underline{x}(t) / \bar{H}_i(t), Z(t)) \tag{A.1}$$

The details are omitted here.

A.2 Derivation of Equation (3.9)

Using the conditional probability relation yields

$$P_j(t+1) = P(H_i(t+1) / \bar{H}_k(t), Z(t+1))\, P(\bar{H}_k(t) / Z(t+1)) \tag{A.2}$$

where $i \in I$ and $k \in I_t$.

Using the Bayes' rule one obtains

$$P(H_i(t+1) / \bar{H}_k(t), Z(t+1))$$

$$= \frac{p(\underline{z}(t+1) / H_i(t+1), \bar{H}_k(t), Z(t))}{p(\underline{z}(t+1) / \bar{H}_k(t), Z(t))} P(H_i(t+1) / \bar{H}_k(t), Z(t)) \tag{A.3}$$

and

$$P(\bar{H}_k(t) / Z(t+1))$$

$$= \frac{p(\underline{z}(t+1) / \bar{H}_k(t), Z(t))}{p(\underline{z}(t+1) / Z(t))} P(\bar{H}_k / Z(t)) \tag{A.4}$$

Substituting (A.3) and (A.4) into (A.2) one obtains (3.9). This completes the derivation.

A.3 Derivation of Equations (3.13), (3.14), and (3.15)

Using the definitions of $\hat{\underline{x}}_i(t/t)$ and $\underline{\Sigma}_i(t/t)$, i.e.,

$$\hat{\underline{x}}_i(t/t) = \int \underline{x}(t)\, P(\underline{x}(t) / H_i(t),\ Z(t))\, d\underline{x} \tag{A.5}$$

$$\underline{\Sigma}_i(t/t) = \int (\underline{x}(t) - \hat{\underline{x}}_i(t/t))\, (\underline{x}(t) - \hat{\underline{x}}_i(t/t))^T\, P(\underline{x}(t) / H_i(t),\ Z(t))\, d\underline{x} \tag{A.6}$$

and the relation

$$p(\underline{x}(t) / H_i(t),\ Z(t))$$

$$= \sum_{j=1}^{N} p(\underline{x}(t) / H_i(t),\ H_j(t-1),\ Z(t))\, P(H_j(t-1) / H_i(t),\ Z(t)) \tag{A.7}$$

equations (3.13) and (3.14) follow immediately. It remains to show equation (3.15). Using the conditional probability and the Bayes' rule yield

$$P(H_j(t-1) / H_i(t),\ Z(t))$$

$$= \frac{P(H_j(t-1),\ H_i(t) / Z(t))}{P_i(t)} \tag{A.8}$$

and

$$P(H_j(t-1),\ H_i(t) / Z(t))$$

$$= \frac{p(\underline{z}(t) / H_i(t),\ H_j(t-1),\ Z(t-1))}{p(\underline{z}(t) / Z(t-1))}\, P(H_i(t) / H_j(t-1))\, P_j(t-1) \tag{A.9}$$

Notice that $p(\underline{z}(t) / Z(t-1))$ is computed by using

$$p(\underline{z}(t)/Z(t-1)) = \sum_{i=1}^{N} \sum_{j=1}^{N} p(\underline{z}(t)/H_i(t), H_j(t-1), Z(t-1))P(H_i(t)/H_j(t-1))P_j(t-1) \tag{A.10}$$

Substituting (A.9) and (A.10) into (A.8) and using (3.18) one obtains (3.15). This completes the derivation.

A.4 Derivation of Equations (3.16) and (3.17)

This derivation involves a straightforward application of the definitions of $\hat{\underline{x}}(t/t)$ and $\Sigma(t/t)$ and

$$p(\underline{x}(t) / Z(t)) = \sum_{i=1}^{N} P_i(t) P(\underline{x}(t) / H_i(t), Z(t)) \tag{A.11}$$

The details are omitted here.

A.5 Derivations of Equation (3.18)

Using the conditional probability yields

$$P(H_i(t) / Z(t))$$

$$= \sum_{j=1}^{N} P(H_i(t) / H_j(t-1), Z(t)) P(H_j(t-1) / Z(t)) \tag{A.12}$$

Using the Bayes' rule one obtains

$$P(H_i(t) / H_j(t-1), Z(t))$$

$$= \frac{p(\underline{z}(t) / H_i(t), H_j(t-1), Z(t-1))}{p(\underline{z}(t) / H_j(t-1), Z(t-1))} P(H_i(t) / H_j(t-1), Z(t-1)) \tag{A.13}$$

and

$$P(H_j(t-1) / Z(t))$$

$$= \frac{p(\underline{z}(t) / H_j(t-1),\ Z(t-1))}{p(\underline{z}(t) / Z(t-1))} P(H_j(t-1) / Z(t-1)) \qquad (A.14)$$

Substituting (A. 13) and (A. 14) into (A. 13) and using (A. 10) one obtains (3. 18). This completes the derivation.

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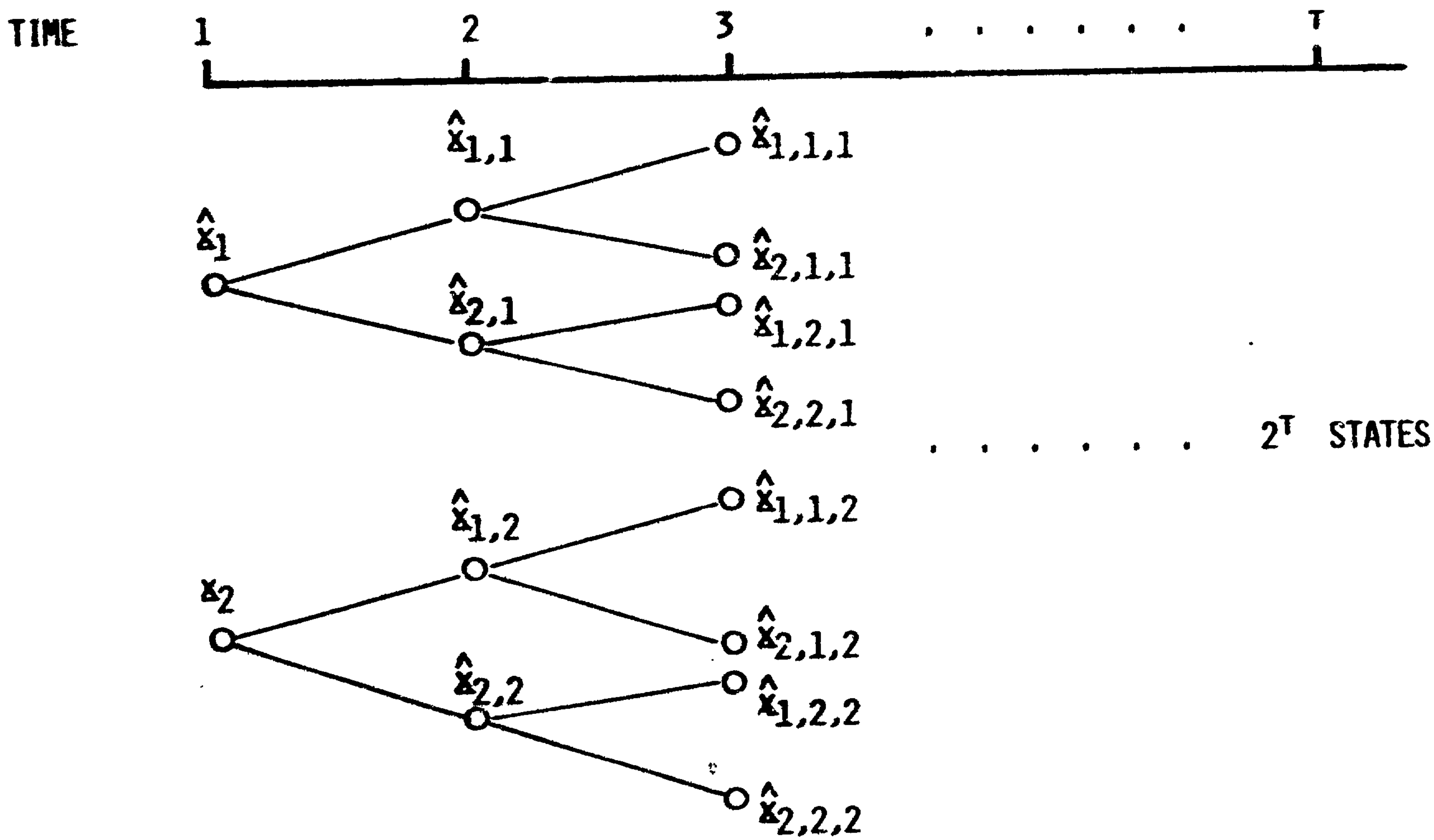


Figure 1 Time Evolution of Elemental Estimates for N=2

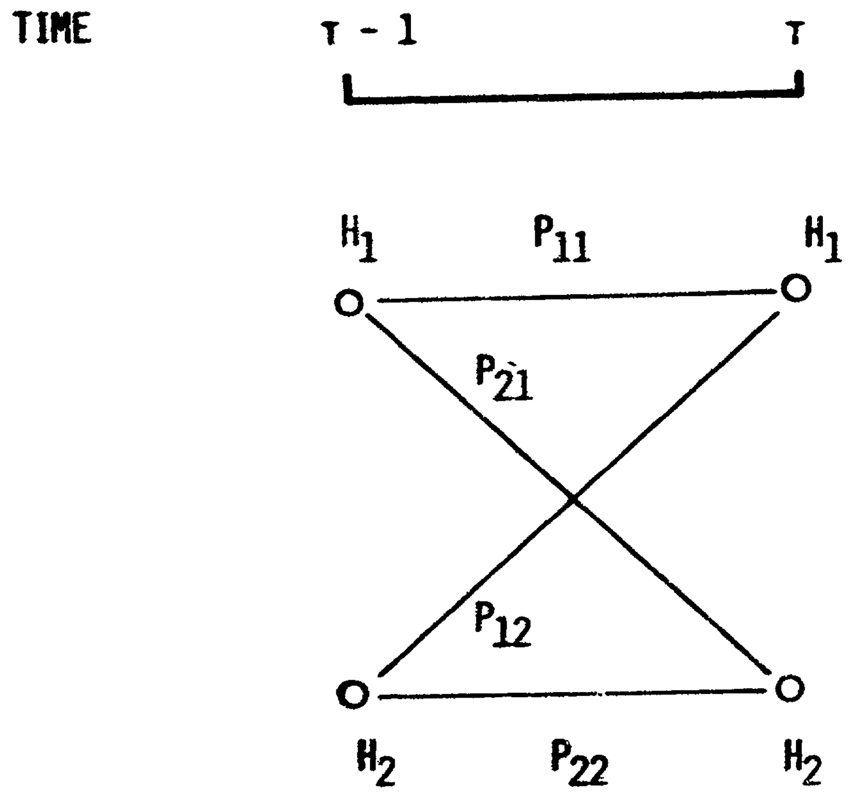
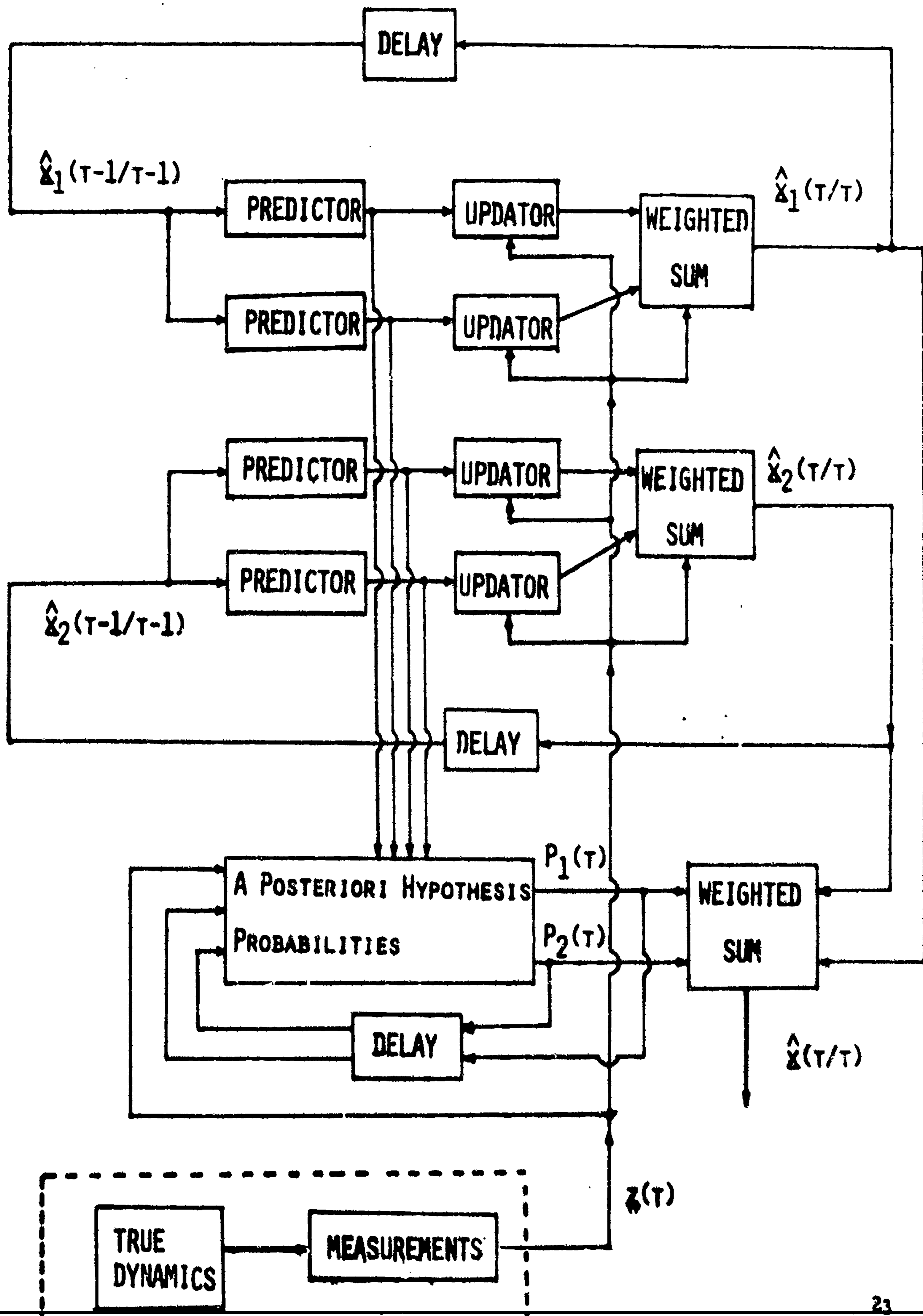


FIGURE 2 TIME EVOLUTION OF HYPOTHESIS PROCESSES FOR MARKOV PARAMETERS WITH N=2

FIGURE 3 STATE ESTIMATOR FOR MARKOV PARAMETERS WITH N=2



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