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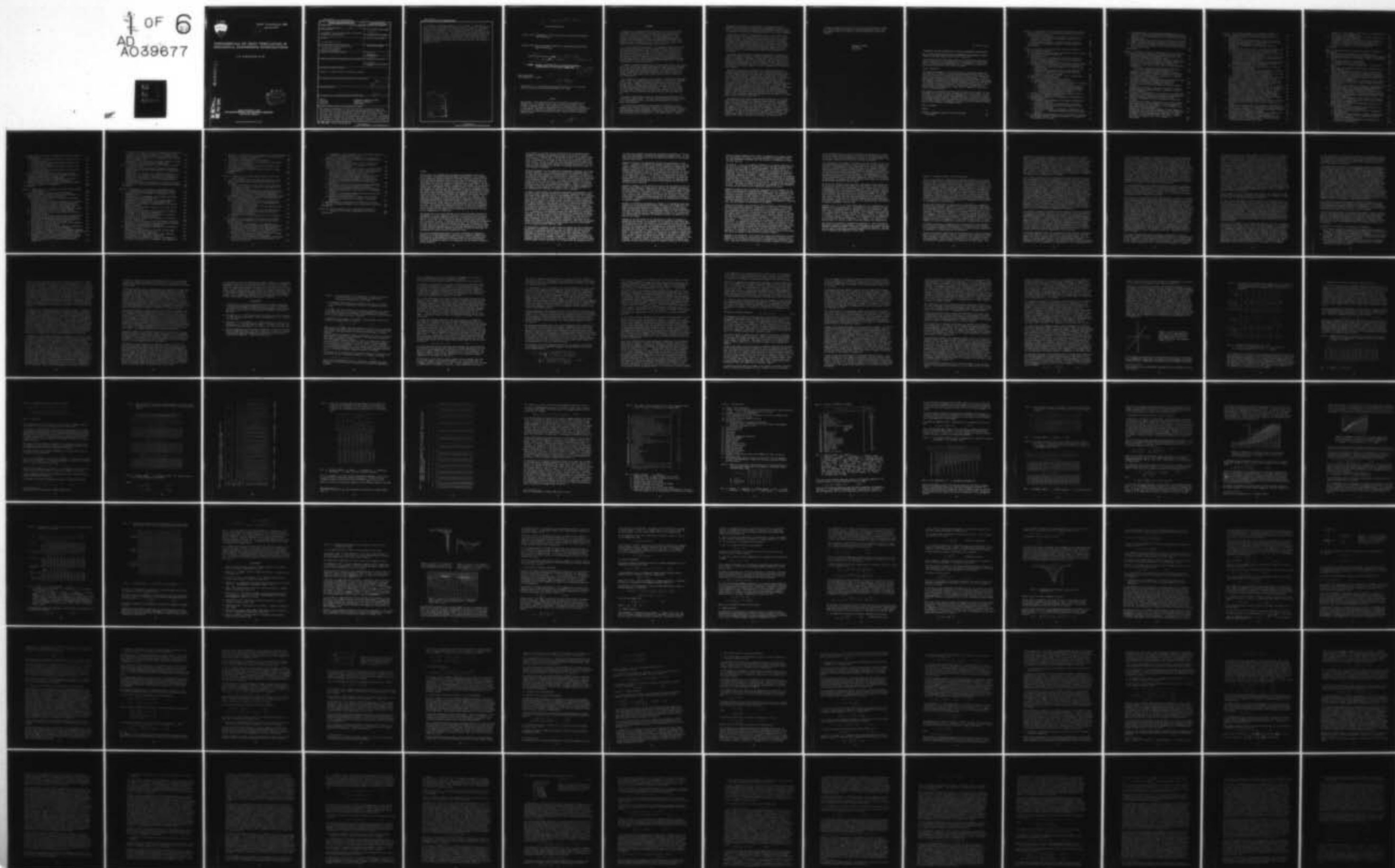

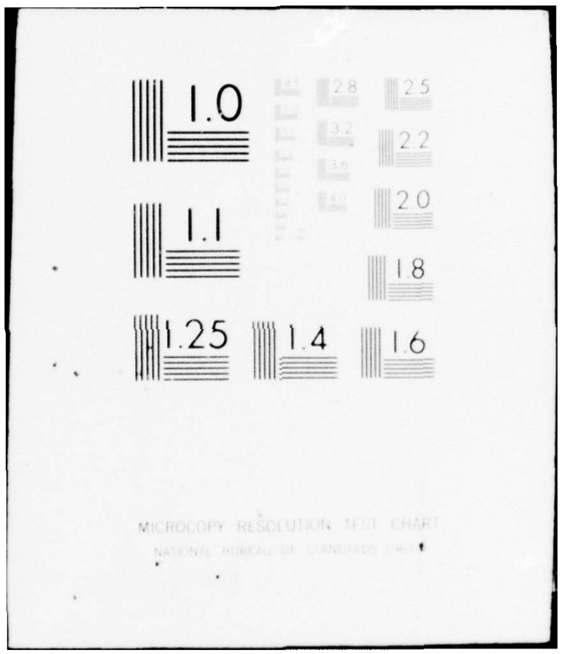
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V.A. Kudryavtsev et al

CORPS OF ENGINEERS, U.S. ARMY
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The textbook "Fundamentals of Frost Forecasting in Geological Engineering Investigations" in regions of seasonally and permanently frozen rocks is the first and still the only contemporary textbook in the Soviet and foreign literature which embraces the main questions of frost forecasting. In it the methodological, mathematical and thermodynamic principles of frost forecasting are examined and methods and calculation procedures are given for determining the influence of various factors and the productive activity of man on frost

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FOREWORD

A majority of the English-speaking readers of this book realize that information about perennially frozen ground has been accumulating in the U.S.S.R. for over a century and that half the territory of the U.S.S.R. is underlain by permafrost. Teaching permafrost is a long tradition in the geology, geography, and engineering departments of several Soviet universities. For these reasons Russian scientists and engineers enjoy a leading position in a field they call, roughly translated, the science of permafrostology (*merzlotovedenie*).

In the early stages of develoment of the permafrost regions the builder and the exploring scientist could not coordinate their interests in a way that would allow them to help each other. A series of spectacular structural failures are on record in the engineering literature, and the scientific literature includes a large amount of observational and descriptive opuses of documentary value. The practical engineer and the scientist reached a mutual understanding at a relatively early time, but not without the all too familiar controversy between basic, applied, "fundamental" and theoretical research.

At present, academic studies of snow, ice and frozen ground in the U.S.S.R. are not in conflict with engineering research. Based on first principles, basic research provides applied researchers with the fundamentals needed to improve design and avoid costly failures. The results of basic research, documented in appropriate publications, currently constitute a foundation for engineering research which finally leads to design practice documentation.

Such a system, in a country one half of which is underlain by permafrost, needs a source for preparation and replacement of its scientific manpower in this field. Leading in permafrost and glaciology teaching is the Geology Faculty of Moscow State University. For more than 20 years a teaching chair in Permafrostology has given an advanced course in what is called "Procedures of Frost Investigation"; this book is the result. Its title means, besides frost penetration forecasting, the prediction of changes in frost conditions and state under all influences including man.

The names of Kudriavtsev, Garagulia, Kondratieva and Melamed are very well known to CRREL readers. For years these authors have produced papers on the fundamental aspects of freezing and the properties of frozen ground and permafrost.

The new book under Kuriavtsev's editorship is a textbook of a very advanced type, suitable for advanced students and professionals in cold regions geology, civil engineering, mining, etc. Based on first principles and invariably rigorous in approach, it "puts meat on the bones" of many subjects pursued up to the present in a less rigorous manner. Besides geologists and engineers, such specialists as geographers and geomorphologists will benefit by applying the fundamentals derived in the book. It will also help to eliminate many repetitious "descriptologies" by suggestion of some novel, rigorous approaches not previously attempted.

Examining the translation (performed by the U.S. Joint Publications Research Service), the reader should keep in mind that it is a rough, unedited version. The translator has preserved all the idiosyncrasies of Russian writing, thus creating a series of ambiguities not readily detectable by a monolinguistic reader. Since the circle of interested readers may be rather limited, publication of an edited version of the book is not justifiable at the present stage (and state) of permafrost study in this country. The book, in its present unedited form, will serve the purpose of familiarizing the reader with Kudriavtsev's way of thinking.

The reader will find that the translated manuscript is sometimes very rough on the English language. Phrases containing "of" proliferations such as "...studies of strength of frozen and of thawing soils" will be found systematically. This is the translator's way of avoiding loss of meaning, and is inevitable in first, unedited, versions. The same can be said for pluralizations and shifts in emphasis. Knowledge of the general subject and careful reading will help to eliminate the difficulties with language.

Among the difficulties in translation are expressions with which we have had some difficulty in the past. One of them is the Russian term "gornaia poroda," which by their definition constitutes any natural solid that occurs below the air- or water-layer, excluding soil in the agricultural sense and, perhaps, organic deposits such as peat or lake bottom ooze. Customarily, it is translated as "rock," which may be correct only in the case of lithic outcrops. In general, the translation of such terms as "soil," "rock," "stratum" and "deposit" produces ambiguities resolvable only by considering the term in context. As is well known to most of us, the Russians use two terms for our "permafrost": "vechnaia merzlota," meaning "eternally frozen ground," and "mhogolethemerzlyi grunt," meaning "ground frozen for many years" (we may use "perennially frozen"), which is actually a hair-splitting procedure. Unfortunately there is also a monstrous ambiguity, "merzlota"; we trantlate it as "permafrost," which is in most cases correct.

The translation also reflects some personal inclinations of the translator which would normally be eliminated through editing. For example, there is the term "permafrozen," which does not exist, and "dispersed soils" or "dispersed rocks" for "unconsolidated deposits." The translator speaks of "cycled heat," meaning "heat exchange." More serious is the ambiguous use of "seasonally thawed" and "seasonally frozen." While less harmful is the not-so-customary use of "equation," "expression" and "formula." The use of "volatilizing" of moisture is unfortunate. The English reader should also come to terms with "positive" and "negative" temperatures - which of course are in terms of the Celsius scale.

Although a textbook, the opus will be useful to the professional reader, and we may end with the observation that by examining the book it might be found that some of our current or planned research has already been completed.

GEORGE K. SWINZOW
USA CRREL
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FUNDAMENTALS OF FROST FORECASTING IN GEOLOGICAL ENGINEERING INVESTIGATIONS

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The textbook "Fundamentals of Frost Forecasting in Geological Engineering Investigations" in regions of seasonally and permanently frozen rocks is the first and still the only contemporary textbook in the Soviet and foreign literature which embraces the main questions of frost forecasting. In it the methodological, mathematical and thermodynamic principles of frost forecasting are examined and methods and calculations procedures are given for determining the influence of various factors and the productive activity of man on frost and geological engineering conditions. The procedure of compilation of a frost forecast is illustrated by a large number of examples of calculations based on concrete material.

The textbook is intended for the study and teaching of the procedure of frost investigations and the procedure of compilation of a frost forecast. At the same time, the problems and questions illuminated in it are of great interest to geologists, hydrogeologists, engineering geologists, builders, road-builders, mining engineers, hydraulic engineers and other specialists participating in the study, exploration, planning, construction and other productive organization of the territory of permafrozen rocks.

Table of Contents

Preface 10

Chapter 1. Principal Aspects of Frost Forecasting 15
  Bibliography 22

Chapter 2. Interconnection of the Geographical, Geological and Thermophysical Conditions of the Formation and Existence of Seasonally and Permanently Frozen Rocks 23
1. The Radiation Heat Balance of the Surface of the Soil as a Condition of the Formation and Dynamics of Seasonally and Permanently Frozen Rocks 23
2. Climatological Calculation of Radiation Balance Components 33
3. Procedure for Calculation of Radiation Balance Components 45
Bibliography 50

Chapter 3. Conductive and Convective Heat Transfer in Rocks and Their Freezing and Thawing 51
1. The Temperature Field of Rocks. Heat Transfer and Heat Cycles 51
2. The Equation of Thermal Conductivity 53
3. Temperature Waves 55
4. Determination of the Configuration of the Frozen Rock Mass and the Temperature Field in It by Solving the Stationary Problem of Thermal Conductivity 59
5. Formulation of the Problem of the Freezing and Thawing of Rocks 62
1) Formulation of the Problem of the Freezing (Thawing) of Homogeneous Soil With the Formation of an Interface (the Stefan Problem) 63
2) Formulation of the Problem of the Freezing (Thawing) of Rocks With Consideration of Unfrozen Water (With the Formation of a Zone of Freezing) 64
6. Brief Survey of Particular Solution of the Stefan Problem 66
7. Solutions of the Stefan Problem 67
1) Solution of the Classical Stefan Problem 67
2) Approximate Solutions of the Stefan Problem 69
A) The Stefan Formula for Determining the Depth of Seasonal and Permanent Freezing (Thawing) of Rocks 69
B) Determination of the Depth of Seasonal and Permanent Freezing (Thawing) of Rocks by the Leybenzon Method 70
C) V. S. Luk'yanov's Formula for Determining the Depth of Seasonal Freezing (Thawing) of Rocks 72
8. Solution of the Problem of Freezing and Thawing of Rocks in the Spectrum of Negative Temperatures 72
9. Investigation of Processes of Freezing With Consideration of Moisture Migration 77
1) Formulation of the Problem of Freezing With Consideration of Moisture Migration 77
2) Generalized Condition of the Stefan Type on a Mobile Boundary During Freezing With Moisture Migration Toward the Front of Freezing 79
3) Heat and Mass Transfer in the Process of Formation of an Ice Layer. Rhythmicity During Freezing With Moisture Migration 83
10. A Very Simple Self-Modeling Solution of the Problem of Freezing With Moisture Migration (Without Consideration of Heaving and the Formation of Ice Layers) 89

11. The Applicability of a Very Simple Self-Modeling Solution of the Problem of Monotonic Freezing With Consideration of Moisture Migration for the Investigation of the Freezing of Moist Finely Dispersed Soils 93
12. Self-Modeling Solution of the Problem of Freezing With Moisture Migration With Consideration of Heaving and Ice Layer Formation 96
13. Method of Solving the Stefan Probelm and the Computer Investigation of Process of Processes of Heat and Mass Transfer During the Freezing of Rocks 108
Bibliography 113

Chapter 4. Approximate Formulas for Determination of Heat Cycles and the Depths of Seasonal and Perennial Freezing (Thawing) of Rocks 115
1. Approximate Formulas for Determination of Heat Cycles and Depths of Seasonally Frozen (Thawed) Layers 115
Calculation of the Depth of Seasonal Thawing (Freezing) in the Case of Equality of the Coefficients of Thermal Conductivity of Rocks in the Frozen and Thawed States (Example 1) 117
Calculation of Heat Cycles in the Layer of Annual Temperature Fluctuations
in an Area of Seasonal Freezing of Rocks (Example 2a) 120
in an Area of Permafrozen Rocks (Example 2b) 121
Calculation of the Depth of Seasonal Freezing (Thawing) at Different Coefficients of Thermal Conductivity of the Rocks in the Frozen and Thawed States. Application of Nomograms for Calculation of ξ (Example 3) 126
2. Approximate Formulas for Determination of Perennial Heat Cycles and Depths of Perennial Freezing (Thawing) of Rocks 133
Calculation of the Depth of Permafrost (Example 4) 136
3. Potential Seasonal Freezing and Potential Seasonal Thawing of Rocks 137
Calculation of the Depths of Potential Seasonal Freezing and Potential Seasonal Thawing of Rocks. Calculation of Intergelosols (Example 5) 138
4. The Nature of the Asymmetry of the Envelopes of the Temperature Fluctuation in Rocks and Its Consideration in Determining the Depths of Seasonal and Perennial Freezing (Thawing) 141
5. The Depth of Propagation of Annual Temperature Fluctuations in Rocks 144
Calculation of the Depth of Propagation of the Annual Temperature Fluctuations (H) in the Region of Seasonal Freezing of Rocks (With and Without Consideration of the Asymmetry of the Temperature Envelopes (Example 6) 146
Calculation of the Depth of Propagation of Annual Temperature Fluctuations (H) in the Region of Permafrozen Rocks (Example 7) 147
Determination of the Average Annual Temperatures of Rocks at the Depth of Annual Zero Amplitudes on the Basis of Single Measurement of Temperature in a Well (Example 8) 149
Bibliography 152

Chapter 5. Forecasting the Change of the Temperature Regime and the Depths of Seasonal Freezing and Seasonal Thawing of Rocks 154
1. Classification of Types of Seasonal Freezing and Seasonal Thawing of Rocks 154
2. Dependence of the Temperature Regime and Depths of Seasonal Freezing and Thawing of Rocks on their Composition, Moisture Content and Thermophysical Characteristics 158
Calculation of the Amount of the Temperature Shift Δt_{λ} (Example 9) 161
3. The Influence of the Snow Cover on the Temperature Regime and the Depth of the Seasonal Freezing and Thawing of Rocks 164
1) The Warming Influence of the Snow Cover With Consideration of Heat Cycles in the Underlying Rocks 165
Calculation of the Warming Influence of the Snow Cover With Consideration of Heat Cycles Passing Through the Surface of the Soil and Snow (Example 10) 168
Calculation of the Warming Influence of the Snow Cover With the Abbreviated Formula of V. A. Kudryavtsev (Example 11) 171
2) The Cooling Influence of the Snow Cover 174
3) Analysis of the Influence of the Snow Cover in Different Freezing Temperature Zones 175
Calculation of the Influence of the Snow Cover on the Depth of Seasonal Freezing of the Ground (Example 12) 176
Calculation of the Influence of the Snow Cover on the Depth of Seasonal Thawing of the Ground (Example 13) 177
4. The Influence of the Plant Cover on the Temperature Regime and the Depth of the Seasonal Freezing and Thawing of Rocks 178
Calculation of the Influence of the Plant Cover on the Formation of the Temperature of the Surface of Rocks (Through the Radiation Heat Balance of the Surface) (Example 14) 181
Calculation of the Influence of the Plant Cover on the Temperature Regime of Rocks as a Layer of Heat Insulation (Example 15) 184
Calculation of the Influence of the Plant Cover on the Depth With Consideration of Latitudinal Zonation (Example 16) 187
5. The Influence of a Water Cover on the Temperature Regime of Bottomset Beds and on Their Seasonal Freezing and Thawing 189
Calculation of the Temperature and Depth of Seasonal Freezing in Bottomset Beds of Shallow Lakes (Example 17) 192
6. Dependence of the Temperature Regime and the Depths of Seasonal Freezing and Thawing of Rocks on the Relief and Exposure of Slopes 194
Calculation of the Influence of the Steepness and Exposure of Slopes on the Formation of the Temperature of Rocks (Example 18) 195
7. The Influence of Swampiness on the Temperature Regime of Rocks and Their Seasonal Freezing and Thawing 197
Calculation of the Influence of Swampiness on the Formation of the Average Annual Temperature of Rocks (Example 19) 198
8. Influence of the Infiltration of Summer Precipitations and Convection of Air on the Temperature of Rocks and the Depth of Their Seasonal Freezing and Thawing 201

Calculation of the Influence of Infiltration of Summer Precipitations on t_ξ and ξ (Example 20) 202
9. The Influence of Latitudinal Zonation and Height Zonation on the Formation of Frost Conditions 205
Calculation of t_ξ and ξ With Consideration of the Influence of Latitudinal Zonation of Natural Factors (Example 21) 207
Analysis of the Summary Influence of the Principal Natural Factors on the Formation of t_ξ and ξ (Example 22) 210
Bibliography 214

Chapter 6. Predicting Change of the Characteristics of Permafrozen Rock Masses 216
1. Principles of the Classification of Permafrozen Rocks 216
2. General Regularities of the Formation of Permafrozen Rock Masses 225
3. Forecasting the Formation of Cryogenic Textures of Permafrozen Rocks 228
1) Cryogenic Textures in Solid Rocks 228
2) Cryogenic Textures of Loose Deposits 229
4. Determination of the Minimal Cryogenic Age of Frozen Rock Masses 235
Calculation of the Minimal Cryogenic Age of Permafrozen Rock Masses (Example 23) 237
5. Influence of the Productive Activity of Man on Change of the Temperature Regime and Thickness of P rmafrozen Rock Masses 238
Approximate Estimate of the Change of Thickness of Permafrozen Rock Masses Under the Influence of Change of the Water Level Under the Frost (Example 24) 243
Calculation of the Thickness of a Thawed Layer of Rocks Separating a Seasonally Frozen Layer from a Permafrozen Rock Mass (Example 25) 245
Calculation of the Thickness of a Permafrost Layer (Example 26) 247
6. Forecasting Change of the Temperature Regime of Permafrozen Rocks in the Layer of Annual Temperature Fluctuations 248
Calculation of Temperatures on the Basis of Depth in Layer h at the Moment of Connection of the Freezing Layer ξ With the Permafrozen Rock Mass (Maximal Temperatures According to SNiP II-B.6-66) (Example 27) 250
Calculation of the Temperature of the Surface of the Ground at the Moment of Joining the Freezing Layer ξ With the Permafrozen Rock Mass (Example 28) 253
7. Regularities of the Change of Thermophysical Properties of Frozen Dispersed Rocks 254
1) Dependence of the Heat Capacity of the Skeleton of the Ground on Composition and Temperature 254
2) Dependence of the Heat Capacity of Soils on Composition and Temperature 255
3) Dependence of the Coefficients of Thermal and Temperature Conductivity of Soils on Their Composition, Moisture Content and Temperature 257
Calculation of Some Thermophysical Characteristics of Sands and Loams (Example 29) 268

8. Regularities of the Change of Mechanical Properties of Frozen Dispersed Rocks 270
1) Dependence of the Mechanical Properties of Frozen Soils on Their Dispersion 271
2) Dependence of the Mechanical Properties of Frozen Soils on Their Moisture Content 276
3) Dependence of the Mechanical Properties of Frozen Soils on Their Structure 280
4) Dependence of the Mechanical Properties of Frozen Soils on Their Temperature 283
9. Forecasting Thermal Subsidences of Soils During Thawing 287
Calculation of the Amount of Thermal Subsidence During the Thawing of Permafrozen Rocks (Example 30) 288
Calculation of the Height of an Embankment Constructed to Prevent Thermal Subsidences of Soils (Example 31) 293
10. Forecasting Changes of the Geological Engineering Properties of Frozen, Freezing and Thawing Rocks 296
Bibliography 299

Chapter 7. Regularities in the Formation of Taliks and Forecasting Their Formation 301
1. Talik Classification by Reasons for Their Formation 301
2. Distinctive Features and the Character of the Influence of Natural Factors on the Formation of Taliks 302
1) The Radiation Heat Balance of the Surface and Its Role in Talik Formation 302
Calculation of the Possibility of Existence of a Radiation Talik on Slopes With a Southern Exposure (Example 32) 302
Calculation of the Influence of the Effective Radiation on the Formation of a Radiation Talik (Example 33) 309
2) The Role of Evaporation and the Positive Temperature Shift in the Formation of Taliks of the Thermal Subtype 311
3) The Role of the Snow Cover in the Formation of Taliks of the Thermal Subtype 312
Calculation of the Critical Height of Snow at Which Taliks of the Thermal Subtype Can Form and Exist (Example 34) 313
Calculation of the Warming Influence of Snow on Sections With Its Close and Loose Adherence (Example 35) 317
4) The Influence of Firn Basins and Glaciers on the Formation of Taliks of the Thermal Subtype 318
5) The Influence of the Flooding and Swampiness of Sections on the Formation of Taliks of the Thermal Subtype 319
Calculation of the Warming Influence of Swampiness Leading to the Formation of Taliks of the Thermal Subtype (Example 36) 321
6) The Influence of Infiltration of Precipitations on the Formation of Taliks of the Radiation-Infiltration Subtype 326
Calculation of the Possibility of Existence of Taliks on Account of Infiltration of Atmospheric Precipitations (Example 37) 326

7) The Influence of Water Covers and the Composition of the Bottom-set Beds on the Formation of Taliks of the Underwater Type 328
Calculation of the Time of Formation of a Talik and Its Configuration (With the Method of D. V. Redozubov) Under a Thermokarst Lake (Example 38) 329
Determination of the Character of the Talik Forming in a Weakly Filtering Alluvium Under a Riverbed (Example 39) 332
8) The Influence of Subsurface Waters on the Formation of Taliks of the Water-Thermal 333
Determination of the Possibility of Forming an Infiltration Talik Through the Runoff of Surface Waters in a Fractured Zone (Example 40) 333
Calculation of the Influence of Long-Period Fluctuations of Temperature on the Surface on the Conditions of Existence of a Talik in Foci of Discharge of Subsurface Waters (Example 41) 337
Calculation of the Minimal Age of a Pressurized-Filtration Talik (Example 41) 339
3. Distinctive Features of the Propagation of Taliks With Consideration of the Latitudinal Zonation and Geostructural Conditions 339
Bibliography 345

Chapter 8. Regularities in the Formation and Prediction of the Development of Geocryological Processes and Phenomena 346
1. Regularities in the Development and the Prediction of the Heaving of Soils 346
Determination of the Probability of Development of Heaving and Its Possible Amount (Example 43) 349
Determination of the Dimensions of a Perennial Hummock of Heaving and the Dynamics of Its Growth (Example 44) 355
2. Regularities in the Formation and Prediction of the Development of Ice Bodies 359
Estimation of the Dynamics of Growth of an Ice Body and Its Thawing (Example 45) 361
Calculation of the Influence of an Ice Body on the Temperature Regime of Basement Rocks (Example 46) 363
Calculation of the Influence of Freezing on the Surface of an Ice Body on Its Thawing (Example 47) 367
Calculation of the Height of the Embankment of an Anti-Ice-Body Belt (Example 48) 369
3. Processes of Frost Cleavage of Rocks and Polygonal Formations 370
Determination of the Conditions of Formation and the Distances Between Frost Cleavage Fissures (Example 49) 373
4. Regularities of the Formation and Prediction of the Development of a Thermokarst 377
Estimation of the Possibility of the Start of Development of a Thermokarst Process (Example 50) 382
Calculation of the Height of Fillings of Soil to Prevent the Development of a Thermokarst (Example 51) 383
Calculation of the Thickness of a Layer of New Formation of Frost After the Drying Up of a Thermokarst Lake (Example 52) 385

5. Processes of Thermal Abrasion and Prediction of Their Development 388
Estimation of the Rate of Thermal Abrasion of Coasts Composed of Strongly Iced Rocks (Example 53) 390
6. Solifluction and Prediction of Its Development 392
Evaluation of the Conditions of the Formation and Development of Solifluction (Example 54) 397
7. Regularities of the Formation of Landslides, Runoffs, Mud Flows, Mud Streams and Avalanches 401
8. The Development of the Process of Differentiation of Large-fragment Material 402
Bibliography 403

Chapter 9. Solution of Some Geocryological Problems of Heat and Mass Transfer 405
1. Solution of the Problem of the Temperature Regime in a Homogeneous Medium With a Periodically Varying Phase State of the Water 405
1) Numerical Calculation of Heat Cycles in Soils With Phase Transitions of Water as a Criterion for Determination of the Temperature Shift 409
2) Investigation of the Dynamics of the Temperature Field in the Upper Layers of the Lithosphere With Consideration of Early-period Fluctuations of the Surface Temperature 412
2. Investigation of the Processes of Freezing and Thawing of Rocks With Consideration of Phase Transitions of Unfrozen Water by Means of a Self-modeling Solution 415
1) Freezing (Thawing) of Porous Bodies With Consideration of the Curve of Unfrozen Water in a Self-modeling Case (the Classical Stefan Problem) 415
2) The Influence of Consideration of the Ice Formation Curve on the Thawing of the Ground 417
3) The Influence of Consideration of the Ice Content Curve on the Process of Freezing and Thawing of Soils With Different Natural Moisture 419
4) Determination of the Effectiveness of the Ice Formation Curve 422
3. Investigation of the Dynamics of Freezing and the Cryogenic Structure of Finely Dispersed Rocks in the General Case by Digital Computer 424
1) Investigation of the Dynamics of the Temperature and Moisture Fields and the Amount of Heaving During Seasonal Freezing in Moisture-saturated Finely Dispersed Soils 424
2) Investigation of the Dependence of the Cryogenic Structure and the Dynamics of Heaving on the Character of the Temperature Conditions on the Surface During the Freezing of Moisture-saturated Finely Dispersed Soils 429
3) Calculation of the Dynamics of the Temperature and Moisture Fields and the Amount of Heaving During Freezing of the Seasonally Thawed Layer 434

4) Investigation of the Freezing of Soil With Moisture Migration by Means of a Self-modeling Solution 438
4. Solution of the Problem of the Thawing of Coarsely Dispersed Rocks With Consideration of the Infiltration of Summer Precipitations (by Digital Computer) 445
5. Examples of the Solution of Problems of Frost Prediction in Connection With the Construction of Structures of Various Types (on an Analog Computer) 448
1) Solution of the Problem of the Dynamics of the Thawing Basin in the Foundation of Buildings Erected on Permafrozen Soils 449
2) Calculation of the Thawing Basin in the Construction of Large Structures With Irregular Heat Release 454
3) Investigation of the Thermal Regime of the Body and Base of an Earthen Dam During Construction and Operation 456
4) Forecast of the Temperature Regime of the Body and Base of a Dam in Rocky Soils, Established in the Process of Operations 464
5) Prediction of Change of Frost Conditions Under Embankments as a Function of the Upper Boundary Temperatures 469
6) Solution of the Problem of Protection of Fluvial Deposits Against Freezing by Means of the Creation of an Artificial Ice Body 473
7) Calculation of Established Temperature Fields Around Underground Ice and Soil Reservoirs 476
8) Prediction of Change of Frost Conditions Around Buried Pipelines With Consideration of Changes of the Surface Temperature 477
6. Approximate Method of Calculating the Thickness of the Layer of Seasonal Thawing (Freezing) of Soils Around Buried Pipelines 479
Bibliography 484

Chapter 10. Basic Principles of Frost Surveying and Mapping for Purposes of a Geological Engineering Frost Forecast 485
Bibliography 489

Preface

In the area of the propagation of seasonally and permanently frozen rocks the geological engineering conditions depend substantially on the character of the permafrost situation because of the sharp change of properties of the rocks both during transition from the frozen into the thawed state and from the thawed into the frozen, and also during change of the temperature of the rocks within the limits of negative or positive values (near zero). The conditions of the bedding and spread of permafrozen rock masses, the properties of frozen rocks, the character of the cryogenic textures, the conditions of the new formation of frost and also the development of various cryogenic processes and effects are determined essentially by heat exchange on the earth's surface. The latter forms in definite geological and geographic conditions, and in connection with that also changes together with change of the natural situation under the influence of the productive organization of the territory. The directivity and character of that change can be taken into consideration by establishing quantitative connections between the components of the natural environment and its change under the influence of man, on the one hand, and the characteristics of the seasonally and permanently frozen rock masses, on the other. The established regularities serve as the basis for the compilation of a frost forecast, the purpose of which is the scientific prediction of the character of the change of frost conditions which sets in in the process of organization of a territory.

The present work is the first textbook of the course "Procedure of Frost Investigations." It was compiled on the basis of a series of lectures and experience in conducting practical classes in a course presented in the Department of Geocryology of the Faculty of Geology of Moscow State University over a period of 20 years. At the same time the latest theoretical developments in frost studies were used in writing the textbook. The textbook is so constructed that in it are given the theoretical principles of forecasting change of frost conditions and measures of the practical application of various ways to solve concrete problems, composed on the basis of field and laboratory investigations of various regions of permafrozen rocks.

The principal methodological aspects of frost forecasting are examined in Chapter 1. By frost forecasting is understood the scientific prediction of the direction of development and degree of change of frost conditions which will occur in the future either in connection with the natural historical course of development of nature or in connection with the economic opening up of the territory.

In the chapter it is shown that the regularities of the formation and development of seasonally and permanently frozen rock masses are concrete expressions of the general laws of development of matter for geological forms of its movement. The concepts of general and concrete forecasts are formulated, corresponding to the tasks of small-scale and specialized large-scale frost surveys. It also is shown that only investigations which make it possible to reveal the particular, general and regional regularities of the formation of frost conditions are the basis of forecasting, and the content of those concepts is revealed.

Examined in Chapter 2 is the connection of geological, geographic and thermophysical conditions of the formation and existence of permafrozen rocks. It is shown that that connection is accomplished in the process of conductive and convective heat transfer in the rock masses. Examined is the role of the structure of the radiative heat balance of the earth's surface for a half-year in the formation of heat cycles in soils. In connection with such a formulation of the question, fairly great attention is given to the factors forming the structure of the radiation heat balance. Stated in a brief and accessible form is the procedure for determination of separate components of the radiation heat balance of the surface, which is of great importance in the compilation of a forecast of changes of frost conditions.

In Chapter 3 which follows the theoretical principles of calculation of heat and mass transfer processes occurring in freezing, thawing and frozen rocks are stated rather completely and on a high mathematical level. It is essential that, side by side with the solution of the problem of freezing in a conductive formulation, which at present is used very widely in frost studies, examined in this chapter are methods recently obtained for solving considerably more complex problems of heat and mass transfer in freezing and thawing rocks. In connection with that, considerable attention is given to the problem of the freezing of rocks with consideration of moisture migration. Phase transformations are considered in the spectrum of negative temperatures in accordance with the curve of freezing water. Of great importance are the solutions of V. G. Melamed of a self-modeling problem with consideration of radiation, which make it possible to effectively investigate the regularities of complex processes of heat and moisture transfer during freezing. Those sections are published for the first time in a logically completed form, and this is of great independent scientific and practical importance. Also given in the chapter is a solution of the problem of the thawing of rocks with consideration of the infiltration of atmospheric precipitations, which is of great importance in studying coarsely dispersed rocks.

The mathematical principles of frost studies in general and frost forecasting in particular, presented in Chapter 3, are very complex and require the application of high-speed computer technology for their solution. Therefore, approximate formulas for the calculation of the basic characteristics of seasonally and permanently frozen rocks, based on the precise solutions presented in Chapter 3, are examined in Chapter 4. The approximate formulas and nomograms constructed on the basis of them make it possible to link the thermophysical aspect of frost conditions with the geological and geographic and by the same token determine the particular and general regularities of the formation and development of seasonal and permanent freezing of rocks. At the same time those formulas can be used as rapid field methods for calculations during a frost survey.

Examined in Chapter 4 are approximate formulas for determination of heat cycles and depths of seasonal and permanent freezing of rocks. Shown on concrete examples for various regions of the propagation of permafrozen rock masses is a procedure for calculating the depth of seasonal freezing and seasonal thawing of rocks (in the case of equality and inequality of the thermal conductivities of rocks in the thawed and frozen states), the depths of potential seasonal freezing and seasonal thawing of rocks and the value of the annual heat cyeles in the layer of seasonal and the layer of annual temperature fluctuations. Such a complex application of calculating formulas for the forecasting of the main characteristics of seasonally and permanently frozen rocks and the study of their dynamics has been made for the first time in this work.

Examined in Chapter 5 is the influence of various natural factors on the formation of the temperature regime and the depth of the seasonal freezing and seasonal thawing of rocks. Presented in the chapter are approximate formulas for determination of the thermal influence on the temperature regime of soils in the layer of their seasonal fluctuations of such factors as snow, vegetation, the water cover, swampiness, the exposure and steepness of slopes, the infiltration of atmospheric precipitations and the composition and moisture of the soils. Calculation of the influence of the listed factors is examined on concrete examples.

Chapter 6 is devoted to forecasting the change of the temperature regime and thickness of the permafrozen rock masses, questions of the formation of cryogenic textures and forecasting the change of properties of frozen soils. Since the distinctive features of forecasting changes of various parameters of frozen rock masses involve to a considerable degree the type of the frozen rock mass, its genesis and the characteristics intrinsic to it, questions of the classification of seasonally and permanently frozen rock masses are examined in the chapter.

Examined in the same chapter is the forecasting of change of thickness of permafrozen rock masses, the forecasting of the thawing of those masses and the forecasting of their new formation. Much attention is given to questions of thermal subsidence of frozen rock masses (subsidence during thawing) and change of the thermophysical and physicomechanical properties of the soils in the thawed and frozen states. An approach to the solution of various problems connected with the need to forecast thermal subsidence is shown on concrete examples. Also presented are examples of forecasting the change of properties of soils in connection with change of their temperature and moisture regimes and other factors. Methods are presented which make it possible to forecast the formation of cryogenic textures in deposits under different conditions of their freezing. Given in the chapter is a method of calculating the maximal temperatures of frozen soils at the moment of complete jointing of the freezing layer of seasonal thawing with the roofs of permafrozen rock masses, needed for calculation of the forces of freezing together in the period in which the most unfavorable conditions exist for the work of foundations in connection with heaving.

The study of taliks in a region of permafrost is of enormous importance in the study of the interaction of masses of frozen rocks and subsurface waters and the solution of tasks of engineering geology. Therefore Chapter 7 is devoted to an examination of the regularities of the formation of taliks in connection with frost characteristics in different geological and geographic conditions. Given in the chapter is a genetic classification of taliks by causes and conditions of existence. Analysis of the reasons for the formation and conditions of existence of taliks in different latitudinal and geological conditions makes it possible to determine the possibility of their origination during the productive organization of territories. In that chapter it is shown that the compilation of a concrete forecast of the origination and development of taliks is based on methods of quantitative estimation of the influence of different natural factors on the temperature regime of the rocks.

Examined in Chapter 8 are the principal regularities of the formation of cryogenic (frost-geological) processes and phenomena, and also a formulation and solution of the tasks in forecasting their development. The dependence of the spread and character of manifestation of various cryogenic processes on the geological conditions of heat transfer on the surface of rocks is investigated. Distinctive features of the spread of processes and phenomena in different frost and temperature zones and geostructural regions are examined in connection with that and examples of calculations are presented.

Chapter 9 is devoted to the solution of various problems in forecasting with electronic and analog computers. Presented in the first part of the chapter are the results of solution of problems both in a self-modeling and in a general formulation, devoted to the quantitative investigation of general and particular regularities of the processes of heat and mass transfer in freezing and thawing soils. Numerical integration of the indicated problems is done on the basis of the algorithms developed in Chapter 3. In particular, one should note here the finding of the course of heat fluxes in time and the amounts of heat cycles in different sections of the ground as a function of the mean amplitude of the temperature fluctuations on the surface, the moisture of the rocks, etc. In addition, presented here are the results of solution of a number of problems relating to freezing with moisture migration which make it possible to quantitatively establish the interconnection between the cryogenic structure and heaving, on the one hand, and the conditions of freezing (in particular, in connection with warming and cooling during winter), on the other. That group of problems is solved with an electronic computer.

Examined at the conclusion of the chapter are complex multidimensional problems arising in the solution of questions of frost forecasting in connection with construction in the region of distribution of seasonally and permanently frozen rocks. One should note here the solution of problems relating to the dynamics of the temperature field in the body and base of an earthen dam, to haloes of thawing around buried pipelines, to the basin of defrosting under structures, etc. Most of those problems were solved with an analog computer (the IGL hydraulic integrator and the EI-12 electrical integrator).

The textbook ends with Chapter 10, devoted to the main aspects of frost surveying and mapping for purposes of frost forecasting. In that chapter it is shown that the forecasting of frost conditions in the process of productive activity is possible only on the basis of study of the regional and zonal regularities of the formation and development of seasonally and permanently frozen rock masses through determination of the quantitative influence of all the main factors of the natural environment on the characteristics of the seasonally and permanently frozen rock masses. The qualitative and quantitative establishment of such bilateral dependences, giving all the calculating parameters, is possibly only while making a frost survey, in the process of which those calculations also are made.

On the whole it should be noted that such a text book has been created for the first time not only in our country but also abroad. It was written with consideration of the latest achievements in frost studies and in essence is a generalizing work in the area of the making of a frost forecast. An original methodical form of presentation of the material, supported by examples of calculations, was found and used in writing the textbook. At the same time it must be noted that since such a textbook has been written for the first time it probably has some shortcomings which will be eliminated in a later edition. We ask that you send your comments and requests relating to the questions examined in it to: Moscow V-234, Leninskiye gori, MGU, Faculty of Geology, Department of Geocryology.

Chapters 1, 2, 4, 5, 6, 7, 8 and 10 were written by V. A. Kudryavtsev, L. S. Garagulya and K. A. Kondrat'yeva. The examples in those chapters were compiled by L. S. Garagulya under the supervision of V. A. Kudryavtsev. Chapters 3 and 9 were written by V. G. Melamed. Participating in the writing of the textbook, besides the main authors, were Ye. I. Nesmelov (sections 2 and 3, Chapter 2), Ye. P. Shusherin (section 8, Chapter 6) and A. A. Ananyan and N. N. Smirnov (section 7, Chapter 6). In the preparation of the manuscript for the press much help was rendered by V. P. Volkov, N. I. Chizhov, M. I. Syritsyn and G. I. Krylov, to whom the authors express their appreciation.

The textbook is intended for students and specialists -- frost specialists, geological engineers, hydrogeologists, cryolithologists and students and specialists of allied directions -- surveyors, planners and builders. The book will be useful to many other specialists working in the area of the distribution of seasonally and permanently frozen rocks.

Chapter 1. Principal Aspects of Frost Forecasting

By frost forecasting is understood the scientific prediction of the development and change of frost conditions which will occur in the future either in connection with the natural historical course of development of nature or in connection with construction and the economic organization of territory. The task of compiling a forecast assumes knowledge in advance of the natural historical conditions of the studied region and especially of all parameters of the natural environment which determine the frost situation. Therefore the forecast of change of frost conditions in geological engineering investigations must be compiled only on the basis of study of general and particular regularities in the formation and development of frozen rock masses and the processes and effects accompanying them. Consequently the preliminary study of those particular and general regularities is an obligatory initial moment in the compilation of the forecast.

By particular regularities is understood the establishment of a regular dependence between separate components of the natural environment (including the anthropogenic factor) such as the character of the radiation-heat balance, the relief (steepness and exposure), covers (snow, vegetative, water and artificial), the composition, complexity and moisture of the ground, etc, and the character of the ffost conditions -- the spread, bedding and thickness of the permafrozen rock masses, the formation of the temperature regime and the depths of the seasonal thawing (freezing), cryogenic textures and iciness, and cryogenic processes and formations. In that case one has in mind the establishment not only of a qualitative but mainly of a /quantitative/ connection.

By general regularities is understood the interconnection of the components of the natural environment, including also the production activity of man, with the general complex of frost conditions. In them particular regularities are manifested in interconnection and interaction.

General and particular regularities are manifested both in separate landscape types and within the limits of regions. Depending on the scale of the survey, local and regional regularities in the formation of frost conditions should be distinguished. Thus the general regularities in the formation of frost conditions can be examined on both the regional and the local levels. The same should be said also in relation to particular regularities. Thus, for example, one can examine the influence of the snow cover both on separate areas and landscape types and within the limits of the entire region of seasonally and permanently frozen rocks. In that case the method of studying particular and general regularities must be based on the assumption that those regularities in the formation and development of permafrozen rock masses are a concrete expression of general laws of the development of matter for geological forms of its movement. Therefore the principal laws of the development of matter, considered from positions of the Leninist theory of cognition, must be the basis of study of the regularities of the formation of permafrozen rock masses for the compilation of a forecast.

It is a matter primarily of a law reflecting the general connection of phenomena and processes in nature, in accordance with which the frost conditions must be considered in a close interconnection with the entire complex of the natural situation, and above all with the geological genetic types of rocks. From the correspondence to definite natural complexes of concrete frost conditions flows a conclusion of the possibility and necessity of application of a landscape indication method. Its methodological basis consists in the fact that the investigated region is subdivided into a number of sections (microregions and landscape types) characterized by definite geological conditions and other elements of a complex of the natural environment. Within the limits of each microregion concrete forms and regularities in the formation and development of seasonally and permanently frozen rock masses are studied. In that case one should have in mind a second general law of the development of matter -- the law of continuity of motion -- of development. Permafrozen rock masses are characterized by a relatively large dynamics of all the main parameters in time. Therefore frost investigations must be completed with a forecast of the change of frost conditions compiled on the basis of comprehensive study of the dynamics of permafrozen rock masses.

The universal connection of phenomena and the universal continuous development of processes and phenomena leads to a need to study the causal connection of of permafrozen rock masses with different elements of the geological and geographic environment. This determines the need to study particular regularities of the formation of frozen rock masses on key sections for each landscape complex which make it possible to extend those regularities to all sections similar in natural conditions. The obtained particular regularities serve as the basis for the establishment of general regularities in the formation of frost conditions of the entire studied territory and also for the compilation of a forecast of the change of frost conditions in connection with construction and the productive organization of the territory.

An extremely important aspect in the compilation of the forecast is determination of the conditions of the transition of rocks from the frozen into the thawed state and the reverse, that is, their qualitative change. That transition is a concrete expression of a law of the development of matter -- transition from quantity to quality. The different state of rocks is connected qualitatively with the appearance or disappearance of the rock-forming mineral, ice, in the rocks. The amount of ice and the character of its distribution in the rock, which determine the cryogenic texture, cause change of the composition and properties of the frozen rock masses, distinctive features of the course of various processes, the formation of phenomena and by the same token engineering geological characteristics of the investigated territory. Therefore it is necessary to thoroughly study the distinctive features of the formation of cryogenic textures for each lithological genetic complex of rocks as a function of all factors of the geological and geographic environment. Thus, in the compilation of the forecast the rocks themselves are the decisive factor. Without their evaluation it is impossible to compile a forecast of the change of the temperature regime of the rocks and all the other characteristics of frozen rock masses. This should be regarded as an expression of the basic position of Marxist-Leninist philosophy of the /primacy of matter/. Permafrozen rock masses must be the main object of investigation in the compilation of a forecast of the change of frost conditions.

In studying permafrozen rocks, several approaches are possible in principle. It is possible to examine concrete frost conditions within the limits of the spread of a given landscape type as a simple correspondence of them without studying the cause and mutual conditionality. It is obvious that such an approach is methodically incorrect. In that case the possibility of forecasting changes of frost conditions is excluded.

Also erroneous is the approach in which only bilateral connections are studied (frozen rocks and climate, frozen rocks and geobotanical conditions, etc) without considering them in an interaction with the entire complex of natural conditions. A scientific forecast can be given only if investigations have been conducted from positions of the Leninist theory of cognition, which assumes /unity of analysis and synthesis/. From those positions the process of a frost survey includes the following steps of cognition: subdivision of the territory into landscape types with the separation of lithological-genetic complexes of rocks, study of the characteristics of the natural conditions within the limits of the distinguished types, the study and analysis of particular regularities in the formation of frozen rocks as a function of each element of the natural complex, especially on the composition, structure and position of rocks, and their generalization -- synthesis as an expression of their interconnection and interaction in the form of general regularities in the formation of permafrozen rock masses over a territory as a function of the joint action of all components of the complex. The last stage in the cognition of the studied object is checking the correctness of the established regularities through practical investigations. This implements in practice the basic position of the Marxist-Leninist theory of cognition, which says: practice is the criterion of truth.

As a result of construction and the productive organization of territory, substantial changes of geological and geographic conditions are occurring which involve changes of the frost situation and all the more so of geological engineering conditions. The character of that connection is expressed in particular and general regularities of the formation and development of seasonally and permanently frozen rocks and is studied in the process of a frost survey. The indicated regularities are determined, on the one hand, by the thermodynamic and thermophysical laws and, on the other, by the geological characteristics of the studied region. Therefore it would be wrong to consider the forecast of change of frost conditions to be a result only of thermophysical processes which can be determined by thermal engineering calculations. Then, for example, the warming influence of the snow cover cannot be regarded as a result of the influence of the thermally insulating layer (snow) with certain characteristics of thermal resistance. The influence of the snow will actually be determined to a considerable degree by the annual heat cycles in the soil. On two sections with different annual heat cycles in the soil but with identical thickness and density the influence of the snow will be different. The greater the amount of the heat cycles the more significantly will the warming influence of a snow cover similar in thickness and density be expressed. If it is taken into account that the heat cycles in the soil depend on the composition, structure and mode of occurrence of the rocks, that is, on the geological conditions of their formation and the existence of frost, and also on the moisture regime of the soil and the character of the radiation heat balance of the surface, then all the complexity of the question under consideration and the impossibility of regarding the warming influence of snow as a thermal engineering effect become obvious.

Similar examples can also be presented with respect to the influence of other geological and geographical factors on the formation of the temperature regime and the depths of freezing and thawing of rocks (§).

As aspect of great importance, as will be shown in the following chapters, is the fact that the heat cycles in the soil depend on the thickness of the layer of seasonal freezing and thawing and on the average annual temperatures of the rocks. Therefore the influence of one and the same factors (snow, vegetation, etc) will have a different effect on the formation of the temperature regime of the rocks and the depths in the case of seasonal freezing and in the case of seasonal thawing of rocks. Therefore in examining the thermodynamic and thermophysical regularities and in compiling calculating procedures it is absolutely necessary to take into account the geological and geographic aspects of the phenomena, so that similar parameters in the calculating procedures can be measured directly in the field and linked with distinctive features of the natural situation.

The basis of the formation of the temperature regime of the rocks, reflecting the character of the heat transfer in the soil and atmosphere, is the radiation heat balance of the surface of the soils. The frost conditions are determined to a considerable degree by the course of the change of the radiation heat balance by seasons of the year. Therefore in compiling a forecast of changes of frost conditions one cannot limit oneself to average data of the radiation heat balance of the surface during the year. Only when the latter is considered for a half-year or a shorter time interval can the connection of the heat cycles in the soil and the remaining components of the radiation heat balance be revealed. Such an approach makes it possible to estimate the role and importance of the radiation heat balance of the surface in the formation of frost conditions as a function of such factors as the subsurface waters, the infiltration of volatile sediments, etc. Without taking those distinctive features into consideration it is impossible to form a correct idea of the change of frost conditions in connection with construction and the productive organization of a territory.

In accordance with the above, the following general scheme of procedure for frost forecasting is presented. In the beginning a frost survey is made in the investigated region, as a result of which the geological and genetic types of frozen rocks and distinctive features of the formation of cryogenic textures within their limits are studied, and also particular and general regularities of the formation of seasonally and permanently frozen rocks and the frost processes occurring in them. In that case it is necessary to establish not only the qualitative connections of the frost characteristics with the geographic and geological factors of the natural environment but also their quantitative expression on the basis of calculating schemes which take into consideration both the thermophysical and geological aspects of the phenomenon in a concrete situation. In addition, taking into account the character of the change of the geological and geographic conditions as a result of proposed or planned construction and economic opening up of the territory, the changes of frost conditions are calculated both for the entire studied territory and for separate sections and structures. The concluding stage of a frost forecast must be the working out of methods of control or purposeful change of the frost and engineering geological conditions to assure optimal working of the equipment in the planned conditions.

The calculating forecasting schemes must include: a) calculations of the radiation heat balance of the surface with consideration of the influence of the geographic and geological factors; b) calculations of the conductive and convective heat transfer in the soil; c) calculations of the freezing of dispersed systems with a front and with a zone of freezing.

A theoretical substantiation of the calculating schemes is given in chapters 3 and 4. Analytical methods of calculation and methods of simulating frost processes are indicated in Chapter 3, and analog and electronic computer methods of calculating frost problems in Chapter 9. On that basis rapid field methods have been developed for the calculation of forecast frost characteristics by means of approximate formulas and nomograms (Chapters 4, 5, 6, 7 and 8).

In making a frost forecast it is necessary to distinguish:

a) the forecast of change of frost conditions connected with the natural course of the dynamics of permafrost in accordance with the natural course of change of the geological and geographic environment;
b) the forecast of change of frost conditions in connection with the concrete productive activity of man within the limits of separate sections and with the construction of specific structures;
c) the forecast of change of frost conditions on wide areas in connection with the summary effect of a complex of structures and the productive organization of large territories.

In that case it is necessary to take into consideration distinctive features of frost forecasting in different stages of the investigation. In a small-scale survey (1:500,000 - 1:200,000, the stage of technical and economic substantiation) a forecast of change of frost conditions is given in very general form. Here one should above all bear in mind the change of frost conditions in connection with change of the influence of various factors of the natural environment, which can either be connected with their natural change in time or be given provisionally for the proposed promising sections of the territory. This includes the influence of the snow and vegetation covers, the composition and moisture of the rocks, the steepness and exposure of slopes, the infiltration of summer atmospheric precipitations, surface and subsurface waters, swampiness, etc. The influence of those factors must be examined from positions of the formation of the radiative heat balance of the surface and the average annual temperatures of the rocks in the layer of their annual fluctuations.

In medium-scale and large-scale surveys (1:100,000 and larger, the stage of the technical plan and working drawings) a specific forecast is compiled of the change of frost conditions for separate sections and construction sites as a function of distinctive features of the planned structures and the designated productive organization of a territory. The basis of that forecast is the general forecast, discussed above, the data of which are determined more precisely for the specific conditions. In addition, artificial transformations of the natural situation are taken into account, such as the leveling of the terrain, the cutting off and filling in of soil, the construction of earthen structures, the draining and irrigation of soils, tree planting and tree felling, the planting of lawns, the construction of artificial covers, etc. The influence of all these transformations is taken into consideration in a forecast through change of the radiative heat balance of the surface and the formation of average annual temperatures of the rocks.

In compiling a forecast much attention must be given to processes of thawing of permafrost rock masses and the formation of them again as a result of construction. Forecasted in that connection are both the changes of depths of the layer ξ and changes of the composition and structure of the thawing, freezing and frozen soils and their properties in accordance with the general forecast of change of frost conditions. In addition, the frost forecast also includes a forecast of the change of properties of the frozen rocks in connection with the change of their temperature regime. The forecasting of the development of cryogenic processes and phenomena also is important. On the basis of everything said above a final frost forecast is compiled for the investigated region. The concluding moment in its compilation during medium- and large-scale surveys is the development of methods of controlling the frost process in order to achieve optimal conditions for the work of the structures. That development involves definite concrete recommendations regarding the productive organization of the territory and the selection of a principle of construction and structural characteristics of the bases and foundations of various structures. Thus an essential difference of the frost forecast is determined in different stages of investigation, exploration and planning, and also the sequence and succession in the scheme -- a general forecast during a small-scale survey and a specific forecast during medium- and large-scale surveys.

A necessary condition of frost forecasting is the application of simple and at the same time sufficiently reliable and precise calculating schemes. In that respect the most advisable are calculating schemes based on precise mathematical solutions of the problems of conductive and convective heat transfer. On the basis of them algorithms are compiled for the solution of heat-transfer problems by electronic computer. In accordance with those schemes series of standard or reference cases of those problems are calculated for extreme and typical conditions. On the basis of them particular and general dependences are determined for the characteristics of heat transfer on different factors and for the further convenience of calculations nomograms are compiled with a precision sufficient for practical purposes.

Heat-transfer problems are solved simultaneously with analog computers and the obtained results are compared with electronic computer solutions. However, numerical solutions obtained by those methods do not always make it possible to reveal the general regularities of heat transfer as a function of each parameter of the natural situation. It is better for that purpose to use approximate calculating formulas which include parameters obtained in the process of a frost survey. In addition, the starting data for a frost forecast can change substantially in the process of planning and construction and it may be necessary to recalculate the forecast. It is convenient to do that by means of a simple procedure suitable for field conditions, rapidly and reliably. Approximate formulas give a very great effect here.

Approximate formulas are calculated and verified on the basis of a precise electronic computer solution and the limits of application of those formulas and their precision are determined. Verified variants of those formulas are recommended for calculations. An obligatory condition for their application is comparison of the obtained results of calculation with full-scale observations on the investigated areas and key sections.

In working up a frost forecast it is necessary to select from the entire variety of approximate formulas those which would make it possible to take into consideration both the thermophysical and the geological-geographic regularities of heat transfer. One should add to that that the calculating formulas include only parameters which can be determined during a frost survey. This makes it possible to obtain in the field by means of rapid calculating methods such data as the average annual temperatures of soils at different depths and the maximal depths of seasonal freezing and thawing, and many others. In the case of any sort of discrepancies both the starting data and the calculations themselves can be verified in the field. The specific forecast for separate areas and structures is refined on the basis of more precise calculating schemes during processing in the office.

The final goal of frost investigations and a frost forecast is an engineering geological evaluation of the investigated region. In the area of spread of permafrozen rocks that evaluation, in spite of the application of an entire complex of engineering geological investigations, can be given only on a frost basis. Thus it is more correct to say that in the region of permafrozen rocks a complex frost engineering geological survey must obligatorily be made, and it must include a complex of investigations, both frost and engineering geological. A less illuminated question in those investigations is the method of frost forecasting, to which the present work is devoted.

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Chapter 2. Interconnection of the Geographical, Geological and Thermophysical Conditions of the Formation and Existence of Seasonally and Permanently Frozen Rocks

1. The Radiation Heat Balance of the Surface of the Soil as a Condition of the Formation and Dynamics of Seasonally and Permanently Frozen Rocks

The thermal state of the surface of the Earth is determined by the amount of arriving short-wave solar energy, its transformation into long-wave energy and the character of the thermal processes connected with the absorption of solar energy by the Earth's surface.

The balance of solar energy on the Earth's surface during a certain time interval is determined with the equation

$$Q_{сум}(1-\alpha)-I=R, \qquad (2.1.1)$$

where Q_{sum} is the summary short-wave radiation of the Sun and sky, representing the sum of the direct solar radiation (Q') arriving on a horizontal surface and the scattered radiation (q) arriving from the vault of heaven, kcal/cm^2;

α is the albedo of the Earth's surface, determined as the ratio of the short-wave radiation reflected from that surface to the total which arrived (in percentages or fractions of unity);

I is the effective long-wave radiation, equal to the difference between the proper radiation of the surface (I_e) and the radiation of the atmosphere (I_a) directed from the atmosphere toward the Earth's surface, kcal/cm^2;

R is the residual radiation or, as it is usually called, the radiation balance, representing the difference between the absorbed short-wave radiation $Q_{sum}(1 - \alpha)$ and the effective long-wave radiation I, kcal/cm^2.

The equation of the heat balance of the Earth's surface is in essence an expression of the law of energy conservation. It is written as follows:

$$R=LE+P+B, \text{ или } R-LE-P-B=0, \qquad (2.1.2)$$

where LE is the expenditure of heat on the process of summary evaporation (evaporation of the soil and the transpiration of moisture by vegetation), kcal/cm^2;

P is the expenditure of heat on the process of turbulent heat transfer between the Earth's surface and the atmosphere, kcal/cm^2;
B is the heat transfer between the Earth's surface and the soil, kcal/cm^2.

The formation and dynamics of the process of seasonal and permanent freezing of rocks are determined by the structure of the radiation heat balance of the surface. To study the connection of the thermal processes taking place in rocks with the thermal processes taking place on the Earth's surface it is necessary to examine the influence of all components of the radiation heat balance on the temperature of the surface and the basement rocks within the annual cycle (during the main seasons of the year).

As is known, with change of the time of year in the middle and high latitudes there is a sharp change in the arrival of direct and scattered radiation, the character of the surface and its albedo and in the final account the amount of absorbed radiation. The effective radiation undergoes considerable changes in connection with change of the surface temperature. The same should also be said with respect to evaporation. All the indicated components vary quantitatively in the course of the year while preserving their sign unchanged. The situation is different as regards turbulent heat transfer and heat cycles in the soil. They vary not only quantitatively but also in their sign. The change of sign, as a rule, is connected with change of the seasons of the year.

The direct and scattered radiations reach large values in the summer and are reduced to a minimum in the winter period. Simultaneously with that, in winter the snow cover, with its albedo of close to 70-80%, reduces the absorbed radiation to almost zero. In the winter period the effective radiation always is substantially smaller than the absorbed radiation, so that the radiation balance is positive and reaches such large values that its sum during the year also remains positive on practically all the territory of the globe. The main amount of that energy is expended on evaporation, turbulent heat transfer and heat cycles in the soil. In the winter period, when the absorbed radiation is sharply reduced, upon transition of the surface temperature through 0^{o} the evaporation and turbulent heat transfer become close to zero and the radiation heat balance for the surface under natural conditions can be written as follows:

$$(Q' + q)(1 - a) = I + B. \qquad (2.1.3)$$

It follows from what has been said that such components and heat expenditures on evaporation and turbulent heat transfer are of great importance in the formation of positive temperatures of the surface of the soil and ground in the spring and summer, in the period of maximal arrival of solar radiation. The effective radiation exerts the main influence on the formation of negative temperatures of the surface of the soil.

The freezing of rocks involves a sharp reduction of the arriving solar radiation and an excess of the effective radiation over the absorbed, that is, a negative radiation balance, as a result of which a temperature of 0^{o} or lower is established on the surface of the soil. It can readily be shown that at that time the negative radiation balance in essence consists of a negative heat cycle in the soils and basement rocks. Therefore the amount of the negative radiation balance is determined essentially by the structure of the annual heat cycles in the soil.

At a temperature of the soil surface of 0°, in the structure of the heat cycles in soils the main importance is acquired by the heat of phase transitions of water into ice. During a certain time that process keeps a zero temperature on the soil surface or close to it (a zero screen). Later (in winter), when the soil is frozen, the surface temperature under the snow remains somewhat higher thanks to heat cycles in the soil. At the same time the amount of absorbed radiation reaches its minimal values. As a result the amount of the radiation balance decreases rapidly, changes its sign and reaches large negative values. The sharp increase of the negative radiation balance is connected mainly with the release of heat from the soil on account of phase transitions of water during freezing. Because of this the negative radiation balance is subjected to latitudinal zonation.

Of great importance to the freezing of soils is the length of existence of the conditions during which the negative radiation balance is kept on the radiating surface. It is precisely the correlation of the length of periods with positive and with negative radiation balance which determines the sign of the annual average temperature of the surface of the soil, in spite of the fact that the sum of the positive radiation balance, as a rule, greatly exceeds the sum of the negative.

Geological and geographic factors are of great importance in the formation of the structure of the radiation heat balance. Included in such factors above all are various natural covers (snow, vegetation and water), the relief and exposure of slopes, the composition and moisture of cover beds and the rocks underlying them, and the hydrological and hydrogeological conditions. All these factors also determine the conditions of insulation of the surface and the value of the albedo, and all the thermal processes, as a result of which, in the latitudinal zones in which the arrival of radiation is great, frozen rock masses can form and exist and, on the contrary, in zones with a small amount of arriving radiation, taliks are widely developed.

The connection of all the enumerated factors with the radiation heat balance of the surface and its temperature regime can be represented in the form of the following equation:

$$T_{\text{ср}} = \frac{1}{2\sqrt[4]{\sigma}\,S}\left(\sqrt[4]{\frac{Q_{\text{п.макс}} LE_{\text{л}} - P_{\text{л}} - B_{\text{ср}} - U + V}{(0,4 - 0,06\sqrt{e_{\text{л}}})(1 - cn_{\text{л}}^2) + 4\left(\frac{T_{\text{макс.л}}}{T_{\text{мин}}}\right)}} + \sqrt[4]{\frac{Q_{\text{п.мин}} - LE - P_{\text{з}} + B_{\text{ср}} - U}{(0,4 - 0,06\sqrt{e_{\text{з}}})(1 - cn_{\text{з}}^2) + 4\left(\frac{T_{\text{мин.з}}}{T_{\text{мин}}}\right)}}\right). \quad (2.1.4)$$

where $Q_{p\text{-max}}$ and $Q_{p\text{-min}}$ are the extreme values of the 10-day sums of absorbed radiation (kcal/cm^2) in the summer and winter half-periods respectively, the subscripts "s" and "w" indicate that the 10-day sums of the corresponding LE and P (kcal/cm^2) are taken at the moment of the summer and winter extreme values of Q_p; B_{av} is the 10-day value of the heat cycle of the soil (kcal/cm^2), equal to its half-year value divided by 18 (the number of 10-day periods in a half-year); e is the absolute atmospheric humidity, mb; n is the coefficient of change of cloudiness by latitude in fractions of unity; U is the quantity of heat lost by rocks in 10 days in accordance with the gradient of the average annual temperature in the layer of seasonal freezing (thawing), determined by the value of the temperature shift (section 2, Chapter 4), kcal//cm^2; V is the amount of heat received by the rocks from infiltrating precipitations in 10 days, kcal/cm^2; T_{av} is the average annual temperature of the surface of the soil, ^oK; $T_{max\text{-}s}$ and $T_{min\text{-}w}$ are the average 10-day temperatures of the surface of the ground at the time of maximal and minimal arrival of solar radiation respectively, ^oK; T_{max} and T_{min} are the average 10-day maximal and minimal air temperatures, ^oK; $\sigma = 0.82 \times 10^{-10}$ is the Stefan-Boltzmann constant.

In each specific region, depending on the specifics of the geological and geographic conditions, their own distinctive features of formation of the components of the radiation heat balance are observed. A difference in structure of the balance within one region can be noted for different landscape complexes. Those differences, the role and importance of each of the elements of the natural environment in the formation of the radiation heat balance must be studied during the conducting of a frost survey. Such a formulation of the investigations provides the necessary material for the compilation of a scientific forecast of the change of frost conditions in connection with construction. On that basis it becomes possible to formulate the question of the admissable change of frost conditions in order to create optimal conditions for the work of structures.

Study of the radiation heat balance during the conducting of frost investigations makes it possible to include the dynamics of the upper boundary conditions for the region. In particular, distinctive features of the distribution and the conditions of occurrence of permafrozen rock masses, the conditions of formation of taliks and many other features can be evaluated. It also is very important that the structure of the radiation heat balance of the surface determines to a great extent the character of the freezing of both seasonally and permanently frozen rocks, which in accordance with the geological structure and composition of the rocks determines the character of the cryogenic textures. There is a no less close connection between the distinctive features of the radiation heat balance and the character of the manifestation of cryogenic processes and phenomena. Thermokarst in essence is a clear expression of the influence of change of the structure of the radiation heat balance on change of the frost conditions. Thus increase of the absorbed radiation through destruction of the plant cover leads to increase of the amplitude of the temperatures on the surface of the soil and increase of the depth of thawing and, consequently, to the formation of thermokarst (in the presence of great iciness of the rocks). The same should be said in relation to solifluction, heaving, crack formation and other processes.

Study of the radiation heat balance and its structure is of great importance also in the examination of height banding and height zonation in frost studies. It is obvious that the amount of direct, scattered and absorbed radiation, as well as the effective radiation, evaporation, turbulent heat transfer and heat cycles in the soil are subject to the height banding and height zonation and are characterized by definite values for different climatic regions and geobotanical zones. Because of this the data of the radiation heat balance and the analysis of its components are, evidently, the only possible scientific basis in the compilation of small-scale areal frost maps.

As a result of the production activity of man the structure of the radiation heat balance changes. For each of the components of the balance that change can be different and be differently expressed in the temperature regime of the rocks, on the depths of the seasonal freezing and thawing, and in the distribution, conditions of occurrence and composition of permafrozen rock masses.

The amount of absorbed radiation

$$Q_n = (Q' + q)(1 - \alpha) \quad (2.1.5)$$

is determined by the amount of arriving direct solar radiation Q', which depends within each region on the steepness and exposure of the terrain. In accordance with that the value of Q' can change substantially when the terrain is leveled. Thus, for example, the smoothing out of the natural slopes of northern exposure with a steepness of 30° and reducing them to a horizontal surface within the limits of $60\text{-}68^{\circ}$ of northern latitude leads to an increase of Q' by 10-20%. When slopes of a southern exposure are smoothed out the reverse regularity is observed, since 20-30% less direct solar radiation falls on a horizontal surface than on southern slopes.

The relative amount of change of Q_p also depends on the amount of scattered radiation q, which remains constant for a given region and does not depend on the change of steepness and exposure of slopes and other characteristics of the surface of the ground. The larger the value of q the smaller the degree to which change of Q_p is reflected in change of the radiation-heat balance.

Change of the albedo of the day surface is of greater importance in the change of the radiation-heat balance in connection with the productive activity of man. The removal of the plant cover, and also the planting of trees, the plowing of the land, the sowing of grass and cereals and the planting of shrubs -- all this leads to a reduction of the albedo of the surface of from 7-10 to 25%. One and the same change in the value of α leads to different changes of the absorbed radiation. Under conditions of a sharply continental climate, where Q' + q reaches large values, even a very small change of α can lead to substantial changes of the temperatures of rocks. Those changes will be less considerable under the conditions of a maritime climate.

In regions of the Far North the change of the quantity of absorbed radiation often is connected with change of α of the surface of snow in the winter.

Thus, for example, in the region of Vokruta the blackening of the surface of the snow because of the deposition of coal dust on its surface leads to an earlier disappearance of the snow, increase of the absorbed solar radiation in the annual cycle and elevation of the average annual temperatures of the rocks.

The change of absorbed radiation leads not only to change of the average annual temperatures but also to change of the annual amplitudes of temperatures on the surface of the ground. Thus, for example, increase of absorbed radiation in the winter leads to increase of the amplitudes of temperature fluctuations in the annual cycle. On account of change of the steepness and exposure of slopes, as calculations and the data of actual observations show, the change of the magnitude of the annual amplitude does not exceed 2-3°C. Considerably greater amplitudes should be expected on account of removal of the plant cover, and also on account of change of the albedo of the surface. In that case the amplitudes of temperatures can vary in the range of 4-5°, and sometimes even more.

Very large changes in the temperature regime of the surface of the ground are noted in the case of the construction of artificial covers, which is connected with change of the absorbed radiation and, especially, with a sharp reduction of heat expenditures on evaporation. Thus, for example, during the creation of concrete and asphalt coverings those changes can lead to elevation of the average annual temperature and increase of the amplitude of annual temperature fluctuations by 2-3° on the surface of a concrete covering and by 3-4° on the surface of asphalt.

Changes in the temperature regime of the surface lead to change of the depths of seasonal freezing and thawing of the ground under coverings. Since the elevation of the average annual temperature occurs simultaneously with increase of the annual amplitude, the depth of seasonal freezing varies inconsiderably and the seasonal thawing increases sharply in comparison with the natural conditions. The structure of the radiation heat blance depends to a great degree on evaporation. For each given region where the atmospheric conditions (the air temperature, atmospheric humidity and the wind rose) remain constant the amount of evaporation on separate sections depends on the moisture of the ground and the character of the vegetation. Such a measure as the drainage of soils on a territory which is being opened up leads to a sharp decrease of the amount of evaporation and in the final account to an elevation of the average annual temperatures of the soil. The same should be said of the removal of the plant cover, which leads to drying of the ground and reduction of the amount of evaporation.

It must be noted that with change of the absorbed radiation there is a substantial change also in the amout of evaporation, since the separate components of the radiation heat balance are closely interconnected. Thus, in particular, increase of Q_p on account of change of the steepness and exposure of slopes or the albedo of the surface leads (all other conditions being equal) to elevation of the average annual temperatures of the ground and increase of evaporation.

Change of the amount of evaporation leads to change of the temperatures and their amplitudes on the surface of the ground. It is obvious that the temperature regime of the grounds in the warm period of the year changes sharply primarily on account of that factor. The increase of evaporation leads to a reduction of the maximal temperatures and, consequently, to a reduction of the annual amplitudes, since in the winter the temperature regime on the surface of the ground is practically independent of the evaporation. On dry sections where there is little evaporation the amplitudes are larger than the ordinary. In connection with that, on sections with great evaporation, other conditions being equal, insignificant changes of depth of the seasonal freezing will be noted as compared with the dry sections. In a region of permafrozen rocks, change of the amount of evaporation leads to a sharp change of the depth of seasonal thawing.

Such regularities will occur during the economic opening up of territory in the case of drainage of ground and regulation of the surface runoff, and also for the case of change of the moisture regime of the soils on account of the head of the subsurface waters or unregulated runoff of technical waters.

Similar results can be noted on account of annihilation of the plant cover or different planting of trees, which leads to a sharp change of the amount of evaporation. The construction of coverings of various kinds (asphalt or concrete) sharply reduces the amount of evaporation, because those coverings prevent the access of subsurface waters to the day surface.

The structure of the radiation heat balance also depends on the turbulent heat transfer (P). It is obvious that its amount is determined by the ground surface temperature (t_0) and the value of the heat-transfer coefficient on the soil-atmosphere boundary (K)*. At higher soil temperatures (all other conditions being equal) the amount of turbulent heat transfer will be greater than at lower temperatures. There also is a direct dependence of turbulent heat transfer on the coefficient K.

The connection of change of the temperature regime of rocks as a function of turbulent transfer is complex and can be examined only as a whole, with consideration of the interaction of all components of the radiation heat balance. We will point out here that elevation of the average annual temperature of rocks while the average annual air temperature remains unchanged always leads to increase of turbulent exchange. Because of that, increase of turbulent exchange involves a decrease of the depth of seasonal freezing and increase of the depths of seasonal thawing of the ground. That dependence can vary in a more detailed examination with consideration of the influence of all components of the radiation heat balance. It is obvious that turbulent heat transfer can vary substantially during change of the various covers, such as, for example, removal of the plant cover or of sowings of various types, covering with concrete or asphalt, etc.

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*The heat-transfer coefficient (K) is included in formula (2.1.6) and corresponds to the coefficient α according to V. S. Luk'yanov and M. D. Golovko, 1957.

Change of the amount of the heat cycles in the ground leads to change of the ground temperature by a different amount, depending on a number of conditions and on the structure of the radiation heat balance. Calculations have shown that such a measure as the draining of ground leads to a decrease of the heat cycles in the ground and reorganization in the structure of the radiation heat balance of the surface in such a way that the average annual temperature of the ground rises. Change of the structure of the radiation heat balance and ground temperature will also be noted in the case of leveling, shearing and filling of ground, as the composition of the ground, its density, conditions of its occurrence and moisture regime can change, and this is reflected in the heat cycles.

In studying the influence of the change of ground conditions on the temperature regime of the surface and the structure of the heat balance it is necessary to bear in mind the character and depth of seasonal freezing and thawing of rocks and their change during the opening up of territory, especially in connection with the fact that the main share of the heat cycles in the ground is determined by phase transitions of water during their freezing and thawing. Of no little importance in that question is the formation of a temperature shift (see 4.1.19 in Chapter 4), the change of which during the economic activity of man can reach 1 or 2°. The change of the annual average rock temperature by such an amount near the southern boundary of the spread of frozen rocks can lead to a sharp change of the depths of the seasonal freezing and thawing and change of the annual heat cycles. The latter leads to change of the structure of the radiation heat balance.

As a result of all the examined complex processes occurring on the earth's surface the temperature regime of soils and grounds forms. The interconnection of those processes and the temperature regime is expressed most completely by the equation of the radiation heat balances, the expenditure components of which depend functionally on the surface temperature. Therefore it would have been natural to determine the surface temperature by solving the equation of the balance in relation to it. However, it is practically impossible to obtain such a solution due to the complex interconnection of all the processes resulting from the large number of factors. At the present time, on the basis of study of the radiation heat balance of the surface only some particular problems are solved, for example, the difference between the air temperature and the temperature of the surface of soil deprived of plant cover (the so-called radiation correction Δt_k) is determined, the influence of the exposure and steepness of slopes on the formation of the surface temperature is determined, the thermal influence of vegetation is determined, etc.

The radiation correction can be calculated for the average annual temperature and the annual amplitude of temperature fluctuation on the surface of the soil by determining the convective component from the equation for the radiation heat balance of the earth's surface. If the coefficient of turbulent heat transfer K, in kcal/m^2 x degree x hour, is known, the difference between the air temperature t_0 and that of the earth's surface t, in $^{\circ}$C, is found from the formula

$$\Delta t_R = t_в - t = \frac{R - LE - B}{K}. \qquad (2.1.6)$$

where R, LE and B are the radiation balance, heat expenditures on evaporation and heat circulations through the soil surface, kcal/m^2 x hr.

Calculation of the radiation correction can be greatly simplified if on the investigated areas the evaporation and heat circulations through the surface are relatively small (for example, on sections composed of fragmental material of various sizes with a deep level of the subsurface waters, with artificial coverings, etc). In that case $\Delta t_R = R/\alpha$ (2.1.7), where α* is the heat-transfer coefficient, considered to be constant and equal to 20 kcal/m^2 x x degree x hr (according to V. S. Luk'yanov and M. D. Golovko, 1957). An application of that formula can be shown in the calculation of the average annual temperature on the ground surface. For example, it is required to determine t_o on a surface which in summer is without plant cover and in winter is covered with snow if the annual course of the air temperature and the radiation balance are known. The section is composed of sand and gravel deposits in which the ground-water level is below 1 meter. The starting data and solution are presented in Table 1, from which it is evident that on the surface of the soil (in winter, on the surface of the snow) the average annual temperature is 2.16^o higher than the average annual air temperature, and the annual amplitude of temperatures is 6.2^o greater than the amplitude of the air temperatures.

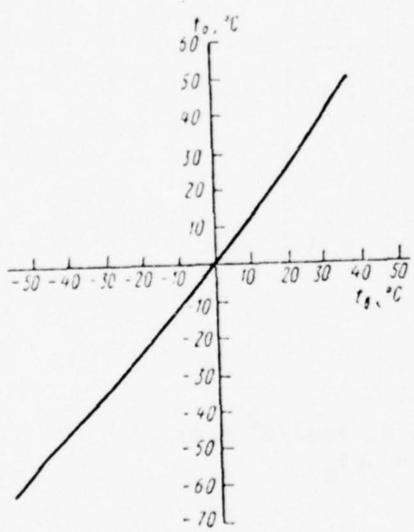

Figure 1. Graph of the interconnection of the average monthly air temperatures (t_a) and ground surface temperatures (t_0) for latitudes of 60^o to 80^o and longitudes of 78^o to 114^o.

In the absence of reliable data on the radiation heat balance of the surface it is recommended in some cases that regional correlation graphs of the interconnection of those temperatures, constructed by the statistical processing

*In the given case the coefficient α corresponds to the coefficient K in formula (2.1.6).

Table 1. Calculation of the average annual temperature of the surface of snow (in winter) and the soil (in summer) with consideration of the radiation correction determined by the method of V. S. Luk'yanov

A Исходные и расчетные данные	I	II	III	IV	V	VI	VII
1 $t_в$, °C	−16,8	−15,0	−8,2	0,7	8,4	15,5	18,2
2 R, ккал/см²мес	− 0,6	− 1,0	1,1	4,3	7,8	9,5	8,4
3 R, ккал/м²час	− 8,2	−13,7	15,1	58,9	106,8	130,1	115,1
4 $\Delta t_R = \frac{R}{\alpha}$; $\alpha =$ =20ккал/м² град·час	− 0,4	− 0,7	0,8	2,9	5,3	6,5	5,8
5 $t = t_в + \Delta t_R$	−17,2	−15,7	−7,4	3,6	13,7	22,0	24,0

A Исходные и расчетные данные	VIII	IX	X	X		B Год	
						t_0	A_0
1 $t_в$, °C	15,2	9,0	0,9	− 8,9	−15,6	0,3	35,0
2 R, ккал/см²мес	6,0	3,6	0,7	− 0,8	− 0,8		
3 R, ккал/м²час	82,2	49,3	9,6	−11,0	−11,0		
4 $\Delta t_R = \frac{R}{\alpha}$; $\alpha =$ =20ккал/м² град·час	4,1	2,5	0,5	− 0,6	− 0,6		
5 $t = t_в + \Delta t_R$	19,3	11,5	1,4	− 9,5	−16,2	2,46	41,2

Key: A - Starting and calculated data B - Year
1 - t_a, °C 2 - R, kcal/ cm² x month 3 - R, kcal/m² x hr
4 - =20 kcal/m x degree x hr 5 - $t = t_a + \Delta t_R$

of the observations of weather stations situated within the given region and representative of the conditions of the investigated section, be used to determine the temperature difference of the air and soil. Presented on Figure 1 is a graph of the interconnection of the average monthly temperatures of the air and soil surface, constructed for the northeastern part of Western Siberia. With that graph, if one knows the air temperature, one can readily determine the temperature of the surface of the snow in winter and the temperature of the surface of soil without plant cover in summer.

2. Climatological Calculation of Radiation Balance Components

The network of special stations conducting actinometric observations (of the regime of solar, terrestrial and atmospheric radiation) and heat balance observations (of the heat and moisture regime near the earth's surface and heat transfer in the soil) is rather sparse at the present time. Therefore in investigating the conditions of the thermal regime of the earth's surface a need arises to calculate the radiation heat balance components. Such a climatological calculation can be made on the basis of data of the network of weather stations with use mainly of the mean values of the meteorological elements for many years, obtained from a long series of observations. However, the use of climatological methods of calculation to obtain the sums of heat for a small number of years or for separate years often leads to large errors.

There are many methods of calculating the actual influx of summary radiation. An overwhelming majority of them is based on the use of the physical connections which exist between the sums of the solar radiation and cloudiness. Such a dependence is expressed in general form as follows

$$Q_{\text{сум}} = Q_0\,(1 - an - bn^2), \qquad (2.2.1)$$

where Q_{sum} is the summary radiation per month under the actual conditions of cloudiness, kcal/cm^2; Q_0 is the summary radiation per month under a cloudless sky, determined by the latitude of the place, the time of year and the conditions of atmospheric transmittance (Table 2), kcal/cm^2; n is the monthly average cloudiness in fractions of unity, published in "Spravochnik po klimatu SSSR" [USSR Climate Manual], Part 5; a and b are numerical coefficients, of which "a" is determined from observations of actinometric stations (Table 3) and "b" is constant and equal to 0.38.

Table 2. Summary radiation under a cloudless sky, kcal/cm^2 x month (calculated for the average length of a month of 30.4 days, according to Budyko et al, 1961)

Широта		II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
90° с.ш.	0,0	0,0	0,1	10.0	21,9	26,0	23,8	12,9	2,4	0,0	0,0	0,0
85	0,0	0,0	0,7	10.2	21,8	25,8	23,4	13,1	3.0	0,0	0,0	0,0
80	0,0	0,0	2,4	10,8	21,4	25,2	23,0	13,4	4,3	0,5	0,0	0,0
75	0,0	0,5	4,0	11,7	21,0	24,5	22,2	13,8	5,8	1,3	0,0	0,0
70	0,0	1,6	6,0	13,1	20,5	23,6	21,2	14,6	7,5	2,7	0,5	0,0
65	0,7	2,8	8,0	14,5	20,1	22,8	21,0	15,6	9,5	4,3	1,4	0,2
60	1,8	4,3	9,9	16,0	20,8	22,9	21,4	16,7	11,3	6,1	2,6	1,1
55	3,1	6,2	11,7	17,3	21,4	23,4	21,9	17,9	12,9	7,8	4,0	2,3
50	4,8	8,2	13,3	18,5	22,2	23,7	22,6	19,1	14,4	9,7	5,8	3,9

Key: A - Latitude B - 90° N. Lat.

Table 3. Coefficients a and c for various latitudes

A φ, °с. ш.	40	45	50	55	60	65	70	75	80
a	0,38	0,38	0,40	0,41	0,36	0,25	0.18	0.16	0,15
c	0,68	0,70	0,72	0,74	0,76	0,78	0,80	0,82	0.84

Key: A - Φ, N. Lat.

Those coefficients show which share of the total flux of summary radiation is retained as a result of its absorption by the cloud cover.

On the basis of climatological calculations and observations of the network of actinometric stations, maps of the radiation balance components have been compiled both for the USSR and for the entire globe ("Atlas teplovogo balansa" [Heat Balance Atlas], 1963; Barashkova et al, 1962). The annual course of summary radiation for the European territory, Western Siberia, Eastern Siberia and the Far East as a function of latitude and time of year is presented in Table 4.

On sections with a mountainous character of the relief a need often arises to estimate the total arrival of solar heat on differently oriented slopes. Such characteristics are usually obtained by calculation.

The flux of direct solar radiation impinging on a sloped surface, Q'_{sl}, can be expressed by the dependence:

$$Q'_{\text{скл}} = Q' \cos i, \qquad (2.2.2)$$

where Q' is the intensity of direct solar radiation on a surface perpendicular to the rays*, kcal/cm^2; i is the angle of incidence of solar rays on the surface of a slope, obtained with the formula

$$\cos i = \cos \alpha \sin h_{\odot} + \sin \alpha \cos h_{\odot} \cos (A_{\odot} - a_{\text{скл}}), \qquad (2.2.3)$$

where α is the steepness of the slope; h_0 is the height of the Sun above the horizon; A_0 is the solar azimuth, and a_{sl} is the azimuth of the slope.

All the enumerated parameters are determined by the laws of spherical trigonometry and expressed in degrees.

The summary solar radiation impinging on slopes is expressed by the sum:

$$Q_{\text{сум.скл}} = Q_{\text{скл}} + q_{\text{скл}} + r_{\text{скл}}, \qquad (2.2.4)$$

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*Data published in "Spravochnik po klimatu SSSR," Part 1.

Table 4 Mean latitudinal values of the summary radiation (kcal/cm^2 x month) (compiled by Ye. I. Nesmelova from observations of the actinometric network)

Таблица 4

Среднеширотные значения суммарной радиации (*ккал/см²мес*) (составлена Е. И. Несмеловой по наблюдениям актинометрической сети)

A Широта, град	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
I Европейская территория												
54	2,0	3,6	8,5	11,5	14,0	16,5	16,0	12,5	8,0	4,5	2,0	1,3
56	1,5	3,1	8,0	11,0	14,2	16,0	15,3	11,5	7,0	3,0	1,8	1,0
58	1,2	2,8	7,4	11,0	13,5	16,0	15,0	11,3	6,5	3,0	1,3	0,8
60	0,8	2,4	6,3	11.0	13,0	15,5	14,5	10,5	5,8	2,7	1,0	0,5
62	0,5	1,8	6,0	10,5	12,0	14,5	14,5	9,5	5,0	2,0	0,6	0,5
64	0,3	1,6	5,5	10,5	12,0	14,2	14,2	9,5	4,5	2,0	0,5	0,4
66	0,2	1,4	5.1	10.0	12,0	14,5	14,0	9,0	4.0	1,5	0,4	0,3
68	0,1	1	4,8	9,5	13,0	14,0	13,5	8,0	3,6	1,2	0,2	0
II Западная Сибирь												
54	2,2	4,0	9,1	11,8	15.0	16,8	15,5	12,8	7,8	4,0	2,2	1,5
56	1,8	3,5	8,8	11,5	14,0	16,3	15,0	12,0	7,0	3,8	1,8	1,0
58	1,5	3,1	8,0	11,5	13,2	16,0	14,7	11,5	6,5	3,0	1,5	0,9
60	1,0	2,7	7,2	11,5	12,8	15,8	14,7	11,0	7,0	2,8	1,1	0,5
62	0,6	2,0	6,1	11,5	12,5	15,8	14,5	10,8	5,2	2,7	0.6	0,4
64	0,5	1,8	5,9	11,3	12,8	15,5	14,5	10,5	4,7	2,5	0,5	0,4
66	0,2	1,5	5,5	10.5	13.0	15,3	14,5	10,2	4,0	2,1	0,4	0.3
68	0,1	1,2	5,0	10,8	13,8	14,5	14,0	9,5	3,7	1,8	0,3	0
III Восточная Сибирь и Дальний Восток												
54	2,2	4,2	9,0	11,0	14,0	15,5	15,0	12,0	8,0	5,0	2,5	1,8
56	1,8	3,8	8,2	11,0	14,0	15,5	14,8	11,3	7,5	4,0	2,0	1,5
58	1,5	3,1	7,8	11,5	14,0	15,5	14,8	11,0	7,0	3,2	1,7	0,9
60	1,0	2,8	7,2	11,5	14,0	15,0	14,0	11,0	6,5	3,2	1,5	0,5
62	0,6	2,2	6,8	11.3	14,0	15,0	14,0	10,5	6,0	3,0	1,0	0,4
64	0,5	2,0	6.1	11,0	14,5	15,0	14,0	10,4	5,5	2,8	1,0	0,4
66	0,2	1,5	5,8	11,5	14,5	15,0	14,0	10,2	4,2	2,4	0,5	0,3
68	0,1	1,3	5,3	11,5	11,5	15,0	14,0	9,0	4,0	2,0	0,3	0

Key: A - Latitude, degrees I - European territory II - Western Siberia III - Eastern Siberia and the Far East

where

$$q_{скл} = q_{гор} \cos^2 \frac{\alpha}{2}; \qquad (2.2.5)$$

$$r_{скл} = r_{гор} \sin^2 \frac{\alpha}{2}. \qquad (2.2.6)$$

Table 5. Daily sums of direct radiation on slopes with different exposure. Q'_{sl} as % of Q'_{hor} (according to A. F. Zakharova, 1958)

Экспозиция	Северная				Южная				Северная				Южная				Северная				Южная			
Широта, град	50								60								70							
Крутизна, град	40	30	20	10	10	20	30	40	40	30	20	10	10	20	30	40	40	30	20	10	10	20	30	40
15/I	0	0	1	37	152	210	264	302	0	0	0	5	252	384	520	668	0	0	0	0	—	—	—	—
15/II	0	0	20	61	138	170	200	214	0	0	0	37	156	208	264	324	0	0	0	0	280	450	700	860
15/III	0	20	48	75	122	140	153	162	0	0	27	64	136	164	188	200	0	0	2	42	150	210	250	300
15/IV	31	50	70	85	112	118	121	124	14	36	60	80	115	129	136	140	16	21	46	74	120	150	150	160
15/V	54	70	82	90	100	104	106	107	42	60	76	89	104	108	113	114	41	59	75	90	110	120	130	140
15/VI	61	77	87	94	102	102	99	93	55	69	81	91	101	104	103	100	52	80	90	96	103	106	108	107
15/VII	58	74	85	92	95	100	102	104	51	66	79	90	100	106	108	108	48	73	85	94	110	120	130	140
15/VIII	42	59	76	89	108	111	114	114	30	49	69	86	110	120	122	124	29	39	63	84	120	120	140	150
15/IX	10	34	58	79	116	130	137	143	1	14	45	73	124	142	160	172	4	4	18	60	140	180	200	210
15/X	0	3	34	69	131	156	180	194	0	0	5	52	150	190	224	256	0	0	0	17	200	320	400	460
15/XI	0	0	4	47	148	193	242	270	0	0	0	14	202	252	344	480	0	0	0	0	840	3100	4330	5400
15/XII	0	0	0	30	155	219	278	320	0	0	0	0	295	480	636	800	0	0	0	0	—	—	—	—

Key: A - Exposure 1 - Northern 2 - Southern B - Latitude, degrees C - Steepness, degrees

Table 6 Daily sums of direct radiation (approximate) Q' in valleys with meridional and latitudinal orientation with different degrees of closure of the horizon (at % of the sums for open terrain) (according to Ye. I. Nesmelova with consideration of G. S. Nikolenko, 1964)

Широта, *град*	Месяц	Ориентировка							
		меридиональная				широтная			
		угол закрытости горизонта, *град*							
		45	30	20	10	10	20	30	45
60	I	14	25	61	71	0	0	0	0
	II	37	48	59	90	90	0	0	
	III	37	50	79	88	99	88	0	
	IV	37	67	77	94	99	98	94	
	V	38	68	86	97	98	97	70	
	VI	39	69	87	97	98	96	64	
	VII	39	69	86	97	98	97	66	
	VIII	39	69	79	95	99,5	97	79	
	IX	37	50	77	86	99	98	0	
	X	34	45	55	88	94	0	0	
	XI	13	38	52	69	0	0	0	
	XII	11	16	56	78	0	0	0	
50	I	38	47	53	68	93	0	0	0
	II	38	49	56	86	96	85	0	0
	III	41	54	81	92	99	99	88	0
	IV	42	71	80	96	99	99	98	0
	V	44	75	92	98	99,6	98	98	96
	VI	62	74	91	98	99	97	96	94
	VII	62	74	91	97	99	97	96	94
	VIII	44	74	82	96	99	98	97	96
	IX	42	73	81	96	99	99	98	0
	X	37	50	73	89	95	88	0	0
	XI	38	51	56	93	94	0	0	0
	XII	42	51	61	94	78	0	0	0

Key: A - Latitude, degrees B - Month C - Orientation a - meridional b - latitudinal 1 - angle of closure of horizon, degrees

Here q_{sl}, kcal/cm^2 is the flux of scattered radiation impinging on the slope; r_{sl} is the flux of short-wave radiation reflected on the slope from the horizontal surface in front of the slope, kcal/cm^2; q_{hor} and r_{hor} are the fluxes of scattered and reflected radiation on the horizontal surface*, kcal/cm^2.

*The values of q_{hor} and r_{hor} were published in "Spravochnik po klimatu SSSR," Part 1.

Table 7. Vertical gradiant of the summary radiation ΔH (kcal/cm^2 x month per 100 meters of height) algebraically added with Q_{sum} of the difference. ϕ = 60-70° N. Lat. Asiatic part of the USSR

ΔH (м)	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
200—300	0,00	0,10	0,25	0,25	0,20	—0,10	0.05	0,05	0.10	0,10	0,00	0.00
300—400	0,00	0,20	0,50	0,50	0,40	—0,20	0,10	0,10	0.20	0,20	0,00	0,00
400—500	0,00	0,30	0,70	0,75	0,60	—0,40	0,15	0,15	0,30	0,30	0,00	0,00
500—600	0,00	0,40	0,90	1,00	0,80	—0,50	0,20	0,15	0,50	0,40	0,00	0,05
600—700	0,00	0,50	1,00	1,20	1,00	—0,50	0,25	0,15	0,70	0,50	0,05	0,10
700—800	0,00	0,55	1,05	1,35	1,25	—0,20	0,30	0,15	0,85	0,60	0,10	0,15
800—900	0,00	0,60	1,10	1,50	1,55	0,20	0,25	0,15	0,90	0,70	0,15	0,20
900—1000	0,00	0,65	1,15	1,65	1,90	0,70	0,20	0,15	0,95	0,80	0,20	0,25
1000—1100	0,00	0,70	1,20	1,80	2,25	1,20	0,05	0,15	1,00	0,90	0,25	0,30
1100—1200	0,00	0,75	1,25	1,95	2,60	1,70	—0,10	0,15	1,05	1,00	0,30	0,30
1200—1300	0,00	0,80	1,30	2,05	2,95	2,20	—0,20	0,10	1,10	1,05	0,35	0,30
1300—1400	0,00	0,85	1,35	2,15	3,25	2,70	—0,10	0,05	1,15	1,10	0,40	0,30
1400—1500	0,00	0,90	1,40	2,20	3,55	3,10	0,00	0,00	1,20	1,15	0,45	0,30
1500—1600	0,05	0,95	1,45	2,25	3,85	3,30	0,10	—0,05	1,25	1,20	0,50	0,30
1600—1700	0,10	1,00	1,50	2,30	4,15	3,50	0.20	—0,10	1,30	1,25	0,55	0,30
1700—1800	0,15	1,05	1,55	2,35	4,45	3,70	0,30	—0,15	1,35	1,30	0.60	0,30
1800—1900	0,20	1,10	1,60	2,40	4,75	3,90	0,35	—0,20	1,40	1,35	0,65	0,30
1900—2000	0,25	1,15	1,65	2,45	5,05	4,10	0,40	—0,25	1,45	1,40	0,70	0,30

The values of q_{sl} and r_{sl} obtained with approximate formulas (2.2.5) and (2.2.6) are characterized by an error of 10-20% for slopes with a steepness of not more than 30-40°. The daily values of the summary radiation are obtained with the formula:

$$\Sigma Q_{сум.скл} = \Sigma Q'_{скл} + \cos^2 \frac{\alpha}{2} \Sigma q_{гор} + \sin^2 \frac{\alpha}{2} \Sigma r_{гор}. \qquad (2.2.7)$$

It is more convenient to calculate the influx of summary radiation on slopes, if we have available the values of the relative daily sums (Table 5) $\Sigma Q'_{sl} / \Sigma Q'_{hor}$. If we know the actual arrival of heat of solar radiation in some region according to the "Spravochnik" and use the indicated ratios, we can always calculate its arrival on the slopes. In the conditions of very rugged ground a need also arises to estimate the reduction of the influx of solar radiation due to the different degree of closure of the horizon in valleys and other negative forms of the relief. In that case one should take into consideration, besides the summary closure of the horizon, expressed in degrees, the orientation of the valleys -- latitudinal or meridional (Table 6).

Of great scientific and practical interest is consideration of the change of the radiation balance components, and above all the summary radiation, with height. Thanks to the increase of atmospheric transmission the influx of direct solar radiation increases with increase of the absolute height of the locality. The intensity of scattered radiation of the sky, on the contrary, decreases with height due to decrease of the moisture content and dustiness of the atmosphere, but to a far lesser degree than that of the direct. Therefore in the mountains, at a considerable height above sea level, the summary radiation under a cloudless sky proves usually to be higher. As observations of actinometric stations in the Asiatic part of the USSR show, a very large gradient of summary radiation is characteristic of the lower 600-800 meters, particularly for the spring and summer months (Table 7).

For calculation of the amount of absorbed radiation one should have available the characteristic reflecting capacity of the earth's surface, that is, the albedo. The spatial variability of the albedo can be very considerable because of the variety of the underlying surface (Table 8). Therefore the amount of absorbed radiation, equal to $Q_{sum}(1 - \alpha)$, will be greatly different for open terrain and that covered by forest. The annual course of the albedo for all regions having a steady snow cover is sharply expressed. Therefore in calculating the monthly sums of absorbed radiation it is necessary to draw in the data on the average dates of the establishment and disappearance of the snow cover, and also the dates of transition of the average daily temperature through 5°, which is assumed to be the start of vegetation of plants*. In calculating the radiation balance for sections on which given structures are being constructed one should use the data on the albedo of uncovered surfaces or artificial coverings (Tables 9 and 10).

*Data of "Spravochnik po klimatu SSSR," Parts 2 and 4.

Table 8. The albedo of natural surfaces of dry land of different landscape zones (data of "Spravochnik po klimatu SSSR")

	A Вид поверхности	B Альбедо, %
1	Устойчивый снежный покров в высоких широтах (севернее 60°)	80
2	Устойчивый снежный покров в умеренных широтах (южнее 60°)	70
3	Лес при устойчивом снежном покрове	45
4	Неустойчивый снежный покров весной	38
5	Лес при неустойчивом снежном покрове весной	25
6	Неустойчивый снежный покров осенью	50
7	Лес при неустойчивом снежном покрове осенью	30
8	Степь и лес в период между сходом снежного покрова и переходом среднесуточной температуы воздуха через 10°	13
9	Тундра в период между сходом снежного покрова и переходом среднесуточной температуры воздуха через 10°	18
10	Тундра, степь, лиственный лес в период от весеннего перехода температур через 10° до появления снежного покрова	18
11	Хвойный лес в период от весеннего перехода температур через 10° до появления снежного покрова	14
12	Лес, сбрасывающий листву в сухое время года, полупустыня в сухое время года	24
13	То же во влажное время года	18
14	Сельскохозяйственные угодья:	
a	а) пойменный лес. Сочная густая трава	21—25
b	б) темнозеленая трава. Разнотравье в первой фазе развития. Земля сухая, темно-серого цвета	17
15	Снег свежевыпавший	85
16	Снег загрязненный	40
17	Снег влажный	43
18	Трава зеленая	28
19	Трава сухая	19
20	Болото с кустарником (марь)	25
21	Заросли кустарников	15—20
22	Лес лиственный	20
23	Лес еловый	10
24	Темнохвойный лес	10—15
25	Светлохвойный лес	15
26	Голубика, багульник	10—11
27	Мхи зеленые влажные	14
28	Осоки (мокрый луг)	22—23
29	Зачерненная поверхность	6
30	Лиственный лес (береза,осина) с примесью сосны. Сомкнутость крон 0,3—0,7	17
31	Сосновые среднетаежные леса, сомкнутость крон 0,5—0,8 с примесью березы, с кустарниками. Поверхность земли покрыта зелеными, бурыми мхами	15

Key: A - Type of surface B - Albedo, %

1 - Steady snow cover in high latitudes (north of 60°)
2 - Steady snow cover in middle latitudes (south of 60°)
3 - Forest under a steady snow cover
4 - Unsteady snow cover in the spring
5 - Forest with unsteady snow cover in the spring
6 - Unsteady snow cover in the autumn
7 - Forest with unsteady snow cover in the autumn
8 - Steppe and forest in the period between disappearance of the snow cover and transition of the average daily air temperature through 10°

Table 8 Continued (Key)

9 - Tundra in the same period
10 - Tundra, steppe and deciduous forest in the period from the spring transition through 10° to the appearance of the snow cover
11 - Coniferous forest in the same period
12 - Forest shedding leaves in the dry period of the year and semiarid land in the dry period of the year
13 - The same in the moist period of the year
14 - Agricultural land
a - Bottom-land forest. Succulent dense grass
b - Dark-green grass. Mixed herbs in the first phase of development. Ground dry, dark-gray in color.
15 - Freshly fallen snow
16 - Contaminated snow
17 - Moist snow
18 - Green grass
19 - Dry grass
20 - Swamp with undergrowth (goosefoot)
21 - Mature undergrowth
22 - Deciduous forest
23 - Spruce forest
24 - Dark coniferous forest
25 - Light coniferous forest
26 - Blueberry and ledum
27 - Moist green mosses
28 - Sedges (moist meadow)
29 - Blackened surface
30 - Deciduous forest (birch, aspen) with admixture of pine. Closure of crowns 0.3 - 0.7
31 - Pine medium-taiga forests, closure of crowns 0.5 - 0.8 with admixture of birch, with undergrowth. Surface of the ground covered with green and brown mosses

Table 9. Albedo (as %) of moist and dry surface of the ground (according to data from "Mikroklimat USSR" [Microclimate of the USSR], Moscow, Gidrometeoizdat, 1968)

	A Поверхность	B Уплотненная		C Свежевспаханная	
		1 сухая	2 влажная	1 сухая	2 влажная
a	Чернозем	12	7	9	5
b	Каштановая почва	14	9	11	6
c	Светлый серозем	32	18	20	13
d	Белый песок	40	18	—	—

Key: A - Surface B - Compacted C - Freshly plowed 1 - dry 2 - moist
a - Chernozem b - Chestnut soil c - Light serozem d - White sand

Table 10 Albedo of artificial coverings

	A Вид поверхности	B Альбедо, %
1	Песчаник	18
2	Цемент	27
3	Известняк	50—65
4	Бетон (светлый)	30—35
5	Гранит (светло-серый)	35—40
6	Мрамор (белый)	45
7	Кирпич красный обыкновенный	30
8	Кирпич силикатный	48—50
9	Сланец (темная глина)	8
10	Туф (гладкотесаная поверхность) голубоватый	50
11	Туф » розово-лиловый	40
12	Туф » темно-розовый	30
13	Туф » красный	25
14	Туф » черный	7
15	Черепица ярко-красная	42—44
16	Железо (ржавое)	25
17	Рубероид светлый	28
18	Рубероид черный	14
19	Толь блестящий	20
20	Стена оштукатуренная голубая	73
21	Стена оштукатуренная розовая	62
22	Стена оштукатуренная желтая	57
23	Стена деревянная некрашеная	40
24	Штукатурка наружная (светлая)	60
25	Краска белая новая	75
26	Краска белая старая	55
27	Щебеночное покрытие	18
28	Гравийное покрытие	13
29	Асфальт	10—30
30	Мостовая, панель (плитками)	17

Key: A - Kind of surface B - Albedo, %
1 - Sandstone 2 - Cement 3 - Limestone 4 - Concrete (light) 5 - Granite (light-gray) 6 - Marble (white) 7 - Ordinary red brick 8 - Silicate brick 9 - Slate (dark clay) 10 - Tuff (smooth-dressed surface) -- bluish 11 - Ditto -- rose-lilac 12 - Ditto -- dark-rose 13 - Ditto -- red 14 - Ditto -- black 15 - Bright red tile 16 - Iron (rusty) 17 - Light roofing felt 18 - Dark roofing felt 19 - Shining tar paper 20 - Plastered blue wall 21 - Plastered rose wall 22 - Plastered yellow wall 23 - Unpainted wooden wall 24 - Light outside plaster 25 - New white paint 26 - Old white paint 27 - Crushed-stone covering 28 - Gravel covering 29 - Asphalt 30 - Pavement, slab in blocks

The value of the effective radiation, which is the discharge component of the radiation balance, is usually calculated with the formula

$$I = I_0 (1 - cn^m) + 4\,\delta\sigma T^3 (T_n - T). \qquad (2.2.8)$$

Here I is the effective radiation under the actual conditions of cloudiness, kcal/cm^2; I_o is the effective radiation under a cloudless sky, kcal/cm^2; T is the air temperature, ^{o}K; T_s is the temperature of the earth's surface, ^{o}K;

m is an empirical coefficient which assumes values of 1 to 3; c is the coefficient of cloudiness, which shows what percentage of the long-wave radiation is absorbed by the cloud cover (see Table 3); δ = 0.85-0.90 is the radiative capacity of the earth's surface; σ= 0.82 x 10^{-10} is the Stefan-Boltzmann constant.

The second term in formula (2.28) represents a correction of the amount of effective radiation caused by inequality of the surface and air temperatures. For monthly sums that correction fluctuates in the range of 0.1-0.5 kcal/cm^2 and is most substantial in the summer.

The effective radiation under a cloudless sky I_o is calculated with the formula

$$I_0 = (0{,}254 - 0{,}0066\, e)\, \delta\sigma T^4. \qquad (2.2.9)$$

Here e is the water vapor tension in mb. The sums I_o presented in Table 11 were calculated with that formula. It should be taken into consideration in that case that for regions with deep ground temperature inversion (Eastern Siberia) the sums I_o prove to be overstated by 30-50%.

Table 11 The effective radiation under a cloudless sky I_o (kcal/cm^2 x months) (according to N. A. Yefimova, 1961)

A Температура воздуха, °C	B Влажность воздуха, мб										
	1	2	3	4	5	6	7	8	9	10	11
40	8,5	8,2	7,8	7,5	7,2	6,8	6,5	6,1	5,8	5,5	4,4
35	8,0	7,7	7,4	7,0	6,7	6,4	6,1	5,8	5,4	5,1	4,2
30	7,5	7,2	6,9	6,6	6,3	6,0	5,7	5,4	5,1	4,8	3,9
25	7,0	6,8	6,5	6,2	5,9	5,6	5,3	5,0	4,8	4,5	3,6
20	6,5	6,3	6,0	5,7	5,5	5,2	5,0	4,7	4,4	4,2	3,4
15	6,1	5,9	5,6	5,4	5,2	4,9	4,7	4,4			
10	5,7	5,5	5,3	5,0	4,8	4,6					
5	5,3	5,1	4,9	4,7	4,5						
0	4,9	4,7	4,5	4,3							
—5	4,6	4,4									
—10	4,3	4,1									
—15	3,9										
—20	3,6										
—25	3,4										
—30	3,1										
—35	2,9										
—40	2,6										

Key: A - Air temperature, °C B - Atmospheric humidity, mb

The mean latitudinal values of the radiation balance of the earth's surface, equal to the difference between the absorbed and effective radiation, for various latitudes within our country are given in Tables 11 and 12. The data of those tables should be used with care for any specific region, monitoring the presented values by means of the data of observations of the actinometric network.

Table 12 Mean latitudinal sums of the radiation balance R (kcal/cm^2) (compiled by Ye. I. Nesmelova according to data of the actinometric network)

A Широта, град	B Месяцы I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	C Год
52	−0.5	−0.2	1,6	5,1	7,6	8,7	8,3	6,4	3,8	1,1	−0.4	−0.7	40,8
54	−0,6	−0.3	1.2	4,7	7,5	8,5	8,2	6,1	3,4	0,8	−0.5	−0.8	38.2
56	−0,6	−0.4	0,7	4,3	7,4	8,4	8,0	5,8	3,0	0.5	−0.6	−0.8	35,7
58	−0.7	−0,5	0,2	3,7	7,2	8,3	7,9	5,6	2.7	0,3	−0,6	−0,8	33,3
60	−0.8	−0,6	−0,2	3,2	6,9	8,3	7,8	5.4	2,4	0.1	−0,7	−0,8	31,0
62	−0.8	−0,6	−0,4	2,2	6,5	8,2	7,8	5,3	2,1	0.1	−0.7	−0,9	28,6
64	−0.7	−0.6	−0,4	1.3	6,0	8,2	7,7	5.1	1,9	−0,2	−0.8	−1,0	26,5
66	−0,7	−0.6	−0,4	0.6	5,5	8,2	7,7	4,9	1,6	−0,4	−0.9	−1.0	24,5
68	−0.7	−0,6	−0,4	0.1	5,0	8,3	7,6	4,5	1,3	−0,7	−0,7	−1,0	22,7

Key: A - Latitude, degrees B - Months C - Year

Table 13 The ratio of the daily sums of the radiation balance on slopes with northern and southern exposures and steepness of 10 and 20° to the sums on a horizontal surface on the 15th of the month (acc to "Mikroklimat SSSR," 1969)

A Широта, град	20° IV	V	VI	VII	VIII	IX	10° IV	V	VI	VII	VIII	IX
						I Северная экспозиция						
56	0,72	0,86	0,88	0,88	0,79	0,58	0,85	0,92	0,94	0,92	0,90	0,78
58	0,70	0,85	0,88	0,87	0,78	0,57	0,84	0,92	0,93	0,92	0,89	0,77
60	0,69	0,85	0,87	0,86	0,77	0,55	0,83	0,91	0,94	0,91	0,89	0,75
62	0,68	0,84	0,86	0,86	0,76	0,53	0,82	0,91	0,94	0,91	0,89	0,72
64	0,67	0,84	0,86	0,85	0,74	0,52	0,82	0,91	0,94	0,92	0,88	0,70
66	0,65	0,83	0,85	0,84	0,73	0,50	0,81	0,90	0,95	0,92	0,87	0,68
68	0,64	0,83	0,84	0,83	0,72	0,48	0,80	0,90	0,96	0,94	0,87	0,65
						II Южная экспозиция						
56	1,26	1,11	1,06	1,08	1,17	1,40	1,13	1,06	1,03	1,04	1,10	1,20
58	1,27	1,11	1,06	1,09	1,17	1,41	1,14	1,06	1,03	1,04	1,10	1,21
60	1,28	1,12	1,07	1,09	1,18	1,42	1,14	1,07	1,03	1,04	1,11	1,21
62	1,30	1,12	1,07	1,10	1,19	1,43	1,15	1,07	1,04	1,05	1,11	1,22
64	1,31	1,13	1,08	1,10	1,19	1,44	1,16	1,07	1,04	1,05	1,12	1,22
66	1,33	1,13	1,08	1,11	1,20	1,45	1,16	1,08	1,04	1,05	1,12	1,23
68	1,34	1,14	1,09	1,11	1,21	1,46	1,17	1,08	1,04	1,06	1,13	1,23

Key: A - Latitude, degree I - Northern exposure II - Southern exposure

Attention is attracted by the sharp decrease of the radiation balance with height, connected mainly with increase of the reflected and decrease of the absorbed radiation due to increase of the length of occurrence of the snow cover, characterized by very high albedo values (Table 8).

3. Procedure for Calculation of Heat Balance Components

/Calculation of the amount of evaporation./ All the presently used methods of determining the summary evaporation (from the surface and transpiration by the plant cover) can be divided into three groups: 1) based on use of the water balance equation; 2) based on use of the equation of turbulent diffusion of water vapor; 3) based on the heat balance equation. Of great importance is direct measurement of evaporation by means of contemporary evaporators but, unfortunately, the equipping of stations with such instruments is not great, especially on the Asiatic territory of our country. Therefore there is special growth of the sole methods of calculating summary evaporation which make it possible to obtain its value on the basis of observations of weather stations.

One of the most accessible approximate methods permitting determination of the amount of evaporation over many years from limited sections of surface is the Tyurk method (1958). For natural surfaces, provided that $r^2/E^2 > 0.1$, the calculating formula has the form:

$$E = \frac{r}{\sqrt{0,9 + \left(\frac{r}{E}\right)^2}} = \frac{1,054\,r}{\sqrt{1 + \left(\frac{1,054\,r}{E_0}\right)^2}}, \qquad (2.3.1)$$

where E is the amount of evaporation (annual sum), mm; r is the sum of the annual precipitation, mm; E_o is the maximal evaporation, which limits the atmospheric humidity, mm; $E_o = 300 + 25t + 0.05t^3$, where t is the mean annual air temperature.

The evaporation from the base surface of the soil can be determined with adequate precision with Tyurk's formula if one introduces the additional coefficient K which takes into account the moisture of the ground, that is,

$$E = \frac{Kr}{\sqrt{1 + \left(\frac{r}{E_0}\right)^2}}, \quad K = \frac{w_e - w_r}{w_n - w_r}; \qquad (2.3.2)$$

where

$$E_0 = \frac{1}{16}(t + 2)\sqrt{Q_{сум}} \quad \text{при } t > 2, \; E_0 = 0 \text{ при } t \leqslant 2°,$$

r is the sum of the precipitations in 10 days, mm; E_o is the maximal evaporation; Q_{sum} is the summary radiation arriving on the surface in 10 days, kcal/cm^2; w_n is the natural moisture content of the ground, in fractions of a unit of volume; w_t is the total moisture content, in fractions of a unit of volume; w_h is the hygroscopic moisture, in fractions of a unit of volume.

For regions of excessive and sufficient moistening of the territory of the USSR a method has been worked out for cauculating the evaporation rate on the basis of the air temperature and humidity by A. R. Konstantinov (1963). The method makes it possible on the basis of the mean values of the air temperature and humidity to calculate the perennial monthly and annual values of evaporation from an area of several square kilometers surrounding the weather station (Figure 2). Weather station data typical for the surrounding territory must be taken in the calculations.

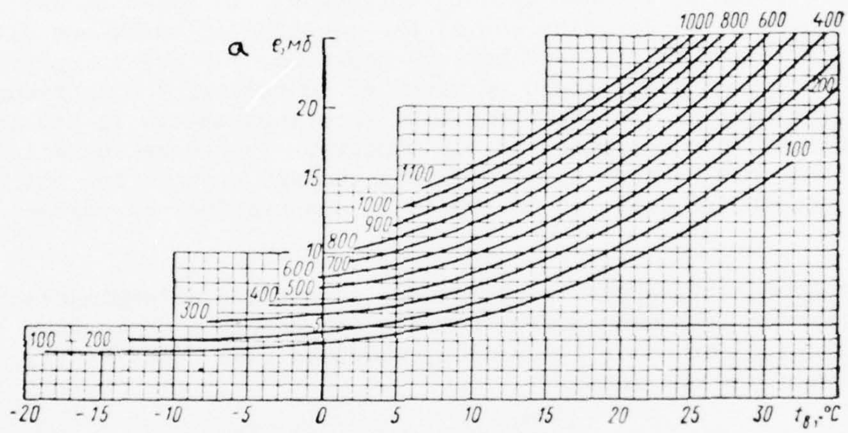

Figure 2. Nomogram for calculating the mean annual perennial evaporation on the basis of the mean annual air temperature (t_a) and humidity (e). a - e, mb

To determine the mean perennial annual amounts of evaporation for sections of dry land limited in size one can also use the equation of M. I. Budyko (1971):

$$E = \sqrt{\frac{R_0 r}{L}\left(1 - L^{-\frac{R_0}{rL}}\right) th \frac{rL}{R_0}}, \qquad (2.3.3)$$

where r is the annual rate of precipitations on the basis of materials obtained in observations at the given point, R_o is the mean perennial annual amount of the radiation balance for a moistened surface.* The ratio R_o/L ($L \approx 600$ kcal/g is the latent heat of evaporation) expresses the evaporability, that is, the maximally possible evaporation from the surface which under the given meteorological conditions occurs from the territory only when it is quite adequately moistened.

The method is applicable for conditions of naturally moistened surfaces of the plains territory of the USSR without limitation. Here the conditions of

*Taken from a map compiled by N. A. Yefimova (1962).

natural moistening of the surface are taken into consideration in formula (2.3.3) by the amount of precipitations r, and the differences in the properties of the basement rocks, including in the plant cover, by the amount of the radiation balance R_o. In calculating the evaporation one can use a nomogram (Figure 3), the mean error in the calculation of which is about 17%.

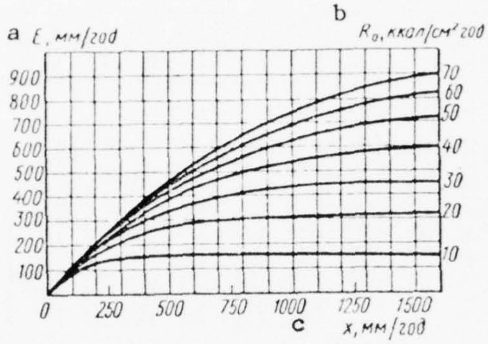

Figure 3. Nomogram for calculation of the mean annual perennial evaporation (E) on the basis of corrected precipitations (x) and the radiation balance of the moistened surface (R_o).
a - E, mm/yr b - R_o, kcal/cm^2 x yr c - x, mm/yr

Table 14 shows the variation of the amount of evaporation as a function of latitude of the place and the type of plant cover on the territory of the USSR.

/Calculation of the amount of turbulent heat transfer of the surface of the ground with the atmosphere/. Calculation of the turbulent heat flow P (cal/cm^2 x days) based on the theory of turbulence was proposed by A. R. Konstantinov (1956). The initial calculating formula has the form

$$P = a \cdot u_{\phi л} \cdot \left(1 + b\,\frac{T_п - T_в}{u_{\phi л}^2}\right)(T_п - T_в). \qquad (2.3.4)$$

Here u_{w-v} is the mean wind velocity for the month at the height of the wind vane, m/sec; T_a is the mean monthly air temperature (in a psychrometric booth), °C; T_s is the mean monthly temperature of the soil surface, °C, measured at weather stations. The coefficients a and b in formula (2.3.4) are assumed to be the following:

1) a = 4.0 and b = 0.1 for the period with snow
2) a = 6.0 and b = 0.9 for the period without snow

The sum of P for a month is obtained by multiplying its daily value by the number of calendar days in the month.

It is recommended that the turbulent heat transfer be calculated in the same way as in calculations of evaporation for representative stations, the observations of which are typical for the surrounding territory. It is inadvisable to use formula (2.3.4) for strongly broken terrain. An approximate distribution of the values of P over the territory of the USSR is presented in Table 15.

Table 14 Distribution of the summary evaporation from the surface of dry land (E, mm) by months*

A Лес	B Широта, град	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	C Год
I Европейская территория СССР														
a Хвойный	68	0	1,3	5,2	15,7	44,5	65,5	57,6	39,3	21,0	10,5	1,3	0	262
	66	0	1,5	6,0	17,9	50,7	74,5	65,5	44,7	23,8	11,9	1,5	0	298
	64	0	1,9	7,9	22,6	63,9	94,0	82,7	56,4	30,1	15,0	1,9	0	376
	62	0	2,2	8,8	26,4	75,0	110,0	96,8	66,0	35,2	17,6	2,2	0	440
	60	0	2,4	9,7	29,2	83	122	107	73	39	19	2	0	486
	58	0	2,5	10	30	85	125	110	75	40	20	2,5	0	500
b Смешанный и лиственный	56	2,5	5	15	44	89	99	89	64	44	25	15	2,5	494
	54	2,4	5	14	44	87	97	87	63	44	24	15	2	484
	52	2,3	5	14	42	84	93	84	60	42	23	14	2	464
II Западная Сибирь														
a Хвойный	0	0	3	15	48	75	72	51	27	9	0	0		300
	66	0	0	4	18	58	20	86	61	32	11	0	0	360
	64	0	0	4	21	67	105	101	71	38	13	0	0	420
	62	0	0	4	23	72	111	108	77	40	14	0	0	450
	60	0	0	5	24	75	118	113	80	42	14	0	0	470
	58	0	0	5	25	78	123	118	83	45	15	0	0	420
c Лиственный	56	0	5	10	28	85	108	94	70	42	23	5	0	470
d Лесостепь	54	0	4	9	26	69	101	88	66	40	22	4	0	440
III Восточная Сибирь и Дальний Восток														
e Северо-таежный	66	0	0	2	8	29	85	74	48	16	3	0	0	265
	64	0	0	3	9	31	91	80	51	17	3	0	0	285
	62	0	0	3	10	33	98	85	55	19	3	0	0	305
	60	0	0	3	10	35	102	90	58	19	3	0	0	320
f Средний и южный таежный	58	0	2	6	19	51	77	74	54	26	10	2	0	320
	56	0	2	6	19	49	74	71	52	25	9	2	0	309
	54	0	2	6	19	49	74	71	52	25	9	2	0	310

Key: A - Forest B - Latitude, degrees C - Year
I - European part of the USSR II - Western Europe III - Eastern Siberia and the Far East
a - Coniferous b - Mixed and deciduous c - Deciduous d - Forest-steppe e - Northern taiga f - Middle and southern taiga
*Compiled by Ye. I. Nesmelova on the basis of: 1) Map of annual evaporation (E, mm). "Materialy mezhduvedomstvennogo soveshcheniya po probleme izucheniya i obosnovaniya rascheta ispareniya" [Materials of the Interdepartmental Conference on the Problem of Study and Substantiation of Calculation of Evaporation]. Valday, 1966; 2) "Trudy GGI," 1968, No 151.

In making a frost survey and compiling a frost forecast it is possible to use the very simple dependence

$$P = \alpha (t_n - t_a), \qquad (2.3.5)$$

Table 15 Turbulent heat transfer and its distribution in the USSR (kcal/ /cm^2 x months) (acc to data of "Atlas teplovogo balansa," 1963)

	Широта, град	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Европейская территория	68	0	0	0	0	0	2	2	1	1	0	0	0
	66	0	0	0	0	0	2	2	1	1	0	0	0
	64	0	0	0	0	1	2	2	1	1	0	0	0
	62	0	0	0	0	1	2	2	1	1	1	0	0
	60	0	0	0	0	1	1	2	1	1	1	0	0
	58	0	0	0	0	1	1	3	1	1	1	0	0
	56	0	0	0	0	1	2	3	2	1	1	0	0
	54	0	0	0	1	1	3	3	3	2	1	0	0
	52	0	0	0	1	2	4	4	3	2	1	0	0
Западная Сибирь	68	0	0	0	0	0	2	2	2	1	0	0	0
	66	0	0	0	0	0	2	2	1	1	0	0	0
	64	0	0	0	0	1	2	1	1	1	0	0	0
	62	0	0	0	0	1	2	1	1	1	1	0	0
	60	0	0	0	0	1	1	1	1	1	1	0	0
	58	0	0	0	0	1	1	2	1	1	1	0	0
	56	0	0	0	1	2	2	3	2	1	1	0	0
	54	0	0	0	1	2	3	3	2	1	1	0	0
	52	0	0	0	2	2	4	4	3	2	1	0	0
Восточная Сибирь и Дальний Восток	68	0	0	0	0	0	2	2	1	1	0	0	0
	66	0	0	0	0	0	2	1	1	1	0	0	0
	64	0	0	0	1	1	2	1	1	1	0	0	0
	62	0	0	1	1	1	2	1	1	1	1	0	0
	60	0	0	1	1	2	2	2	1	1	1	0	0
	58	0	0	1	1	2	2	2	1	1	1	0	0
	56	0	0	1	1	2	2	2	2	1	1	0	0
	54	0	0	1	1	2	2	2	2	1	1	0	0
	52	0	0	1	1	2	2	2	2	1	1	0	0

Key: A - Latitude, degrees I, II and III -- As for Table 14

where α is the coefficient of turbulent heat transfer between the soil and the atmosphere (kcal/m^2 x degrees x hrs).

The value of α can be determined in a survey from the equation of the radiation-heat balance if its components are obtained by direct measurements in the field, that is,

$$\alpha = \frac{R - LE - B}{t_п - t_в}. \qquad (2.3.6)$$

The coefficient α should be determined for each type of landscape (see example 14).

Calculation of the amount of cycled heat in the soil and rocks. The heat cycled in the soil in a half-year, according to V. A. Kudryavtsev, is determined with the equation [see (4.1.11) in Chapter 4, where the cycled heat is designated by the letter Q)

$$B = \sqrt{2}\, A_0 \sqrt{\frac{\lambda T C}{\pi}} \div \frac{(2A_{cp}C\,\xi_{2c} + Q_\Phi \xi)\, Q_\Phi \sqrt{\frac{\lambda T}{\pi C}}}{2A_{cp}C\,\xi_{2c} + Q_\Phi \xi + \sqrt{\frac{\lambda T}{\pi C}}\,(2A_{cp}C + Q_\Phi)},$$

where A_o is the amplitude of the annual fluctuations of temperature on the surface of the ground (its physical value in ^oC); T is the period, equal to a year, in hours; λ is the thermal conductivity of the soils in the grounds, kcal/m^3 x degrees; Q_{ph} is the temperature of the phase transformations of water in soils and grounds, kcal/m^3; ξ is the depth of the seasonal freezing (thawing) of the soil, m; A_{av} is the average amplitude in the layer of seasonal freezing (thawing), ^oC, determined with formula (4.1.3); ξ_{2c} is determined with formula (4.1.5).

It is evident from the presented equation that the value of B depends on the continental climate (through A_o), the height zonation and the latitudinal zonation (through t_ξ). It is very important that the heat cycles depend to a great degree on the thermophysical characteristics of the ground and the phase transitions of the water during freezing and thawing, which is connected with the influence of geological factors (the genesis, structure and composition of the rocks and the hydrogeological conditions) on the formation of the heat cycles.

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Chapter 3. Conductive and Convective Heat Transfer in Rocks and Their Freezing and Thawing

1. The Temperature Field of Rocks. Heat Transfer and Heat Cycles

In the upper layers of rocks subjected to very sharp thermal effects the temperature serves as a very active and variable parameter which characterizes the thermal state of the rocks and its changes.

The temperature distribution in rocks is called their temperature field.

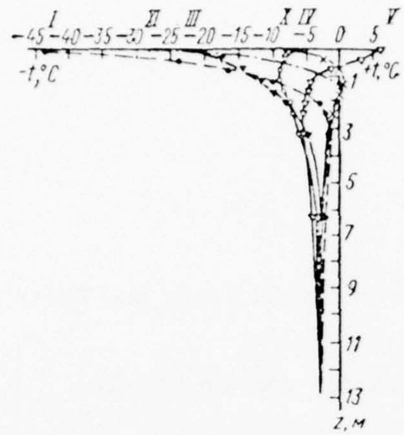
The temperature field in rocks is determined completely if the temperatures (t) are known at all points in the rock at each given moment of time (τ), that is, if the function $t(x, y, z, \tau)$ is known.

If the position of surfaces of equal temperatures does not vary in time, the temperature field is called stationary. But if $t'_{\tau}(x, y, z, \tau) \neq 0$ the position of surfaces of equal temperature varies in space and time and the temperature field is called nonstationary.

In practice the temperature field in rocks is judged most often on the basis of the data of observations in wells in which the temperature is measured through definite intervals in depth (z) and at given moments of time (τ). Three kinds of temperature curves can be constructed: 1) of temperature as a function of depth at different moments of time ($t = f(z)_{\tau = \text{constant}}$) (Figure 4); 2) of the variation of temperature as a function fo time at a given depth ($t = f(\tau)_{z = \text{constant}}$) (Figure 5); 3) of the variation of depth of a given isotherm as a function of time ($z = f(\tau)_{t = \text{constant}}$) (Figure 6). The last-mentioned kind of curves often is called a thermoisoplet.

It is possible to judge unequivocally the distribution and variation of temperatures in the volume of rocks on the basis of measurements of temperature in a single vertical well only when it is known that the isothermic surfaces are arranged horizontally (perpendicular to the well or in parallel with the day surface). In that case the problem of the temperature distribution becomes unidimensional.

However, if the isothermic surfaces are bent and not perpendicular to the wells, then to determine them it is necessary to measure the temperature in at least three wells close to one another and not lying on a straight line.



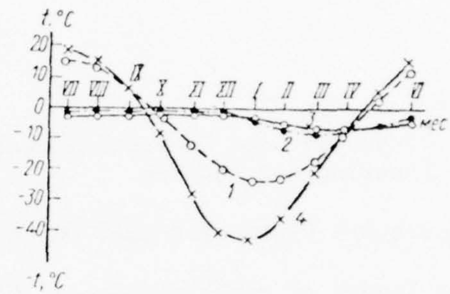


Figure 4. Curves of the variation of rock temperatures as a function of depth at different moments of time.

Figure 5. Curves of the variation of rock temperatures as a function of time at depths of 1 - 0.4 m, 2 - 2 m, 3 - 2.4 m, and 4 - the air

a

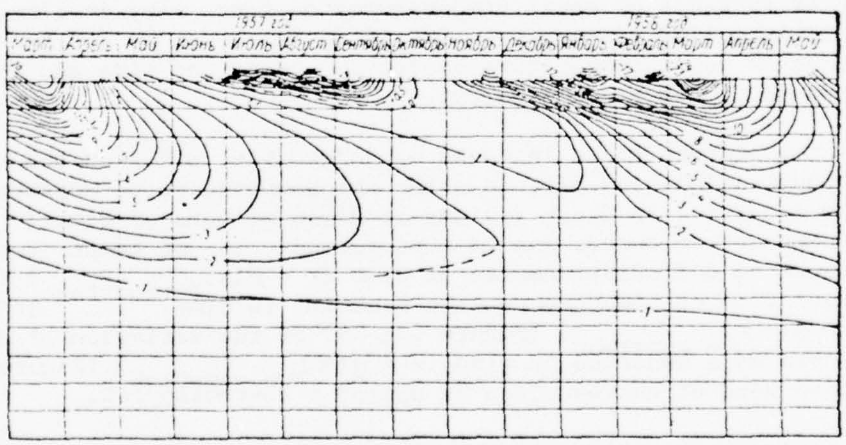


Figure 6. Curves of the variation of rock temperatures over a profile in time (thermoisoplets). a - Months (March, April, etc).

The definitive selection of the necessary number of wells is generally determined by the concrete geological and geographic conditions on each given section. The intensity of change of temperature in the direction of the normal to the isothermic surfaces ($\partial t/\partial n$, where n is the normal to the isosurface) is called the temperature gradient. The temperature gradient caused by the heat flux going from the depths of the Earth toward the surface is called the geothermal gradient. The reciprocal of the geothermal gradient is called the geothermal degree. The geothermal degree shows in what vertical distance the temperature changes by 1°C.

Temperature fields are established in rocks as a result of heat exchange of the latter with the surrounding medium (atmosphere). The heat exchange is expressed by the heat balance equation which connects the energy influx, transformation and expenditure. The heat exchange is expressed numerically by the amount of energy transformed from one form to another in a given volume of rock during the time interval under consideration.

In the climatic zones of the Earth in which there are seasonal variations of the arrival and outflow of energy, the annual period of variation of temperatures in rocks is subdivided into two parts: the cooling and the heating half-periods. The amount of heat arriving in the rock during the heating half-period and leaving it during the cooling half-period is called the heat cycled in the rock.

Very closely connected with those heat cycles in middle and northern latitudes are the processes of rock freezing and thawing described by nonlinear equations of heat and mass transfer.

2. The Equation of Thermal Conductivity

Processes of thermal conductivity (diffusion and filtration) are described by equations with parabolic second-order partial derivatives. For simplicity we will consider the process of heat propagation in a rod thermally insulated on the sides and sufficiently thin that at any moment of time the temperature can be considered identical at all points of the cross section (a unidimensional problem of thermal conductivity). To find an equation satisfying $t(z, \tau)$ we will formulate the physical laws defining the processes of heat propagation.

/The property of thermal conductivity/. If the temperature of a body is irregular, heat flows directed from points with higher to points with lower temperatures arise in it. The amount of heat flowing through the cross section z in the time interval $(\tau, \tau + d\tau)$ is equal to:

$$dQ = qF\, d\tau, \tag{3.2.1}$$

where $q(z, \tau) = -\lambda(z) \frac{\partial t}{\partial z}$ is the density of the heat flux equal to the amount of heat passing per unit of time (hour) through a unit of area F, in m^2. Here λ is the thermal conductivity coefficient of the rod, which depends on the material. The minus is explained by the fact that the heat flux is directed toward a reduction of temperature. Consequently the amount of heat Q, in kcal, passing in the time interval (τ_1, τ_2) through the cross section z is equal to

$$Q = -F \int_{\tau_1}^{\tau_2} \lambda(z) \frac{\partial t(z, \tau)}{\partial z}\, d\tau.$$

/The property of heat capacity/. The amount of heat needed for the heating of a body by $\Delta t^{o}C$ is equal to $Q = C_{o}\gamma v\Delta t$, where C_{o} is the specific heat, in kcal/kg x degree, γ is the density, kg/m^{3}, and v is the volume, in m^{3}.

If the temperature variation is different for different sections or the rod is not homogeneous, then

$$Q = F\int_{z_1}^{z_2} C_0(z)\,\gamma(z)\,\Delta t(z)\,dz.$$

/Heat sources/. Within a body heat can be released or absorbed (for example, during radioactive decay, chemical reactions, etc), which is characterized by the density of the heat sources $w(z, \tau)$ at any point z at the moment τ. As a result of the effect of sources on the section (z_1, z_2) in the time (τ_1, τ_2) the quantity of heat

$$Q = F\int_{\tau_1}^{\tau_2}\int_{z_1}^{z_2} \omega(z, \tau)\,dz\,d\tau.$$

is released (absorbed).

The Fourier equation describing the process of thermal conductivity in a unidimensional problem has the form

$$C_0(z, t)\,\gamma(z, t)\frac{\partial t(z, \tau)}{\partial \tau} = \frac{\partial}{\partial z}\left[\lambda(z, t)\frac{\partial t}{\partial z}\right] - \omega(z, \tau). \qquad (3.2.2)$$

In the particular case of a homogeneous medium without internal sources, equation (3.2.2) is written in the very simple form:

$$\frac{\partial t(z, \tau)}{\partial \tau} = a^2\frac{\partial^2 t}{\partial z^2},$$

where $\alpha^2 = \lambda/C_{o}\gamma$ is the coefficient of thermal conductivity. Henceforth we will use C to designate the volumetric specific heat $C_{o}\gamma$.

The equation of heat propagation in space (a three-dimensional problem) is written similarly. In that case

$$C\frac{\partial t(x, y, z, \tau)}{\partial \tau} = \frac{\partial}{\partial x}\left[\lambda(x, y, z, t)\frac{\partial t}{\partial x}\right] + \frac{\partial}{\partial y}\left[\lambda(x, y, z, t)\frac{\partial t}{\partial y}\right] + \frac{\partial}{\partial z}\left[\lambda(x, y, z, t)\frac{\partial t}{\partial z}\right] + \omega(x, y, z, \tau),$$

or in the very simple form

$$\frac{\partial t(x, y, z, \tau)}{\partial \tau} = a^2\,\Delta t, \qquad (3.2.3)$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ —

is a Laplace operator.

/The formulation of boundary-value problems/. To single out the only solution of a thermal conductivity equation it is necessary to add to the equation the initial and terminal boundary conditions. The initial condition consists in assigning the values of the function $t(z, \tau)$ at the initial moment τ_o. The boundary conditions differ as a function of the temperature regime on the boundaries. In frost studies three types of boundary conditions are usually considered:

1) type I, in which the temperature on the boundaries is given, for example, $t(\ell, \tau) = \phi_1(\tau)$, where $\phi_1(\tau)$ is given in a certain interval $\tau_o \leqslant \tau \leqslant \tau_1$; $\tau_1 - \tau_o$ is the time of investigation of the process;

2) type II, in which the value of the derivative

$$\frac{\partial t}{\partial z}(l, \tau) = \varphi_2(\tau).$$

is given on the boundary. This condition occurs if the value of the heat flux in the course of time is given;

3) type III, in which a linear combination of the function and the derivative is given

$$\frac{\partial t}{\partial z}(l, \tau) + at(l, \tau) = \varphi_3(\tau).$$

This condition corresponds to heat exchange in accordance with Newton's law on the surface of a body with a surrounding medium the temperature of which $\phi_3(\tau)$ is known.

An important particular case is the heat exchange of a body with a surrounding body through radiation. In that case the heat flux obtained by the surface is proportional to the difference of the fourth powers of the absolute temperatures of the surfaces participating in the heat exchange:

$$\frac{\partial t}{\partial z}(0, \tau) = \sigma\varkappa\,[t^4(0, \tau) - t^4(z, \tau)].$$

Here σ is the Stefan-Boltzmann constant; $\varkappa$ is a constant which depends on the ability of the body to absorb radiant energy and on the mutual disposition of the radiating and irradiated bodies. The boundary conditions on the different boundaries can be of different kinds, so that the number of boundary-value conditions is large. Also often used is the condition of complete thermal contact of two media (a multilayered medium) with fixed boundaries, which consists in continuity of temperature and the heat flux

$$t(z_1 - 0, \tau) = t(z_1 + 0, \tau),$$

$$\lambda(z_1 - 0)\left.\frac{\partial t}{\partial z}\right|_{z_1 - 0} = \lambda(z_1 + 0)\left.\frac{\partial t}{\partial z}\right|_{z_1 + 0},$$

where z_1 is the depth of contact of the layers.

3. Temperature Waves

The problem of the propagation of temperature waves in the ground without consideration of phase transitions is one of the first examples of the application of the mathematical theory of thermal conductivity developed by Fourier to the study of natural phenomena (Tikhonov and Samarskiy, 1953).

The temperature on the surface of the ground has a clearly expressed periodicity (daily, annual or perennial). The problem of conductive propagation of periodic temperature fluctuations in the ground, that is, determination of a periodically established temperature regime, is a task without initial conditions, since during multiple repetition of the temperature course on the surface the influence of the initial temperature drops and becomes much smaller than other factors which are neglected (for example, heterogeneous ground).

We will examine a solution of the problem of a periodically established regime for a homogeneous semirestricted rod in the region z > 0 (Tikhonov and Samarskiy, 1966) provided that given on the surface is

$$t(0, \tau) = t_0 + A\cos\omega\tau, \qquad (3.3.1)$$

or

$$t(0, \tau) = t_0 + A\sin\omega\tau, \qquad (3.3.2)$$

where t_o is the mean temperature during the period of oscillations, $\omega = 2\pi/T$ is the frequency, A is the amplitude of temperature oscillations on the surface of the ground and T is the period.

A limited solution of the posed problem under the conditions (3.3.1) or (3.3.2) has the respective forms

$$t(z, \tau) = t_0 + Ae^{-\sqrt{\frac{\omega}{2a^2}}z}\cos\left(\omega\tau - \sqrt{\frac{\omega}{2a^2}}z\right).$$

$$t(z, \tau) = t_0 + Ae^{-\sqrt{\frac{\omega}{2a^2}}z}\sin\left(\omega\tau - \sqrt{\frac{\omega}{2a^2}}z\right).$$

The posed problem is solved similarly in the case of a restricted rod when a constant temperature is given on the lower boundary. In that case the solution has a more complex form. If the boundary function represents a combination of harmonics of different frequencies or amplitudes, the solution because of the linearity of the problem is obtained by superposition of the solutions corresponding to the separate harmonics. In precisely the same way, in the case where a constant temperature gradient is given in the medium, for example, the geothermal gradient g, degrees/meter, a solution of the problem is obtained in the form

$$t(z, \tau) = t_0 + gz + Ae^{-\sqrt{\frac{\pi C}{\lambda T}}z}\sin\frac{2\pi}{T}\left(\tau - \frac{z}{2}\sqrt{\frac{CT}{\pi\lambda}}\right).$$

On the basis of the obtained solution Fourier derived the following dependences for the process of temperature wave propagation in the ground. During periodic temperature oscillations on the surface, in the course of a long time temperature oscillations with the same period are established in the ground, where:

1) the amplitude of oscillations decreases exponentially with depth

$$A(z) = A_0 e^{-z\sqrt{\frac{\pi C}{\lambda T}}} \qquad (3.3.4)$$ (Fourier's first law)

that is, when the depth changes according to an arithmetic progression the amplitude changes according to a geometric one;

2) the temperature oscillations in the ground occur with a shift of phases proportional to the depth

$$\delta = \frac{1}{2} z \sqrt{\frac{CT}{\pi\lambda}} \qquad \text{(Fourier's second law)}$$

3) the depth of penetration of temperature into the ground depends on the period of oscillations on the surface. For temperature oscillations with the periods T_1 and T_2 the depths z_1 and z_2 at which an identical relative temperature variation occurs are connected by the correlation

$$\frac{z_2}{z_1} = \sqrt{\frac{T_2}{T_1}} \qquad \text{(Fourier's third law)}$$

That law makes it possible to find the depth of penetration of oscillations with an identical amplitude as a function of the period.

From the presented solution of the thermal conductivity equation during periodic temperature oscillations on the surface of the ground flows an additional series of regularities of great importance in frost studies.

It follows from (3.3.4) that

$$\xi = \sqrt{\frac{\lambda T}{\pi C}} \ln \frac{A_0}{A_\xi}, \qquad (3.3.5)$$

where ξ is the thickness of the layer of ground on the surface of which the amplitude of temperature oscillations is equal to A_0 and on its basement to a certain $A_\xi = A(\xi)$.

In the case of annual temperature fluctuations at $A_\xi = \varepsilon$, where ε is the precision of measurement (usually $\varepsilon \leq 0.1\ ^{\circ}C$), within the limits of which it can be assumed that the oscillations at $z > \xi$ are practically damped, and ξ is the depth of propagation of annual temperature oscillations (the depth of "zero" annual amplitudes). It follows from (3.3.5) that the depth of the zero annual amplitudes increases with increase of A_0, λ and T and decreases with increase of C.

Expression (3.3.4), which describes the damping of amplitudes with depth, makes it possible to determine the annual heat cycled in a layer of soil with the thickness ξ. It follows from (3.3.3) that during periodic oscillations the temperature in a layer of ground with the thickness ξ varies in the range from minimal to maximal or by the doubled value of the amplitude of oscillations (Figure 7). Consequently, in a layer with the thickness ξ the heat cycled during the half-period Q_0 is equal to

$$Q_0 = 2(A_0 - A_\xi) \sqrt{\frac{\lambda T C}{\pi}}.$$

If we introduce the concept of the mean amplitude (A_m) for a layer with the thickness ξ, the expression for Q_0 can be written in the form

$$Q_0 = 2CA_{cp}\xi. \qquad (3.3.6)$$

Consequently, if we use (3.3.5) we obtain

$$A_{cp} = \frac{A_0 - A_\xi}{\ln \frac{A_0}{A_\xi}}. \qquad (3.3.7)$$

To determine the annual heat cycles it was assumed that the temperature during the year at each depth in the layer under consideration with the thickness ξ varies by twice the value of the temperature oscillations $2A_\xi$, that is, in accordance with the envelopes depicted on Figure 7. It is obvious that the envelopes represent fictitious curves, since the maximal and minimal temperatures reach different depths at different times. Because of that the heat cycles according to the envelopes are substantially overstated in comparison with those passing through the surface into the soil Q_3.

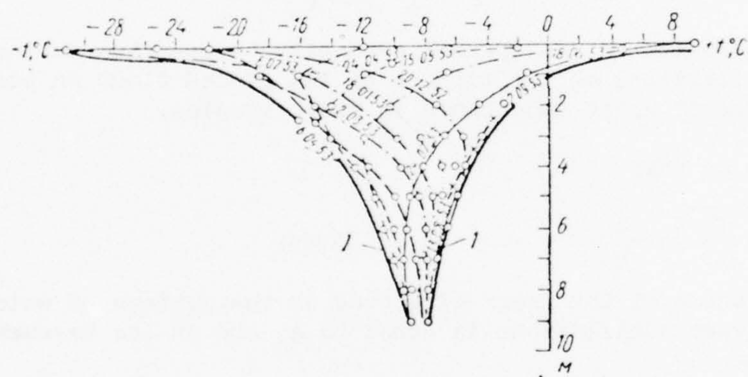


Figure 7. Temperature curves during the year and their envelopes (1).

Formula (3.3.3) is used to determine the latter.

During periodic oscillations the arrival of heat is noted during one half-period and the discharge of heat during the other. In that case the heat fluxes change signs. In the annual balance, when there is a periodically established regime, the inflow is equal to the outflow.

We will find an expression for the temperature gradient on the surface at any moment of time. To do that, differentiating (3.3.3) with respect to depth and substituting z = 0, we obtain (without consideration of the geometric gradient)

$$\left.\frac{\partial t}{\partial z}\right|_{z=0} = -\sqrt{\frac{\pi C}{\lambda T}}\, A_0 \left(\cos\frac{2\pi}{T}\tau \mp \sin\frac{2\pi}{T}\tau\right). \qquad (3.3.8)$$

If we equate (3.3.8) to zero we obtain for boundary conditions (3.3.2) two moments of inversion of sign of the heat fluxes during the annual cycle:

$$\tau_1 = \frac{3}{8} T \text{ и } \tau_2 = \frac{7}{8} T.$$

In the case of (3.3.1) we have analogously

$$\tau_1 = \frac{1}{8} T \text{ и } \tau_2 = \frac{5}{8} T.$$

Finally the inflow (outflow) part of the heat fluxes during the complete cycle will be equal to

$$Q_п = \sqrt{2}\,(A_0 - A_\xi)\sqrt{\frac{\lambda TC}{\pi}}.$$

If we compare the obtained expressions for the heat cycled Q_o according to the envelopes with those passing through the surface Q_s, it becomes evident that Q_o is greater than the real heat cycles Q_s by $\sqrt{2}$ times.

The number of heat cycles during an arbitrary time interval is obtained similarly during corresponding substitution of the integration limits.

By means of A_m the expression for the heat cycles through the surface of the soil for the layer ξ assumes the form

$$Q_п = \sqrt{2}\, A_{ср} C \xi. \qquad (3.3.9)$$

It is obvious that the described theory is valid during heat propagation in dry homogeneous ground, without consideration of phase transitions of water transformation.

4. Determination of the Configuration of the Frozen Rock Mass and the Temperature Field in It by Solving the Stationary Problem of Thermal Conductivity

The configuration of the rock mass and its thermal regime are in a definite regular connection with the conditions of heat transfer on the surface and the heat flux from the depths of the earth. The main difficulty in solving the given problem, especially in the multidimensional case, consists in the substantial nonstationary character of the temperature field, primarily in the layer of seasonal freezing and thawing. At the same time, in connection with perennial temperature fluctuations on the surface, the temperature field below the layer of annual fluctuations also is nonstationary and varies at any depth in any time segment. However, the amplitude of the perennial temperature fluctuations is considerably smaller than the amplitudes of their annual fluctuations. To simplify the problem, D. V. Redozubov proposed that below the layer of annual fluctuations the temperature field is practically stationary.

Starting from that, D. V. Redozubov (1959) proposed a simple method of "thermal prospecting for frost," based on solution of the Dirichlet problem (the first boundary-value problem), that is, solution of the stationary problem of thermal conductivity under type I boundary conditions. That method makes it possible to determine approximately the configuration of the permafrozen rock mass and the temperature field in it, using data on the basis of shallow wells and the known temperature at a fairly great depth (on the known geothermal gradient in the given region). The geothermal gradient is taken into consideration, as in Section 3, in accordance with the principle of superposition, as the problem is linear in such a formulation.

Exclusion of the layer with a sharply nonstationary temperature field leads to a need to introduce a bounding surface where a temperature distribution is given which is invariable in time (but depends on the coordinates in the case of a 2- or 3-dimensional problem). The bounding surface is selected as a function of the relief of the surface. A very simple case of the form of the bounding surface is a plane, and, in the two-dimensional case, a straight line bounding a half-plane. In the latter case the stationary temperature field for the homogeneous half-plane $-\infty < x < \infty$ and $z > 0$ is determined by solving the Laplace equation

$$\frac{\partial^2 t(x, z)}{\partial x^2} + \frac{\partial^2 t(x, z)}{\partial z^2} = 0 \qquad (3.4.1)$$

upon the condition $$t(x, 0) = \varphi(x), \qquad (3.4.2)$$

where $z = 0$ is the equation of the bounding surface and $\phi(x)$ is the given temperature distribution on it.

As is known, the solution of that problem is written with the Poisson integral

$$t(x, z) = \frac{z}{\pi} \int_{-\infty}^{\infty} \frac{\varphi(s)\, ds}{(s - x)^2 + z^2}. \qquad (3.4.3)$$

In the case of consideration of the heat flux from the depths of the earth, using superposition of solutions we have

$$t(x, z) = gz + \frac{z}{\pi} \int_{-\infty}^{\infty} \frac{\varphi(s)\, ds}{(s - x)^2 + z^2}.$$

In a number of cases, instead of the geothermal gradient it is more convenient to give the temperature at the depth h, which ought to be considerably greater than the depth of propagation of negative temperatures. That condition is satisfied by the data of deep wells: in practice, t(x, h) depends little on the form of the relief and the conditions on the bounding surface.

As a result the task is reduced to solution of equation (3.4.1) for the band bounded by the lines $z = 0$ and $z = h$ (Figure 8a) under condition (3.4.2), and also

$$t(x, h) = \psi(x). \qquad (3.4.4)$$

The last-mentioned task is solved by means of conformal transformation in a complex region.

At first the band with the height h is conformally depicted in a band with the width π by means of the transformation $\omega = p + iq = \frac{\pi}{h}(x + zi)$ and then, using the depicting function $v = a + ib = e^{\omega} = e^{\frac{\pi}{h}(x + iz)}$, in a half-plane with a positive value of b.

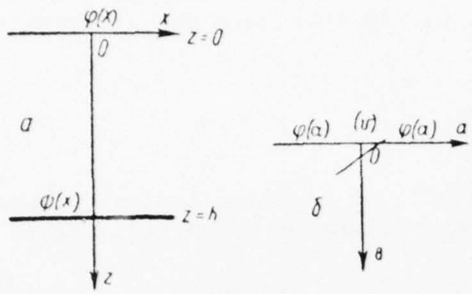


Figure 8. To solve the Dirichlet problem: a - for a layer with the thickness h in the coordinates x and z; b - for a half-plane in the coordinates a and b.

The correlation between the initial and final contours is established with the formulas:

$$a = e^{\frac{\pi}{h}x} \cos \frac{\pi}{h} z,$$

$$b = e^{\frac{\pi}{h}x} \sin \frac{\pi}{h} z. \quad (3.4.5)$$

As can readily be seen from (3.4.5), the boundaries of the initial band in that case go over into the real axis of the plane v (Figure 8b), with the line z = 0 going over into the half-line b = 0, a > 0 and the line z = h into the half-line b = 0, a < 0.

As a result, in the region τ a solution of the posed problem will be

$$t(a, b) = \frac{b}{\pi} \int_{-\infty}^{\infty} \frac{\mu(s)\, ds}{(s-a)^2 + b^2}, \quad (3.4.6)$$

where $\mu(a)$ is the representation of the functions $\phi(x)$ and $\psi(x)$ after transformation in the corresponding intervals of the axis b = 0. If we find t(a, b) and replace a and b by x and z we obtain a formula for calculation of the initial stationary temperature field.

In the case where $\phi(x)$ is described by different formulas on different sections, it is necessary to find with formulas (3.4.5) the new intervals of those sections after representation in (2) and divide (3.4.6) by the sum of the corresponding integrals. In that case, (3.4.6) is integrated without difficulty and uniformly for all sections with an analogous type of $\mu(a)$. Calculation of such a problem is more complex if the bounding surface is broken and is done with use of a Christoffel-Schwartz transform.

Two-dimensional schemes are used directly, in particular, to investigate stationary fields in vertical sections perpendicular to rivers and extended runoff belts. In the same cases, where the boundary conditions on the bounding surface (plane) depend on both coordinates and, consequently, determine the three-dimensional temperature field, the solution of the problem is made much more difficult. Very attainable is the case where the temperature on the bounding plane is invariable within concentric circles with the radii R_i and temperatures T_i between them, i = 1, 2, ..., n. In that case the solution at any point of the vertical axis has the form

$$t(0, 0, z) = gz + T_1 + \sum_{i=2}^{n} \frac{(T_i - T_{i-1}) z}{\sqrt{R_{i-1}^2 + z^2}}.$$

Such temperature distributions are characteristic of circular bodies of water, closed depressions and separate elevations on sections of convergent frost.

Among the shortcomings of the scheme proposed by D. V. Redozubov is disregard of the heterogeneity of the medium and the internal heat sources. In addition, the abstraction connected with the adopted stationary scheme does not take into consideration the perennial variation of climate, as a result of which "thermal prospecting for frost" can only approximately indicate the disposition of the lower boundary of the frozen rocks without consideration of its dynamics in time (aggradation or degradation of the frost).

D. V. Redozubov's method has been successfully used recently for individual cases in the calculation of the limiting basin of thawing under structures where there are permafrozen rock masses.

5. Formulation of the Problem of the Freezing and Thawing of Rocks

The freezing and thawing of moist ground is a complex thermodynamic process which takes place in a heterogeneous capillary-porous medium. The problem of the dynamics of that process in time is among the most complex of mathematical physics. The main difficulty in solving that problem is the need to take into consideration variation of the aggregate state and thermophysical characteristics of the medium, as a result of which the problem becomes nonlinear. In addition, during freezing, simultaneously with change of the temperature field there is mass transfer caused by the movement of moisture. In a considerable number of cases, when during freezing there is an absence of intensive frost heaving connected with moisture transfer, for practical purposes it is possible to limit the investigation of the process of freezing of moist ground by calculating its thermal regime with consideration of the phase transitions of water. By virtue of the fact that, depending on the physical properties of the ground, phase transitions can occur both completely at the freezing temperature (coarsely dispersed soils) and also in the spectrum of temperatures (finely dispersed), two formulations of the problem of freezing without consideration of migration are possible.

We will examine separately the mathematical formulation of each of those problems. For simplicity we will examine a two-phase medium (one mobile interface).

Here and henceforth by a phase is understood a zone, the number of mobile interfaces always being assumed to be one less than the number of phases. Therefore by a single-phase problem will be understood a problem in a region with immobile boundaries in the absence of phase transitions. A case where there is one phase but one or two boundaries of the region of investigation are mobile (for example, the problem of ablation), is a particular case of the corresponding multiphase problem.

1) Formulation of the Problem of the Freezing (Thawing) of Homogeneous Soil With the Formation of an Interface (the Stefan Problem)

During transition of the ground temperature through the critical value there is a sudden change of the physical state of the ground. On the surface where phase transitions occur (the mobile interface) the melting (solidification) temperature is always preserved which without limitation of generality can be considered equal to zero.

It is assumed that during movement of the interface the heat of phase transformations of water (Q_{ph}) is completely released. In each of two zones (the upper is limited to the plane z = 0 and the lower from below to the plane $z = \ell$) the sought temperature distribution functions in the two zones $t_i(z, \tau)$ i = 1 and 2, satisfy the Fourier condition, the boundary-value conditions being given.

The thermophysical conditions in the two zones -- the thermal conductivity λ, the thermal conductivity a^2, the moisture w and the density γ -- are constant and given. During transition through the interface the thermophysical constants change suddenly. The change of density of the heat-conducting medium is disregarded and processes of convection and radiation are not taken into consideration.

The mathematical formulation of a unidimensional Stefan problem in the case a two-phase homogeneous medium is as follows (Tikhonov and Samarskiy, 1966).

In each zone the process is described by a Fourier equation:

$$\left.\begin{aligned} \frac{\partial t_1(z,\tau)}{\partial \tau} &= a_1^2 \frac{\partial^2 t_1(z,\tau)}{\partial z^2} \text{ для } 0 < z < \xi(\tau), \\ \frac{\partial t_2(z,\tau)}{\partial \tau} &= a_2^2 \frac{\partial^2 t_2(z,\tau)}{\partial z^2} \text{ для } \xi(\tau) < z < l \end{aligned}\right\} \quad (3.5.1)$$

with the boundary conditions (in the case of the first boundary-value problem)

$$\left.\begin{aligned} & t_1(0,\tau) = \Phi_1(\tau), \\ & t_1(z,0) = \varphi_1(z), \text{ при } 0 < z < \xi(0), \\ & t_2(l,\tau) = \Phi_2(\tau), \\ & t_2(z,0) = \varphi_2(z), \text{ при } \xi(0) < z < l, \\ & \text{причем } \xi(0) = \xi_0 > 0. \end{aligned}\right\} \quad (3.5.2)$$

The following conditions are fulfilled on the mobile interface $z = \xi(\tau)$

$$t_1[\xi(\tau),\tau] = t_2[\xi(\tau),\tau] = t_0 = 0, \quad (3.5.3)$$

$$\lambda_1 \frac{\partial t_1(z,\tau)}{\partial z}\bigg|_{z=\xi(\tau)} - \lambda_2 \frac{\partial t_2(z,\tau)}{\partial z}\bigg|_{z=\xi(\tau)} = Q_\Phi \xi'(\tau). \quad (3.5.4)$$

$Q_\phi = Q_{ph}$

The subscripts 1 and 2 relate to the solid and liquid zones respectively, and Q_{ph} is the heat of phase transformations of water in one cubic meter of ground.

Condition (3.5.4), which determines the rate of advance of the front, does not depend on the course of the process of freezing or thawing if on the left side the first term corresponds to the flux in the frozen zone. In calculations of the seasonal freezing or thawing the depth of the annual temperature fluctuations usually is considered to be ℓ.

In such a formulation, the problem, in spite of the linearity of the equations of thermal conductivity in each of the zones, is classed as nonlinear in the sense of conditions on the mobile interface (Stefan, 1889).

This is of enormous importance from the point of view of finding a solution in various conditions, since in the nonlinear problem it is impossible to apply the method of superposition of solutions. In connection with that, any change in the boundary conditions leads to a need for repeated solution of the problem in complete volume. Thus, for example, the problem of freezing at the temperature of the surface, representing a superposition of several oscillations, cannot be solved in the form of a linear combination of corresponding solutions for each of the oscillations.

In calculations of seasonal or permanent freezing (thawing) the process, as a rule, is reduced to a multifront problem. In that case the Stefan problem is written in the following manner. Let at $\tau = 0$ there be an n-phase medium, $n \geqslant 2$. In that case the number of mobile boundaries is n - 1. In each of the zones (i = 1, 2, ..., n) the following equations are examined

$$\frac{\partial t_i(z,\tau)}{\partial \tau} = a_i^2 \frac{\partial^2 t_i}{\partial z^2}, \quad \xi_{i-1}(\tau) < z < \xi_i(\tau), \quad \tau > 0 \qquad (3.5.5)$$

under the given boundary conditions (on immobile boundaries $t_1(0, \tau) = \phi_0(\tau)$, $t_n(\ell, \tau) = \phi_n(\tau)$ and the initial conditions $t_i(z, 0) = \phi_i(z)$; $\xi_{i-1}(0) < z < \xi_i(0)$, where $\xi_0(\tau) \equiv 0$, $\xi_n(\tau) \equiv \ell$, and $\xi_{i-1}(0) < \xi_i(0)$.

The following conditions are fulfilled on mobile interfaces

$$t_i(\xi_i(\tau), \tau] = t_{i+1}[\xi_i(\tau), \tau] = 0, \qquad (3.5.6)$$

$$\lambda_i \frac{\partial t_i}{\partial z}\Big|_{z=\xi_i(\tau)} - \lambda_{i+1} \frac{\partial t_{i+1}}{\partial z}\Big|_{z=\xi_i(\tau)} = (-1)^{i+1} Q_i \xi_i'(\tau). \qquad (3.5.7)$$

If the first is a zone of thawing, then in the last-mentioned condition the right side is taken with the opposite sign.

2. Formulation of the Problem of Freezing (Thawing) of Rocks With Consideration of Unfrozen Water (With the Formation of a Zone of Freezing)

In real soils free moisture freezes at a certain temperature near $0^{\circ}C$, and combined freezes in a certain range of negative temperatures in accordance with the curve of unfrozen water (Figure 9). Without limitation of generality one can consider $0^{\circ}C$ to be the temperature of the start of freezing.

During freezing of the ground a zone of freezing forms in which, side by side with water unfrozen at the given temperature, there also are ice crystals.

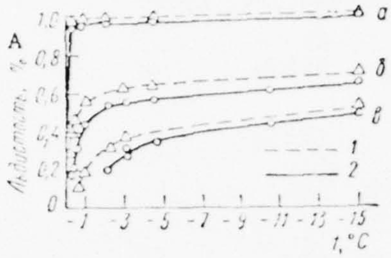


Figure 9. Curves of freezing (ice-content) (1) and thawing (2) of typical dispersed rocks according to Z. A. Nersesova: a - sand; b - loam; c - clay A - Ice-content, %

The interface of that zone with thawed rocks is the zero isotherm, that is, the mobile interface. In the zone of freezing there is a continuous release of the heat of phase transitions of water in a quantity proportional to the tangent of the angle of inclination of the curve of ice formation $i(t)$ to the axis of temperatures and the rate of cooling.

Thus in the freezing zone there are continuously distributed heat sources:

$$\omega(z, t) = \mu \frac{\partial i(z, t)}{\partial \tau} = \mu \frac{\partial i(z, t)}{\partial t} \frac{\partial t}{\partial \tau}$$

(the derivation of the formula is presented in the book of V. S. Luk'yanov and M. D. Golovko, 1957). Here μ = 80,000 kcal/m^3 is the freezing heat of 1 cubic meter of water.

The situation is similar during thawing. It is essential that, since the ice content increases with lowering of the temperature, $i'(t) < 0$.

At the same time, on the interface, where the temperature is always constant and equal to that of the start of phase transitions, there are phase transformations of free water similar to that described earlier during consideration of the Stefan problem. If the natural moisture of the ground is equal to w, the amount of free water is equal to $w - w_{un}(0)$, where $w_{un}(t)$ is the unfrozen moisture at the temperature t, when $t \leq 0$.

Thus, in the case of freezing (thawing) of the ground in the spectrum of temperatures the thermophysical characteristics in both zones change suddenly during transition through the interface. However, in the freezing (thawing) zone in the given case the thermophysical characteristics depend substantially on the temperature*.

Thus, in the case of freezing (thawing) of the ground with consideration of phase transformations of the unfrozen water in freezing zones, the temperature field is described by a quasilinear equation.

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*In that case it is assumed that during change of the ice content the amount of unfrozen water remains practically unchanged.

For the case of freezing of the ground in the spectrum of negative temperatures the mathematical formulation of the problem has the form (the first zone is assumed to be frozen) (Dostovalov and Kudryavtsev, 1969)

$$C_1(z, t)\frac{\partial t_1(z, \tau)}{\partial \tau} = \frac{\partial}{\partial z}\left[\lambda_1(z, t)\frac{\partial t_1}{\partial z}\right], \quad 0 < z < \xi(\tau), \qquad (3.5.8)$$

$$C_2(z)\frac{\partial t_2(z, \tau)}{\partial \tau} = \frac{\partial}{\partial z}\left[\lambda_2(z)\frac{\partial t_2}{\partial z}\right], \quad \xi(\tau) < z < l. \qquad (3.5.9)$$

On the mobile interface

$$t_i[\xi(\tau), \tau] = 0, \quad i = 1, 2, \qquad (3.5.10)$$

$$\lambda_1[\xi(\tau), 0]\frac{\partial t_1(z, \tau)}{\partial z}\bigg|_{\xi(\tau)} - \lambda_2[\xi(\tau)]\frac{\partial t_2(z, \tau)}{\partial z}\bigg|_{\xi(\tau)} = Q_\phi(\xi)\,\xi'(\tau). \quad (3.5.11)$$

Here $\lambda_1(z, t)$ is the thermal conductivity of the ground in the freezing zone at the temperature $t^\circ C$ and the summary moisture $\overline{w}(t)$, $\overline{w}(t) = v[w - w_{un}(t) + w_{un}(t)$, v is the coefficient of volumetric expansion during the freezing of free water, $Q_{ph} = \mu[w - w_{un}(0)]$ is the heat of phase transitions of free water during the freezing (thawing) of 1 m^3 of rock, $C_1(z, t)$ is the effective heat capacity in the freezing zone and $\lambda_2(z)$ and $C_2(z)$ are the thermal conductivity and heat capacity respectively of the thawed ground in the Stefan problem. Here and henceforth one has in mind the moisture content of the ground by volume.

6. Brief Survey of Particular Solutions of the Stefan Problem

The first attempt to solve the problem of thermal conductivity with consideration of release of the heat of phase transitions on a mobile interface was made by Lame and Clapeyron (1831). They examined the problem of the cooling of an originally fused homogeneous sphere at zero temperature on the surface with reference to the solution of the question of the solidification of the earth. The depth of freezing of the ground was calculated for the first time by Zahlschutz (1862), who obtained a very simple formula in the case of a zero initial ground temperature. That formula later became known as the "Stefan formula."

The Austrian mathematician Stefan (1889) made an enormous contribution to the solution of the problem which later was named for him. In particular, he is credited with a rigorous solution of the self-modeling problem for a semi-restricted homogeneous medium ("the classical Stefan problem").

Later, starting from practical needs, approximate methods of solving the Stefan problem received intensive development. An important role was played here by the first method of L. S. Leybenzon (1931), used by him to solve the problem of the time of freezing of an oil pipeline. It was used later by S. S. Kovner in calculating the time of freezing of spheres, A. A. Charnyy (1940) to calculate the freezing of the ground around a well, and also by M. M. Krylov (1934) and others.

Of great importance from the point of view of practical use is the formula proposed by V. S. Luk'yanov and nomographed by M. D. Golovko (1955), in which heat cycles on account of heat capacity in the frozen zone and warming of the surface of the ground by means of insulation were taken into account.

In the Department of Geocryology formulas have been developed for calculating the depths of freezing and heat cycles during periodic temperature fluctuations on the surface of the ground (Kudryavtsev and Melamed, 1963, 1965), which are examined in detail in Chapter 4.

An enormous influence was exerted on the development of the question and the solution of theoretical and practical problems connected with the freezing and thawing of the ground by analog computers, of which one should note first of all the hydraulic integrator of the system of V. S. Luk'yanov.

The solution of single- and multifront Stefan problems in general form under arbitrary boundary conditions has been widely developed -- by L. I. Rubenshteyn (1947), V. G. Melamed (1957), F. P. Vasil'yev (1964), A. B. Uspenskiy (1968), etc. In addition, methods of solving the self-modeling problem in the case of freezing and thawing of ground in a range of temperatures have been substantially developed (Melamed, 1963). However, because of the complexity of the problem it can be solved with the enumerated algorithms only by using electronic computers.

7. Solutions of the Stefan Problem

1. Solution of the Classical Stefan Problem

The classical Stefan problem is the name given a very simple self-modeling problem of freezing and thawing in a homogeneous isotropic medium under constant boundary conditions. It is assumed that the temperature on the surface at $\tau = 0 + 0$ changes instantaneously and becomes equal to a constant different in sign* from the initial distribution. In that case a mobile interface appears, the rate of advance of which (side by side with the temperature fields in both zones) is subject to determination.

Under the indicated conditions the problem of freezing (thawing) is reduced to solution of equations (3.5.1), (3.5.3) and (3.5.4) under the boundary conditions

$$t_1(0, \tau) \equiv T_1 = \text{const}, \tag{3.7.1}$$

$$t_2(z, 0) \equiv T_2 = \text{const}, \ z > 0, \ \xi(0) = 0. \tag{3.7.2}$$

In the case of freezing $T_1 < 0$ and $T_2 \geqslant 0 + 0$, and in the case of thawing $T_1 > 0$ and $T_2 \leqslant 0 - 0$.

A solution of the posed single-front problem is found in the form (Tikhonov and Samarskiy, 1953)

*The temperature of the phase transformations of water is assumed to be zero.

$$t_1(z, \tau) = A_1 + B_1 \operatorname{erf}\left(\frac{z}{2\sqrt{a_1^2\tau}}\right),$$

$$t_2(z, \tau) = A_2 + B_2 \operatorname{erf}\left(\frac{z}{2\sqrt{a_2^2\tau}}\right),$$

where A_i and B_i, $i = 1$ and 2, are unknown constants, and is the integral of errors.

It is obvious that the functions $t_i(z, \tau)$, $i = 1$ and 2, satisfy the Fourier equation. If we substitute $t_i(z, \tau)$ in (3.5.3) we obtain

$$A_1 + B_1 \operatorname{erf}\left(\frac{\xi(\tau)}{2\sqrt{a_1^2\tau}}\right) = 0, \quad A_2 + B_2 \operatorname{erf}\left(\frac{\xi(\tau)}{2\sqrt{a_2^2\tau}}\right) = 0. \qquad (3.7.3)$$

The conditions (3.7.3) are fulfilled at all values of τ, which is possible only upon the condition

$$\xi(\tau) = \beta\sqrt{\tau}, \qquad (3.7.4)$$

where β is a certain constant.

In the final account the solution of the problem in the case of freezing is reduced to seeking the root of the equation, which flows from (3.5.4):

$$\lambda_1 \frac{T_1 e^{-\frac{\beta^2}{4a_1^2}}}{a_1 \operatorname{erf}\left(\frac{\beta}{2a_1}\right)} + \lambda_2 \frac{T_2 e^{-\frac{\beta}{4a_2^2}}}{a_2\left[1 - \operatorname{erf}\left(\frac{\beta}{2a_2}\right)\right]} = -Q_\phi \beta \frac{\sqrt{\pi}}{2}. \qquad (3.7.5)$$

The existence of the positive root β of that equation when the signs of T_1 and T_2 are different follows from the fact that during change of β from 0 to ∞ its left side continuously varies from $-\infty$ to $+\infty$ and its right from 0 to $-\infty$. The singularity of the root flows from the fact that both the left and right sides of (3.7.5) are monotonic functions of β. On the basis of that, β can be readily found by the method of selection.

The self-modeling multifront problem is examined quite analogously. Proof of the existence and singularity of the solution of that problem were presented in L. I. Rubenshteyn's well-known book, "Problema Stefana" [The Stefan Problem] (1967).

In addition, if it is assumed that the curve of unfrozen water has a linear form, the problem is reduced to the classical. Upon such an assumption the effective heat capacity remains constant. It is obvious, however, that such an attempt to take into consideration the phase transformations of water is restrictive from the practical point of view. At the same time, approximation of the curve of unfrozen water by several links of a broken line leads to a system of transcendental equations. the numerical solution of which is extremely difficult in practice.

2. Approximate Solutions of the Stefan Problem

A. The Stefan Formula for Determining the Depth of Seasonal and Permanent Freezing (Thawing) of Rocks

In connection with the fact that the numerical solution of transcendental equation (3.7.5) presents some difficulties, for rough approximate calculations a formula often is used which in the literature is called the Stefan formula (first derived, as indicated above, by Zahlschutz).

We will examine the freezing of a semirestricted homogeneous medium present at the temperature of phase transitions $t(0) = 0$. At the initial moment of time the temperature $T_1 < 0$ is given instantaneously on the surface and is later kept constant.

For maximal simplification it is assumed that the temperature distribution in the upper zone is subject to a rectilinear law. Hence it follows that at any point of it (including on the interface from the direction of the upper zone) the heat flux is equal to $\lambda_1(T_1/\xi(\tau))$, where $\xi(\tau)$ is the depth of freezing at the arbitrary moment of time τ.

The assumption that in the lower zone the temperature is constant and equal to 0°C has the result that the heat flux from below toward the interface at any values $\tau > 0$ is equal to 0. Then the Stefan condition will assume the form

$$\pm \lambda_1 \frac{T_1}{\xi(\tau)} = Q_{\phi} \xi'(\tau).$$

The minus sign applies to the case of freezing and the plus sign to thawing. Consequently, if we integrate with respect to τ from 0 to an arbitrary τ_0 (in that case ξ varies from 0 to $\xi(\tau)$), we obtain

$$\xi(\tau_0) = a\sqrt{\tau_0}, \qquad (3.7.6)$$

where

$$a = \sqrt{\frac{2\lambda_1 |T_1|}{Q_{\phi}}}.$$

The obtained expression can be rewritten differently in the form

$$\xi(\tau) = \sqrt{\frac{2\lambda_1 \Omega}{Q_{\phi}}}, \qquad (3.7.7)$$

where $\Omega = \tau|T_1|$ is the sum of the heat or frost degree-hours.

It is obvious that in practical calculations, especially when $T_2 \neq 0$, the results obtained with formula (3.7.6) will be considerably overstated. Nevertheless, because of its exceptional simplicity that formula is often used in practice in estimates even in the case of a variable temperature at the surface. For that it is necessary only to determine the value of Ω on a seasonal temperature diagram. The calculations with (3.7.6) can be simplified somewhat if data are available on the depth of freezing (thawing) for a definite year on a specific area bare of snow cover. Actually, let $\Omega = \Omega_1$ and $\xi = \xi_1$.

be known for that year. Then for any other year when $\Omega = \Omega_2$ the corresponding depth of freezing (thawing) on the same area can be found in accordance with (3.7.7) in the form of $\xi_2 = \xi_1 \sqrt{\frac{\Omega_2}{\Omega_1}}$.

B. Determination of the Depth of Seasonal and Permanent Freezing (Thawing) of Rocks by the Leybenzon Method

The idea of the method, based on variational principles, is that the temperature distribution in both zones is given in a very simple form but one satisfying the boundary conditions of the problem. Then by means of the Stefan condition an equation is found which describes the change of the interface $\xi(\tau)$.

The method under consideration is especially effective with reference to the Stefan problem because the heat cycles occurring on the phase transitions considerably exceed the imprecisions in the heat cycles arising during distortion of the temperature field within the zones. The temperature distribution functions in the two zones are selected in a very simple form so that the Stefan condition may be integrated in an explicit form.

We will examine an approximate solution of the classical Stefan problem, described by (3.5.1) under the conditions (3.5.3), (3.5.4), (3.7.1) and (3.7.2) by the Leybenzon method for a semirestricted rod. The temperature distribution in the two zones is assumed to be the following:

$$t_1(z, \tau) = T_1\left(1 - \frac{z}{\xi}\right),$$

$$t_2(z, \tau) = T_2 \operatorname{erf} \frac{z - \xi}{2\sqrt{a_2^2 \tau}}.$$

It is obvious that when the value of ξ is fixed the runctions $t_i(z, \tau)$, i = = 1 and 2, satisfy both the equation of thermal conductivity and the corresponding boundary conditions. It also is obvious that

$$\left.\frac{\partial t_1}{\partial z}\right|_{z=\xi} = -\frac{T_1}{\xi}, \quad \left.\frac{\partial t_2}{\partial z}\right|_{z=\xi} = \frac{T_2}{\sqrt{\pi a_2^2 \tau}}.$$

As a result the Stefan condition in that case will assume the form (the upper sign corresponds to freezing and the lower to thawing)

$$\pm\left(-\lambda_1 \frac{T_1}{\xi} - T_2 \sqrt{\frac{\lambda_2 C_2}{\pi \tau}}\right) = Q_\phi \xi'. \quad (3.7.8)$$

A solution of equation (3.5.1) under the condition $\xi(0) = 0$ is

$$\xi(\tau) = \beta\sqrt{\tau}, \quad (3.7.9)$$

where β -- a constant -- is a positive (in the physical sense) root of the quadratic equation to which (3.7.8) is reduced upon substitution of (3.7.9):

$$\mp \frac{\lambda_1 T_1}{\beta} \mp T_2 \sqrt{\frac{\lambda_2 C_2}{\pi}} = \frac{1}{2} Q_\phi \beta.$$

As a result, if we take into consideration the signs of T_1 and T_2 during freezing and thawing, we have

$$\beta = \sqrt{\mp \frac{2\lambda_1 T_1}{Q_\phi} + \frac{T_2^2 \lambda_2 C_2}{\pi Q_\phi^2}} \mp \frac{T_2}{Q_\phi} \sqrt{\frac{\lambda_2 C_2}{\pi}}. \qquad (3.7.10)$$

It is obvious that the obtained solution is a generalization of the above-considered Stefan formula. Therefore, as was to be expected, in the particular case at $T_2 = 0$ the Stefan formula (3.7.6) follows from (3.7.10). At $T_2 \neq 0$ the value of β is known to be smaller than that of α determined with (3.7.6). This agrees with the physical picture, since at $T_2 \neq 0$ movement of the interface is hindered by the flow of heat from the lower zone, which is a function of T_2.

Using the Leybenzon method, one can obtain analogously an approximate solution of the problem of the rate of freezing (thawing) and the dynamics of the temperature field around an infinitely circular cylinder, on the walls of which a constant temperature T_1 is maintained. This task is of considerable interest in the approximate calculation of the radius of freezing columns, basins of thawing around underground pipelines, etc. For simplicity we will examine the case where the temperature of the ground around the cylinder at the initial moment is equal to the temperature of the start of phase transformations T_o. As is known, the dynamics of the temperature field around an infinite cylinder in the axisymmetric case (in that case the temperature depends on only one coordinate -- the radius r) is described by the equation

$$a^2 \left(\frac{\partial^2 t(r, \tau)}{\partial r^2} + \frac{1}{r} \frac{\partial t(r, \tau)}{\partial r} \right) = \frac{\partial t(r, \tau)}{\partial \tau}. \qquad (3.7.11)$$

in the region $r_o < r < R(\tau)$, where $R(\tau)$ is the radius of freezing (thawing) of the ground around the cylinder and r_o is the cylinder radius. It can readily be seen that in the given case the Stefan condition has the same form as in the above-considered case of a rod. As in the preceding problem, we will assume $t(r, \tau)$ in the following form

$$t(r, \tau) = \frac{T_0 \ln \frac{r}{r_0} - T_1 \ln \frac{r}{R(\tau)}}{\ln \frac{R(\tau)}{r_0}}.$$

In that case $t(r, \tau)$ satisfies (3.7.11) at $R(\tau)$ = constant, and also the boundary conditions $t(r_o, \tau) = T_1$, $t(R(\tau), \tau) = T_o$. The sought connection between the time τ and the position of the interface $R(\tau)$ has the form

$$\tau = \frac{Q r_0^2}{2\lambda (T_0 - T_1)} F\left(\frac{R(\tau)}{r_0} \right), \qquad (3.7.12)$$

where

$$F(s) = s^2 \ln s - \frac{1}{2}(s^2 - 1).$$

The solution of transcendental equation (3.7.12) presents no difficulties. As a result, for any specific conditions, (that is, at given values of r_o, λ, Q and T_1) we obtain from (3.7.12) at each fixed time of freezing (thawing) the corresponding value of the basin of freezing (thawing) ground. It can readily be seen that the use of the obtained approximate solution, because of neglect of the second phase, gives under specific conditions a result with some reserve, the amount of which increases with increase (in the absolute value) of the initial temperature of the ground. Let us note that in the case of freezing or thawing around a spherical source the problem is readily reduced to the above-considered problem for a rod with substitution of $v = rt(r, \tau)$. In that case, however, the boundary conditions become proportional to the value of r. In practice in (3.17.12) it is more convenient to give R and find τ.

C. V. S. Luk'yanov's Formula for Determining the Depth of Seasonal Freezing (Thawing) of Rocks

An important shortcoming of the above-considered formulas is neglect of heat capacity in the upper zone. V. S. Luk'yanov has proposed an approximate formula which considers both the indicated factor and the presence of spontaneous warming of the surface of the ground (snow cover and various insulating coverings). The formulation of the problem in that case differs from that considered above also by the fact that the lower zone is discarded and its influence is replaced by the heat flux q from the bottom toward the interface. The thermal insulation of the surface of the ground is replaced by the introduction of the thermal resistance of the insulation (without considering the heat capacity).

In that case both the value q and the thermal resistance of the insulation are assumed to be constant during the time interval under consideration and equal to the average values. In addition, taken into consideration is the coefficient of heat transfer from the surface, assumed to be constant (equal to 20 $kcal/m^2 \times hr \times degree$). In connection with that, in the calculations use is made of the thickness of the layer of ground S, in meters, the thermal resistance of which is equal to the sum of the thermal resistances of the insulation layer and heat transfer from the surface. The temperature distribution in the upper zone is assumed to be rectilinear and the surface temperature to be constant.

Since under those assumptions the problem is reduced to a transcendental equation, M. D. Golovko prepared a grid nomogram which makes it possible to readily find its solution with a precision adequate for practice. Usually in calculations with the formula of V. S. Luk'yanov, in designating the heat flux averaged for the period, from the lower-lying layers to the interface, a map of isolines compiled by the authors on the basis of the processing of long-term data for a large territory of the USSR is used. The procedure of calculations with that formula was examined in detail in the work of V. S. Luk'yanov and M. D. Golovko (1957). In spite of all the value of the formula of V. S. Luk'yanov and M. D. Golovko for engineering calculations, some indefiniteness in the designation of the flux from below to the interface is an important shortcoming.

8. Solution of the Problem of Freezing and Thawing of Rocks in the Spectrum of Negative Temperatures

Because of the dependence of the thermophysical characteristics of the medium on the temperature the solution of that problem during arbitrary boundary conditions encounters serious mathematical difficulties and requires considerable expenditures of machine time. A widespread and very effective means of investigating such nonlinear problems of mathematical physics is self-modeling solutions. The latter are important not so much as particular solutions of the class of problems under consideration but mainly as an instrument for the investigation of the main regularities of the process, determination of the degree of influence of given parameters on the procedure of solution, etc.

A very simple self-modeling solution of the problem of freezing and thawing is the above-examined (section 7.1) solution of the classical Stefan problem.

In the formulation of a self-modeling problem of the Stefan type (3.5.8-3.5.11) with consideration of phase transformations of water in the temperature range, as in section 5, the following conditions necessary for the fulfilment of self-modeling are assumed:

a) the examination is conducted on a homogeneous semirestricted rod $z > 0$;
b) the boundary conditions are constant.

We will consider a self-modeling solution of the following problem of freezing in the negative temperature range:

$$C(t)\frac{\partial t(z,\tau)}{\partial \tau} = \frac{\partial}{\partial z}\left[\lambda(t)\frac{\partial t}{\partial z}\right],\ z>0,\ \tau>0,\ t\neq 0, \qquad (3.8.1)$$

$$t(z,0)=T_1,\ z>0;\ t(0,\tau)=T_0,\ \tau>0, \qquad (3.8.2)$$

$$\lambda(0-0)\frac{\partial t}{\partial z}\bigg|_{z=\xi(\tau)\mp 0} - \lambda(0+0)\frac{\partial t}{\partial z}\bigg|_{z=\xi(\tau)\pm 0} = Q\xi'(\tau), \qquad (3.8.3)$$

$$t[\xi(\tau),\tau]\equiv 0. \qquad (3.8.4)$$

Here $C(t)$ is the effective heat capacity of the medium, $C(t) = C_1(t) + F(t)$, $C_1(t) \geqslant$ constants > 0 is the additive heat capacity, $F(t)$ is the intensity of internal sources on account of phase transitions of water, $F(t) \geqslant 0$, $\lambda(t) \geqslant$ constant > 0 is the thermal conductivity of the medium, where $C(t)$ and $\lambda(t)$ are continuous and continuously diffused functions respectively; T_0 and T_1 are constants, and $T_0 \neq 0$. At $t > 0$ by virtue of $F(t) \equiv 0$ we have $\lambda(t) \equiv \lambda_0$ and $C(t) \equiv C_0$, where λ_0 and C_0 are constants > 0 are the characteristics of thawed ground. The upper sign on the left side of (3.8.3) corresponds to the case where the first phase is frozen and the lower phase is the thawed phase.

The problem under consideration also has meaning at $T_1 = 0$ if $T_1 = 0 - 0 \operatorname{sgn} T_0$ (in the opposite case the Stefan problem is reduced to the equation of thermal conductivity*). In that case the problem degenerates into a single-phase (one-zone) problem of the Stefan type and its examination is correspondingly simplified in comparison with the case $T_1 \neq 0$.

In certain works (Melamed, 1963; Bachelis and Melamed, 1964) the existence and singularity of the self-modeling solution (3.8.1) - (3.8.4) are demonstrated, such that

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*sgn T_0 designates the sign of T_0. In frozen ground $T_0 < 0$ and sgn $T_0 = -1$.

$$\operatorname{sgn} t(z,\tau) = \operatorname{sgn} T_0,\ 0 < z < \xi(t);$$
$$\operatorname{sgn} t(z,\tau) = \operatorname{sgn} T_1,\ z > \xi(t).$$

The examination is made separately for the cases of thawing and freezing. As a result of the investigation, effective algorithms of numerical integration are proposed which make it possible to obtain, with slight limitations of the characteristics of the medium, a solution of the corresponding problems with any prescribed precision. It follows directly from the singularity of solution of the initial problem of the Stefan type under consideration (3.8.1 - 3.8.4), demonstrated, in particular, by F. P. Vasil'yev (1964), that the thus-found self-modeling solution is also the sought solution of the system (3.8.1 - 3.8.4) under the given boundary conditions. Similarly to section 5 we seek the function $\phi(p)$ and α = constant > 0 such that $t(z, \tau) = \phi(p)$, where $p = z\tau^{-1/2}$ and also $\xi(\tau) = \alpha\sqrt{\tau}$ are a solution of (3.8.1 - 3.8.4). In that case the system (3.8.1 - 3.8.4) assumes the form

$$\varphi''(p) + a(\varphi)\varphi'^2(p) + b(\varphi)\varphi'(p)p = 0,\ \varphi \neq 0, \tag{3.8.5}$$
$$\varphi(0) = T_0,\ \varphi(\alpha) = 0, \tag{3.8.6}$$
$$\varphi(\infty) = T_1, \tag{3.8.7}$$
$$\lambda(0-0)\varphi'(\alpha \mp 0) - \lambda(0+0)\varphi'(\alpha \pm 0) = \frac{1}{2}Q\alpha. \tag{3.8.8}$$

Thus the problem under consideration is reduced to a nonlinear limiting (in the sense of the condition for ∞) boundary problem, for the numerical integration of which it is necessary to investigate the asymptotic behavior of the integral curves.

In connection with the fact that $\phi'(p)$ undergoes a discontinuity, determined with (3.8.3) during the transition of ϕ through zero, the procedure of solution of (3.8.1) must be examined separately at $\phi < 0$ and $\phi > 0$. In finding the solution it is necessary that only values of $\phi'(0)$ be given which satisfy the condition

$$\operatorname{sgn} \varphi'(0) \operatorname{sgn} T_0 = -1.$$

In considering the reverse function one can readily be convinced that $\phi(\alpha)$ is a monotonically increasing continuous function of α. An important role is played in the investigation and solution of system (3.8.5 - 3.8.8) by the majorant and minorant (3.8.5).

The equation of the curve ϕ (p) majorizing (3.8.5) under the conditions $\phi(0) = \tilde{\phi}(0)$, $\tilde{\phi}'(0) = \tilde{\phi}'(0)$ has the form

$$\tilde{\varphi}''(p) + \tilde{a}\tilde{\varphi}'(p) + \tilde{b}\tilde{\varphi}'(p)p = 0, \tag{3.8.9}$$

where a and b are constants, $a = \max_t \frac{\lambda'(t)}{\lambda(t)}$ and $b \in (0, \min_t \frac{\lambda(t)}{2C(t)})$. In the case of the minorant $a = \min_t \frac{\lambda'}{\lambda}$, $b = \max_t \frac{C(t)}{\lambda(t)} + \delta$ and $\delta > 0$.

Since (3.8.9) is integrated in explicit form it is possible to determine directly the deviation $\tilde{\alpha}(0)$ at which the minorant (majorant) satisfies conditions (3.8.6) and (3.8.7). It is obvious that the integral curve (3.8.5) of $\phi(p)$, which satisfies $\phi(0) = T_0$ and $\phi_0(0) = \tilde{\phi}'_0(0)$, is known to intersect the straight line $\phi = T_1$. To find a solution of (3.8.5 - 3.8.7) with the prescribed precision it is sufficient to construct the integral curve (3.8.5), (2.8.6) and (3.8.8) $\phi(p)$, satisfying the condition*

$$\bar{\varphi}\left(Y\left(\frac{\varepsilon}{2}\right)\right) = T_1, \text{ где } Y\left(\frac{\varepsilon}{2}\right) = \left\{\frac{C(T_1)}{4\lambda(T_1)}\varepsilon\,|\varphi_0'(T_1 \pm \varepsilon)|\right\}^{-1}.$$

This is explained by the fact that $\phi(p)$ at $p > Y(\varepsilon/2)$ does not intersect either $\phi = T_1$ or $\phi = T_1 \pm \varepsilon/2$. Therefore, since the integral lines (3.8.5) at $\phi < 0$ and $\phi > 0$ diverge with growth of p, the sought solution is known to differ from $\phi(p)$ by less than $\varepsilon/2$. In practice the solution of (3.8.5 - 3.8.8) is found with the precision ε by changing the value of α until the corresponding solution of (3.8.5), (3.8.6) and (3.8.8) does not satisfy the condition

$$\varphi\left(Y\left(\frac{\varepsilon}{2}\right)\right) \in \left[|T_1| - \frac{\varepsilon}{2}, |T_1|\right]$$

The region in which there is the single sought value of α is limited to the points of intersection of the majorant and minorant respectively of (3.8.5 - 3.8.8) with the axis of abscissas. In that case if

$$\left|\varphi\left(Y\left(\frac{\varepsilon}{2}\right)\right)\right| < \left|T_1 - \frac{\varepsilon}{2}\right| \text{ или } \left|\varphi\left(Y\left(\frac{\varepsilon}{2}\right)\right)\right| > |T_1|,$$

the value of α decreases or increases respectively. The proposed algorithm for numerical integration is readily accomplished with any class of electronic computer, and also with continuous analog computers. Let us note that if we find as a result the solution of $\phi(z/\sqrt{\tau})$ and α with the prescribed precision we obtain $t(z, \tau)$ and $\xi(\tau)$ directly at any values $\tau > 0$ and $z > 0$.

Let us note that the algorithm under consideration has been used for the general problem of hardening, when besides discrete phase transformations at $t = 0$ phase transformations occur continuously at all $t \neq 0$. Such a problem is of interest, for example, in examining fusion with consideration of metamorphic reactions, problems with recrystallization, etc. However, in the case of thawing and freezing of finely dispersed rocks the solution is simplified somewhat, since at $t > 0$ equation (3.8.5) is integrated in explicit form. This applies especially to freezing, where a quasilinear equation of thermal conductivity occurs in a limited region $p \in (0, \alpha)$. If the simplicity of finding a self-modeling solution of the problem is taken into consideration, by means of it it is possible to readily determine the different regularities of the influence of phase transitions of unfrozen water on the course of the process of freezing (thawing) of finely dispersed rocks and on the basis of them correct the results of calculations of the corresponding Stefan problem.

However, the obtained results of investigation of the process of freezing and thawing in the range of negative temperatures by means of a self-modeling

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*Here, as above, the upper sign corresponds to freezing and the lower to thawing.

problem do not in any way exclude the need to solve such a problem under arbitrary boundary conditions. This applies primarily to the case of periodic temperature fluctuations on the surface, since under natural conditions the process of freezing is replaced by the process of thawing. Such a sequence, in accordance with the indicated regularities of freezing and thawing in the temperature range, in the course of time leads to increase or divergences in comparison with the solution of the Stefan problem.

At the present time a solution of the quasilinear Stefan-type problem under consideration under arbitrary boundary conditions and parameters of the problem is possible only with electronic computers. There is a considerable number of different methods of solving the unidimensional problem of freezing (thawing) with consideration of phase transitions in the temperature range. One of the most effective algorithms for the solution of a quasilinear problem of the Stefan type is the method of front capture in a network grid (Vasil'yev, 1964). The idea of it is that by means of iterations a time interval is sought during which the front is shifted under the given conditions by one step of the three-dimensional grid. Unfortunately, that algorithm is in principle inapplicable in the case of a not strictly monotonic motion of the front (in particular, when it is in its limiting position), and also in a multifront problem. The solution of the latter problem, under the condition that the number of fronts is invariable, can be obtained by the method of front rectification (Uspenskiy, 1968). In that case each zone in each step in time is transferred into the segment (0, 1), in connection with which the thermal conductivity equations are made considerably more complicated. Solution of the multifront problem by smoothing out the coefficients is somewhat more convenient. In that case the discrete phase transitions which occur at the temperature of the start of freezing are "spread" over a certain temperature range (in that case the Stefan condition is excluded). In practice in such an approach the entire process will be described by a simple quasilinear equation of thermal conductivity with complex coefficients (substantially dependent on temperature). This leads to a need to use iterations, the convergence of which worsens when the fronts are brought closer together.

A rather simple and effective method of solving a quasilinear task of the Stefan type is the method of straight lines (the Rote method) (Bachelis and Melamed, 1971). In that case the investigated time interval is subdivided into time layers with a definite step and the derivative in time on each layer is replaced by a difference ratio. As a result the thermal conductivity equations are reduced to ordinary differential equations with respect to the coordinate. In the final account the solution of the problem is reduced to finding a method of tests of the position of the front in which the Stefan condition is fulfilled with the given precision. It is essential that in that case the movement of the interface can be arbitrary.

The considered algorithms for the solution of a unidimensional problem of the Stefan type are extended completely to the case of axial (in the case of cylindrical bodies) and spherical symmetry.

9. Investigation of Processes of Freezing With Consideration of Moisture Migration

The above-considered methods of investigating the freezing of grounds within the framework of a conductive problem, that is, by solving the Stefan problem, in a number of cases prove to be inadequate for the complete description of the process. In particular, in the investigation of freezing in moisture-saturated dispersed rocks it is necessary simultaneously with heat transfer to examine mass transfer in connection with moisture migration toward the front of freezing. In that case the processes of freezing and moisture migration are closely interconnected.

Together with the redistribution of moisture during freezing, under certain conditions moisture migration can lead to heaving and the formation within the freezing ground of ice interlayers of different thickness, which sharply complicates the solution of that problem in comparison with the Stefan problem. The only possible way to investigate freezing with consideration of moisture migration consists in the simultaneous solution of a system of equations describing the heat and mass transfer in the presence of a mobile interface. The additional difficulty, which substantially complicates the solution of such a problem, is the fact that in dispersed media the heat and mass transfer characteristics of the medium (in particular, the coefficients of thermal and potential conductivity) are sharply variable functions of the total moisture. For example, the coefficient of potential conductivity (diffusivity) varies by several orders of magnitude during variation of the moisture within the range from the moisture of build-up to the total moisture content. In addition, in connection with the presence of unfrozen moisture in dispersed media, in calculations, similarly as in section 8, it is necessary to take into consideration the effective heat capacity, which depends substantially on the temperature in accordance with the curve of unfrozen water.

By virtue of what has been said above the known approximate methods of calculating frost heaving in which the processes of freezing in essence do not depend on moisture migration and, in addition, all the coefficients given are constant, are extremely approximate.

1. Formulation of the Problem of Freezing With Consideration of Moisture Migration

As has been shown in a number of works (for example, by G. A. Martynov, 1959), during the freezing of finely dispersed grounds moisture migrates toward the front of ice formation mainly in the liquid phase.

On the basis of the theory of transfer in colloidal capillary-porous bodies very great development has been obtained by the potential theory of film and capillary mechanisms of moisture movement, developed mainly in the works of A. V. Lykov and his school. According to that theory, on the front of ice formation (on the side of the thawed zone, at a temperature of $0 + 0^{\circ}$) a gradient of the potential of film moisture, causing migration, forms.

The process of moisture migration in the solid zone, accomplished much more slowly than in the liquid zone, has in essence not been studied up to now. It is only known that for a wide class of grounds (except heavy clays, perhaps) moisture migration in the frozen zone in open systems is small in comparison with mass transfer in the thawed ground. At the same time, moisture migration in frozen ground mainly leads only to a redistribution of the ice content over the section, whereas its total quantity is determined by the moisture migration from the thawed zone toward the front of freezing. In connection with that, in the present work moisture migration in the solid zone is excluded from consideration. It must be noted, however, that this limitation, which simplifies somewhat the technical difficulties in solving the problem, is not a principal one when the algorithm proposed below is used and involves only the absence of reliable data on mass transfer in the frozen zone. In addition, as usual, deformation of the skeleton of the ground during heaving and also the dependence of the change of physical characteristics of the medium on pressure are disregarded.

At the same time, together with the above-stated, sufficiently substantiated assumptions in works devoted to quantitative examination of the process of freezing with moisture migration, a number of assumptions also are made which considerably simplify the solution but physically are not completely justified. Thus it is assumed everywhere that the coefficients of heat and mass transfer do not depend on the moisture w (a a rule they are generally considered constant). The roughness of the indicated assumption follows, in particular, from the fact that the coefficient of potential conductivity increases by several orders of magnitude during increase of w. In addition, it is assumed that the phase transformations occur completely on the interface. As indicated above (section 6) under certain conditions this can be assumed in calculations of the Stefan problem without moisture migration. However, the latter is important precisely in finely dispersed grounds, where the phase transitions of combined water, occurring in the range of negative temperatures, are considerable. In the present formulation those assumptions are not used because they are sharply negative.

In connection with different change of the dimensions of the frozen and thawed zones, the motion of their mobile boundaries can conveniently be examined separately, assuming $z = 0$ as the immobile boundary and relating the heaving to the mobile boundary of the frozen zone. Henceforth we will designate the coordinates of the mobile boundaries of the frozen and thawed zones by $\xi(\tau)$ and $y(\tau)$ respectively. It is obvious that within the framework of a unidimensional problem the process under consideration formally includes both freezing properly speaking (in that case $\xi'(\tau) \geqslant y'(\tau) \geqslant 0$) and thawing from below ($\xi'(\tau) \leqslant < y'(\tau) < 0$). Henceforth by freezing will be assumed only the case $y' \geqslant 0$, and the case of thawing from below will be examined separately. The amount of heaving $h(\tau)$ which will occur during freezing, obviously, is equal to $h(\tau) \equiv \xi(\tau) - y(\tau)$.

We will examine first the case where at the initial moment $\tau = 0$ $\xi(0) \geqslant y(0) > 0$ (the formation of the frozen zone in the presence of moisture migration toward the front of freezing will be examined below). Let the first zone

($0 > z > \xi(\tau)$) be frozen. During the freezing of grounds, as has been shown by G. A. Martynov (1959), one can neglect the transfer of moisture under the influence of a temperature gradient and the intensity of phase transformations on account of evaporation and condensation. Then in the presence of the above-cited considerations the known system of equations describing heat and mass transfer in homogeneous capiallayr-porous media (Lykov, 1954) can be reduced to the following system of quasilinear equations of a parabolic type ($0 < \tau < < T$):

$$\bar{C}(\bar{t}, \bar{w}) \frac{\partial \bar{t}(z, \tau)}{\partial \tau} = \frac{\partial}{\partial z}\left[\bar{\lambda}(\bar{t}, \bar{w}) \frac{\partial \bar{t}}{\partial z}\right], \ 0 < z < \xi(\tau), \qquad (3.9.1)$$

$$C(w) \frac{\partial t(z, \tau)}{\partial \tau} = \frac{\partial}{\partial z}\left[\lambda(w) \frac{\partial t}{\partial z}\right], \ y(\tau) < z < l, \qquad (3.9.2)$$

$$\frac{\partial w(z, \tau)}{\partial \tau} = \frac{\partial}{\partial z}\left[K(w) \frac{\partial w}{\partial z}\right], \ y(\tau) < z < l. \qquad (3.9.3)$$

Here $t(z, \tau)$, $w(z, \tau)$, $C(w)$ and $\lambda(w)$ are the temperature, moisture (by volume, in fractions of unity), heat capacity and heat-transfer coefficient in the thawed zone, $\bar{t}(z, \tau)$, $\bar{w}(z, \tau)$, $\bar{C}(\bar{t}, \bar{w})$ and $\bar{\lambda}(\bar{t}, \bar{w})$ are the same in the frozen zone, and $K(w)$ is the coefficient of potential conductivity. It is obvious that $\bar{w}$ is the summary moisture (ice + unfrozen water).

On the mobile boundaries of the solid and liquid zones there occurs the obvious condition

$$\bar{t}[\xi(\tau), \tau] = t[y(\tau), \tau] = 0. \qquad (3.9.4)$$

The remaining conditions on the mobile boundaries [the condition for (3.9.3) and also the condition analogous to (3.5.11) which characterizes the rate of movement of the mobile boundaries and connects the processes of heat and mass transfer], which complete the given problem, as will be shown below, vary substantially as a function of the course of the process.

2. Generalized Condition of the Stefan Type on a Mobile Boundary During Freezing With Moisture Migration Toward the Front of Freezing

It is characteristic of the process of freezing that the structure of the frozen zone depends essentially on how the correlation between $\xi'(\tau)$ and $y'(\tau)$ varies in the course of freezing. Actually, let the arbitrary time interval (τ, τ_2), $\tau \geqslant 0$ and $\tau_2 - \tau_1 = \Delta\tau$ be given and let $\xi(\tau_1) > y(\tau_1) > 0$. We will use Δy and $\Delta\xi$ to designate the corresponding movements of the mobile boundaries and $\bar{w}_{\Delta\xi}$ to designate the summary moisture in the layer $\Delta\xi$.

Then during freezing with consideration of moisture migration in the course of Δt the following conditions are possible:

a) $\Delta\xi = \Delta y > 0$. In that case, in the course of $\Delta\tau$ heaving is absent ($\Delta h = 0$) and, consequently, we have $w_{\Delta\xi} < w_s$, where w_s is the degree of saturation of mineral aggregates (the total moisture content). In that case a massive cryostructure forms in the process of freezing;

b) $\Delta\xi = \Delta y > 0$. Then the process of hardening is accompanied by buckling, and because $\Delta y > 0$ we have $1 > w_{\Delta\xi} > w_s$. Within the limits of the segment $\Delta\xi$ form schlieren (microlayers) divided by sections of the enclosing medium, the relative disposition of which within $\Delta\xi$ has not been determined because of averaging with respect to τ. In practice in that case a finely layered microschlieren cryostructure forms;

c) $\Delta\xi > 0$ and $\Delta y = 0$. In this case a macrolayer forms which fills the entire segment $\Delta\xi$, and $w_{\Delta\xi} = 1$. It is obvious that in that case the heaving will be maximal.

Thus the cryogenic structure of the freezing zone can be greatly different, depending on the correlation between $\xi'(\tau)$ and $y'(\tau)$.

In addition, if in the course of an arbitrary fixed interval case "a" or "b" occurs (that is, $y'(\tau) > 0$), we will call such a process monotonic freezing (in the presence or absence of heaving respectively), and in case "c" -- freezing with formation of an interlayer. It is obvious that during heat and mass transfer under the conditions describing movement of the mobile boundary of the liquid zone (in contrast with the Stefan problem in a "conductive" problem) to flow of moisture toward the front of crystallization must be taken into account. However, the mass transfer to $z = y(\tau)$ and, consequently, the indicated "generalized" Stefan condition varies substantially as a function of which of the freezing processes take place. The latter is connected with the fact that the behavior of $w[y(\tau), \tau]$ depends sharply on the type of freezing. Actually, the available experimental data indicate that in case "a" (and also, with sufficient precision, in case "b") it can be assumed that $w(y(\tau), \tau) = w_o =$ constant. In that case it is noted that w_o is close to the critical mass of the absorbed substance (that is, to a value of the moisture content below which migration is practically impossible. On the basis of the processing of the experimental data by M. N. Goldshteyn (1948), V. P. Titov (1959) proposed the following empirical expression for w_o:

$$w_0 = \tilde{w}_0^* + 0{,}37\,(n_1 - 1{,}5),$$

where $\tilde{w}_o^*$ is the moisture content of the limit of plasticity; n_1 is the plasticity number. With respect to tenacious soils, w_o surpasses by several percentages the limit of plasticity or the moisture of rolling into a cord. It is essential that $w_o > w_{un}(0)$.

The constancy of $w[y(\tau), \tau]$ will be strictly substantiated below for case "a". At the same time, as will be shown, in case "b" $w[y(\tau), \tau]$ is determined by the heat fluxes on the mobile boundary [at $z = \xi(\tau)$ and $z = y(\tau)$] and in principle must vary in the process of formation of a macrolayer. However, case "b" is introduced purely formally. Actually, in that case the freezing is in essence reduced to alternating cases of "a" and "b". Therefore the interval $\Delta\tau$, within which case "b" occurs, can, generally speaking, be broken down into a finite number of segments within which either "a" or "b" occurs. Since the latter is known to be smaller than $\Delta\tau$, then in practice during fairly few fixations of $\Delta\tau$ in case "b" it can be considered that $w(y(\tau), \tau) \simeq w_o$.

Thus henceforth during monotonic hardenining we assume that

$$w[y(\tau), \tau] \simeq w_0. \qquad (3.9.5)$$

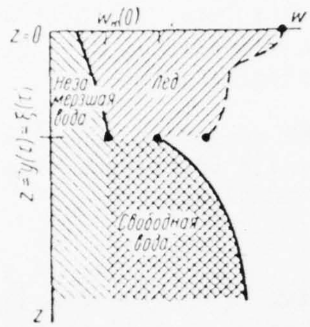

Figure 10. Schematic distribution of moisture content in rock in the case of monotonic freezing. a - Unfrozen water; b - Ice; c - Free water

The distribution of moisture in the frozen (solid line) and thawed states in the case of monotonic freezing is presented schematically on Figure 10. During transition across the interface $z = y(\tau)$ the amount of moisture in the liquid phase changes suddenly from w_o to $w_{un}(0)$; that moisture, being free and immobile, crystallizes in the process of freezing just as in an ordinary Stefan formulation. At the same time the total moisture in the solid zone also undergoes a discontinuity of the first kind at the point $z = \xi(\tau)$, the amount of the jump being determined by the influx of moisture migrating toward the front of freezing.

The condition describing, similarly to the Stefan condition, the rate of movement of the mobile boundaries of the liquid and solid zones in the case of monotonic hardening is readily obtained by compiling an equation of the heat balance for the elementary volume (with a cross sectional area of 1 x 1). We know from (3.9.5) that in connection with the crystallization of the "free" immobile moisture, in a unit of time in the layer $y'(\tau)$ (in the case under consideration, $\xi'(\tau) > y'(\tau) > 0$) heat amounting to $A(\tau) = \mu[w_o - w_{un}(0)]\, y'(\tau)$ is released, where μ is the heat of freezing of 1 m^3 of water. In connection with moisture migration toward the mobile boundary of the liquid zone in accordance with (3.9.3) under the condition (3.9.15), $K(w_o)w_z\big|_{z=y(\tau)}$ of water arrives and crystallizes. In that case a quantity of heat is released which is equal to

$$B(\tau) = \mu K(w_0) \frac{\partial w}{\partial z}\bigg|_{y(\tau)}, \quad w|_{y(\tau)} = w_0. \qquad (3.9.6)$$

It can readily be seen that the transfer of the heat of moisture migrating toward the front of crystallization on account of heat capacity is a small value of a higher order of magnitude than $A(\tau)$ or $B(\tau)$. To make up the heat balance $A(\tau) + B(\tau)$ must be equal to the differences of the heat fluxes on the mobile boundaries of the liquid and solid zones

$$D(\tau) = \bar{\lambda} \frac{\partial \bar{t}}{\partial z}\bigg|_{\xi(\tau)} - \lambda(w_0) \frac{\partial t}{\partial z}\bigg|_{y(\tau)}.$$

As a result we obtain a condition generalizing the Stefan condition, which determines the dynamics of the interfaces during monotonic hardening

$$D(\tau) - B(\tau) = A(\tau). \qquad (3.9.7)$$

It is obvious that the examination of the problem of freezing with consideration of moisture migration in a certain interval $[\tau_1, \tau_2]$ makes sense when the the following necessary conditions are fulfilled: a) $w(z, \tau_1) \geqslant w_o$ and $y(\tau) \leqslant z \leqslant L$ (the necessary condition for moisture migration); 2) $D(\tau) > 0$ and $\tau \in [\tau_1, \tau_2]$ (the necessary condition for freezing).

Under those conditions, for monotonic hardening with consideration of migration toward the front of crystallization it is necessary and sufficient that during the time interval under consideration there be the condition

$$F(\tau) \equiv D(\tau) - B(\tau) > 0. \tag{3.9.8}$$

It is essential that the formally introduced function $F(\tau)$ be a continuous function of the time and that during monotonic freezing, as follows from (3.9.7), $F(\tau) \equiv A(\tau)$.

The amount of the total moisture in the freezing zone during monotonic freezing at $\tau > \tau_1$ is determined for all values of $z \in [\xi(\tau_1), \xi(\tau_2)]$ with the correlation

$$\begin{aligned} \bar{\omega}(z, \bar{t}) = [\xi'(\tau^*)]^{-1}\{\nu[\mu^{-1}B(\tau^*) + (\omega_0 - \omega_н(0)y'(\tau^*)] + \\ + \omega_н(0)y'(\tau^*)\} + (\nu - 1)[\omega_н(0) - \omega_н(\bar{t})], \end{aligned} \tag{3.9.9}$$

where ν = constant > 1 is the coefficient of volumetric expansion during the transition of water into ice, and τ^* is the moment of time at which $\xi(\tau) = z$ and $\xi(\tau_1, \tau_2)$. In the general case, when in the time interval under consideration, monotonic freezing alternates with thawing from below, τ^* is the root of the equation

$$\int_{\tau_1}^{\tau_2} \xi'(s)\, ds = z - \xi(\tau_1). \tag{3.9.10}$$

It is obvious that by the moment τ_2 the boundary of the freezing zone can intersect the point z under consideration an odd number of times. Therefore in the general case it is necessary to take as the sought value of τ^* the largest of the roots of (3.9.10), which corresponds to the last period of freezing (before the moment τ_2 under consideration). In (3.9.9) the first term in the curly brackets characterizes the moisture in the solid phase (the ice content) and the second, the same in the liquid phase, and the last term the expansion during the transition of the unfrozen water into ice. The rate of bulging during monotonic freezing is determined in the course of the process from the correlation

$$h'(\tau) = \begin{cases} \nu\mu^{-1}B(\tau) - [\omega_n - \nu\omega_0 + (\nu - 1)\omega_н(0)]\, y'(\tau) \equiv \Phi, & \Phi > 0, \\ 0, & \Phi \leqslant 0. \end{cases} \tag{3.9.11}$$

Here the first term in the expression for Φ characterizes the influx of moisture in connection with migration in a unit of time, and the second is the amount of water necessary to fill the pores in the hardening medium.

3. Heat and Mass Transfer in the Process of Formation of an Ice Interlayer. Rhythmicity During Freezing with Moisture Migration

We will now examine the process of heat and mass transfer during freezing when condition (3.9.8) does not occur. By virtue of what was said above, the value of $B(\tau)$ determined from (3.9.6) is found from a solution of (3.9.3) under condition (3.9.5). Therefore, when the condition necessary for moisture migration is fulfilled, $B(\tau) \geqslant 0$ at all values of $\tau \in (\tau_1, \tau_2)$. In particular, violation of condition (3.9.8) can occur in both a closed and all the more so in an open system in connection, for example, with increase of $B(\tau)$ (since the value of $w_z|_{z=y(\tau)}$ under condition (3.9.5) can increase unlimitedly with increase of $y'(\tau)$) or decrease of $D(\tau)$.

Let the following occur at a certain value $\tau = \tau_3 \in (\tau_1, \tau_2)$

$$F(\tau) = 0; \; F'(\tau) \leqslant 0, \qquad (3.9.12)$$

and the conditions necessary for hardening and migration are in general fulfilled. Then, if in the left half-neighborhood τ_3 $F(\tau) > 0$, then condition (3.9.12) means that at $\tau = \tau_3$ the freezing of thawed ground ceases and the removal of heat from the interface $D(\tau)$ is sufficient only to crystallize the moisture migrating in accordance with (3.9.3) under condition (3.9.5). In that case $y'(\tau_3) = 0$, whereas $\xi'(\tau_3) = h'(\tau_3) > 0$, that is, the process is changed qualitatively. Thus upon fulfilment of the necessary conditions of freezing and migration the condition (3.9.12) is necessary for the formation of an ice interlayer to start after monotonic freezing. It can readily be seen that from physical considerations the condition $F(\tau) \leqslant 0$ at $\tau > \tau_3$ is sufficient for the continuous formation of an ice interlayer. In that case condition (3.9.5), generally speaking, does not occur. In particular, (3.9.5) is known not to be fulfilled at $F(\tau) < 0$. Actually, as is known from numerous experiments, at $D(\tau) > 0$ the existence of a layer of water in the thawed zone near the mobile boundary $z = y(\tau)$ is excluded. However, if (3.9.5) occurred at $F(\tau) < 0$, then in accordance with (3.9.7) the moisture migration toward $z = y(\tau)$ would exceed the amount which could be crystallized at the given $C(\tau) < 0$. In connection with that $w[y(\tau)\tau] \neq$ constant at $F(\tau) < 0$ and varies, assuring self-regulation of the flow of moisture toward $z = y(\tau)$, so that at the given $D(\tau)$ the inflow of moisture can be completely crystallized:

$$K(w)\, w'_z|_{z=y(\tau)} = \mu^{-1} D(\tau). \qquad (3.9.13)$$

Let us note that $B(\tau)$, determined from (3.9.6) under the condition (3.9.5), in the case of a macrolayer is calculated formally and plays a purely auxiliary role -- in finding the sign of $F(\tau)$. In addition, the value of $\left.\frac{\partial w}{\partial z}\right|_{z=y(\tau)}$ at $w|_{z=y(\tau)} = w_0$, entering $B(\tau)$ at $F(\tau) < 0$, majorizes the actual moisture gradient at the point $y(\tau)$. It is obvious that in that case, because $y'(\tau) = 0$, $\bar{w}|_{z=\xi(\tau)} = 1$, whereas the moisture migration toward $z = y(\tau)$ varies as a function of the value of $D(\tau)$ in accordance with (3.9.13). When $D(\tau)$ is reduced to zero $w|_{z=y(\tau)}$ increases until in the right half-neighborhood

$z = y(\tau)$ the moisture distribution becomes gradientless*. During subsequent increase of $D(\tau)$, by virtue of (3.9.13), $w(y(\tau), \tau)$ again decreases until the value w_o is achieved at a certain value $\tau_4 > \tau_3$ is reached. In that case, obviously, $F(\tau) = 0$, that is, $D(\tau) = B(\tau)$. If $F'(\tau) > 0$, then in a certain right half-circle τ_4 (3.9.5) again occurs and monotonic freezing starts. At the same time, in the process of formation of the ice interlayer the thawed ground near the mobile interface is dried. In connection with that, in closed systems for which the indicated circumstance is very characteristic, monotonic freezing after an interlayer, as a rule, leads to the formation of ground containing little ice.

Freezing from the surface is described similarly if at the moment of the temperature inversion $w\big|_{z=0} > w_o$. In that case, as in that examined earlier, the well-known fact of ice accumulation near the surface of moisture-saturated ground during gradual cooling is readily substantiated.

If at $\tau \in [\tau_1^*, \tau_2^*]$ the conditions necessary for freezing with migration are fulfilled but (3.9.8) does not occur, then the thickness of the macrolayer forming in that case is determined from the correlation

$$H = \frac{\nu}{\mu} \int_{\tau_1^\bullet}^{\tau_2^\bullet} D(s)\, ds. \qquad (3.9.14)$$

The above-applied approach to investigation of the interaction of the temperature and moisture fields during the freezing of finely dispersed grounds has the direct result that the motion of the interface occurs differently and the distribution of the total moisture over the profile of the frozen zone has an oscillational character. The examination of this phenomenon, typical during freezing with migration, we will conduct upon certain assumptions natural for finely dispersed grounds regarding the coefficients of thermal and potential conductivity and the curve of the unfrozen water:

$$\bar{\lambda}'_{\bar{w}} \geqslant 0,\ K'(w) > 1 \text{ при } w_п \geqslant w \geqslant w_0,\ w'_н(\bar{t}) \geqslant 0 \text{ при } t < 0;\ w_н(0) > 0.$$

Actually, let $D(\tau)$ at any moment τ so exceed $B(\tau)$ that within the limits of some selected value of $D(\tau)$ there is an intensive advance of the fronts of the frozen and thawed zones with the formation of a low-ice layer $\Delta\xi$ ($\Delta\xi = \Delta y$) By virtue of $w'_{un}(0) > 0$ the effective heat capacity in a certain left half-neighborhood $\tau = 0$ is essentially an increasing function of temperature and, consequently, the layer $\Delta\xi$ has considerable thermal inertia. Therefore during the indicated increase of the frozen zone, regardless of the boundary conditions (under the condition of their smoothness) $D(\tau + \Delta\tau) < D(\tau)$. At the same time, since $K(w)$ is a sharply increasing function of w at $w \in [w_o, w_{un}]$, where $K(w_o)$ is relatively small ($\approx 10^{-5} - 10^{-4}$ m^2/hr), the corresponding reduction of the thawed zone in the course of $\Delta\tau$ causes a strong local disturbance of the moisture field near $z = y(\tau + \Delta\tau)$. Hence in the case under consideration we have

*This explains, for example, intensive ice accumulation on the base of a layer of seasonal freezing in sufficiently moistened soils.

$$\left.\frac{\partial\omega(z,\tau+\Delta\tau)}{\partial z}\right|_{z=y(\tau+\Delta\tau)} > \left.\frac{\partial\omega(z,\tau)}{\partial z}\right|_{z=y(\tau)},$$

that is, $F(\tau+\Delta\tau) < F(\tau)$ and, consequently, $y'(\tau+\Delta\tau) < y'(\tau)$. In particular, in that case $F(\tau+\Delta\tau) < 0$ is possible and, consequently, the formation of an ice interlayer.

Thus, after rapid freezing (with formation of a low-ice layer of frozen ground) the freezing rate is slowed down, whereas the ice content increases correspondingly. If it is taken into consideration that the thermal conductivity of the frozen ground is a non-decreasing function of the total moisture, that deceleration of freezing under the given conditions has the result that $F(\tau + 2\Delta\tau) > F(\tau + \Delta\tau)$. As a result, after deceleration the freezing rate is again intensified (in that case the ice accumulation decreases), etc. It must be noted that the very fact of rhythmicity in ice accumulation occurs independently of the selection of $\Delta\tau$, whereas the quantitative aspect of the question evidently depends to a certain degree on the size of $\Delta\tau$. In addition, the oscillating character of the cryogenic structure of the freezing layer does not distort the general tendency toward variation of the ice content over the profile, which is determined by the specific boundary conditions. The noted rhythmicity during freezing is manifested in essence in fluctuations of the ice content values around a certain smooth curve which reflects on the average the above-indicated general course of change of the ice content with depth.

Thus the conducted examination of the interaction of the temperature and moisture fields agrees qualitatively with the known theory of rhythmic formation of ice layers, which describes the mechanism of cyclic formation of ice lenses, starting with the formation of crystallization nuclei.

In the mathematical formulation of the two-phase problem under consideration it is necessary to indicate the time interval T_1 during which the given problem has meaning under the two-phase initial condition (that is, there is no formation of a new front from the surface or disappearance of a zone). We will examine the boundary conditions in the very general form

$$\bar{\lambda} t'_z|_{z=0} = \bar{q}[0, \tau, t(0, \tau)], \quad \lambda t'_z|_{z=l} = q[l, \tau, t(l, \tau)].$$

If at the initial moment the first zone is frozen, then $\bar{q}'_{\bar{t}} \geqslant 0$ and $q'_t \leqslant 0$ is a physically sufficient condition that a new front is not forming from the surface, and also that the values of $\bar{t}^*(0, \tau)$ and $t^*(l, \tau)$, which are the roots of the equations $\bar{q} = 0$ and $q = 0$ respectively (the physical meaning of the roots is the ambient temperature), must be smaller and larger than zero respectively. The requirements imposed on the signs of the ambient temperature are evident. As for the conditions $\bar{q}'_{\bar{t}} \geqslant 0$ and $q'_t \leqslant 0$, they designate that with elevation of the temperature on the upper boundary the outflow of heat into the environment also increases (on the immobile boundary of the thawed zone -- the reverse), which excludes the possibility of inversion of sign of $t(0, \tau)$ and $t(l, \tau)$ respectively. If the first zone is the thawed zone, however, it is sufficient to require that $\bar{q}'_t \leqslant 0$ and $q_t \geqslant 0$ and the roots $\bar{q} = 0$ have opposite signs to those considered above.

Disappearance of the zone means that $y(\tau) = 0$ or $y(\tau) = \ell$. In that case, if $0 < \delta < y(0) < \beta < \ell$, there exists a finite time, determined by the boundary conditions, in the course of which the front is known not to emerge on the mobile boundaries. If that time is smaller than the given integration interval, then, by solving the problem in the course of the indicated time interval and making a decision by the end of it in accordance with the initial condition, it is possible to continue the solution until $y(\tau)$ (within the limits of the given precision) does not emerge beyond the boundary of the region under consideration. After that the problem is considerably simplified and reduced to the solution of equations of thermal conductivity and moisture diffusion in all the region under consideration.

If the above-considered cases of monotonic freezing and the formation of a macrolayer are combined, the problem of freezing with consideration of migration can be presented in the following form. It is necessary to determine the distribution of temperatures $\bar{t}(z, \tau)$ and $t(z, \tau)$ respectively in the frozen ($0 < z < \xi(\tau)$) and the solid zone ($y(\tau) < z < \ell$), of moisture $w(z, \tau)$ in the thawed zone, and also the velocity of advance of the boundaries of those zones $\xi(\tau)$ and $y(\tau)$ and, consequently, the heaving $h(\tau) \equiv \xi(\tau) - y(\tau)$, satisfying the system (3.9.1 - 3.9.4) under arbitrary (with consideration of what was said above) initial and boundary conditions on the mobile boundaries.

In that case the ice content forming on the front of freezing is described by the expression

$$\bar{w}[\xi(\tau), \tau] = \frac{\mu^{-1}\nu E(\tau) + \nu[w_0 - w_n(0)]\, y'(\tau) + w_n(0)\, y'(\tau)}{\xi'(\tau)}, \quad (3.9.15)$$

where

$$E(\tau) = \begin{cases} B(\tau), & \text{если } A(\tau) > B(\tau), \\ A(\tau), & \text{если } 0 \leqslant A(\tau) \leqslant B(\tau). \end{cases}$$

The heaving rate is determined in the process of solution with the formula

$$h'(\tau) = \bar{w}(\xi(\tau), \tau)\, \xi'(\tau) - w_n y'(\tau) + |\bar{w}(\xi(\tau), \tau)\, \xi'(\tau) - w_n y'(\tau)|. \quad (3.9.16)$$

The conditions on the mobile boundaries under the freezing conditions under consideration are combined in the form

$$ak(w)\, w'|_{y(\tau)} + bw(y(\tau), \tau) = \chi(\tau), \quad (3.9.17)$$

where $a = 0$, $b = 1$, $\chi = w_0$ at $A(\tau) > B(\tau)$ (monotonic freezing), and $a = 1$, $b = 0$ and $\chi = \mu^{-1}A(\tau)$ at $0 < A(\tau) < B(\tau)$ (formation of an ice layer).

The rate of advance of the boundaries of the frozen and thawed zones is determined from the correlations

$$\xi'(\tau) = \begin{cases} y'(\tau) + h'(\tau), & \text{если } A(\tau) > B(\tau), \\ h'(\tau), & \text{если } 0 < A(\tau) \leqslant B(\tau). \end{cases} \quad (3.9.18)$$

$$y'(\tau) = \begin{cases} [A(\tau) - B(\tau)]\,\mu^{-1}, & \text{если } A(\tau) > B(\tau), \\ 0, & \text{если } 0 < A(\tau) \leqslant B(\tau). \end{cases} \quad (3.9.19)$$

The problem under consideration obviously is of interest in the case where $w(z, 0) > w_0$ and $z \in (y(0), \ell)$ and there is no flow of moisture out through the lower boundary, that is, $K(w) \frac{\partial w}{\partial z}\Big|_{z=\ell} \geq 0$. In that case from (3.9.13) $w(z, \tau) > w_o$, $\tau \in (0, T_1]$ and $\frac{\partial w}{\partial z}\Big|_{y(\tau)} \geq 0$ and, consequently, $B(\tau) \geq 0$ and $\tau \in [0, T_1]$ under any freezing conditions. In addition, it follows directly from (3.9.15), (3.9.18) and (3.9.19) that $\nu(w_o - w_{un}(0)) + w_{un}(0) < \bar{w}(z, \tau) < 1$ and $z \in [0, \xi(\tau)]$, that is, the total moisture of the frozen zone during monotonic freezing satisfies the condition $w(z, \tau) \in (w_o, 1]$.

The application of known methods of solving the Stefan problem (the method of capturing the front in a network grid, of smoothing of coefficients and of straightening of fronts) presents considerable difficulties in the solution of the "systems" problem under consideration.

At the same time, a solution of the indicated problem under arbitrary boundary conditions with consideration of heaving and the formation of ice layers can be obtained with sufficient precision by the straight-line method (the Rote scheme), which has been recently proposed and substantiated as an algorithm for the solution of quasilinear problems of the Stefan type (Bachelis and Melamed, 1972). In accordance with that method the time interval in the course of which the process is examined (upon the condition that the number of phases does not change) is broken down into N layers with a step in time $\Delta\tau$, in each of which the time derivative is replaced by a difference ratio. In that case on each time layer (at the time $\tau = n\Delta\tau$, $n = 1, 2, ..., N$) the equations (3.9.1-3.9.3) and (3.9.15-3.9.17) are reduced to ordinary differential equations in relation to the functions $\bar{t}_n(z)$, $t_n(z)$ and $w_n(z)$, which are approximate values of the functions $\bar{t}(z, \tau)$, $t(z, \tau)$ and $w(z, \tau)$ respectively. The position of the boundary on the new time layer y_n and ξ_n is found by solving the transcendental equation into which (3.9.18) and (3.9.19) turn.

In accordance with the principle of the maximum, for a problem of the Stefan type the values of the heat fluxes within the region $\bar{\lambda}\bar{t}'_n(z)$, $z \in (0, \xi_n]$, $\lambda t'_n(z)$ and $z \in [y_n, \ell)$ are limited by the values of the heat fluxes on the boundaries, and also at the initial moment. It follows from this* that in the problem under consideration of heat and mass transfer during freezing during the transition from one time layer to another it is possible to indicate an interval with respect to z with its center at the point y_{n-1} (on the preceding layer), within which on the given time layer a single positive root of the equation (3.9.18) or (3.9.19) must be found. At a fixed value of y_n, partial solutions are readily found by means of trial runs, after which the values of the fluxes of heat and moisture at the points y_n, and ξ_n are determined with the given precision by known methods of numerical differentiation, that is, the values of $A(\tau_n)$ and $B(\tau_n)$. If it is taken into consideration that in the given problem the requirements for precision of calculation of the heat and moisture fluxes on the interface must be extremely high, it is advisable to calculate them by means of the method of "interpolating conjugation." The

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*It is obvious that moisture migration toward the freezing front leads to curtailment of the interval determined by the thermal problem because $B(\tau) > 0$.

latter assures approximation, very "smooth" in the sense of a minimum of linearized mean-square curvature of the first derivative function, given in a table.

As a result, at sufficiently small values of $\Delta\tau$ the method of straight lines makes it possible to determine with the prescribed precision the movement of the upper boundary of the thawed zone and also, with consideration of heaving, of the upper boundary of the frozen zone. That algorithm, being rather simple, makes it possible during practically any limitations imposed on the boundary conditions and parameters of the problem to solve the "systems" problem under consideration with consideration of phase transitions in the range of negative temperatures, insulation on the surface, heterogeneity of the ground, etc. Starting from the algorithm for numerical integration, during freezing the ice content is found as the average in the course of a step in time (from this also follows heaving during monotonic freezing), since with difference methods it is not possible to determine small ice layers so much as is desired. In the nonlinear case, when the heat and mass transfer characteristics depend substantially on the temperature and humidity (which themselves must be determined in the process of solving the problem), the method of iterations must be used. This applies especially to the finding of the moisture in the thawed zone. Let us note that in that case, instead of partial solutions, solution of boundary-value problems for nonlinear ordinary differential equations in the corresponding zones can prove to be more effective.

In the multifront case, when there is a stratification of thawed and frozen rocks, the problem is examined similarly. In that case, in the process of thawing from the surface it is necessary for the flow of moisture, determined by its evaporation, to be given as the upper boundary condition for the equation of moisture conductivity. In internal thawed zones, however, bounded on both sides by frozen layers, the condition of insulation of the moisture is given as the boundary condition during freezing. In essence the only complication of the solution of the problem in the multiphase case is the fact that the values of y_n and ξ_n are found in each time layer as a result of solution of a system of transcendental equations to which expressions of the type of (3.9.18) and (3.9.19) are reduced for each of the mobile fronts.

The above-considered algorithm is valid provided that within the integration interval the number of zones remains unchanged, their dimensions being sufficiently large for the use of numerical methods of the equations of thermal and moisture conductivity. But if the zone disappears or forms from the surface, certain natural simplifications must be introduced. It is obvious, for example, that when the mobile interfaces are brought closer together in the process of bilateral freezing of an internal thawed layer the values of the flows of heat and moisture from the disappearing zone toward the boundaries become negligibly small.

Comparison of the results of calculations in accordance with the indicated algorithm with the corresponding self-modeling solution (sections 8 and 13 of this chapter) has shown that the proposed procedure assures obtaining a solution with a sufficiently high degree of precision.

10. A Very Simple Self-Modeling Solution of the Problem of Freezing With Moisture Migration (Without Consideration of Heaving and the Formation of Ice Layers)

The above-indicated difficulties in solving the problem of the freezing of moisture-saturated finely dispersed rocks in a general formulation unavoidably led to an examination of self-modeling solutions. Various approaches to the finding of self-modeling solutions of the given problem and the evaluation of their applicability (primarily of a very simple problem very widespread in frost studies) for investigating the problem as a whole will be described in detail below. Here we will note once more that, in spite of all the effectiveness of self-modeling solutions, their use in the investigation of complex natural phenomena is possible only when a very large number of factors is taken into consideration and in that connection extreme caution is required. This applies especially to the conjugated problem under consideration, since the coarsening of given elements in the formulation of the problem (in particular, neglect of heaving in the equation for the frozen zone in the very simple self-modeling solution, linearization of the corresponding equations as a result of averaging of the heat and moisture exchange characteristics, etc) inevitably leads to distortion of the entire picture.

We will examine the following Stefan-type problem on a half-line for a system of two quasilinear parabolic equations, to which under constant boundary conditions the above-described problem of thermal and mass transfer is reduced in the case of monotonic freezing without heaving. It is required to find the functions (z, τ) and $\xi(\tau)$ satisfying the equations:

$$C(t, w)\frac{\partial t}{\partial \tau} = \frac{\partial}{\partial z}\left[\lambda(t, w)\frac{\partial t}{\partial z}\right], \quad z>0, \quad \tau>0, \quad t \neq t_0, \qquad (3.10.1)$$

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial z}\left[M(t, w)\frac{\partial w}{\partial z}\right], \quad z>0, \quad \tau>0, \quad t\neq t_0, \qquad (3.10.2)$$

$$t[\xi(\tau), \tau] = t_0, \quad w[\xi(\tau)+0, \tau] = w_0, \quad \tau>0, \qquad (3.10.3)$$

$$\lambda(t_0-0, \overline{w})\frac{\partial t}{\partial z}\Big|_{z=\xi(\tau)-0} - \lambda(t_0+0, w)\frac{\partial t}{\partial z}\Big|_{z=\xi(\tau)+0} -$$

$$-\mu M(t_0+0, w)\frac{\partial w}{\partial z}\Big|_{z=\xi(\tau)+0} = Q\xi'(\tau), \quad (\tau)>0, \qquad (3.10.4)$$

under the boundary conditions $t(0, \tau) = T_0$ = constant $< t_0$, $t(z, 0) = T_1$ = = constant $> t_0$, $w(z, 0) = w_1$ = constant $\geqslant w_0 > w^*(t_0)$, $z > 0$ and $\xi(0) = 0$. Here $M(t, w) = K(w)$ at $t > t_0$ and $M(t, w) = 0$ at $t < t_0$, and $\lambda(t, w)$, $K(w)$, $C(t, w)$ and $w^*(t)$ are continuous functions of their own arguments, excepting $t = t_0$, where λ and C experience a discontinuity of the first kind: $\lambda(s, p)$, K(s) and $w^*(s)$ are continuously differentiable, $\lambda \geqslant$ constant > 0, K $\geqslant$ constant > 0, C $\geqslant$ constant > 0, $w^* \geqslant$ constant $\geqslant 0$. At $t > t_0$ we have $C(t, w) \equiv$ $\equiv C_0$, $\lambda(t, w) \equiv \lambda_0$, where λ_0 and C_0 -- constant > 0, $w^*(t) \equiv w_{un}(t)$ and $t < t_0$.

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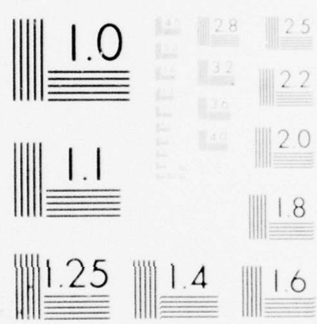

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We will prove below the existence of a solution of the system (3.10.1-3.10.4) satisfying the above-indicated conditions, such that $t(z, \tau) < t_0$, $0 < z < \xi(\tau)$, $t(z, \tau) > t_0$ and $z > \xi(\tau)$. On the basis of a conducted investigation an algorithm will be proposed for finding a solution of the problem under consideration with any prescribed degree of precision.

Since $M(t, w) = 0$ at $t < t_0$ (moisture migration in the frozen zone is not considered), then at $0 < z < \xi(\tau)$ equation (3.10.2) degenerates. Since in the given case it is assumed that the freezing occurs without heaving, the total moisture at each point of the frozen zone in accordance with (3.10.6) and (3.10.9) is written in the form

$$\bar{w}(z, \tau) = \nu \left\{ K(w_0) \frac{\partial w}{\partial z} \Big|_{z=\xi(\tau^*)+0} [\xi'(\tau^*)]^{-1} + w_0 - w^*(t) \right\} + w^*(t)(\nu - 1),$$

$$t < t_0,$$

where $\tau^* \in (0, \tau)$ also is determined from the correlation $z = \xi(\tau)$. The existence of a single $\tau^* = \tau^*(z)$ for each $z \in (0, \xi(\tau))$ follows from the fact that, as will be shown below, under the adopted boundary conditions $\xi'(\tau) > 0$ at all values of $\tau > 0$.

As before, it can be assumed without limitation of generality that $t_0 = 0$. It is obvious that at $w_1 = w_0$ the problem (3.10.1-3.10.4) is reduced to a generalized classical Stefan problem (Section 8).

We will briefly examine the course of proof of the existence and singularity of the self-modeling solution of system (3.10.1-3.10.4) at $w_1 > w_0$, and also indicate an algorithm which makes it possible to obtain that solution with any prescribed degree of precision. In that case the existence and singularity of the self-modeling solution is assured only when certain additional limitations are imposed on the thermophysical characteristics of the medium, limitations connected with the dependence of the latter on the total moisture in the frozen zone. It must be emphasized, however, that those requirements, which naturally arise from the interaction of the temperature and moisture fields, as will be shown below, are realized practically everywhere.

Similarly to section 8, we will seek the functions $\phi(p)$, $\psi(p)$ and α = constant > 0 such that $t = \phi(p)$, $w = \psi(p)$, where $p = z\tau^{-1/2}$, and also $\xi(\tau) = \alpha\sqrt{\tau}$ are a solution of (3.10.1) and (3.10.2) under conditions (3.10.3) and (3.10.4). In that case (3.10.1) is reduced to a nonlinear second-order ordinary differential equation in relation to $\phi(p)$ for $[0, \infty]$ with a "joined" solution at $\phi = 0$ in accordance with (3.10.4).

The total moisture in the frozen zone $\bar{w}(z, \tau]$, $t \leqslant 0$ is transformed into the form

$$\psi(p) = \nu [2K(w_0)\psi'(\alpha + 0)\alpha^{-1} + w_0 - w^*(0)] + w^*(0) + $$
$$+ [w^*(0) - w^*(\varphi)](\nu - 1) \equiv N(\alpha, \varphi), \quad \varphi < 0.$$

It is obvious that $N(\alpha, \phi)$ can be presented in the form $N(\alpha, \phi) = n(\alpha) + I(\phi)$, where $n(\alpha) = N(\alpha, 0)$, $I(\phi) = [w^*(0) - w^*(\phi)](\nu - 1)$.

Thus in the self-modeling case at $\phi \leqslant 0$ we have $\lambda(\phi, \Psi) \equiv \lambda[\phi, n]$ and $C(\phi, \Psi) = C[\phi, n]$, where $n = n(\alpha)$ is a parameter. If we substitute $t = \phi(p)$ in (3.10.1) we obtain

$$\varphi'' + r[\varphi, n(\alpha)]\varphi'^2 + g[\varphi, n(\alpha)]\varphi' p = 0, \quad \varphi < 0, \qquad (3.10.5)$$

$$\varphi'' + g_0\varphi' p = 0, \quad \varphi > 0, \qquad (3.10.6)$$

where $r(\phi, n) = (\ln \lambda(\phi, n))_\phi$, $g(\phi, n) = C(\phi, n)/2\lambda(\phi, n)$ and $g_o = C_o/2\lambda_o$, where $g(\phi, n) \geqslant$ constant < 0 and $g_o \geqslant$ constant > 0,

In that case the boundary conditions for (3.10.1) will assume the form

$$\varphi(0) = T_0 < 0, \quad \varphi(\infty) = T_1 \geqslant 0, \quad \varphi(\alpha) = 0. \qquad (3.10.7)$$

Demonstrated in the work of Melamed (1969) was the existence of a solution of (3.10.5) - (3.10.7) satisfying the conditions $\phi(p) < 0$, $0 < p < \alpha$, $\phi(p) > 0$ $p > \alpha$. In that case equations (3.10.5) and (3.10.6) occur at $0 < p$ and $p >$ respectively and equation (3.10.2) is reduced to a nonlinear second-order ordinary differential equation in relation to $\phi(p)$ for $[\alpha, \infty]$. If we substitute $w = \Psi(p)$ in (3.10.2) we obtain

$$\psi'' + a(\psi)\psi'^2 + b(\psi)\psi' p = 0, \quad p > \alpha, \qquad (3.10.8)$$

where $\alpha(\Psi) = K'(\Psi)K^{-1}(\Psi)$, $b(\Psi) = [2K(\Psi)]^{-1}$, where $b(\Psi) \geqslant b_o > 0$ at $\Psi \geqslant$ $\geqslant w_o$. The boundary conditions for (3.10.2) will assume the form

$$\psi(\alpha) = \omega_0, \quad \psi(\infty) = \omega_1. \qquad (3.10.9)$$

At $\phi \neq 0$, as has been pointed out, (2.10.5) and (3.10.6) have continuous coefficients and the solution must be continuous together with the first and second derivatives. At $\phi = 0$ the coefficients of (3.10.5) and (3.10.6) lose discontinuity of type I, the solution must be continuous, and $\phi'(p)$ in accordance with (3.10.4) has a discontinuity of the first kind, determined with the correlation

$$\lambda(0-0, n)\varphi'(\alpha - 0) - \lambda_0\varphi'(\alpha + 0) - \mu K(\omega_0)\psi'(\alpha) = \frac{1}{2}Q\alpha. \qquad (3.10.10)$$

In connection with the need to fasten together the solution of (3.10.5) and (3.10.6) (at $\phi < 0$ and $\phi > 0$), which, as was pointed out above (section 8), is possible only under certain conditions, and also with the fact that the value of n and, consequently, of the coefficient of (3.10.5) are determined by the flow of moisture, that is, $\Psi(\alpha)$, at first it is necessary to examine the behavior of the integral curves of (3.10.8). The investigation of (3.10.8) and construction of an algorithm for its solution with the prescribed precision are carried out similarly to the examination of equation (3.8.1) at $\phi < 0$ in the case of freezing in the range of temperatures.

For transition to the thermal part of the problem, in connection with the dependence of the thermophysical characteristics in the frozen zone on the total moisture $N(\alpha, \phi)$ it is necessary to examine in advance the connection between the ice content and the freezing rate. As a result of examination of the behavior of $\psi'(\alpha)\alpha^{-1}$ it was demonstrated (Melamed, 1969) that $n(\alpha)$ is a strictly monotonically decreasing function of α, from which follows the continuity of $\psi'(\alpha)$.

If we now examine the integral curves (3.10.5) - (3.10.7) similarly as in section 8 we arrive at the following theorem.

Theorem. Let 1) $\lambda(\phi, n) \geqslant$ constant > 0, $C(0, n) \geqslant$ constant > 0 and $K(\psi) \geqslant$ $\geqslant$ constant > 0;
2) λ, C and K be continuous functions of their own arguments, λ and K being continuously differentiable;
3) $\lambda(0\text{-}0, n)$ is non-decreasing and $r(\phi, n)$ and $g(\phi, n)$ are non-increasing functions of n;
4) $w_1 > w_0 > 0$, $T_o < 0$ and $\operatorname{sgn} T_1 = -\operatorname{sgn} T_o$.

Then the solution of (3.10.5) - (3.10.10) which satisfies the condition $\phi(p) <$ < 0, $0 \leqslant p < \alpha$, $\phi(p) > 0$ and $p > \alpha$ exists also uniquely.

From this directly follows the existence of a solution of the initial system (3.10.1 - 3.10.4). If we substitute in the found solution $p = z\tau^{-1/2}$ we obtain a solution of the initial system (3.10.1-3.10.4) which satisfies the condition $t(z, \tau) < 0$, $0 \leqslant z < \xi(\tau)$, $t(z, \tau) > 0$ and $z > \xi(\tau)$. The algorithm for numerical integration of the problem under consideration with any given precision consists of the following stages. First, using (3.10.6), (3.10.7) and (3.10.10), by means of the majorant and minorant of (3.10.5) we find the interval containing the sought value of α. Being given its current value, we find the corresponding value of $\psi'\alpha$ and, consequently, $n(\alpha)$, after which we integrate (3.10.5) from right to left. Depending on the sign of $p(T_o)$ obtained in that case (more or less than zero), we correspondingly reduce or increase α within the indicated range until we obtain $p(T_o) = 0$ with the prescribed precision. It follows from the divergence of the integral curves of (3.10.5) that if the found solution at $p = 0$ falls into ε -- the neighborhood of $\phi =$ $= T_o$, then at $p > 0$ it is known to differ from the precise solution by less than ε.

The proposed algorithm can readily be accomplished with any electronic computer, and for that it is sufficient to use only standard programs for the numerical integration of ordinary (second-order) differential equations and solution of the transcendental equation. In that case the coefficient of thermal conductivity of the frozen ground, the curve of the unfrozen water and also the coefficient of moisture transfer can be given in both functional and in tabular form.

11. The Applicability of a Very Simple Self-Modeling Solution of the Problem of Monotonic Freezing With Consideration of Moisture Migration for the Investigation of the Freezing of Moist Finely Dispersed Soils

As was found in the preceding section, in a formulation sufficiently similar to real conditions (the phase transitions occur in the range of negative temperatures and the heat and mass transfer characteristics depend on the total moisture) the finding of a very simple self-modeling solution, without consideration of heaving, of the problem of freezing with moisture migration under the theorem conditions is reduced to solution of a limiting boundary-value problem for a system of nonlinear differential equations. At the same time, in a linearized formulation, with constant heat and mass transfer characteristics of the ground, the problem is greatly simplified, and equations (3.10.1-3.10.3) are integrated in explicit form. In that case a solution of the problem is sought as a result of solution of the transcendental equation to which (3.10.4) is reduced*. The exceptional simplicity of finding a self-modeling solution of precisely the linearized problem had the result that it began to be widely used in frost studies (Zolotar', 1964; Fel'dman, 1969).

It can readily be seen, however, that the use of such a self-modeling solution to investigate freezing with moisture migration, in contrast with the Stefan problem, is far from always possible. Actually, as follows from section 10, if the conditions indicated in the formulation of the theorem are fulfilled, then for any values $T_o < 0$ and $T_1 > 0$ there is a self-modeling solution formally existing during monotonic freezing. In that case the interface in the course of the time τ moves according to the law $\xi(\tau) = \alpha\sqrt{\tau}$, where α = constant > 0. During increase of T_o and T_1, and also increase of the initial moisture w_1, the freezing rate (that is, the value of α) will diminish, while always remaining strictly positive, however.

Thus formally during any selection of $T_o < 0$, $T_1 > 0$ and $w_1 > w_o$ in the problem under consideration for all $\tau > 0$ there occurs an advance of the interface in coordinates connected with the thawed ground. However, this does not correspond to reality, since at T_o values close to 0^o and also rather large values of T_1 and $w_1 - w_o$, in the process of freezing there can be intensive heaving with the formation of ice layers, during which the freezing of thawed ground does not occur at all**. It must be noted that the impossibility of using a very simple self-modeling solution under those conditions is readily seen also from the solution itself. As indicated in section 10, in the self-modeling case the moisture in the frozen zone is determined by the value $\psi'(\alpha)\alpha^{-1}$, where $\psi'(\alpha)$ characterizes the influx of migrating moisture toward the front of freezing at the given $\alpha > 0$. In that case it is shown that the total moisture in the frozen zone monotonically decreases with increase of α, and $\psi'(0) > 0$. It follows from this that in the problem under consideration, when the freezing rate decreases ($\alpha \to 0$), the moisture content (by volume)

*In that case the conditions of the theorem which assure the existence of a unique such solution, evidently, are automatically fulfilled.

** This will be demonstrated rigorously below, in section 12.

in the frozen zone increases unlimitedly, which is impossible. As will be shown below, that contradiction is connected with the exclusion of heaving from the formulation of the problem.

Thus the formal application of such a self-modeling solution to investigate the process of freezing with moisture migration can lead to absurd results. The appearance of works devoted to specific calculations of practical importance of the amount of heaving in the process of heat and mass transfer during the freezing of dispersed media by means of a very simple solution has the result that the investigation of the possibility of application of such a self-modeling case becomes essentially necessary.

We will examine the area of applicability of a very simple self-modeling solution for monotonic freezing to investigate the influence of moisture migration on the process of freezing under real conditions. For convenience we will turn to the symbols of section 8 t, $\bar{t}$, $\xi(\tau)$, $y(\tau)$, w and $\bar{w}$. As was shown in section 9, in different cases of freezing with consideration of moisture migration (monotonic freezing occurring without heaving of the freezing ground or accompanied by slight heaving with the formation of a microschlieren texture, and also during the formation of ice layers of finite thickness) the behavior of $w[y(\tau),\tau]$ is essentially different. In connection with that the determination of the area of applicability of a very simple self-modeling solution is reduced primarily to examination of the question of the cases in which there is fulfilment of the condition $w[y(\tau),\tau] \equiv w_o$, which is taken as a boundary condition for the equation of moisture conductivity in a very simple self-modeling solution at any $T_o < 0$ and $T_1 > 0$. In that case, naturally, it is assumed that the conditions assuring self-modeling of an ordinary Stefan problem (for example, T_o, T_1 and w_1 are constants and, consequently, the absence of water-bearing horizons, etc), do not occur.

At the present time it is generally considered that the potential $P(z,\tau)$ of the forces acting on moisture at each point of the ground in the process of its freezing is an unequivocal function of the moisture content. In that case in the frozen zone the potential, generally speaking, depends on the ice content at the given point. Thus in a very simple self-modeling formulation it is necessary that $P[y(\tau),\tau]$ = constant and $\tau > 0$. From obvious physical considerations that condition has the result that, regardless of whether the potential is a continuous function or varies abruptly during transition across the interface, it happens* that $P[\xi(\tau),\tau]$ = constant and $\tau > 0$. By the same token, in the self-modeling case $w[\xi(\tau),\tau]$ = constant, that is, the water content by volume in the liquid phase on the mobile boundary of the frozen zone in the process of freezing must remain unchanged. Obviously this is equivalent to constancy of the content by volume of the unfrozen moisture $w^*(t)$ at $z = \xi(\tau)$.

It can readily be seen, however, that $w\big|_{z=\xi(\tau)} = w^*(0)$ does not depend on the moisture saturation of the ground only at $w[\xi(\tau),\tau] < dw_p$. Actually,

*If P varies continuously during the transition from the thawed into the frozen zone, then $P[y(\tau),\tau] = P[\xi(\tau),\tau]$.

we will use $w^{**}(\bar{t})$ and $\bar{t} \leqslant 0$ to designate the content of the unfrozen water by weight. As is known, at a fixed temperature w^{**} is practically independent of $\bar{w}$ and is a characteristic of the given ground. If in a certain volume of frozen ground at t = 0 the total moisture volume is equal to $\bar{v}$, the volume of the skeleton of the ground per unit of volume is equal to 1 - v at $\bar{v} > w_p$ and $1 - w_p$ at $\bar{v} > w_p$. Consequently,

$$w^{*}(0) = \begin{cases} w^{**}(0)(1-\bar{v})\beta, & \text{если } \bar{v} > w_{п}, \\ w^{**}(0)(1-w_{п})\beta, & \text{если } \bar{v} \leqslant w_{п}, \end{cases}$$

where β = constant < 0 is the correlation of the densities of the skeleton of the ground and water. By the same token $w^{*}(0)$ is the characteristic of the ground only at $\bar{v} \leqslant w_p$.

Thus constancy of $w[\xi(\tau), \tau]$ and, consequently, of $w[y(\tau), \tau] = w_o$ is possible only when the total moisture forming during freezing with moisture migration satisfies the condition $w[\xi(\tau), \tau] < w_p$, that is, there is no heaving. If, however, $w[\xi(\tau), \tau] > w_p$, then $w[y(\tau), \tau]$ varies in the course of time, being an increasing function of the difference $w[\xi(\tau), \tau] - w_p$. At the same time (Melamed, 1969) the requirements enumerated in the theorem conditions (section 10) for $\bar{\lambda}(\bar{t}, \bar{w})$ and $C(\bar{t}, \bar{w})$ at $t \leqslant 0$, which assure finding a unique solution of the problem of freezing with migration, are practically always realized and are not limiting.

Summing up the above, one can say that if the solution of (3.10.5 - 3.10.10) obtained in accordance with section 10 satisfies condition $\bar{\psi}(p) < w_p$ and $p \in [0, \alpha]$ it actually is a unique self-modeling solution of the problem of monotonic freezing with migration. Because of that the corresponding solution of (3.10.1-3.10-4) can serve as an effective and fairly reliable instrument for investigating that problem which makes it possible to determine the quantitative regularities and estimate the role of each of the numerous factors of the process, and also forecast and optimize the process for purposes of control. At the same time the use of such a self-modeling solution for calculating the amount of heaving of the ground during freezing is impossible. The application of that solution for the investigation of a real process in that case inevitably leads to errors which in the course of time increase substantially (because of disruption of the formulation of the problem).

Thus, in studying the influence of migration on the process of freezing under real conditions by means of a very simple self-modeling solution it is possible to investigate only those grounds the initial moisture content of which does not surpass the heaving threshold under the given conditions. Let us note that finding the value of the latter as a function of the boundary conditions of the heat and mass transfer characteristics also is readily accomplished by means of that solution (see Chapter 9). If one takes into consideration the complexity of the problem under consideration in the general formulation and also the fact that the value of the heaving threshold is of enormous practical importance, it is difficult to overestimate the role of the obtained very simple self-modeling solution in investigating the freezing of finely dispersed systems.

Along with that it must be emphasized that attempts to use in a similar manner the self-modeling solution of a linearized problem (when the characteristics of the medium are constant) even in the absence of heaving can considerably distort the picture. Actually, in finely dispersed grounds $C(t, w)$ is a sharply varying function of temperature with a maximal value greater by an order of magnitude than the value of the additive heat dapacity. Therefore the replacement of the effective heat capacity by a constant one changes the solution of the thermal part of the problem and, consequently, also the problem as a whole. Still greater error is involved by replacement of the constant coefficient of thermal conductivity, which with increase of the moisture content increases by several orders of magnitude. Finally, the results of the solution are substantially affected by variation of the coefficient of thermal conductivity of the frozen background as a function of the degree of moisture saturation.

Thus the use of a very simple self-modeling solution of the problem of monotonic freezing with consideration of moisture migration toward the mobile front to investigate the process under real conditions is possible only under certain conditions, when in the process of freezing a massive or microschlieren cryogenic texture forms. In addition, in the given case, examination of the problem in an essentially nonlinear formulation is far more obligatory than when a self-modeling solution is used in an ordinary Stefan problem, since use of the solution of a linearized problem even in a qualitative investigation requires preliminary determination of the values of characteristics of the medium effective for the given conditions.

12. Self-Modeling Solution of the Problem of Freezing With Moisture Migration With Consideration of Heaving and the Formation of Ice Layers

The above-examined self-modeling solution, in spite of all its simplicity and effectiveness, cannot be used to solve questions, very important from the point of view of historical and engineering frost studies, connected with heaving and schlieren formation. At the same time, some generalization of the method described in section 10, connected with the introduction into the equation of thermal conductivity for the frozen zone of a term describing the movement of the frozen zone in connection with heaving, makes it possible to obtain the corresponding self-modeling solutions both in the case of monotonic freezing and during the formation of ice layers. Let us note that during the examination of freezing with migration in the general formulation (section 9) the movement of the frozen zone during heaving is taken into consideration through corresponding change of the start of read-off in the frozen zone in comparison with the thawed.

1. The problem of the freezing of finely dispersed rocks with consideration of heaving and ice layers in a self-modeling case has the following form. It is required to find the functions $t(z, \tau)$, $\bar{t}(z, \tau)$, $w(z, \tau)$, $h(\tau)$ and $y(\tau)$ satisfying the conditions*:

*For simplicity the volumetric expansion of unfrozen moisture is excluded from consideration.

$$\frac{\partial \bar{t}}{\partial \tau} = \bar{C}^{-1}(\bar{t}, \bar{w}) \frac{\partial}{\partial z}\left[\bar{\lambda}(\bar{t}, \bar{w}) \frac{\partial \bar{t}}{\partial z}\right] + h'(t) \frac{\partial \bar{t}}{\partial z}, \quad -h(\tau) < z < y(\tau),$$
$$\tau > 0, \qquad (3.12.1)$$
$$\frac{\partial t}{\partial \tau} = a_0^2 \frac{\partial^2 t}{\partial z^2}, \quad z > y(\tau), \quad \tau > 0, \qquad (3.12.2)$$
$$\frac{\partial w}{\partial \tau} = \frac{\partial}{\partial z}\left[K(w) \frac{\partial w}{\partial z}\right], \quad z > y(\tau), \quad \tau > 0, \qquad (3.12.3)$$
$$\left.\begin{array}{l} t[y(\tau), \tau] = \bar{t}[y(\tau), \tau] = t_0, \quad \bar{t}[-h(\tau), \tau] = T_0, \quad \tau > 0, \\ t(z, 0) = T_1, \quad w(z, 0) = w_1, \quad z > y(0), \quad h(0) = y(0) = 0, \end{array}\right\} \qquad (3.12.4)$$
$$\bar{w}(y(\tau), \tau) = \frac{1}{h' + y'}\left[\nu\left(K(w) \frac{\partial w}{\partial z}\right)_{z=y(\tau)} + my'(\tau)\right], \quad \tau > 0, \qquad (3.12.5)$$
$$h'(\tau) = \begin{cases} \nu\left(K(w) \frac{\partial w}{\partial z}\right)_{z=y(\tau)} + (m - w_n)\, y'(\tau) \equiv F(\tau), & F(\tau) \geqslant 0, \quad \tau > 0, \\ 0, & F(\tau) < 0, \end{cases} \qquad (3.12.6)$$

On the mobile interface (at $z = y(\tau)$) one of the following conditions is fulfilled:

$$I[\tau, w(y(\tau), \tau)] = Qy'(\tau), \quad w(y(\tau), \tau) = w_0, \quad \text{если} \quad I(\tau, w_0) \geqslant 0, \qquad (3.12.7)$$
$$I[\tau, w(y(\tau), \tau)] = 0, \quad y'(\tau) = 0, \quad \text{если} \quad I(\tau, w_0) < 0. \qquad (3.12.8)$$

Here

$$I[\tau, w(y(\tau), \tau)] \equiv \left(\bar{\lambda}(\bar{t}, \bar{w}) \frac{\partial \bar{t}}{\partial z}\right)_{z=y(\tau)} - \lambda_0 \left.\frac{\partial t}{\partial z}\right|_{z=y(\tau)} -$$
$$-\mu\left(K(w) \frac{\partial w}{\partial z}\right)_{z=y(\tau)}; \quad \bar{\lambda}_1 \geqslant \bar{\lambda} \geqslant \bar{\lambda}_2 > 0, \quad \bar{C}_1 \geqslant \bar{C} \geqslant \bar{C}_2 > 0,$$
$$K_1 \geqslant K \geqslant K_2 > 0,$$

where $\bar{C}$ is continuous and $\bar{\lambda}$ and K are continuously differentiable functions of their arguments, T_o, T_1, λ_i, $\bar{C}_i$, K_i, $i = 1, 2$, w_o, w_1, w_p, m and t_o are constants, $T_o < t_o$, $T_1 > t_o$, $0 < w^*(0) < w_o < w_1 < w_p < 1$, λ_o, Q, μ, ν and κ_o^2 -- constants > 0, $m = \nu(w_o - w^*(0)) + w^*$. As above, without limitation of generality we will assume that $t_o = 0$. Henceforth we will assume* that $0 < m < w_p$.

It is obvious that at $w_o = w_1$ system (3.12.1-3.12.8) is reduced to the known self-modeling Stefan problem (section 8). In addition, problem (3.12.1-3.12.7)

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*At $m > w_p$, when heaving occurs at any surface temperatures, the problem is simplified.

at $h(\tau) = 0$ and $\tau > 0$ was examined in section 10. Finally, the solution of (3.12.1-3.12.7) at $I(t, w_o) = 0$ also is a solution of (3.12.1-3.12.6 and 3.12.8) at $w(y(\tau),\tau) = w_o$. Demonstrated below is the existence of a unique self-modeling solution of the problem under consideration which satisfies, depending on the boundary conditions, (3.12.1-3.12.7) or (3.12.1-3.12.6 and 3.12.8) at all $\tau > 0$ and such that $t(z,\tau) < 0$, $z \in [-h(\tau), y(\tau)]$, $t(z,\tau) > 0$ and $z > y(\tau)$. In that case the requirements for the given problems remain the same as in the proposed $h'(\tau) = 0$ and $\tau > 0$ (theorem, section 10). As a result of the investigation an algorithm was proposed for finding the indicated solution with any prescribed precision.

2. In contrast with section 10, where a self-modeling solution formally exists under any boundary conditions, in the given formulation, besides finding the solution itself, the question arises of the conditions of existence of self-modeling solutions describing the freezing and formation of an ice layer respectively.

We will examine a condition assuring monotonic freezing, that is, the existence a self-modeling solution of (3.12.1-3.12.7) at all $\tau > 0$.

We will introduce the designation $p = z/\sqrt{\tau}$. We will seek the function $\phi(p)$, $\bar{\Phi}(p)$ and $\Psi(p)$ and also the constants $\alpha \geq 0$ and $\beta \geq 0$ such that $t = \phi(p)$, $\bar{t} = \bar{\Phi}(p)$, $w = \Psi(\tau)$, $y(\tau) = \alpha\sqrt{\tau}$ and $h(t) = \beta\sqrt{t}$ are a solution of (3.12.1-3.12.7). From (3.12.5) and (3.12.6) we have

$$\beta \equiv \beta(\alpha) = \begin{cases} 2\nu K(w_0)\psi'(\alpha) + (m - w_n)\alpha, & \psi'(\alpha) \geqslant -\dfrac{(m-w_n)\alpha}{2\nu K(w_0)}, \\ 0, & \psi'(\alpha) < -\dfrac{(m-w_n)\alpha}{2\nu K(w_0)}, \end{cases} \tag{3.12.9}$$

$$\bar{w}(p) \equiv n(\alpha) = \frac{1}{\alpha+\beta}[2\nu K(w_0)\psi'(\alpha) + m\alpha], \quad -\beta \leqslant p \leqslant \alpha. \tag{3.12.10}$$

As a result, system (3.12.1-3.12.7) is reduced to the following system of nonlinear ordinary differential equations

$$\bar{\varphi}''(p) + r(\bar{\varphi}, n)\bar{\varphi}'^2 + g(\bar{\varphi}, n)\bar{\varphi}'^2(p+\beta) = 0, \quad \beta < p < \alpha, \tag{3.12.11}$$

$$\varphi'' + g_0\varphi' p = 0, \quad p > \alpha, \tag{3.12.12}$$

$$\bar{\varphi}(-\beta) = T_0, \quad \bar{\varphi}(\alpha) = \varphi(\alpha) = 0, \quad \varphi(\infty) = T_1 \tag{3.12.13}$$

$$\psi'' + a(\psi)\psi'^2 + b(\psi)\psi' p = 0, \quad p > \alpha, \tag{3.12.14}$$

$$\psi(\alpha) = w_0, \quad \psi(\infty) = w_1, \tag{3.12.15}$$

where $r = (\ln\bar{\lambda})_{\bar{\varphi}}, \quad g = \bar{C}/2\bar{\lambda}, \quad g_0 = (2a_0^2)^{-1}, \quad a = (\ln K)_{\psi}, \quad b = (2K)^{-1}.$

In the future we will sometimes use the designation

$$\widetilde{\varphi}(p) = \begin{cases} \bar{\varphi}(p), & -\beta(\alpha) \leqslant p \leqslant \alpha, \\ \varphi(p), & p \geqslant \alpha. \end{cases}$$

In that case at the point $p = \alpha$ $(\widetilde{\phi} = 0)$ the coefficients of (3.12.11-3.12.12) lose discontinuity of the first kind, $\phi(p)$ is continuous, whereas $\widetilde{\phi}'(p)$ in accordance with (3.12.7) has a discontinuity determined with the correlation

$$\bar{\lambda}(0,\ n(\alpha))\widetilde{\varphi}'(\alpha - 0) - \lambda_0\widetilde{\varphi}'(\alpha + 0) - \mu K(\omega_0)\psi'(\alpha) = \frac{1}{2}Q\alpha. \tag{3.12.16}$$

It follows directly from the indirect uniqueness theorem that, if $\widetilde{\phi}(p)$ exists, then $\widetilde{\phi}(p) \leqslant 0$, $p \in [-\beta, \alpha]$, $\widetilde{\phi}(p) \geqslant 0$ and $p > \alpha$. Proven below upon the assumptions of the theorem (section 10) with respect to r and g is the necessary and sufficient condition of existence of a unique solution of (3.12.11-3.12.16) such that $\widetilde{\phi}(p) < 0$, $p \in [-\beta, \alpha]$, $\widetilde{\phi}(p) > 0$ and $p > \alpha$.

3. The problem under consideration, described by (3.12.11-3.12.16), represents a nonlinear limiting (in the sense of conditions for ∞) boundary-value problem, for the numerical integration of which the asymptotic behavior of the integral curves must be investigated. In connection with the need in the examination of the solutions of (3.12.11) and (3.12.12) to take (3.12.16) into consideration, and also with the fact that $n(\alpha)$ and $\beta(\alpha)$ depend on $\psi'(\alpha)$, at first, quite analogously to the case $\beta(\alpha) \equiv 0$ (section 10) an investigation is made of the integral curves (3.12.14). As was found in section 10, at any $\alpha \geqslant 0$ there is a unique solution (3.12.14) under conditions (3.12.15), and $\psi'(\alpha) > 0$ and finally is a continuous monotonically increasing function of α, $\psi'(p) > 0$ and $p > \alpha$. In the same place a simple algorithm is presented for solving (3.12.14) and (3.12.15) with any prescribed precision at any $\alpha \geqslant 0$. In addition, in section 10 it was demonstrated that $\psi'(\alpha)\alpha^{-1}$ is a monotonically decreasing function of α.

We will now examine $\beta = \beta(\alpha)$ and $n = n(\alpha)$. For that purpose we will examine the equation

$$\frac{\psi'(\alpha)}{\alpha} = \frac{\omega_n - m}{2\nu K(\omega_0)}. \tag{3.12.17}$$

It is obvious that at $\alpha \geqslant 0$ there exists not more than one root α^* of (3.12.17).

The equation of the curve $\psi^*(p)$ majorizing (minorizing) the solution of (3.12.14) at $\psi^*(\alpha) = \psi(\alpha) = w_0$, $\psi^{*\prime}(\alpha) = \psi'(\alpha) = \psi'_0 > 0$ has the form

$$\psi^{*\prime\prime}(p) + a^*\psi^{*\prime 2} + b^*\psi^{*\prime}p = 0, \tag{3.12.18}$$

where a^* and b^* are certain constants which satisfy the conditions $a \leqslant a(\psi)$, $0 < b^* < b(\psi)$ $(a^* > a(\psi),\ b^* > b(\psi))$.

From (3.12.18) under conditions (3.12.15) we obtain directly

$$\psi^{*'}(\alpha) = \frac{\exp\{a^*(w_1 - w_0)\} - 1}{a^* \sqrt{\frac{\pi}{2b^*}} \exp\left\{\frac{1}{2} b^* \alpha^2\right\} \left(1 - \operatorname{erf} \sqrt{\frac{b^*}{2}} \alpha\right)} \qquad (3.12.19)$$

At any fixed value of $\alpha \geqslant 0$ the value of $\psi'(\alpha)$ is bounded from below and above by the corresponding values of the slopes of the majorant and minorant, determined with (3.12.19). Hence it follows that during change of α from 0 to ∞ the left side of (3.12.17) monotonically and continuously decreases from ∞ to a certain positive value bounded from below and above by the values $(a^*)^{-1}b^*[\exp\{a^*(w_1 - w_0)\} - 1]$ respectively at

$$a^* = a_1^* = \text{мин}\, a(\psi),\ b^* = b_1^* = \text{мин}(\psi) - \delta \ \text{и}\ a^* = a_2^* = \text{макс}\, a(\psi),$$

$$b^* = b_2^* = \text{макс}\, b(\psi) + \delta,\ \delta \in (0,\ \text{мин}\, b(\psi)),\ \psi \in [w_0,\ w_1].$$

Hence one can readily obtain a sufficient condition for the ground to be in danger of heaving at the given natural moisture content. Actually, if we take (3.12.17) into consideration, we find that if

$$\frac{w_n - m}{2\nu K(w_0)\, b_1^*} < \frac{\exp\{a_1^*(w_1 - w_0)\} - 1}{a_1^*},$$

then a^* does not exist, $\beta(\alpha) > 0$ at any $\alpha \geqslant 0$ and, consequently, in the given ground heaving will occur under any conditions of freezing. But if

$$\frac{w_n - m}{2\nu K(w_0)\, b_2^*} \geqslant \frac{\exp\{a_2^*(w_1 - w_0)\} - 1}{a_2^*},$$

then $\alpha^* > 0$ is known to exist, that is, $\beta(\alpha) \equiv 0$ at all $a > \alpha^*$, $\beta(\alpha) > 0$ at $\alpha \in [0, a^*]$. In that case a "threshold" temperature of the surface T^* will be found at which at all $T_o \leqslant T^*_o$ heaving will be absent, whereas at $T_o \in {}^o(0, T^*_o)$ heaving occurs. Since $\psi'(\alpha)$ is a monotonically increasing continuous function of α, then by virtue of $m + 1 > w_p$ from (3.12.9) it follows that $\alpha + \beta(\alpha)$ monotonically and continuously increases with increase of α. Hence in accordance with (3.12.10) we find* that $n(\alpha)$ is a monotonically decreasing function of α at $a \geqslant 0$, where $n(0) = 1$.

4. Now we will examine the integral curves (3.12.11) and (3.12.12). From (3.12.11), using the uniqueness theorem, we find indirectly that if $\bar{\Phi}_1(p)$ is a certain solution of (3.12.11) then $\bar{\Phi}_1'(p)$ does not change sign at $p \in [-\beta(\alpha), \alpha]$. Since $\bar{O}$ = constant satisfies (3.12.11) at $\bar{\Phi}'(-\beta) = 0$, then from (3.12.13) we have $\bar{\Phi}'(-\beta) > 0$ and, consequently, $\bar{O}'(p) > 0$ and $p \in [-\beta, \alpha]$. From (3.12.12), if we designate that $q_o = (2a_o)^{-1}$, we have

$$\tilde{\varphi}'(p) = \tilde{\varphi}'(\alpha + 0) \exp\{q_0^2(\alpha^2 - p^2)\},$$

$$\tilde{\varphi}(p) = \tilde{\varphi}'(\alpha + 0) \int_\alpha^p \exp\{q_0^2(\alpha^2 - s^2)\}\, ds, \quad p > \alpha. \qquad (3.12.20)$$

*If α^* exists, then $n(\alpha)$ is examined separately at $\alpha > \alpha^*$ (in that case $n(\alpha) \leqslant w_p$) and $\alpha \in [0, \alpha^*)$.

from which it is evident that $\tilde{\Phi}'(p)$ does not change sign even at $p > \alpha$. From this follows the existence of $p = p(\Phi)$ -- the inverse function of $\tilde{\Phi}(p)$, where $p(0) = \alpha$. In addition, it follows from (3.12.20) that $\phi(\infty)$ exists also finitely, and

$$T_1 = \frac{\sqrt{\pi}}{2q_0} \varphi'(\alpha)[1 - \mathrm{erf}(q_0\alpha)\exp\{q_0^2\alpha^2\}]. \tag{3.12.21}$$

Since, as in the cases examined earlier (sections 8 and 10), the condition $\tilde{\Phi}'(-\beta) > 0$ by virtue of (3.12.16) is not sufficient for fastening at $p = \alpha$ of the integral curves (3.12.11) and (3.12.12) satisfying (3.12.13), the examination of $\tilde{\Phi}(p)$ will be conducted by integration from right to left.

We will prove that the integral curves (3.12.11) and (3.12.12) under the condition $\tilde{\Phi}(\infty) = T_1$ do not intersect during movement "backward." We will examine $\tilde{\Phi}_i(p)$, i = 1 and 2, satisfying $\phi_i(\infty) = T_1$ and $\phi_i(\alpha_i) = 0$. For definiteness let $\alpha_1 > \alpha_2$. It follows from (3.12.20) and (3.12.21) that $\tilde{\Phi}_1(p) < \tilde{\Phi}_2(p)$ at $p > \alpha_1$.

Since $\tilde{\Phi}_1'(\alpha_1 + 0) > 0$ and, in accordance with (3.12.16), $\tilde{\Phi}'_1(\alpha_1 - 0) > 0$, then $\tilde{\Phi}_1(p) < 0$ at $p \in (\alpha_2, \alpha_1)$, from which that $\phi_1(p) < \phi_2(p)$ in the given interval is known.

We will designate that $z = p \pm \beta(\alpha)$ and $v(z) = \bar{\Phi}(z - \beta)$. Then from (3.12.11) and (3.12.13) we have

$$v''(z) + r(v, n)z'^2 + g(v, n)z'z = 0,$$
$$0 < z < \alpha + \beta(\alpha), \quad v(0) = T_1, \quad v(\alpha + \beta(\alpha)) = 0. \tag{3.12.22}$$

We will transform (3.12.5), taking into consideration that $z = z(v)$ (existence follows from $\Psi'(p) \geq 0$ and $p > -\beta$). Then

$$z''(v) = r(v, n)z' + g(v, n)z'^2 z. \tag{3.12.23}$$

Lemma 1 occurs. Let $z_1(v)$, 1 = 1 and 2, be solutions of (2.16.23) under the given initial conditions, where $z_2(0) \geqslant z_1(0) > 0$, $z_1(0) \geqslant z_2'(0) > 0$, and $[z_1(0) - z_2(0)]^2 + [z_1'(0) - z_2'(0)]^2 \neq 0$. Then, if r and g are nonincreasing functions of n, then at all $v < 0$ at which $z_2 \geqslant 0$ we have $z_1(v) < z_2(v)$ and $z_1'(v) > z_2'(v)$.

The given lemma is proven quite similarly to lemma 1 in the work of V. G. Melamed (1969).

From (3.12.16) and (3.12.21) we have

$$z'(0) = \bar{\lambda}(0, n)\left\{\frac{2T_1 q_0 \lambda_0}{\sqrt{\pi}} \frac{\exp\{-q_0^2\alpha^2\}}{1 - \mathrm{erf}(q_0\alpha)} + \mu K(w_0)\psi'(\alpha) + \frac{1}{2}Q\alpha\right\}. \tag{3.12.24}$$

Since $\exp\{\alpha^2\}[1 - \mathrm{erf}(\alpha)]$ is a monotonically decreasing function of α, then from (3.12.20) $\phi'(\alpha)$ is a monotonically increasing continuous function of α at a given T_1. Hence at $\bar{\lambda}_n' \geq 0$, as follows from point 3, $z'(0)$ decreases monotonically and continuously with increase of α at the given values of T_1, w_o and w_1, remaining larger than zero.

5. We will examine the family E_1 of the integral curves (3.12.23) corresponding to the given values of $\alpha \geq 0$ and those of $z'(0)$ obtained from (3.12.24). Let $v_i(z)$, i = 1 and 2, be the curves of the family E_1 corresponding to $\alpha_1 < \alpha_2$. In that case, as indicated above, $\alpha_1 + \beta(\alpha_1) < \alpha_2 + \beta(\alpha_2)$. Then, as $z_o'(0) > z_2'(0)$, it follows from lemma 1 that during decrease of v the curves of E_1 diverge. In that case, since $z(v) \equiv 0$ is a solution of (3.12.23) at $z'(0) = 0$, for curves of E_1 we have $v(z) < 0$ and $v'(z) > 0$ at $0 < z < \alpha + \beta(\alpha)$.

The equation of the curve $z^*(v)$ majorizing the corresponding curve E_1 under the conditions $z^*(0) = \alpha + \beta(\alpha)$ and $z^{*\prime}(0) = z'(0)$ has the form $z^{*\prime\prime}(v) = Rz^{*\prime} + Gz'^2 z^*$, where R and G are certain constants, where $R \leq r(v, n)$, $0 < G < g(v,n)$ at all $v < 0$, and $z'(0)$ is determined in accordance with (3.12.24). In that case the furnction

$$v^*(z) = \frac{1}{R}\ln\left[1 - \frac{R}{z'(0)}\int_z^{\alpha+\beta(\alpha)} \exp\left\{\frac{1}{2}G\left[(\alpha+\beta)^2 - s^2\right]\right\} ds\right] \tag{3.12.25}$$

majorizes the corresponding curve of (3.12.22). Similarly ($R \geq r$ and $G > g$) the majorant of (3.12.23) and minorant of (3.12.29) are determined. It can readily be seen that, in contrast with the case $\beta(\alpha) = 0$, solutions of (3.12.11-3.12.16) do not exist at any values of the initial data.

It follows directly from lemma 1 that a necessary condition of the existence of solution (3.12.11-3.12.16) is condition $z(T_o) \leq 0$ (that is, $p(T_o) \leq \beta(\alpha)$ at $\alpha = 0$. Henceforth for brevity this condition will be called condition 1. We will prove that under condition 1 the curve $v^*(z)$, determined with (3.12.25) at $\alpha = 0$, $R \leq r$ and $0 < G < g$, satisfies the condition $T_o < v^*(0) < 0$. We will examine (3.12.25) with the indicated values of R and G under the condition $v^*(0) = T_o$. It is obvious that in that case (3.12.25) is reduced to a transcendental equation with respect to α. Since $\alpha + \beta(\alpha)$ and $v'(\alpha + \beta(\alpha))$ are monotonically increasing functions of α, the right side of (3.12.25) at $z = 0$ is a continuous monotonically decreasing function of α, larger than T_o at $\alpha = 0$ under condition 1 and not limited from below. Consequently, under condition 1 a solution of the indicated equation $\bar{\alpha}$ exists and is unique, where $\bar{\alpha} > 0$. We will examine the set E_2 of those values α at which the integral lines (3.12.22) intersect the axis $z = 0$ at $v \geq T_o$. That set is not empty because of condition 1. At the same time, E_2 is limited from above by $\bar{\alpha}$, since $z(T_o) > 0$ at $\alpha = \bar{\alpha}$. It follows from the continuous dependence of the solution of (3.12.23) on the initial conditions that the precise upper boundary $\hat{\alpha}$ of the set E_2 belongs to E_2.

We will prove that the solution of (3.12.22) $v(z)$ corresponding to $\alpha = \hat{\alpha}$ cannot intersect the axis $z = 0$ at $v > T_o$. Actually, let $v(0) > T_o$. Since $\hat{v}'(0) > 0$, in a certain left half-neighborhood of the point we have $T_o < \hat{v}(z) < \hat{v}(0)$.

From this is found $\alpha < \hat{\alpha}$ such that the corresponding integral curve of (3.12.22) also intersects the axis $z = 0$ at $v > T_o$, which is impossible.

And so $0 < \hat{\alpha} < \bar{\alpha}$ and $\hat{v}(0) = T_o$. Since at the given value of T_1 from (3.12.22) $z'(0)$ is a continuous and monotonic function of α, then under condition 1 from the divergence of the integral curves of (3.12.22) it follows that for any $T_1 > 0$ there exists a unique value $\alpha = \alpha_2 > 0$ such that $v_o(0) = T_o$. Thus the following theorem 1 has been proven.

/Theorem 1/. Let at $\alpha = 0$ the solution of (3.12.11-3.12.16) satisfy condition 1. Then, if $r(\phi, n)$ and $g(\phi, n)$ are nonincreasing functions of n, $\bar{c}(\phi, n) >$ constant > 0 and $\bar{\lambda}(\phi, n) \geqslant$ constant > 0, where $\bar{c}$ is continuous, λ and K are continuously differentiable functions of their arguments and $\bar{\lambda}'_0 > 0$, then a solution of (3.12.11-3.12.16) exists and is unique under the given boundary conditions. However, if $p(T_o) > -\beta(\alpha)$ at $\alpha = 0$, no solution of (3.12.11-3.12.16) exists.

6. The conducted investigation makes it possible to propose a simple algorithm for numerical solution of the problem under consideration if $p(T_o) < -\beta(\alpha)$ at $\alpha = 0$*. At first from (3.12.25) at $z = 0$ and $v^* = T_o$ is found $\bar{\alpha}$, after which the solution of (3.12.11-3.12.16) is readily determined by the method of dividing the segment $(0, \bar{\alpha})$ in two. Assuming at first that $\alpha = 1/2\bar{\alpha}$, we integrate (3.12.14) under the conditions (3.12.15) to find the corresponding value of $\psi'(\alpha)$ and then, having determined $\beta(\alpha)$, $n(\alpha)$ and $\bar{\phi}'(\alpha)$ respectively in accordance with (3.12.19), (3.12.10) and (3.12.16), we will integrate (3.12.10) from right to left, assuming that $p = p(\bar{\phi})$.

Depending on the sign of $p(T_o) + \beta(\alpha)$ obtained in that case (larger or smaller than zero), we correspondingly reduce or increase α until we obtain $p(T_o) + \beta(\alpha) = 0$ with the given degree of precision.

By virtue of the continuous dependence on the initial conditions on the finite segment $T_o \leqslant \bar{\phi} \leqslant 0$ the solution of interest to us can thus be calculated with the prescribed precision. In that case it follows from lemma 1 that if at $p = -\beta(\alpha)$ the found solution enters ε -- the neighborhood of $\bar{\phi} = T_o$, then at $p = -\beta$ it will differ from the true known value by less than ε.

7. We will now examine the self-modeling solution of the initial problem in the case where condition 1 is not fulfilled and, consequently, monotonic freezing cannot occur. We will prove that when condition 1 is not fulfilled there is a unique self-modeling solution of (3.12.1-3.12.6) and (3.12.8) satisfying $\bar{t}(z, \tau) < 0$, $z \in [-h(\tau), 0]$, $t(z,\tau) > 0$ and $z > 0$, where $w[0, \tau] \in (w_o, w_1)$ and $\tau > 0$.

Similarly to point 2 we will seek functions $\psi(p)$, $\phi(p)$ and $\bar{\phi}(p)$ and the constant $\beta > 0$ such that $t = \phi(p)$, $\bar{t} = \bar{\phi}(p)$, $w = \psi(p)$ and $h(\tau) = \beta\sqrt{\tau}$ are a solution of (3.12.1-3.12.6) and (3.12.8).

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*It is known that if $P(T_o) = \beta(\alpha)$ at $\alpha = 0$ then the solution corresponding to $\alpha = 0$ is the sought one.

Since at $y'(\tau) = 0$ from (3.12.5) and (3.12.6) we have $\bar{w}(z) = 1$, $z \in [-h(\tau), 0]$ and $h'(\tau) = \nu(K(w)\, \partial w/\partial z)_{z=0}$, where $w(0, \tau)$ has not been determined in advance, then (3.12.1-3.12.6) and (3.12.8) are reduced to the following system of differential equations:

$$\bar{\varphi}''(p) + r(\bar{\varphi}, 1)\bar{\varphi}'^2(p) + g(\bar{\varphi}, 1)\bar{\varphi}'(p)(p+\beta) = 0, \quad -\beta \leqslant p \leqslant 0, \tag{3.12.26}$$

$$\varphi''(p) + g_0\varphi'(p)p = 0, \quad p > 0, \tag{3.12.27}$$

$$\bar{\varphi}(0) = \varphi(0) = 0, \quad \varphi(\infty) = T_1, \quad \bar{\varphi}(-\beta) = T_0, \tag{3.12.28}$$

$$\psi''(p) + a(\psi)\psi'^2(p) + b(\psi)\psi'(p)p = 0, \quad p > 0, \tag{3.12.29}$$

$$\psi(\infty) = w_1, \tag{3.12.30}$$

$$\bar{\lambda}(0, 1)\bar{\varphi}'(0) - \lambda_0\varphi'(0) = \mu(K(\psi)\psi'(p))_{p=0} = \frac{\mu\beta}{2\nu}. \tag{3.12.31}$$

It is essential that in (3.12.26-3.12.31), in contrast with (3.12.11-3.12.16) $\beta = \beta(\psi(0))$. It is obvious that the investigation of (3.12.26-3.12.31) at an arbitrary fixed value of $\psi(0) \in (w_0, w_1)$ is conducted quite similarly to the examination of (3.12.11-3.12.16). In particular, the integral curves of (3.12.27) and (3.12.28) at the indicated values of $\psi(0)$ satisfy the condition $\psi'(p) > 0$ and $p \geqslant 0$. In addition, $\psi(\infty)$ also exists finitely, and a solution of the corresponding limiting boundary-value problem exists and is unique.

We will introduce the designation $K(\psi_i)$, $\psi_i'(p) = v_i(p)$ and $p \geqslant 0$. From (3.12.29) we have

$$v_i'(p) = -\frac{1}{2}\psi_i'(p)p, \quad p > 0. \tag{3.12.32}$$

Lemma 2 occurs. Let $\psi_i(p)$, $i = 1$ and 2, be the solution of (3.12.19) under the conditions

$$\psi_2(0) \geqslant \psi_1(0), \quad v_2(0) \geqslant v_1(0), \quad [\psi_1(0) - \psi_2(0)]^2 + [v_1(0) - v_2(0)]^2 \neq 0. \tag{3.12.33}$$

Then $\psi_2(p) > \psi_1(p)$ and $p > 0$.

Proof. Let us assume the opposite, that is, that there exists a point $p_1 > 0$ very close to $p = 0$ of intersection $\psi_1(p)$ and $\psi_2(p)$. It can readily be seen that there exists a half-neighborhood $p = 0$ in which $\psi_2(0) > \psi_1(0)$. Actually, if $\psi_2(0) = \psi_1(0)$, then from (3.12.33) we have $\psi_2(0) > \psi_1(0)$. But if $\psi_2 > \psi_1(0)$, the statement is obvious.

Thus, $\psi_1(p) < \psi_2(p)$, $p \in (0, p_1)$, $\psi_1(p_1) = \psi_2(p_1)$ and $\psi_1'(p) > \psi_2'(p)$ and, consequently, $v_1(p_1) \geqslant v_2(p_1)$. It follows from (3.12.32) that

$$v_2'(p) - v_1'(p) = \frac{1}{2}p[\psi_1'(p) - \psi_2'(p)]. \tag{3.12.34}$$

If we integrate (3.12.34) in the range from 0 to p_1, by virtue of (3.12.33) we have

$$0 \geqslant v_2(p_1) - v_1(p_1) - v_2(0) + v_1(0) = \frac{1}{2}\int_0^{p_1} s\,[\psi_1'(s) - \psi_2'(s)]\,ds =$$

$$= \frac{1}{2}\int_0^{p_1} [\psi_1(s) - \psi_2(s)]\,ds > 0,$$

which is impossible. Lemma 2 has been proven.

8. Let $\Psi^0 \in (w_0, w_1)$. We will use $\Psi(p, \Psi^0)$ to designate the integral curves of (3.12.29) and (3.12.30) satisfying the condition $\Psi(0) = \Psi_0$. We will also designate that $f(p, \Psi^0) = K(\Psi(p, \Psi^0)\Psi'(p, \Psi^0)$. It is obvious that $(p, \Psi^0) > 0$ and $p \geqslant 0$.

Lemma 3 occurs: $f(0, \Psi^0)$ is a strictly monotonic decreasing continuous function of Ψ^0. Proof. Let Ψ_i^0, i = 1 and 2, satisfy the condition $w_0 < \Psi_1^0 < \Psi_2^0 < w_1$. We will examine the set F_1 of values of v(0) such that the corresponding integral curves of (3.12.29) $\Psi_2(p)$ and $\Psi_2(0) = \Psi_2^0$ intersect with $\Psi(p, \Psi^0)$. That set is not empty, since at v(0) = 0 from (3.12.29) we have $\Psi_2(p) = \Psi_2^0$ and according to lemma 2 is limited from above $f(0, \Psi_1^0)$. By virtue of the continuous dependence of the solutions of (3.12.29) on the initial conditions the precise upper boundary $\bar{v}(0)$ of the set F_1 belongs to F_1. It is obvious that $\bar{v}(0) < f(0, \Psi_1^0)$.

We will prove that the integral curve of (3.12.19) $\Psi_2(p)$ and $\Psi_2(0) = \Psi_2^0$ corresponding to $\bar{v}(0)$ satisfies the condition $\Psi_2(p) < \Psi(p, \Psi_2)$ and $p > 0$. Actually, let p_2 be the point closest to p = 0, where $\Psi_2(p, \Psi_1^0)$. Physically, $f(p, \Psi^0)$ is the flow of moisture at the point p at $w(0, t) = \Psi^0$. It is obvious that $p_2 > 0$, where $\Psi_2'(p_2) < \Psi'(p_2, \Psi_1^0)$. From the ordinary uniqueness theorem $\Psi_2'(p_2) < \Psi'(p_2, \Psi_1^0)$ and, consequently $\Psi_2(p) < \Psi(p, \Psi_1^0)$ in a certain right half-neighborhood of the point p_2. However, then, by virtue of the continuous dependence of the solutions of (3.12.29) on the initial conditions v(0) is found, belonging to F_1 and such that v(0) > v(0), which is impossible.

Thus $\Psi_2(\infty) \geqslant w_1$. We will show that $\Psi_2(\infty) = w_1$. Actually, let $\Psi_2(\infty) > w_1$. Then by virtue of the continuous dependence of the solutions of (3.12.29) on the initial conditions one finds $\tilde{v}^*(0) < \bar{v}(0)$ such that the corresponding integral curve of (3.12.29) $\Psi^*(p)$ and $\Psi^*(0) = \Psi_2^0$ also intersects $\Psi = w_1$, which is impossible.

Thus, $\Psi_2(p) = \Psi(p, \Psi_2^0)$, $p > 0$, $\bar{v}(0) = f(0, \Psi_2^0) \leq f(0, \Psi_1^0)$. We will now prove that $f(0, \Psi_2^0) < f(0, \Psi_1^0)$. Actually, let $f(0, \Psi_2^0) = f(0, \Psi_1^0)$. From $\Psi_1^0 < \Psi_2^0 < w_1$ follows the existence of the point $p_0 > 0$ such that $\Psi(p_0, \Psi_1^0) = \Psi_2^0$, where $\Psi_1'(p, \Psi_1^0) > 0$ and $p \in [0, p_0]$. Then from (3.12.32) we have $f(p_0, \Psi_1^0) = \Psi_2^0 < f(0, \Psi_1^0)$ and, consequently, $\Psi'(p_0, \Psi_1^0) < \Psi'(0, \Psi_2^0)$.

Similarly to lemma 1 it can readily be found that the integral curve of (3.12.29) $\psi_0(p), \psi_0(p_0) = \psi_2^0, \psi_0'(p_0) = \psi'(0, \psi_2^0)$ satisfies the condition

$$\psi(p, \psi_2^0) > \psi_0(p) > \psi(p, \psi_1^0), \quad p > 0. \qquad (3.12.35)$$

It follows from the uniqueness of the solution of thellimiting boundary problem for equation (3.12.29) that $\psi_0(\infty) > w_1$. Then from (3.12.35) we find that $\psi(\infty, \psi_2^0) > w_1$, which is impossible.

Thus it has been proven that $f(0, \psi^0)$ is a decreasing function of ψ^0. From (3.12.32) at the values of ψ^0 under consideration it follows that $\psi'(p, \psi^0)$ and, in accordance with (3.12.29), $\psi''(p, \psi^0)$ are limited to a certain right half-neighborhood $p = 0$. As a result, from the contrary we find directly that $f(0, \psi^0)$ is a continuous function of ψ^0. Lemma 3 has been proven.

9. We will examine the integral curves of (3.12.26) at a fixed value of ψ^0. We will transform (3.12.26) and (3.12.31), considering that $p = p(\bar{\phi})$ (the existence of $p = p(\phi)$ follows from point 4). Then

$$p''(\bar{\varphi}) = \bar{r}(\bar{\varphi}, 1)p' + g(\bar{\varphi}, 1)p'^2[p + \beta(\bar{\psi}^0)], \quad T_0 \leqslant \varphi \leqslant 0, \quad (3.12.36)$$

$$p'(0-0) = \bar{\lambda}(0, 1)[\lambda(0)\varphi'(0) + \mu(2\nu)^{-1}\beta(\psi^0)]^{-1}. \qquad (3.12.37)$$

We will examine the family F_2 of integral curves of (3.12.36) at $p(0) = 0$ and the values of $p'(0 - 0)$ obtained from (3.12.37) at the given values of ψ^0. We will use $p(\bar{\phi}, \psi^0)$ to designate them. Let $p(\phi, \psi_i^0)$, $i = 1$ and 2, be curves of the family F_2 corresponding to

$$\psi_i^0, \; \omega_0 \leqslant \psi_1^0 < \psi_2^0 < \omega_1. \text{ Из } (3.12.31) \; \beta(\psi_1^0) > \beta(\psi_2^0), \; p'(0, \psi_1^0) < p'(0, \psi_1^0).$$

Similarly to the proof of lemma 1 in that case we find that $p(\phi, \psi_1^0) > p(\phi, \psi_2^0)$, $p'(\phi, \psi_1^0) < p'(\phi, \psi_2^0)$ at all $\bar{\phi} < 0$ at which $p_1(\phi_0, \psi_1^0) \geqslant -\beta(\psi_1^0)$.

Thus it has been proven that the curves of the family F_2 diverge for the given ψ^0. Since $p = 0$ is the solution of (3.12.26) at $p'(0) = 0$, then for the curves F_2 we have $p(\bar{\phi}, \psi^0) < 0$ and $p'(\phi, \psi^0) > 0$ at any $\phi \in [T_0, 0]$ and $\psi^0 \in [w_0, w_1]$.
We will examine the set F_3 of the values of ψ^0 at which the corresponding curves of F_2 intersect the straight lines $p = \beta(\psi^0)$ at $\bar{\phi} \geqslant T_0$. That set is not empty (since $\beta(w_1) = 0$ and $p'(0, w_1) > 0$) and is limited from below by $\psi^0 = w_0$, since $p(T_0, w_0) < -\beta(w_0)$ by virtue of condition 1. From the continuous dependence of the solutions of (3.12.36) on the initial conditions the precise lower boundary ψ^0 of the set F_3 belongs to F_3. From the contrary, similarly to point 8, we find that the F_2 curve $p(\bar{\phi}, \psi^0)$ corresponding to ψ^0 satisfies the condition $\phi(-\beta(\psi^0)) = T$. Since $\beta(\psi^0)$ and $p'(0, \bar{\phi}^0)$ are monotonically decreasing and monotonically increasing continuous functions of ψ^0 at the given values of β and w_1, then, when condition 1 is not fulfilled, from the divergence of the curves of the family F_2 follows the existence of a unique value of $\psi^*(0) \in (w_0, w_1)$ such that $p(T_0, \psi^*(0)) = -\beta(\psi^*(0))$.

Thus theorem 2 has been proven.

/Theorem 2/. Let the functions $\bar{\lambda}$, $\bar{C}$ and K satisfy the conditions of theorem 1. Then, if condition 1 does not occur under the given boundary conditions, a solution of (3.12.26-3.12.31) exists and is unique, where $\Psi(0) \in (w_o, w_1)$.

10. A solution of (3.12.26-3.12.31), when condition 1 is not fulfilled, is found similarly to that examined in section 10. The difference is that here (instead of searching for the value of α) the value of $\Psi(0)$ is sought by dividing the segment $[w_o, w_1]$ in two. Assuming at first that $\Psi^o = 1/2\ (w_o + w_1)$, we integrate (3.12.29) under the condition (3.12.30) in order to find the corresponding values of $K(\Psi^o)$, $\Psi'(0, \Psi^o)$ and $\beta(\Psi^o)$. Then, finding $p'(0 - 0, \Psi^o)$ from 3.12.37), we integrate (3.12.36) at $p(0) = 0$ from right to left, depending on the sign obtained in that case for $p(T_o, \Psi^o)$ (more or less than zero), we correspondingly increase or decrease Ψ^o until we obtain $p(T_o, \Psi^o) = -\beta(\Psi^o)$ with the prescribed precision. By virtue of the continuous dependence on the initial conditions on the segment $T_o \leq \phi \leq 0$ the solution thus found can be calculated with any given precision.

Thus the self-modeling solution of the problem under consideration makes it possible during slight limitations on the characteristics of the medium to investigate the process of heat and mass transfer during the freezing of finely dipsersed grounds with consideration of heaving and ice layer formation. The proposed algorithm for finding that solution with any prescribed precision is rather simply accomplished with any electronic computer. At the same time it must be noted that the examination of the process under self-modeling conditions in no way excludes the need to solve the problems in the general case. This happens because under arbitrary boundary conditions and in a limited area of investigation the character of freezing can vary in the course of time, whereas that is excluded in the self-modeling case. In particular, if in the self-modeling case the conditions are such that a layer forms during freezing, then that formation occurs from the surface and is not limited in time.

Comment 1. As was found in the examination of monotonic freezing with heaving (point 3) in the presence of arbitrary fixed moisture-exchange characteristics $[w_o, w_p$ and $K(w)]$, the existence of α depends on the amount of initial moisture w_1. If the indicated parameters are such that a solution of (3.12.17) does not exist at $\alpha \geq 0$ (in particular, at rather large values of w_1 and T_o), freezing will be accompanied by heaving. However, if α^* exists then, other conditions being equal, a value of $T_o^* < 0$ is found at which α^* will be a solution of the entire problem. The value of w_1 corresponding to that can serve as a threshold of heaving, but only at $T_o \leq T_o^1$. On the other hand, it follows from the above considerations that even at values of w_1 rather close to w_o a value of $T_o < 0$ can be found at which freezing will be accompanied by heaving. Thus the cited proof once more emphasizes the fact that the moisture content of the heaving threshold is a function of the boundary conditions. This is valid to a still greater degree in the general case of a surface temperature which varies arbitrarily.

13. Method of Solving the Stefan Problem and the Computer Investigation of Processes of Heat and Mass Transfer During the Freezing of Rocks

The problem of the dynamics of freezing in the course of time, even in a very simple Stefan formulation (with limitation of the conductive aspect of the phenomenon) belongs to the class of very complex problems of mathematical physics. At the present time a solution of the Stefan problem in explicit form can be obtained only in the self-modeling case of a unidimensional problem (for a homogeneous half-space with constant boundary conditions, heat and mass transfer characteristics of the ground in the thawed and frozen states and a linear dependence of the curve of unfrozen water on temperature). It is obvious that the practical use of the indicated self-modeling solution is difficult.

When there are any changes in the situation as compared with the self-modeling task, and also in the multidimensional case, only a numerical solution of the corresponding Stefan-type problem can be obtained with electronic or analog computers. Still greater difficulties arise during an attempt to take into consideration moisture transport together with conductive heat transfer.

The above-indicated has the direct result that a solution of the problem of freezing even in a unidimensional formulation is impossible in general without using modern computer technology. Together with that it should be emphasized that up to now a number of the input parameters have been determined with insufficient precision. If one takes into consideration also the complexity of finding the solution itself, and also the fact that for purposes of forecasting and control of the freezing process it usually is necessary to obtain only approximate results, obtaining approximate solutions of the problem of freezing remains extremely urgent and must be developed in every possible way.

It also should be borne in mind that there recently has been a rapid development of experimental technology for determination of the heat and mass transfer characteristics of the medium and the refinement (as a rule, in the direction of complication) of known concepts of varied phenomena accompanying freezing and thawing. On the other hand, the formulation and solution of the problem of freezing in a very complete form, inevitably leading to the use of electronic computers, stimulate in turn the development of experimental work. Thus the realization of the possibilities being opened up in the use of electronic computers is in a certain sense connected with the planning of experiments, and the reverse. A characteristic example of such interaction is the mathematical study of the process of moisture migration during freezing, when the formulation of the corresponding problem required from experimenters answers to a number of questions connected with the behavior of moisture on the contact of the thawed and frozen zones. In addition, as calculations showed a substantial dependence of the dynamics of the process on the value of the coefficient of potential conductivity (diffusivity), the investigation of freezing in moisture-saturated finely dispersed grounds by means of electronic computers led to a need to develop new procedures for more precise determination of that parameter, etc.

Taking into consideration what has been said, one can predict with confidence that the application of computers in the solution of both multidimensional and unidimensional problems connected with freezing (thawing) will become practically universal in the very near future. Even at present a considerable number of planning and prospecting organizations are widely using digital and analog computer calculations of the Stefan problem in solving various questions. At the same time, the expansion (both quantitative and qualitative) of the area of application of computers in the solution of various problems in frost studies is advancing a number of problems connected with the effectiveness of realization of existing algorithms for the numerical integration of a Stefan-type problem with a digital computer. The main difficulty arising in that case consists in obtaining by computer reliable results of computation. All this leads to a need for analysis of available numerical methods of solving unidimensional and multidimensional problems of the Stefan type and presents important requirements from the point of view of obtaining and accomplishing given algorithms for solution with the prescribed degree of precision. The fact is that during the accomplishment of algorithms for the computer solution of a Stefan-type problem, besides the accumulation of various errors arising at each step of integration, the result depends to an enormous degree on the stability, convergence, order of approximation and other qualities of the difference scheme which must be verified in all cases. As is known, in the numerical integration of equations of the parabolic type both explicit and implicit difference schemes are used. The former are far simpler and the volume of work [of the order of $O(1/h^2)$, where h is the step on the coordinate] is far smaller than in implicit schemes, where the volume of work is of the order of $O(1/h)$. However, implicit schemes at a weight of not less than 0.5 are unconditionally stable and can be used at any values of h and steps in time τ, whereas explicit (a weight equal to zero) are stable only at $\tau \leqslant \frac{1}{2\alpha^2} h^2$, where α^2 is the maximal value of the thermal conductivity. In an unidimensional case an explicit scheme is provisionally stable at $\tau \leq \frac{1}{2\alpha^2} h^2$. It must be borne in mind that with respect to frost study problems assuring stability in explicit schemes is, as a rule, difficult, because of the small values of τ (in relation to the total integration time).

It would appear, since formally the precision of a solution increases with decrease of the integration step, it would be advisable to select them as small as possible. However, with increase of the number of steps the volume of calculations* increases sharply and, consequently, the total error increases for the entire interval of integration. Recently, in connection with that, so-called economic schemes are used more and more often, schemes which combine in themselves the best qualities of explicit (small volume of work) and implicit (unconditional convergence) schemes.

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*It is obvious that in problems with mobile boundaries the rate of movement of which is not known in advance, in the process of finding the movement of the front the corresponding equations in each zone must be solved multiply in each time layer.

Finally, it must be borne in mind that until convergence of a difference solution toward a solution of the initial problem has been demonstrated and an estimate is made of the rate of convergence, it is not at all clear whether a digital computer solution is true. In addition, if the question of convergence is to be made clear, as is often done in practice, by contraction of the grids, an incorrect conclusion can be drawn about its convergence (the scheme "converges" but not toward the solution of the initial problem).

Thus the use of electronic computers for the numerical integration of problems of the Stefan type presents a number of rigid requirements both for the algorithms themselves (from the point of view of their software) and for their realization. As was assumed in the theory of equations in partial derivatives, the Stefan problem can be conventionally divided into the linear and the quasilinear. The difference between them is that in the linear case the coefficients of the corresponding equations are functions only of the coordinates and time (in particular, piecewise constants), whereas in the quasilinear case they depend also on the unknown functions. A characteristic example of the quasilinear problem is the freezing of finely dispersed grounds, where in the frozen zone, instead of the additive heat capacity, it is necessary to examine the effective heat capacity, which varies by an order of magnitude in the region of the main phase transitions. It must be borne in mind that even in the linear case the Stefan problem is in principle nonlinear because of the presence of a mobile interface. That complicates extremely the investigation of particular and general regularities of the processes connected with freezing and thawing, since during any changes of the boundary conditions it leads to a need to make repeated calculations. In particular, nonlinearity of the Stefan problem, even in the case of linear equations, excludes the possibility of applying the principle of superposition of solutions to it, etc. This applies to a still greater degree to the quasilinear problem, where in the process of solution it is necessary to use the method of iterations.

At the present time algorithms for the solution of heat and mass transfer during freezing in the unidimensional case have been very greatly worked out and substantiated. A comparative classification of methods of numerical solution of an unidimensional problem of the Stefan type is as follows. One of the most effective and simply realized algorithms for solutions of a linear Stefan problem is the method of reducing it to a system of ordinary differential equations. Among its merits, besides simplicity of realization, is the fact that that method permits with the prescribed precision finding a solution of the multifront problem with a varying phase state of the water in the case of the first, second and mixed (with geothermal heat taken into consideration) boundary-value problems in both annual and perennial cycles. A substantial advantage of that method from the computational point of view is the fact that the determination of the rate of advance of the interface, which is the main characteristic of the process, in contrast from any difference methods is made without extremely approximate numerical differentiation.

In the case of a single-front quasilinear problem, if the process of freezing is examined within the limits of the time interval in which the mobile boundary moves monotonically, the difference method of "capturing the front in a grid node." Its idea consists in finding by means of iterations that time segment in the course of which the interface falls from one coordinate grid node into another. In the case of a multifront quasilinear problem, if the number of mobile boundaries remains unchanged, the method of straightening of fronts can be used. In that case, as a result of transformation at each step in time, the corresponding zones are brought to the segment (0.1), and in each phase the equation of thermal conductivity is substantially complicated. However, if in the process of freezing there can be the formation or disappearance (degeneration to a point) of phases, it is advisable to find a solution of a quasilinear problem of the Stefan type by the method of smoothing of coefficients. In that case the phase transitions occurring on the zone interface are "smeared" in a certain temperature interval with its center at the critical point.

Finally, the last problem, and also conjugate problems (in particular, when mass exchange is taken into consideration in connection with moisture migration during freezing or the infiltration of volatile sediments during thawing) can be solved by the straight-line method (the Rote scheme). In that case only the derivative in time can be replaced by a difference ratio and on each time layer the process is described by a system of nonlinear ordinary differential equations. It should be emphasized that it was precisely that method which made it possible to obtain a numerical solution of the problem of freezing with moisture migration with consideration of heaving and the formation of ice layers.

The methods of trapping the front in a "grid node," of "straightening fronts" and of "smoothing of coefficients" have been developed and are now used in the computer of Moscow State University. The finding of a solution of problems of the Stefan type by reduction to a system of ordinary differential equations and the straight-line method has been substantiated and is being realized in practice at the Department of Geocryology of MSU. It must be borne in mind that in all the enumerated difference methods an implicit scheme is used which assures stability of solution practically independent of the step in time, where the numerical solution of boundary-value problems for the control of thermal conductivity is accomplished by means of trial runs with iterations. Those algorithms make it possible to find a solution in the presence of any nonlinearities in the coefficients of equations and any types of boundary conditions.

The situation is much worse in the case of multidimensional problems. In essence the only fairly well substantiated algorithm for the solution of multidimensional problems of the Stefan type is the method of smoothing of coefficients. In that case the corresponding equations of thermal conductivity in the frozen and thawed zones are solved by the Pis'men-Recorde method or in accordance with the economic locally unidimensional scheme of Samarskiy with iterations.

However, both the programing and computation of multidimensional problems on modern model M-20 computers present considerable difficulties. It suffices to say that even when there is limitation of the number of iterations the machine time required for computation of a single variant of a two-dimensional problem with a model M-20 is 5-10 hours. The worst convergence usually occurs upon the disappearance or appearance of new zones.

Thus at present the numerical realization of algorithms for the computer solution of multidimensional problems is extremely laborious. In connection with that, attempts have been recently made to approximately solve multidimensional problems by methods in a certain sense similar to the method of balance (the integral-interpolation method) (L. N. Khrustalev, M. A. Minkin, A. I. Levkovich and Yu. S. Pal'kin). The idea of such methods is loaded with the method of hydraulic analogies, widely applied in the investigation of problems with phase transformations. The hydraulic integrator of the system of V. S. Luk'yanov (IGL) is a computing device very well known in frost studies, by means of which a considerable number of various problems connected with freezing and thawing have been investigated. It must be emphasized that a solution obtained by modeling is known to be stable, and the only complexity here consists in estimating the precision of such a solution. The successes achieved by means of the IGL and also the simplicity and accessibility of the procedure for solution by means of the method of hydraulic analogies, on the one hand, and the growth of computer technology, on the other, have caused various attempts to "replace" the hydraulic integrator by electronic computer calculations to some degree equivalent (with precision with respect to the discretization of the time* and increase of the number of nodal points).

It is unconditional that in assuring stability in some cases the results of such computer calculations can give not only a qualitatively but also a quantitatively sufficiently correct result. However, because of the above-indicated difference between an unconditionally stable solution on an IGL and a conditionally stable solution with an electronic computer, and also in connection with the accumulation of errors and, as a result of that, the impossibility either of assuring the prescribed precision or estimating the size of the error in the obtained solution, the indicated approximate computer methods of solution cannot give confidence in the result. Therefore during any changes, in essence each such calculation must be verified by comparison with the solution of a similar problem on a model or by means of the above-considered laborious but substantiated methods. In addition, during the computer implementation of approximate methods it is necessary to monitor the stability of the method itself by making several calculations of a given problem with different correlations between the steps in time and space. Nevertheless such monitoring is only a necessary but not a sufficient condition of applicability of the scheme.

Thus, approximate methods of solving Stefan-type problems by computer, especially in the unidimensional case, when there is a large volume of computations and an absence of software, cannot give complete confidence in the correctness of the calculation. In addition, the coincidence of some data thus obtained with the solutions known in separate cases cannot serve as a criterion of the applicability of the algorithm in general when there are any changes

*In the method of hydraulic analogies discretization is carried out only with respect to space.

in the formulation of the problem or in the boundary conditions. Needed in such an approach is systematic verification of computer results by means of the same computer, an analog computer or by full-scale modeling, which is assured to a considerable degree by the use of electronic computers.

Of course, all that has been said in connection with the examination of numerical realization of the scheme does not remove the need for analysis of the formulation of the problem from the point of view of rather complete description of the phenomenon under consideration. However, if all the conditions assuring the obtaining of reliable computer results are fulfilled, then series calculations of Stefan-type problems can readily be made by computer with variation of the input parameters and boundary conditions in a wide range. In the final account the use of electronic computers makes it possible on the basis of the obtained approved reference data to determine the general regularities of the formation and development of processes of freezing and thawing of rocks as a function of different elements of the geological and geographic medium and the conditions of freezing and, as a result of that, approach a solution of the question of optimization in the forecasting and control of freezing processes. In addition, it is precisely the series computer calculations which must become a reliable tool for the construction of practically adequately precise and simple formulas for the calculation of extremely complex problems of frost studies.

Thus the use of electronic computers in frost studies constitutes one of the links of general complex frost investigations which include field survey work, stationary investigations at test areas and stations, full-scale and mathematical modeling and laboratory work on the determination of the composition and properties of rocks, and also data on the development of simplified methods (approximate formulas and nomograms) needed in conducting a frost survey under field conditions.

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Chapter 4. Approximate Formulas for Determination of Heat Cycles and the Depths of Seasonal and Perennial Freezing (Thawing) of Rocks

1. Approximate Formulas for Determination of Heat Cycles and Depths of Seasonally Frozen (Thawed) Layers

The solutions of the Stefan problem presented in Chapter 3 make it possible to illuminate many questions of frost studies. However, they can be calculated only with electronic computers, which are still not available to all institutions studying questions of frost studies. Still more important is the fact that calculations must be made in the field during the making of a frost survey, when the use of electronic computers is completely excluded. Simultaneously with that the Stefan problem solutions presented in Chapter 3 do not reflect the clear connection of the thermophysical aspect of the freezing (thawing) of rocks with the geographical geological nature of that phenomenon. That linkage is possible only through determination of the importance in the formation of the depths of seasonal freezing (thawing) of rocks of characteristics which, being principal and leading, permit establishing the particular and general regularities in the formation of that phenomenon and classifying it. Such basic classificational characteristics, as will be shown in Chapter 5, are the annual amplitude of temperatures on the surface of the ground, the mean annual temperature on the base of the layer of seasonal freezing (thawing) and the composition and moisture content of the ground (Kudryavtsev, 1959).

/Approximate formulas for determining the depths of seasonal freezing (thawing) of rocks/. To reveal particular and general regularities in the formation of seasonal and perennial freezing (thawing) or rocks it is necessary to obtain a calculating formula which would make it possible to link the seasonal (ξ) or perennial (ξ_{per}) freezing (thawing) of rocks with the main classificational characteristics. The results of calculation with that formula ought to give a close convergence with the known precise methods of solving the Stefan problem.

The indicated four parameters enter the Fourier equation of thermal conductivity for periodic oscillations on the surface of the earth, but without consideration of the phase transitions of water during freezing of the ground (3.3.4). We will use that equation, adding to it the heat cycles connected with phase transitions of water. In that case the heat cycles for the envelope of the temperature curves (Q_0) will be equal to:

$$Q_0 = Q_c + Q_w,$$

where Q_c is the heat cycles connected with the heat capacity of the ground and Q_w is the heat cycles connected with phase transitions of water in the ground during its freezing (thawing).

For a layer with the thickness z we have $W_w = zQ_\phi^*$. As a result, if Q_0 is expressed in accordance with (3.3.6), the total heat cycles on the envelopes for a layer of seasonal freezing (thawing) with the thickness ξ assume the form

$$Q_0 = (2A_{cp}C + Q_\Phi)\xi. \qquad (4.1.1)$$

It is advisable to write that expression in the following form:

$$Q_0 = 2\xi A_{np}C,$$

where A_{red} is the reduced mean amplitude for the layer ξ, which also includes the heat of phase transitions of water:

$$A_{np} = A_{cp} + \frac{Q_\Phi}{2C}. \qquad (4.1.2)$$

Expression (4.1.2) determines the effective amplitude, which includes the additional term $Q_\phi/2C$, representing the conditional variation of amplitude in connection with phase transitions of water in the layer of freezing (thawing).

For determination of the values of A_{red} and A_m in the layer ξ with consideration of the phase transitions of water, assuming that $\lambda_T = \lambda_m$, we will use formula (3.3.7), introducing in accordance with out assumption, instead of A_o and A_ξ the expressions $A_o + Q_\phi/2C$ and $t_\xi + Q_\phi/2C$, where t'_ξ is the average annual temperature on the base of the layer of seasonal freezing (thawing). Then we have

$$A_{np} = \frac{A_0 - t_\xi}{\ln \dfrac{A_0 + \dfrac{Q_\Phi}{2C}}{t_\xi + \dfrac{Q_\Phi}{2C}}},$$

from which by means of (4.1.2) we obtain

$$A_{cp} = \frac{A_0 - t_\xi}{\ln \dfrac{A_0 + \dfrac{Q_\Phi}{2C}}{t_\xi + \dfrac{Q_\Phi}{2C}}} - \frac{Q_\Phi}{2C}. \qquad (4.1.3)$$

The final expression for determination of the depth of seasonal freezing (thawing) of the ground at $\lambda_T = \lambda_m$ during the period T is presentdd in the following form (Kudryvtsev and Melamed, 1965):

$$\xi = \frac{2(A_0 - t_\xi)\sqrt{\dfrac{\lambda TC}{\pi}} + \dfrac{(2A_{cp}C\xi_{2c} + \xi Q_\Phi)Q_\Phi\sqrt{\dfrac{\lambda T}{\pi C}}}{2A_{cp}C\xi_{2c} + Q_\Phi\xi + \sqrt{\dfrac{\lambda T}{\pi C}}(2A_{cp}C + Q_\Phi)}}{2A_{cp}C + Q_\Phi}. \qquad (4.1.4)$$

*Calculation of Q_ϕ with formula (4.1.8) is presented in example 1.

where

$$\xi_{2c} = \frac{2(A_0 - t_\xi)\sqrt{\frac{\lambda TC}{\pi}}}{2A_{cp}C + Q_\phi}. \qquad (4.1.5)$$

Formula (4.1.4) constitutes a quadratic equation with respect to ξ. It can readily be seen that its roots are real and of different signs; from physical considerations the negative root is discarded. To estimate the precision of the proposed formula the values of ξ were determined by computer (Kudryavtsev and Melamed, 1965). The maximal relative error in a rather large range of moisture contents and amplitudes does not exceed 5%; when the moisture content decreases the divergences decrease.

Calculation of the Depth of Seasonal Thawing (Freezing) in the Case of Equality of the Coefficients of Thermal Conductivity of Rocks in the Frozen and Thawed States (Example 1)

The following initial parameters are necessary for calculation of the depth of the seasonal thawing (freezing): the average annual temperature (t_o) and the physical amplitude of the annual temperature oscillations (A_o) on the surface of thawing (freezing) rocks, the volumetric heat capacity of thawed (frozen) rock (C_{vol-t} and C_{vol-f}) and its coefficient of thermal conductivity (λ), and also the heat expenditures on phase transitions of water in the rock during freezing (Q_{ph}).

The temperature conditions on the surface of rocks are determined in the process of field frost investigations with consideration of the influence on them of various natural or artificial covers. The coefficient of thermal conductivity of rock with an undisturbed structure and natural moisture, in which freezing is proceeding, is determined by special laboratory or field methods. The volumetric heat capacity and heat expenditures on the phase transitions of water in nature are calculated on the basis of calorimetric determinations of the specific heat capacity of the rocks (C_{spec}) and the quantity of unfrozen water in the rock (w_{un}) at different negative temperatures, and also of field determinations of the unit weight of the skeleton of the rocks (γ_{sk}) and its natural moisture* (w).

To calculate the volumetric heat capacity of frozen rock and the heat of the phase transitions it is necessary to know the quantity of unfrozen water in the rock which remains after freezing and does not participate in the phase transformations. For that purpose a graph is constructed of the variation of the quantity of unfrozen water in the rock (w_{un}, %) as a function of its negative temperature (-t). Then the average winter temperature on the surface of the freezing rock is determined for the given case and the value of w_{un} corresponding to the obtained temperature is taken from the graph of the

*The moisture content by weight (total) in relation to a dry weighted portion.

dependence of w_{un} on -t. The calculations of the volumetric heat capacity and the quantity of heat of the phase transformations of water are made with the following formulas:

$$C_{об.м} = C_{уд}\gamma_{ск} + 0{,}5\,\frac{(\omega - \omega_н)\,\gamma_{ск}}{100} + 1{,}0\,\frac{\omega_н\gamma_{ск}}{100},\ ккал/м^3град; \quad (4.1.6)$$

$$C_{об.т} = C_{уд}\gamma_{ск} + 1{,}0\,\frac{\omega\gamma_{ск}}{100},\ ккал/м^3град; \quad (4.1.7)$$

$$Q_ф = 80\,\frac{(\omega - \omega_н)\,\gamma_{ск}}{100},\ ккал/м^3. \quad (4.1.8)$$

The thus obtained initial parameters permit going over directly to calculation of the depth of freezing (thawing) of rock with formula (4.1.4), which for more convenient calculation under field conditions can be written in the following form:

$$\xi = \frac{-B + \sqrt{B^2 + 4DE}}{2D}, \quad (4.1.9)$$

where

$$D = \alpha\delta; \quad B = \alpha v + \alpha^2\sigma - \beta\delta - \sigma\delta^2; \quad E = v\beta + \alpha\beta\sigma + v\sigma\delta;$$

$$\alpha = A_{ср} + \delta; \quad \beta = (A_0 - t_\xi)\,\sigma; \quad v = A_{ср}\xi_{2c};$$

$$\sigma = \sqrt{\frac{\lambda T}{\pi C_{об}}}; \qquad \delta = \frac{Q_ф}{2C_{об}}; \qquad A_{ср} = \frac{A_0 - t_\xi}{\ln\frac{A_0 - \delta}{t_\xi + \delta}} - \delta;$$

$$\xi_{2c} = \frac{\beta}{\alpha};$$ T is a year, or 8760 hours.

For example, it is required to calculate the depth of seasonal thawing of alluvial sandy loam if the following are given: $t_0 = -2^\circ$; $A_0 = 12^\circ$; $\gamma_{sk} = 1250$ kg/m^3; w = 23%; C_{spec} = 0.18 kcal/(kg)(degree); $\lambda_f = \lambda_t$ = 0.9 kcal/(m)(degree)(hr). The given determinations of w_{un} are presented on Figure 11.

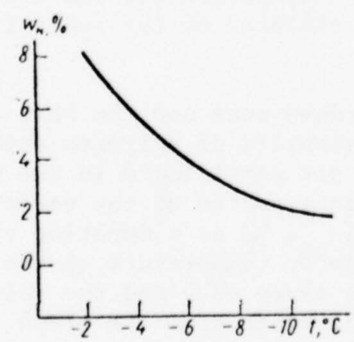


Figure 11. Graph of w_{un} as a function of temperature for sandy loam.

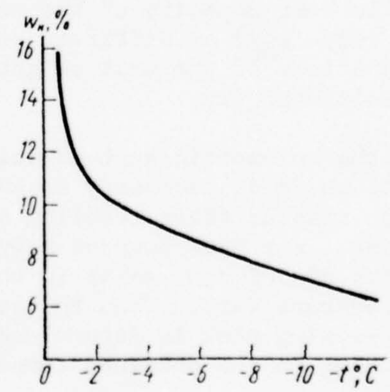


Figure 12. Graph of w_{un} as a function of temperature for loam.

Solution. 1. We will determine the volumetric heat capacity of thawed loam and the heat of phase transformations of water, for which we determine w_{un} and the average winter temperature on the surface t_{wtr}:

$$t_{зим} = \frac{t_{жин}}{2} = \frac{|t_0| + A}{2},$$

$$t_{зим} = \frac{2+12}{2} = -7^{\circ}. \qquad (4.1.10)$$

According to the graph (Figure 11) we find that at $t_{wtr} = -7^{o}$, $w_{un} = 3\%$

$$C_{об\,т} = 0{,}18 \cdot 1250 + 1{,}0\,\frac{23 \cdot 1250}{100} = 512{,}5\ ккал/м^3град;$$

$$Q_{\Phi} = 80\,\frac{(23-3)\,1250}{100} = 20\,000\ ккал/м^3.$$

2. We will calculate the sought depth of the seasonal thawing, making the corresponding substitutions in expression (4.1.9):

$$A_{cp} = 6{,}7;\quad \xi_{2c} = 0{,}84;\quad \delta = 19{,}5;\quad \sigma = 2{,}21;\quad \upsilon = 5{,}63;$$

$$\beta = 22{,}1;\quad \alpha = 26{,}2;\quad E = 1646{,}63;\quad B = 393{,}24;\quad D = 510{,}9.$$

$$\xi = \frac{-393{,}24 + \sqrt{393{,}24^2 + 4 \times 510{,}9 \times 1646{,}63}}{2 \times 510{,}9} = \frac{-393{,}24 + 1876}{1021{,}8} = 1{,}45\ м.$$

/Approximate formulas for determination of heat cycles in the layer of annual temperature fluctuations/. Also determined from (4.1.4) is the formula for the heat cycles passing across the surface of the soil in a half-year with consideration of phase transitions of the water in grounds during freezing (thawing). The heat cycles across the surface for the layer of seasonal freezing are equal to

$$Q_{\xi} = \sqrt{2}\,(A_0 - t_{\xi})\sqrt{\frac{\lambda TC}{\pi}} + \frac{(2A_{cp}C\xi_{2c} + Q_{\Phi}\xi)\,Q_{\Phi}\sqrt{\frac{\lambda T}{\pi C}}}{2A_{cp}C\xi_{2c} + Q_{\Phi}\xi + \sqrt{\frac{\lambda T}{\pi C}}\,(2A_{cp} + Q_{\Phi})}. \qquad (4.1.11)$$

In thawed grounds underlying the layer of seasonal freezing the annual heat cycles passing across the surface in accordance with (3.3.9) are

$$Q_h = \sqrt{2}\,t_{\xi}\sqrt{\frac{\lambda TC}{\pi}}. \qquad (4.1.12)$$

Summarizing Q_h and Q_{ξ}, we obtain an expression for the heat cycles acrosss the surface in the layer of annual temperature fluctuations $H = h + \xi$ with consideration of the phase transitions of water during seasonal freezing in the following form:

$$Q = \sqrt{2}A_0\sqrt{\frac{\lambda TC}{\pi}} + \frac{(2A_{cp}C\xi_{2c} + Q_{\Phi}\xi)\,Q_{\Phi}\sqrt{\frac{\lambda T}{\pi C}}}{2A_{cp}C\xi_{2c} + \sqrt{\frac{\lambda T}{\pi C}}\,(2A_{cp}C + Q_{\Phi})} \qquad (4.1.13)$$

Control calculations of the heat cycles with the given formula made by computer gave a convergence good enough for practical purposes (1-5%) (Kudryavtsev and Melamed, 1965).

The annual heat cycles in the region of seasonal freezing of rocks are formed of three component parts: a) the portion of the heat cycle expended on change of the temperature of the layer of seasonal freezing (determined by the heat capacity of the rocks of the layer); b) the portion of the heat cycle expended on the phase transitions of water into the layer of seasonal freezing (determined by the moisture content of the rocks), and c) the portion of the heat cycle expended on change of the temperature in the layer of its annual fluctuations below the base of the seasonal freezing (determined by the heat capacity of the rocks).

In the region of propagation of permafrozen rocks the annual heat cycles are formed of four component parts: the first three, corresponding to those enumerated above, and a fourth -- the portion of the heat cycle expended on the phase transitions of water in permafrozen rocks lying in the layer of annual temperature fluctuations below the base of the layer of seasonal freezing. In connection with that the calculation of the annual heat cycles in permafrozen rocks and their seasonal thawing has certain specifics in comparison with calculation of the annual heat cycles in the region of propagation of thawed rocks. Therefore we will examine the simpler case first.

Calculation of Heat Cycles in the Layer of Annual Temperature Fluctuations in the Area of Seasonal Freezing of Rocks (Example 2a)

For convenience of calculation formula (4.1.13) can be written in the following form:

$$Q = \sqrt{2}A_0 \sqrt{\frac{\lambda T C_{\text{об}}}{\pi}} + \frac{Q_{\Phi}}{\frac{1}{\sigma} + \frac{\alpha}{v + \xi\delta}}, \qquad (4.1.14)$$

where

$$\alpha = A_{cp} + \delta; \quad v = A_{cp}\xi_{2c}; \quad \sigma = \sqrt{\frac{\lambda T}{\pi c_{\text{об}}}}; \quad \delta = \frac{Q_{\Phi}}{2C_{\text{об}}}.$$

The parameters t_o, A_o, C_{rev}, λ and Q_{ϕ} necessary for calculation of the heat cycles are determined in the same manner as in example 1.

/Example of calculation/. Calculate the annual heat cycle in loams which passes across the surface of the soil under the following conditions: $A_o = 17^o$, $\gamma_{sk} = 1300$ kg/m^3; w = 30%, $C_{spec} = 0.19$ kcal/(kg)(degree) and $\lambda_f = \lambda_t$ °= = 1.0 kcal/(meter)(degree)(hr).

In a calorimetric test of a sample of loam to determine w_{un} at a negative temperature the following data were obtained (Figure 12).

Solution. 1. According to (4.1.10) we determine t_{wtr} to be -7.8°, at which w_{un} = 8%. We determine the volumetric heat capacity of loam and the heat of phase transformations of water in loam during its freezing and thawing with formulas (4.1.6), (4.1.7) and (4.1.8):

$$C_{об\,м} = 247 + 0,5\,\frac{22\cdot 1300}{100} + 1,0\,\frac{8\cdot 1300}{100} = 494\ ккал/м^3град;$$

$$C_{об\,т} = 0,19\cdot 1300 + 1,0\,\frac{30\cdot 1300}{100} = 637\ ккал/м^3град;$$

$$Q_{\Phi} = 80\cdot\frac{(30-8)\,1300}{100} = 22\,880\ ккал/м^3.$$

2. We determine the depth of seasonal freezing of loam with formula (4.1.9);

$A_{cp}=8,6$; $\xi_{2c}=1,16$; $\delta=23,16$; $\sigma=2,38$; $v=9,98$; $\beta=36,89$; $\alpha=31,76$; $E=3706,75$; $B=586,71$; $D=735,56$; $\xi=1,88$ м.

3. We calculate the sought value of the annual heat cycle in loam, making the corresponding substitutions in formula (4.1.14):

$$Q = \sqrt{2}\cdot 17\sqrt{\frac{1,0\cdot 637\cdot 8760}{3,14}} + \frac{22\,880}{\dfrac{1}{2,38} + \dfrac{31,76}{9,98+1,88\cdot 23,16}} =$$

$$= 31\,952 + 22\,577 = 54\,529\ ккал/м^2.$$

Calculation of Heat Cycles in the Layer of Annual Temperature Fluctuations in a Region of Permafrozen Rocks (Example 2b)

Annual temperature fluctuations in a permafrozen rock mass are caused by changes of w_{un} in the layer beneath the base of seasonal thawing. Therefore, as has already been noted, the annual heat cycles in the region of propagation of permafrozen rocks include the heat of phase transitions of water both in the layer of seasonal thawing of rocks and in the lower-lying layer of frozen deposits. In that case formula (4.1.13) for determination of the annual heat cycles in the layer h + ξ of the region of permafrozen rocks assumes the form

$$Q = \sqrt{2}A_0\sqrt{\frac{\lambda T C_{об}}{\pi}} + \frac{Q_{\Phi}}{\dfrac{1}{\sigma} + \dfrac{\alpha}{v+\xi\delta}} + \frac{\Delta\omega}{\dfrac{1}{\sigma} + \dfrac{S}{U+h\psi}}, \quad (4.1.15)$$

where

$$\psi = \frac{\Delta\omega}{2C_{об}};\quad S = B_{cp} + \frac{\Delta\omega}{2C_{об}};\quad U = B_{cp}\cdot h_{2c};$$

$$h_{2c} = \frac{2t\sqrt{\dfrac{\lambda T C_{об}}{\pi}}}{2B_{cp}C_{об} + \Delta\omega};\qquad B_{cp} = \frac{t}{\ln\dfrac{t+\psi}{\psi}} - \psi;$$

h is the thickness of the layer of annual temperature fluctuations in permafrozen rocks below the base of the layer of seasonal thawing; B_m is the mean amplitude of annual temperature fluctuations in the layer h; Δw is the heat of phase transitions of water in the layer h.

If the grounds of the layer of seasonal thawing differ substantially from the grounds in the layer h in their thermophysical properties, it is advisable to present the first term in formula (4.1.15) in the form of two terms, that is:

$$\sqrt{2}A_0\sqrt{\frac{\lambda TC_{o6}}{\pi}} = \sqrt{2}(A_0 - t_\xi)\sqrt{\frac{\lambda_1 TC_{o6}}{\pi}} + \sqrt{2}\,t_\xi\sqrt{\frac{\lambda_2 TC_{o6}}{\pi}}.$$

Calculation of the annual heat cycles in permafrozen rocks is complicated by the fact that it is also necessary to determine h and Δw. The values of h and Δw depend on the amplitude of the annual temperature fluctuations on the base of the layer of seasonal thawing of rocks (A_ξ), the character of the change of w_{un} in the rocks during those fluctuations and the thermophysical properties of the rocks (λ and C). If w_{un} in rocks at high negative temperatures (from 0.0 to -1.0°) is practically equal to zero, then the annual heat cycles in permafrozen rocks should be calculated with the same formula as for the region of propagation of thawed rocks. Such calculations are correct for all coarsely dispersed frozen rocks (sands, gravel and gravel-rubble soils).

If the annual heat cycles are calculated for finely dispersed frozen rocks (sandy loams, loams and clays), then it is necessary to take into consideration the phase transitions in the permafrozen zone. In that case, in order to determine Δw it is necessary to know the mean amplitude of the annual temperature fluctuations on the layer h. We will assume* that on the base of the layer of seasonal thawing of the rocks the amplitude of the annual temperature fluctuations is numerically equal to the average annual temperature, and on the base of the layer h is practically equal to zero. The temperature envelopes in the layer h can be provisionally replaced by straight lines. Therefore with a small allowance the mean amplitude of temperatures on the layer h can be determined as the arithmetic mean, equal to $1/2\,A_\xi$. Then the average maximal temperature on the layer h will be equal to $1/2\,t_\xi$ and the average minimal as $3/2t_\xi$ (Figure 13). The amount of water participating in phase transitions in a permafrozen rock mass should also be determined on the basis of those points. For that purpose, on the curve of the dependence of w_{un} on -t is taken the quantity of w_{un} at the temperature $1/2t_\xi$ and $3/2t_\xi$. The latter is deducted from the former and the quantity of water participating in phase transitions in the frozen rock mass is obtained. The quantity of heat expended on phase transitions of water in the layer h is

$$\Delta w = 80\,\frac{\left(w_{н}\ \text{при}\ \frac{1}{2}t_\xi - w_{н}\ \text{при}\ \frac{3}{2}t_\xi\right)\gamma_{ск}}{100},\ \text{ккал/м}^3. \qquad (4.1.16)$$

The thickness of the layer h should be determined as the thickness of the layer of seasonal freezing (according to 4.1.4) under the condition that $A_o = t_\xi$ and A_h on the base of the layer of annual fluctuations is equal to 0.1° (precision of measurement). In that case the formula assumes the form

*Without consideration of the asymmetry of the temperature envelopes. See consideration of the latter in section 4 of the present chapter.

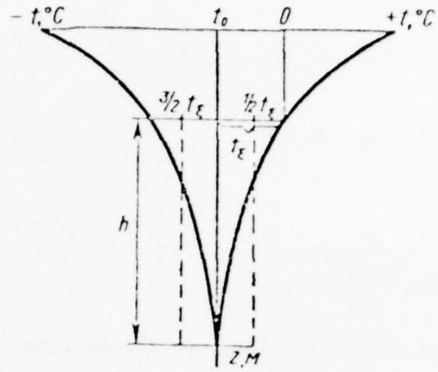


Figure 13. For determination of w_{un} in a permafrozen rock mass.

$$h = \frac{2t\sqrt{\frac{\lambda T C_{o6}}{\pi}} + \frac{(2B_{cp}C_{o6}h_{2c} + \Delta w \cdot h)\,\Delta w \sqrt{\frac{\lambda T}{\pi C_{o6}}}}{2B_{cp}C_{o6}h_{2c} + \Delta w \cdot h + \sqrt{\frac{\lambda T}{\pi C}}\,(2B_{cp}C_{o6} + \Delta w)}}{2B_{cp}C_{o6} + \Delta w}. \quad (4.1.17)$$

If Δw and h have been determined in that manner, one can readily calculate the value of the annual heat cycles.

For example, we will calculate Q on a section composed of permafrozen loams for which $t_o = -2^o$ and $A_o = 13^o$ are known. The rocks are characterized by the following properties: $\gamma_{sk} = 1300$ kg/m^3; w = 30%, $C_{spec-sk} = 0.19$ kcal/(kg)(degree); $\lambda_f = \lambda_t = 1.5$ kcal/(m^2)(degree)(hr). A graph of the dependence of w_{un} on -t is presented on Figure 12.

Solution. 1. On the graph of Figure 12 we find the quantity of unfrozen water in loams at temperatures of -1^o and -3^o and determine Δw in the layer h; at $t = -1^o$, $w_{un} = 12\%$ and at $t = -3^o$, $w_{un} = 10\%$.

$$\Delta w = 80\,\frac{(12-10)\,1300}{100} = 2080 \text{ ккал/м}^3$$

2. Assuming that $A_\xi = |t_\xi| = |t_o| = 2^o$, from equation (4.1.17) we find h, having preliminarily calculated with formula (4.1.6) that $C_{vol-m} = 435$ kcal/(m^3)(degree):

$$h=\frac{2\cdot 2\sqrt{\frac{1,5\cdot 8760\cdot 435}{3,11}}+\frac{(2\cdot 0,9\cdot 435\cdot h_{2c}+2080\cdot h)\,2080\,\times}{2\cdot 0,9\cdot 435\cdot h_{2c}+2080h+}}{2\cdot 0,9\cdot 435+2080}\times$$

$$\times\frac{\times\sqrt{\frac{1,5\cdot 8760}{3,14\cdot 435}}}{+\sqrt{\frac{1,5\cdot 8760}{3,14\cdot 435}}\,(2\cdot 0,9\cdot 435+2080)} \Bigg/ \left(2\cdot 0,9\cdot 435+2080\right);$$

$$B_{cp}=\frac{2}{\ln\frac{2+2,4}{2,4}}-2,4=0,9;\qquad h_{2c}=\frac{2\cdot 2\cdot\sqrt{\frac{1,5\cdot 8760\cdot 435}{3,14}}}{2\cdot 0,9\cdot 435+2080}=1,88.$$

After all the substitutions we determine that h = 2.92 m.

3. We calculate the depth of the seasonal thawing of loam on the condition that λ_t = 1.5 kcal/(m)(degree)(hr), $C_{vol\text{-}t}$ = 637 kcal/(m^3)(degree), $Q_\phi \approx$ ≈ 25,000 kcal/m^3, $t_o = -2^o$ and $A_o = 13^o$. With the nomogram (Figure 17) we find that $\xi = \sqrt{1.5}\times 1.5$ m ≈ 1.8 m.

4. We calculate the annual heat cycle in accordance with equation (4.1.15) at the following values of the expressions:

$$\alpha=A_{cp}+\delta=26,74;\quad \delta=\frac{Q_\phi}{2C_{o6.\tau}}=\frac{25\,000}{2\cdot 637}=19,62;$$

$$\psi=\frac{\Delta\omega}{2C_{o6.м}}=\frac{2080}{2\cdot 435}=2,4;$$

$$A_{cp}=\frac{A_0-t_0}{\ln\frac{A_0+\delta}{t_0+\delta}}-\delta=\frac{13-2}{\ln\frac{13+19,62}{2+19,62}}-19,62=7,12;$$

$$\xi_{2c}=\frac{2(A_0-t_0)\sqrt{\frac{\lambda T}{\pi C}}}{2A_{cp}C+Q_\phi}=\frac{2\cdot 11\sqrt{\frac{1,5\cdot 8760}{3,14\cdot 500}}}{2\cdot 7,12\cdot 637+2500}=0,005;$$

$$V=A_{cp}\xi_{2c}=0,39;\quad S=B_{cp}+\frac{\Delta\omega}{2C_{o6.м}}=0,9+2,4=3,3;$$

$$U=B_{cp}h_{2c}=0,9\cdot 1,88=1,69;$$

$$Q=\sqrt{2}\cdot(13-2)\sqrt{\frac{1,5\cdot 8760\cdot 637}{3,14}}+$$

$$+\frac{25\,000}{\frac{1}{\sqrt{\frac{1,5\cdot 8760}{3,14\cdot 637}}}+\frac{26,74}{0,39+0,39\cdot 1,7\cdot 19,62}}+$$

$$+\sqrt{2}\cdot 2\sqrt{\frac{1,5\cdot 8760\cdot 435}{3,14}}+\frac{2080}{\frac{1}{\sqrt{\frac{1,5\cdot 8760}{3,14\cdot 435}}}+\frac{3,3}{1,69+2,92\cdot 2,4}}=$$

$$=53\,167\ ккал/м^2.$$

Thus under the given concrete conditions the annual heat cycle through the surface of the soil is equal to 53,167 kcal/m^2.

It is evident from the calculation that in the case of seasonal thawing of the ground in the layer h the heat cycles are expended not only on the heat capacity (in connection with temperature variation) but also on the phase transitions of water in permafrozen grounds. Therefore the annual heat cycles in the region of seasonal thawing usually exceed the annual heat cycles in the region of seasonal freezing. However, it should be taken into consideration that the phase transition of water in the layer h leads to some reduction of the layer h itself, which under certain conditions can lead to decrease of the heat cycle in comparison with its values in the region of seasonal freezing.

Determination of the temperature shift and depths of seasonal freezing (thawing) of rocks at $\lambda_f \neq \lambda_t$. If the thermophysical characteristics of the frozen and thawed grounds are different, that is, $\lambda_f \neq \lambda_t$, then during periodic temperature fluctuations on its surface a temperature shift (Δt_λ) is noted in the layer of seasonal freezing (thawing). The nature of that shift is explained by the fact that the heat cycles connected with phase transitions of water in the ground, and partially the heat cycles expended on change of the ground temperature (heat cycles on account of heat capacity), at temperatures of different sign proceed at different thermophysical characteristics of the frozen and thawed grounds. In that case in calculating ξ it is necessary to take into consideration the temperature shift, which is determined with the expression (Kudryavtsev and Melamed, 1965)

$$\Delta t_\lambda = \frac{\xi}{T\lambda_{\text{пр}}}\left(1 - \frac{\sqrt{\lambda_{\text{т}}}}{\sqrt{\lambda_{\text{м}}}}\right)\left[\sqrt{2}\, A_0 \sqrt{\frac{\lambda TC}{\pi}} + \frac{(2A_{\text{ср}}C\,\xi_{\text{сс}} + Q_\Phi\,\xi)\,Q_\Phi\sqrt{\frac{\lambda T}{\pi C}}}{2A_{\text{ср}}C\,\xi_{\text{сс}} + Q_\Phi\,\xi + \sqrt{\frac{\lambda T}{\pi C}}\,(2A_{\text{ср}}C + Q_\Phi)} - \frac{1}{2}\, nA_{\text{ср}}C\,\xi K\right]. \quad (4.1.18)$$

where λ_{red} is the reduced coefficient of thermal conductivity, which can be determined approximately with the formula

$$\lambda_{\text{пр}} = \frac{\lambda_{\text{т}}(A_0 + t_0) + \lambda_{\text{м}}(A_0 - t_0)}{2A_0}. \quad (4.1.19)$$

Having a formula for calculation of the temperature shift Δt_λ we can calculate the depth of the seasonal freezing (thawing) at different thermophysical characteristics of the thawed and frozen grounds. In that case one should substitute $t_\xi - \Delta t_\lambda$ instead of t_ξ in formula (4.1.4), λ_{red} instead of λ and A'_m instead of A_m:

$$A'_{\text{ср}} = \frac{A_0 - t_\xi + \Delta t_\lambda}{\ln \dfrac{A_0 + \dfrac{Q_\Phi}{2C}}{t_\xi - \Delta t_\lambda + \dfrac{Q_\Phi}{2C}}} - \frac{Q_\Phi}{2C}. \quad (4.1.20)$$

Thus the general form of the formula for calculation of the depth of both the seasonal freezing and the seasonal thawing is one and the same for different thermal conductivities. There is a difference only in the concrete values of Δt_{λ}, A_m and λ_{red}.

Calculations of the depths of seasonal freezing (thawing) with formula (4.1.4) at $\lambda_f \neq \lambda_t$ have been verified by computer calculations. The convergence of the results at ξ is equal to 0.5 m and deeper is within the range of 1-3% (Kudryavtsev and Melamed, 1965).

Calculation of the Depth of Seasonal Freezing (Thawing) at Different Coefficients of Thermal Conductivity of the Rocks in the Frozen and Thawed States. Application of Nomograms for Calculation of ξ (Example 3).

As shown above, in the case of inequality of the thermophysical characteristics of rocks during their transition from the thawed into the frozen state and the reverse, in the layer of seasonal freezing or thawing a temperature shift Δt_{λ} forms which leads to change of the average annual temperature on the base of the layer of seasonal freezing (thawing) in comparison with the average annual temperature on the surface of the soil (the calculation of Δt_{λ} is presented in example 9 of Chapter 5). In that case, a correction equal to Δt_{λ} is introduced into formula (4.1.4) for the calculation of ξ, as a result of which a transcendental equation is obtained. Because of the complexity of solution of the system of obtained equations for determination of the depth of freezing or thawing it is advisable to use nomograms, which can be obtained by means of electronic computers for a wide range of input parameters.

The nomograms (Figures 14-19) represent series of curves of the variation of the depth of seasonal freezing (thawing) as a function of the variation of four parameters: 1) the volumetric heat capacity of frozen rocks C_{vol-f} (for the case of seasonal freezing) or thawed rocks C_{vol-t} (for the case of seasonal thawing); 2) the quantity of heat expended on phase transitions of water during the freezing (thawing) of 1 cubic meter of rock, Q_{ϕ}; 3) the amplitude of annual fluctuations on the surface of the soil A_o (the physical value of the amplitude of the air with consideration of the influence of all factors forming the temperature regime of the surface -- see chapter 5); 4) the average annual temperature of the rocks at the base of the layer of seasonal freezing or thawing -- t_{ξ} (with consideration of the influence of the temperature shift Δt_{λ}). For all cases in calculations of ξ in the compilation of the nomograms the coefficient of thermal conductivity of the rocks was taken to be constant and equal to 1.0 kcal/(m)(degree)(hr). This is explained by the fact that, as follows from equations (4.1.4), ξ is directly proportional to $\sqrt{\lambda}$. Therefore any sought value of ξ for rocks with a coefficient of thermal conductivity different from unity will be equal to the value of ξ found on a nomogram and multiplied by the square root of the true value of the coefficient of thermal conductivity of the given rock.

Depending of the variation of C_{vol}, three groups of series of graphs have been calculated for the following values of C_{vol}: 300, 500 and 800 kcal/(m^3)(degree).

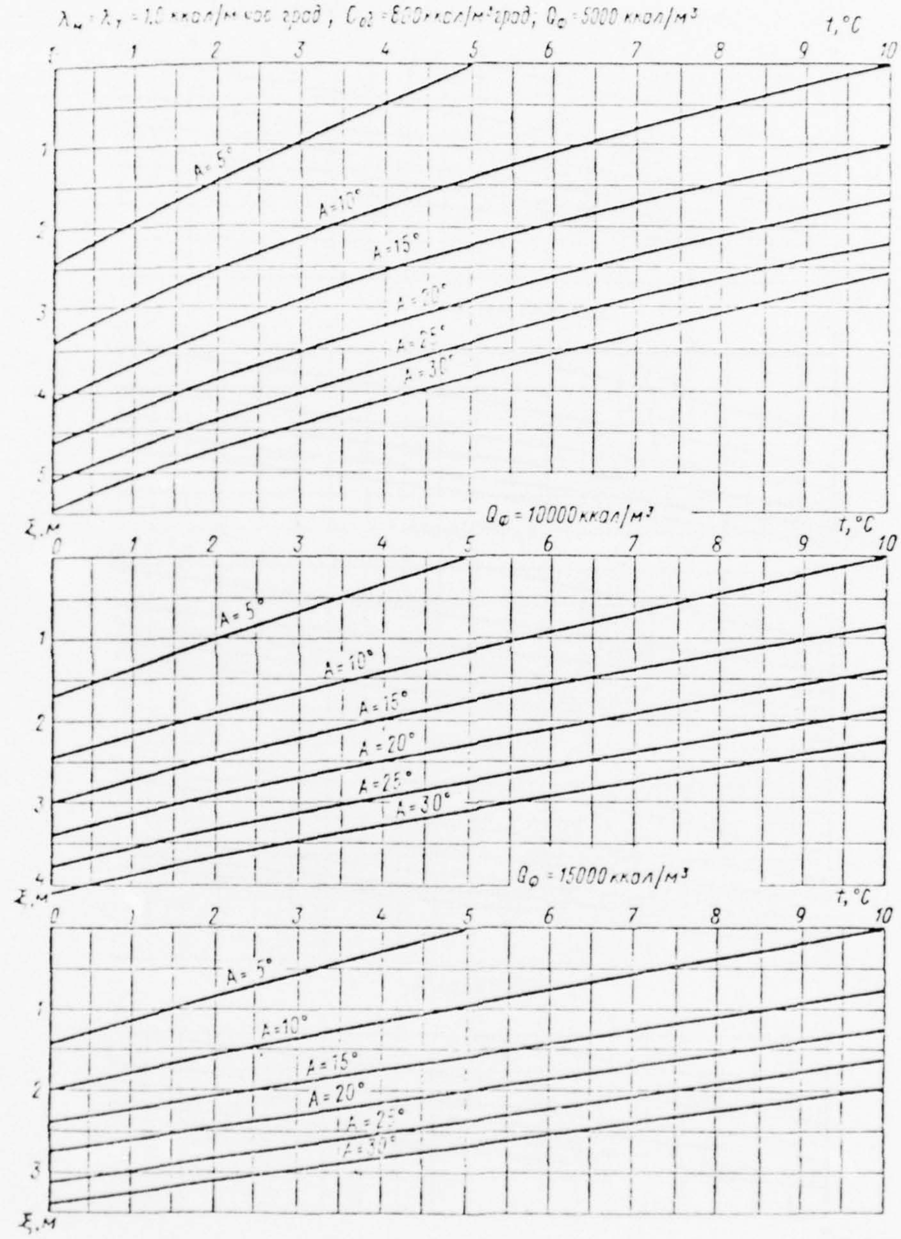


Figure 14. Nomograms for calculation of $\xi_{seasonal}$ at:
$\lambda_{f} = \lambda_{t} = 1.0$ kcal/(m)(hr)(degree);
$C_{vol} = 800$ kcal/(m^3)(degre);
$Q_{\phi} = 5{,}000$, 10,000 and 15,000 kcal/m^3

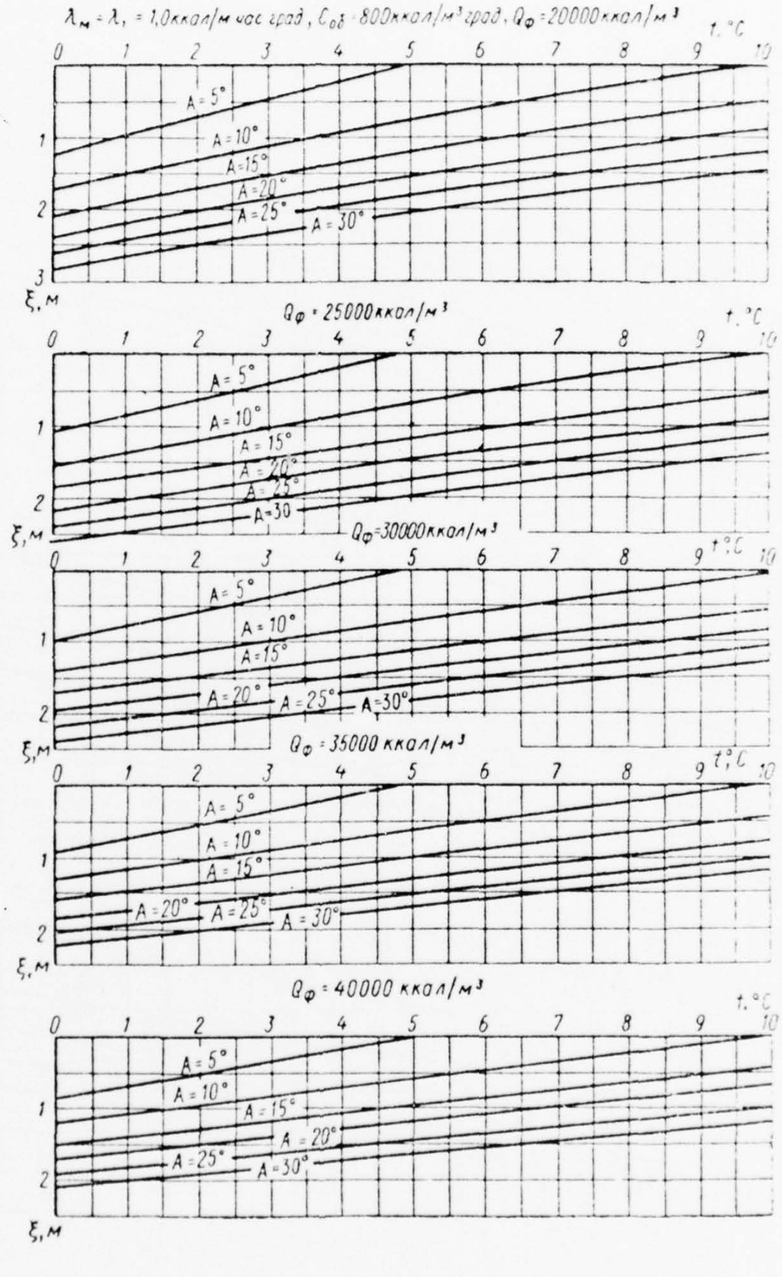


Figure 15. Nomograms for calculation of $\xi_{seasonal}$ at:
$\lambda_f = \lambda_t = 1.0$ kcal/(m)(hr)(degree);
$C_{vol} = 800$ kcal/(m^3)(degree);
$Q_\phi = 20{,}000$, 25,000, 30,000 and 40,000 kcal/m^3

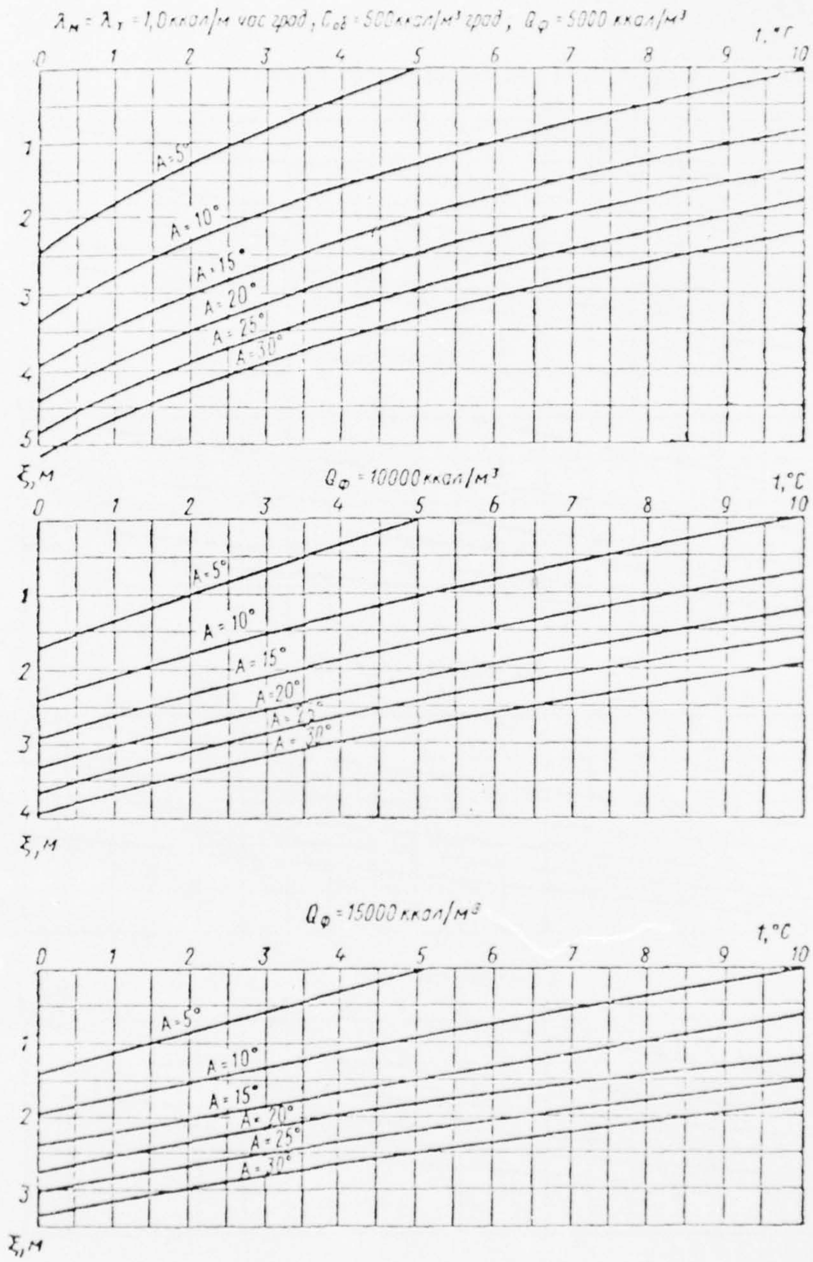


Figure 16. Nomograms for calculation of $\xi_{seasonal}$ at:
$\lambda_f = \lambda_t$ = 1.0 kcal/(m)(hr)(degree);
C_{vol} = 500 kcal/(m^3)(degree);
Q_ϕ = 5,000, 10,000 and 15,000 kcal/m^3

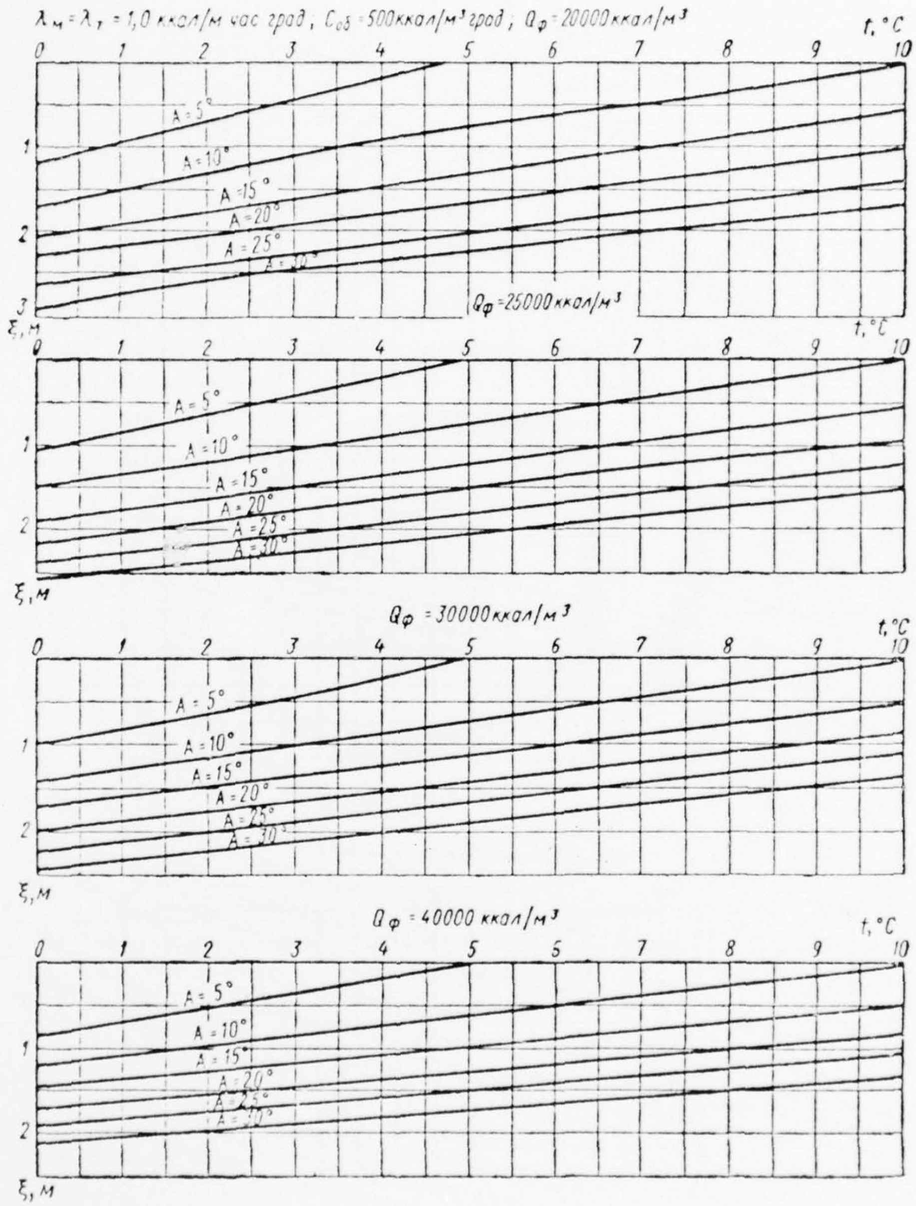
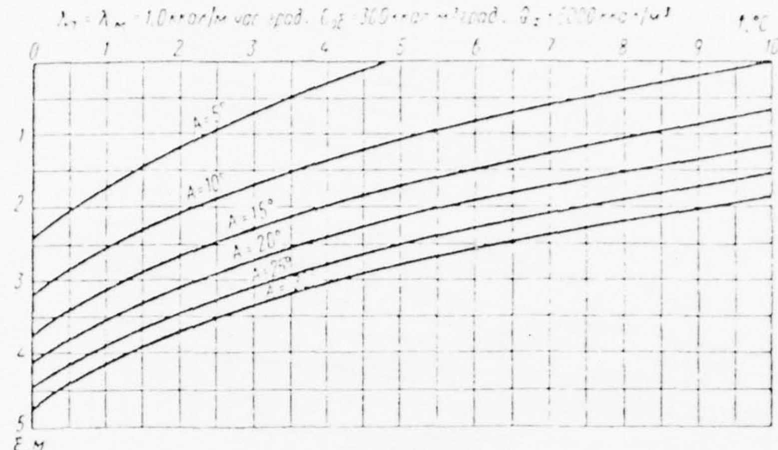
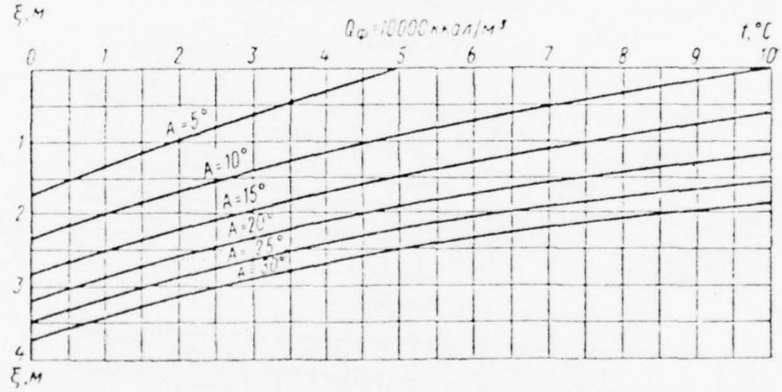


Figure 17. Nomograms for calculation of $\xi_{seasonal}$ at:
$\lambda_f = \lambda_t$ = 1.0 kcal/(m)(hr)(degree);
C_{vol} = 500 kcal/(m^3)(degree);
Q_ϕ = 20,000, 25,000, 30,000 and 40,000 kcal/m^3





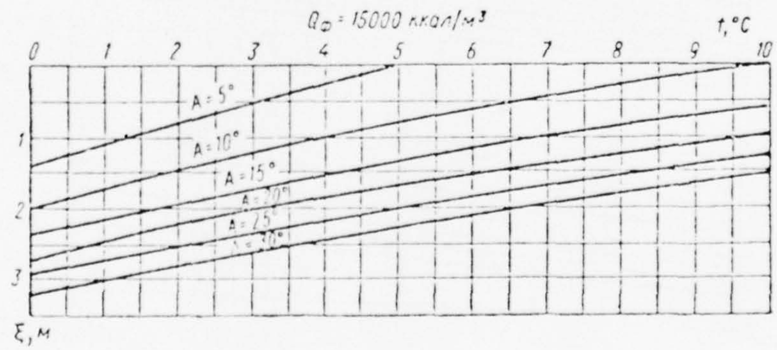


Figure 18. Nomograms for calculation of $\xi_{seasonal}$ at:
$\lambda_f = \lambda_t = 1.0$ kcal/(m)(hr)(degree);
$C_{vol} = 300$ kcal/(m^3)(degree);
$Q_\phi = 5{,}000$, 10,000 and 15,000 kcal/m^3

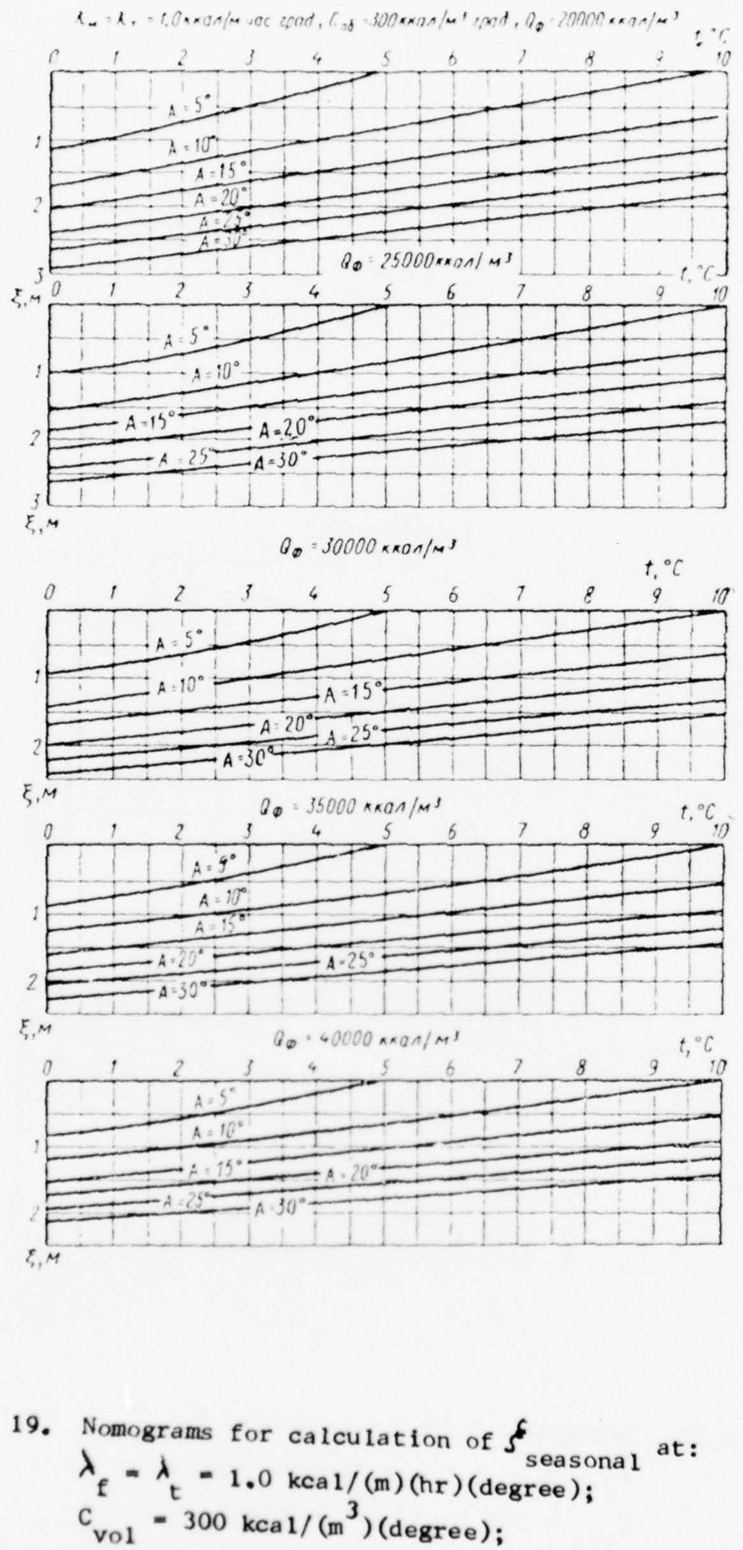


Figure 19. Nomograms for calculation of $\xi_{seasonal}$ at:
$\lambda_f = \lambda_t = 1.0$ kcal/(m)(hr)(degree);
$C_{vol} = 300$ kcal/(m^3)(degree);
Q_ϕ = 20,000, 25,000, 30,000, 35,000 and 40,000 kcal/m^3

The series of graphs were calculated as a function of the variation of Q_ϕ for the following values of it: 5,000, 10,000, 15,000, 20,000, 25,000, 30,000, 35,000 and 40,000 kcal/m^3.

Each graph represents a set of curves expressing the variation of ξ (along the ordinate) as a function of the variation of t_ξ (along the abscissa) for the following constant values of A_o (written under each curve): 5, 10, 15, 20, 25 and 30^o.

In order to determine on the basis of the nomogram the depth of seasonal freezing or thawing of rocks, it is necessary to know the following five initial parameters: C_{vol}, Q_ϕ, t_ξ, A_o and λ. Then a series of graphs should be selected for the value of Q_ϕ, a series which characterizes the given ground. In that series a graph is selected which corresponds to the given value of Q_ϕ. On the graph a curve is sought which corresponds to the value of A_o (the physical meaning). On that curve the depth ξ_{nom} is determined as a function of t_ξ. Finally, to obtain the sought value of it is necessary to multiply the value ξ_{nom} by $\sqrt{\lambda}$. For example, it is necessary to calculate the depth of the seasonal thawing of rocks at a construction site if the following data are known: C_{vol-t} = 500 kcal/(m^3)(degree); Q_ϕ = 20,000 kcal/m^3; λ = 1.2 kcal/(m)(degree)(hr); $t_\xi = -3.0^o$; $A_o = 15^o$. For that purpose we will take a series of graphs obtained for $C_{vol} \cong 500$ kcal/(m^3)(degree). In that series we select a graph corresponding to the value Q_ϕ = 20,000 kcal/m^3 and find a curve giving the dependence of the variation of the depth of thawing ξ on the average annual temperature on the base of the layer of seasonal thawing t_ξ for the value $A_o = 15^o$. Then on the axis of abscissas we find the value of t_ξ equal to -3^o, and from that point we seek the perpendicular to the intersection with the given curve ($A_o = 15^o$). The ordinate of the point of intersection also is the sought depth of thawing ξ_{nom} and is 1.65 meters. Finally, we multiply the obtained depth by $\sqrt{1.2}$ and find that the depth of the seasonal thawing at the construction site is 1.73 meters.

In the case where the initial values of C_{vol}, Q_ϕ and A_o are different from those presented on the graphs, the depth ξ must be found by means of corresponding interpolations.

In cases where C_{vol}, Q_ϕ and A_o go beyond the limits of the values adopted on the nomograms, the calculating formulas presented in section 1 of this chapter should be used.

2. Approximate Formulas for Determination of Perennial Heat Cycles and Depths of Perennial Freezing (Thawing) of Rocks

The perennial freezing and thawing of rocks does not differ in principle from the seasonal. The difference consists only in the length of the period of temperature fluctuations on the surface of the earth and the need to take into consideration the influence of the geothermal gradient. For determination of the regularities of the formation of permafrozen rock masses as a function of geographic geological conditions, such as: the influence of lithological features, thermophysical characteristics and the moisture content of rocks, the temperature conditions on the surface, the geothermal gradient and other factors affecting the process of perennial freezing and thawing, an approximate formula is presented below. The basis of that formula is the scheme for calculation of the depths of seasonal freezing (thawing) of rocks (4.1.14) in which it is necessary to change the length of the period of temperature fluctuations on the surface, and also take into account the influence of the geothermal gradient.

The geothermal gradient is taken into account in the following manner. On the base of the permafrozen rock mass at the maximal depth of freezing (thawing) during the period T the amplitude of temperature oscillations (physical) must be equalized with the average temperature during that period $t_{\xi_{perm}}$. Then, as follows from Figure 20

$$t_{\xi,\text{мн}} = t_{0,\text{мн}} + g\,\xi_{\text{мн}},$$

where $t_{0\text{-perm}}$ is the average temperature during the period T on the surface of the earth, $^{\circ}$C; ξ_{perm} is the maximal depth of the permanent freezing (thawing), meters; g is the geothermal gradient (degrees/meter).

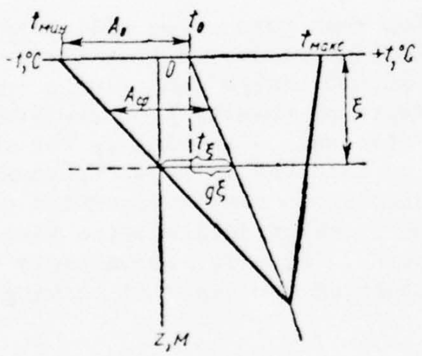


Figure 20. Derivation of the formula for the depth of permafrost.

If we substitute (4.1.18) in (4.1.4) we obtain a formula for calculating the depths of permafrost with consideration of g:

$$\xi_{\text{мн}} = \frac{2\,(A_{0,\text{мн}} - t_{0,\text{мн}} - g\xi_{\text{мн}})\sqrt{\dfrac{\lambda TC}{\pi}} +}{2A_{\text{cp}}\,C + Q_{\text{ф}}}$$

$$+\frac{(2A_{\text{cp}}\,C\xi_{2c} + Q_{\text{ф}}\,\xi_{\text{мн}})\,Q_{\text{ф}}\sqrt{\dfrac{\lambda T}{\pi C}}}{\dfrac{(2A_{\text{cp}}C\,\xi_{2c} + Q_{\text{ф}}\,\xi_{\text{мн}}) + \sqrt{\dfrac{\lambda T}{\pi C}}\,(2A_{\text{cp}}C + Q_{\text{ф}})}{2A_{\text{cp}}C + Q_{\text{ф}}}}, \qquad (4.2.1)$$

where

$$\xi_{2c} = \frac{2\,(A_{0,\text{мн}} - t_{0,\text{мн}} - g\,\xi_{\text{мн}})\sqrt{\dfrac{\lambda TC}{\pi}}}{2A_{\text{cp}}C + Q_{\text{ф}}}. \qquad (4.2.2)$$

With that formula calculations were made of the permafrost at T = 100,000 years for different values of $A_{o\text{-perm}}$, Q_{ϕ}, λ, $t_{o\text{-perm}}$ and g. Simultaneously control calculations were made by computer of the Stefan problem for the same values of those parameters, which show that the divergence in depths does not exceed 8% (Kudryavtsev and Melamed, 1965, 1967).

In formula (4.2.1) the numerator represents the heat cycles along the envelopes. If we replace the coefficient 2 in the first term in the numerator of the right side by $\sqrt{2}$ we obtain the heat cycles passing through the surface of the ground for a layer of permanent freezing (thawing):

$$Q_{\xi\,\text{мн}} = \sqrt{2}\,(A_{0,\text{мн}} - t_{0,\text{мн}} - g\,\xi_{\text{мн}})\sqrt{\frac{\lambda TC}{\pi}} + \frac{(2A_{\text{cp}}C\,\xi_{2c} + Q_{\phi}\,\xi_{\text{мн}})\,Q_{\phi}\sqrt{\frac{\lambda T}{\pi C}}}{2A_{\text{cp}}C\,\xi_{2c} + Q_{\phi}\,\xi_{\text{мн}} + \sqrt{\frac{\lambda T}{\pi C}}\,(2A_{\text{cp}}C + Q_{\phi})}. \tag{4.2.3}$$

In the layer of thawed rocks underlying the permafrozen rock mass the heat cycles during the period T are equal to

$$Q_{h,\text{мн}} = \sqrt{2}\,(t_{0,\text{мн}} + g\,\xi_{\text{мн}})\sqrt{\frac{\lambda TC}{\pi}}. \tag{4.2.4}$$

If we sum up (4.2.3) and (4.2.4) we finally obtain

$$Q_{H,\text{мн}} = Q_{\xi,\text{мн}} + Q_{h,\text{мн}} = \sqrt{2}\,A_{0,\text{мн}}\sqrt{\frac{\lambda TC}{\pi}} + \frac{(2A_{\text{cp}}C\xi_{2c} + Q_{\phi}\,\xi_{\text{мн}})\,Q_{\phi}\sqrt{\frac{\lambda T}{\pi C}}}{2A_{\text{cp}}C\,\xi_{2c} + Q_{\phi}\,\xi_{\text{мн}} + \sqrt{\frac{\lambda T}{\pi C}}\,(2A_{\text{cp}}C + Q_{\phi})}. \tag{4.2.5}$$

The calculations of Q_{perm} with formula (4.2.5) were verified by corresponding computer calculations. Comparison of those results showed that the error of calculations with the formula amounts to 1 - 2% (Kudryavtsev and Melamed, 1965, 1967).

To facilitate calculations with formulas (4.2.1) and (4.2.5), nomograms were compiled (Kudryavtsev and Melamed, 1967). A distinctive feature of the nomograms, connected with the presence of the geothermal gradient, is that with increase of the period of oscillations the maximal depth of freezing for the given ground tends asymptotically toward the limiting value. The latter is completely determined from a solution of the corresponding steady-state problem, that is, depends only on the average temperature on the surface during the period and the thermal conductivity of the ground. Let us note that the time of achievement of the limiting value increases with increase of the geothermal gradient.

Calculation of the Depth of Permafrost (Example 4)

Determine the thickness of permafrozen rocks if it is known that the perennial freezing of rocks occurred under the influence of 300-year temperature fluctuations on the surface of the ground. The amplitude of those fluctuations is 6° and the average perennial temperature is 0°. Frozen were alluvial deposits (alQ_4) represented by sandy loams and loams. Their properties are characterized by the following weighted average values: λ_f = 1.0 kcal/(m)(degree)(hr); C_{vol} = 500 kcal/(m^3)(degree) and Q_{ϕ} = 20,000 kcal/m^3. The geothermal gradient (g) in the frozen rock mass (below the layer of annual temperature fluctuations) is 0.01 degree/m.

Solution. For calculation of the depth of the permafrost, formula (4.2.1) can be written in the following form:

$$\xi_{MH} = \frac{A_{0\cdot MH} - t_{0\cdot MH} - g\,\xi_{MH} + \dfrac{(A_{cp}\,\xi_{2c} + a\xi_{MH})\,a}{A_{cp}\,\xi_{2c} + a\xi_{MH} + \beta\,(A_{cp} + a)}}{A_{cp} + a}\,\beta, \qquad (4.2.6)$$

where

$$a = \frac{Q_{\Phi}}{2C}, \quad \beta = \sqrt{\frac{\lambda T}{\pi C}}, \quad A_{cp} = \frac{1}{2}\,(A_{0\cdot MH} + t_{0\cdot MH} + g\xi_{MH}),$$

$$\xi_{2c} = \frac{(A_{0\cdot MH} - t_{0\cdot MH} - g\,\xi_{MH})\,\beta}{A_{cp} + a}.$$

The presented equation is transcendental, since the temperature regime on the base of the layer of permafrost is determined as a function of ξ_{perm}. Therefore the problem is solved by the method of trial and error -- identity is found at several values of ξ_{perm}.

1. We assume that ξ_{perm} is 20 m. Then all the intermediate calculations for substitution in the equation will be as follows:

$$a = \frac{20\,000}{1000} = 20{,}0; \quad \beta = \frac{1\cdot 8760\cdot 300}{3{,}14\cdot 500} \cong 42;$$

$$A_{cp} = \frac{1}{2}\,(6 + 0{,}01\cdot 20) = 3{,}1; \quad \xi_{2c} = \frac{(6 - 0{,}2)\,42}{3{,}1 + 20} = 10{,}5;$$

$$\xi_{MH} = \frac{6 - 0{,}2 + \dfrac{(3{,}1\cdot 10{,}5 + 20\cdot 20)\,20}{3{,}1\cdot 0{,}5 + 20\cdot 20 + 42\,(3{,}1 + 20)}}{3{,}1 + 20}\cdot 42 = 33{,}13 \text{ м}.$$

Identity was not obtained, as 20 ≠ 33.13 m.

2. We assume that ξ_{perm} = 50 m. Then we obtain

$$\xi_{MH} = 42\cdot\frac{6 - 0{,}5 + \dfrac{(3{,}25\cdot 9{,}9 + 20\cdot 50)\,20}{3{,}25\cdot 9{,}9 + 20\cdot 50 + 42\,(3{,}25 + 20)}}{3{,}25 + 20} = 27{,}6 \text{ м}.$$

In this case also, 50 ≠ 27.6 m.

3. By a method of construction similar to that which will be analyzed in detail in examples 9, 10 and other, we find (Figure 21) that ξ_{perm} = 31.2 m.

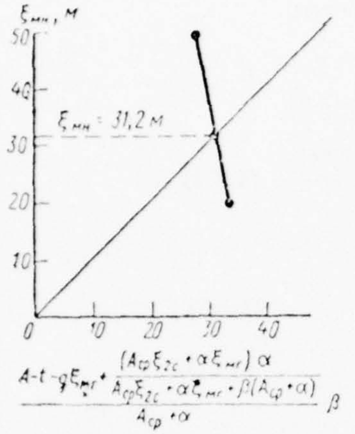


Figure 21. For determination of the depth of permafrost.

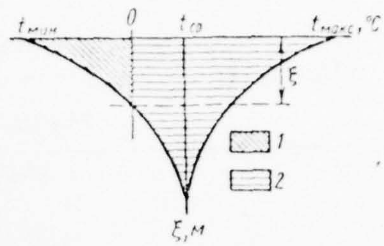


Figure 22. For determination of the potential seasonal thawing: 1 - heat cycles at negative temperatures in the layer of seasonal freezing; 2 - heat cycles at positive temperatures, determining the depth of potential seasonal thawing (if t_m was 0^o).

3. Potential Seasonal Freezing and Potential Seasonal Thawing of Rocks

At positive annual average temperatures of the rocks the heat cycles in them are maximally used in the half-period of cooling for the freezing of the upper layer of the rocks. In the half-period of warming, however, the heat cycles are used to thaw the seasonally frozen layer only partially, and the remaining portion of them is used to warm the rocks above zero degrees. This is obvious, since the heat cycles at positive and negative temperatures are not equal and the former considerably exceed the latter (Figure 22).

Thus during seasonal freezing as much freezes as can freeze at the given heat cycles; but only that thaws which has frozen, regardless of the heat cycles at temperatures above zero degrees. The thawing of of the upper horizon of the frozen mass which would be observed if there were complete use of the heat cycles at positive temperatures is called the potential seasonal thawing.

A similar situation occurs during seasonal thawing. In that case the upper horizon of the frozen mass thaws which can thaw during complete use of the heat cycles at positive temperatures. At negative temperatures, however, the heat cycles are not completely used for the freezing of the thawed layer; the remaining portion of them is used to lower the temperatures of the underlying frozen rocks. Consequently, in that case one can speak of the potential seasonal freezing of rocks.

One can use for calculating the depths of potential seasonal freezing (thawing) formula (4.1.4) for the seasonal freezing (thawing), in which the following substitutions are made: 1) instead of the amplitude of temperatures on the surface of the ground A_o the sum of that amplitude with the average annual temperature on the surface ($A_o + t_o$), that is, in that case, instead of A_o, its actual value is substituted; 2) zero is substituted instead of t_ξ.

In that case the formula will assume the following form:

$$\xi_n = \frac{2(A_0 + t_0)\sqrt{\frac{\lambda TC}{\pi}} + \frac{(2A_{cp}C\,\xi_{2c} + Q_\Phi\,\xi)\,Q_\Phi\sqrt{\frac{\lambda T}{\pi C}}}{2A_{cp}C\,\xi_{2c} + Q_\Phi\,\xi + \sqrt{\frac{\lambda T}{\pi C}}\,(2A_{cp}C + Q_\Phi)}}{2A_{cp}C + Q_\Phi}, \quad (4.3.1)$$

where

$$\xi_{2c} = \frac{2(A_0 + t_0)\sqrt{\frac{\lambda TC}{\pi}}}{2A_{cp}C + Q_\Phi}; \quad (4.3.2)$$

$$A_{cp} = \frac{A_0 + t_0}{\ln\frac{A_0 + t_0 + \frac{Q_\Phi}{2C}}{\frac{Q_\Phi}{2C}}} - \frac{Q_\Phi}{2C}. \quad (4.3.3)$$

The following scheme (see Figure 22) is used to explain the correctness of that substitution. It is evident on that diagram that at a certain positive average annual temperature of the ground only that portion of the heat cycles is expended on freezing which is completed at negative temperatures (corresponding to the part with the slanted hatching). The remainder of the heat cycles (the horizontal hatching) must determine the depth of potential seasonal thawing of the rocks. Those heat cycles can be used for thawing the rocks only if the entire rock mass is frozen. In the region of seasonal thawing that can be only at a zero average annual temperature of the rocks. In that case the amplitude on the surface of the rocks will be equal to the segment on the axis of temperatures from t_{max} to 0^o, that is, $A_o + t_o$ at $t_m = 0$. The depths of both the potential seasonal thawing and the potential seasonal freezing of rocks can be calculated with the indicated formula.

Calculation of the Depths of Potential Seasonal Freezing and Potential Seasonal Thawing of Rocks. Calculation of Intergelosols (Example 5)

Problems in the determination of the depths of potential seasonal freezing and thawing of rocks arise in connection with the dynamics of transitional and semi-transitional types of seasonal freezing and thawing, which leads to the formation of intergelisols of frozen rocks or to separation of permafrozen rock masses. The formation of frozen intergelisols, which exist for a number of years on sections on which tabetisols or taliks are widespread, and also exist for several years on sections on which permafrozen rocks are widespread, considerably complicate the conditions of construction and the working of minerals in many regions of Siberia and the Far East. Therefore in doing frost and engineering geological work a need arises for determination of the conditions of formation and the thickness of intergelosols of frozen rocks or layers of thawed deposits.

The first part of the problem consists in analyzing the concrete natural conditions determining the temperature regime of the deposits and determining the character of the dynamics of those conditions. In that case it is necessary to start from the following.

Intergelosols of frozen rocks form and exist when the average annual temperature on the surface of the soil (t_o) drops by a value exceeding the average annual temperature of the rocks on the base of the layer of seasonal freezing t_ξ, that is, $t_o - t_{o\text{-int}} > t_\xi$, where $t_{o\text{-int}}$ is the average annual temperature on the surface of the soil at which the formation of the intergelosol starts; t_o and t_ξ are the perennial average annual temperatures on the surface of the soil and at the depth of seasonal freezing.

Correspondingly, the formation of a thawed layer of rocks separating the layer of winter freezing from the permafrozen rock mass is determined by the condition $t_o = t_{\theta\text{-int}} < t_\xi$.

After investigation of the possibility of formation of intergelosols of frozen rocks (or a separated thawed layer), their thickness forming in the course of a year or a number of years is determined. For that purpose, in accordance with the temperature regime established on the surface of the rocks and also the composition and properties of the rocks, the depth of the potential seasonal freezing (or potential seasonal thawing) of the rocks is calculated with formula (4.3.1), presented in the form

$$\xi_n = \frac{-B' + \sqrt{(B')^2 + 4D'E'}}{2D'}. \qquad (4.3.4)$$

where $$D' = \alpha'\delta;\ B' = \alpha'V' + (\alpha')^2\sigma - \beta'\delta - \sigma\delta^2;\ \ E' = V'\beta + \alpha'\beta'\sigma + V'\sigma\delta;$$

$$\alpha' = A'_{cp} + \delta;\ \ \beta' = (A_0 + t_{0,\text{nep}})\,\sigma;\ \ V' = A'_{cp}\cdot\xi_{2c};$$

$$\sigma = \sqrt{\frac{\lambda T}{\pi C_{o6}}};\quad \delta = \frac{Q_\Phi}{2C};$$

$$A'_{cp} = \frac{A_0 + t_{0,\text{nep}}}{\ln\dfrac{A_0 + t_{0,\text{nep}} + \delta}{\delta}} - \delta;\quad \xi'_{2c} = \frac{\beta'}{\alpha'}.$$

Then with formula (4.1.4) the depth of the seasonal thawing (freezing) corresponding to $t_\xi = 0^\circ$ and an amplitude of $A_o - t_{o\text{-int}}$ is calculated for the same period T. The difference between the depth of the potential seasonal freezing and the depth of the seasonal thawing of rocks constitutes the thickness of the intergelosol, and the difference between the depth of the potential seasonal thawing and the depth of the seasonal freezing of rocks -- the thickness of the separating thawed layer.

It must be noted that a zero or very close to zero average annual temperature on the base of the layer of seasonal thawing (freezing) will exist only in the first year of the formation of a frozen intergelosol (separating thawed layer). If, however, the new temperature conditions on the surface of the soil remain unchanged in the course of a number of subsequent years, then on the base of the layer of seasonal thawing (freezing) a negative (positive) average annual temperature is established which corresponds to the new steady-state regime. The latter also must be taken into account in calculations.

For example, it is required to calculate the thickness of the intergelosols forming on the lower terrace of a river if it is known that the perennial average annual air temperature (t_a) is -6.2^o, the annual amplitude of air temperature fluctuations* (A_a) is -22^o, and the average perennial thickness of the snow cover (at the end of February) is 0.7 meter, and in individual years decreases to 0.5 m. The terrace is composed of silty sandy loam containing pebbles, with a moisture content of 25% on the average. In addition, the following data were obtained experimentally: $\gamma_{sk} = 1350$ kg/m^3; $\lambda_f = 1.7$ and $\lambda_t = 1.2$ kcal/(M)(degree)(hr), and $C_{vol-f} = 456$ and $C_{vol-t} = 580$ kcal/(m^3)(degree) and $Q_\phi = 21,600$ kcal/m^3 were calculated with formulas (4.1.6), (4.1.7) and (4.1.8).

The surface of the terrace is covered with spruce and deciduous forest with a moss cover 0.1-0.15 meter thick which has a cooling effect, reducing the average annual temperature on the surface of the soil by 1.0^o as compared with t_a and reducing A_o (its physical value) by the same amount; t_ξ amounts to $+0.2^o$.

Solution. 1. We will investigate the possibility of forming intergelesols in the given region during change of the thickness of the snow cover. For that purpose we determine the temperature regime on the surface of the rocks under the snow and plant covers (this question is examined in greater detail in examples 10, 11 and 14 of Chapter 5).

If the thickness of the snow is 0.7 meter at a density of 0.22 g/cc, its warming influence amounts to 7.4^o, calculated with the empirical formula of V. A. Kudryavtsev (see example 11, Chapter 5). The average annual temperature on the surface of the soil will be $t_o = t_q + \Delta t_{snow} - \Delta t_{plant}$, where Δt_{plant} is the temperature correction for the influence of the plant cover.

Having calculated t_o ($t_o = -6.2 + 7.4 - 1.0 = +0.2^o$) and obtained a positive average annual temperature on the surface of the rocks, it can be concluded that at a snow thickness of 0.7 m or more intergelesols do not form under the conditions under consideration. However, in years when the thickness of the snow cover is reduced to 0.5 m the average annual temperature on the surface of the soil is lowered, as the warming influence of the snow is reduced to 5.7^o. As a result of that the annual average temperature on the surface becomes negative, that is, $t_o = -6.2+5.7-1.0 = -1.5^o$, from which it is evident that in years with winters with little snow intergelesols will form in the thawed deposits of the terrace.

The temperature regime on the surface of the soil at which the thickness of the intergelesol should be calculated is characterized by: $t_o = -1.5^o$,

$$A_0 = A_в - \Delta t_{сн} - \Delta t_{раст} = 22 - 5,7 - 1,0 = 15,3^\circ.$$

*Here and henceforth a physical value of the amplitude is assumed which is equal to half of its meteorological value determined on the basis of the monthly average air temperatures.

2. With formula (4.3.2) and the nomograms (Figures 15 and 17) the depth of the potential freezing is calculated, assuming that $t_\xi = 0^\circ$, $A_o = 15.3 + 1.5 = 16.8^\circ$ and substituting the following initial parameters characterizing the frozen rock

$$(\lambda_м, C_{об.м}, Q_ф): A'_{cp} = 7{,}66;\ \xi'_{2c} = 1{,}73;\ \delta = 23{,}68;\ \sigma = 3{,}22;\ V' = 13{,}25;\ \beta' = 54{,}1;\ \alpha' = 31{,}34;\ B = 329;\ D' = 742{,}13;\ E' = 7186{,}63;$$

$$\tilde{\xi}_{п,пр} = \frac{-329 + \sqrt{329^2 + 4\cdot 742{,}13\cdot 7186{,}63}}{2\cdot 742{,}13} = \frac{-329 + 4619}{1484{,}26} = 2{,}89\ м.$$

3. We calculate the depth of seasonal thawing in sandy loams under the conditions:

$$t_\xi = 0^\circ;\ A_0 = 15{,}3 - 1{,}5 = 13{,}8^\circ;\ A_{cp} = 6{,}2;\ \xi_{2c} = 1{,}3;\ \delta = 18{,}62;\ \sigma = 2{,}4;\ V = 6{,}2\cdot 1{,}3 = 8{,}06;\ \beta = 33{,}12;\ \alpha = 24{,}82;\ E = 2600{,}03;\ B = 229{,}75;\ D = 462{,}15;$$

$$\tilde{\xi}_{от} = \frac{-229{,}75 + \sqrt{229{,}75^2 + 4\cdot 462{,}15\cdot 2600{,}03}}{2\cdot 462{,}15} = \frac{-229{,}75 + 2205}{924{,}3} = 2{,}14\ м.$$

4. Having determined the depth of the potential seasonal freezing and the depth of the seasonal thawing, we calculate the thickness of the intergelesol of frozen rock formed per year: $M_{int} = 2.89 - 2.14 = 0.75$ m.

4. The Nature of the Asymmetry of the Envelopes of the Temperature Fluctuation in Rocks and Its Consideration in Determining the Depths of Seasonal and Perennial Freezing (Thawing)

The annual or perennial temperature fluctuations on the surface of the ground can be represented with sufficient exactness in the form of a harmonic function. In the absence of phase transformations those fluctuations in a periodically steady-state regime propagate in accordance with the Fourier laws. In the presence of a vertical axis of oscillation (without consideration of the geothermal gradient) the deviations from it on both sides are identical and the envelopes represent a symmetric figure.

In the presence of a seasonally freezing (thawing) layer the course of the variation of temperature on its base can with some approximation be approximated by a harmonic function with the amplitude A_ξ and an average ground temperature t_ξ. At different values of λ_t and λ_f, t_ξ coincides with the average annual temperature on the surface (t_o), and in the contrary case differs by the amount of the temperature shift.

In accordance with that, a number of approximate formulas have been proposed for the conditions of a periodically steady-state regime for calculation of the depths of seasonal and perennial freezing and thawing of the soil, and also of annual and perennial heat cycles of the soil, the temperature shift, the warming influence of the snow cover, etc (Kudryavtsev and Melamed, 1961-1967). Such an approach to the investigation of seasonal and perennial freezing has proven to be rather effective and has made it possible to solve a considerable number of problems of frost prediction with a precision high for practical purposes.

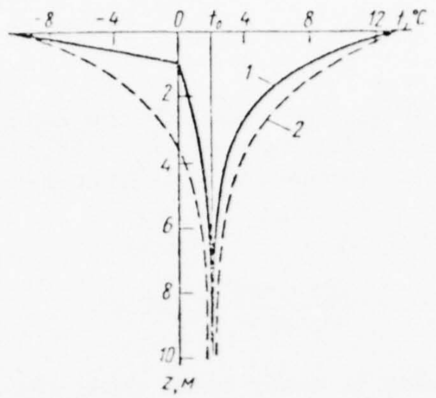


Figure 23. Asymmetry of the temperature envelopes during seasonal freezing with consideration of the phase transitions of water (1) and in the case of "dry" ground (2).

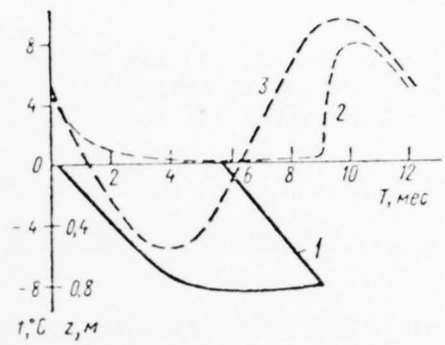


Figure 24. Dynamics of the depth of seasonal freezing (thawing) of the ground (1) and the temperature on the base of the layer of seasonal freezing (thawing) in the annual cycle (2) and in the absence of phase transitions of water (3).

At the same time, in examining the data of weather stations on observation of the soil temperature regime, in a number or cases an essential asymmetry of the temperature envelopes is noted. In particular, during seasonal freezing (thawing) of the soil the one of the envelopes which is in a region of negative (positive) temperatures is disposed much closer to the mean than the other envelope (Figure 23). A very sharply indicated asymmetry is manifested when there is a considerable moisture content of the rocks. The asymmetry of the envelopes during freezing (thawing) of rocks is completely regular and is completely explained by processes of phase transformations of water and with adequate precision can be taken into consideration in calculations according to the procedure proposed in the present work (see exampel 6, Chapter 4).

The physical meaning of the asymmetry of the temperature envelopes in the presence of a seasonally freezing (thawing) layer at $\lambda_t = \lambda_f$ is well illustrated by Figure 24. Depicted on it is the course of change of the depth of seasonal freezing (thawing) of the ground ξ and the temperature on the base of the layer of freezing (thawing) t_ξ in the time T in the annual cycle. Well visible on the drawing is the presence of the so-called "zero screen," noted over 60 years ago by M. I. Sumgin, connected with the phase transitions of water in the layer of freezing (thawing). Presented in the same place is the course of the variation of temperature in time at the same depth for the case of "dry" ground, when phase transitions of water are absent. In that case the thermoisoplet under consideration obeys a harmonic law, the envelopes are symmetric in relation to the average annual temperature of the ground and the amplitude A_z at any depth z is determined in accordance with (3.3.4) from the following expression:

$$A_z = A_0 e^{-z\sqrt{\frac{\pi}{KT}}}, \qquad (4.4.1)$$

where A_o is the amplitude of temperature fluctuations on the surface of the ground, $^{\circ}$C, K is the coefficient of thermal conductivity, m^2/hr, and T is the period (year), hrs.

During seasonal freezing with phase transitions of water there can be a more sudden damping of temperature with depth, which is manifested clearly for the branch embracing the region of negative temperatures (see Figure 24). In that case at $\lambda_t = \lambda_f$ it is of great importance that, in spite of the asymmetry the average annual temperature of the soil at the depth ξ can be assumed to be equal to the average at the surface, that is, $t_\xi = t_o$ (in the absence of a temperature shift) and the average temperature at the depth of the base of the layer of annual temperature fluctuations, that is, $t_\xi = t_o = t_m$ (at small values of g). Actually, by the average annual soil temperature is understood the value in relation to which there is equality of the annual heat cycles of the ground. By virtue of the fact that during phase transformations of water the course of the temperature on the base of the layer of freezing (thawing) is not described by a harmonic function, t_m cannot be regarded as the arithmetic mean between the maximal and minimal temperatures at that depth in the course of the year. Therefore as t_ξ it is necessary, generally speaking, to take the mean integral value on the graph of the course of temperatures at that depth during the year with consideration of the zero screen (Figure 24). However, as follows from the results of numerous calculations of the depths of seasonal freezing (thawing) with analog and electronic computers, at $\lambda_t = \lambda_f$ the indicated mean integral value practically coincides with the average annual temperature on the surface and on the base of the layer of annual fluctuations.

Thus, in calculating the depths of seasonal freezing (thawing) with consideration of the asymmetry of the envelopes the average annual soil temperature (t_m) in depth can be assumed to be invariable. It is precisely because of this fact in the seasonal freezing (thawing) for the branch of envelopes disposed correspondingly on the left (right) of the axis of oscillations, that the amplitude of oscillations on the base of the layer of freezing (thawing) is equal to t_o.

The matter is somewhat different for the second branch. The thawing of the layer of seasonal freezing which froze during the winter starts from the moment of inversion of sign of the temperature on the surface of the ground in the direction of positive temperatures and occurs during only a portion of the "warm" half-period. In the remaining time the ground is warmed on account of the heat capacity. That change of the ground temperature can be described with adequate precison in accordance with the Fourier law with an amplitude equal to A_o but a period smaller than a year. The latter is determined by the time interval from the moment of conclusion of thawing to the achievement of the maximal temperature on the surface. In that case for calculation of A_ξ with consideration of asymmetry it is necessary to find the amount of curtailment of the period T_c, which can be determined if the time necessary for thawing the seasonally frozen layer is known. It can be calculated with the formula for the potential seasonal freezing (thawing) of the ground (4.3.1)).

Since the depth of the seasonal freezing (thawing) ξ is a portion of the depth of the potential thawing (freezing) ξ_p, where the value of ξ_p is directly proportional to $\sqrt{T}$, then from the ratio ξ/ξ_p the time necessary for thawing (freezing) of the layer of seasonal freezing (thawing) is calculated directly.

During the thawing of the seasonally frozen layer the heat cycles on its base, proceeding through heat capacity, will certainly be smaller than for "dry" ground. In the latter case, as is known, the heat cycles in the ground are directly proportional to the amplitude of the surface temperature and the square root of the length of the period. The number of heat cycles on the base of the layer of seasonal freezing (thawing) can be reduced by reducing either the amplitude or the length of the period. Consequently,

$$A_\xi \sqrt{T} = A'_\xi \sqrt{T_c},$$

where A_ξ is the sought amplitude on the base of the layer of seasonal freezing (thawing). A_ξ is determined from (4.4.1) for the depth $z = \xi$.

With consideration of the above, the final expression for the amplitude A_ξ in the region $t > t_o$ (as well as the region $t < t_o$) can be presented in the following form:

$$A_\xi = A'_\xi \sqrt{1 - \left[\frac{\xi}{\xi_n} - \left(1 - \frac{t_\xi}{A_0 + t_\xi}\right)\right]^2}. \qquad (4.4.2)$$

Since the values of ξ and ξ_p entering (4.4.2) have been nomographed (see Figures 14-19), A_ξ is calculated for concrete conditions without labor (see example 7).

The results of calculation of A_ξ with the proposed procedure for different values of A_o, t_o and the ground moisture at $\lambda = 0.69$ kcal/(m)(hr)(degree) and $C_\xi = 500$ kcal/(m^3)(degree) were verified by comparison with the data obtained with an analog computer. The divergences in the values of A_ξ amount to less than 1% (Kudryavtsev and Melamed, 1972). See example 7 for an example of consideration of asymmetry in determining the depth of propagation of annual temperature fluctuations.

5. The Depth of Propagation of Annual Temperature Fluctuations in Rock Masses

According to Fourier's first law (3.3.4) the depth of propagation of annual temperature fluctuations in rocks is determined by the two factors of (4.4.1) -- the coefficient of thermal conductivity and the amplitude on the surface of the ground. The validity of the law is observed for regions where there is neither perennial nor seasonal freezing of rocks. During freezing the phase transitions of water greatly distort that regularity. In that case the depth of propagation of the annual temperature fluctuations (H) should be considered the sum: $H = \xi + h$, where h is the depth of propagation of the annual temperature fluctuations, counting from the base of the layer ξ*. The depth ξ is determined with formula (4.1.4). The depth h in the region of

*Henceforth for brevity we will use the following expressions: the depth ξ and the layer ξ are the depth and layer of seasonal freezing (or thawing).

seasonal freezing can be calculated with formula (4.4.1), since the phase transitions of water are absent in that layer. According to the law indicated in (4.4.1) the depth h will be determined by the amplitude of the annual temperature fluctuations on the base of the seasonally frozen layer (A_ξ) and the thermophysical characteristics of the rocks in the layer h.

In the absence of asymmetry of the temperature curves A_ξ must be equal to t_ξ. In that case in calculating h t_ξ is substituted in (4.4.1) instead of A_0 (the precision of measurement of temperatures).

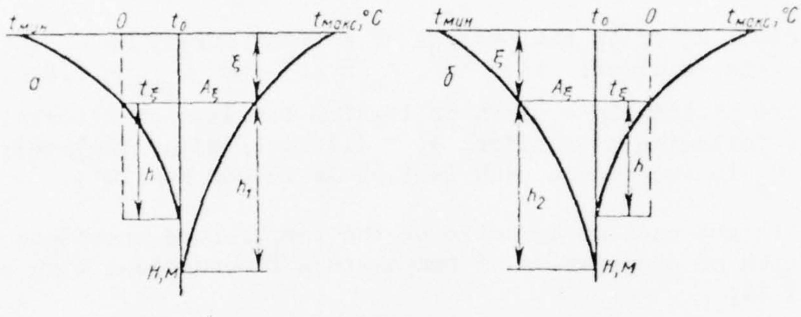


Figure 25. For calculation of H: a - for the case of seasonal freezing of rocks; b - for the case of their seasonal thawing.

In the presence of asymmetry of the temperature field the depth of propagation of annual fluctuations should be examined separately for each branch of the envelopes (Figure 25). For the case of seasonal freezing in the winter period the amplitude of temperature fluctuations on the base of the layer ξ on the branch of the envelope will equal t_ξ (Figure 25a). In accordance with that the depth of propagation of temperature fluctuations in the winter period will be equal to h. For the summer period, because of asymmetry, the amplitude on the base of the layer of seasonal freezing A_ξ will be greater than t_ξ and must be calculated with formula (4.4.2). The depth of propagation of annual fluctuations h_1 obtained for the summer period will be substantially greater than h and it is obvious that it is precisely h_1 which must be taken into consideration in determining H: $H = \xi + h_1$.

In the region of permafrozen rocks asymmetry of the temperature field also is observed in the layer with annual temperature fluctuations (Figure 25b). Calculation of the depth of propagation of fluctuations here has a certain distinctive feature in connection with the fact that in frozen rocks underlying the layer of summer thawing there is a change in the amount of unfrozen water and, consequently, there are phase transitions of water. The depths h and h_2 are found with formula (4.1.17) obtained from (4.1.14) for calculation of ξ. In finding h the amplitude on the base ξ is assumed to be equal to $|t_\xi|$; heat expenditures on phase transitions of water in the layer h are determined in accordance with the curve of the ice content and the variation of temperatures with depth. The amplitude of annual temperature fluctuations in the layer h varies from t_ξ on the surface of the layer h to zero at its base. In the calculation of h_2 the amplitude at the depth ξ is equal to $\bar{A}_\xi$, which is calculated with (4.4.2). In that case the phase transitions are determined on the basis of the change of the quantity of unfrozen water in the temperature range from $t_{\xi\,min} = t_\xi - A_\xi$ to t_ξ. It is obvious that $H = \xi + h_2$ must be taken as the depth of propagation of the annual temperature fluctuations. Examples of the calculation of H are presented below.

Calculation of the Depth of Propagation of the Annual Temperature Fluctuations (H) in the Region of Seasonal Freezing of Rocks (With and Without Consideration of the Asymmetry of the Temperature Envelopes (Example 6)

Calculate the depth H, if in the process of a frost survey on a section the following data were obtained: $t_\xi = +3^\circ$, $A_o = 15^\circ$ and $\xi_{max} = 1.8$ m. The upper part of the profile to a depth of 18-20 m consists of alluvial sandy loams, with the following properties: $\lambda_f = 1.2$ kcal/(m)(hr)(degree); $K^* = 0.003\ m^2/hr$; Q_ϕ in accordance with (4.1.4) is 18,000 kcal/m^3.

Solution. 1. In the case of symmetry of the temperature envelopes, when $A_\xi = t_\xi$, the depth of propagation of temperature fluctuations with consideration of (4.4.1) is:

$$H = \xi + h;\ h = \ln \frac{t_\xi}{0,1} \sqrt{\frac{KT}{\pi}}; \qquad (4.5.1)$$

$$h = \ln \frac{3,0}{0,1} \sqrt{\frac{0,003 \cdot 8760}{3,14}} \approx 9,8\ м.$$

Consequently, H = 1.8 + 9.8 = 11.6 m.

2. In the case of asymmetry of the temperature curves instead of t_ξ, A_ξ is taken and the depth H is determined in the following manner:

$$H = \xi + h_1;\ h_1 = \ln \frac{A_\xi}{0,1} \sqrt{\frac{KT}{\pi}}, \qquad (4.5.2)$$

where in accordance with (4.4.2)

$$A_{\xi,лет} = A'_\xi \sqrt{1 - \left[\frac{\xi}{\xi_n}\left(1 - \frac{t_\xi}{A_0 + t_\xi}\right)\right]^2},$$

where $A_{\xi\text{-sum}}$ is the amplitude of temperature fluctuations at the depth ξ in the summer period and A'_ξ is calculated with (4.4.1). For the conditions characterizing the given section we find that

$$A'_\xi = 15 \cdot e^{-1,8\sqrt{\frac{3,14}{0,003 \cdot 8760}}} = 7,2^\circ;$$

$\xi_p = 2.6$ m (according to the nomogram on Figure 19 at $t = 0^\circ$, $A_o = 15 + 3 = 18^\circ$). If we substitute the values of A_ξ, A_o and ξ_p, we find $A_{\xi\text{-sum}}$:

$$A_{\xi,лет} = 7,2 \sqrt{1 - \left[\frac{1,8}{2,6}\left(1 - \frac{3}{15+3}\right)\right]^2} \approx 5,8^\circ.$$

*$K = \lambda/C\rho$; in Chapter 3 K corresponds to a^2.

Then $h_1 = \ln \frac{5.8}{0,1} \cdot 2,88 \approx 11,7$ м, а $H = 1,8 + 11,7 = 13,5$ м.

Thus it is obvious that for the case under consideration the depth of propagation of the annual temperature fluctuations in the ground, calculated with consideration of the asymmetry of the temperature envelopes, exceeds by almost 2 meters the depth calculated without consideration of asymmetry.

Calculation of the Depth of Propagation of Annual Temperature Fluctuations (H) in the Region of Permafrozen Rocks (Example 7)

Calculate the depth H in the region of propagation of permafrozen rocks if the following data are known: $t_\xi = -3^o$, $A_o = 15^o$; the layer of seasonal thawing, equal to 1.6 m, is underlain by frozen loams with $\gamma_{sk} = 1100$ kg/m^3; w = 35%, $\lambda_f = 1.5$ kcal/(m)(hr)(degree); the content of w_{un} in the loam as a function of -t is shown on the graph (Figure 26). According to formulas (4.1.6) and (4.1.8) $C_{vol-f} \cong 460$ kcal/m^3 and $Q_\phi = 26,400$ kcal/m^3 respectively.

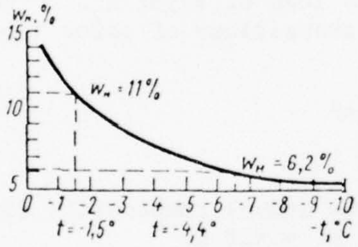


Figure 26. Graph of w_{un} as a function of temperature for loam.

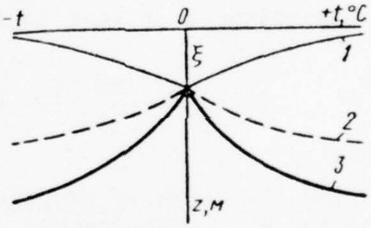


Figure 27. Latitudinal zonation: 1 - depth of the seasonal freezing and thawing of rocks; 2 - depth of the potential seasonal freezing and thawing; 3 - depth of the propagation of annual temperature fluctuations.

Solution. 1. We calculate with formula (4.4.2) the value of A_ξ (with consideration of the asymmetry of the temperature curves), for which we find in accordance with (4.4.1)

$$A'_\xi = 15 \cdot e^{-1,6 \sqrt{\frac{3,14}{0,003 \cdot 8760}}} = 8,6^\circ;$$

$\xi_p = 2.4$ m (according to the nomogram on Figure 19 at $t = 0^o$ and $A_o = 15 + 3 = 18^o$);

$$A_\xi = 8,6 \sqrt{1 - \left[\frac{1,6}{2,4}\left(1 - \frac{3}{18}\right)\right]^2} \approx 7,2^{\circ 1}.$$

*In formula (4.4.2) the absolute value of t_ξ is taken in all cases.

2. We determine the amount of heat expended on phase transitions of water in the frozen rock mass in the layer h_2 (see Figure 25b). Since the amplitude on the base of the layer of seasonal freezing is determined by the value of $|t_\xi|$ (for the envelope the minimal temperatures by depth), then it is obvious that on the surface of the layer h_2 the phase transitions of the unfrozen water proceed in the temperature range from 0^o to $t = t_\xi - A_\xi$. In the given case it is from 0 to -10.2^o.

On the base of the layer h_2 practically no phase transitions of water occur in the annual cycle of temperature fluctuation. Therefore it can be assumed that on the average for the layer h_2 the phase transitions of unfrozen water are determined by the temperature range from $1/2t_\xi$ to $t_\xi - 1/2A_\xi$. In the case under consideration that range is in the temperature range of 1.5-6.6^o.

According to the graph of the dependence of the amount of unfrozen water in loam on the temperature (Figure 26) we find that at $t = -1.5^o$ $w_{un} = 11\%$, and at $t = -6.6^o$, $w_{un} = 6.2\%$. Consequently, 4.8% of the moisture participates in the phase transitions of water in the frozen loam of layer h_2. From this we find the expenditures of heat on the phase transitions of water (Δw) in layer h_2:

$$\Delta w = 80 \frac{1100 \cdot 4.8}{100} = 4224 \text{ ккал/м}^3.$$

3. We find the depth of propagation of the annual temperature fluctuations in the layer h_2 with formula (4.1.17): $h_2 = 3.8$ m.

Consequently, the entire layer of annual temperature fluctuations (H) with consideration of the layer of seasonal melting (ξ) is: $H = 3.8 + 1.6 = 5.4$ m.

The depths of propagation of the annual temperature fluctuations calculated with the given procedure are used for calculation at the depth of the zero annual amplitude of the average annual temperature of rocks on the basis of single measurements of them in wells, for the calculation of the annual heat cycles, for determination of the depth of core-drilling wells dug during a frost survey, etc.

The depth of propagation of annual temperature fluctuations in rocks is subject to geographic latitudinal zonation. The latter is presented on Figure 27. During movement from southern regions northward seasonal freezing of the soil appears, the depth of which increases in proportion to approach to the southern boundary of the region of propagation of permafrozen rocks, where it reaches its maximum. There the seasonal freezing changes into seasonal thawing. During further advance northward the depth of the latter diminishes.

The depth of potential thawing in the region of seasonal freezing is maximal in the south and diminishes in proportion to movement northward. On the southern boundary of the propagation of permafrozen rocks it becomes equal to the depth of the seasonal freezing. An analogous dependence is noted for the depth of the potential freezing of soils (see Figure 27).

Qualitatively the same regularity is also observed for the depth of propagation of annual temperature fluctuations. In the south and in the north, where the average annual temperatures differ sharply from zero, the depths of propagation of annual fluctuations are great and reach 15-20 or more meters. Near the southern boundary of propagation of permafrozen rocks, at temperatures close to zero, the depth of propagation of annual fluctuations is limited to 2 or 3 meters beneath the base of the layer of seasonal freezing (thawing). This can be illustrated by data for the region of the city of Salekhard (see Figure 6 and Chapter 3).

From analysis of the regularities in the propagation of temperature fluctuations in the ground mass it follows that in a region of permafrozen rocks the depth H, other conditions being equal, will be smaller than in a region of seasonal freezing. This is connected with the phase transitions of water occurring under the influence of the annual temperature fluctuations in the layer of permafrozen rock masses underlying the layer of seasonal thawing.

Determination of the Average Annual Temperatures of Rocks at the Depth of Annual Zero Amplitudes on the Basis of Single Measurement of Temperature in a Well (Example 8)

Limitation of t_h without consideration of the geothermal gradient. In a frost survey in a well 14 meters deep a profile of deposits (alQ_3) was studied and samples of rocks were taken for determination of their moisture content, density and other properties. The characteristics of the rocks in the profile of the well are presented in Table 16. The depth of the seasonal thawing of rocks reaches 2.9 meters on the profile. After the well had stood the rock temperatures were measured, the variation of which by depth is presented on Figure 28.

Table 16 Characteristics of rocks over the profile in a well

A Интервал глубин, м	B Порода	C Суммарная влажность w, %	D Объемный вес скелета породы γ, кг/м³	E Коэффициент теплопроводности мерзлой породы λ, ккал/м·град·час	F Объемная теплоемкость породы C, ккал/м³·град
0,0—5,7	1 Песок мелкозернистый кварцевый	15	1800	2,1	500
5,7—8,1	2 Супесь тяжелая пылеватая	28	1460	1,6	400
8,1—14,0	3 Супесь легкая	23	1580	1,7	450

Key: A - interval of depth, m B - Rock C - Total moisture content w, % D - Volume weight of rock skeleton γ, kg/m³ E - Coefficient of thermal conductivity of frozen rock λ, kcal/(m)(degree)(hr) F - Volume specific heat of rock C, kcal/(m³)(degree 1 - Fine-grained quartz sand 2 - Heavy silty sandy loam 3 - Light sandy loam

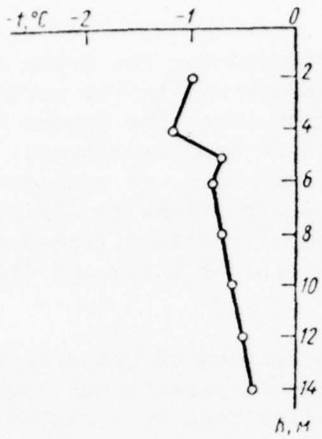


Figure 28. For determination of t_m on the basis of a single measurement of temperatures in a well.

Solution. To find the average annual temperature of rocks at the depth of the zero annual amplitude on the basis of a single measurement of temperatures in a well it is advisable to use Fourier equation (3.5.1), applying it for the layer of annual temperature fluctuations (h) below the layer ξ. The problem is solved by the method of trial and error. From a temperature curve constructed on the basis of the data of measurements are taken the values of the temperature (t) at the proposed depth (h), read off from the base of the layer ξ. The values of t and h are substituted in formula (3.5.1). The calculation is repeated until the selected values assure identity. In that case it is assumed that $A_\xi = t_\xi$ and phase transitions in layer h are not taken into consideration.

1. Since the profile of the deposits has a three-layered structure, to find t_H it is necessary to determine the weighted mean values of λ and C on the basis of which the coefficient of thermal conductivity K of layer h is calculated later. In the calculation of the weighted mean values for the layer h, layer ξ is excluded:

$$\lambda_{\text{ср.вз}} = \frac{2,1 \cdot 2,8 + 1,6 \cdot 2,4 + 1,7 \cdot 5,9}{11,1} \approx 1,8 \text{ ккал/м} \cdot \text{град} \cdot \text{час},$$

$$C_{\text{ср.вз}} = \frac{500 \cdot 2.8 + 400 \cdot 2,4 + 450 \cdot 5,9}{11,1} \approx 450 \text{ ккал/м}^3 \cdot \text{град},$$

$$K = \frac{\lambda}{C_{\text{об}}} = \frac{1,8}{450} = 0,004 \text{ м}^2/\text{час}.$$

2. We will assume that H = 10 meters, and then t_H at that depth in accordance with Figure 28 is equal to -0.6°. We make the necessary substitutions in equation (3.5.1) and the transformation with respect to H:

$$H = \xi + \sqrt{\frac{KT}{\pi}} \ln \frac{t}{0,1};$$

$$10 = 2,9 + \sqrt{\frac{0,004 \cdot 8760}{3,14}} \ln \frac{0,6}{0,1}.$$

We find that 10 $\neq$ 2.9 + 3.35 x 1.8, that is, the depth of propagation of the annual fluctuations is not 10 meters and the average annual temperature does not correspond to -0.6°.

3. We will assume that H = 9 meters. At that depth t = -0.65°. Upon substitution we find that 9 = 2.9 +3.35 x 1.87, that is, 9 = 9.1, which can be taken as identity.

Consequently, in the given case the depth of propagation of the annual temperature fluctuations in frozen sands and sandy loams reaches 9 meters, and the average annual temperature at that depth is -0.65°.

In cases where $t_\xi \neq A_\xi$ and it is impossible to neglect the phase transitions of water in the frozen rock mass, the average annual temperature of the rocks at the depth of zero annual amplitudes should be determined on the basis of single measurements of temperature in wells only after the depth H has been found with the calculating methods analyzed in examples 6 and 7.

/Determination of t_h with consideration of the geothermal gradient/. The geothermal gradient, obviously, can be determined from the equation:

$$g = \frac{t_{h_1} - t_h}{h_1 - h},$$

where t_{h_1} is the temperature of the rocks at a certain depth h_1 (H_1 must be at least 10 meters greater than h) and t_h is the average annual temperature on the base of the layer of annual temperature fluctuations h.

In accordance with the gradient the average annual temperature on the base of the layer of seasonal thawing at the known value of t_{h_1} will be:

$$t_\xi = t_{h_1} - (t_{h_1} - t_h)\frac{h_1}{h_1 - h}.$$

If we assume the condition that on the base of the layer ξ $A_\xi = t_\xi$, we can write:

$$h = \sqrt{\frac{KT}{\pi}} \cdot \ln \frac{\left| t_{h_1} - (t_{h_1} - t_h)\frac{h_1}{h_1 - h} \right|}{0{,}1}.$$

Therefore, if we have the temperature curve at the moment of investigation in the well, we can by matching the values of t and h find the value of t_h.

Thus, for example, in conducting a frost survey on section 1 of a terrace above a flood plain composed to a depth of 37 meters of a mass of alluvial sandy loams with a coefficient of thermal conductivity (α) of 0.0036, the following temperature distribution (Table 17) was obtained in a well during single measurements. On the basis of Table 17 we will assume that h_1 = 25 m, and then t_{h_1} = -0.8°. We will then assume that h = 10 m and t_h = -1° and verify whether those values give identity when substituted in the equation for h:

$$10 = \sqrt{\frac{0.0036 \cdot 8760'}{3,14}} \ln \frac{\left| -0,8 - (-0,8 + 1,0)\frac{25}{15} \right|}{0,1};$$

$$10 = 3,2 \ln \frac{|-0,8 - 0,2 \cdot 1,7|}{0,1}; \quad 10 = 3,2 \ln \frac{1,14}{0,1},$$

that is

$$10 \neq 3,2 \cdot 2,43.$$

Table 17 Temperature distribution in a well

A Глубина, отсчитанная от подошвы слоя ξ, м	1	2	3	4	5	6	8	10	15	20	25
B Температура, °C	−0,1	−0,3	−0,6	−0,7	−0,9	−1,1	−1,1	−1,0	−1,0	−0,9	−0,8

Key: A - Depth, counted from the base of the layer ξ, m B - Temperature, °C

Let us be given new values of h in accordance with Table 17: h = 8 m, when t_h = -1.1°. We verify the presence of identity:

$$8 = 3,2 \cdot \ln \frac{\left| -0,8 - 0,3\frac{25}{17} \right|}{0,1}; \quad 8 = 3,2 \ln \frac{1,25}{0,1};$$

8 $\approx$ 8.1, that is, we practically have identity.

Thus we find that the average annual temperature of the rocks at the depth h is -1.1°.

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Chapter 5. Forecasting the Change of the Temperature Regime and the Depths of Seasonal Freezing and Seasonal Thawing of Rocks

1. Classification of Types of Seasonal Freezing and Seasonal Thawing of Rocks

The study of regularities of the formation of the temperature regime of rocks in the layer of annual temperature fluctuations is inseparably connected with study of regularities in the formation of the layer of seasonal freezing and the layer of seasonal thawing, since it is precisely in those layers that the main part of the thermal processes is realized which includes above all the annual heat cycles (conductive heat exchange) and heat and moisture transport (convective heat exchange). The annual heat cycles in rocks are a component of the radiation heat balance of the earth's surface and can be represented as a function of the temperature regime of the surface and the thermophysical properties and moisture content of the rocks.

In studying seasonal freezing and thawing it is necessary to take into consideration both the thermophysical aspect of the process and the geological medium and geographic situation in which it proceeds, and also their mutual connection. For fulfilment of that main condition it is necessary to distinguish the most generalized factors or signs determining the conditions and character of the seasonal freezing and thawing, and classify the studied phenomenon on the basis of them. Distinguished as such signs (Kudryavtsev, 1959) are the average annual temperature of the rocks, the amplitude of the annual temperature fluctuations on the surface of the soil and the composition and moisture content of the rocks.

The values of the annual heat cycles of the soil, and in particular that portion of them which is connected with seasonal freezing and thawing, are determined by the aggregate of those four signs. Established in the classification for each of those signs are the limits of their changes, starting from the connection of the quantitative values which give a new quality of the phenomenon under consideration.

The first two signs, the average annual temperature of the rocks and the annual amplitude of oscillation of the average monthly temperatures on their surface are geographic; they are readily mapped on both large and small scales. The average annual temperature of the rocks is subject to latitudinal and height zonations. The other two signs (the lithological composition and the moisture content of the rocks) are geological and vary regionally in accordance with the structure of the earth's crust and the principal forms of the relief.

The zonal signs are made the basis of the classification and determine the difference between the seasonal freezing and seasonal thawing of the soil.. The seasonal freezing represents the freezing of thawed rocks having an average annual temperature above 0°. The layer of seasonal freezing is underlain by unfrozen rocks and is formed on account of heat cycles proceeding at negative rock temperatures. The seasonal thawing represents the thawing of frozen rocks having an average annual temperature below 0°. The layer of seasonal thawing is underlain by permafrozen rocks and is formed on account of heat cycles proceeding at positive rock temperatures.

As a result of that, classification consists of two parts. Adopted as the boundary separating seasonal freezing from seasonal thawing in the classification is an average annual rock temperature of 0°. But the average annual temperature of the air and rocks does not remain constant from year to year, but fluctuates continuously. The most frequent deviations form in the range of $\pm 1^\circ$. In separate years they reach values of $\pm 2^\circ$. In accordance with that in the range of 0 to $\pm 1^\circ$ the average annual temperature will periodically pass through 0° and assume negative and positive values. At average annual temperatures of rocks of from +1 to +2° and from -1 to -2° such a course will also be traced, but episodically, in separate warm and cold years. This determines the need to distinguish the transitional and semitransitional types of seasonal freezing and thawing of rocks respectively.

The transition of the average annual temperatures of rocks in the range from +2 to +5° into the negative, or in the range from -2 to -5° into the positive is connected with the long periods or sharp changes of heat transfer on the surface of the ground. Therefore in the given intervals of the average annual temperature of rocks long stable types of seasonal freezing and types of seasonal thawing of rocks respectively are distinguished.

In the temperature intervals from + 5 to +10° and from -5 to -10°, stable types of seasonal freezing and types of seasonal thawing of rocks respectively are distinguished. At a temperature above +10°, southern, subtropical and tropical types of seasonal freezing of rocks are established, and at a temperature lower than -10°, arctic and polar types of seasonal thawing of rocks.

Depending on the correlation of the amplitude and the average annual temperature, also determined are four unstable types of seasonal freezing and four unstable types of seasonal thawing of the soil (episodically and periodically manifested and periodically and episodically disappearing).

The following types are distinguished on the basis of the amplitudes of temperature on the surface of rocks: the marine type with amplitudes of temperature of less than 7.5°, characteristic of the seacoasts of middle latitudes; the moderately marine, with amplitudes of temperature of 7.5 to 11° -- on the northern seacoasts; the moderately continental with amplitudes of temperature of 11 to 13.5° -- in the European part of the USSR; the continental, with amplitudes of temperature of 13.5 to 17° -- in the Western Siberian lowland; the more intensively continentalwwith amplitudes of temperature of 17 to 21° -- on the Central Siberian highland; the sharply continental with amplitudes of temperature of 21 to 24° and the especially sharply continental with amplitudes of temperature above 24° -- in the Northeast and in Zabaykal'ye.

A total of 85 general geographic types of seasonal freezing and types of seasonal thawing of rocks have been distinguished, within each of which it is necessary to distinguish varieties in composition and moisture content. In accordance with that, with respect to composition one can distinguish the following main varieties of rocks: 1) rocky and semi-rocky, fissured and weathered; 2) gravel-pebble; 3) rock debris; 4) coarse to fine sands; 5) fine sands; 6) sandy loams; 7) loams; 8) clays; 9) peat.

With respect to moisture content it is necessary to distinguish four gradations as a function of the quantity of water participating in the phase transitions during the freezing and thawing of rocks. The first gradation at $w < w_{un}$ is characterized by an absence of phase transitions during the freezing of grounds. In the following three gradations the phase transitions increase from 0 at $w = w_{un}$ to a maximum value at $w = w_t$. It is advisable to divide that range of phase transitions into three gradations. For the second gradation $w_{un} < w < w_{un} + 1/3(w_t - w_{un})$; for the third gradation $w_{un} + 1/3(w_t - w_{un}) < w < w_{un} + 2/3(w_t - w_{un})$ and for the fourth gradation $w > w_{un} + 2/3(w_t - w_{un})$. Here w is the natural moisture content of the ground, determined at the moment of freezing (thawing), w_{un} is the amount of unfrozen water and w_t is the total moisture content.

Thus the number of combinations of different values of classification parameters can reach a large value. From this, naturally, also flows a great variety of depths of seasonal freezing and thawing of rocks observed in nature. One and the same depth of seasonal freezing (or thawing) of rocks at different points is explained by different combinations of natural factors and, on the contrary, under identical natural conditions different depths of seasonal freezing (thawing) form. As a result of the complexity of the influences of those factors the change of one of them in these two cases leads to very different results.

Within each type for all the distinguished varieties the depth of seasonal freezing and thawing of rocks can be calculated with any of the existing calculating formulas. The only requirement in that case is expression of the depth ξ through the average annual temperature of the rocks and the amplitude of temperatures on their surface with consideration of distinctive features of the composition and moisture content of the rocks.

A very important aspect in that approach is the possibility of determining the dynamics of change of both the types of seasonal freezing and thawing of the grounds and the depths corresponding to them in time as a function of the change of concrete conditions.

A map of the types of seasonal freezing and thawing of rocks compiled on the basis of the classification under consideration makes it possible to reflect the regularities in the development of the studied phenomenon and permits determining how at each point a definite depth of freezing (or depth of seasonal thawing) forms. If one knows the character of the change of each factor of a given complex of conditions one can determine how the depth of the seasonal freezing (or thawing) or rocks also changes. In addition, both the maximal mean perennial depths and the limits of their fluctuations are given on such maps and the character of their variation in the process of the construction and subsequent operation of structures is pointed out. Therefore nomograms for the calculation of depths must obligatorily be attached to such maps (see Figures 14-19).

The average annual temperature of rocks and the amplitude of temperatures on their surface, and also the composition, moisture content and thermophysical characteristics of the rocks are determined as a result of field survey work. In calculations with the formulas (Chapter 4) completely determined values of the parameters are taken, and so the results obtained with the calculations are unambiguous. Such an approach excludes the possibility of free selection of the values of parameters from reference manuals. The results of calculations on the basis of parameters concretely determined in the field make it possible to verify the correctness of the calculations by comparing the obtained data with the actual depths of the seasonal freezing and thawing of the rocks.

It is an important fact that through the main characteristics adopted in the classification the seasonal freezing and thawing of rocks are linked with the general frost situation of the region. Therefore each type of seasonal freezing and thawing is connected both with the general character of the type of permafrozen rock masses and with distinctive features of the cryogenic phenomena developed within its limits. In addition, the temperature conditions of the rocks and the depths of seasonal freezing and thawing are connected with the subfrostal waters and leakage water enclosed in them. By virtue of this the map of types of seasonal freezing and thawing or rocks, besides its direct task, makes it possible to judge the general frost situation.

The interconnection of geological, geographic and thermophysical regularities in the formation and development of the temperature of rocks (t_ξ and A_o), the depths of their seasonal freezing (ξ_f) and thawing (ξ_t) and all other characteristics of frost conditions is expressed through the influence of different elements of the natural environment on t_ξ, A_o and ξ. By virtue of this, in compiling a frost forecast it is necessary to determine the qualitative and quantitative influence of the principal components of the natural environment on the indicated parameters.

For a characterization of the temperature regime of the rocks in the layer of seasonal freezing (thawing) it is necessary to have available the following basic data: the average annual temperature (t_o) and the amplitude of the annual temperature fluctuations under covers (on the surface of the soil (A_o) and the average annual temperature at the depth ξ (t_ξ). The first two characteristics are formed under the influence of the radiation heat balance of the surface of the ground and the thermal effect of different natural covers (snow, plant and water). The third characteristic (t_ξ) is formed under the influence of the temperature regime on the surface of the soil, distinctive features of the composition and moisture regime of the rocks and thermal processes (conductive and convective) taking place in the layer ξ. Connected with convective heat exchange in the layer ξ is the formation of a temperature shift on account of the separating effect of the infiltered volatile atmospheric precipitations or ground and subsurface waters (Δt_{prec}). Examined below is the influence of the principal natural factors on the formation of the average annual temperature and the depth of the seasonal freezing (thawing) of rocks.

2. Dependence of the Temperature Regime and Depths of Seasonal Freezing and Thawing of Rocks on Their Composition, Moisture Content and Thermophysical Characteristics

The dependence of the change of the temperature regime and the depths of the seasonal freezing and thawing of rocks on their lithological characteristics and moisture content is presented in the following form.

Change of the lithological characteristics of the composition of rocks leads primarily to change of their thermophysical properties -- thermal conductivity and heat capacity. It follows from formula (4.1.4) that ξ is directly proportional to the square root of the thermal conductivity and has a somewhat more complex dependence upon the heat capacity. It is evident from Table 18 and Figure 29 that when the dispersion of the rocks increases the coefficient of thermal conductivity decreases. Therefore, other conditions being equal, the greatest depths of seasonal freezing (thawing) form in coarsely dispersed rocks and the smallest in finely dispersed.

Table 18 Change of the thermal conductivity of rocks as a function of their mechanical composition

A Порода	B Гранулометрический состав, мм								C Удельный вес, т/м³	D Полная влагоемкость (в весов) $w_п$, %	E w при определ. λ, %	F λ, ккал/м·час·град	
	1,0	1,0—0,5	0,5—0,25	0,25—0,10	0,10—0,05	0,05—0,01	0,01—0,005	0,005				a мерзлого	b талого
1 Песок кварцевый, мелкозернистый	—	0,2	20,8	77,4	1,51	0,09	—	—	2,65	23,2	18,6	2,04	1,68
2 Супесь легкая, пылеватая	—	1,95	0,41	6,87	25,13	44,80	6,72	10,27	2,69	30,4	30,4	1,56	1,30
3 Суглинок легкий, пылеватый	6,11	14,8	2,43	21,45	11,93	19,49	6,14	13,56	2,70	37,2	34,5	1,07	0,90

Key: A - Rock B - Granulometric composition, mm C - Specific gravity, tons/m^3 D - Total moisture content w_t, % by weight E - w in determination of λ, % F - λ, kcal/(m)(hr)(degree) a - frozen b - thawed 1 - Quartz fine sand 2 - Light silty sandy loam 3 - Light silty loam

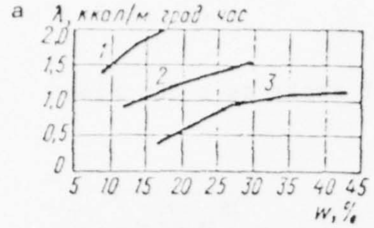

Figure 29. Change of the coefficient of thermal conductivity of rocks as a function of moisture content: 1 - sand; 2 - sandy loam; 3 - loam
a - λ, kcal/(m)(degree)(hr)

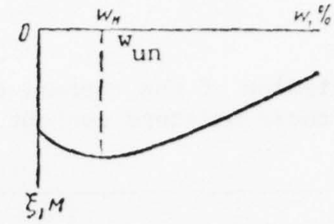

Figure 30. Change of the depth of seasonal freezing and thawing of the ground as a function of moisture content.

It also is generally known that the thermophysical properties of rocks vary substantially with variation of their density, porosity and mineralogical composition. Denser rocks have greater thermal conductivity and heat capacity.

In examining the lithological characteristics of the composition of rocks and its influence on the depth ξ it is necessary to simultaneously examine also the moisture content of the rocks, since with variation of it their thermophysical characteristics also vary substantially. As follows from Table 19 and Figure 29, the coefficient of thermal conductivity of dispersed rocks increases with increase of the moisture content. Very sharp increase of the thermal conductivity of thawed rocks is observed in the ranges of low moisture contents (up to the maximal molecular moisture content) and continues to increase with increase of the moisture content to the total moisture capacity. Further increase of the moisture content leads to disturbance of the contacts between the mineral particles, to reduction of the density of the rocks and therefore to reduction of the thermal conductivity.

The variation of the thermal conductivity of frozen rocks in connection with variation of the moisture oontent is characterized by a somewhat different dependence. Decrease of the thermal conductivity of frozen rocks is observed only at low moisture contents (up to the maximal molecular moisture capacity), when the forming separate ice crystals worsen the thermal contacts. In all other cases an increase of the moisture (ice) content of frozen dispersed rocks leads to increase of their thermal conductivity. In accordance with the latter, increase of the moisture content of the rocks ought to lead to increase of the depth of seasonal freezing (thawing). But the moisture content of the rocks has a very strong influence on the depth ξ through the phase transitions of water, the percentage of the participation of which in the total annual heat cycles of the rocks reaches 50% or more. In that case the larger the moisture content of the rocks the more heat is expended on phase transitions of water in them and the smaller the depth of the seasonal freezing (thawing).

The general dependence of the change of depth of the seasonal freezing (thawing) of rocks on their moisture content is depicted on Figure 30. It is evident from the presented curve that when the moisture content increases from

Table 19 Variation of the thermal conductivity of rocks as a function of their moisture content

	A Порода	B Объемн. вес скелета породы $\gamma_{ск}$, т/м³	C Объемн. вес влажн. породы γ, т/м³	D Полная влагоемкость $w_п$, %	E w при определении λ, % к сухой навеске	F λ, ккал/м·час×град	
						a мерзлого	b талого
1	Песок мелкозернистый	1,61	1,76	23,2	9,4	1,48	1,49
2	То же	1,64	1,87	23,2	14,1	1,83	1,59
	» »	1,58	1,87	23,2	18,6	2,04	1,68
3	Супесь легкая, пылеватая	1,58	1,75	30,4	10,6	0,91	0,86
2	То же	1,63	1,95	30,4	19,8	1,27	1,34
	» »	1,48	1,93	30,4	30,4	1,56	1,30
4	Суглинок легкий, пылеватый . . .	1,34	1,56	37,2	16,4	0,4	0,57
2	То же	1,44	1,84	37,2	27,4	0,95	0,84
	» »	1,36	1,83	37,2	34,5	1,07	0,92
	» »	1,16	1,67	37,2	43,5	1,10	0,73

Key: A - Rock B - Volume weight of rock skeleton γ_{sk}, t/m³ C - Volume weight of moist rock, γ, ton/m³ D - Total moisture capacity, w_t, % E - w during determination of λ, % of dry weighed portion F - λ, kcal/(m)(hr)(degree) a - frozen b - thawed
1 - Fine-grained sand 2 - Ditto 3 - Light, silty sandy loam 4 - Light, silty loam

zero to w_{un} the depth ξ increases. This is explained by the fact that in that case the thermal conductivity increases more than the heat capacity. In that range of moisture content all the moisture in the rocks remains in the liquid state at negative temperatures. In that case the rocks are not frozen.

At natural moisture contents of the rocks exceeding w_{un} at temperatures below 0^o a portion of the water freezes. With increase of the moisture content in that case, in the total heat cycle of the rock there is a sharp increase of the percentage of phase transitions, in connection with which the depth ξ increases.

The composition and moisture content of the rocks substantially determine the depth of the seasonal freezing (thawing) also in connection with the temperature shift which changes t_ξ in comparison with t_o.

In Chapter 4 it was pointed out that the temperature shift (Δt_λ) arises on account of change of the coefficient of thermal conductivity of the rock during its transition from the melted to the frozen state in the process of seasonal freezing (thawing). The value of Δt_λ is proportional to the difference of the square roots of the thermal conductivities of the frozen and thawed ground, and also the value of the annual heat cycles. It is known that the greater the ice content of the rocks the more the coefficient of thermal conductivity of the frozen rock differs from that of the thawed rock (this dependence is traced in Table 19). On the basis of that it can be concluded that with increase of the ice (moisture) content of the rocks the temperature shift (other conditions being equal) increases and, consequently, the average annual temperature on the base of the layer of seasonal freezing (thawing) decreases. Reduction of the average annual temperature of the rocks leads to increase of the depth of seasonal freezing (reduction of the depth of seasonal thawing). Therefore the total influence of the moisture content of the rocks on the depth of seasonal freezing proves to be somewhat less than on the depth of seasonal thawing. In the former case the increase of the temperature shift during increase of the moisture content of the rocks compensates somewhat the reduction of the depth of seasonal freezing which occurs in connection with increase of heat expenditures on phase transitions of water in the rock. In the latter case the influence of the moisture content proves to be maximal, since the increase of the temperature shift and the increase of the heat of phase transitions in connection with the increase of the moisture content of the rocks lead to reduction of the depth of seasonal thawing.

Calculation of the Amount of the Temperature Shift Δt_λ (Example 9)

Calculate Δt_λ and t_ξ on the base of the layer of seasonal freezing of rocks if as a result of a frost survey the following data were obtained. The area with a surface in the layer of annual temperature fluctuations is composed of alluvial loams with the characteristics: γ_{sk} = 1100 kg/m^3; w = 35%; λ_f = = 1.3; λ_t = 1.0 kcal/(m)(hr)(degree); the quantity of unfrozen water in the loam as a function of the negative temperature varies according to the graph (Figure 31). The temperature regime of the surface is determined with the values $t_o = 1.8^\circ$ and $A_o = 10^\circ$.

Solution. For calculation of the amount of the temperature shift we use the following calculating equation:

$$\Delta t_\lambda = \frac{\xi^2 (Q_\Phi + A_{cp}C)}{T} \cdot \frac{\sqrt{\lambda_M} - \sqrt{\lambda_T}}{\lambda_{np}}, \qquad (5.2.1)$$

which represents a particular case of equation (4.1.18) under the condition $t_o < A_o/2$.

That equation is transcendental and is solved by trial and error. For that purpose some values are given for Δt_λ, for example, -0.5, -1.0 and 1.5°. Assuming those values, all the initial parameters (ξ and A_m) are determined for calculation of the right side of the equation and then depicted graphically are the two equations $x = \Delta t_\lambda$ and

$$x = \frac{\xi^2 (Q_\Phi + A_{cp}C)}{T} \cdot \frac{\sqrt{\lambda_M} - \sqrt{\lambda_T}}{\lambda_{np}},$$

where the assumed values for Δt_λ, that is, -0.5, -1.0 and -1.5°, are plotted on the y axis. The first equation represents the bisector of the angle and the second is a certain curve obtained in calculation of the right side of the equation with substitutions corresponding to the given x. At the point of intersection of the straight lines condition (5.2.1) is accomplished, and so the value of Δt_λ at that point also is the unknown value. To solve that problem we perform the following actions.

1. We determine the thermophysical characteristics of the loam. For calculation of C_{vol-f} and Q_ϕ it is necessary to determine the amount of unfrozen water in the loam. The average winter temperature in the layer of seasonal freezing of the ground, at which w_{un} should be determined, is assumed to be equal to the average winter temperature at the surface (t_{o-wtr}). The latter can be considered approximately equal to 1/2 t_{o-min} (t_{o-min} is the minimal temperature at the surface of the ground):

$$t_{0,\text{зим}} = \frac{1}{2}(t_0 - A_0) = -4,1^\circ.$$

With the graph (Figure 31) we find that w_{un} at $t = -4.1^\circ$ is $\sim 8\%$. Then with formulas (4.1.6), (4.1.8) and (4.1.k9) we determine that:

$$C_{\text{об м}} = 0,18 \cdot 1100 + 0,5\,\frac{(35-8)\,1100}{100} + 1,0\,\frac{8 \cdot 1100}{100} = 434\ \text{ккал/м}^3\text{град};$$

$$Q_\text{ф} = 80\,\frac{(35-8)\,1100}{100} = 23\,760 \approx 24\,000\ \text{ккал/м}^3;$$

$$\lambda_\text{пр} = \frac{1,3\,(10-1,8) + 1,0\,(10+1,8)}{20} = 1,12\ \text{ккал/м}\cdot\text{час}\cdot\text{град}.$$

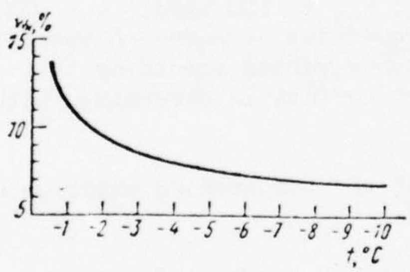

Figure 31. Graph of the dependence of w_{un} on the temperature in the loam.

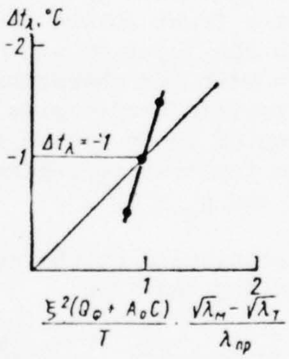

Figure 32. Graph for finding the value of Δt_λ.

2. We find A_m with (4.1.3) and ξ with the nomogram (Figure 17) and calculate the right side of equation (5.2.1) successively for the assumed values of Δt_λ. For $\Delta t_\lambda = -0.5^\circ$ we find A_m under the condition that t is taken with consideration of Δt_λ:

$$t_\xi = t_0 + \Delta t_\lambda = 1,8 - 0,5 = 1,3^\circ.$$

$$A_\text{ср} = \frac{10 - 1,3}{\dfrac{10 + 27,65}{1,3 + 27,65}} - 27,65 = \frac{8,7}{0,263} - 27,65 \approx 5,4^\circ.$$

We find ξ with the nomogram (Figure 17) in accordance with the following initial parameters: $C_{vol-f} = 434$ kcal/(m^3)(degree), $Q_\phi = 24{,}000$ kcal/(m^3); t = = 1.3°; $A_o = 10^\circ$; $\lambda_f = 1.3$ kcal/(m)(hr)(degree). Then

$$\xi_{\text{ном}} = 1{,}33\ м; \quad \xi_{\lambda=1,3} = 1{,}33 \cdot \sqrt{1{,}3} \approx 1{,}5\ м.$$

Now we calculate the right side of equation (5.2.1):

$$\frac{(1{,}5)^2\,(5{,}4 \cdot 434 + 24\,000)}{8760} \cdot \frac{1.3 - 1.0}{1{,}13} = \frac{2{,}25 \cdot 26\,343{,}6}{8760} \cdot 0{,}124 = 0{,}84.$$

The calculations are made similarly for $\Delta t_\lambda = -1.0^\circ$ and $\Delta t_\lambda = -1.5^\circ$. The obtained data are presented in Table 20 and on the graph (Figure 32).

Table 20 Calculating data for determining Δt_λ.

Расчетные данные	Δt_λ, °C		
	−0,5	−1,0	−1,5
$\frac{\xi^2 (A_0 C_{c6} + Q_\phi)}{T} \cdot \frac{\sqrt{\lambda_м} - \sqrt{\lambda_т}}{\lambda_{np}}$	0,84	~0,997	1,13

Key: A - Calculating data

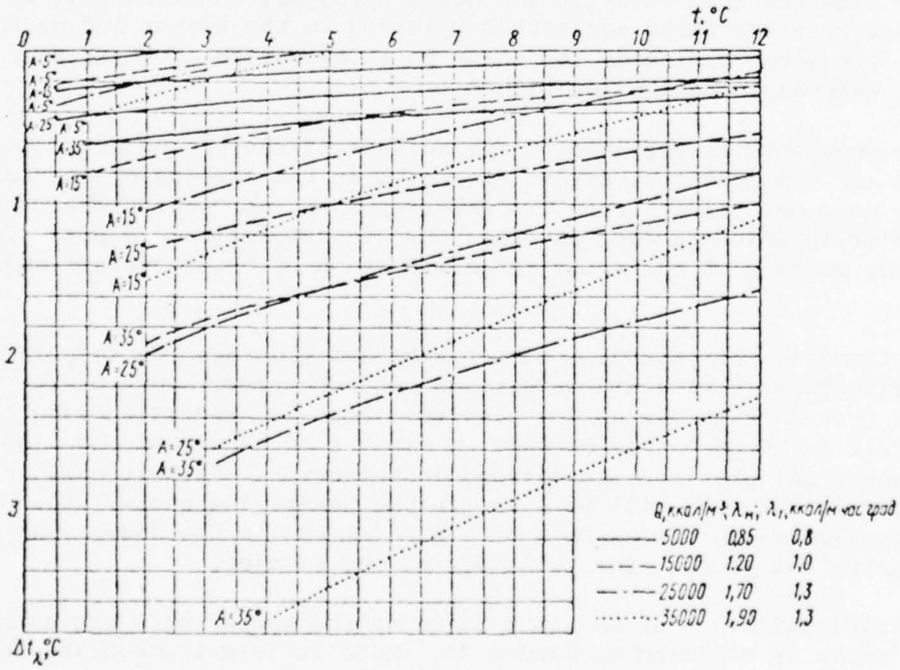

Figure 33. Nomogram for calculation of Δt_λ.
a - Q, kcal/m^3 b - λ_f
b - λ_t, kcal/(m)(hr)(degree)

It is evident from Table 20 and the graph (Figure 32) that the sought value of the temperature shift is -1.0°.

Consequently the average annual temperature on the base of the layer of seasonal freezing is $t_{\xi} = t_{o} + \Delta t_{\lambda} = 1.8 - 1.0 = 0.8^{\circ}$ and the depth of the seasonal freezing (ξ) according to the nomogram (Figure 17) is 1.63 m.

In connection with the complexity of solving such problems to determine the temperature shift a nomogram was obtained by computer. The input parameters in that case were selected by starting from the most frequently encountered cases. A nomogram for determination of Δt_{λ} is presented on Figure 33.

3. The Influence of the Snow Cover on the Temperature Regime and the Depth of the Seasonal Freezing and Thawing of Rocks

A snow cover leads to a change of the heat exchange on the surface of the ground. And in that sense its importance is varied. A white snow cover increases the albedo of the earth's surface. That leads to decrease of the absorption of radiant energy and a decrease of the average annual temperatures of the rocks.

At the same time the snow cover, which has low thermal conductivity, as a heat insulator protects the rocks against heat losses in the winter and at the same time as it were warms the rocks and leads to an elevation of their average annual temperatures in comparison with t_a.

In the case when snow is retained on the surface of the ground after the onset of positive air temperatures, delays are noted in the thawing of the rocks. The melting snow maintains a zero temperature on the surface of the rocks for a certain time in spite of the fact that the air temperature is positive. This leads to some cooling of the rocks and a reduction of their average annual temperatures.

During the formation of perennial firn basins and glaciers the temperature of the underlying rocks varies (in comparison with the temperature of the rocks on sections free of firn basins and glaciers) both in connection with both the change of heat exchange on the surface on account of change of the albedo and with the geothermal gradient established in the cover. In that case an increase of temperature with depth will be noted in the underlying rocks. The difference in temperatures on the surface of a glacier and in the underlying rocks will be greater the greater the thickness of the glacier.

All this varied influence of the snow cover on the temperature regime of the underlying rocks is depicted on Figure 34, where it is evident that with change of the thickness of the snow cover its influence changes to the opposite several times. Up to a thickness of the snow equal to h_1 a cooling influence on account of increase of albedo is noted. Then, in the range of thicknesses of h_1 to h_2, the warming effect of snow as a heat insulator dominates. At thicknesses of h_2 to h_3 the cooling influence of snow gradually increases on

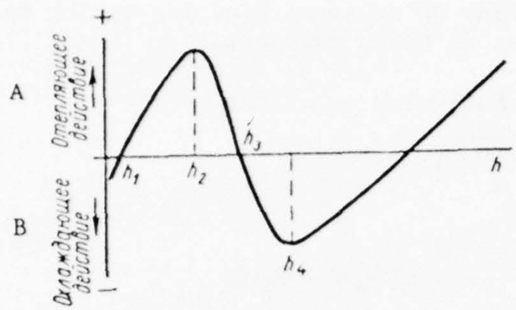

Figure 34. Variation of the influence of the snow cover on the temperature regime of the underlying rocks as a function of its thickness.
A - Warming effect; B - Cooling effect

account of delay of its disappearance in the summer. At a thickness of h_3 that cooling influence is equal to the warming influence of the snow as a heat insulator. At a thickness of from h_3 to h_4 the cooling effect of snow exceeds the warming effect. During the formation of permanent firn basins and glaciers simultaneously with their cooling influence on account of change of the albedo of the surface a warming influence on the underlying rocks is noted which gradually increases with increase of their thickness. Here we have a clear example of manifestation of the dialectical law of transition of quantity into quality.

Thus, in the examination of regularities in the formation of the temperature regime of rocks, besides qualitative evaluation it is necessary to determine the quantitative connections of those regularities. The latter in turn serve for a better understanding of the qualitative connection, that is, examination of the essence of the question.

1. The Warming Influence of the Snow Cover With Consideration of Heat Cycles in the Underlying Rocks

The warming influence of the snow cover, which leads to increase of the annual average temperatures of the rocks, can be quantitatively estimated by means of the heat cycles passing from the ground through the snow covers into the atmosphere in the winter. It is obvious that those heat cycles will be equal to the heat cycles of the soil passing through the surface of the ground in the same period. Those heat cycles are determined with formula (5.3.1) if they relate to the period of time from the moment of onset of negative temperatures on the surface of the ground until inversion of the sign of the heat cycles, that is, the moment when heat emission by the ground ceases and its warming starts:

$$Q_{\text{гр}} = \left[\frac{n}{2}(A_{\text{ср}} - t_\xi)C + Q_\Phi\right]\xi + t_\xi\sqrt{2}\cdot\sqrt{\frac{\lambda TC}{\pi}}\cdot\frac{A_{\text{ср}} - t_\xi}{A_{\text{ср}}}, \quad (5.3.1)$$

where $n/2\ (A_m - t_\xi)\ C_\xi$ are the heat cycles of the layer of seasonal freezing, connected with the heat capacity of the rocks at negative temperatures;

$Q_\phi \xi$ are the heat cycles on account of phase transitions of water in the layer of seasonal freezing $t_\xi \sqrt{2} \sqrt{\frac{\lambda TC}{\pi}} \cdot \frac{A_{cp} - t_\xi}{A_{cp}}$ are the heat cycles proceeding in thawed rocks underlying the layer of seasonal freezing during the time of existence of negative temperatures on their surface.

The coefficient n is determined from the following equation:

$$\sqrt{2}\,(A_{cp} - t_\xi)\sqrt{\frac{\lambda TC}{\pi}} + \frac{(2A_{cp}C\,\xi_{2c} + \xi Q_\Phi)\,Q_\Phi \sqrt{\frac{\lambda T}{\pi C}}}{2A_{cp}C\,\xi_{2c} + \xi Q_\Phi + \sqrt{\frac{\lambda T}{\pi C}}\,(2A_{cp}C + Q_\Phi)} = n\,(A_{cp}C + Q_\Phi)\,\xi.$$

If we solve that equation with respect to n and substitute it in (5.3.1) we obtain an expression for determination of the heat cycles passing through the surface of the rocks during the time of existence of negative temperatures on the surface of the ground before inversion of the sign of Q_{gr} in the following form:

$$Q_{rp} = \frac{1}{2}\left(1 - \frac{t_\xi}{A_{cp}}\right)\sqrt{2}\,A_0 \sqrt{\frac{\lambda TC}{\pi}} + \frac{(2A_{cp}C\,\xi_{2c} + Q_\Phi\,\xi)\,Q_\Phi \sqrt{\frac{\lambda T}{\pi C}} \cdot \frac{1}{2}\left(1 - \frac{t_\xi}{A_{cp}}\right)}{2A_{cp}C\,\xi_{2c} + Q_\Phi\,\xi + \sqrt{\frac{\lambda T}{\pi C}}\,(2A_{cp}C + Q_\Phi)} + \frac{1}{2}Q_\Phi\,\xi\left(1 + \frac{t_\xi}{A_{cp}}\right). \qquad (5.3.2)$$

In that equation the values of A_m, ξ_{2c} and ξ are functions of the average annual temperature of the rocks (t^m). Therefore the heat cycles in the layer ξ also are functions of t_ξ at the given values of A_o, λ, C, T and Q_ϕ. The number of heat cycles passing through the snow cover is determined in general in the following manner:

$$Q_{cH} = \lambda_{cH} \sum_{\tau_1}^{\tau_2} \frac{t_0 - t_{cH}}{z_{cH}}\,\tau, \qquad (5.3.3)$$

where λ_{sn} is the thermal conductivity of the snow, t_o is the temperature on the surface of the rocks, t_{sn} is the temperature on the surface of the snow cover, z_{sn} is the thickness of the snow cover, τ_1 is the time, counted from the onset of negative temperatures on the surface of the rocks, and τ_2 is the time corresponding to the inversion of sign of the heat cycles on the surface of the rocks.

If we use the average values of the negative temperatures on the surface of the snow cover ($t_{sn\text{-}wtr}$ and on the surface of the rocks $t_{0\text{-}wtr}$, and also the mean values of the thickness of the snow cover z_{sn} ($0.5\, z_{max}$), during the time segment under consideration expression (5.3.3) can be re-written in the following form:

$$Q_{сн} = \lambda_{сн} \frac{t_{0,зим} - t_{сн\,зим}}{z_{сн}} (\tau_2 - \tau_1). \qquad (5.3.4)$$

If we know the course of variation of temperatures of the air and snow cover during the winter we can determine t_{sn-wtr} and z_{sn} in accordance with the data of factual observations. In the absence of those data t_{sn-wtr} can be assumed to be approximately two-thirds of the minimal average monthly temperature of the air or surface of the snow during the winter with consideration of a radiation correction. The value of z_{sn} can be assumed to be one-half of the thickness of the snow cover corresponding to the moment of time of inversion of the sign of the heat cycles on the surface of the rocks. However, the latter can be determined by linear interpolation on the basis of the data of maximal thickness of the snow cover during the winter for the given geographic point.

It follows from the essence of the thermophysical process that the heat cycles Q_{gr} and Q_{sn} must be equal. If we equate (5.3.2) and (5.3.4) we finally obtain the following equations:

$$\frac{1}{2}\left(1 - \frac{t_\xi}{A_{cp}}\right)\sqrt{2}\,A_0\sqrt{\frac{\lambda TC}{\pi}} + \frac{(2A_{cp}C\,\xi_{2c} + Q_\Phi\,\xi)\,Q_\Phi\sqrt{\frac{\lambda T}{\pi C}}\cdot\frac{1}{2}\left(1 - \frac{t_\xi}{A_{cp}}\right)}{2A_{cp}C\,\xi_{2c} + Q_\Phi\,\xi + \sqrt{\frac{\lambda T}{\pi C}}\,(2A_{cp}C + Q_\Phi)} + \frac{1}{2}Q_\Phi\,\xi\left(1 - \frac{t_\xi}{A_{cp}}\right) = \lambda_{сн}\frac{t_{0,зим} - t_{сн,зим}}{z_{сн}}(\tau_2 - \tau_1). \qquad (5.3.5)$$

When that equation is used the values of A_o, A_m, ξ_{2c} and t_{o-wtr} should be represented as a function of t_ξ in the following form:

$$A_0 = t_{макс} - t_\xi; \qquad (5.3.6)$$

$$A_{cp} = \frac{t_{макс} - 2t_\xi}{\ln\dfrac{t_{макс} - t_\xi + \dfrac{Q_\Phi}{2C}}{t_\xi + \dfrac{Q_\Phi}{2C}}} - \frac{Q_\Phi}{2C}; \qquad (5.3.7)$$

$$\xi_{2c} = \frac{2\,(t_{макс} - 2t_\xi)\sqrt{\frac{\lambda TC}{\pi}}}{2A_{cp}C + Q_\Phi}; \qquad (5.3.8)$$

$$t_{0,зим} = \frac{2}{3}(t_{макс} - 2t_\xi). \qquad (5.3.9)$$

When those expressions are substituted in (5.3.5) the latter will assume the form of a functional dependence on t_ξ, which is the sought value. The equation is transcendental and complex and is not solved with respect to t_ξ. A solution is readily found graphically.

When the thickness of the snow cover is large (> 0.5 m) and the thermal conductivity of the snow is low, formula (5.3.2) gives overstated values of Δt_{sn}. In those cases it is advisable to use for calculation of Δt_{sn}, instead of (5.3.2), the following formula:

$$\sqrt{\frac{2\lambda TC}{\pi}}(A_в - \Delta t_{сн}) + \frac{Q_ф}{\sqrt{\frac{\lambda T}{\pi C}} + \frac{A_{ср} + \frac{Q_ф}{2C}}{A_{ср}\xi_{2c} + \frac{Q_ф}{2C}\xi}} = \frac{3}{4}\frac{\lambda_{сн}}{z_{сн}} T \cdot \Delta t_{сн}\left(1 + \frac{\sqrt{1 - \left(\frac{t_в}{A_в}\right)^2}}{\frac{\pi}{2} - \arcsin\frac{t_в}{A_в}}\right).$$

The derivation of that formula and nomograms for calculation of Δt_{sn} with it will be published in "Merzlotnyye issledovaniya," No 16 (1976). We will only note that the change of height of the snow cover is assumed to be according to a parabolic law. In addition, the cooling influence of the snow in spring is neglected.

Calculation of the Warming Influence of the Snow With Consideration of Heat Cycles Passing Through the Surface of the Soil and Snow (Example 10)

Calculate the warming influence of the snow cover on the temperature regime of the surface of rocks* represented on the investigated section by sandy loams characterized by the following data: w = 22%, γ_{sk} = 1320 kg/m^3, and w_{un} in the layer of seasonal thawing is 4.4% on the average. The average annual air temperature is -10.5^o and A_a is 22^o (on the basis of average monthly values). The thickness of the snow cover on the section at the moment of the spring inversion of sign of the heat cycles (at the end of March or start of April) is 0.2 m. Its average density is 0.29 g/cc. The time τ, starting from the moment of stable transition of temperature on the surface of the soil through 0^o (it is assumed that it coincides with the moment of establishment of the snow) to the moment of the spring inversion of sign of the heat cycle through the snow is 4750 hours.

Solution 1. We examine Q_{gr} and Q_{sn} at $\Delta t_{sn} = 2^o$. Then $t_\xi = t_o = t_a + t_{sn} = -10.5 + 2.0 = -8.5^o$; $A_o = A_a - \Delta t_{sn} = 22 - 2 = 20^o$. If we assume that $t_{0\text{-}wtr} = 2/3\, t_{0\text{-}min}$ ($t_{0\text{-}min}$ is the minimal temperature on the surface of the soil under snow), that is, $t_{0\text{-}min} = 2/3\,(t_o - A_o)$, we find that $t_{0\text{-}wtr} = 2/3\,(-8.5 - 20) = -19^o$. For the calculation of A_m we determine $C_{vol\text{-}t}$ with (4.1.7)

*In calculations of A_m and ξ_{2c}, Q_{gr} and Q_{sn}, $t_o = t_\xi$ and is taken on the basis of the absolute value, without considering the sign.

and Q_ϕ with (4.1.8): $C_{vol-t} \approx 550$ kcal/(m^3)(degree) and Q_ϕ = 18,600 kcal/m. On the basis of meteorological data we calculate the average monthly temperature on the surface of the snow t_{sn-wtr} = -21°. For calculation of Q_{gr} we find

$$A_{cp} = \frac{A_0 - t_\xi}{\ln \dfrac{A_0 + \dfrac{Q_\Phi}{2C}}{t_\xi + \dfrac{Q_\Phi}{2C}}} - \frac{Q_\Phi}{2C} = \frac{20 - 8,5}{\dfrac{20 + 16,9}{8,5 + 16,9}} - 16,9 = 14^\circ,$$

$$\xi_{2c} = \frac{2(A_0 - t_\xi)\sqrt{\dfrac{\lambda TC}{\pi}}}{2A_{cp}C + Q_\Phi} = \frac{2(20 - 8,5)\sqrt{\dfrac{8760 \cdot 550 \cdot 1}{3,14}}}{2 \cdot 14 \cdot 550 + 18\,600} \approx 0,84 \text{ м}.$$

We find the depth of the seasonal thawing ξ with nomograms (Figures 15 and 17) at the following initial data: $C_{vol-t} \approx 550$ kcal/(m^3)(degree); Q_ϕ = 18,600 kcal/m^3; λ = 1.0 kcal/(m)(degree)(hr); $t_\rho = t$ = -8.5°; A_ρ = 20°; ξ_{nom} = 1.25 m. Then on the basis of the left side of equation (5.3.5) we find:

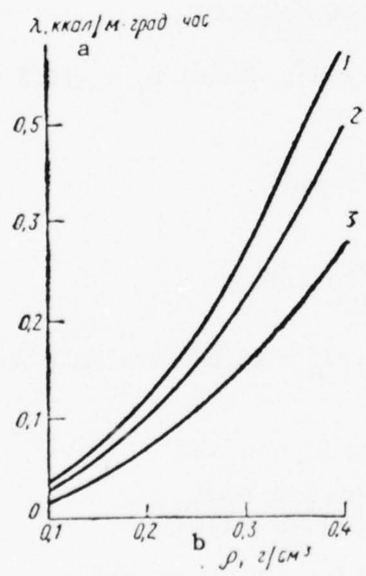

Figure 35. Graph of the dependence of the thermal conductivity of snow on its density according to the formulas of: 1) Kondrat'yev, $\lambda = 0.0085\,\rho^2$; 2) Abel's, $\lambda = 0.0068\,\rho^2$; 3 - Brakht, $\lambda = 0.0049\,\rho^2$. a - λ, kcal/(m)(degree)(hr); b - ρ, g/cc

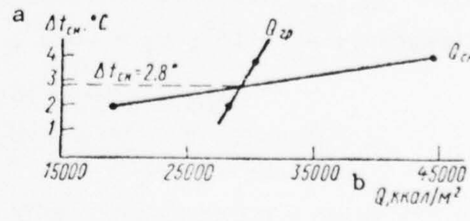

Figure 36. Graph for determination of Δt_{sn}.

a - Δt_{sn}, °C; b - Q, kcal/m^2

$$Q_{гр} = 20\,\frac{\sqrt{2}}{2}\left(1 - \frac{8,5}{14}\sqrt{\frac{8760\cdot 550\cdot 1}{3,14}}\right) +$$

$$+ \frac{(2\cdot 14\cdot 550\cdot 0,84 + 18\,600\cdot 1,25)\,18\,600\sqrt{\frac{1\cdot 8760}{3,14\cdot 550}}\cdot\frac{1}{2}\left(1 - \frac{8,5}{14}\right)}{2\cdot 14\cdot 550\cdot 0,84 + 18\,600\cdot 1,25 + \sqrt{\frac{8760\cdot 1}{3,14\cdot 500}}\,(2\cdot 14\cdot 550 + 18\,600)} +$$

$$+ \frac{1}{2}\,18\,600\cdot 1,25\left(1 + \frac{8,5}{14}\right) =$$

$$= 6938,4 + \frac{36076\cdot 18\,600\cdot 2,25\cdot 0.2}{36076 + 2,25\cdot 3400} + 18\,600 \approx 28220\ ккал/м^2.$$

Now we will find Q_{sn} at $\Delta t_{sn} = 2.0^{\circ}$. To do that it is necessary to know the coefficient of thermal conductivity of the snow. We will use the following empirical formula of Abel's which gives the dependence between the density and thermal conductivity of snow (Figure 35):

$\lambda_{sn} = -2.4 \times \rho^2$ (kcal/m x degree x hr) where ρ is the mean winter density of snow, g/cc. In our case $\lambda_{sn} = 2.4 \times (0.29)^2 = 0.2$ kcal/(m)(degree)(hr). On the basis of the right side of equation (5.3.5) we calculate

$$Q_{сн} = \frac{0,2\,(-19-(-21))}{0,1}\cdot 4750 = 19\,000\ ккал/м^2.$$

2. Similarly we calculate Q_{gr} and Q_{sn} at $t_{sn} = 4^{\circ}$. Then: $t_{\xi} = -10.5 + 4 = -6.5^{\circ}$; $A_o = 22 - 4 = 18^{\circ}$; $t_{0\text{-}wtr} = -16.3^{\circ}$:

$$A_{ср} = \frac{18 - 6,5}{\ln\frac{18 + 16,9}{6,5 + 16,9}} - 16,9 = 11,7^{\circ};$$

$$\xi_{2c} = \frac{2\,(18 - 6,5)\cdot 1239}{23,4\cdot 550 + 18\,600} = \frac{28\,497}{31\,470} = 0,9^{\circ};$$

$\xi_{nom} = 1.4$ m (at $C_{vol\text{-}t} \approx 550$ kcal/(m^3)(degree); $Q_{\phi} = 18{,}600$ kcal/m^2; $\lambda = 1$ kcal/(m)(hr)(degree); $A_o = 18^{\circ}$; $t_{\xi} = -6.5^{\circ}$.

$$Q_{гр} = 0,7\cdot 0,45\cdot 18\cdot 1239 +$$

$$+ \frac{(23,4\cdot 550\cdot 0,9 + 18\,600\cdot 1,4)\ 18\,600\cdot 2,25\cdot 0,225}{23,4\cdot 550\cdot 0,9 + 18\,600\cdot 1,4 + 2,25\,(23,4\cdot 550 + 18\,600)} +$$

$$+ \frac{1}{2}\,18\,600\cdot 1,4\cdot 1,55 = 7025 + 3270 + 20\,181 = 30\,476\ ккал/м^2;$$

$$Q_{сн} = \frac{0,2\,(21 - 16.3)}{0.1}\cdot 4750 = 44\,364\ ккал/м^2.$$

3. To find the true value of Δt_{sn}, as has already been said, a graph is constructed on the axis of ordinates of which the values of Δt_{sn} are plotted, and on the axis of abscissas, Q_{gr} and Q_{sn} (Table 21). Those data proved to be

quite sufficient for determination of the point of intersection and, as is evident from Figure 36, $Q_{gr} = Q_{sn}$ at $\Delta t_{sn} = 2.8^{o}$. Consequently, on an investigated section a snow cover with a height of 0.2 m increases the average annual temperature of the surface of the soil by 2.8^{o} above the average annual air temperature.

Table 21 Calculating data for determination of Δt_{sn}

$\Delta t_{сн}$, °C	$Q_{гр}$, ккал/м²	$Q_{сн}$, ккал/м²
2,0	28 220	19 000
4,0	30 476	44 364

a - t_{sn}, ^{o}C

b - Q_{gr}, kcal/m^2

c - Q_{sn}, kcal/m^2

Calculation of the Warming Influence of the Snow Cover with the Abbreviated Formula of V. A. Kudryavtsev (Example 11)

Under field conditions the abbreviated formula of V. A. Kudryavtsev (1954), derived on the basis of statistical processing of factual data, can be used for approximate calculations of the thermal influence of the snow. The formula has the form

$$\Delta t_{сн} = \Delta A_{сн} = \frac{A_в}{2}\left(1 - e^{-z\sqrt{\frac{\pi}{KT}}}\right), \qquad (5.3.10)$$

where ΔA_{sn} is the decrease in amplitude of the annual temperature fluctuations (physical value) under snow; ^{o}C; z is the height of the snow cover, m; K is the coefficient of thermal conductivity of the snow, m^2/hr; T is the period, equal to one year, hours; A_a is the meteorological amplitude of the annual air temperature fluctuations, ^{o}C.

For convenience of calculations with the indicated formula of V. A. Kudryavtsev (1954) a table of changes of the value of

$$\left(1 - e^{-z\sqrt{\frac{\pi}{KT}}}\right),$$

has been compiled, written in the form of $(1 - 1/f)$ as a function of different height and density of the snow. That method is widely used in the practice of various frost investigations as a quick method. For example, it is necessary to determine the warming influence of snow cover with a height of 0.5 m and a density of 0.19 g/cc, which it exerts on the temperature of the surface of the soil in a region with the following climatic conditions: $t_a = -10.6^{o}$ and $A_a = 21.7^{o}$.

In Table 22 we find that for snow cover with a height of 0.5 m and a density of 0.19 g/cc the coefficient $(1 = 1/f)$ is equal to 0.274. Consequently,

$$\Delta t_{сн} = \Delta A_{сн} = 21{,}7 \cdot 0{,}274 = 5{,}9^{\circ}.$$

Table 22 Value of $(1 - 1/f)$ as a function of the thickness, density and coefficient of thermal conductivity of the snow cover (according to V. A. Kudryavtsev, 1954)

Плотность снежного покрова ρ, г/см³	Коэфф. температуропроводности снега K, м²/час	Мощность (z) снежного покрова в м									
		0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
0,075	0,010	0,094	0,181	0,259	0,329	0,398	0,451	0,503	0,551	0,597	0,632
0,110	0,0015	0,081	0,155	0,224	0,288	0,345	0,400	0,447	0,491	0,532	0,572
0,150	0,0020	0,071	0,136	0,197	0,253	0,306	0,355	0,400	0,442	0,482	0,518
0,190	0,0025	0,064	0,123	0,178	0,230	0,279	0,324	0,367	0,407	0,445	0,480
0,225	0,0030	0,058	0,113	0,164	0,213	0,259	0,302	0,343	0,381	0,416	0,450
0,250	0,0035	0,054	0,105	0,153	0,198	0,242	0,282	0,321	0,357	0,392	0,425
0,300	0,0040	0,051	0,098	0,143	0,186	0,227	0,267	0,303	0,338	0,371	0,403
0,340	0,0045	0,048	0,093	0,136	0,178	0,216	0,254	0,289	0,323	0,356	0,386
0,380	0,0050	0,045	0,088	0,130	0,169	0,206	0,242	0,277	0,309	0,341	0,371
0,415	0,0055	0,043	0,081	0,124	0,161	0,197	0,232	0,265	0,297	0,327	0,355

Key: a - Density of the snow cover ρ, g/cc b - Coefficient of thermal conductivity of snow K, m^2/hr

From which we find that under the snow

$$t_0 = t_a + \Delta t_{сн} = -10,6 + 5,9 = -4,7°,$$
$$A_0 = A_a - \Delta A_{сн} = 21,5 - 5,9 = 15,6°.$$

For comparison we will calculate Δt_{sn} with the data of example 10, in which the warming influence of the snow cover was calculated with consideration of the heat cycles of the underlying rocks: $t_a = -10.5°$; $A_a = 44°$; $z_{sn} = 0.2$ m; $\rho = 0.29$ g/cc. With Table 22 we find that $(1 - 1/f) = 0.1$. Consequently, $\Delta t_{sn} = \Delta A_{sn} = 22 \times 0.1 = 2.2°$, and according to formula (5.3.5) $\Delta t_{sn} = 2.8°$.

The difference of $0.6°$ is explained by the influence of heat cycling in the layer of seasonal thawing, which is not considered in the abbreviated formula. Because of this it is advisable to use the complete formula, which makes it possible to take heat cycling in rocks into consideration.

Besides a purely quantitative estimate of the warming influence of the snow cover, equation (5.3.5) makes it possible to trace also general regularities in the formation of the temperature regime of rocks under a snow cover.

The main regularity is that the warming influence of the snow cover depends on the heat cycles of the soil for the given region. The greater the heat cycling of the soil, then, other conditions being equal, the greater the warming influence of the snow, and the reverse. Consequently, all the factors and conditions which determine the heat cycles of rocks or exert a given influence on them also determine the amount of the warming influence of the snow cover. Hence the warming influence of an identical snow cover (in thickness, density, thermophysical and other characteristics) on different sections within a single region will be different, depending on the composition and moisture content of the rocks. It will be smaller on dry grounds and greater on water-saturated grounds.

The annual heat cycles can, on account of phase transitions in rocks, vary by 1.5-2 times in a given region. Consequently, the warming influence of the snow cover can also vary by 1.5-2 times on account of that. It is interesting that under the conditions of a sharply continental climate the heat cycles in rocks are always larger than under the conditions of a maritime climate, and therefore the warming effect of the snow cover also must be greater in the former case.

The warming influence of the snow cover is different for the seasonal freezing and seasonal thawing of rocks.

All other conditions being equal, the heat cycles of rocks in the case of seasonal thawing will be larger than during seasonal freezing. This is explained by the fact that in frozen rocks underlying a seasonally thawed layer, during annual temperature fluctuations, besides heat cycles connected with heat capacity there will also be heat cycles expended on the phase transitions of water in frozen rocks. In the case of a seasonally frozen layer there will be no such heat cycles. By virtue of that, when there is a seasonally thawed layer the warming influence of snow, all other conditions being equal, will be greater than when there is a seasonally frozen one.

As is known (Kudryavtsev, 1965), the heat cycles of rocks are maximal in the southern boundary of the propagation of permafrost, that is, at average annual rock temperatures of 0°. With change of the latter in the direction of their increase and decrease (south and north of the southern boundary) the heat cycles decrease all the more, the more the average annual rock temperatures differ from 0°. In accordance with that, the warming influence of the snow cover will also be maximal on the southern boundary of the region of propagation of permafrozen rocks and will decrease toward the south and north of that boundary. Manifested in that is latitudinal zonation of the warming influence of the snow cover, connected with the formation of the temperature regime of the rocks.

A similar regularity is also noted in relation to height zonation. The maximal warming influence of the snow cover diminishes where the average annual rock temperatures are 0°. At higher and lower levels that influence will decrease all the more, the more the average annual rock temperatures differ from 0°. It should be mentioned that the indicated manifestation of latitudinal and height zonation in the phenomenon under consideration is valid only for moist rocks in which, during freezing, of considerable importance in heat cycles is the portion of them completed on account of phase transitions of water in freezing rocks.

In dry rocks (for example, hard rocks) that regularity will not occur. In that case the number of heat cycles of the rocks and, consequently, the warming influence of the snow will be determined by the annual amplitude of temperatures on the surface of the soil and will not depend on the average annual temperatures. The amplitudes do not have latitudinal zonation and depend in each concrete case on the distance of the studied region from the sea.

The amplitude can decrease with height as a result of temperature inversion, change of the atmospheric humidity and the arrival of solar radiation. The influence of the snow cover, evidently, also will vary as a function of that.

Besides general regularities of latitudinal and height zonations it is necessary to also note a number of particular regularities, such as, for example, the difference in the warming influence of the snow cover on slopes with different exposure and steepness, for sections with different plant cover, as a function of the character of the manifestation of winter temperature inversion, etc. In all those cases the warming influence of the snow cover will be greater the greater the heat cycles of the rocks, and the reverse.

As a result of everything explained above it follows that the warming influence of the snow cover not only is determined by the character of that cover but also to no small degree depends on the entire complex of the natural situation, starting from the composition of the freezing rocks and their moisture content and ending with the plant cover and the character of the terrain.

2. The Cooling Influence of the Snow Cover

For a complete characterization of the influence of the snow cover on the formation of the temperature regime of rocks it is necessary to dwell once more on the cooling influence of that cover when the disappearance of snow is completed after the onset of positive temperatures on the surface of the soil. During the time the snow is lying after the onset of positive air temperatures the temperature on the surface of the rocks will be 0° and will become positive only after the snow has disappeared.

When the snow is retained to the second half of the summer or to the autumn the maximal temperature on the surface of the soil will be correspondingly lower and the amplitude of temperatures on the surface will be reduced in comparison with ordinary conditions. In that case, when the snow is retained, but disappears before the onset of maximal air temperatures (mid-July), the maximal values of temperatures on the surface of the ground will not differ from those on sections where the snow disappeared without delay. Delay of the disappearance of the snow in that case does not affect the amplitude of temperatures on the surface of the ground (Figure 37). Consequently, the temperature fluctuations in rocks (at positive values) will be completed just as if that period on the surface of the rocks were equal to

$$\tau = T\left(\frac{T_+ - T_-}{T_+}\right), \qquad (5.3.11)$$

where τ is the curtailed period of temperature fluctuations, connected with delay of disappearance of the snow, T_+ is the length of existence of positive air temperatures, T_- is the length of delay of disappearance of the snow from the surface of the soil and T is a period equal to one year.

In that case the depth of the seasonal thawing will be determined with formula (4.1.4) but with substitution of the value of τ from equation (5.3.11). It is obvious that ξ_1 will be somewhat smaller than ξ at T equal to one year. The ratio of those depths will be directly proportional to the square root of the periods

$$\frac{\xi_1}{\xi} = \frac{\sqrt{\tau}}{\sqrt{T}}.$$

That dependence follows from the main formula (4.1.4), but the obtained decreased depth of thawing (ξ_1) must be related to the total period of fluctuations (T) in order to obtain the average annual temperature of the rocks with consideration of the cooling effect of the snow cover. That temperature can be obtained from the same equation (4.1.4), where it will be the unknown value, and ξ_1 must be substituted in place of ξ. The value of ξ_{2c} must be obtained with consideration of the changed average annual temperature. The cooling influence of the snow cover can be thus obtained in relation to both the change of the average annual temperatures of the rocks and the change of the depths of their thawing.

A similar situation is noted also for the seasonal freezing of rocks, but in that case the ratio $\sqrt{\tau} / \sqrt{T}$ will be a considerably smaller value and therefore the change of the depths of the seasonal freezing and the average annual temperature of the rocks will also be considerably smaller than during seasonal thawing (all other conditions being equal).

3. Analysis of the Influence of the Snow Cover in Different Freezing Temperature Zones

Thus, in determining the influence of the snow cover on the seasonal freezing and seasonal thawing it is necessary to take into account all aspects of that phenomenon: change of the albedo of the surface, the warming influence on account of the heat insulating effect in the winter period and the cooling influence on account of delay of the disappearance of snow in the spring and summer. In the middle latitudes the summary influence of the snow cover usually remains warming.

Qualitatively the influence of the snow cover on the depth of the seasonal freezing and seasonal thawing can be determined by means of the following schematic diagram (Figure 38), consisting of a diagram of the dependence of the depths of the seasonal freezing and thawing on the average annual temperature of the rocks (t_ξ) and the amplitude of temperatures (A_o) on the surface.

The warming influence of the snow cover, which changes the depths of the seasonal freezing and thawing, is determined with that diagram in the following manner.

Examining first the seasonal freezing (the left side of Figure 38), we will assume that in the presence of a snow cover on the surface the average annual rock temperature was t_1 and the temperature amplitude on the surface was A_2, and then the depth of the seasonal freezing in that case is determined by the position of the point m_1. Removal of the snow cover leads to two consequences:

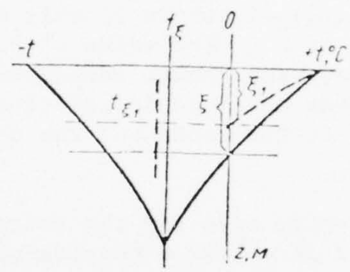

Fig. 37 Diagram of yearly temperature fluctuations in deposits under normal conditions (solid line) and under conditions of delayed snow-covering removal (dotted line)

firstly, the average annual temperature is reduced from t_1 to t_2 and, secondly, the amplitude on the surface increases from A_2 to A_3.

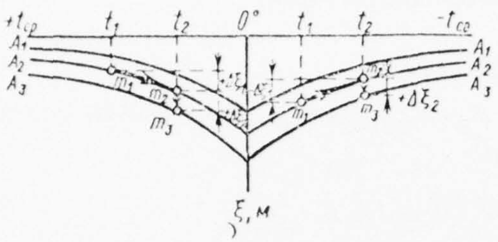

Figure 38. Schematic diagram of the influence of the snow cover on the depth of seasonal freezing and thawing of rocks (ξ): t_1 -- the average annual rock temperature in the presence of snow cover; t_2 -- the same during the removal of the snow cover; $A_1 < A_2 < A_3$ -- the amplitudes of the average monthly temperatures on the surface of the rock.

As is evident on Figure 38, reduction of the average annual temperature to the value of t_2 increases the depth of seasonal freezing to a value marked by the point m_2, that is, give a positive increment of that depth ($+\Delta\xi_1$). If we examine the influence of increase of the amplitude of temperatures during removal of the snow cover from the value A_2 to the value A_3, it is evident that it increases the depth of freezing to the value marked by the point m_3, that is, also leads to a positive increment ($+\Delta\xi_2$) of that depth.

Thus when the snow cover in the region of seasonal freezing is removed the depth of the latter increases both on account of lowering of the average annual temperature and on account of increase of the amplitude. Those influences are added and as a result a sharp increase of the depth of seasonal freezing is obtained when the snow cover is removed ($\Delta\xi_1 + \Delta\xi_2$). This can be shown on the example of calculation of the depth of the seasonal freezing.

Calculation of the Influence of the Snow Cover on the Depth of Seasonal Freezing of the Ground (Example 12)

Calculate how ξ_f varies at a construction site where the snow cover is regularly scraped off in the winter as compared with ξ_f under natural conditions, if the following data were obtained during a frost survey. The ground conditions are characterized by the propagation of sandy loam rocks with γ_{sk} = = 1300 kg/m^3; w = 18%, $\lambda_f = \lambda_t$ = 1.3 kcal/(m)(degree)(hr), and there is practically no unfrozen water at a temperature of -0.5°. The climatic conditions are characterized by the following data: $t_a = 3°$; $A_a = 19°$; z_{sn} = 0.5 m; ρ_{sn} = = 0.19 g/cc.

Solution. 1. We determine the temperature regime on the surface of the soil under snow under natural conditions (there is no plant cover and we neglect the radiation correction). To calculate the warming influence of the snow we use abbreviated formula (5.3.10) and Table 21:

$$\Delta t_{сн} = \Delta A_{сн} = 19 \cdot 0,274 = 5,2°;$$

$$t_0 = t_в + \Delta t_{сн} = 3 + 5,2 = 8,2°;$$

$$A_0 = A_в - \Delta A_{сн} = 19 - 5,2 = 13,8°.$$

2. We determine C_{vol-f} and Q_ϕ of the ground with (4.1.6) and (4.1.8):

$$C_{об.м} = 0,19 \cdot 1300 + 0,5 \frac{18 \cdot 1300}{100} = 364 \text{ ккал/м}^3 \cdot \text{град};$$

$$Q_\phi = 80 \cdot \frac{18 \cdot 1300}{100} = 18\,720 \text{ ккал/м}^3.$$

3. On the basis of the obtained data t_o, A_o, C_{vol-f}, Q_ϕ and λ with the nomogram (Figure 17) we find the value of ξ_{nom}:

$$\xi_{ном} = 0,87 \text{ м}; \; \xi_{\lambda=1,3} = 0,87 \cdot \sqrt{1,3} \simeq 1,0 \text{ м}.$$

4. When the snow cover is removed the temperature regime on the surface of the soil changes in comparison with the natural conditions. If we neglect the radiation correction, then t_o will be equal to t_a, that is, 3^o, and A_o will be equal to A_a, that is, 19^o. In the problem the calculation of the temperature regime of the surface of the soil and the ground is simplified, since the main goal of the problem is to show the influence of the snow cover. Therefore the depth of the seasonal freezing of the ground in the given case is found at the following initial parameters: $t_o = 3^o$; $A_o = 19^o$; $C_{vol-f} = 364$ kcal/(m^3)(degree); $Q_\phi = 18{,}720$ kcal/m^3; $\lambda = 1.3$ kcal/(m)(hr)(degree). We obtain: $\xi_{nom} = 1.95$ m; $\xi_{\lambda=1.3} = 1.95 \times 1.14 = 2.2$ m.

Thus the removal of a snow cover with a height of 0.5 m and a density of 0.19 g/cc under the indicated conditions leads to increase of the depth of seasonal freezing of the ground by more than 100%.

The situation is different in the region of seasonal thawing (the right side of Figure 38).

Let in the presence of a snow cover the annual average rock temperature be t_1, the temperature amplitude A_2 and the depth of seasonal thawing be determined respectively by the point m_1. Then when the snow cover is removed tha annual average temperature will be reduced to the value t_2, and this will lead to a reduction of the depth of seasonal thawing $(-\Delta\xi_1)$. At the same time the amplitude of temperature changes on the surface increases from the value of A_2 to the value of A_3, and this increases the depth of seasonal thawing by the value $(+\Delta\xi_2)$.

Thus in the case of seasonal thawing the influences of changes of the average annual rock temperature and the temperature amplitude on the surface at the depth ξ when the snow cover is removed compensate one another and as a result very slight changes of the depth of seasonal thawing $(\Delta\xi_2 - \Delta\xi_1)$ are obtained, which also is well illustrated by the example.

Calculation of the Influence of the Snow Cover on the Depth of Seasonal Thawing of the Ground (Example 13)

Calculate how ξ_t varies on an area where the snow cover is regularly scraped off in winter as compared with natural conditions. The grounds on the area are the same as in example 12 and are characterized by the following thermophysical properties: $C_{vol-t} = 480$ kcal/(m^3)(degree); $Q_\phi = 18{,}720$ kcal/m^3; $\lambda = 1.3$ kcal/(m)(degree)(hr). The climatic conditions were determined to be: $t_a = -8^\circ$; $A_a = 19^\circ$; $z_{sn} = 0.5$ m; $\rho_{sn} = 0.19$ g/cc. In the given case we still neglect the radiation correction.

Solution. 1. Under natural conditions in the presence of snow with a height of 0.5 m the temperature regime on the surface of the soil is determined as in example 12: $\Delta t_{sn} = 5.2^\circ$; $t_o = -8 + 5.2 = -2.8^\circ$; $A_o = 19 - 5.2 = 13.8^\circ$. The depth of the seasonal thawing of the ground is $\xi_{nom} = 1.64$ m and $\xi_{\lambda=1.3} = 1.9$ m.

2. In the case of removal of the snow cover the temperature conditions on the surface of the soil are similar to the temperature regime of the air. We assume that $t_o = -8^\circ$ and $A_o = 19^\circ$. The depth of the seasonal thawing of the ground in that case is $\xi^o_{nom} = 1.33$ m; $\xi_{\lambda=1.3} = 1.5$ m.

Thus the removal of snow in the region of seasonal thawing leads to an insignificant reduction of the depth ξ even as a result of sharp lowering of the average annual temperature of the surface of the soil. In the presented example that reduction amounts to $\sim 20\%$ of the depth forming under natural conditions.

4. The Influence of the Plant Cover on the Temperature Regime and the Depth of the Seasonal Freezing and Thawing of Rocks

The influence of the plant cover on the depth of the seasonal freezing and thawing and on the temperature regime in rocks represents only one aspect of the complex, comprehensive and important problem of the interaction of the vegetation with seasonally and perennially frozen rocks.

Even in a brief examination of the indicated question it is necessary to bear in mind the following dependences and correlations.

1. The plant cover influences the development of frozen rocks through the changes of heat exchange between the soil and atmosphere caused by it.

The plant cover in the summer period partially retains direct and scattered solar radiation, which leads to some cooling of rocks. In the winter period, on the contrary, the plant cover is a heat insulator which prevents the release of heat from soils. In that case the plant cover exerts a warming effect on rocks. These two effects of the plant cover on the input and output parts of the radiant energy balance can be very different. The relative importance of of their influence -- reduction of the irradiation of the soil in summer and radiation of long-wave energy in winter -- varies in the transition from north to south.

Actually, in the north the winter is longer and the summer shorter than in the south, and so the heat insulating, as it were the warming, effect of plant cover on the temperature of soils can predominate. Such a comparison is possible, of course, only when other conditions are equal.

For different plant covers (for example, moss cover and a pine forest) the indicated summer and winter effects on heat exchange will greatly differ. Differences of vegetation also cause a difference in the albedo of surfaces, which in turn creates differences in the reflection and absorption of radiant energy and in the temperature regime of soils and rocks.

2. Plant covers affect the temperature regime of soils also through their effect on heat exchange between the atmosphere and the soil. Those effects of vegatation on heat exchange occur in different ways:

a) firstly, different types of vegetation volatilize differently the moisture they take from the soil, thus influencing the heat balance of the air and soil through their moisture content (the process of transpiration);

b) different types of vegetation retain and preserve differently the snow cover against drifting and melting, which also exerts a great influence on the course of the seasonal freezing and thawing. The influence of the plant cover proves by this to be complexly connected with the influence of the snow cover;

c) finally, different types of vegetation retain moisture in the soil differently, thus influencing the thermophysical characteristics of the soil and through them the course of heat exchange between the soil and atmosphere.

3. Under natural conditions not only the effect of plant covers on the development of frozen rock masses occurs, but also the reverse influence of the latter on plant covers (Tyrtikov, 1963).

Thus frozen rock masses and plant covers, as a rule, develop in parallel, reacting to changes of each other. This circumstance, among others, is a main one for judging the correspondence between frost and geobotanical conditions and is used in frost surveying.

The influence of vegetation on the formation and development of frozen rock masses is manifested, in particular, in changes of the depths of freezing and thawing during the replacement of plant communities.

The importance of the replacement of some types of vegetation by others in the formation of permafrozen rock masses increases during movement from north to south and is especially great in regions adjacent to the southern boundary of the permafrost region. The degree of concentration of plant communities in types of seasonal freezing and thawing increases in the same direction, that is, the indicative importance of the plant cover increases.

The data of A. P. Tyrtikov presented in Table 23 confirm the position on the cooling influence of the plant cover in regions near the southern boundary of

Table 23 Change of the depth of seasonal freezing and temperature of the soils and subsoils in the process of replacement of plant communities in the flood plain of the Yarudya River (according to A. P. Tyrtikov, 1963)

	A Последовательные стадии развития растительности	B Состав почв и подпочв	C Мощность, см		D Глубина протаивания, м	E Температура (°С) на глубине, м										F Мощность многолетнемерзлой толщи, м
			a мха	b торфа		1	2	3	4	5	6	7	8	9	10	
1	Березовые леса с травяным покровом	a Супеси, пески; в верхней части прослои суглинка	d нет	d нет	e сезонное промерзание	f положительная										d нет
2	Березово-лиственничный лес с примесью ели на зеленомошно-сфагновом покрове	b До 3,5 м суглинок, ниже песок	10	10	1,0	--	—	—0,1	—0,1	—0,2	—0,1	—0,1	—0,1	—	—	g больше 8
3	Редкостойный угнетенный сфагновый лиственнично-березовый лес	c До 0,4 м суглинок с супесью, ниже песок	10	30	0,4	—0,2	—0,2	—0,2	—0,2	—0,2	—0,2	—0,3	—0,3	—0,2	—0,2	h больше 10

Key: A - Successive stages of development of vegetation B - Composition of soils and subsoils C - Thickness, cm a - mosses b - peat D - Depth of thawing, m E - Temperature (^{o}C) at depth, m F - Thickness of permafrozen rock mass, m 1 - Birch forests with grassy cover a - Sandy loams, sands; in the upper part, layers of loam 2 - Birch-deciduous forest with admixture of spruce on green moss-sphagnum cover b - Up to 3.5 m of loam, sand below 3 - Thin depressed sphagnum deciduous-birch forest c - Up to 0.4 m of loam with sandy loam, sands below d - none e - seasonal freezing f - positive g - over 8 h - over 10 permafrozen rocks stated at the beginning of the section. However, it should be recalled that the influence of the plant cover on the soil temperature is varied and complex and requires further study. Under different conditions, especially in combination with the effect of snow, the plant cover can have a warming or a cooling influence. Thus, for example, it has been established that in the Far East at a snow thickness of up to 20 cm annihilation of the forest leads to cooling of the soil, and at greater thicknesses of the snow, to its warming.

It can be said that in a continental climate with a long summer and a severe winter with a snow cover of 10-20 cm the plant cover identically protects the soil against both summer heating and winter cooling. When the snow cover is thicker the warming effect of the plant cover in winter will be greater than the cooling effect in summer.

In northern regions the presence of a moss-grass cover and brush causes increase of the average annual temperature of the soil by 1 - 2^{o}. The grass-moss cover, brush and forest reduce the annual amplitudes by 15-25% separately and 30-50% taken together.

Moist mossy and peat covers, because of their great moisture capacity, sharply increase their thermal conductivity in the frozen state. This causes a reduction of the average annual soil temperature by 1 - 3%, except in regions with a thick snow cover, where the reverse effect is observed. The reduction of annual amplitudes of the soil temperature under moss and grass covers can reach 50-60%, and when there is a thick moss cover and peat -- even 80-90%. The quantitative influence of vegetation on the temperature regime of the soil surface can be determined by means of the calculating methods proposed below, depending on the aspect of tha influence. By one method an estimate is obtained of the influence of vegetation on the radiation heat balance and through it on the air temperature directly on the plant cover on the soil. That method has already been examined by us as a method of determining the radiation correction, representing the difference between the temperature of the surface on the level of 2.0 (1.5) m (see the example on page 31). The influence of vegetation as a heat insulating layer is determined by other methods.

Calculation of the Influence of the Plant Cover on the Formation of the Temperature of the Surface of Rocks (Through the Radiation Heat Balance of the Surface)((Example 14)

In the process of a frost survey, data of microclimatic observations were obtained at several specially equipped weather points. The first point is a station on the second terrace above the flood plain, composed of deluvial-solifluctional, detrital, heavy, silty, sandy loams (γ_{sk} = 1500 kg/m^3; w = 25-30%; λ_t = 1.3-1.5 kcal/(m)(degree)(hr). The point was set up on a felling area where the vegetation consisted of brush (wild rosemary and mountain cranberry), infrequent herbage and green-moss cover with a height of 3-5 cm. The second point was situated on a swampy surface of the first terrace above a flood plain, in a dense deciduous forest where the surface of the soil is covered by a solid moss cover with a height of about 10 cm. The layer of seasonal thawing at that point was well decomposed mineralized peat (γ_{sk} = 800 kg/m^3; w varied from top to bottom in the layer from 50 to 80%; λ_t = 0.25-0.5 kcal/(m)(hr)(degree). The third site was in a swamp of sedge and moss within terrace 1 above the flood plain, where to a depth of 0.35 m is peat (γ_{sk} = 700 kg/m^3; w = 43%; λ = 1.0-1.2 kcal/(m)(hr)(degree).

The investigations at those places included radiation, gradient (at heights of 0.1, 0.5 and 1.5 m) and geothermal (to a depth of 1.5-2.0 m) observations, and the composition and properties of rocks of the layer of seasonal thawing and the upper part of the underlying frozen rock mass also were studied. On the basis of the results of observations the following data were obtained: R, LE, B and $\Delta t_R = t_{plant} - t_{air}$, where $t_{plant} - t_{air}$ is the difference between the air temperature on plant cover near the soil and at a height of 1.5 m. The heat expenditures on turbulent heat exchange and the coefficient of heat transfer from the surface α were calculated with those data. The starting data and values of α are presented in Table 24.

Table 24. Components of the radiation heat balance (sum for 10 days) and calculation of the coefficient α according to data obtained in actinometric and gradient observations on 20-31 July 1964

A Составляющие радиационно-теплового баланса	B № площадок 1	2	3
R, ккал/см² 1	2,65	2.4	2,71
LE, ккал/см²	1,65	2.15	1,86
B, ккал/см²	0,52	0.14	0.4
$P = R - LE - B$, ккал/см²	0,48	0,11	0,45
$\Delta t_R = t_{\text{раст}} - t_{\text{в}}$, °C 2	2,3	1,5	1,7
$\alpha = \frac{R - LE - B}{\Delta t_R}$, ккал/м²·град·час 3	9	3	11

Key: A - Components of radiation heat balance B - For points
1 - kcal/cm^2 2 - $\Delta t_R = t_{plant} - t_{air}$, °C
3 - kcal/(m^2)(degree)(hr)

Assuming that the correlation between the radiation balance at station point 1, where the regime observations were made, and points 2 and 3, obtained in the last 10 days of July, was preserved during the entire summer, the summary values of the components of the radiation heat balance during the summer Ω_R, Ω_{LE} and Ω_B were calculated. On the basis of those data the difference of the sums of the temperature of the surface of the plant cover ($\Omega_{t_{plant}}$) and air ($\Omega_{t_{air}}$) during the same summer period was determined with the formula

$$\Omega_{t_{\text{раст}}} = \Omega_{t_{\text{в}}} + \frac{1}{\alpha_{\text{раст}}}(\Omega_R - \Omega_{LE} - \Omega_B).$$

If the difference of the sums of the temperatures t_{plant} and t_{air} during the year is divided by 12 (the number of months in a year), it is possible to obtain the average annual value of that difference. The calculating data for a year are presented in Table 25.

Table 25. Components of the radiation heat balance of the soil surface (total for a year) and determination of the difference of the average annual temperature of the surface of the plant cover and air as a function of the character of the vegetation

A Составляющие радиационно-теплового баланса и Δt_R	B № площадок		
	1	2	3
R, ккал/см² 1	18,9	17,4	19,7
LE, ккал/см²	12,5	15,4	13,2
B, ккал/см²	2,8	1,5	2,6
P, ккал/см²	3,6	0,5	3,9
α, ккал/м²·час·град 2	9	3	11
$\Omega_{t_0} - \Omega_{t_в}$, °C	5,5	2,3	5,0
$\Delta t_R = t_{раст} - t_в$, °C 3	0,5	0,2	0,4

Key: A - Components of the radiation heat balance and Δt_R B- For the points
1 - kcal/cm^2 2 - kcal/(m^2)(hr)(degree) 3 - $t_{plant} - t_{air}$, °C

It follows from Table 25 that the average annual air temperature on the surface of vegetable cover near the soil (moss, grassy cover, forest underbrush, etc) is 0.2-0.5° higher than the average annual air temperature at a height of 1.5 m.

The presented example shows that in estimating the influence of the plant cover on the temperature of the surface it must be taken into consideration that the correction for the factors of solar radiation in the summer period and of different microregions is differentiated as a function of the character of the cover. On swampy sections with a sedge-moss cover that correction is 0.1-0.2°, and it reaches its greatest values (0.5°) on felled areas and dry sections with a thinned plant cover.

The following approximate dependences, derived by E. D. Yepshov (1971), can be proposed for calculation of the thermal influence of the plant cover (as a layer of heat insulation) on the temperature of the soil surface:

$$\Delta A_{раст} = \frac{\Delta A_1 \tau_1 + \Delta A_2 \tau_2}{T}, \qquad (5.4.1)$$

$$\Delta t_{раст} = \frac{\Delta A_1 \tau_1 - \Delta A_2 \tau_2}{T} \cdot \frac{2}{\pi}, \qquad (5.4.2)$$

where ΔA_1 and ΔA_2 are the difference in the average daily air temperatures on the surface of the plant cover and under it respectively during the cold and warm times of year, °C, and τ_1 and τ_2 are the length of existence of the negative and positive air temperatures respectively, hr; T is a period equal to a year, in hours.

$$\Delta A_1 = A_1 \left(1 - e^{-z \sqrt{\frac{\pi}{K_{м} \cdot 2\tau_1}}}\right), \quad A_1 = A_{раст} - t_{раст}; \qquad (5.4.3)$$

$$\Delta A_2 = A_2 \left(1 - e^{-z \sqrt{\frac{\pi}{K_{т} \cdot 2\tau_1}}}\right), \quad A_2 = A_{в} + t_{в}, \qquad (5.4.4)$$

where z is the height of the plant cover, m; K_f and K_t are the coefficients of thermal conductivity of the plant cover in the frozen and thawed states respectively; A_{plant} and t_{plant} are the annual amplitude of air temperature and the annual average air temperature on the surface of plant cover under snow respectively, that is, with consideration of Δt_{sn}. In calculating A_1 and A_2 the values of t_{plant} and t_{air} are taken with consideration of the sign.

Calculation of the Influence of the Plant Cover on the Temperature Regime of Rocks as a Layer of Heat Insulation (Example 15)

Calculate the thermal influence of moss cover with a height of 0.1 m on the temperature regime of the soil surface on a section of a lake-alluvial plain if $t_{air} = -15.4^{o}$, $A_{air} = 26.6^{o}$; $z_{sn} = 0.5$ m; $\rho_{sn} = 0.19$ g/cc. In addition, during field survey work the following data were obtained which characterize the reduction of the daily amplitude on account of the moss cover in the summer (Table 26).

The length of the period with positive average daily temperatures is 106 days, and with negative is 259 days.

Table 26. Initial data on reduction of the daily amplitudes of temperatures on account of moss cover

A Место закладки термометров	B Термометр макс., мин.	C Показания термометров по срокам, °C			D Среднее из трех значений амплитуды, °C
		23/VII 10—50	24/VII 11—05	25/VII 11—10	
1 На поверхности мха	a макс.	22,5	18,1	18,9	c $A_{раст} = 18,3$
	b мин.	—0,8	2,4	2,9	
2 Под мхом на поверхности почвы	a макс.	6,3	5,8	6,0	$A_0' = 0,5$
3 Высота мха 0,1 м	b мин.	5,7	5,4	5,5	

Key: A - Placement of thermometers B - Thermometer, max & min C - Thermometer readings for periods, ^{o}C D - Mean of 3 values of amplitudes, °C
a - max b - min c - A_{plant}
1 - On the surface of the moss
2 - Under the moss on the soil surface
3-- Moss height of 0.1 m

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FUNDAMENTALS OF FROST FORECASTING IN GEOLOGICAL ENGINEERING INV--ETC(U)
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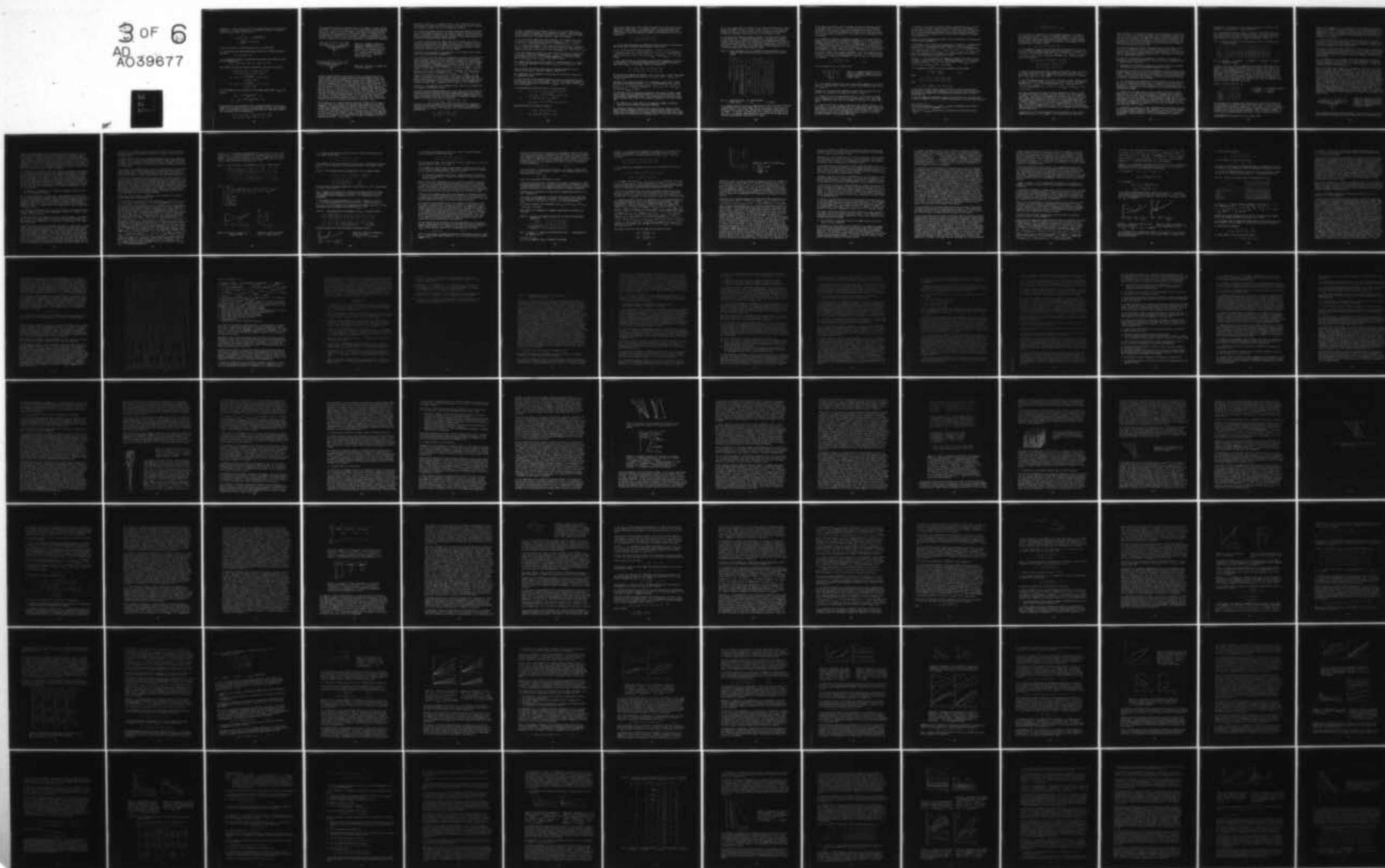

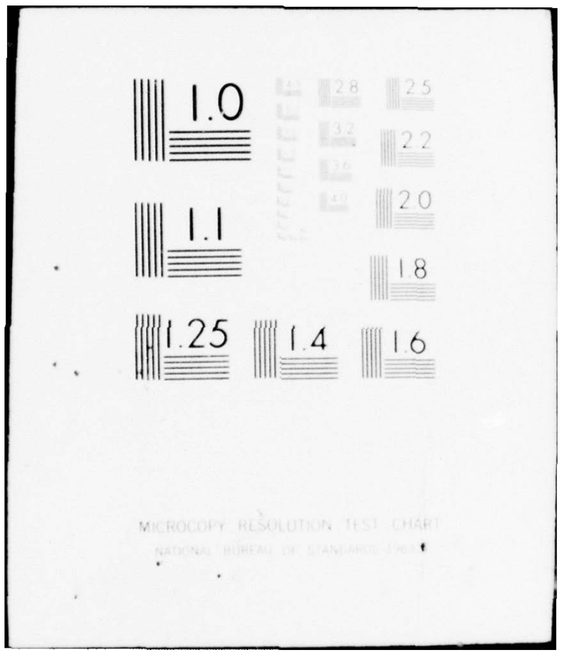
1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6
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Solution. 1. On the basis of data of the temperature point we calculate the coefficient of thermal conductivity of thawed moss, using for that purpose equation (4.4.1), which we solve for K:

$$K_т = \frac{\pi \cdot z^2_{раст}}{T' (\ln A_{раст} - \ln A'_0)^2} = \frac{3{,}14 \cdot 0{,}01}{24 (\ln 9{,}15 - \ln 0{,}25)^2} =$$

$$= \frac{0{,}0314}{24 \cdot (3{,}597)^2} \approx 0{,}0001 \ м^2/час;$$

On the basis of data of similar calculations K_f = 0.005 m^2/hr.

2. We calculated with abbreviated formula (5.3.10) the thermal influence of the snow:

$$\Delta t_{сн} = 26{,}6 \cdot 0{,}271 = 7{,}2°.$$

3. The temperature regime on the surface of the moss cover (under the snow) will be determined as:

$$t_{раст} = -15{,}4 + 7{,}2 = -8{,}2°; \quad A_{раст} = 26{,}6 - 7{,}2 = 19{,}4°.$$

With formulas (5.4.3) and (5.4.4) we calculate A_1 and A_2 and ΔA_1 and ΔA_2:

$$A_1 = 19{,}4 + 8{,}2 = 27{,}6°; \quad A_2 = 26{,}6 - 15{,}4 = 11{,}2°;$$

$$\tau_1 = 259 = 6216 \ час; \quad \tau_2 = 106 \cdot 24 = 2544 \ час;$$

$$\Delta A_1 = 27{,}6 \left(1 - e^{-0{,}1 \sqrt{\frac{3{,}14}{0{,}0005 \cdot 2 \cdot 6216}}}\right) = 27{,}6 (1 - e^{-0{,}071}) =$$

$$= 27{,}6 \cdot 0{,}07 = 1{,}93°;$$

$$\Delta A_2 = 11{,}2 \left(1 - e^{-0{,}1 \sqrt{\frac{3{,}14}{0{,}0001 \cdot 2 \cdot 2544}}}\right) = 11{,}2 (1 - e^{-0{,}248}) =$$

$$= 11{,}2 \cdot 0{,}22 = 2{,}46°.$$

4. With formulas (5.4.1) and (5.4.2) we find the sought values ΔA_{plant} and Δt_{plant}:

$$\Delta A_{раст} = \frac{1{,}93 \cdot 6216 + 2{,}46 \cdot 2544}{8760} = 2{,}1°;$$

$$\Delta t_{раст} = \frac{2}{3{,}14} \cdot \frac{1{,}93 \cdot 6216 - 2{,}46 \cdot 2544}{8760} = 0{,}4°.$$

Thus the plant cover, represented on the investigated section by moss with a height of 0.1 m, reduces A_o by 2.1° and increases t_o by 0.4°. With consideration of the influence of the snow and plant covers the temperature regime on the soil surface will be determined as:

$$t_0 = t_в + \Delta t_{сн} + \Delta t_{раст} = -15{,}4 + 7{,}2 + 0{,}4 = -7{,}8°,$$

$$A_0 = A_в - \Delta t_{сн} - \Delta A_{раст} = 26{,}6 - 7{,}2 - 2{,}1 = 17{,}3°.$$

The qualitative influence of the plant cover on the depth of seasonal freezing and thawing of rocks can be traced on the following schematic diagram (Figures 39 and 40). Shown on it are the changes of the depth of seasonal freezing and thawing as a function of changes of the average annual rock temperature and the amplitude of temperatures on its surface. Therefore to determine the influence of the plant cover on the depth ξ it should be determined how it influences the change of the average annual temperatures and their annual amplitudes.

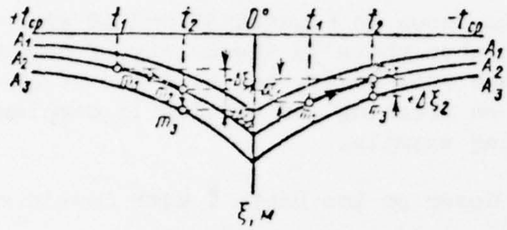

Figure 39. Schematic diagram for qualitative determination of the influence of the plant cover on the depth of seasonal freezing and thawing of rocks in northern regions: t_1 -- average annual temperature of soil with cover; t_2 -- the same without plant cover.

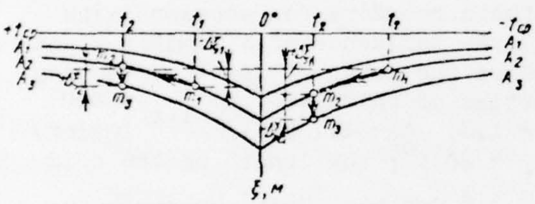

Figure 40. The same as on Figure 39, in southern regions.

Since the cooling influence of the plant cover has a stronger influence in southern regions and the warming influence in northern regions, those influences should be examined separately for southern and northern regions. The removal of the plant cover in northern regions leads to reduction of the average annual temperature of the soil and increase of the annual amplitude. On sections on which unfrozen rocks propagate (left side of Figure 39) it is shown that annihilation of the plant cover reduces the average annual temperature from the value t_1 to the value t_2, as a result of which the depth ξ_f increases from the point m_1 to the point m_2 in the positive direction $\Delta\xi_1$, and the annual amplitude increases from the value A_2 to the value A_3, which in turn increases the depth of seasonal freezing from the point m_2 to the point m_3, that is, also leads to a positive increase of $\Delta\xi_2$. As a result of such addition of the influence in northern regions the removal of the plant cover considerably increases the depth of seasonal freezing by the value $\Delta\xi_1 + \Delta\xi_2$.

On the contrary, as is evident from the right side of Figure 39, the removal of the plant cover on sections of seasonal thawing changes its depth inconsiderably. This occurs because the indicated operation, firstly, lowers the average annual temperature from the value t_1 to the value t_2 and as a result of that reduces the depth of thawing from the point m_1 to the point m_2, that, is, leads to a negative increment $-\Delta\xi_1$. Secondly, increase of the amplitude from the value A_2 to the value A_3 increases the depth of thawing from point m_2 to point m_3, creating $\Delta\xi_2$. Thus the first and second influences as it were compensate each other $(\Delta\xi_2 - \Delta\xi_1)$.

Presented on Figure 40 is a diagram similar to that of Figure 39 which shows the influence of removal of the plant cover on the depth of seasonal freezing and thawing in southern regions of the permafrost territory.

It is evident from Figure 40 that in southern regions the removal of the plant cover has the opposite influence on the depth of seasonal freezing (the left side of the diagram) in comparison with northern regions. Here when the plant cover is removed the depths of seasonal freezing of rocks change slightly and the depths of seasonal thawing increase greatly.

It should be recalled that the regularities shown on Figures 39 and 40 are valid when other conditions are equal, and when there is inequality of any of the natural factors at all, for example, the snow cover, can also not be observed. The influence of the plant cover on freezing and thawing is complex and varied, as can be shown on the following example.

Calculation of the Influence of the Plant Cover on the Depth ξ With Consideration of Latitudinal Zonation (Example 16)

In the northern part of the region of permafrost (on sections with long-stable types of seasonal thawing) and near its southern boundary (on sections with semi-transitional types of seasonal thawing) the influence of a similar plant cover is manifested differently. In both cases the layer of seasonal thawing consists of loams, the thermophysical properties of which are: C_{vol-t} = 620 kcal/(m^3)(degree); Q_ϕ = 25,000 kcal/m^3; λ_t = 1.3; λ_f= 1.7 kcal/(m)(hr)(degree). In the north: t_{air} = -15.4°; A_{air} = 26.6°; the length of the cold period is 8.4 and that of the warm period is 3.5 months. Near the southern boundary of the region of permafrozen rocks t_{air} = -6.6°; A_{air} = 21°; the length of the warm period is 5 and that of the cold is 7 months. The snow cover in the northern and southern zones of the region of permafrozen rock masses is identical: z_{sn} = 0.5 m and ρ_{sn} = 0.19 g/cc. The plant cover consists of moss with a thickness of 0.1 m; the coefficient of thermal conductivity of the moss in summer (K_t) is 0.0001 m^2/hr and in winter -- K_f = 0.0005 m^3/hr.

Solution. A. /Calculation of the influence of moss cover in the northern zone of the region of permafrozen rocks/.

It follows from the preceding example that a moss cover in the north under the indicated conditions increases the annual average temperature of the surface of the soil by 0.4° and reduces the annual amplitude of temperatures by 2.1°. Under the influence of the snow and plant covers the temperature regime of the surface of the soil in the north is as follows:

$$t_0 = t_в + \Delta t_{сн} + \Delta t_{раст} = -15,4 + 7,2 + 0,4 = -7,8°;$$
$$A_0 = A_в - \Delta t_{сн} - \Delta A_{раст} = 26,6 - 7,2 - 2,1 = 17,3°.$$

When the moss cover is removed and the snow cover preserved the temperature conditions on the surface of the soil with consideration of the radiation correction, equal here to Δt_R = 0.5° and ΔA_R = 1.3°, will be determined correspondingly as:

$$t_0 = -15,4 + 7,2 + 0,5 = -7,7°;$$
$$A_0 = 26,6 - 7,2 + 1,3 = 20,7°.$$

In order to determine the influence of the moss cover on the depth of the seasonal thawing we calculate, using nomograms, the depths of thawing forming in the indicated two temperature conditions of the surface of the soil (under natural conditions and on sections with the plant cover removed).

1. We will find the temperature shift according to the nomogram (see Figure 33) at the following initial data (natural conditions): a) $t_o = -7.8^o$, $A_o = 17.3^o$, C_{vol} - 620 kcal/(m^3)(degree); Q_ϕ = 26,000 kcal/m^3; λ_t^o = 1.3 and λ_f = 1.7 kcal/(m)(hr)(degree). Under those conditions $\Delta t_\lambda \cong 0.7^o$. Consequently, $t_\xi = -7.8 - 0.7 \cong -8.5^o$. b) $t_o = -7.7^o$; $A_o = 20.7^o$; the remaining parameters are the same as in "a"; $\Delta t_\lambda{}^o \cong 1.0^o$. Consequently, $t_\xi = -7.7 - 1.0 = -8.7^o$.

2. We will determine the depth of seasonal thawing of rocks on a section with a moss cover. The initial parameters are: C_{vol-t} = 620 kcal/(m^3)(degree); Q_ϕ = 26,000 kcal/m^3; λ_t = 1.3 kcal/(m)(hr)(degree); $A_o = 17.3^o$; $t_\xi = -8.5^o$. We will find ξ_{nom} = 0.9 m, $\xi_{\lambda=1.3}$ = 0.9 x 1.14 = 1.08 m.

On a section without a moss cover the depth of seasonal thawing at $A_o = 20.7^o$; $t = -8.7^o$; (the remainder are the same) is: ξ_{nom} = 1.05 m, $\xi_{\lambda=1.3} \cong$ 1.05 x 1.14 = 1.20 m.

Thus the removal of the moss cover under the indicated conditions leads to increase of the depth of seasonal thawing of loam by 12 cm.

B. /Calculation of the influence of moss cover near the southern boundary of the permafrozen rocks/.

1. First we calculate the warming influence of the snow with abbreviated formula (5.3.10): Δt_{sn} = 21 x 0.274 = 5.8^o. From which the temperature conditions on the surface of the moss are determined: $t_{plant} = -6.6 + 5.8 = -0.8^o$; $A_{plant} = 21 - 5.8 = 15.2^o$.

2. Using equations (5.6.3) and (5.6.4), we find the values:

а) $A_1 = 15{,}2 + 0{,}8 = 16°$; $\tau_1 = 7 \cdot 30 \cdot 24 = 5040$ *час*;

$$\Delta A_1 = 16\left(1 - e^{-0{,}1\sqrt{\frac{3{,}14}{0{,}0005 \cdot 2 \cdot 5040}}}\right) = 16 \cdot 0{,}08 = 1{,}28°.$$

б) $A_2 = 21 - 6{,}6 = 14{,}4°$; $\tau_2 = 3720$ *час*;

$$\Delta A_2 = 14{,}4\left(1 - e^{-0{,}1\sqrt{\frac{3{,}14}{0{,}0001 \cdot 2 \cdot 3720}}}\right) = 14{,}4 \cdot 0{,}19 = 2{,}74°$$

Whence with (5.6.1) and (5.6.2) we obtain

$$\Delta A_{раст} = \frac{1{,}28 \cdot 7 + 2{,}74 \cdot 5}{12} \approx 1{,}9°;$$

$$\Delta t_{раст} = 0{,}64\,\frac{1{,}28 \cdot 7 - 2{,}74 \cdot 5}{12} \approx -0{,}3°.$$

Thus in the southern part of the region of permafrost the plant cover (in the given case the moss cover is a cooling factor; it lowers the average annual temperature of the surface of the soil by 0.3°. With consideration of the snow and plant covers the temperature regime on the surface of the soil is determined as:

$$t_0 = -6{,}6 + 5{,}8 - 0{,}3 = -1{,}1^\circ;$$
$$A_0 = 21 - 5{,}8 - 1{,}9 = 13{,}3^\circ.$$

3. We will find successively the temperature shift and depth of seasonal thawing of the loam on a section with and without a moss cover.

a. On a section with moss and snow covers: $t_o = -1.2^\circ$, $A_o = 13.3^\circ$, $C_{vol-t} = 620$ kcal/(m^3)(hr); $Q_\phi = 26{,}000$ kcal/m^3; $\lambda_t = 1.3$; $\lambda_f = 1.7$ kcal/(m)(hr)(degree); $\Delta t \cong 1.0^\circ$; $t = -1.2 - 1.0 = -2.2^\circ$; $\xi_{nom} = 1.30$ m; $\xi_{\lambda=1.3} = 1.3 \times 1.14 = 1.44$ m.

b. On a section with snow and without a moss cover the temperature conditions on the surface of the soil, with consideration of the radiation correction, where here is $\Delta t_R = 1.0^\circ$ and $\Delta A_R = 2.9^\circ$, are determined as:

$$t_0 = t_в + \Delta t_{сн} + \Delta t_R = -6{,}6 + 5{,}8 + 1{,}0 = 0{,}2^\circ;$$
$$A_0 = A_в - \Delta t_{сн} + \Delta A_R = 21 - 5{,}8 + 2{,}9 = 18{,}1^\circ.$$

We will find the temperature shift Δt_λ. At $t = +0.2^\circ$ and $A_o = 18.1^\circ$ (the remaining initial parameters are the same as in "a") $\Delta t_\lambda = 1.6^\circ$. Then $t_\xi = 0.2 - 1.6 = -1.4^\circ$.

The depth of the seasonal thawing on the area without a moss cover is determined by the following conditions: $C_{vol-t} = 620$ kcal/(M^3)(hr); $Q_\phi = 26{,}000$ kcal/m^3; $\lambda_t = 1.3$ kcal/(m)(hr)(degree); $A_o = 18.1^\circ$; $t_\xi = -1.4^\circ$. Then $\xi_{nom} \cong 1.80$ m and $\xi_{\lambda=1.3} = 2.0$ m.

In conclusion it can be noted that the depth of the seasonal thawing of loam when the plant cover is removed increases both in the north and in the south of the region of permafrozen rocks. In the former case that increase is about 10% of the depth which forms in the presence of vegetation, and in the latter 40%.

5. The Influence of a Water Cover on the Temperature Regime of Bottomset Beds and on Their Seasonal Freezing and Thawing

The temperature regime of freshwater lakes without outlets depends on their depth. Since the thickness of the ice in lakes during very severe winter conditions does not exceed 2-2.5 m, bottomset beds in bodies of water with a depth of more than 2.5 m always are in an unfrozen state. In that case, if the width of the lake exceeds twice the thickness of the frozen rock masses in the given region, then under the lake, as a rule, forms a tabetisol or talik which goes through, and when the width is smaller and the thickness of the permafrozen rock masses is greater a talik which does not pass through can form.

To characterize the temperature regime of water in bottomset beds in winter we will present data on temperature measurements of the water in Lake Pereval'noye (Table 27), situated on the divide of the Kebyume-Khandig rivers at a height of 1418 meters; the lake is 400 meters wide, 800 meters long and 17.9 meters deep (Shvetsov, 1951). The average annual temperature of the rocks in the region is -10°. Presented in the same table are the temperatures in a lake on the Yano-Indigirskaya lowland (Chizhov, 1973).

Table 27. Water temperature (°C) in Lake Pereval'noye (according to P. F. Shvetsov, 1951) and in Lake Glubokoye (according to A. B. Chizhov, 1973).

Озеро Перевальное		Озеро Глубокое					
Глубина, м	28/IV 1948	Глубина, м	31/VII 1973	29/VIII 1973	13/IX 1973	20/IX 1973	6/XI 1973
1,7 (лед)	0,0	0,5 (лед)		9,2	7,8		0,0
2,7	1,2	1,0	12,4			6,6	1,3
3,7	1,3	2,0	12,4		7,7		1,7
4,7	1,9	2,5		9,2		6,6	
5,7	2,7	3,0	12,4				1,8
6,7	3,0	3,5		9,1		6,6	
9,7	3,1	4,0	12,2				2,1
11,7	3,2	4,5		9,3	7,7	6,7	
13,7	3,4	5,0					2,2
15,7	3,5	5,5		9,0	7,5	6,7	
17,7	3,6	6,0	8,2				2,4
дно	4,6	6,5	7,4	8,7	7,4	6,5	
		7,0	6,8				2,6
		7,5	6,5	8,9	7,5	6,6	
		8,0	6,4				2,6
		8,5		8,6	7,4	6,5	
		9,0	6,0				2,8
		9,5		8,6	7,2	6,4	
		10,0					2,8
		10,5	6,5	8,7	7,4	6,5	
		11,0					3,0
		11,5 (дно)		8,6	7,5		

Key: A - Lake Pereval'noye B - Lake Glubokoye
1 - Depth, meters 2 - ice 3 - bottom

It is evident from Table 35 that the temperature in winter increases with depth to approximately 4°, at which temperature the fresh water is very dense and tends to descend to the bottom. That process of convective mixing occurs in the summer. In addition, in summer there also is mixing of the water by the wind, and so its maximal summer temperature in shallow northern lakes can be considered approximately identical over its depth.

If the depth of the body of water is smaller than the ice thickness possible in the given region or close to it, the lake can freeze to the bottom and both negative and positive average annual temperatures can form in the bottomset beds. In that case, depending on the depth of the water in the lake, the annual amplitudes of temperature on the surface of the bottomset beds also will change.

It is convenient to examine the temperature regime of the bottomset beds in shallow lakes freezing to the bottom by using Figure 41, on which a schematic diagram is presented of the distribution of minimal (t_{min}), maximal (t_{max}) and average annual (t_m, equal to t_o and t_ξ) temperatures on the ice cover as a function of the depth of the body of water (h). It can be assumed with some approximation that in winter the minimal temperature in the ice cover with the thickness H_{ice} varies linearly from t_{min} on the surface of the ice to 0^o on its base, and the summer maximal temperature of the water remains constant in depth as a result of convective mixing of the water. It is evident on Figure 41 that the average annual temperature of the ice (t_m) at a certain depth H is equal to:

$$t_H = \frac{\left(\frac{H_л - H}{H_л}\right) t_{мин} + t_{макс}}{2} \qquad (5.5.1)$$

and at the depth $H = H_1$ it is equal to 0^o.

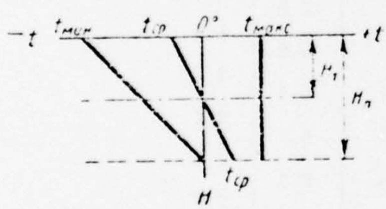

Figure 41. Schematic diagram of the distribution of minimal, maximal and annual average temperatures in the ice cover of a body of water.

At an ice thickness smaller than H_1 the average annual temperature on the surface of the bottomset beds is below 0^o, and at an ice thickness larger than H_1 it is above 0^o.

Therefore the following three temperature regimes of the bottomset beds are possible as a function of the depth of the body of water H.

1. A depth of the body of water smaller than H_1. In that case the average annual temperature on the surface of the bottomset beds is below zero; they are in a permafrozen state and only in summer do they thaw to a certain small depth.

2. A depth of the body of water H larger than H_1 but smaller than H_{ice}. In that case, as is evident from Figure 40, the average annual temperature on the lower surface of the ice and on the surface of the bottomset beds is above 0^o, but the body of water and the bottomset beds freeze in winter as a result of the negative winter values of the temperature in tha layer. In that case seasonal freezing of the bottomset beds occurs.

3. Finally, if the depth of the body of water H is greater than H_{ice} the body of water does not freeze to the bottom and the temperature on the surface of the bottomset beds always remains positive. In that case, under the lake there must be a talik which passes through or does not pass through, depending on the correlation of the dimensions of the lake and the thickness of the permafrozen rock masses.

Calculation of the Temperature and Depth of Seasonal Freezing in Bottomset Beds of Shallow Lakes (Example 17)

In conducting a frost survey the following data were obtained: a maximal depth of the lake of 1.0 m and bottomset beds composed of loams, for which it is known that γ_{sk} = 1100 kg/m^3; w = 40%; λ_t = 1.0; λ_f = 1.3 kcal/(m)(degree)(hr). The climatic conditions were: t_{air} = -8.5^o; A_{air} = 23^o; z_{sn} = 0.3 m; ρ_{sn} = 0.25 g/cc. The thickness of the ice cover H_{ice} on deep bodies of water formed in winter under the regional conditions is 1.6 m.

Solution. 1. We will determine the temperature regime on the surface of ice (water) in a body of water (t_{0-ice}). The warming influence of snow according to formula (5.3.10) is Δt_{sn} = 23 x 0.153 = 3.5^o. Then t_{0-ice} = -8.5 + + 3.5 = -5.0^o * and A_{0-ice} = 23 - 3.5 = 19.5^o.

2. We determine the depth of the zero isotherm H_1 with the formula

$$H_1 = H_л \left(1 + \frac{t_{макс.л}}{t_{мин.л}}\right), \qquad (5.5.2)$$

where

$$t_{макс.л} = A_{о.л} + t_{о.л} = 19{,}5 + (-5{,}0) = 14{,}5°;$$

$$t_{мин.л}{}^{2} = A_{о.л} - t_{о.л} = 19{,}5 - (-5{,}0) = 24{,}5.$$

Then

$$H_1 = 1{,}6\left(1 + \frac{14{,}5}{-24{,}5}\right) = 1{,}6\,(1 - 0{,}59) = 0{,}66 \text{ м}.$$

And so, under the body of water, where its depth is ~0.7m, passes the zero isotherm of the average annual temperature on the surface of the bottomset beds. The boundary of propagation of the permafrozen rock masses under the lake coincides with it.

3. We will examine the temperature regime on the surface of the bottomset beds H within the limits of the talik, where the lake depth is 1.0 m, with formula (5.5.1):

---

*In precise calculations t_{0-ice} and A_{0-ice} must be determined with consideration of the radiation correction.

**The value of $t_{min-ice}$ obtained in the calculation is taken with the minus sign.

$$t_H = \frac{\frac{1,6-1,0}{1,6}(-24,5)+14,5}{2} = 2,6°.$$

If it is taken into consideration that the maximal temperature in summer on the surface of the water is equal to the maximal temperature on the surface of the bottomset beds (as a result of convective mixing in a shallow body of water) and the average annual temperature is known, one can readily determine the amplitude of the annual temperature fluctuations on the surface of the bottomset beds at a depth of 1.0 m:

$$A_H = t_{макс_в} - t_0 = 14,5 - 2,6 = 11,9°.$$

4. To determine the depth of seasonal freezing of bottomset beds we will calculate the initial thermophysical parameters, assuming that the quantity of w_{un} in loam at an average winter temperature (t_{wtr}) of -4.65° (t_{m-wtr} = 1/2 t_{min} = = 1/2 (11.9 - 2.6) = -4.65° is about 8% (see Figure 31). Then with formula (4.1.6) we calculate

$$C_{об\,м} = 0,18 \cdot 1100 + 0,5\frac{(40-8)\,1100}{100} + 1,0\frac{8 \cdot 1100}{100} =$$

$$= 456 \text{ ккал/м}^3 \text{ град, а по формуле (4.1.8)}$$

$$Q_\Phi = 80 \cdot \frac{33 \cdot 1100}{100} = 29\,040 \text{ ккал/м}^3.$$

5. Under the established temperature regime of the bottomset beds in the layer of seasonal freezing of rocks a temperature shift forms. Its value in the given case is close to 1° (see example 9 with similar conditions). Consequently, $t = t_H + \Delta t_\lambda = 2.6 - 1.0 = 1.6°$.

6. With the nomogram (Figure 17) we find the depth of seasonal freezing of the bottomset beds on the section of lake where its depth is 1.0 m. The initial parameters are: C_{vol-f} = 456 kcal/(m³)(degree); Q_ϕ = 29,040 kcal/m³; λ_f = = 1.3 kcal/(m)(degree)(hr); A_o = 11.9°; t_ξ = 1.6°; ξ_{nom} = 1.3; $\xi_{\lambda=1.3}$ = 1.3 x x 1.14 = 1.5 m.

Salt lakes, especially when there are high concentrations of brine, can have the temperature regime of bottomset beds not subject to the above-indicated regularity. In that case the bottomset beds can freeze even at a depth of the body of water (H) exceeding the thickness of the ice in open bodies of water. Investigations have shown that on the bottom of strongly mineralized lakes a negative temperature of the saline water can be preserved in winter and summer. This is explained by the fact that the maximal density of the saline water is observed at temperatures of -15 to -20°. Thus, in salt lakes convective currents do not develop in the summer and heat exchange occurs only through thermal conductivity.

A temperature of -5° has been observed in the bottom layer of such lakes in summer and of down to -20° in winter. Clear from this is the role of salt bodies of water in the cooling of bottomset beds and formations of permafrozen rocks under them. In that case the change of the hydrochemical regime of the lake can lead to change of the temperature regime of the bottomset beds. With increase of the salinity of the water the seasonal freezing of bottomset beds can change into their seasonal thawing and the formation of permafrozen rock masses and, on the contrary, when bodies of water are freshened a permafrozen rock mass existing under them can melt.

The seasonal freezing and thawing of bottomset beds and their mutual transitions can be accomplished also on a shallow sea bottom along the coast of northern seas. This is possible because sea water freezes at -1.9^{o} and, being itself in liquid form, can freeze bottomset beds if the latter are saturated with less mineralized moisture.

The question of the formation of permafrozen rock masses on the coasts of northern seas during the interaction of salty sea and fresh continental waters is examined later in Chapter 8.

6. Dependence of the Temperature Regime and the Depths of Seasonal Freezing and Thawing of Rocks on the Relief and Exposure of Slopes

The position of a section in the terrain determines to a great extent the temperature regime of the soil and is in that respect a strong and variably acting factor. The variety and complexity of the influences of a given position of a section in the relief of the locality on the depth of the seasonal freezing and thawing is expressed in the following.

1. The average annual rock temperatures, and consequently also the depths of the seasonal freezing and thawing, vary as a function of the height levels of the locality. With increase of the height by 213 m the average annual rock temperature is lowered by 1.0^{o} and the depths of the seasonal freezing and thawing change accordingly.

2. Variation of the height leads not only to change of the annual average temperatures of the rocks and annual amplitudes of temperatures on their surfaces, but often in that case the lithological composition and the moisture content of the rocks also change. In other words, with height there is a complex change of all the classificational characteristics of the types of seasonal freezing and thawing, and also of such factors as the snow and plant covers. As a result the difference in temperature of the rocks in mountains and at sea level can reach $10-20^{o}$ or more.

3. In addition, the exposure of slopes influences the heat exchange and depth of seasonal freezing and thawing. Also important in that case is the steepness of the slopes, which determines the angle of incidence of solar rays and so the amount of absorbed radiation. That influence was examined in detail in Chapter 2.

Calculation of the Influence of the Steepness and Exposure of Slopes on the Formation of the Temperature of Rocks (Example 18)

On an investigated section the slopes are composed of block debris material with a small content (20-30%) of sandy loam filler. The annual course of the radiation balance of the surface on slopes of different exposure and steepness obtained in the process of a survey is presented in Table 28.

Table 28 Radiation balance (kcal/(cm^2)(month) on slopes of different curvature and exposure

Экспозиция	Крутизна, град	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Год
Южная	10	−2,5	−1,9	−2,1	0,1	8.6	9,3	8,4	6,0	2,0	−1,0	−2,9	−2,6	21.4
Южная	20	−2,5	−1,8	−1.8	0,4	8,7	9,5	8,4	6,4	2,5	−0,4	−2,8	−2,6	24.0
Северная	10	−2,5	−1,9	−3,2	−3,0	−1,9	−0,6	5,9	6,8	4,2	0,0	−2,3	−2,5	0.0
Северная	20	−2,5	−2,1	−3,2	−3,1	−2 1	−0,6	5,5	7,2	3,9	−0,5	−2,3	−2,5	−2,3
Западная	10—20	−2,5	−2.0	−3,0	−0,8	7.2	9,5	9,0	5,8	1,4	−1,9	−2,8	−2,5	17,4
Восточная	10—20	−2,5												

Key: A - Exposure a - Southern b - Northern c - Western d - Eastern
B - Curvature, degrees

Solution. Since the slopes are composed of block debris formations it can be assumed that the amount of evaporation from the surface of the slopes is not great. It can be assumed that the coefficient of heat transfer from the surface α is 20 kcal/(m^2)(degree)(hr)*. Then the radiation correction for the temperature of the surface of the slopes can be determined from the equation $\Delta t_R = R/\alpha$. The calculations should be made as shown in the example on page 30. The results of the calculations showed that the radiation correction for the average annual temperature of the surface of the rocks for different slopes varies in the following manner (Table 29).

Table 29 Table of values of t_R

Экспозиция	Крутизна, град	Δt_R, °C
Южная	10	+1,2
Южная	20	+1,4
Северная	10	0,0
Северная	20	−0,1
Западная	10—20	+0,9
Восточная		

A - Exposure B - Curvature, degrees
a - Southern b - Northern
c - Western c - Eastern

The difference of the average annual temperatures on slopes with southern and northern exposures is explained mainly by the difference on them of the summer amplitude of temperature fluctuations on the surface of the rock. Actually, in winter, when there is very little radiant energy arriving, the northern and southern slopes are cooled almost identically. But, as the data of Tables 6 and 13 show, the summer heating of the rocks on them is different and far more

*According to V. S. Luk'yanov and M. D. Golovko (1957) intense on the southern slopes. The decrease of the summer and annual amplitudes on the northern slopes causes a reduction of the average annual rock temperatures on them. The maximal difference between the average annual rock temperatures on slopes with northern and southern exposures is observed at a steepness of the slopes of 15-20° and decreases during both further increase and decrease of steepness.

In the high latitudes the influence of exposure of the slopes on seasonal thawing and on the temperature regime of the rocks diminishes somewhat, since there in the course of the entire summer day the sun moves in a circle above the horizon, warming slopes with all exposures.

The change of the average annual temperatures of rocks and their amplitudes on slopes with northern and southern exposures is shown on Figure 42. The depths of the seasonal freezing of rocks on slopes with northern and southern exposures differ far less than the depths of seasonal thawing on slopes with the same exposures.

Let us consider first, how the depths of seasonal freezing of rocks differ under the condition that the average annual rock temperature is higher and the annual amplitude of temperature on the surface is greater on slopes with southern than with northern exposures.

Let at the average annual temperature t_1 (Figure 42, left side) and the amplitude A_2 on a slope with a northern exposure (N) the depth of seasonal freezing of the ground is marked by the point m_1. On a slope with a southern exposure (S) the average annual rock temperature under corresponding conditions is t_2, but the depth of the seasonal freezing here is smaller and is marked by the position of the point m_2. In that case the depth of freezing would be smaller by the value $-\Delta\xi_1$. But on a slope with a southern exposure as compared with a northern exposure the amplitude of the temperatures increases from the value A_2 to the value A_3, as a result of which the depth of freezing also increases by $+\Delta\xi_2$ from the value at point m_2 to the value at point m_3. Thus during seasonal freezing the influence of the exposure of the slopes on the depth of freezing through change of the average annual temperatures and amplitudes of temperatures compensate one another $(\Delta\xi_2 - \Delta\xi_1)$. Consequently the exposure of the slopes has little influence on the depth of the seasonal freezing of rocks.

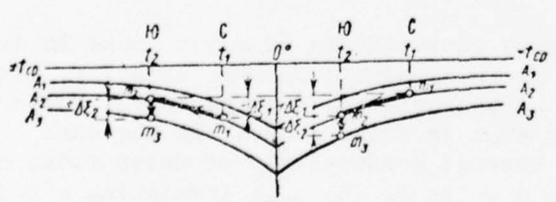

Figure 42. Change of the average annual rock temperatures and amplitudes of average monthly temperatures on slopes with northern and southern exposures.

Let us examine in the same way the influence of the exposure of slopes on the depth of seasonal thawing (right side of Figure 42). It is evident from the figure that in comparison with the slope with the northern exposure with the average annual rock temperature t_1 and the amplitude of temperatures A_2, on the slope with a southern exposure (S) the depth of the seasonal thawing increases from the point m_1 to the point m_2 on account of increase of the average annual temperature and from point m_2 to point m_3 on account of increase of the amplitude. In the given case both influences lead to increase of the depth of thawing ($\Delta\xi_1 + \Delta\xi_2$). It is evident from a comparison of the two cases that the exposure of the slopes has a stronger influence on the depth of seasonal thawing than on the depth of their seasonal freezing.

The influence of the exposure on the depth of seasonal freezing and thawing can also be complicated by other factors, for example, irregularity in the distribution of the snow cover. Typical in that respect is the region of Vorkuta, where the influence of the exposure of the slopes on the temperature of the soil is very small. This occurs as a result of the blowing and redeposition of snow in winter from the southern to the northern slopes by the southern and southwestern winds which prevail there in winter. As a result of that the influence of the exposure of the slopes is compensated by the opposite influence of the snow cover, and the temperatures of the rocks are equalized on slopes with all exposures.

7. The Influence of Swampiness on the Temperature Regime of Rocks and Their Seasonal Freezing and Thawing

Often on swampy sections the average annual rock temperature is 0.5-1° lower than on dry, well-drained sections. Such a dependence has been observed in the Far East, in Zabaykal'ye, in Southern Yakutiya and in other regions with small thickness of the snow cover. For the regions of Vorkuta, Western Siberia, Igraka and others with a thick snow cover of up to 0.8-1.0 m the reverse dependence is noted -- on swampy sections the temperature of rocks is higher than on drained sections.

The following considerations can be presented to explain these different influences of swampiness on the average temperatures of rocks as a function of the thickness of the snow cover.

A larger quantity of heat accumulates in moist rocks and swamps in the summer than in dry. At the same time, as a result of the greater heat capacity of moist grounds their temperature is not elevated so considerably as the temperature of dry rocks.

In winter (without snow) the great thermal conductivity of moist rocks in swamps permits them to cool almost as rapidly as those with less heat capacity, dry rocks with less thermal conductivity. As a result the average annual temperature of the grounds in a swamp is lower than in drier rocks. In the case of a thick snow cover the increase of the thermal conductivity of moist rocks during freezing already does not play such a role, as the heat insulating effect of the snow has the main importance. In that case their great heat capacity proves to be the deciding factor. Therefore under snow moist rocks with great heat capacity cool more slowly than dry rocks with less heat capacity, and as a result the average annual temperature of moist rocks becomes higher than that of dry rocks. For example, during the growth of bodies of water in Western Siberia the process of formation of permafrozen rock masses proceeds in the following manner.

As long as there is water, which strongly accumulates heat, on the surface of the body of water, the layer of ground which froze in winter completely thaws in summer, and a permafrozen rock mass does not form. Therefore in the northern taiga of Western Siberia the lowland and transitional swamps are underlain by thawed rocks.

In proportion to the accumulation of peat the separate sections of swamp emerge from under the water, raising themselves above the lowered sections of the swamp by 50-60 cm. In that case the sedges and cotton grasses on them are replaced by peat mosses, in places by green-moss cover with low-growing shrubs.

In winter those peat mounds freeze more rapidly than the lower and moister sections, the snow is blown away more strongly from them, their temperature is reduced and there is increase of their iciness and volume as a result of moisture migration and heaving. At first under such peat bogs through-flows start to form; then their thickness increases and they gradually change into permafrozen rock masses. In proportion to increase of the height of the peat bog its moisture and ice content increases, which leads to change of the average annual rock temperatures and to corresponding changes of the depths of the start of seasonal freezing, and then of seasonal thawing.

Calculation of the Influence of Swampiness on the Formation of the Average Annual Temperature of Rocks (Example 19)

In a frost survey the following data were obtained: $t_{air} = -11.7^{\circ}$, $A_{air} = 22^{\circ}$; $z_{sn} = 0.4$ m; $\rho_{sn} = 0.28$ g/cc; $\lambda_{sn} = 0.25$ kcal/(m)(degree)(hr). The time from the moment of establishment of snow to the moment of the spring inversion of the sign of heat cycling through the surface $\tau = 4700$ hours. Swampiness of the surface is manifested in the form of sedge-green-moss swamps with open mirrors of water only in the period of snow melting and falling rains. In the rest of the year water is observed between mounds in the form of small depressions and the moss cover is saturated with water. The soils are silty loams containing a large quantity of organic matter. Their characteristics are: $\gamma_{sk} = 800$ kg/m^3; $w_{vol} = 68\%$; $\lambda_t = 0.7$; $\lambda_f = 1.5$ kcal/(m)(degree)(hr); $C_{vol-t} = 800$ kcal/(m^3)(degree); $Q_{\phi} = 54{,}400$ kcal/m^3. In addition, as a result of investigations it also has been established that on swampy sections the radiation correction for the temperature of the surface is: $t_R = 1.5^{\circ}$ and $A_R = 2.5^{\circ}$. The vegetation increases the average annual temperature and reduces the amplitude of temperatures on the surface of the soil respectively: $\Delta t_{plant} = 0.7^{\circ}$; $\Delta A_{plant} = 2.0^{\circ}$. On well-drained sections in the same region there are sandy loams, with the following properties: $\gamma_{sk} = 1200$ kg/m^3; $w_{vol} = 20\%$; $w_{un} = 3\%$; $C_{vol-t} = 428$ kcal/(m)(degree); $\lambda_t = 1.0$; $\lambda_f = 1.2$ kcal/(m)(degree)(hr); $Q_{\phi} =$ $= \Delta t_R = 0.8^{\circ}$; $\Delta A_R = 1.8^{\circ}$. The plant cover, consisting of a sparse deciduous forest with underbrush, increases the average annual temperature of the soil by 0.5° and reduces the annual amplitude by 1.5°. The snow cover, just as on swampy sections, reaches a height of 0.4 m with a density of 0.28 g/cc ($\lambda_{sn} = 0.25$ kcal/(m)(degree)(hr)).

Solution. 1. We determine the warming influence of snow cover with a height of 0.4 m and λ_{sn} = 0.25 kcal/(m)(degree)(hr) on swampy sections just as in example 10, it being given that Δt_{sn} = 2.4 and 6° in accordance with formula (5.5.5). The data of the calculation are presented in Table 30. If we construct a graph (Figure 43) we find that Δt_{sn} = 5.1°.

Table 30 Calculating data for determination of Δt_{sn} on swampy sections

1 $\Delta t_{сн}$, °C	2 $t_о = t_в + \Delta t_R + \Delta t_р + \Delta t_{сн} = -11,7 + 1,5 + 0.7 + \Delta t_{сн}$, °C	3 $A_о = A_в + \Delta A_R - \Delta A_р - \Delta t_{сн} = 22 + 2,5 - 2 - \Delta t_{сн}$, °C	ξ, м	4 $A_{ср}$, °C	ξ2C, м	5 $Q_{гр}$, ккал/м²	6 $t_{сн. зим}$, °C	7 $t_{о. зим}$, °C	8 $Q_{сн}$, ккал/м²
2	−7,5	20,5	0,7	13,8	0,47	43 211	−23,1	−18,7	25 850
4	−5,5	18,5	0.8	11,6	0,50	47 687	—	−16.0	41 712
6	−3.5	16,5	0,85	9,6	0,52	49 715	—	−13,3	54 638

Key:
1 - Δt_{sn}, °C
2 - $t_o = t_{air} + \Delta t_R + \Delta t_r + \Delta t_{sn}$ = 11.7 + 1.5 + 0.7 + Δt_{sn}, °C
3 - $A_o = A_{air} + \Delta A_R - \Delta A_r - \Delta t_{sn}$ = 22 + 2.5 - 2 - Δt_{sn}, °C
4 - A_m, °C
5 - Q_{gr}, kcal/m²
6 - $t_{sn\text{-}wtr}$, °C
7 - $t_{o\text{-}wtr}$, °C
8 - Q_{sn}, kcal/m²

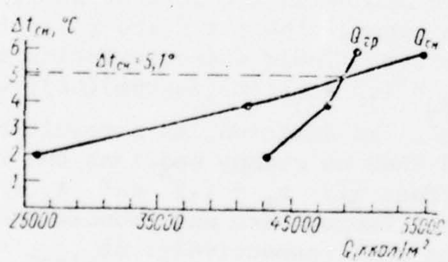

Figure 43. Graph for finding Δt_{sn} on a swampy section.

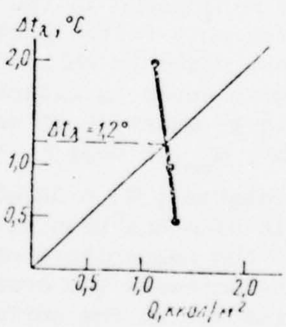

Figure 44. Graph for finding $\Delta t\lambda$ on a swampy section.

2. We find the temperature shift in the layer of seasonal thawing of rocks on a swampy section, where

$$t_0 = -11{,}7 + 1{,}5 + 0{,}7 + 5{,}1 = -4{,}4°;$$

$$A_0 = 22 + 2{,}5 - 2{,}0 - 5{,}1 = 16{,}4°.$$

Using equation (5.3.1) by the method of trial and error, being given the values for Δt_λ of 0.5, 1.0 and 2.0°, we calculate both sides of the equation (Table 31).

Table 31. Calculating values for determination of Δt_λ on swampy sections

Δt_λ, °C	t_ξ, °C	ξ, м	A_{cp}, °C	$\frac{\xi^2 (Q_{гр} + A_{cp} C)}{T} \cdot \frac{\sqrt{\lambda_м} - \sqrt{\lambda_т}}{\lambda_{пр}}$
0,5	−4,9	0,82	10,2	1,34
1,0	−5,4	0,80	10,5	1,28
2,0	−6,4	0,75	11,0	1,13

Having constructed the graph (Figure 44), we find that $\Delta t_\lambda = 1.2°$. Consequently, $t_\xi = -4.4 - 1.2 = -5.6°$.

Thus on swampy sections under the conditions of the given region the average annual temperature of the ground as a result of the combined action of snow and temperature shift increases by 3.9° (5.1 - 1.2) in relation to the air temperature.

3. We determine the warming influence of snow with a height of 0.4 m and $\lambda_{sn} = 0.25$ kcal/(m)(degree)(hr) on drained sections. Being given values of $\Delta t_{зн}$ of 1, 2 and 4° we obtain the following calculating data (Table 32). Having constructed the graph (Figure 45) we find that $\Delta t_{sn} = 1.8°$.

Table 32 Calculating data for determination of Δt_{sn} on drained sections

$\Delta t_{сн}$, °C	$t_0 = t_в + \Delta t_R + \Delta t_p + \Delta t_{сн} =$ [illegible] $.7 + 0.8 +$ [illegible] $+ \Delta t_{сн}$, °C	$A_0 = A_в + \Delta A_R - \Delta A_p - \Delta t_{сн} = 22 + 1.8 - 1.5 - \Delta t_{сн}$, °C	ξ, м	A_{cp}, °C	$\xi_{зс}$, м	$Q_{гр}$, ккал/м²	$t_{сн.зим}$, °C	$t_{о.зим}$, °C	$Q_{сн}$, ккал/м²
1	−9,4	21,3	1,40	14,5	1,0		−23,0	−20,2	16 450
2	−8,4	20,3	1,45	13,7	1,06	22 820	—	−18,9	24 088
4	−6,4	18,3	1,60	11,7	1.14	25 997	—	−16,2	39 950

(Column headings as for Table 30, except for numerical values)

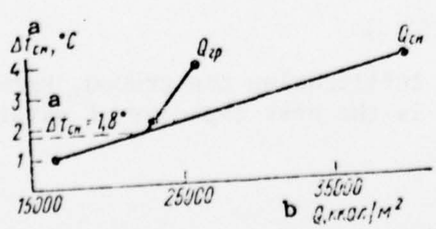

Figure 45. Graph for finding Δt_{sn} on drained section.
a - Δt_{sn}, °C b - Q, kcal/m²

4. We examine the temperature shift in the layer of seasonal thawing on sections composed of sandy loams, where

$$t_0 = -11{,}7 + 0{,}8 + 0{,}5 + 1{,}8 = -8{,}6°;$$
$$A_0 = 22{,}0 + 1{,}8 - 1{,}5 - 1{,}8 = 20{,}5°.$$

With the nomogram (Figure 33) we find that for those conditions $\Delta t_\lambda = 0.5^{\circ}$ and, consequently, $t_\xi = -8.6 - 0.5 = -9.1^{\circ}$.

If we compare the obtained values of t_ξ on swampy and drained sections we see that in the given region swampiness increases the average annual rock temperature on account of influence on Δt_{sn} and Δt_λ by 2.6°.

8. The Influence of Infiltration of Summer Precipitations and Convection of Air on the Temperature of Rocks and the Depth of Their Seasonal Freezing and Thawing

Rocks can vary their temperature not only on account of thermal conductivity but also as a result of convective heat exchange. Convective heat exchange in rocks can be accomplished basically by three methods: 1) by moisture migration to the front of ice separation during the freezing of rocks (see Chapter 6), 2) by infiltration into the rocks of warm or cold waters, bringing heat or cooling rocks, and 3) by the entry of warm or cold air into the rocks.

Convective heat exchange caused by moisture migration was examined in Chapter 3. Here we will dwell on examination of the influence of the infiltration of precipitations and convective air flows into rocks on the formation of the temperature regime and the depth of seasonal freezing and thawing.

The intensity of heat transport into a rock through the infiltration of surface waters (precipitations), firstly, as a function of temperature and the filtration capacity of soils and underlying rocks. The temperature regime forming under the influence of the infiltration of precipitations depends also on the thermophysical characteristics of the filtering rocks. For example, in the region of the Aldanskoye highland the divides are composed of strongly weathered bedrocks covered with large-block eluvia and deluvia. More than half of the relatively small quantity of annual precipitations falls there in the summer period and, being filtered, warms the layer of deluvia and eluvia and also the fractured zone of the bedrocks.

The heat balance can be determined in the following manner for the layer of seasonal thawing through the transport of heat into the ground by infiltrating summer precipitations. It is obvious that the additional arrival of heat (Q_{add}) in the seasonally thawed layer is determined with the formula

$$Q_{доп} = Vt_{oc}C, \qquad (5.8.1)$$

where V is the quantity of summer precipitations infiltrating the ground, kg/m^2; t_{prec} is their average summer temperature, $^{\circ}C$; C is the heat capacity of water, assumed to be 1.

Some sort of heat flux from the ground to the atmosphere, equal to that additional heat arrival, must be simultaneously established. This will be assured by a temperature shift into the layer of seasonal freezing (thawing), on the base of which the temperature will be higher than on the surface of the ground. Through this a heat flow also will be established from the layer of seasonal freezing (thawing) into the atmosphere, the amount of which is determined in the following manner:

$$Q = \frac{\Delta t_{oc}}{\xi} \lambda_{np} \tau. \qquad (5.8.2)$$

If we add (5.8.1) to (5.8.2) and solve for Δt_{prec} we obtain a formula which makes it possible to quantitatively estimate the warming influence of the the infiltrating waters:

$$\Delta t_{oc} = \frac{V t_{oc} \xi}{\lambda_{np} \cdot \tau}, \qquad (5.8.3)$$

where λ_f is calculated with formula (4.1.19).

It follows from (5.8.3) that the elevation of the ground temperature by infiltrating warm precipitations is directly proportional to the quantity of infiltrating precipitations, their temperature and the thickness of the layer ξ and inversely proportional to the reduced coefficient of thermal conductivity.

Calculation of the Influence of Infiltration of Summer Precipitations on t_ξ and ξ (Example 20)

As a result of investigations on a section of the second terrace above the flood plain the following data are known: the sands have γ_{sk} = 1500 kg/m^3; C_{spec} = 0.18; w = 12%; w_{un} = 0; $\lambda_f = \lambda_t$* = 1.5 kcal/(m)(hr)(degree).

The climatic conditions are determined to be: t_{air} = 6.8°; A_{air} = 20°; z_{sn} = = 0.3 m; ρ_{sn} = 0.19 g/cc.

The conditions and temperature of summer precipitations are presented in Table 33.

Table 33 Conditions of summer precipitations and their average monthly temperature

A Месяцы	V	VI	VII	VIII	IX
1 Количество осадков, мм	11	20	75	65	30
2 Среднемесячная температура воздуха, °C	0,3	10,0	17,0	9,3	3,5

Key: A - Months 1 - Quantity of precipitations, mm 2 - Average monthly air temperature, °C

*$\lambda_t = \lambda_f$ is assumed in order to simplify the problem.

Solution. 1. We determine the temperature conditions on the surface of the soil with consideration of the influence of the snow cover (Δt_{sn} = 20 x x 0.178 = 3.6°) and the radiation corrections, equal to Δt_R = 0.8° and ΔA_R = 2.3°:

$$t_0 = t_в + \Delta t_R + \Delta t_{сн} = -6,8 + 0,8 + 3,6 = -2,4°;$$
$$A_0 = A_в + \Delta A_R - \Delta t_{сн} = 20 + 2,3 - 3,6 = 18,7°.$$

2. We determine the thermophysical properties of the sand with (4.1.7) and (4.1.4)

$$C_{об.т} = 0,18 \cdot 1500 + 1,0 \frac{12 \cdot 1500}{100} = 450 \text{ ккал/м}^3 \text{ град};$$
$$Q_ф = 80 \cdot \frac{12 \cdot 1500}{100} = 14\,400 \text{ ккал/м}^3.$$

3. The summary heat flux ($Q_{prec} = C_{air} \times V_{prec} \times t_{prec}$) arriving in the layer ξ with the precipitations can be determined as the sum of the products of the monthly (10-day) sums of the precipitations times the average monthly (average 10-day) air temperature during the entire warm period of the year.

The sum of the precipitations in mm cited in climatic reference books corresponds to the quantity of water in kg arriving per m^2 of area. Therefore in the given case Q_{prec} = 1.0 (11 x 0.3 + 20 x 10.0 + 75 x 17.0 +65 x 9.3 + 30 x x 3.5) = 2217 kcal/m^2. The heat capacity of water C_w = 1.0 kcal/kg x deg.

4. Calculating equation (5.8.3) for determination of the warming influence of infiltration (applicable only for the region of seasonal thawing of rocks) is transcendental, since it contains two interrelated unknowns: Δt_{prec} and ξ. Therefore it is solved by the method of trial and error, for which certain values of Δt_{prec} are given, for example, a) Δt_{prec} = 0.3° and b) Δt_{prec} = 0.5°. We find t_ξ and ξ in accordance with those values. We will examine these cases: a) Δt_{prec} = 0.3°; $t = t_o + \Delta t_{prec}$ = -2.4 + 0.3 = -2.1°; we find ξ with a nomogram (see Figure 16) at the following initial data: C_{vol-t} = 450 kcal/(m^3) (degree); Q_ϕ = 14,400 kcal/m^3; λ = 1.5 kcal/(m)(hr)(degree); A_o = 18.7°; t_ξ = = -2.1°. We find that ξ_{nom} = 2.15 m; $\xi_{\lambda=1.5}$ = 2.15 x 1.22 = 2.6 m. b) Δt_{prec} = = 0.5°; t = -2.4 + 0.5 = 1.9°; we find ξ for the given value of t_ξ; the remaining initial parameters are the same as in case "a": ξ_{nom} = 2.2 m; $\xi_{\lambda=1.5}$ = 2.2 x 1.22 = 2.7 m.

For cases "a" and "b" we find the right side of equation (5.8.3):

$$\frac{Q_{oc}\xi}{\lambda\tau} = \frac{2217,5 \cdot 2,6}{1,5 \cdot 8760} = 0,44;$$
$$\frac{Q_{oc}\xi}{\lambda\tau} = \frac{2217,5 \cdot 2,7}{1,5 \cdot 8760} = 0,46.$$

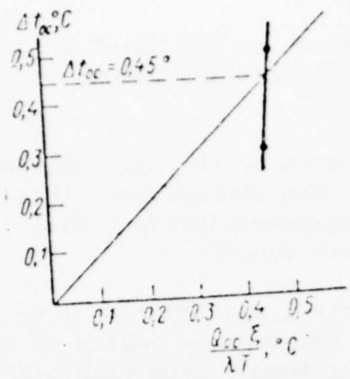

Figure 46. Graph for finding Δt_{prec}.
a - Δt_{prec}, ^{o}C
b - $\Delta t_{prec} = 0.45^{o}C$
c - $\frac{Q_{prec}\xi}{\lambda_T}$, ^{o}C

Similarly to how it was done in finding the temperature shift Δt_{λ} (example 9) we will construct a graph (Figure 46) on which the straight line $y = \Delta t_{prec}$ is intersected by the line $y = Q_{prec}/\lambda_T$. The point of intersection gives the sought value of Δt_{prec} (on the axis of abscissas) of $\sim 0.45^{o}$. Thus the temperature regime in the layer ξ, consisting of sandy deposits, is characterized by elevation of the average annual temperature on the base ξ in relation to the average annual temperature on the surface of the soil of 0.45^{o} on account of the warming effect of infiltration of atmospheric precipitations.

Calculation of the warming influence of infiltrating summer precipitations for the Aldan region, where V = 300 mm in the summer, $t_{prec} = 15^{o}$; $\xi \simeq 3$ m; $\lambda_f = 0.9$, gives a value of Δt_{prec} of 1.6^{o}. In that calculation it was assumed that all summer precipitations fall on a horizontal surface where there is no surface runoff. The precipitations infiltrate the ground, releasing all the store of heat. It follows from that approximate calculation that even under such favorable conditions as, for example, in the region of Aldan, the value of Δt_{prec} is relatively small. At the same time, for that region in the literature a warming influence of the infiltrating summer precipitations is usually estimated with figures of the order of $4\text{-}6^{o}$. It follows from formula (5.8.3) that those figures are greatly overstated. Actually, such an elevation of the average annual temperature in comparison with the temperature on the surface is explained by other factors, in particular by the arrival of heat on account of the condensation of vapor contained in the air during its circulation in rock pores. This is evident from the fact that the same increase of the ground temperature from 1.5 to 2.0^{o} which is caused by the infiltration of 300 mm of summer precipitations can be caused by the condensation of 100 dm^3 of water per m^2 of surface of the ground (or 10 mm of a layer). But that will occur provided that all the heat of condensation of the vapor goes for the additional heating of the rock. There is no question that that will not occur. Some heat will be lost to the atmosphere. But if even 90% of that heat is lost and only 10% goes for the heating of rocks, in that case the condensation of water in a quantity of 100 mm will be equivalent to 300 mm of infiltrating precipitations if all the heat content of those precipitations (in relation to 0^{o}) is used for the heating of rocks.

Actually, even in the latter case only a portion of the heat will be assimilated by rocks, and a portion will be lost to the atmosphere. This follows even from the fact that not all the impinging precipitations will infiltrate the ground. Some of them will go into surface runoff.

In the light of those positions the relatively great importance of the condensation of vapors in the formation of the temperature regime of soils is evident. However, the influence of infiltrating summer warm precipitations, as calculations with formula (5.8.13) show, evidently does not go beyond the limits of 1.5-2.0^{o} and rarely reaches greater values.

Convective flows of air can play a considerable role in the formation of average annual temperatures of porous rocks. In porous soils and grounds, evidently, there constantly is an exchange of gases with the atmosphere, caused by fluctuations of air pressures and temperatures on the surface of the soil.

If the rocks are very porous, such gas exchange can become very intense. The process in that case proceeds as follows: cold (heavy) atmospheric air displaces the warmer and lighter air from the cavities of the rocks and cools the latter. In that case it is itself warmed somewhat and is again displaced by colder air. Winter ventilation of that sort is very noticeable in systems of shafts and galleries and works in porous rocks, intensively cooling the latter to a considerable depth.

In addition, separate volumes of air are often trapped in fissures and pores of rocks and compressed by the pressure of subsurface waters. Upon emergence on the surface the temperature of those masses decreases sharply as a result of adiabatic expansion. In the region of Aldan, for example, in summer at an air temperature on the surface of about 20^{o} flows of cold air from slits and fissures in rocks with a temperature of about -11 to -14^{o} (Epshteyn, 1964).

Thus summer infiltrating warmings and winter convective coolings, combined or separately, can occur in porous or fractured rocks. Those influences can considerably change the average temperatures of rocks and the depths of their seasonal freezing and thawing.

9. The Influence of Latitudinal Zonation and Height Zonation on the Formation of Frost Conditions

The influence of various factors on the formation of the temperature regime of rocks and their seasonal freezing and thawing and other frost characteristics are manifested differently in different geological and geographic conditions. In particular, in that question of great importance are the composition of the rocks and the annual heat cycles. The composition of the rocks belongs among the regional factors, and the heat cycles are subject to geographic latitudinal zonation and height zonation.

As is known (see Figure 22), from south to north in the region of seasonal freezing of rocks the annual heat cycles of the soil increase with reduction of the average annual temperature of the rocks on account of increase of the depth of freezing. On the southern boundary of the region of permafrost, where the depths of seasonal freezing and thawing reach a maximum, the heat cycles in the soil have very great importance. Further in the region of propagation of permafrozen rocks from the southern boundary northward the average annual temperatures decrease, as do the depth of the seasonal thawing and the annual heat cycles in the soil and grounds.

In accordance with the character of the change of heat cycles, the influence of different factors on the formation of frost conditions should be examined as a function of the geographic latitudinal zonation and the height zonation. Let us examine on that level the influence of heat cycles on the structure of the radiation heat balance, in accordance with which the temperature regime of the rocks is formed. It is evident from formula (4.1.11) that in the half-period of heating under the same conditions of insulation of the surface with equal amounts of absorbed radiation the components of the heat balance can have different values as a function of the heat cycles in the soil. The increase of heat cycles at equal amounts of absorbed radiation leads to reduction of the temperature of the surface, which in turn leads to a decrease of the effective radiation, expenditures of heat on evaporation and turbulent heat transfer. Since the greatest changes of heat cycles are noted near the southern boundary of the propagation of permaffozen rocks, and they dampen from south to north, then accordingly the maximal changes of the components of the radiation heat balance on account of heat cycles will also be noted near the southern boundary. Large changes of heat cycles are also observed on sections with very icy soils in the layer of seasonal freezing where during the entire summer, thanks to the "zero screen," very large temperature gradients are observed in the layer. On such sections the role of heat cycles of the soil also is large in the structure of the radiation heat balance.

In the half-period of cooling the heat cycles in the soil exert a very great influence on the structure of the radiation heat balance. In connection with the sharp reduction of the arriving radiation the heat cycles in the soil become the main source of thermal processes occurring on the surface.

In view of the fact that the quantity of absorbed radiation regularly increases during movement from north to south, and the annual heat cycles, reaching their maximum near the southern boundary of propagation of permafrozen rocks, decrease toward both the north and the south, the latitudinal zonation in the structure of the radiation heat balance is very clearly traced north of the southern boundary of the permafrost and is smoothed out to the south in accordance with the decrease of the share of the heat cycles. All this will have a substantial influence on the formation of the temperature regime, the seasonal freezing and thawing of the ground and the cryogenic processes. By virtue of that, in compiling a forecast of the changes of frost conditions in connection with change of the conditions on the surface, leading to change of the structure of the radiation heat balance, it must be taken into consideration in which frost-temperature, latitudinal and height zone the investigations are being conducted.

Directly connected with heat cycles in the soil is the formation of the temperature regime on account of a temperature shift. It is known that the maximal amount of temperature shift is noted at the southern boundary of propagation of permafrozen rocks. It diminishes to the south and north. The difference in the amount of shift in that case can reach 1.5-2.0° for different frost-temperature zones. It is obvious that the influence of that factor must be taken into consideration as a function of the latitudinal and height zonation differently for different zones.

Still greater importance in the formation of the temperature regime of the soil is acquired by the latitudinal and height zonation in the evaluation of the warming influence of snow. The difference in the warming influence of the snow cover between different frost-temperature zones can reach 3-5° at the same height of the snow and an identical state and moisture content of the rocks. In that case, maximal values of the warming influence of snow will be observed near the southern boundary of the permafrost. North and south of that it will diminish.

A similar situation is noted in examining the influence of the exposure and steepness of slopes, the plant cover and the infiltration of atmospheric precipitations.

In examining the influence of surface bodies of water the latitudinal and height zonation will be expressed primarily in variation from south to north of the critical depth of the bodies of water at which taliks start to form under the riverbeds and lakes. The configuration of the through and non-through taliks under riverbeds and lakes will also change substantially as a function of the frost-temperature zone in which they are situated.

Finally, it is very important to take into consideration the influence of the latitudinal and height zonation in examining the transforming role of man's productive activity. The same measures (removal of the snow cover, drainage of soils, change of the plant cover, the enswamping and drainage of territory, asphalting and leveling of the terrain, etc) in different frost-temperature zones can lead to substantially different results.

Calculation of t_ξ and ξ with Consideration of the Influence of Latitudinal Zonation of Natural Factors (Example 21)

Calculate how the temperature regime and depth of seasonal thawing of the ground change during the removal of the plant and snow cover on sections near the southern boundary of the propagation of permafrozen rocks (section I) and on a section in frost-temperature zone IV (section II). In the process of frost investigations the following data were obtained on section I: $t_{air} = -6.3^\circ$, $A_{air} = 19^\circ$, $z_{sn} = 0.5$ m; $\rho_{sn} = 0.25$ g/cc; $\lambda_{sn} = 0.28$ kcal/(m)(degree)(hr); $\tau_{Q_{sn}} = 3150$ hours; $\Delta A_2 = 7.0^\circ$; $\Delta A_1 = 2.5^\circ$; $\tau_2 = 3720$ hours; $\tau_1 = 5040$ hours see 5.4.3 and 5.4.4). The soils in the layer ξ are composed of sandy loam with the characteristics $\gamma_{sn} = 1250$ kg/m^3; $w_{vol} = 28\%$; $C_{vol\text{-}t} \approx 500$ kcal/(m^3)(degree); $Q_\phi = 22{,}400$ kcal/m^3; $\lambda_f = 1.3$ kcal/(m)(degree)(hr).

On section II the conditions are characterized by $t_{air} = -11.7^{o}$; $A_{air} = 44^{o}$; $z_{sn} = 0.5$ m; $\rho_{sn} = 0.25$ g/cc; $\lambda_{sn} = 0.28$; $\tau_{Q_{sn}} = 4330$ hrs; $\Delta A_2 = 6^{o}$; $\tau_2 = 2400$ hrs; $\Delta A_1 = 3^{o}$; $\tau_1 = 6360$ hrs. In composition the soils are similar to ghose on section I: $\gamma_{sk} = 1200$ kg/m^3; $w_{vol} = 30\%$; $C_{vol-t} = 500$ kcal/(m^3)(degree); $\lambda_f = 1.3$ kcal/(m)(degree)(hr); $Q_\phi = 24{,}000$ kcal/m^3.

Solution. 1. We determine the influence of the plant cover on the temperature regime of the surface with formulas (5.4.2) and (5.4.1), in which case, for calculation of A_1, Δt_{sn} was determined with (5.3.10).

a) on section I
$$\Delta t_{раст} = \frac{2}{3,14} \cdot \frac{2,5 \cdot 5040 - 7,0 \cdot 3720}{8760} = -1,0^{\circ};$$

$$\Delta A_{раст} = \frac{2,5 \cdot 5040 + 7,0 \cdot 3720}{8760} = 4,4^{\circ};$$

b) on section II
$$\Delta t_{раст} = \frac{2}{3,14} \cdot \frac{3 \cdot 6360 - 6 \cdot 2400}{8760} = 0,3^{\circ};$$

$$\Delta A_{раст} = \frac{3 \cdot 6360 + 6 \cdot 2400}{8760} = 3,8^{\circ}.$$

2. We find the warming effect of the snow cover on section I. Using calculating equation (5.3.5) by trial and error we find that $\Delta t_{sn} = 4.7^{o}$. The initial and calculating data are presented in Table 34, and the graph for finding Δt_{sn} is on Figure 47.

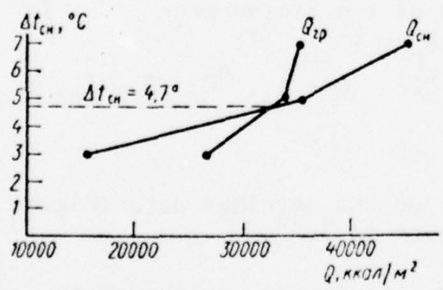

Figure 47. Graph for finding Δt_{sn} near the southern boundary.
a = Δt_{sn} b - Q_{gr} c - Q_{sn} d - Q/kcal/m^2

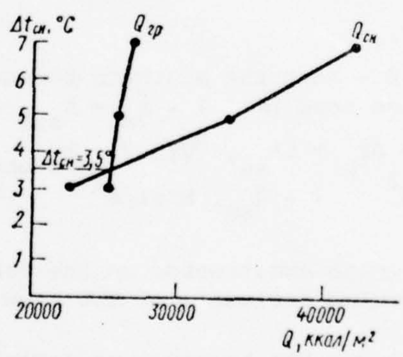

Figure 48. Graph for finding Δt_{sn} in frost-temperature zone IV.

3. We determine how the temperature regime on the surface of the soil and the depth ξ change during removal of the snow and plant covers on a section near the southern boundary:

a) under natural conditions:

$$A_0 = 19 - 4{,}4 - 4{,}7 = 9{,}9°;$$

$$t_0 = -6{,}3 - 1{,}0 + 4{,}7 = -2{,}6°;\ \xi = 1{,}30\ м;$$

b) after removal of plant and snow covers:

$$A_0 = 19°;\ t_\xi = -6{,}3°;\ \xi = 1{,}56\ м.$$

Upon removal of the covers the temperature regime of the grounds in the layer of seasonal thawing becomes more severe but the depth ξ increases by 0.26 m.

4. We find the warming influence of the snow cover on the section in temperate zone IV. The initial and calculated data presented in Table 34.

Table 34 Initial and calculated date for determination of Δt_{sn}

	A $\Delta t_{сн}$, °C	B Вблизи южной границы мерзлой зоны			C В IV мерзлотно-температурной зоне		
		3	5	7	3	5	7
1	$A_0 = A_в - \Delta A_p - \Delta t_{сн}$, °C	11,6	9,6	7,6	15,2	13,2	11,2
2	$t_0 = t_в + \Delta t_p + \Delta t_{сн}$, °C	−4,3	−2,3	−0,3	−8,4	−6,4	−4,4
3	$t_{0,\ зим}$, °C	13,6	8,0	5,3	15,7	13,2	10,4
	ξ, м	1,14	1,43	1,54	0,9	0,97	1,0
4	$A_{ср}$, °C	8,0	5,7	2,7	11,8	9,6	7,6
5	$\xi_{ср}$, м	0,6	0,7	0,8	0,34	0,58	0,62
6	$Q_{гр}$, ккал/м²	26 454	33 692	34 943	25 252	25 871	26 776
7	$Q_{сн}$, ккал/м²	15 523	35 280	44 806	22 794	33 546	42 148

A - Δt_{sn}, °C B - Near the southern boundary of the frozen zone C - In frost temperature zone IV 1 - $A_o = A_{air} - \Delta A_p - \Delta t_{sn}$, °C; 2 - $t_o = t_{air} + \Delta t_p + \Delta t_{sn}$, °C; 3 - $t_{o\text{-}wtr}$, °C; 4 - A_m, °C; 5 - ξ_f, m 6 - Q_{gr}, kcal/m² 7 - Q_{sn}, kcal/m²

We find from a graph constructed on the basis of the obtained data (Figure 48) that the warming influence of the snow cover is 3.5°.

5. We determine how the temperature regime varies on the surface of the soil and the depth ξ upon the removal of the snow and plant cover on section II:

a) under natural conditions:

$$t_0 = -11{,}7 + 0{,}3 + 3{,}5 = -7{,}9°;$$

$$A_0 = 22 - 3{,}8 - 3{,}5 = 14{,}8°;\ \xi = 1{,}0\ м.$$

b) after removal of the plant and snow covers:

$$t_0 = -11{,}7°;\ A_0 = 22°;\ \xi = 1{,}0\ м.$$

Upon removal of the covers in that zone the temperature regime becomes, just as on section I, more severe, but the depth of the seasonal thawing in that case is practically unchanged.

It should also be noted that the influence of the plant and snow covers varies substantially in accordance with the latitudinal zonation. Near the southern boundary of the permafrost the plant cover is a cooling factor and lowers the average annual temperature of the soil surface by 1.0^{o}; on the sections in frost temperature zone IV that same cover (moss-shrub) is a warming factor and increases the average annual ground temperature by 0.3^{o}. The influence of the snow is unequivocal in both cases -- it increases the soil temperature -- but its quantitative influence is not identical. Calculations have shown that snow of the same height and density under identical ground conditions elevates the temperature of the surface near the southern boundary by 4.7^{o}, and in frost-temperature zone IV by 3.5^{o}. This difference is connected with the latitudinal zonation of the annual heat cycles in the soils and with reduction of them during movement from south to north.

The presented analysis of the influence of different factors on the temperature regime of the soil surface and in the layer ξ must be used in concrete calculations of the annual heat cycles in the soil (Q) and depths of the seasonal and potential freezing or thawing of the rocks (ξ and ξ_f) necessary for forecasting different changes of frost conditions.

Analysis of the Summary Influence of the Principal Natural Factors on the Formation of t_ξ and ξ (Example 22)

In making a frost survey, studies are made by means of various field methods of the composition, cryogenic structure, properties, temperature regime, thickness, propagation of seasonally and permanently frozen rocks and corresponding frost processes and phenomena. To study regularities of the change of frost conditions in connection with change of the natural situation, an analysis is made of the bilateral dependences between the indicated characteristics of the types of seasonally and permanently frozen rocks and separate natural factors. First a quantitative estimate is given of the influence of natural factors on the temperature regime of soils and the depth of their seasonal thawing or freezing. It is advisable for that purpose to compile a summary table in which it is necessary to give a characterization of each type of locality according to geomorphological and geological structure, the hydrological and geobotanical conditions, the state of the snow cover and other conditions determining heat exchange on the surface of the soil and in the thickness of deposits, and also the thermophysical properties of soils (Table 35). Later, for all the principal microregions distinguished on the scale of the survey, a series of calculations should be made of the influence of each factor on t_ξ and ξ, as was shown in the examples of chapters 4 and 5, and the obtained results tabulated. In summing the influence of all factors we find the calculated values of t_ξ and ξ, which must be compared with the natural values obtained during a frost survey. Comparison of the calculated and natural values is necessary for verification of the correctness of the adopted calculating scheme. However, in the comparison it should be borne in mind that the calculated data, as a rule, characterize the average annual values of the parameters, and the natural -- their values for a specific year. In addition, divergence of the calculated and natural data can be observed if all the components of heat exchange on the surface of the soil and in the rocks have not been taken into consideration. For example, rather often in calculations the convective component of heat exchange is not taken into consideration, and that leads in individual cases to large divergences. Since the calculation of convective heat exchange in soils usually presents considerable difficulties, it is advisable to determine the influence of the convection of water or air on t_ξ and ξ by field observations.

The correctness of the calculated data is determined to a considerable degree by the procedure in calculating the influence of natural factors on t_ξ and ξ. It must be done as follows. First, corrections are determined for the temperature regime of the surface on account of the radiation balance, then the influence of the snow cover is determined, and later -- that of the plant and water cover and swampiness -- Δt_{sw}. After calculation of the indicated corrections the temperature regime of the surface of the soil is determined according to the scheme:

$$t_0 = t_в + \Delta t_R + \Delta t_{сн} \pm \Delta t_{раст};$$

$$A_0 = A_в + \Delta A_R - \Delta A_{сн} - \Delta A_{раст}.$$

Then one proceeds to calculate t_ξ with consideration of the temperature shift and infiltration of warm precipitations in the layer of seasonal thawing, that is,

$$t_\xi = t_0 - \Delta t_\lambda + \Delta t_{ос}.$$

After the table is completed, an analysis is made of the regularities of change of frost conditions on the investigated territory as a function of a complex of natural factors. The calculated data not only help analysis of distinctive features of the formation of different types of seasonally and permanently frozen rocks, but make it possible to predict their changes in connection with the natural dynamics of the natural factors or their variation during construction, and also to purposefully vary the frost situation.

Presented below is an example of the compilation of such a table for a section of the Yenisey River where a large-scale frost survey has been made.

The investigated section is on the left bank in the lower reaches of the Yenisey and includes an in-shore shoal, a low and a high flood plain complicated by numerous lakes and a first terrace above the flood plain. The alluvial lower flood plain is composed of beds of alternating sands, sandy loams and loams, with a subordinate number of layers of the latter. The high flood plain is composed of silty loams and sandy loams with layers and lenses of clay, sand and peat. The first terrace above the flood plain was formed in Sartanian time. The profile of the deposits is characterized by good uniformity over the area: from the surface to a depth of 1-1.5 m lie sandy loams, below, to a depth of 5-8 m, sands containing up to 10-15% of well-rounded pebbles. A formation of alternating sands, loams and pebbles underlies the profile. The thermophysical properties of those deposits in the layer of seasonal thawing are presented in Table 35.

A Элемент рельефа		B Микрорельеф и расчет		C Состав и генезис пород слоя ξ	w, %	D $\gamma_{об}$, г/см³	E $\gamma_{ск}$, об г/см³	F $C_т$, ккал/м³·град	G $\lambda_т$, ккал/м·град·час	H $Q_ф$, ккал/м³	I Температурный режим воздуха, °C: a $A_в$	I b $t_в$	Δt_R, °C	J Влияние снега: a $\frac{h, м}{\rho, г/см^3}$	J b $\Delta t_{ск}$, °C	K $\Delta t_{раст}$, °C: a на $A_в$	K b на $t_в$	Температурный режим на поверхн., °C: $A_о$	$t_о$	Δt_λ, °C	L $\Delta t_{ос}$, °C	M $\Delta t_{таз}$, °C	$t_ξ$, °C: a расч.	$t_ξ$ b факт.	ξ, м
I. I. надпойменная терраса	a	Траншея	1	Супесь с линзами песка и растительными осадками	27,5	1,71–2,27	1,22–1,53	560–690	1,18–1,76	23 000–28 000	23,5	–10,5	1,2	1,2/0,35	8,2	—	—	16,3	–1,1	–0,8– –1,1	0,3	0,6–0,8	–1,6– –1,8	–2,0, –2,4	—
	b	Просека		al Q_{III}^3 sr	20,3	1,6–1,97	1,29–1,62	490–620	1,05–1,37	16 000–21 000	23,3	–10,5	1,2	1,9/0,35	8,4	—	—	16,1	–0,9	–0,7– –0,9	0,3	—	–1,3– –1,5	–1,3, –1,5	1,3
	c	Естественные условия — редкий кустарник и кустарничек на мохово-лишайниковом покрове	2	Супесь с прослоями песка, с 1,5 м — песок м/з al Q_{III}^3 sr	23	1,83	1,53	630	1,38	21 000	23,5	–10,5	—	1,5–2,0/0,35	8,0 9,0	0,7	0,3	13,6 14,6	–1,8 –2,8	–0,7– –0,9	0,3	—	–2,2– –3,2	–2,3, –3,2	1,4–1,6
II. Высокая пойма	a	Траншея	3	Суглинок пылеватый, легкий, с прослоями супеси тяжелой и легкой	63,7	1,39–1,73	0,88–1,06	730–880	1,2–1,5	37 000–45 000	23,3	–10,5	1,2	0,5/0,28	7,3	—	—	17,2	–2,0	–1,6	—	–	–3,6	1,1, –0,6	—
	b	Просека	4	Суглинки тяжелые с прослоями пылеватых супесей	51	1,4–2,2	0,96–1,67	670–1170	1,15–1,53	31 000–54 000	23,3	–10,5	1,2	0,4/0,3	6,9	—	—	17,6	–2,4	–1,6	—	—	–4,0	[illegible]	1,2–1,5
	d	Естественные условия — густой ольховник с примесью ивы		al Q_{IV}	61,5	1,68–1,41	0,81–1,12	680–950	1,2	34 000–44 000	23,3	–10,5	—	2,0/0,25	12,4	0,7	–0,3	10,2	+1,6	–1,6	—	—	0	–0,1, –0,2	1,2
III. Озерная котловина на высокой пойме	a	Траншея	5	Суглинок пылеватый, тяжелый, с 0,7 м супесь тяжелая	21,6	1,51–2,19	0,95–1,32	800–1200	1,53	50 000–69 000	23,3	–10,5	1,2	0,35/0,28	4,8	—	—	19,7	–4,5	–1,5	—	—	–6,0	—	—
	b	Просека	6	Супесь тяжелая, с прослоями суглинка	45,5	1,08–1,38	0,56–0,84	300–540	1,1	17 000–26 000	23,3	–10,5	1,2	0,4/0,28	5,1	—	—	19,1	–3,9	–0,8	—	—	–4,7	—	1,4–1,7
	e	Естественные условия — редкий травяной покров		al Q_{IV}	59,7	1,50–1,51	1,07–0,96	680–760	1,15	35 000–39 000	23,5	–10,5	—	0,55/0,25	7,3	0,5	0,2	17,0	–3,4	–1,3	—	—	–4,7	—	1,3
IV. Низкая пойма	a	Траншея	7	м/з песок	32,5	1,62	1,21	610	1,25	28 000	23,3	–10,5	1,2	3,3–1,2/0,3	4,0–7,0	— —	—	17,5–20,5	–5,3– –2,3	–0,8	0,3	—	–2,8– –5,8	–3,4, –4,6	—
	f	Просека и естественные условия — редкий травяной покров	8	Супесь с прослоями песка и суглинка alQ_{IV}	30,1	1,89	1,53	740	1,64	35 000	23,3	–10,5	1,2	0,5/0,28	5,9	—	—	18,6	–3,4	–1,2	0,3	—	–4,3	[illegible]	1,5

Table 35. Table of initial and calculated data for region of the lower reacher of the Yenisey

Table 35 (Continued) Key

A - Element of relief B - Microrelief and calculation C - Composition and genesis of rocks of layer D - γ_{vol}, g/cc E - $\gamma_{sk\text{-}vol}$, g/cc

F - C_T, kcal/(m)(degree)(hr) G - $_T$, kcal/(m)(degree)(hr) H - Q_ϕ, kcal/m^3

I - Air temperature regime, ^oC - a - A_{air} b - t_{air} J - Influence of snow

a - h, m/ρ, g/cc b - t_{sk}, ^oC K - t_{plant}, ^oC a - for A_{air} b - for t_{air}

L - Δt_{prec}, ^oC M - Δt_{gas}, ^oC N a - calc b - actual

I - First terrace above flood plain II - High flood plain III - Lake basin on high flood plain IV - Low flood plain

a - Trench b - Cut-through c - Natural conditions -- sparse brush and low bush on moss-lichen cover d - Natural conditions -- dense alder scrub with admixture of willow e - Natural conditions -- sparse grassy cover f - Cut-through and natural conditions -- sparse grassy cover

1 - Sandy loam with lenses of sand and plant sediments
2 - Sandy loam with layers of sand, with 1.5 m of fine-grained sand al Q^3_{III} sr
3.- Silty loam, light, with layers of heavy and light sandy loam
4 - Heavy loams with layers of silty sandy loams
5 - Silty loam, heavy, with 0.7 m heavy sandyloam
6 - Heavy sandy loam, with layers of loam
7 - Fine-grained sand
8 - Sandy loam with layers of sand and loam al Q_{IV}

The climate of the region is characterized by low average annual air temperatures (t_{air} = -10.5^o), great amplitude of their annual temperature fluctuations (A_{air} = 23.3^o) and a large amount of precipitations (400 mm). The thickness of the snow cover is extremely irregular on account of drifts and varies from 0.2 to 0.5 m on the shore and the lower flood plain to 0.7 m and more on the higher flood plain.

Permafrozen rocks are spread completely over the lower flood plain and their thickness is about 100 m. On the high flood plain taliks are developed under the bodies of water, and in the part near the riverbed -- frost which does not flow together. The complex of factors for each element of the relief with which the formation of frost conditions (t_ξ and ξ) is mainly connected is presented in the left part of Table 35.

Calculations were made of t_ξ and ξ under natural conditions and during their variation as a result of construction for the indicated initial data with the above presented formulas. It is evident from Table 35 that one and the same measure, for example, the laying out of a trench, on different elements of the relief characterized by their own ground and moisture conditions, leads to the formation of different depths of thawing and rock temperatures.

To verify the correctness of the procedure of forecast calculations, on the sections observations were made of the change of frost conditions in the process of construction and operation of structures. In a comparison of the calculated values of t in the belt of cut-throughs and trenches and the data obtained in observations in the first two years after the start of construction, divergences of up to 2-3° were obtained. This is explained by the fact that the cut-throughs and trenches on all elements of the relief had existed not more than a year and, consequently, were characterized by an unsteady temperature regime at the moment of observation, whereas the calculated data correspond to steady-state conditions. Comparison of data characterizing natural conditions with forecast data within the limits of various elements of the relief makes it possible to give a geological engineering evaluation of the section with respect to the conditions of construction and also to designate a system of measures to control the frost process.

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Chapter 6. Predicting Change of the Characteristics of Permafrozen Rock Masses

In the area of the propagation of permafrozen rock masses the geological engineering evaluation of an investigated region is determined to a considerable degree by the frost conditions. In the planning, construction and operation of structures and in the productive opening up of a permafrost region it is necessary to know the frost conditions not only at the moment of the investigation but also the character of their variation during construction and operation. In that case the forecast must include the characteristics of change of the areas of propagation of permafrozen rocks and tabetisols (taliks), the conditions of their occurrence, the seasonal freezing and seasonal thawing of rocks, the temperature regime, the heat cycles, cryogenic textures and structures, composition and thickness, the thermophysical and physicomechanical properties of frozen, freezing and thawing grounds, and also cryogenic processes and phenomena. The compilation of such a forecast must be based on the study of particular, general and regional regularities in the formation and development of permafrozen rock masses. It must be taken into consideration that the formation and development of permafrozen rocks is inseparably connected with the development of the earth's crust in the investigated region: with geological-structural and lithological-facial features, with the geomorphological structure, the neotectonics and the character of the hydrogeological structures, and also with the character of the surface conditions and processes, primarily with climatic features and the conditions of freezing of permafrozen rock masses. That connection is reflected very completely in the scheme for classification of types of permafrozen rock masses. According to that classification, to compile a forecast of the change of frost conditions (in the sense of the dynamics of frozen rock masses) in the process of a frost survey the necessary information about the principal classification signs should be gathered together.

1. Principles of the Classification of Permafrozen Rocks

The genetic classification of permafrozen rocks must reflect the objective regularities of their formation and development.

It is known in permafrost studies that the character of frozen rock masses is determined by a geographic complex of natural conditions manifested in definite geomorphological and geological-structural conditions. As a function of such a combination of the complex of natural conditions, frozen rock masses have their own history of development and in connection with a specific situation have given qualitative and quantitative characteristics. In connection with that it is advisable to classify into three main groups the entire variety of possible classification characteristics somehow or other determining the distinctive features of the formation and development of permafrozen rock masses: I -- geological-tectonic medium and a geomorphological situation, to which the rocks of a definite composition and age correspond; II -- distinctive features of heat transfer in connection with geological and geographic conditions and III -- distinctive features of the frozen rock masses themselves, with all the characteristics intrinsic to them.

Each of those groups must include a large number of characteristics. It is advisable to examine, evidently, only the most significant of them and those which have been most studied and are most readily linked with the character of the frozen rock masses themselves.

1. Classification Signs of Subdivision of Frozen Rock Masses According to Geological and Geographic Conditions

1. The subdivision of frozen rock masses according to the principal geological-structural elements of the earth's crust. It is known that the development of rocks, including the permafrozen, is inseparably connected with the development of the earth's crust. In that case a special role is played by neotectonic processes, which are comparable in time with the formation of frozen rock masses and which determined the character of unconsolidated formations and the principal features of the contemporary relief. Therefore it is advisable to distinguish three types of frozen rock masses, concentrated in the contemporary structural forms of the earth's crust, which are characterized by a definite type of neotectonic development (Nikolayev, 1962):

a) frozen rock masses of platform regions with weak manifestation of neotectonics. These regions are divided into continental and those arranged within the limits of the continental shelf of the ocean. According to the directivity of the neotectonic movements regions are distinguished in which upthrusts have predominated and regions within whose limits subsidences (plates) have played an important role;

b) frozen rock masses of regions of continental mountain building. In intensity of the processes of mountain building those regions are divided into regions with weak manifaestation of mountain building processes, regions of mountain building of moderate intensity and regions with very intensive mountain building processes;

c) frozen rocks of regions of contemporary geosynclines. More detailed subdivisions of the classification characteristics with respect to geological-structural and neotectonic features of a territory can be adopted in accordance with the existing classification of N. N. Nikolayev (1962).

2. The enumerated structural forms and the character of their development determine to a considerable degree the distinctive features of the geomorphological conditions in which the formation of frozen rock masses occurs. On

the basis of that characteristic it is advisable to classify frozen rocks according to the relief with the general type of geological structures into three groups:

a) permafrozen rock masses within the limits of accumulation plains;
b) permafrozen rock masses within the limits of denudation plains;
c) permafrozen rock masses within the limits of mountains and highlands.

It is advisable to make a more detailed subdivision of the classification of permafrozen rock masses on the frost-geomorphological principle on the basis of the difference in features of the structure of the relief, and also of the directivity and intensity of neotectonic movements which stipulated the regularities of the frozen or thawed state and the formation of thicknesses of frozen rock masses. For example, permafrozen rocks within the limits of the accumulation plains, relatively stable in a tectonic respect, with a deep rock bedding (or with a shallow rock bedding), etc.

To clarify the regularities in the formation of frozen rock thicknesses and their structure, the formation of the temperature regime and the frost physical geological (cryogenic) processes and formations accompanying them, it is advisable to classify permafrozen rock masses further by types and relief elements. For example, permafrozen rocks within river basins, permafrozen rocks of steep and gentle slopes, etc.

Existing geomorphological classifications (for example, that of N. V. Bashenina et al, 1959) can be used in constructing a particular classification of permafrozen rocks on the frost-geomorphological principle.

3. The character of the hydrogeological structure along with other components of the geological and geographical conditions and with processes of heat exchange determines the concrete form of interaction of the subsurface waters with the frozen rocks, which finds reflection in the principal characteristics of frozen rock masses. Types of water-pressure systems derived by A. M. Ovchinnikov (1960) can be used for the classification of frozen rocks according to hydrogeological conditions. On that basis four types of frozen rock masses are distinguished:

a) frozen rock masses of artesian basins of the platform type;
b) frozen rock masses of artesian intermountain areas and piedmont troughs;
c) frozen rock masses of water-pressure systems of fissure waters of old crystalline massifs without neotectonic deformations (or complicated by neotectonic movements and faults);
d) frozen rock masses of jointed basins of subsurface waters of mountain structures without manifestation of young magmatism (with manifestation of young magmatism, with recent volcanic activity, etc).

The mineralization of subsurface waters has a great influence on the formation of frozen rocks within the limits of distinguished hydrogeological structures. Therefore it is advisable to distinguish frozen rocks forming during the freezing of water-bearing horizons with fresh waters (a mineralization of up to 1 g/liter), salty (1-10 g/liter) and saltier waters (10-50 g/liter) and brines (more than 50 g/liter). Permafrozen rock masses are classified in greater detail on the basis of the main distinctive features of their interaction with different types of subsurface waters of the frozen zone (Romanovskiy, 1972) and the main reasons for the formation of tabetisols (see Table 45).

4. Frozen rock masses are subdivided on the basis of genesis, composition and age in accordance with generally accepted geological classifications. It also is advisable to classify unconsolidated sedimentary frozen rocks of the Cenozoic on the basis of the lithological facies to which they belong. On the basis of lithological characteristics permafrozen rocks can be subdivided in detail on the basis of the corresponding geological engineering classifications.

II. Classification Characteristics for the Subdivision of Frozen Rock Masses According to the Character of the Heat Transfer

Of great importance in the formation of frozen rock masses is the total amount of absorbed shortwave solar radiation impinging on the earth's surface, and also the structure of the radiation balance of the ground. The connecting links between the radiation-heat balance and thermal processes in the soil are the effective radiation and the heat cycles of the soil. In the structure of the radiation-heat balance the place of those two characteristics is essentially different at different latitudes and is determined to a great extent as a function of the continental climate (see Chapter 2). In connection with that it is advisable to distinguish the following types of frozen rock masses:

1. On the basis of geographic latitude one can distinguish the southern, middle and northern types of frozen rock masses, characterized by a definite range of amounts of absorbed solar radiation and effective radiation.

2. On the basis of a continental climate it is necessary to distinguish the types of frozen rock masses characteristic of a maritime (with an air temperature amplitude of up to 11°), continental (with amplitudes of 11 to 17°) and extremely continental climate (with an amplitude larger than 17°). Each of those types of frozen rock masses has its own conditions of formation of the temperature regime of rocks.

3. On the basis of the average annual temperatures of rocks it is advisable to divide frozen rock masses into five frost-temperature zones, each of which is characterized by a definite qualitative complex of frost conditions: the first zone with average annual temperatures of rocks of from 0 to -1°, the second -- from -1 to -3°, the third -- from -3 to -5°, the fourth -- from -5 to -10°, and the fifth-- below -10°. Depending on the character of the change of the main complexes of surface conditions which regularly form the average annual temperature of the rocks within the limits of the distinguished geomorphological elements, the temperature gradations can be smaller. For example, in the second zone temperatures of -1 to -2° can be distinguished as sufficiently dynamic for the opening up of territory; the fourth and fifth zones can be given in gradations from -5 to -7°, from -7 to -9°, from -9 to -11° and below -11°, which is characteristic of the coasts of arctic seas and corresponds to certain geological-structural conditions (Kondrat'yeva et al, 1972).

4. On the basis of the length of the period of temperature fluctuations on the surface of the ground the following gradations of frozen rock masses are distinguished:

a) rock masses in the frozen state during the contemporary period (Q_{IV}). It is advisable to distinguish among them short-period frozen rock masses which have existed in the frozen state for tens of years, medium-period -- for hundreds of years, and long-period -- for thousands of years;

b) rock masses which have been in the frozen state since Upper Quaternary time (Q_{III});

c) since Middle Quaternary time (Q_{II});

d) since Lower Quaternary time (Q_{I});

e) since pre-Quaternary time.

5. On the basis of correlation of the average annual (t_m) and extreme (t_{min} and t_{max}) temperatures of rocks, three gradations of frozen rock masses can be distinguished: a) when $t_{max} < 0$; b) when $t_m < 0$; and c) when $t_{min} < 0$ and $t_m > 0$.

In the first case, permafrozen rock masses during the entire period of temperature fluctuations flow together, but their lower surface is periodically shifted and their thicknesses vary. In the second case the frozen rock mass periodically freezes and thaws but from above and from below, and in some periods of time either nonconfluent or layered frost forms. The central part of the frozen rock mass in that case exists for the entire period. In the third case, during a large part of the period of fluctuation of temperature on the surface of the ground there is no frozen rock mass. It appears periodically only in the "cold" part of the period.

6. On the basis of the quantity of the heat flux from below, toward the base of the frozen rock masses, it is advisable to distinguish three gradations of frozen rock masses: a) with a small, b) medium, and c) large heat flux, corresponding to the geothermal gradients of from 0 to 0.02, from 0.02 to 0.04 and more than 0.04 degree/m at a coefficient of thermal conductivity of 2 kcal/(m)(hr)(degree). Under those conditions the heat fluxes are determined as follows: from 0 to 250 kcal/(yr)(m^2), from 250 to 500 and more than 500 kcal/(yr)(m^2). The amount of the heat flux from the depths of the earth is expressed above all in the thicknesses of the frozen rock masses, on their dynamics, and also on the rate of freezing or thawing, which in turn determines the distribution of the ice content of the frozen rock masses and their cryogenic textures.

7. On the basis of the ice content of the frozen rock masses and the corresponding perennial heat cycles in rocks it is advisable to distinguish the following three gradations:

a) frozen rock masses without inclusions of ice (frost), with minimal heat cycles, connected with only one heat capacity (without phase transitions);

b) frozen rock masses with little ice, with a monolithic cryogenic texture (with a moisture content not greater than the total moisture capacity), with heat capacities of medium value, forming mainly on account of the heat capacity of the rocks and partially on account of phase transitions of the water;

c) frozen rock masses with large amounts of ice, with ice layers and lenses (the moisture content of which is greater than the total moisture capacity), with large heat cycles which form mainly on account of phase transitions of water and only partially on account of the heat capacity of the rocks.

These gradations make it possible to characterize not only the heat cycles but also the composition of the frozen rock masses, their genesis, and the conditions of freezing, and will even indirectly characterize the history of the formation of the frozen rock masses. It is obvious that those gradations will relate only to epigenetic frozen rock masses. Syngenetic rock masses, evidently, can differ mainly in the total ice content with consideration of the history of their formation and the dynamics of their freezing (see section 3 of the present chapter). Hard fractured rocks freezing epigeneticallly should be classified with consideration of the filling of fissures with ice.

8. On the basis of the character of the convective heat transfer participating in the formation of the frozen rock masses, the following three gradations should be distinguished:

a) frozen rock masses with convective heat transfer on account of the circulation of the waters above the frost;

b) frozen rock masses with interrupted spread with convective heat exchange on talik zones;

c) frozen rock masses with convective heat transfer on their lower surface on account of the circulation of waters below the frost.

The influence of convective heat exchange on account of waters below the frost is determined by the depth of their occurrence and the character of the circulation. A very great influence is noted when the subsurface waters contact frozen rock masses with intensive circulation. With increase of the depth of subsurface waters from the lower surface of frozen rock masses and with increase of their velocity the warming influence of the subsurface waters diminishes.

d) frozen rock masses with convective heat transfer on account of the circulation of air masses through the cracks and karst cavities. Included in the former type are frozen rock masses and ice of caves, the formation of which involves the flow of cold winter air into cavities of the earth's crust under conditions of hindered air exchange. Frozen rocks of that type are often widespread in various mine workings both in and outside a region of permafrost.

The circulation of air masses is a phenomenon fairly widespread in rocks composing denudation plains with a thin and coarse unconsolidated cover. The air circulation occurs in the zone of aeration and is directly connected with with atmospheric pressure fluctuations and the regime of the ground water level. Frozen rocks of that type are characterized by great dynamicity.

III. Classification Characteristics for the Subdivision of Frozen Rock Masses According to Distinctive Features of their Propagation, Occurrence and Main Characteristics

1. On the basis of the character of their distribution, frozen rock masses are subdivided into the following varieties:

a) frozen rock masses with a continuous distribution in which through taliks are developed only under large rivers and lakes and on sections of discharge of waters under frost;

b) frozen rock masses with an interrupted distribution with islands of thawed rocks, the formation of which can be caused both by the warming influence of surface waters and by other distinctive features of heat exchange on the surface of the ground. The frozen rocks occupy more than 50% of the area;

c) frozen rock masses with a continuous distribution, representing large blocks, developed on a general background of thawed or unthawed deposits. The area occupied by the frozen rocks amounts to less than 50%;

d) frozen rock masses with an island-like distribution, developed locally. Their formation or preservation is possible when there is a combination of a number of favorable factors and conditions. In area they occupy not more a few percentage points.

2. On the basis of the character of their structure frozen rock masses can be divided (vertically) into:

a) continuous frozen rock masses without thawed layers;
b) layered frozen rock masses, where in the profile one observes an alternation of permafrozen rocks and thawed layers or bodies of a different form.

3. On the basis of the interaction of a frozen rock mass with a layer of seasonal thawing and freezing the following are distinguished:

a) confluent frozen rock masses in which the layer of seasonal thawing is the roof of the latter;
b) nonconfluent, where between the base of the layer of seasonal freezing and the upper surface of the permafrozen rock mass remains a thawed layer which is preserved in the course of the winter;

4. On the basis of genesis, permafrozen rock masses are divided into:

a) the epigenetic, that is, those which froze after the accumulation and epigenesis of rocks;

b) the syngenetic, that is, those which accumulated and froze in the geological sense simultaneously. Syngenetic frozen rocks in composition and age are unconsolidated Quaternary deposits;

c) polygenetic, that is, in the character of the freezing having a two-level, and more rarely a multilevel, structure. The lower level of two-level rock masses is formed of epigenetic frozen rock masses, and the upper, of syngenetic.

5. On the basis of cryogenic structure (the cryogenic textures of the rocks) frozen rock masses are divided into:

a) epigenetic, having inherited (according to A. I. Popov) cryogenic textures in which the spatial differentiation of the ice inclusions is caused by the initial fracturing, porosity or cavernosity of the rocks. Among them one can distinguish cryogenic textures: inherited primary textures in which the ice volume does not exceed the volume of open porosity (or fracturing) of the rock before its freezing, and inherited expanded textures, that is, those in which the volume of the ice inclusions is greater than the volume of the open porosity (or fracturing) which occurred before the start of freezing of the rocks. Inherited cryogenic textures are encountered in solid and semi-solid rocks;

b) epigenetic, having migrational-segregational or congelational cryogenic textures. In such frozen rocks the moisture in the process of freezing could migrate or freeze in cavities, creating a spatial disposition of ice which could not correspond to the initial composition of the rock. Included in rocks having that type of cryogenic textures are loose rocks (Quaternary and pre-Quaternary deposits) and also a portion of the semi-solid weathered rocks;

c) syngenetic frozen rocks having cryogenic textures which arise as a result of redistribution of moisture in the seasonally thawed layer during its freezing. A large portion of the cryogenic textures of syngenetic frozen deposits is created thanks to freezing of the seasonally thawed layer from below and transition of the lower part of the latter into the permafrost state. In connection with that the ice content by volume of the deposits usually exceeds their total moisture capacity in that state;

d) epigenetic and syngenetic frozen rock masses with large ice accumulations in the form of syngenetic and epigenetic multiple-vein ice, injection ice, hydrolaccoliths, cave ice and buried firn basins and glaciers.

6. On the basis of structure and composition, permafrozen rock masses are divided into those:

a) having a single-level structure, that is, composed completely from roof to base of either loose or solid rocks;

b) having a two-level structure, that is, composed in the upper part of loose and in the bottom of solid or semi-solid permafrozen rocks. Within the limits of loose frozen rock masses syngenetic and epigenetic freezing of the deposit can be distinguished.

7. On the basis of the number of freezing cycles, permafrozen rocks are divided into:

a) those which have frozen once, that is, those which have existed continuously in the permafrozen state from the start of their freezing to the present;

b) those which have frozen and thawed repeatedly, that is, those which since the start of their perennial freezing to the present have frozen at least and thawed once either completely or from above or from below.

8. On the basis of thickness of the frozen rock masses, depending on their composition, other conditions being equal, the following four gradations can be distinguished:

a) frozen rock masses of limiting thickness, composed of frozen solid rocks with a large coefficient of thermal conductivity;

b) frozen rock masses of great thickness, composed of loose frozen rocks;

c) frozen rock masses of medium thickness, composed of loose deposits with a moisture content (ice content) not greater than the total moisture capacity ($w < w_{m-c}$);

d) frozen rock masses of small thickness, composed of loose deposits with a moisture content (ice content) greater than the total moisture capacity. In that case in the process of formation of frozen rock masses it is assumed that there is a possibility of inflow of moisture from below toward the front of freezing from lower-lying thawed water-bearing layers.

9. On the basis of the dynamics of frozen rock masses the generally accepted directions of development of the frost process should be distinguished: degradational, stable and aggradational. It is known that there exists an infinite number of such directions in both time and space (in depth). Therefore it is advisable to examine aggradational, degradational and stable directions in the uppermost levels of the frozen rock masses, on its upper boundary and also on the lower boundary and in the central part of the frozen rock mass. In that case the following frozen rock masses can be distinguished: a) degrading over the entire thickness; b) degrading in the upper part and aggrading on the lower boundary, and the reverse; c) aggrading upward and degrading downward; d) aggrading over the entire thickness, etc. Similar gradations can be derived as a function of time: short-period degradations, medium-period aggradations, long-period aggradations, Upper Quaternary degradations, etc.

Very great interest usually is aroused by determination of the contemporary state of frozen rock masses and also the history of their development. In each concrete case the classification scheme of the dynamics and the history of the development of frozen rock masses in a given region must be its own and be closely connected with the general course of the geological development of the investigated region.

In examining degradation and aggradation one should distinguish varieties connected only with change of the temperature field and varieties connected with the thawing of frozen rock masses and their formation again. In that case the classification characteristics with respect to the dynamics and history of frozen rock masses are closely interwoven with the characteristics of their composition and structure.

2. General Regularities of the Formation of Permafrozen Rock Masses

The thermal state of permafrozen rocks is connected with heat exchange on the surface of the ground between the lithosphere and atmosphere. That heat exchange is completed under definite geological and geographic conditions which determine the composition of the frozen rock masses and character of their development during the period of time under consideration.

Frozen rock masses have very varied composition, distribution, temperature regime, thickness and length of existence.

/The composition, structure and texture of frozen rock masses/ reflect distinctive features of the processes of freezing and their dynamics and are factors determining the properties of permafrozen rocks. The main component of frozen rocks is ice. The ice content of frozen rock masses, their structure and cryogenic textures depend mainly on: 1) the geological genetic type of the rocks (genesis and stratification of deposits) and facies they belong to (flood-plain, oxbow, lake facies, etc); 2) their dispersion and mineralogical composition; 3) the moisture content of deposits and their aqueous properties; 4) the character of the freezing of the rocks (syngenetic and epigenetic rock masses); 5) the rates of processes of freezing of rocks and the character of the temperature fluctuations on the surface. In that case it should be borne in mind that different in characteristic features in that respect are loose moist rocks in which ice appears as a rock-forming mineral. Magmatic and sedimentary dense rocks with low porosity and moisture content change their composition, texture and properties insignificantly during freezing.

Syngenetic rock masses, the freezing of which occurred during sedimentation, are widespread in alluvial, alluvial-lake, lake-swamp, deluvial, proluvial and seacoast sediments. They usually have a layered cryogenic texture connected with the fact that the seasonally thawed layer freezes in winter not only from above but also from below, in connection with which ice schlieren form in both the upper part of that layer and in its base. That phenomenon is connected with the dynamics of the seasonal thawing of the rocks and has latitudinal zonation (see Figure 28). Syngenetic rock masses with a fine-layered cryogenic texture are very widespread in permafrozen rocks, the t_f of which is below -3°. In regions where permafrozen rocks are prevalent and where t_f is above -3°, monolithic and lens-shaped cryogenic textures are mainly developed. Loose deposits which have frozen epigenetically are characterized by layered and layered-reticular cryogenic textures and their formation is connected with certain aqueous properties and the moisture content of the thawed rocks which are freezing and lying below, and also with the rate of freezing and, consequently, with the thermophysical properties of the rocks, the temperature regime on the surface and the geothermal gradient.

The optimal conditions of ice formation are connected with slow freezing, with an adequate constant flow of moisture toward the front of freezing and with large heat cycles in the freezing rocks. When there are harmonic fluctuations of temperature on the surface the rate of epigenetic freezing of the rocks diminishes with depth. However, when there is a constant water-bearing level the flow of moisture toward the front of freezing increases with depth. By virtue of these two circumstances the ice content of the frozen rock mass could increase with depth.

But the heat cycles in a frozen rock mass diminish with depth in an almost geometric progression, and that leads to a sharp reduction of the possible ice formation with depth. This is decisive and the general picture of the distribution of ice by depth usually is characterized by the presence of a layer with a larger ice content within the limits of the upper third of the frozen rock mass. Superposed on that general regularity is the influence of the geological structure and lithological features of the separate levels, and also their physical and aqueous properties.

/The influence of the upper boundary conditions on the formation of frozen rock masses and their temperature regime/. When there are harmonic fluctuations of temperature on the surface of the ground the upper boundary conditions are determined by the period of perennial temperature fluctuations on the surface T_{per}, the amplitude of temperature fluctuation on the surface during that period A_{o-per} and the average temperature on the surface during that period t_{o-per}.

Figure 49. Damping of amplitudes with depth as a function of T_{per}: 1 - 10 yrs; 2 - 40 yrs; 3 - 300 yrs; 3a - shift of phases -- maxima and minima -- in depth during fourth of a period.

In a number of works (Milankovich, 1939; Kudryavtsev, 1953; Shnitnikov, 1957; et al) the existence of perennial fluctuations of the conditions of heat exchange on the surface of the ground with the periods $T_1 = 11$ years, $T_2 = 40$ years, $T_3 = 300$ years and $T_n = n \times 10^5$ years, that is, relativaly short-period and long-period oscillations, was substantiated. In connection with that the actual changes of the thermal state of rocks have a complex oscillatory character which Figure 49 can illustrate. The influence of the length of the period (T_{per}) on the depth of permafrost (ξ_p) flows from formula (4.2.1); ξ_p is directly proportional to $\sqrt{T_{per}}$. Other conditions being equal, the thickness increases with decrease of t_{o-per} from south to north. That increase proceeds according to a complex law as a result of imposition of the influence of a large number of geographic and geological factors and conditions. As a result of that at the depth of the frozen rock mass there can simultaneously be aggradational and degradational processes, and the change of thickness of the frost will be determined by the intensity of the process predominating at the given depth. For example, on account of short-period oscillations thawing of the frozen rock mass can proceed from the surface, and at the same time on account of long-period oscillations of the temperature the base of the frozen rock mass can be lowered.

The influence of $A_{o\text{-}per}$ and $t_{o\text{-}per}$ on ξ_{per} is similar to that of the corresponding parameters on ξ_{seas}. However, it is evident from formula (4.2.1) that the depth ξ_{per} depends substantially on the heat flow from below and the counted value of the geothermal gradient g in the underlying thawed rocks. It has been found with computer calculations (Kudryavtsev and Melamed, 1967) that when g increases from 0 to 0.03 degree/m, ξ_{per} decreases by a factor of 1.5-2. In accordance with that, an especially great influence of g is noted in regions of high tectonic activity.

In the case of close bedding to the base of the frozen rock mass of a water-bearing horizon the influence of g on the thickness and temperature regime of the frozen rock mass is complicated and often overlapped by the invluence of convective heat transfer. In a number of cases that influences causes a thawed state of the rocks on those sections, where without it a frozen rock mass of a given thickness would have existed.

/The influence of the lithological characteristics of the composition and ice content of rocks/ on the dynamics of ξ_{per} is expressed by λ, C and Q_{ϕ} (formula (4.2.1). The thickness of the frozen rock mass is approximately proportional to $\sqrt{\lambda}$ (with an accuracy of within 10-15%). Therefore in solid rocks (λ_f = = 2.5 kca./(m)(hr)(degree) the thicknesses of the frozen rock mass, other conditions being equal, is 1.4-1.6 times as great as in loose rocks (λ_f = = 1.2 kcal/(m)(hr)(degree). Hence moist loose frozen rock masses with a small thickness can have an older cryogenic age that masses of solid frozen rocks with a great thickness of the frost.

A substantial influence is exerted on the formation of ξ_{per} by the thermophysical properties, namely the difference between λ_f and λ_t. In that case, during permafrost an effect of temperature shift similar to that during seasonal freezing is noted. The difference between λ_f and λ_t is noted mainly in the case of moist loose rocks.

The character of the geological structure of the region also has an influence on the formation of the thickness of frozen rock masses, especially when there are substantial differences in the thermal conductivity of the rocks. When the crystalline foundation or dense sedimentary rocks are close the thickness of the frozen rock mass will always be greater than in loose rocks.

/The influence of subsurface waters/ on the formation of the thickness of a frozen rock mass is of great importance, as in the process of perennial freezing the water-bearing layers of rocks become pressurized. In that case the physical properties of the rocks (water permeability, heat capacity, thermal conductivity, moisture content, etc) change.

The development of frozen rock masses is completed in a dynamic thermal interaction with subsurface waters through convective heat exchange between them. The character and intensity of the interaction of subsurface waters on the thickness of the frozen rock mass is manifested differently in different hydrogeological structures, since the feeding and discharge conditions, the regime, dynamics, chemism and temperature of the subsurface waters are connected with the latter. A very great thermal effect of subsurface waters on frozen rock masses is noted during their direct contact. This applies especially to thermal waters concentrated in deep faults and also artesian waters arising from great depths. Connected with them, as a rule, is the formation of talik zones or frozen rock masses of small thickness. Subsurface waters with high mineralization and brines contribute to the formation of a deeper zone of cooling of rock masses as compared with sections of rocks with fresh subsurface waters. In addition, during perennial freezing subsurface waters determine to a considerable degree the cryogenic structure of the frozen rock masses and also their cryogenic texture.

3. Forecasting the Formation of Cryogenic Textures of Permafrozen Rocks

The cryogenic texture characterizes distinctive features of the distribution of ice over the profile and the character of ice inclusions in rock as a function of the composition and distinctive features of freezing, which is of great importance for the development of the process of thermokarst and settlement of the surface during thawing, which develop both under natural conditions and especially as a result of their violation during the production activity of man.

The formation of cryogenic textures is determined mainly by three factors: 1) lithological facial characteristics of deposits (the composition and character of addition) and the properties of rocks; 2) their moisture content and the possibility of the flow of moisture toward the front of freezing and 3) conditions of freezing -- the character of the upper boundary conditions (harmonic or sudden changes of temperature on the surface) and the character of the lower boundary conditions (the geothermal gradient and heat flow from the depths of the earth).

1. Cryogenic Textures in Solid Rocks

In solid rocks the cryogenic textures are inherited. Usually in that case the ice inclusions are confined to the fissures (disjunctive, plicate, stratification and weathering). Also noted are ice inclusions connected with the filling of the karst cavities. The character of the ice inclusions of hereditary cryogenic textures in solid rocks is determined by the character of the fissures and cavities and is connected with general geological conditions, above all with the history of the geological development of the region. Simultaneously with that the character of the inherited cryogenic textures in solid rocks is determined to a considerable degree by the hydrogeological conditions -- the presence of fissure, karst-fissure and stratal-karst waters of water-bearing complexes subjected to perennial freezing. The latter circumstance determines the chemism of the ice of inherited cryogenic textures in solid rocks. During freezing of solid rocks, fracturing increases on account of additional opening of existing fissures as a result of the freezing of water and as a result of processes of thermal cracking.

On the basis of distinctive features of the formation of ice in solid rocks the following types of ice can be distinguished (Krivonogova, 1975):

1 - cement ice, forming in cracks filled with water before the freezing of the rock mass (below the level of the ground waters);
2 - injection ice forming in fissures when water is introduced into them under pressure as a result of irregular freezing of fractured flooded strata;
3 - infiltration ice forming during the infiltration into a frozen rock mass of waters of talik zones or surface waters;
4 - sublimation ice forming on account of vapor-phase moisture entering the fissures and cavities of the frozen rock mass;
5 - segregation ice forming as a result of unpressurized migration of moisture toward the front of freezing.

During the freezing of solid rocks with the indicated types of ice, ice-saturated sections form which are characterized by different distention. In that case the greatest distention arises during the formation of segregation ice, and the least during the formation of sublimation ice.

2. Cryogenic Textures of Loose Deposits

/Regularities in the formation of cryogenic textures during epigenetic freezing of rocks/. The cryogenic textures of epigenetic frozen rock masses form during either harmonic or abrupt variations of the upper boundary conditions. Dependent on the character of the change of temperatures on the surface of the ground is the distribution (thickness, frequency and direction) of ice schlieren in the freezing rock.

During harmonic change of the upper boundary conditions in coarse soils (gravel-pebble and sandy deposits), if they are water-saturated, form monolithic (massive) cryogenic textures. In that case all the pores usually are filled with ice and the mineral particles of the skeleton of the ground are not displaced very much -- by the amount of the volumetric expansion of ice in comparison with the volume of water during freezing (9%). In that case schlieren textures are absent (with the exception of injection ice). The thickness of the latter and the conditions of occurrence are determined by distinctive features of the frost hydrogeological situation.

In finely dispersed rocks the cryogenic textures form in all their variety in accordance with the general laws examined in section 3 of Chapter 9. When the temperature change on the surface in a perennial profile has a harmonic course and in the absence of an influx of water from water-bearing levels the following regularity is noted. In the initial period of freezing of moisture-saturated finely dispersed rocks in the uppermost level an abundance of ice schlieren of small thickness is noted. This is connected with the greater moisture content of the ground at the front of freezing, larger temperature gradients and high rates of freezing. Gradually with depth there is a substantial decrease of the ice schlieren and microschlieren and infrequent fine-schlieren cryogenic textures form, which is connected with decrease of the moisture content of the rocks on the front of freezing. Since large temperature gradients and high rates of freezing are still preserved in the layer, the moisture does not succeed in extending to the front and making up for the deficit of moisture which formed on account of migration into higher-lying rocks. Under the icy level there is drying of the rocks, during the freezing of which fine vertical fissures often form, already filled with ice after freezing.

During further increase of the depth of freezing the temperature gradients and rates of freezing diminish, moisture succeeds in extending to the front of freezing in a large quantity and infrequent horizontal medium and thick schlieren of ice start to form. This regularity ought to be intensified with depth, except for one fact. During harmonic oscillations at the surface the heat cycles are damped with depth in accordance with a law similar to geometric progression (Figure 50a). By virtue of this, in the lower part of a permafrozen rock mass during its freezing the heat cycles become so insignificant that they already cannot assure the formation of thick ice schlieren. Thus according to the heat cycles the maximal thicknesses of the segregation schlieren of ice are confined to the part of a permafrozen rock mass of medium depth. Frequently, near the base of frozen rock masses the heat cycles are sufficient only for the formation of a thin concluding lens of ice.

During repeated freezing and thawing a portion of permafrozen rock masses can thaw both from below and from above and again freeze. In that case the newly formed cryogenic textures will correspond to the new freezing conditions. Thus, for example, during the partial tnawing and second freezing of loose deposits a profile can be obtained as on Figure 51, where the distribution of cryogenic textures is shown in accordance with the ice content during primary freezing -- a broken line for the upper part, to the depth h, and a solid line to deeper levels. As a result of partial thawing of permafrozen rock masses to the depth h and subsequent freezing of rocks, in that layer form new cryogenic textures designated by a solid line. This is connected with the fact that during secondary freezing, as a result of drawing water to the front of freezing, there is a redistribution of moisture in the layer h and at the top of the layer form frequent fine and medium schlieren cryogenic textures, and in the middle part of the layer h, infrequent medium and thick schlieren ones. In that case a relative dehydration of the lower part of that layer to a moisture close to w_{un} will be noted. It is interesting to note that in the absence of waters above frost the total moisture balance of the layer h must be equal to the total moisture content of the rocks of that layer, since the permafrozen rock mass underlying the layer h is water-pressurized, as a result of which the inflow of moisture from the depths is excluded.

Of great importance in the formation of cryogenic textures in a frozen rock mass are the conditions of moisture exchange on its base. In particular, in the presence of waters under the frost the size of the ice schlieren increases with depth, in connection with which the total ice accumulation considerably surpasses the stock of moisture in the layer ξ_{per} by the start of freezing. During the slow freezing of moisture-saturated finely dispersed rocks there

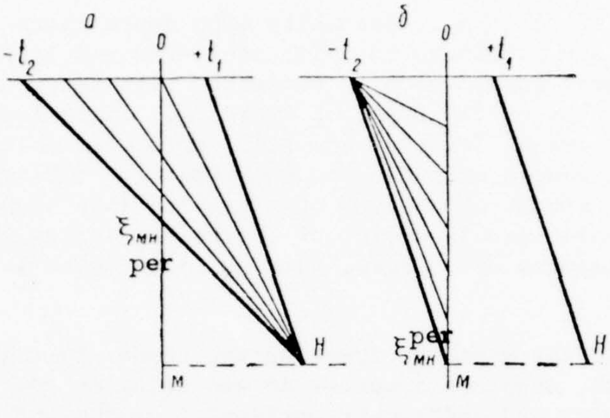


Figure 50. Character of the temperature field during sudden (a) and harmonic (b) variation of t on the surface of the ground.

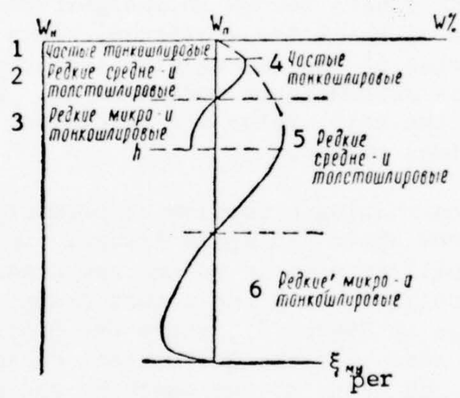


Figure 51. Schematic diagram of the formation of cryogenic textures in a permafrozen rock mass, the upper part of which to the depth h has thawed and refrozen.
1 - Frequent fine schlieren 2 - Infrequent medium and thick schlieren 3 - Infrequent micro and fine schlieren 4 - Frequent fine schlieren 5 - Infrequent medium and thick schlieren 6 - Infrequent micro and fine schlieren

occurs the formation of thick schlieren textures, since a rather large amount of moisture from the lower-lying levels succeeds in reaching the front of freezing. This leads to dehydration of those levels, as a result of which, as a rule, the grounds of mineral layers acquire a moisture content close to the maximal molecular. In that case the ground shrinks and small vertical cracks form in it. In the process of freezing of those mineral layers, along the vertical cracks formed moisture is drawn to the front of freezing and thin vertical ice schlieren form. As a result, during relatively slow epigenetic freezing form thick schlieren cryogenic textures with thin vertical veins.

The possibility of forming in finely dispersed rocks a given type of cryogenic textures can be predicted with the method described in section 3 of Chapter 9. In particular, it is pointed out there that the application of oscillations of different periods substantially affects the formation of cryogenic textures even during single freezing. Thus, for example, in the process of long-period freezing and the formation of a permafrozen rock mass, medium- and short-period warmings can be superposed, which lead to deceleration of the rates of freezing and the formation of thick-schlieren cryogenic textures. On the contrary, the application of medium- and short-period coolings on long-period ones leads to acceleration of the rates of freezing, halting of the formation of thick-schlieren cryogenic textures and their transition into thin- and micro-schlieren and even monoliths.

A similar situation is also noted in the seasonal freezing of rocks. In that case the short-period thaws in the course of the winter also lead to the formation of thick-schlieren textures in the layer of seasonal freezing and by the same token to increase of the rock heaving. Explained by the same considerations is the difference in the process of heaving in regions with sharply continental and maritime climates. Under the conditions of a sharply continental climate the freezing rate is high enough, in connection with which in the upper level of seasonal freezing form monolithic and micro- and thick-schlieren cryogenic textures. In the central part of the freezing layer form thin-schlieren cryogenic textures, through which heaving of the ground mainly proceeds. During freezing of the lower part of the layer ξ, where the thermal cycles are sharply reduced, form their schlieren and monolithic cryogenic textures.

Under the conditions of a maritime climate the freezing rates are very low even in the uppermost part of the layer ξ and therefore the formation of thick-schlieren textures, even when there is an adequate inflow of moisture from below toward the front of freezing, can be noted within the limits of the entire freezing layer.

Near the southern boundary of the region of permafrozen rocks, where t_ξ is close to zero, the annual thermal cycles are damped within the layer of seasonal freezing. By virtue of that the lower level of the layer ξ, as a rule, is characterized by a monolithic and thin-schlieren cryogenic texture.

During a sudden change of temperatures on the surface of the ground the change of the temperature field proceeds in accordance with the schematic diagram on Figure 50. For case "b" the thermal cycles remain constant in depth, since at any depth the temperature must change at most by the value $(t_2 - t_1)$. Therefore the thermal cycles have no influence on the formation of cryogenic textures, as they are constant. By virtue of that the regularities in the formation of cryogenic textures for case "b" are determined only by the rate of freezing in accordance with the temperature gradient and hydrogeological conditions (moisture content of the ground and the possibility of moisture flow toward the front of freezing from lower-lying levels). The temperature gradient and rate of freezing in case "b" decrease with depth and, consequently, the probability of the formation of ice schlieren increases with depth. As a result of that the ice content in permafrozen rocks which froze during a sudden change of temperature on the surface of the ground increases with depth.

During the partial thawing of such permafrozen rock masses and their subsequent freezing, already as a result of harmonic temperature oscillations on the surface complex superposed cryogenic textures can form, with a regular distribution of them by depth characteristic of both harmonic and sudden changes of the upper boundary conditions. The concrete conditions of the formation of cryogenic textures can be calculated with the method presented in Chapter 9.

/Regularities of the formation of cryogenic textures during syngenetic freezing of deposits/. The main regularities and character of formation of cryogenic textures of syngenetically freezing deposits are connected with the freezing from below of the layer of summer thawing and with regularities of the process of frost cleavage. Under the conditions of sedimentation of flood-plain, coastal-maritime and slope facies in a region of permafrost, frost cleavage is accompanied by the formation of reopened-vein (wedge-vein) ices or primordially ground veins, depending on the correlations of the minimal and average annual temperatures of rocks on the surface, their amplitude, the layer thickness ξ and the depth of cracking. As a result of that form cryogenic textures of syngenetic rock masses with a polygonal lattice of reopened-vein ices, ground or mixed ice-ground veins separated by masses of mineral ground. The dimensions and conditions of occurrence of the reopened-vein ices and ground seams and the dimensions of the polygons are determined by the composition of the ground and its properties, the temperature regime of the grounds at the moment of total freezing of the layer of summer thawing, and also the conditions of sedimentation. The lattice size of the reopened-vein ices can be determined with the formula of B. N. Dostovalov (1952). The probability of formation of icy, ground or mixed veins can be determined with the method of N. N. Romanovskiy (1971, 1972). He also established the signs of different types of vein formations (Figure 52) and their connection with the temperature zonation (see Figure 119 in section 3, Chapter 8).

The cryogenic textures of the mineral masses of ground between ice veins are determined by the following conditions. In the process of sedimentation, during silt deposition in the period of high water, the surface marker is elevated by a certain amount. As a result of that the marker of the upper boundary of the permafrozen rocks must simultaneously rise. This occurs because the depth of the seasonal thawing of the following year does not reach the depth of thawing of the previous year. As a result a permafrozen rock mass forms which represents the sum of elementary layers which are residual levels of the freezing layer of seasonal thawing. It is obvious that in that case the cryogenic textures of syngenetically freezing permafrozen rocks can be only either thin-schlieren or monolithic. The latter is determined by the conditions of freezing of the layer of summer thawing. In that case, when the layer of summer thawing freezes only from above, all the moisture is drawn toward the front of freezing into the upper levels, the lower are dehydrated and only a massive texture forms. This occurs near the southern boundary of the permafrozen rock masses, where all the annual thermal cycles are extinguished in a layer of seasonal thawing. In more northern regions of the permafrost region, especially in zones IV and V (according to V. A. Kudryavtsev, 1954) the thermal cycles on the base of the layer of seasonal thawing reach large values. As a result of that the freezing of the layer of summer thawing proceeds not only from above but also from below. In that case the moisture in the layer of summer thawing is drawn toward both

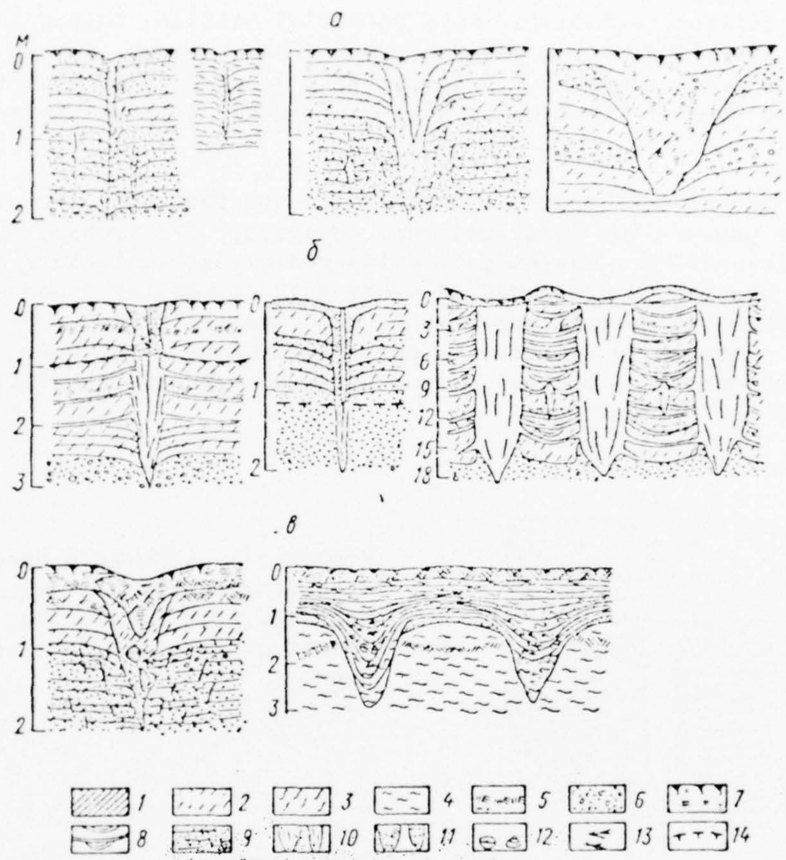


Figure 52. Types of polygonal-vein formations and cryogenic textures of enclosing deposits (schematicized according to N. N. Romanovskiy, 1972). a - primordially ground veins forming in seasonally thawed and seasonally frozen layers; b - ice veins in a frozen rock mass; c - pseudomorphoses on ice veins; 1 - loams; 2 - sandy loams; 3 - loesses; 4 - loessial rocks and aleurites; 5 - peat; 6 - sand and gravel; 7 - layer of soil and vegetation and humified rocks; 8 - ice schlieren and "bands" in syngenetically freezing deposits; 9 - lamination of rocks and minor faults; 10 - re-opened vein ices; 11 - ground veins; 12 - tests of freshwater mollusks; 13 - allochthonic plant residues; 14 - upper surface of permafrozen rocks (STS boundary).

the upper and lower fronts of freezing, as a result of which ice schlieren form both in the upper part and in the base of the layer of seasonal thawing. In that case, in the process of sedimentation, form residuary layers of the freezing layer of summer thawing, infrequently represented by ice with a small admixture of mineral particles. In a perennial profile, thinly layered cryogenic textures of frozen rock masses form in that manner. During multiple freezing of such ice schlieren on one another is accordance with the short-period oscillations of temperature banded cryogenic textures form (Maksimova, 1972).

A characteristic feature of those deposits is the fact that the summary ice content often exceeds the total moisture capacity. The probability of formation of permafrozen rock masses with a layered cryogenic texture can be calculated with the method presented in section 12 of Chapter 3 and section 3 of Chapter 9. Thus the general appearance of syngenetic frozen rock masses in the presence of reopened vein ice is characterized by a polygonal lattice of vein ices with masses of mineral ground between them with a thinly layered cryogenic texture (Figure 53).

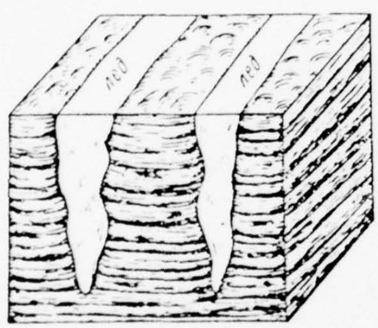


Figure 53. Schematic cross-section of a permafrozen rock mass with reopened vein ice during syngenetic freezing. a - ice b - Block diagram.

Knowledge of the regularities in the formation of cryogenic textures and the character of their distribution, and also of the conditions of occurrence of underground ices during epigenetic as well as during syngenetic freezing of rocks is the basis for compiling a forecast of the change of frost conditions. Calculation of the possible amount of heaving in connection with the formation of cryogenic textures or layers as a result of thermokarst processes determine the geological engineering evaluation of territory, the conditions and principles of construction and the character of the measures necessary for the creation of optimal working conditions of a structure (see section 3 of Chapter 9).

4. Determination of the Minimal Cryogenic Age of Frozen Rock Masses

The question of determination of the length of existence of rocks in the frozen state is of great importance in determining the genesis of frozen rock masses and their dynamics in Quaternary time. For syngenetic permafrozen rock masses the question of determination of the age of the frost is solved unequivocally with determination of the geological age of the frozen deposits. During epigenetic freezing of rocks the cryogenic age of the frozen rock masses can be determined with formula (4.2.1), by means of which one can take into consideration the different conditions of freezing of the upper layers of the lithosphere and their composition. In that case one bears in mind that, other conditions being equal, greater thicknesses of frozen rock masses are always connected with larger periods of perennial oscillations of temperature on the surface of the ground. Therefore for each given thickness of the frozen rock mass (according to formula 4.2.1) the smallest possible period of temperature oscillations and the cryogenic age of that rock mass corresponding to it can be determined. The data necesary for that, the thickness of the frozen rock mass (ξ_{per}), the thermophysical properties of the rocks (λ, C and Δw) and also the geothermal gradient (g) in thawed rocks (below the base of the frozen rock mass) are determined as a result of field investigations. The unknown size of the period of oscillation T_{per} is selected so that the minimal value of the period corresponds to the maximal thicknesses of the frozen rock mass. This condition is valid when the average temperature of the rocks (t_{o-per}) during the period T_{per} on the surface of the frozen rock mass will be equal to the amplitude of the temperature oscillations A_{o-per}.

The lower boundary conditions are determined in the following manner (Kudryavtsev, 1968). At large periods and small amplitudes of temperatures on the lower surface of the frozen rock masses, and also in the presence of phase transitions of water accompanying the freezing and thawing of rocks, the minimal and maximal values of g can be taken as the envelopes of temperature characteristic of harmonic long-period oscillations.

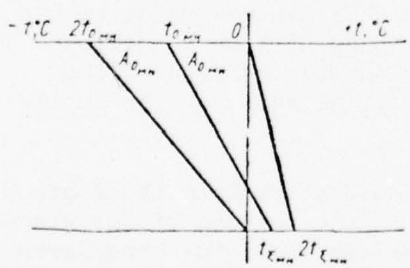


Figure 54. For determination of the minimal cryogenic age of permafrozen rock masses.

In accordance with that one can readily obtain $t_{\xi-max}$ during the period T_{per} on the lower surface of the frozen rock mass and t_{o-min} on its upper surface. The two values are obtained graphically by the construction of straight lines drawn from the point 0^o on the upper surface of the frozen rock mass with the angle of inclination g_{min} and from the point 0^o on the lower boundary of the frost with the angle of inclination t_{max} (Figure 54). It follows from formula (4.2.1) that with increase of the thickness of the frozen rock masses the minimal cryogenic age increases in a quadratic dependence. The change of the minimal age of frozen rock masses depends substantially on their composition and moisture content, that is, to freeze loose deposits to a definite depth twice as much time is required as for solid rocks. If a ratio of the coefficients of thermal conductivity of 1:2 is taken for loose Quaternary deposits and solid pre-Quaternary rocks, then 200-meter frozen loose rock masses will have a minimal cryogenic age similar to frozen rock masses with a thickness of 400 m in solid rocks. The moisture content of the deposits also has a substantial influence: when the heat expended on phase transitions of water is doubled the minimal cryogenic age of the frozen rock mass increases by about 1.4-1.5 times while their thickness remains unchanged. The dependence of the thickness of the frozen rock masses and their minimal cryogenic age on g is complex. However, it can be assumed that the minimal cryogenic age of the frozen rock mass will vary in proportion to g^2. It also can be noted that with increase of $A_{o\text{-}per}$ and reduction of $t_{\xi\text{-}per}$ the thicknesses of the frozen rock masses will increase and the minimal cryogenic age will decrease.

Calculation of the Minimal Cryogenic Age of Permafrozen Rock Masses (Example 23)

Determine the minimal cryogenic age of permafrozen alluvial deposits if in the process of a frost survey the following data were obtained: the thickness of the frozen rock mass is 100 m, composed of interlayered silty sandy loams and loams. The properties of the deposits are characterized by: $\lambda_f = 1.5$, $\lambda_t = 1.0$ kcal/(m)(degree)(hr); $C_{vol\text{-}f} = 500$ kcal/(m^3)(degree); $Q_{\phi\text{-}per} = 20{,}000$ kcal/(m^3) ($Q_{\phi\text{-}per}$ is the heat of phase transitions during the perennial freezing of rocks). In the underlying thawed deposits (loams glmQ_{II}) $\lambda_f = 1.25$ kcal/(m)(degree)(hr) and g = 0.03 degree/m.

Solution. It is necessary first to determine the conditions under which the maximal thicknesses of permafrozen rock masses form at a minimal value of the period of oscillation $T_{min\text{-}per}$. It is obvious that they will be conditions in which the average temperature of the rocks $t_{o\text{-}per}$ on the surface of the permafrozen rock mass during the period $T_{min\text{-}per}$ will be equal to the amplitude of oscillations of temperatures on the surface.

In determining the lower boundary conditions one should start from the fact that the long-period oscillations of heat exchange on the surface of the ground, which determine the formation of permafrozen rock masses, as a rule propagate not only in the frozen rock mass but also in the underlying thawed rocks. Therefore the geothermal gradient in the underlying thawed rocks varies in a certain range from the maximal to the minimal values.

In accordance with that the maximal temperature during the period of fluctuations ($2t_{\xi\text{-}per}$) can be obtained on the lower boundary of the permafrozen rock mass and the minimal ($2t_{o\text{-}per}$) on its surface. The former is obtained by constructing a straight line which starts from $t = 0^o$ on the surface of the permafrozen rock mass and has an angle of inclination corresponding to the minimal value of the geothermal gradient. The latter ($2t_{o\text{-}per}$) is also obtained by constructing a straight line, but one passing through the point $t = 0^o$ on the base of the frozen rock mass and having an angle of inclination corresponding to the maximal value of the geothermal gradient. If we connect the middle of the obtained segments on the lower and upper surfaces of the permafrozen rock mass (points corresponding to $t_{o\text{-}per}$ and $t_{\xi\text{-}per}$) we obtain the axis of long-wave oscillations, the angle of inclination of which corresponds to the average value of the geothermal gradient.

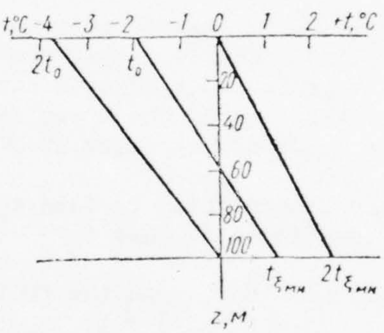


Figure 55. Determination of minimum cryogenic age of permafrost strata.

In accordance with the indicated construction a calculation also is made of the minimal cryogenic age of a permafrozen rock mass. In that case the complete length of the period (T_{per}) is determined at which the uppermost levels of the frozen rock mass are in a constantly frozen state. The equation for permafrost (4.2.1), which is solved for T, is used in the calculation.

In accordance with what has been said, the procedure in solving the problem must be the following:

1. We find the values of the maximal and minimal gradients in a 100-meter rock mass. It is obvious that the maximal gradient will be observed at λ_t and the minimal at λ_f, since $\lambda_f > \lambda_t$. It is known that the gradient in underlying thawed rocks is 0.003 degree/meter when their thermal conductivity is 1.25 kcal/(m)(degree)(hr). Provided that the heat flux passing through the lower boundary of the frozen rock mass is equal to the flow through the surface of the soil, the gradient in the upper 100-meter rock mass will vary from 0.038 at λ = 1.0 to 0.025 degree/meter at λ = 1.5 kcal/(m)(degree)(hr).

2. We make the necessary construction to find t_o and $t_{\xi\text{-per}}$ as indicated above. It is evident from Figure 55 that t_o = -1.9^o and $t_{\xi\text{-per}} = 1.25^o$.

3. From equation (4.2.1) we find T when the following initial data are substituted: $A = t_o = 1.9^o$; $(t - g\xi_{per}) = t_{\xi\text{-per}} = 1.25^o$; ξ_{per} = 100 m; α = = 20 kcal/(m^2)(hr)(degree);

$$A_{cp} = \frac{1}{2}(|t_0| + t_{\xi мн}) \simeq 1{,}6^{\circ}.$$

We substitute all the data in formula (4.2.1)

$$100 = \frac{1{,}9 - 1{,}25 + \dfrac{\left[1{,}6 \cdot \dfrac{(1{,}9 - 1{,}25)\sqrt{\dfrac{1{,}5 \cdot T}{3{,}14 \cdot 500}}}{1{,}6 + 20} + 20 \cdot 100\right] \cdot 20}{1{,}6 \cdot \dfrac{(1{,}9 - 1{,}25)\sqrt{\dfrac{1{,}5 \cdot T}{3{,}14 \cdot 500}}}{1{,}6 + 20} + 20 \cdot 100 + \sqrt{\dfrac{1{,}5 \cdot T}{3{,}14 \cdot 500}}(1{,}6 + 20)}}{1{,}6 + 20} \times \sqrt{\frac{1{,}5 \cdot T}{3{,}14 \cdot 500}}.$$

When we solve the equation for T we find that $T \approx$ 67,250 years.

5. Influence of the Productive Activity of Man on Change of the Temperature Regime and Thickness of Permafrozen Rock Masses

The economic opening up of territory leads to a sharp change in the natural situation, and consequently also of the frost conditions. On sections of structures those changes are so considerable that the frost conditions differ substantially from those noted before construction in the course of a frost survey. As a rule, at construction sites the plant cover is removed and a leveling of the terrain is made which changes the steepness of the slope and sometimes the exposure (by cutting away and filling in), and the composition and moisture content of the soils are changed. As a result of the uncovering of basins, soils are drained or flooded. The construction of drainage structures and sewer systeme, bodies of water and reservoirs also leads to change of the moisture regime of the soils. Various artificial coverings (asphalt, concrete, etc) and the arrangement of lawns and tree plantings lead to a sharp reduction of the surface of the ground and change of the temperature regime of the soils. Directly under industrial and public structures, depending on their thermal regime, one can note either a lowering of the average annual temperatures and preservation of the frozen state of the soils (under cold structures) or thawing of frozen rocks with the formation of a thawing basin (under structures with great heat release into the ground).

Construction on large areas (settlements of a city type and cities) in a permafrost region causes important changes of front conditions on the entire territory of the settlement or city, including even the sections where the natural conditions remain undisturbed, if their area is small enough in comparison with the neighboring sections of the construction site. In that case the change of frost conditions is connected not only with change of the natural conditions on the area of the construction site but also with change of the microclimatic conditions on adjacent sections. In the case of linear structures (railroad lines, highways, gas and oil pipelines, electric power transmission lines, underground communications, etc) very important changes of the natural conditions, including the frost conditions, are noted within the limits of the lines and rights of way. For airport construction those changes extend to a far larger territory, including air strips and the airport facilities. In the case of hydraulic engineering construction the change of natural conditions, and especially of frost conditions, includes enormous territories of artificial reservoirs and adjacent regions. The same changes are observed in the regions of the opening up and working of minerals and in regions of the development of agriculture in a region of permafrozen rocks.

It is obvious that all changes of frost conditions can be studied only on the basis of knowledge of regularities in the formation of permafrozen rock masses in connection with the character of the natural situation and its changes during construction. Forecasting the change of frost conditions during different kinds of economic assimilation, having a single common basis -- regularities in the formation of permafrozen rock masses, is characterized by specifics connected both with distinctive features of change of the natural environment during construction and with the character of the interaction of structures with frozen ground during operations.

We will examine the formulation of some problems in forecasting change of frost conditions, for example, during the construction of housing settlements or cities. As has already been pointed out, in some cases a need arises for construction while preserving the frost conditions of the ground at the base of the structures. Then, obviously, in the compilation of the forecast problems must be solved in the determination of the dynamics of the temperature regime of permafrozen rocks and its influence on the properties and thickness of the frozen rock mass. The temperature regime of the ground under buildings in that case must be in a small definite range of negative temperatures, and this is achieved by various measures provided for in the construction of the foundations of definite structures. On adjacent sections the temperature regime of the rocks and the depth of the seasonal thawing must be calculated with consideration of concrete changes on the surface of the soil, as was shown in Chapter 4 (examples 1 and 3). In other cases, in forecasting change of front conditions it can be a matter not only of change of the temperatures of rocks within the range of negative values but also of the thawing of permafrozen rock masses and the thermal precipitations arising in that case or of new formation of frozen rock masses and the formation of their cryogenic textures. The formulation of the problems in the compilation of a forecast is determined in all cases by the complexity of the interaction of the structure and environment or complex of different structures and the environment. In the case where a forecast is compiled for small areas or separate structures in uniform widespread conditions, the problem is assumed to be unidimensional. It is reduced mainly to forecasting the change of the radiation thermal balance of the surface of the soil and its temperature regime (section 1, Chapter 2 and example 14, Chapter 5), determination of the thawing rates of permafrozen rocks under a structure, determination of the contours of the thawing basin (according to D. B. Redozubov, section 4, Chapter 3 and examples 38 and 39, Chapter 8) and determination of the thermal precipitation of the soils (section 9, Chapter 6).

In compiling a forecast of the change of frost conditions for large territories with a different development the matter is much more complex. In that case it is necessary to take into account the complex thermal interaction of buildings of different size with heat release and cold, the walks separating them, lawns, etc. In that case the temperature field at the points of observation is often determined by two- and even three-dimensional problems. And in spite of the fact that the composition of a forecast will be presented with the same listing as for separate buildings, the solution of the problems is extremely complex. What has been said can be illustrated by the following scheme. Presented on Figure 56 is a cross-section of the area of a development within which are buildings with heat release. Under them the thawing basins are shown. On the remainder of the area the conditions are favorable for the existence of permafrozen rock masses. On the schematic diagram it is shown that the thickness of the frozen rock mass considerably exceeds the depth of the thawing basins. In that case, development of the area leads only to thawing of the frozen rock masses under the structures and does not change the general picture of the bedding of permafrozen rock masses over the area, but changes its thickness on separate sections. However, on the whole, development on a large area can lead to reduction of the thickness of the frozen rock mass in comparison with natural conditions in accordance with the integral temperature for the territory of the development corresponding to the new surface conditions. A solution of such a problem can be obtained by simulating the problem on an electric integrator. A schematic diagram of the solution follows (Figure 57).

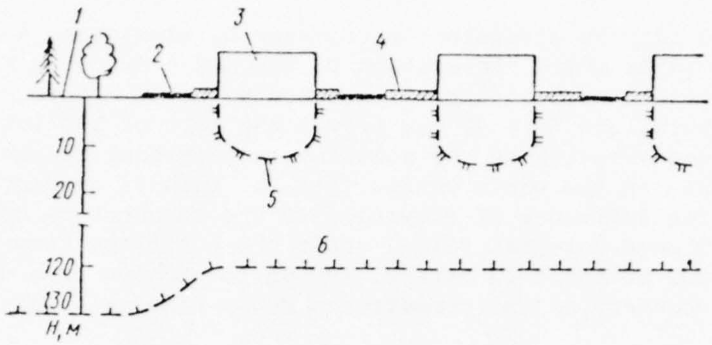


Figure 56. Formation of thawing basins under buildings when the frozen rock mass is very thick: 1 - sections with natural conditions; 2 - traveled parts of paths; 3 - buildings; 4 - lawns; 5 - boundary of thawing basin under building; 6 - lower boundary of permafrozen rock mass after development of area.

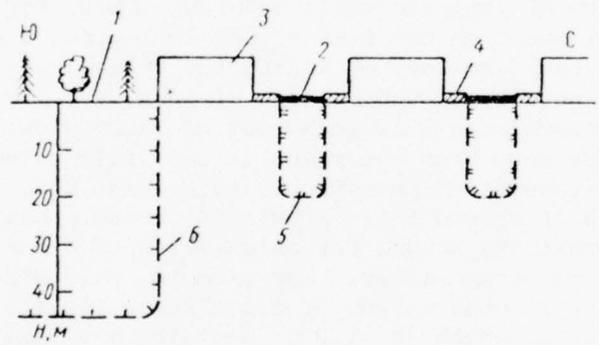


Figure 57. Formation of taliks (tabetisols) on a developed section when the frost is not thick: 1 to 4 - the same as on Figure 56; 5 - boundary of islands of permafrozen rocks on the area of the development; 6 - boundary of permafrozen rock mass after development of the area.

For example, a section characterized by a continuous expanse of permafrozen rock mass consisting of alluvial-lake sandy loam and loam deposits, the average annual temperature on the surface of which is -3°, is developed with public buildings. The thickness of the frozen rock mass is 130 meters and in the underlying thawed rocks $g = 0.03$ degree/m. After development of the section, under the thawing buildings t_o rose to $+2^{\circ}$. On the lawns adjacent to the buildings the temperature of the surface of the ground also increases and depends on the exposure of the walls: on the southern side of the building it is -0.5° and on the north, -1.2°. On the traveled part of the paths t_o decreases to -5.5°. It must be determined how the thickness of the frozen rock mass varies on the area of the development.

The problem is solved with an electrical integrator by simulating a steady temperature field in soils which corresponds to the new conditions on developed territory. The width of buildings is assumed to be 20 meters, that of the lawns to be 5 meters, and that of the travelling part of the paths to be 10 meters. The obtained results of the solution testify that the thickness of the frozen rock mass on the whole varies little. This is evident from the the fact that under the influence of elevation of the temperature of the surface of the soil (its mean-integral value) under the buildings form thawing basins with a thickness of about 13 meters, and on the entire area of the development the lower boundary of the permafrozen rocks rises a total of more than 10 meters.

Presented on Figure 57 is the same case of economic opening up of territory but at a small thickness of the frozen rock masses, when the thawing basins under heat-releasing buildings form penetrating taliks. Under those conditions the continuous extent of permafrozen rock masses is destroyed. Depending on the correlation of areas with surface conditions favorable and unfavorable for the preservation of the frozen state of rocks, a talik can form on a large area of the territory of a development and frozen rocks will be preserved only in the form of separate small islands. Thus, for example, in the case of an urban development in the form of parallel streets with an arrangement of boulevards on the territory of a city the following picture can occur. Under buildings, as a result of much release of heat, a talik forms. On lawns planted with shrubs accumulates a large amount of loose snow, the height of which increases because snow from the roads is partially piled on the lawns. All this leads to the thawing of permafrozen rocks also under the lawns (see Figure 57). In a mass of thawed soils a belt of frozen rocks can be preserved under a road. An approximate scheme for calculation of a forecast in such cases is made in the following manner. For example, used under the development is a section of continuous extent of permafrozen rocks with a thickness of 43-45 meters, the t_o of which is -1.0^o. Deposits consists of alluvial sands and sandy loams with pebbles and gravel and layers of sandy loam. The geothermal gradient in the underlying thawed rocks is 0.03 degree/meter. As a result of the development the temperature regime of the soils also varies and under the heated buildings $t_o = 5.0^o$ is established. Under the lawns on the southern side of the buildings t_o rises to 1.0^o, and on the northern to 0.5^o, while on the travelled part of the roads it decreases to -4.5^o. The width of the buildings is 20 meters, of the lawns 5 meters, and of the travelled part of the roads 10 meters. It must be determined how the thickness of the permafrozen rock mass varies on the area of the development.

The problem was solved on an electrical integrator by simulating the steady temperature regime of the soils corresponding to the new conditions. The results of the solution are shown on Figure 57 and testify that under the indicated conditions on the territory of the development the entire mass of frozen soils thaws. Permafrozen rocks are preserved only under the travelled part of the road, but their thickness is reduced to 20 meters.

In compiling a forecast of the change of frost conditions it is necessary to turn attention to the following. If the construction is done on sections which have permafrozen rock masses of great thickness, the thawing basins under

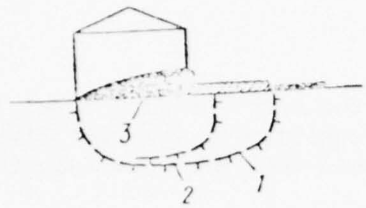


Figure 58. Change of the contour of the thawing basin during change of conditions on the surface: 1 - original boundary of thawing basin; 2 - boundary of thawing basin after widening of road at the expense of part of the lawn; 3 - plot formed on account of freezing of part of the original thawing basin.

the area of the development does not emerge beyond the limits of the frozen rock mass. In that case the following can occur. For example, when the road is widened at the expense of the lawn (Figure 58) the total thawing basin on the developed area under the building and lawns must partially freeze, and in the closed volume of the basin a large hydrodynamic pressure must arise, with an outflow of ground waters within the building.

A similar situation can occur on account of change of lower boundary conditions. When the frozen rock mass is not thick the general thawing basin (see Figure 57) can go beyond the limits of the permafrozen rock masses, and under the area of the development forms a penetrating talik. We will assume that in that region the small thickness of the frozen rock masses is connected with the warming influence of the subsurface waters. When the waters of that level are used and there is a sharp lowering of their level that warming influence is removed, which can be expressed in a decrease of the geothermal gradient by a factor of 1.5-2, which in turn leads to increase of the thickness of the frozen rock masses also by approximately 1.5-2 times.

Approximate Estimate of the Change of Thickness of Permafrozen Rock Masses Under the Influence of Change of the Water Level Under the Frost (Example 24)

Determine how the thickness of a permafrozen rock mass on the area of a development changes if it is established that as a result of use of waters below the frost their level on that territory will drop in 10 years 15 meters below the base of the frozen rock mass, recorded at a depth of 24 meters in a frost survey conducted in the stage of searches according to a plan task and working drawings.

The territory is characterized by the following ground-frost conditions. From the surface to a depth of 18-20 meters lies a formation of frozen alluvial deposits consisting of loams with thin layers (about 5 cm) of sandy loams and sand. Below (to a depth of about 50 meters) lie water-bearing sands and coarse gravels of a riverbed facies. The permafrozen rocks have an average annual temperature ($t_o = t_\xi = t_m$) of -1.5^o and are characterized by the following indicators of properties: for loams $\gamma_{sk} = 1.2$ g/cc, $w_{vol} = 30\%$, and $\lambda = 1.3$ kcal/(m)(degree)(hr); for sands $\gamma_{sk} = 1.78$ g/cc, $w_{vol} = 28\%$ and $\lambda = 1.8$ kcal/(m)(degree)(hr), and the filtration coefficient of the sands is ~ 10 m/hr.

The waters below frost are contacting. As a result of their influence the temperature gradient in the frozen rock mass below the layer of annual fluctuations (7 meters) reaches 0.09 degree/meter. When the temperature regime of the rocks is steady the geothermal gradient in the water-bearing level is $\sim$ 0.06 degree/meter (found from the condition of equality of the heat fluxes directed toward the lower boundary of the permafrozen rock mass and away from it).

During use of the water-bearing level for 10 years the heat flux toward the lower boundary of the frozen rocks decreases by a factor of 1.5 on account of a considerable reduction (by 15 meters) of the level of the subsurface waters (the level loses contact with the frozen rock mass). The moisture content of the sand diminishes to 7% by volume when the subsurface water level drops.

Solution. 1. We determine how the temperature gradient in the frozen rock mass varies in the new steady temperature regime corresponding to a reduction of the subsurface water level by 15 meters and a decrease of the heat flux toward the frozen rock mass by a factor of 1.5.

The heat flux toward the lower boundary of the permafrozen rocks before start of use of the level, obviously, was $q = \lambda g = 1.8 \times 0.06 = 0.108$ kcal/(m^2)(hr).

After 10 years of use it became

$$q' = \frac{0{,}108}{1{,}5} = 0{,}07 \text{ ккал/м}^2\text{час.}$$

Consequently, in the new steady regime the temperature gradient in the frozen rock mass must become

$$g'_м = \frac{q'}{\lambda_м} = \frac{0{,}07}{1{,}3} = 0{,}054 \text{ град/м.}$$

2. We find the thickness of the permafrozen rock mass corresponding to the new steady regime provided that the average annual temperature of the rocks remains unchanges and equal to -1.5°. Then

$$H = 7 + \frac{|-1{,}5|}{0{,}054} = 7 + 28 = 35 \text{ м.}$$

Consequently, as a result of use of the water-bearing level the thickness of the permafrozen rock mass will increase by 11 meters.

3. We find the time τ in the course of which the thickness of the permafrozen rock mass increased by 10 meters. To do that we use the Stefan formula (3.7.7), from which we obtain $\tau = H_f^2 Q_\phi / \lambda t$ 8700 years.

Since the moisture content by volume of the freezing sand is 7%, then, consequently, $Q_\phi = 70 \times 80 = 5600$ kcal/m^3. If it is assumed that the mean temperature t on the lower boundary of the frozen rock mass (at a depth of 24 meters) in the period of freezing is -0.3°, that is,

$$t = \frac{1}{2}\left(t_{\xi+h} + g'_м(24-7)\right) = \frac{1}{2}(-1{,}5 + 0{,}054 \cdot 17) = -0{,}3^\circ,$$

then we obtain

$$\tau = \frac{11^2 \cdot 5600}{1{,}8 \cdot 0{,}3 \cdot 8760} = 143 \text{ года.}$$

Upon increase of the thickness of the frozen rock mass the general thawing basin on the development area can prove to be within the contour of the permafrozen rocks. Before use of the water-bearing level and in the presence of a non-through talik, widening of the roads at the expense of the lawns would lead to the formation of frozen rock masses under them and would not lead to icing effects under the buildings. Icing in that case, often complicated by hummocks of heaving, can form on the lawn areas.

The development of icings on the development area can occur not only when the thickness of the permafrozen rocks changes as a result of change of the lower boundary conditions. Sometimes leading to the formation of icing is change of the upper boundary of permafrozen rocks, especially in the case of the formation of a thawed layer separating a seasonally frozen layer from a permafrozen layer, which is characteristic of transitional types of seasonal thawing. Often in that case the thawed layer of rocks becomes a collector of water below frost. The latter also leads to the formation of icing. Therefore prediction of the separation of the layer ξ from a frozen rock mass on sections of a development is a very important task, especially for such regions as Zabaykal'ye, where icing is widespread.

Calculation of the Thickness of a Thawed Layer of Rocks Separating a Seasonally Frozen Layer from a Permafrozen Rock Mass (Example 25)

In the course of a frost survey it is established that within the limits of an investigated region high-temperature frozen rock bodies, the average annual temperature of which varies in the range of -0.5 to -1°, are widely developed. In the upper part of the profile the deposits consist of alluvial sands, underlain by loams from a depth of 15 meters. The sands have the following characteristics: γ_{sk} = 1700 kg/m^3; w = 15%, λ_f = 1.8 and λ_t = 1.5 kcal/(m)(degree)(hr); C_{vol-f} = 434 and C_{vol-t} = 561 kcal/(m)(degree); Q_{ϕ} = 20,400 kcal/m^3. The average 10-year air temperature in the region t_{air} = -3.7°, the amplitude of air temperature fluctuations is 21° and the height of the snow cover on the average reaches 0.3 meter at a density of 0.2 g/m^3. In separate years the average annual air temperature increases to -1.5° and in that case an increase of atmospheric precipitations is noted in the winter, as a result of which the height of the snow increases to 0.4 meter.

Solution. 1. For determination of the conditions of formation of the thawed layer between a seasonally frozen layer and the surface of a frozen rock mass it is necessary to investigate regularities of the change of the temperature regime of the surface of the soil in the layer ξ in the course of a number of years (the period of use). Elevation of the temperature of the surface under natural conditions usually is connected with elevation of the air temperature and increase of the snow height. Elevation of the ground temperature in the layer ξ as compared with the temperature of the surface often occurs under the influence of the infiltration of summer atmospheric precipitations. In the investigated region it has been established that elevation of the t_{o} of the rocks on account of infiltration of precipitations amounts to +0.5$^{\circ\circ}$(see the calculation of Δt_{prec} in example 20, Chapter 5). This does not lead to the formation of non-confluent frost in years when t_{air} is close in its value to the 10-year average, and the height of the snow (h_{sn}) does not exceed 0.3 meter, since according to (5.3.10) $\Delta t_{sn} = 21 \times 0.176 \approx 3.7^{o}$; $t_o = t_{air} + \Delta t_{sn} = -3.7^{o} + 3.7^{o} = 0^{o}$. We determine that $t_{\xi} = t_o - \Delta t_{\lambda} + \Delta t_{prec}$. We find Δt_{λ} for the given conditions with the nomogram (Figure 33), equal to -1.4^{o}. Then $t_{\xi} = 0 - 1.4 + 0.5 = -0.9^{o}$ (for calculation of t_o and t_{ξ} in detail, see Chapters 4 and 5, examples 24, 21, etc). Having obtained by means of such calculations t_o and t_{ξ} at different combinations of t_{air} and h_{sn}, the values of which are taken on the basis of meteorological data, it is possible to readily determine the sought conditions. Thus, in the investigated region in years with $t_{air} = -1.5^{o}$ and $h_{sn} = 0.4$ m we get $\Delta t_{sn} = 0.225 \times 21 \approx 4.7^{o}$ and $t_o = -1.5 + 4.7 = 3.2^{o}$, that is, very favorable conditions for separation of the layer of winter freezing from the surface of the permafrozen rock mass.

2. In order to determine to what depth the roof of permafrozen rocks descends in warm years with $t_{air} = -1.5^{o}$, obviously it is necessary to calculate the depth of the potential thawing of rocks (see Chapter 4, section 3 and example 5). The following are the starting data for that: $t_{\xi} = 0^{o}$; $A_o = 21 - 4.7 + 3.2 = 19.5^{o}$; $Q_{\phi} = 20{,}400$ kcal/m^3; $C_{vol-t} = 561$ kcal/(m^3)(degree) and $\lambda_t = 1.5$ kcal/(m)(degree)(hr). With the nomogram (Figure 17) we find that $\xi_{p-0t} = \sqrt{1.5} \times 2.4 \approx 3$ meters. To that one must add the increase of the depth of thawing on account of infiltration of summer precipitations.

The depth of seasonal thawing is ~ 2.6 meters at $t_{\xi} = -0.9^{o}$, $A_o = 21 - 3.7 = 17.3^{o}$ and the same values of Q_{ϕ}, C_{vol-t} and λ_t. Consequently, in a warm year the roof of a permafrozen rock mass can drop by 0.4-0.5 meter (with consideration of convective heat exchange) as compared with its average position over the years.

3. We find the thickness of the thawed layer of rocks which is preserved in the course of the entire winter period above the roof of the permafrozen rock mass. The thickness of the layer is equal to the difference between the depth of the potential thawing of rocks and their seasonal freezing in a warm year. With a nomogram (Figures 17 and 19) we determine the depth of seasonal freezing at the following initial data: $t_o = +3.2^{o}$; $A_o = 21 - 4.7 = 16.3^{o}$; $Q_{\phi} = 20{,}400$ kcal/m^3; $C_{vol-f} = 434$ kcal/(m^3)(degree); $\lambda_f = 1.8$ kcal/(m)(degree)(hr); $\xi = \sqrt{1.8} \times 1.7 = 2.3$ meters. Thus the thickness of the thawed layer preserved to the end of the winter period is not less than 0.7 meter ($\xi_{p-0t} - \xi_{fr}$).

In all calculations of the change of thickness of frozen rock masses it is necessary to take into consideration that that change occurs in the course of time and that it is necessary to estimate the importance of that factor.

In particular, the rate of change of the thickness of the frozen rock mass or the rate of new formation of frozen rocks should be compared with the time of existence of buildings and other structures. For approximate estimation of the rate of perennial thawing of frozen rock masses or their new formation the very simple Stefan formula (3.7.7) can be used.

For analysis of the regularities in the formation of the thickness of permafrozen rocks, made in the course of a frost survey, and also for calculation of the new formation of passages and thin frozen rock masses, during the forecast it is necessary to calculate the depth of the permafrost, assuming that the temperature regime on the surface of the soil varies according to a periodic law.

The forecast of the change of thickness of permafrozen rock masses is of great importance in any type of economic opening up of territory. In all cases the problem is reduced to calculation of the temperature fields and dynamics of the front of freezing of rocks. Presented in example 26 is a method of approximate estimation of the change of thickness of frozen rock masses. More precise solutions in the determination of the thickness of permafrost are given in section 1, Chapter 9.

Calculation of the Thickness of a Permafrost Layer (Example 26)

In a frost survey the presence of thin (20-25 meter) frozen rock masses in river valleys was established. Presumably this can be connected with the formation of frozen rocks in the cooling period observed in the 300-year fluctuations of the air temperature. To prove that connection, one should calculate the depth of the permafrost if it is known that the average annual temperature of the surface of the soil is 0^{o} and the amplitude of fluctuations of the average annual temperatures during 300 years reaches 8^{o}. The frozen rock mass is composed of alluvial sandy loams containing loams of similar composition, the specific gravity of the skeleton of which can be assumed to be 1200 kg/m^3. The average moisture of rocks for the layer is close to the total moisture capacity and is 33%. The thermal conductivity of the frozen sandy loams and loams varies in the range of 1.2 to 1.35 kcal/(m)(degree)(hr) and for calculations can be assumed to be 1.3 kcal/(m)(degree)(hr). If it is taken into account that about 5% of the water in rocks did not participate in the phase transitions during freezing (taken in accordance with the temperature regime of the rocks and the curve of unfrozen water -- see example 7) we determine that $Q_{\phi} \approx 27{,}000$ kcal/m^3 and $E_{vol\text{-}f} \approx 470$ kcal/(m^3)(degree).
The geothermal gradient in the frozen rock mass is 0.02 degree/meter.

Solution. We calculate the depth of permafrost with fromula (4.2.1), which in a form more convenient for calculation is written as follows:

$$H = H_{2c} + \frac{(AH_{2c} + \alpha H)\alpha\beta}{[AH_{2c} + \alpha H + \beta(A + \alpha)](A + \alpha)},$$

where

$$\alpha = \frac{Q_\Phi}{2C};\ \beta = \sqrt{\frac{\lambda T}{\pi C}};\ A = \frac{1}{2}(A_0 + t);$$

$$t = t_0 + gH;\ H_{2c} =$$

$$= \frac{2(A_0 - t_0 - gH)\sqrt{\frac{\lambda TC}{\pi}}}{2AC + Q_\Phi}.$$

The given equation is transcendental, since the lower boundary -- the average perennial temperature and the amplitude of temperature fluctuations on the base H -- are determined as a function of the depth of the permafrost H. And so it is solved by trial and error, that is, several values of H are given and identity of both its parts is achieved upon substitution in the equation.

1. We will assume that H = 10 m. Then we have: $t = 0 + 0.02 \times 10 = 0.2^\circ$, $A = 0.5\ (8 + 0.2) = 4.1^\circ$; $\alpha = 28.7$; $\beta = 48$;

$$H_{2c} = \frac{2(8 - 0,2)\sqrt{\frac{1,3 \cdot 300 \cdot 470 \cdot 8760}{3,14}}}{2 \cdot 4,1 \cdot 470 + 27\,000} = 11,16\ м.$$

When all the starting values are substituted in the equation, identity is not obtained, since the right side of the equation, that is,

$$H_{2c} + \frac{(AH_{2c} + aH)\,a\beta}{[AH_{2c} + aH + \beta(A + a)]\,(A + a)}$$

is equal to ~ 19 m and on the left side H = 10 m.

2. We are given other values of H, for example, H = 30 cm. Upon corresponding substitution we find that

$$H_{2c} + \frac{(AH_{2c} + aH)\,a\beta}{[AH_{2c} + aH + \beta(A + a)]\,(A + a)} = 26\ м.$$

Consequently, identity also is not obtained in that case.

To find the unknown value of H we use graphic construction, as was done in finding Δt_{sn} and Δt_{prec} (see Chapter 5, examples 10, 20, etc). It is evident from Figure 59 that the depth of permafrost of the indicated rocks in 300-year fluctuations of temperature on the surface of the soil is 24 m, which corresponds to the thickness of the frozen rock masses on the investigated section.

6. Forecasting Change of the Temperature Regime of Permafrozen Rocks in the Layer of Annual Temperature Fluctuations

In solving a number of critical problems (for example, in determining the depth of bedding of foundations) a need arises for classification of the dynamics of change of the temperature regime of permafrozen rocks in the layer of annual fluctuations of temperature. A solution of this problem in the general case with consideration of phase transitions in the range of temperatures can be found only by means of a computer. Simultaneously with that there exist approximate methods of solving that problem. In the latter case it is assumed initially that the phase transitions of water in the underlying permafrozen soils during temperature variation in the range of 0 to -10 or -15^{o} are relatively small in comparison with phase transitions in the layer of seasonal thawing. Because of this it is advisable to base the approximate calculation scheme on the introduction of the effective heat capacity, which takes phase transitions into account.

One should proceed as follows in determining the effective heat capacity. By means of a formula for determining the depth of seasonal thawing (freezing) of rocks (4.1.4) the depth of the extent of annual temperature fluctuations in a permafrozen rock mass is calculated with consideration of the phase transitions of the unfrozen water in the temperature range (see example 7). Then the obtained depth h below the layer ξ should be substituted in the Fourier equation (3.3.4) and from it the effective heat capacity C_{eff} should be determined. In finding C_{eff} instead of A_{o} we should introduce into the formula A_{ξ} (without consideration of asymmetry), that is, $A_{\xi} = |t_{\xi}|$.

The character of the temperature field in a permafrozen rock mass can with adequate approximation be written with equation (4.2.1) if C_{eff} is substituted instead of C.

Determination of the maximal temperatures in layer h is of great practical importance. According to the SNiP (Construction Norms and Specifications) for calculation of the depths of foundations a diagram of forces is determined for freezing in the layer of seasonal thawing and in the underlying layer of permafrozen rocks. For that purpose it is necessary to have the temperature distribution in the soils and a characterization of their properties for the most unfavorable moment in relation to the stability of the foundation during heaving. Such a moment is the time of connection of the freezing seasonally thawed layer with the permafrozen rock mass. It is precisely at that time that the maximal forces of freezing and the maximal forces of heaving develop in the seasonally thawed layer, causing bulging of the foundation. The temperatures in the permafrozen rock mass (in the layer h), which determine the characteristics of the properties of permafrozen rocks and the diagram of forces of freezing of the ground to the foundation, which restrain the latter from bulging, must be calculated for that moment.

The temperature distribution by depth in the layer h at the moment of connection of the freezing layer ξ with the frozen rock mass, evidently, must be determined under the condition $t = 0^{o}$ on the base of the layer ξ. In connection with the fact that a steady-state process is examined periodically, calculation without consideration of a concrete moment of time can be done with formula (4.2.1), in which the term corresponding to the geothermal gradient also is not taken into consideration.

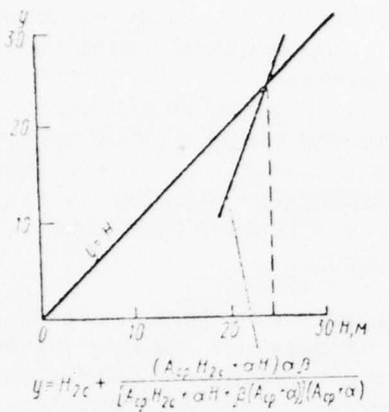


Figure 59. Graph for finding the depth of permafrost.

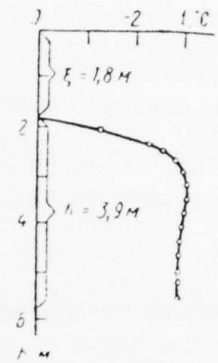


Figure 60. Temperature distribution in the layer h at the moment of time T/4.

Calculation of Temperatures on the Basis of Depth in Layer h at the Moment of Connection of the Freezing Layer ξ With the Permafrozen Rock Mass (Maximal Temperatures According to SNiP II-B.6-66) (Example 27)

Given: an average annual ground temperature of -3°; the following thermophysical properties of the ground: $\lambda_f = 1.5$ kcal/(m)(degree)(hr); $C_{vol-f} = 460$ kcal/(m^3)(degree); $Q_\phi = 3168$ kcal/m^3 (see calculation of Q_ϕ in the layer h in example 7).

Solution. 1. We determine the thickness of the layer of annual temperature fluctuations in the permafrozen rock mass with formula (4.14). As follows from example 7, for the given conditions h = 3.9 m. At $\xi = 1.8$ m the entire layer of annual temperature fluctuations will be: H = 1.8 + 3.9 = 5.7 m.

2. From equation (3.3.4), making the appropriate substitutions, we find the effective heat capacity:

$$C_{э\phi} = \frac{\lambda T \left(\ln \frac{|t_\xi|}{0.1} \right)^2}{h^2}, \quad (6.6.1)$$

$$C_{э\phi} = \frac{1,5 \quad 8760 \cdot 11,56}{15,21} \approx$$

$$\approx 9987 \text{ ккал/м}^3.$$

3. We determine the temperature distribution for the depth h at the moment of its joining with the layer ξ. It is obvious that under boundary conditions on the base of the layer ξ of the type $t_\xi \left(1 - \sin \frac{2\pi}{T} \tau\right)$ the joining of the freezing layer ξ and the frozen layer h sets in at the moment of

time $\tau = T/4$. In our case the calculation of the maximal temperatures in the layer h is made from the depth of 1.8 m to the depth of 5.7 m with the step z = 0.3 m with the formula

$$t(h_{0.3}, \tau) = t_\xi \left(1 - \exp\left\{-z\sqrt{\frac{\pi}{aT}}\right\}\cos z\sqrt{\frac{\pi}{aT}}\right) \qquad (6.6.2)$$

At $\tau = \frac{T}{4}$, $a = \frac{\lambda}{C_{эф}}$, $t_\xi = -3°$.

The results of the calculations are presented in Table 36 and on the graph (Figure 60). In case of need the calculation can be made for any depth within the limits of the layer h.

Table 36. Calculation of t_{max} at the depth h with formula (6.6.2)

a Глубина h с $z = 0.3$ м	0,3	0,6	0,9	1,2	1,5	1,8	2,1	2,4	2,7	3,0	3,3	3,6	3,9
b Глубина слоев $\xi + z$	2,1	2,4	2,7	3,0	3,3	3,6	3,9	4,2	4,5	4,8	5,1	5,4	5,7
c $t_{макс}$ по глубине h . .	−1,3	−2,3	−2,9	−3,1	−3,2	−3,2	−3,1	−3,1	−3,0	−3,0	−3,0	−3,0	−3,0

Key: a - Depth h with z = 0.3 m b - Depth of layer $\xi + z$ c - t_{max} at depth h

For calculation of the diagrams of forces of freezing together in the layer ξ it is necessary simultaneously with determination of t_{max} in the layer h to determine also the temperature distribution in the layer at the moment of joining. The change of temperature in the layer ξ at that time can be taken according to a linear law with sufficient accuracy.

Thus the problem consists in finding the temperature of the surface at the moment of time under consideration and in determining the length of freezing of the seasonally thawed layer. The latter can be found with the very simple Stefan formula (3.7.7). In that case the time must be read from the moment of inversion of sign of the temperatures on the surface (τ_o). The value of the sum of the frost-degree-hours Ω as a function of τ entering formula (3.7.7) is determined during harmonic oscillations of the type $t_o + A \sin 2\pi/T$ from the correlation

$$\Omega(\tau_{см}) = t_0(\tau_{см} - \tau_0) + \frac{A_0 T}{2\pi}\sin\frac{\pi}{T}(\tau_{см} + \tau_0)\sin\frac{2\pi}{T}(\tau_{см} - \tau_0). \qquad (6.6.3)$$

where $\tau_o = T/2 + T/2\pi \arcsin t_\xi/A_o$ is the moment of start of freezing of the seasonally thawed layer.

The moment of time τ_j at the given value of ξ is readily determined by solving transcendental equation (6.6.3) if the value of Ω from the Stefan equation is substituted in it:

$$|\Omega_{(\tau_{cM})}| = \frac{\xi^2 Q_\Phi}{2\lambda_M}. \qquad (6.6.4)$$

To simplify finding the temperature of the surface at the moment of joining, the results of solution of equation (6.6.4) at different values of A_o and Q_ϕ are reduced to the corresponding nomograms. The indicated graphs (Figure 61) illustrating the dependence of the temperature of the surface at the moment of joining (t_j) on the average annual temperature of the ground t_ξ at given values of Q_ϕ in a broad range of values of A_o, were constructed in accordance with the type of nomograms of the depths of seasonal thawing (see Figures 14-19). Each nomogram represents curves obtained at C_{xa1-f} = 500 kcal/(m^3)(degree) for A_o = 5, 10, 20 and 30^o at Q_ϕ = 5000, 10,000, 20,000 and 40,000 kcal/m^3 respectively. It is obvious that in the approximation under consideration neither t_j nor the values of the time of joining are dependent on the value of the thermal conductivity of the ground.

All curves of t_j (see Figure 61) start from the origin of the coordinates (since at $t_\xi = 0$ the depth of seasonal thawing ξ coincides with the potential thawing and, consequently, $t_j = 0$) and ends on the t_ξ axis at the point $t_\xi = A_o$ (since in that case $\xi = 0$).

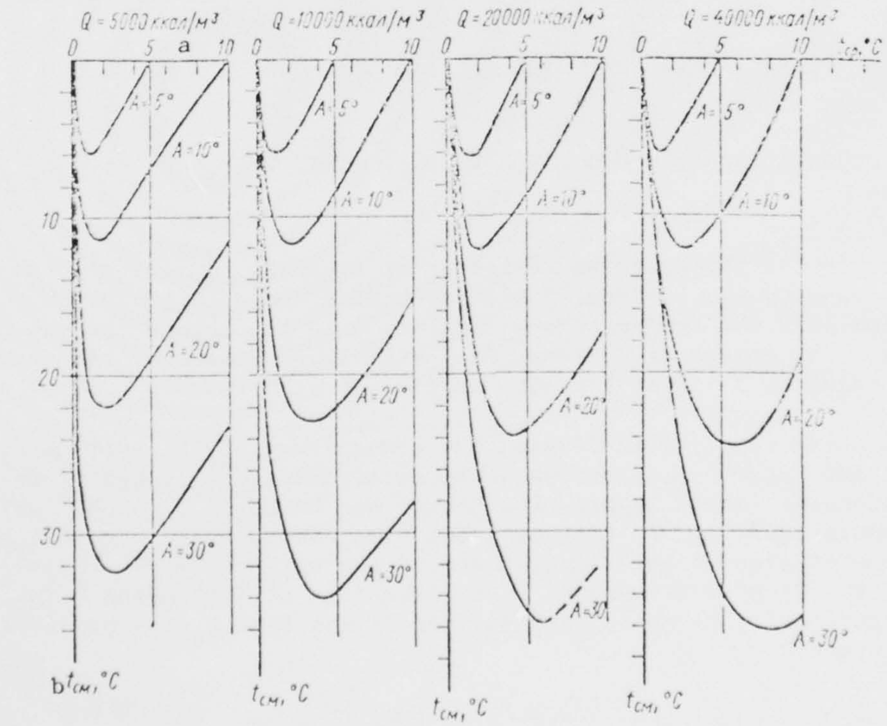


Figure 61. Nomograms for finding t_{max} at the moment of joining of the layer ξ with the permafrozen rock mass.
Key: a - kcal/m^3 b - t_j, ^{o}C

Independently of A_o and Q_ϕ the form of the curves of t_j as a function of t is identical: a sharp drop of t_j at values of t_ξ close to zero and a somewhat slower increase after reaching a minimum. It is characteristic that the values of t_ξ at which the minimal values of t_j are reached decrease slightly, although A_o varies in a wide range in that case. Since it is evident on the nomograms (see Figure 61) that at Q_ϕ = 10,000 kcal/m^3 for $A_o = 5^o$ the minimal t_j of -6^o corresponds to $t_\xi = -^o$, whereas at $A_o = 30^o$ the minimal t_j of -34^o is achieved at a value of t_ξ of only -4^o. The calculations show that at different values of Q_ϕ and different values of C the values of t_j for different values of A_o differ very little in practice, and this makes it possible to use the diagram for calculation of t_j at C = 500 kcal/(m^3)(degree) for other values of C.

Calculation of the Temperature of the Surface of the Ground at the Moment of Joining the Freezing Layer ξ With the Permafrozen Rock Mass (Example 28)

On the basis of frost survey data it is known that a construction section within the limits of a layer with annual temperature fluctuations (15-18 m) is composed of loams characterized by γ_{sk} = 1200 kg/cm^3; w = 27%; $\lambda_f = \lambda_t$ = 1.5 kcal/ /(m)(degree)(hr); Q_ϕ = 21,120 kcal/m^3 at w_{un} = 5%. The average annual temperature on the surface of the soil and the average annual temperature of the ground is -3.5^o ($t_o = t_\xi$), and the amplitude of the annual temperature fluctuations on the surface of the soil, provided that the snow is removed in the winter, is 20^o. Determine the time of joining the freezing layer ξ with the surface of the permafrozen rock mass and the temperature of the surface of the soil at that moment.

Solution. 1. We determine with the nomogram (see Figure 17) the depth of seasonal thawing of the ground at the construction site under the following starting conditions: $t_o = t_\xi = -3.5^o$; $A_o = 20^o$; Q_ϕ = 21,120 kcal/m^3; C = 500 kcal/(m^3)(degree); λ_t = 1.5 kcal/(m)(degree)(hr). We find that ξ_{nom} = 1.8 m and, consequently, the unknown value of ξ is ~ 2.2 m ($\xi = \xi_{nom} \times \sqrt{\lambda}$).

2. We determine the sum of the frost-degree-hours necessary to freeze the layer ξ under the given conditions. In accordance with (6.6.4)

$$\Omega_{(\tau_{см})} = \frac{(2.2)^2 \cdot 21\,120}{2 \cdot 1.5} = 34\,073{,}6 \text{ град} \cdot \text{час}.$$

3. We find the duration of freezing ($\tau_j = \tau$) of the layer ξ by solving transcendental equation (6.6.3) at $\Omega_{(\tau_j)}$ = 34,074 degree-hrs and

$$\tau_0 = 4380\left(1 + \frac{1}{\pi}\arcsin\frac{-3.5}{20}\right) = 4135 \text{ час}.$$

By determining τ by trial and error (see Table 37) we find the sought value of $\tau_j \approx 6849$ hours under the given conditions. Consequently the length of freezing of the layer ξ before joining with the permafrozen rock mass is 2354 hours.

Table 37. For finding the temperature of freezing of the layer to the permafrozen rock mass

a № п/п	b $\tau_{см}$, час	c $\Omega(\tau_{см})$ град·час
1	6000	22 896,9
2	7000	45 946,4
3	6500	34 332,5
4	6300	29 665,7
5	6490	33 098,1

Key: a - Expt No   b - τ_j, hr   c - $\Omega_{(\tau_j)}$, degree-hours

4. With the nomogram (Figure 61) we find the temperature on the surface of the soil at the moment τ_j, when complete freezing of the layer ξ occurs (freezing from below is not taken into account in that case). The starting data are: Q_ξ = 21,120 kcal/m^3; t = -3.5^o and A_o = 20^o. We find on the diagram that for those conditions t_j = -23.4^o.

7. Regularities of the Change of Thermophysical Properties of Frozen Dispersed Rocks

1. Dependence of the Heat Capacity of the Skeleton of the Ground on Composition and Temperature

The average heat capacity of the skeleton of the ground was determined by the accepted method in the range from +22^{o}C to t_{inv}, where the latter varied from 0 to -120^{o}C.

The results of experimental determination of the heat capacity of the skeleton of soils as a function of temperature (Figure 62) make it possible to consider that the heat capacity of the skeleton of kaolin is practically constant in the entire range of temperatures from +20 to -90^o (C_{sn} = 0.209 kcal/(g)(degree)). The heat capacity of the skeleton of heavy loams decreases with reduction of the temperature. That dependence is expressed with the formula

$$C_{ск} = 0,193 - 0,000138\,(22,6 - t_{обр}) \qquad (6.7.1)$$

and represents the linear change of the heat capacity over the entire range of temperatures from +20 to -120^{o}C.

Determinations of the heat capacity of different fractions of alluvial quartz sand give the general curve of the temperature dependence of the heat capacity, which is a linearly increasing function in the same temperature ranges;

$$C_{ск} = 0,183 - 0,00020\,(22,6 - t_{обр}). \qquad (6.7.2)$$

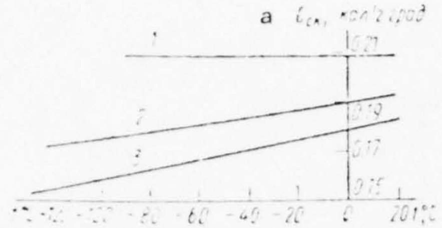


Figure 62. Dependence of the average heat capacity of the skeleton of the ground on temperature: 1 - kaolin; 2 - loam; 3 - quartz sand. a - C_{sk}, kcal/(g)(degree)

The conducted investigation permits drawing the conclusion that the temperature coefficient of heat capacity of the skeleton of the rocks is sufficiently small and should be taken into consideration only in considering broad temperature ranges. Thus the change of the temperature of a sample by 100°C reduces the heat capacity of loams by 7% and of sands by 10%.

2. Dependence of the Heat Capacity of Soils on Composition and Temperature

On the basis of the obtained values of the heat capacity of the skeleton of soils and the amount of subfrozen water as a function of temperature the specific heat of moist ground is calculated in the thawed and frozen states:

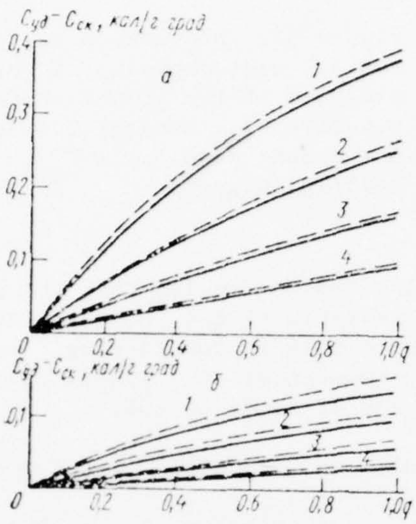
$$C_{т} = \frac{100 \cdot C_{ск} + C_{в} w}{100 + w}, \qquad (6.7.3)$$

$$C_{м} = \frac{100 \cdot C_{ск} + C_{в} w_{н} + C_{л}(w - w_{н})}{100 + w}. \qquad (6.7.4)$$

Presented on the diagrams (Figures 63 and 64) are the dependences of the calculated values of the heat capacity on the degree of moisture saturation and the specific weight of the skeleton for clays and sands. In the frozen state the heat capacity of clays was calculated without consideration of the amount of unfrozen water.

The specific heat of the ground (Figure 63) increases with increase of the degree of moisture saturation and is greater the smaller the specific weight of the skeleton. The change of the specific heat is greater in the thawed than in the frozen state. All the curves presented on Figure 63 converge at the zero value of the degree of moisture saturation (dry ground). The increase of the specific heat with increase of the degree of moisture saturation at large values of the specific weight of the skeleton can be assumed to be linear, and at small values, the increase is greatly decelerated in comparison with the linear dependence.

The volumetric heat capacity (Figure 64) increases linearly with increase of the degree of moisture saturation. The increase is steeper the smaller the specific weight of the skeleton of the ground. Diagrams 63 and 64 were constructed in a wide range of values of the specific weight of the skeleton, from 0.8 to 2.4 g/cm^3, covering practically all encountered values, and therefore they can be used to determine heat capacity instead of calculations with



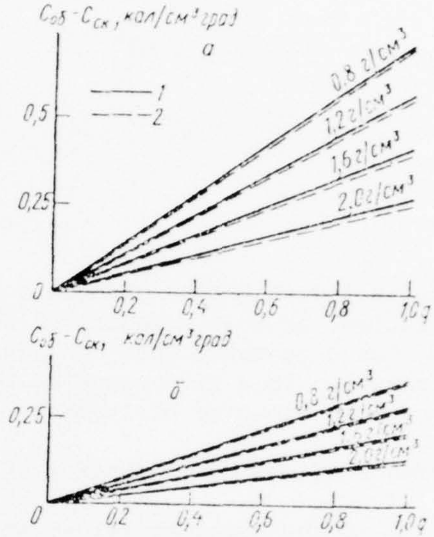


Figure 63. Dependence of the specific heat ($C_{spec} - C_{sk}$) of sand (broken line) and clay on q in the thawed (a) and frozen (b) states at γ_{sk}: 1 - 0.8; 2 - 1.2; 3 - 1.6; 4 - 2.0 g/cm^3. 1 - $C_{spec} - C_{sk}$, kcal/(g)(deg)

Figure 64. Dependence of the volumetric heat capacity ($C_{vol} - C_{sk}$) on q and γ_{sk} (g/cm^3) in the thawed (a) and frozen (b) states. 1 - clay; 2 - sand. 1 - $C_{vol} - C_{sk}$, (cm^3)(deg)

the presented formulas (6.7.3 and 6.7.4). The diagrams were constructed without consideration of the amount of unfrozen water and in the frozen state the values of the heat capacity are slightly understated.

Reduction of the temperature in frozen grounds has far less influence on the heat capacity than the transition from the thawed to the frozen state. Thus, for example, for alluvial sand with 2 = 22% the change of temperature from 0 to -20°C reduces the specific heat from 0.34 to 0.25 kcal/(g)(degree), and further reduction of the temperature to -120°C reduces that value to 0.21 kcal/(g)(degree).

According to the obtained experimental data the temperature coefficient of the specific heat of the skeleton is maximal for sand and amounts to 0.2% per degree. The specific heat of moist sand also diminishes at the same rate. Probably that value should be considered the upper limit of possible reduction of heat capacity with reduction of temperature. That change is substantial only in the consideration of a wide range of temperatures.

3. Dependence of the Coefficients of Thermal and Temperature Conductivity of Soils on Their Composition, Moisture Content and Temperature

Under natural conditions soils are encountered with a moisture equal to the hygroscopic and higher. In soils used for study of the coefficient of temperature conductivity, the hygroscopic moisture corresponds to a degree of moisture saturation of 0.05 for sands and $\sim$ 0.16 for heavy loams.

In the process of investigations the degree of moisture saturation given is from 0.1 - 0.2 to 0.9 - 1.0, that is, practically from the hygroscopic moisture content to total moisture saturation. Within those limits of moisture all the obtained dependences of the coefficients of thermal and temperature conductivity are valid. The thermophysical characteristics of dry ground were not taken into consideration in the calculations. This is especially important for sands, in which a very sharp change of the coefficients of thermal and temperature conductivity are observed at low moisture contents (Chudnovskiy, 1955).

Presented below are the results of the processing and generalization of the coefficients of thermal and temperature conductivity obtained in recent years for each type of ground. Distinguished separately is the influence of the transition of ground from the thawed to the frozen state on the coefficients of thermal and temperature conductivity, which are of special practical interest.

/The coefficient of temperature conductivity of soils (α)/. For samples with a destroyed structure, experimental determinations have been made of:

a) the dependence of the coefficient of temperature conductivity on the degree of moisture saturation;
b) the change of the above-indicated dependence when the temperature is lowered to -120°C;
c) the dependence for loams on porosity at two values of the degree of moisture saturation in the thawed and frozen states.

The dependence of α on the degree of moisture saturation of soils. In loams and clays the results of determination of the dependence of the coefficient of temperature conductivity on the degree of moisture saturation at the same porosity (about 0.42), close to the average value for those soils, are presented on Figure 65. The divergence of the experimental points from the averaged curves does not exceed $\pm 5\%$. In the thawed state (Figure 65a) α increases with increase of the degree of moisture saturation, reaching a maximum at values of the latter of 0.6 - 0.8, and then decreases.

Curves corresponding to different soils are disposed fairly close to one another, and therefore a single general dependence of them can be expressed, the error of which does not exceed $\pm 10\%$. The equation of that generalized dependence of α on the degree of moisture saturation (q) has the form

$$a_T = 0{,}00078 + 0{,}00334q - 0{,}00252\,q^2 \quad (м^2/час). \qquad (6.7.5)$$

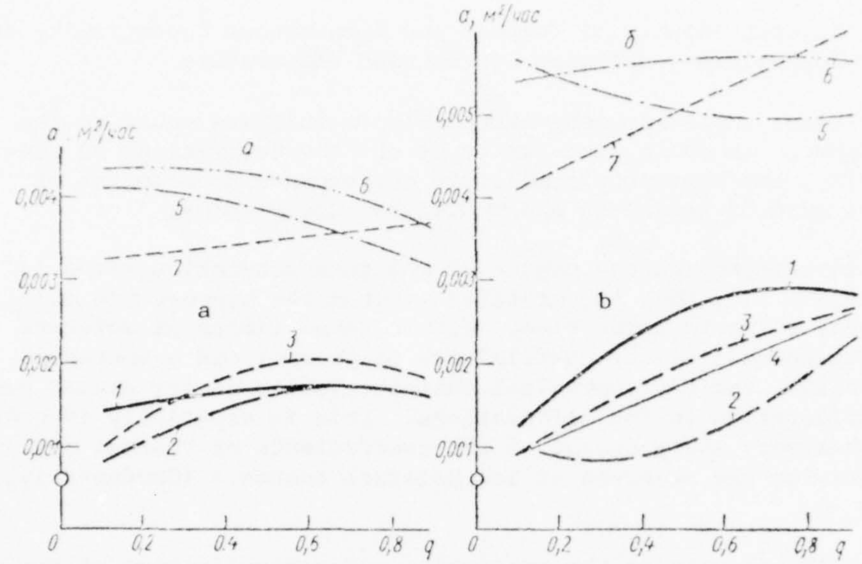


Figure 65. Dependence of the coefficient of temperature conductivity of soils on q in the thawed (a) and frozen (b) states: 1 - kaolin; 2 - heavy loam; 3 - medium loam; f - average value of α for kaolin and loams; 5 - quartz sand, 0.1-0.25 mm fraction; 6 - quartz sand, 0.25-0.50 mm fraction; 7 - silty sand. a - α, m^2/hr

In the frozen state (Figure 65b) α also increases with increase of the degree of moisture saturation, but the form and position of the curves corresponding to different soils differ much more than in the thawed state and, as is evident on Figure 65, generalized curve (4) for heavy loams differs from the curve for kaolin and medium loam. Whereas for medium loam and kaolin the dependence of the coefficient of temperature conductivity represents a monotonic increase with increase of the degree of moisture saturation, one which decelerates after $q \simeq 0.8$, and can be expressed by a quadratic equation, for heavy loams the analogous dependence has a more complex form.

Those two dependences can be expressed by a single averaged curve, but with great error. An equation expressing the dependence of the coefficient of temperature conductivity of frozen loams has the form

$$a_м = 0{,}00072 + 0{,}00228q - 0{,}00022q^2 \quad (м^2/час), \qquad (6.7.6)$$

with a mean-square error of about 20%. It should be noted that the above-presented averaged dependences of the coefficient of temperature conductivity of loams and clays are valid in the entire range of values of q from 0 to 1.

In studying sands, determinations of the dependence of the coefficient of temperature conductivity on the degree of moisture saturation were made for samples also with a single value of the porosity (about 0.37). The coefficient of temperature conductivity of quartz sand decreases and that of silty sand increases with increase of q in the entire range from 0.1 to 0.9, and in the region of values of q of 0.6-0.9 the values of α of all sands practically coincide (Figure 65.). Comparison with values of the coefficient of temperature conductivity of dry sand permits drawing the conclusion that the greatest change occurs in the very first stage of moistening.

Thus for approximate calculations it can be assumed that the coefficient of temperature conductivity of sands varies almost not at all at values of q greater than 0.1. The established regularity is also valid to a considerable degree for soils in the frozen state. However, the scattering of values here is much larger, and samples of the same sand can differ by 20-25% in values of the coefficient of temperature conductivity (see Figure 65).

The obtained dependences of the coefficient of temperature conductivity of sands on the degree of moisture saturation (at a value of q greater than 0.1) can be represented by generalized equations for the thawed state within $\pm10\%$, and for the frozen state within $\pm25\%$:

$$a_{т} = 0{,}00415 - 0{,}00077q \ (м^2/час), \quad (6.7.7)$$

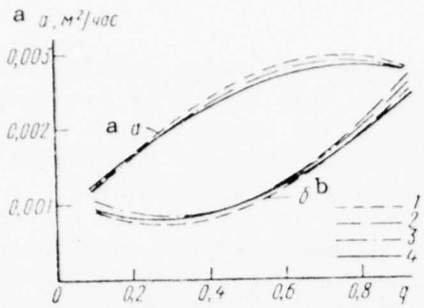
$$a_{м} = 0{,}00488 + 0{,}00054q \ (м^2/час). \quad (6.7.8)$$

/The dependence of α on the temperature of soils in the frozen state/. The coefficient of temperature conductivity of loams and clays in the frozen state has been determined in several temperature ranges from -20 to -120°C. Comparison of the obtained values for all the loams and clays shows that the reduction of temperature has practically no effect on the values and form of the dependence of the coefficient of temperature conductivity on the degree of moisture saturation.

Presented on Figure 66 as an illustration are the dependences of the coefficient of temperature conductivity on the degree of moisture saturation in three temperature ranges for kaolin and heavy loam. The separate curves for each temperature range and the mean from -20 to -80°C practically completely coincide.

The influence of the temperature on the coefficient of temperature conductivity of frozen sand is manifested much more strongly than in loams and clays, but no clear regularity has as yet been established in that change. Thus, in some cases α decreases with reduction of the temperature, and in others it remains almost constant (Figure 67), while in still others it decreases in the range of -40 to -60°C and increases at lower temperatures. This is observed in practically identical formations, in the best case different in the time of stay at a given negative temperature, which, as is known, does not have to cause any sort of changes in the structure of samples.

The change of the coefficient of temperature conductivity with reduction of the temperature in some cases reaches 80%, and so it is very desirable to make clear what causes that change. Possibly the reason for change of α is the presence of microfissures and change of the quantity of them when the temperature is reduced.



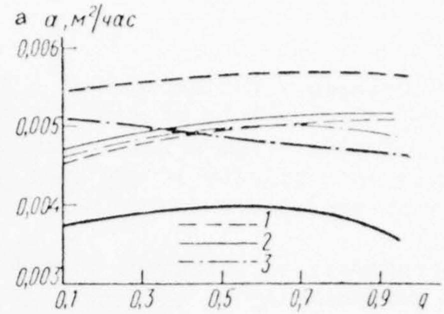


Figure 66. Dependence of the coefficient of temperature conductivity of kaolin (a) and heavy loam (b) on q and temperature: 1 - from -20 to -40°; 2 - from -40 to -60°; 3 - from -60 to -80°; 4 - from -20 to -80° (average value). a - α, m²/hr

Figure 67. Dependence of the coefficient of temperature conductivity of quartz sand of the 0.25-0.50 mm fraction (two lots) on q and temperature: 1 - from -20 to -40; 2 - from -40 to -60°; 3 - from -60 to -80°. a - α, m²/hr

/The dependence of α on the porosity of the ground (n)/has been verified only for loams. The experiments were conducted at two values of q, 0.6 and 0.9, and the porosity varied from 0.35 to 0.65. The results are presented on Figure 68.

In the thawed state the character of the dependence of α on the porosity and the values of α themselves at q = 0.6 coincide completely with the values of α at q = 0.9. The coincidence of values of α at degrees of moisture saturation of 0.6 and 0.9 is completely regular (see Figure 65).

At the indicated values of the degree of moisture saturation in the thawed state increase of the porosity from 0.35 to 0.50 leads to a decrease, and its further increase to 0.65 has practically no effect on the value of α. That dependence can be expressed with the equation

$$a_{т} = 0{,}00607 - 0{,}0163n + 0{,}0140n^2 \ (м^2/час). \quad (6.7.9)$$

In the frozen state (Figure 68) the dependence of the coefficient of temperature conductivity on the porosity of loam is different for the degrees of moisture saturation of 0.6 and 0.9. Thus, at q equal to 0.6 α increases with increase of n from 0.35 to 0.48 and from then on remains almost unchanged. At q = 0.9, α is practically independent of the porosity, but the scattering of values reaches 20%.

Such dependences evidently can be explained by the fact that in the process of heat transfer ice has an essential role. At q = 0.6 the replacement of a portion of the ground by ice increases the "monolithic character" of the system in the sense of heat transfer and, as a result, increases the conductivity. At q = 0.9 all the pores must be filled by ice and the replacement of part of the ground by ice has no effect on the "monolithic character" of the system or, consequently, on the conductivity. In the frozen state the

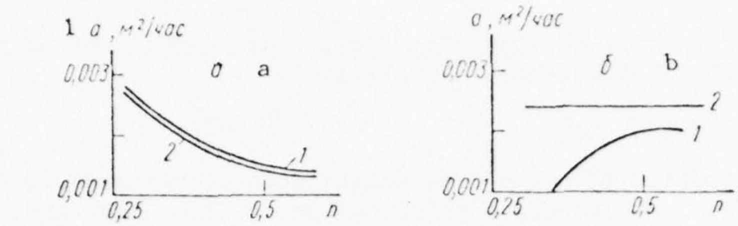


Figure 68. Dependence of the coefficient of temperature conductivity of loams on the porosity in thawed (a) and frozen (b) states at values of q of 0.6 (1) and 0.9 (2). 1 - α, m^2/hr

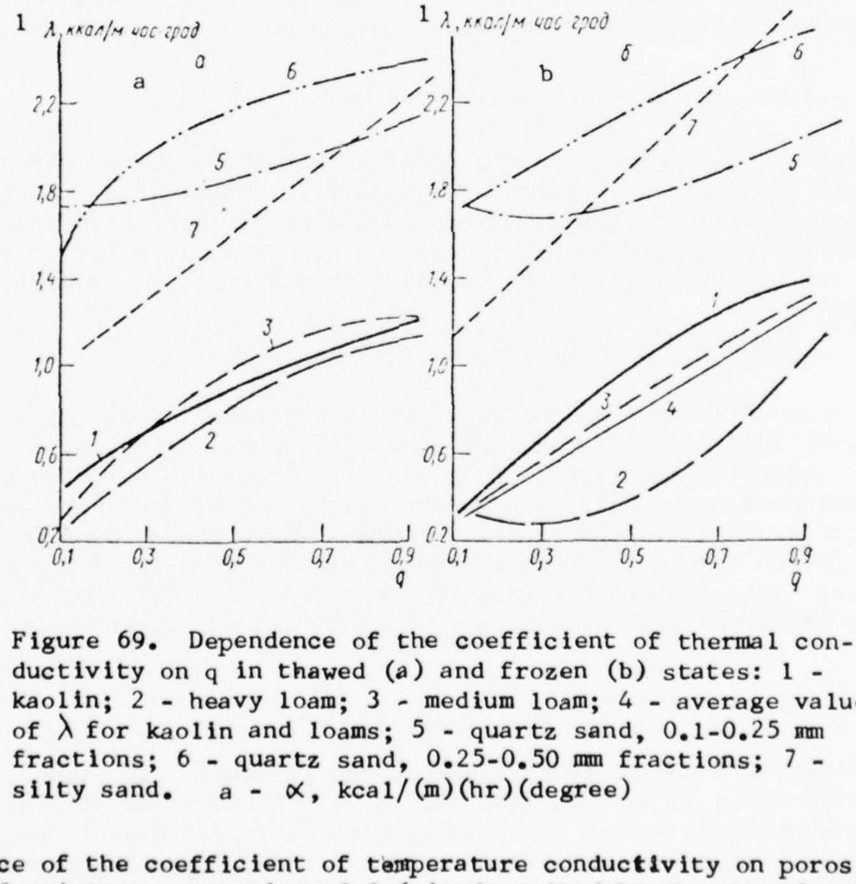


Figure 69. Dependence of the coefficient of thermal conductivity on q in thawed (a) and frozen (b) states: 1 - kaolin; 2 - heavy loam; 3 - medium loam; 4 - average value of λ for kaolin and loams; 5 - quartz sand, 0.1-0.25 mm fractions; 6 - quartz sand, 0.25-0.50 mm fractions; 7 - silty sand. a - α, kcal/(m)(hr)(degree)

dependence of the coefficient of temperature conductivity on porosity at a degree of moisture saturation of 0.6 is described by the equation

$$a_м = -0{,}00747 + 0{,}0365n - 0{,}0349n^2,\ (м^2/час), \qquad (6.7.10)$$

and at a degree of moisture saturation of 0.9, as has already been noted, does not depend on the porosity.

/The coefficient of thermal conductivity (λ)/. The coefficient of thermal conductivity is calculated with the correlation'

$$\lambda = aC_{уд}\gamma_{об}, \qquad (6.7.11)$$

in which the experimental values of the coefficient of temperature conductivity, the specific heat and the specific weight of the soils are substituted. It is quite natural that the character of the change of the coefficient of thermal conductivity reminds one to a great extent of similar regularities obtained for the coefficient of temperature conductivity.

The dependence of λ on the degree of moisture saturation (q) of soils. In loams and clays the dependence of λ on q at a single value of the porosity is presented for all four investigated soils on Figure 69. In the thawed state (Figure 69a) λ increases almost linearly to the value q = 0.8, and then saturation sets in. The curves for the soils under consideration are rather close together and they can be expressed by a common dependence with an error of $\pm$10%:

$$\lambda_т = 0,141 + 2,16q - 1,04q^2 \ (ккал/м \cdot град \cdot час). \qquad (6.7.12)$$

In the frozen state (Figure 69b) the divergence between curves for different soils is more considerable, although the general tendency for λ to increase with increase of q is preserved. Averaged curves of loams with an accuracy of about 20% are presented on Figure 69. An averaged curve for frozen loam represents a monotonic increase of λ with increase of q, the equation for which has the form

$$\lambda_м = 0,183 + 1,06q - 0,209q^2 \ (ккал/м \cdot град \cdot час). \qquad (6.7.13)$$

The coefficient of thermal conductivity of investigated sands (Figure 69) increases with increase of q. As has already been noted for α, a very large change of λ occurs in the region of values of q from 0 to 0.1. On that section λ increases 6 times, and upon further change of q, λ increases, and in the region of q = 0.8-1.0 the rate of growth of λ is slowed down in both the thawed and frozen states. The scattering of values of λ for different sands in the frozen state is larger than in the thawed. The values of λ in terms of q can be expressed with an accuracy of $\pm$25% with a similar dependence:

$$\lambda_т = 1,32 + 1,05q, \qquad (6.7.14)$$

$$\lambda_м = 1,38 + 1,05q. \qquad (6.7.15)$$

/The dependence of λ on the temperature of the ground/. In loams and clays when the temperature is reduced from -20 to -120°C the value of λ remains practically unchanged, as was noted early for α (Figure 70). In sands, lowering the temperature causes a different change of λ, repeating the above-noted distinctive features in the change of α.

/The dependence of λ on the porosity/, as noted above, has been determined only for loams. The results are presented on Figure 71. In the thawed state a decrease of λ has been obtained with increase of the porosity, similar to that existing in the literature (Chudnovskiy, 1962).

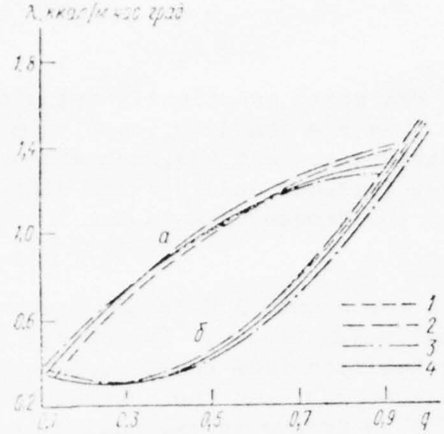


Figure 70. Dependence of the coefficient of thermal conductivity of kaolin (a) and heavy loam (b) on q and temperature: 1 - from -20 to -40°; 2 - from -40 to -60°; 3 - from -60 to -80°; 4 - from -20 to -80° (average value).
1 - λ, kcal/(m)(hr)(degree)

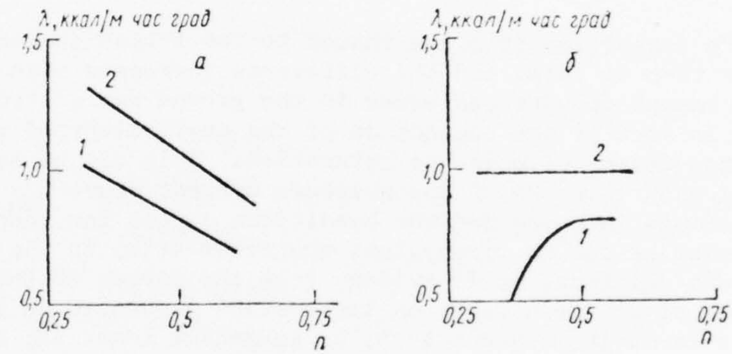


Figure 71. Dependence of the coefficient of thermal conductivity of loams on the porosity in the thawed (a) and frozen (b) states at values of q of 0.6 (1) and 0.9 (2).
a - λ, kcal/(m)(hr)(degree)

In the frozen state (Figure 71b), as has already been noted for α, the dependende of λ at q = 0.6 and 0.9 is different. In the former case λ increases with increase of the porosity, and in the latter it remains practically constant.

/Change of α and λ during the transition of soils from the thawed to the frozen state/. The transition of soil from the thawed to the frozen state substantially changes the coefficients of thermal and temperature conductivity. The thermophysical characteristics are practically equalized at +10 and $-20^{\circ}C$. The interval of intensive phase transitions of water into ice is not considered.

For all the investigated soils the correlation between the indicated coefficients in the thawed and frozen states depends on the degree of moisture saturation. The experimentally determined coefficients of thermal and temperature conductivity of thawed and frozen dry soils practically coincide, but it must be noted that in the frozen state they are steadily lower than in the thawed. However, with increase of the moisture content and, consequently, also of the degree of moisture saturation, they increase according to different laws, and on some sections more rapidly in the frozen than in the thawed state, and as a result the curves intersect.

The coefficients of temperature conductivity in the thawed and frozen states differ far more than the coefficient of thermal conductivity. The change of the form of dependence of α on q during the transition from the thawed to the frozen state is sharply expressed in loams (Figure 72), and the intersection of curves, accompanied by a sharp increase of α in frozen soils, occurs at different values of q; thus, in kaolin this is observed at $q \simeq 0.2$, in medium loam at $\simeq 0.5$, and in heavy loams at $\simeq 0.6$-0.7. In sands the described curves intersect in the region of values of q from 0 to 0.1 and the observed dependence reminds one somewhat of kaolin in the region $q = 0.7$-0.9.

However, in the transition from the thawed to the frozen states α in sands increases more than in loams and the difference increases with increase of q. Evidently the amount of unfrozen water in the ground has a strong influence on change of the form of the dependence of the coefficient of temperature conductivity on the degree of moisture saturation. This can be explained by the fact that only upon increase of the moisture content above w_{un} will a portion of the water change into ice and the conditions appear for change of the form of the dependence of the thermophysical characteristics on the degree of moisture saturation. In fact, as is evident from the curves of the dependence of the quantity of unfrozen water on temperature presented on Figure 73, at -20°C in heavy loams its content is 6.5%, in medium loams 4%, in kaolin it is not detected at all, and in sands according to calorimetric measurements there is none even at -0.5°C. The total dependence for loams and clays is presented on Figure 74.

The coefficients of thermal conductivity in the thawed and frozen states are very similar in value. It should be noted, however, that at small degrees of moisture saturation the coefficient of thermal conductivity in the thawed state is higher than in the frozen, and when there is complete moisture saturation, the reverse. In sands the curves of λ in the frozen and thawed states intersect at $q < 0.1$, and therefore in the entire investigated range $\lambda_f > \lambda_t$, although their values are very similar.

In loams and clays the curves of the dependence of λ in the thawed and frozen states intersect at different degrees of moisture saturation, but always at larger values than in α. All investigated loams and kaolins can be put in a series by order of increase of moisture saturation in which the curves of λ and α intersect in the thawed and frozen states (see Figure 72).

The curves of the coefficients of temperature conductivity of loams and clays in the thawed and frozen states intersect at $q \simeq 0.5$-0.6, which corresponds approximately to the maximal molecular moisture capacity, and the curves of the coefficient of thermal conductivity intersect at $q \simeq 0.8$, which is close to the lower limit of plasticity. At a moisture content smaller than the

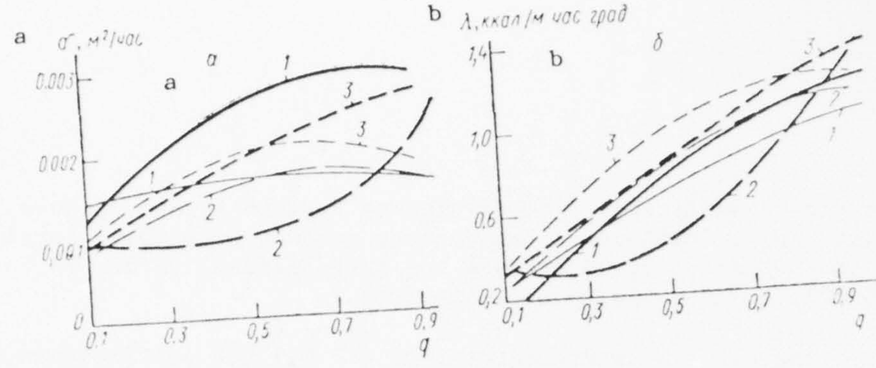
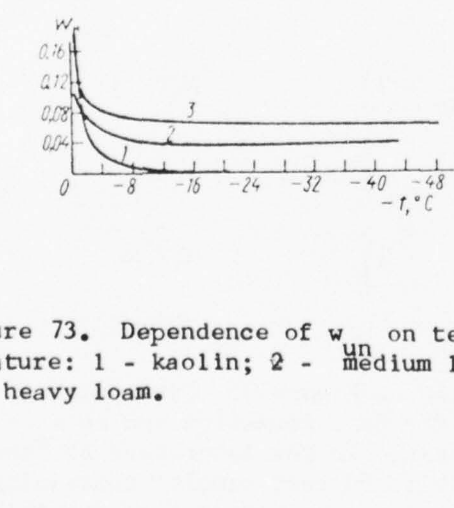


Figure 72. Dependence of the coefficient of temperature conductivity (a) and thermal conductivity (b) on q in the thawed (finellines) and frozen states: 1 - kaolin; 2 - heavy loam; 3 - medium loam. a - α, m^2/hr b - λ, kcal/(m)(hr)(degree)



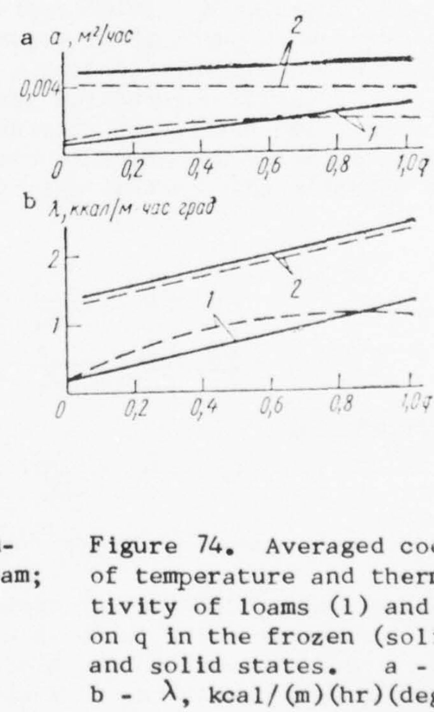


Figure 73. Dependence of w_{un} on temperature: 1 - kaolin; 2 - medium loam; 3 - heavy loam.

Figure 74. Averaged coefficients of temperature and thermal conductivity of loams (1) and sands (2) on q in the frozen (solid lines) and solid states. a - α, m^2/hr b - λ, kcal/(m)(hr)(degree)

maximal molecular moisture capacity α is smaller in the frozen than in the thawed, and at larger moisture contents, on the contrary, is larger in the frozen than in the thawed state.

At moisture contents below the lower limit of plasticity λ is smaller in the frozen than the thawed state, and at larger moisture contents larger in the frozen than the thawed state. The divergence between λ_t and λ_f does not exceed 20%, and the coefficient of temperature conductivity near total moisture saturation in the frozen state exceeds the corresponding values in the thawed state by 30-50%, and for kaolin by up to 100%.

In sands at a degree of moisture saturation $q > 0.1$ the coefficients of thermal and temperature conductivity in the frozen state are larger than in the thawed. However, if α of frozen sand is 30-100% larger than that of thawed, then λ differs by no more than 20%, and in some cases practically coincides (as, for example, for quartz sands).

The influence of organic residues on λ and α. The influence of peat formation on the thermophysical characteristics of soils has been investigated by A. A. Konovalov and L. T. Roman (1969). For the characteristics of peatized soils, besides the ordinary physical properties, they introduced the degree of peat formation K_p, which represents the weight ratio of the peat and mineral components per unit of volume. The variety of composition of peatized soils did not permit finding a simple indicator of the physical properties of the ground determining its thermophysical characteristics. However, for peatized soils, the moisture content of which fluctuates very greatly and is close to the absolute moisture capacity, a dependence of λ on K_p has been established. In that case for peatized sands

$$\lambda_т = 1,97 - \ln\left(\frac{K_т}{0,225K_т + 0,08} + 1\right), \qquad (6.7.16)$$

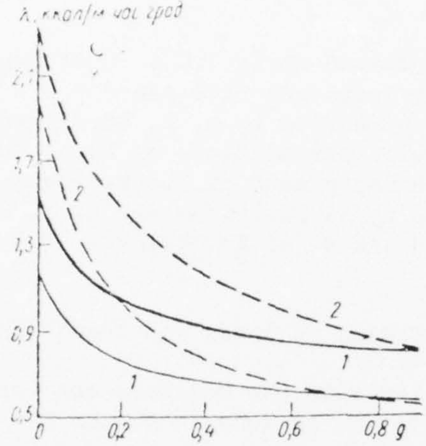
$$\lambda_м = 2,35 - \ln\left(\frac{K_т}{0,15K_т + 0,11} + 1\right), \qquad (6.7.17)$$

and for peatized clays

$$\lambda_т = 1,15 - \ln\left(\frac{K_т}{1,02K_т + 0,21} + 1\right), \qquad (6.7.18)$$

$$\lambda_м = 1,50 - \ln \frac{K_т}{0,74K_т + 0,20} + 1\Big). \qquad (6.7.19)$$

The dependences presented above are depicted on Figure 75, from which it is evident that λ decreases with increase of the peat formation and at K_p = 0.6-0.7 is practically equal to the λ of peat. In the laboratory of the Department of Geocryology it has been established that samples containing organic residues have coefficients of thermal and temperature conductivity more steadily lower than those obtained for mineral soils of the same type. Data of various authors on thermal conductivity in thawed and frozen ground are presented in conclusion in Table 38.



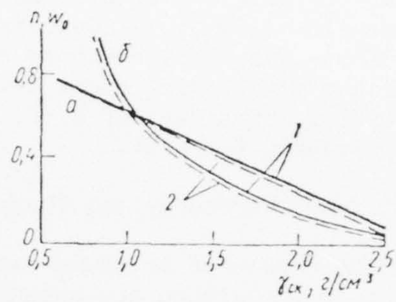


Figure 75. Dependence of the coefficient of thermal conductivity of loams (1) and sands (2) on the degree of peat formation in the thawed (fine lines) and frozen states (acc. to L. G. Roman and L. A. Konovalov, 1969). a - λ, kcal/(m)(hr)(degree)

Figure 76. Dependence of the porosity (a) and absolute moisture capacity (b) on the specific weight of the skeleton of clays (1) and sands (1). 1 - γ_{sk}, g/cm^3

Table 38. Data on thermal conductivity of frozen and thawed rocks (kcal/(m)(hr)(degree)

A Степень влагонасыщенности	B λ талых пород				C λ мерзлых пород		
	1	2	3	4	1	2	3
1 Глины и суглинки							
0	0,16		0,14 (0,18)		0,16		0,18 (0,17)
0,2	0,38	0,54	0,54	0,77	0,49	0,54	0,40
0,4	0,61	0,86	0,83	1,11	0,80	0,92	0,64
0,6	0,84	1,05	1,07	1,31	1,09	1,31	0,89
0,8	1,08	1,18	1,20	1,45	1,38	1,70	1,16
1,0	1,34	1,28	1,26	1,56	1,68	2,09	1,45
2 Супеси и мелкие пески							
0	0,28		0,26		0,28		0,24
0,2	0,72	1,19	1,53	1,09	0,97	0,73	1,59
0,4	1,17	1,49	1,74	1,52	1,47	1,20	1,80
0,6	1,48	1,57	1,95	1,77	1,91	1,68	2,01
0,8	1,75	1,79	2,16	1,95	2,34	2,15	2,22
1,0	2,00	1,89	2,37	2,09	2,77	2,63	2,43
3 Торф*							
0	0,05				0,05		
0,2	0,12				0,16		
0,4	0,21				0,29		
0,6	0,33				0,46		
0,8	0,52				0,71		
1,0	0,85				1,10		

Table 38 (Continued)
A - Degree of moisture saturation B - of thawed rocks C - of frozen rocks 1 - Clays and sandy loams 2 - Sandy loams and fine sands 3 - Peat*
Notes: 1 - Data of SNiP and V. P. Ushkalov, processed by L. T. Roman and A. A. Konovalov (1969); 2 - Calculated with correlations of M. S. Kersten (1955); 3 - Data of laboratory of the Department of Geocryology; 4 - Calculated with correlations proposed by N. S. Ivanov and R. I. Gavril'-yev (1965). *The data of L. T. Roman and A. A. Konovalov (1959) have been presented for peat.

Calculation of Some Thermophysical Characteristics of Sands and Loams (Example 29)

1. Determine the degree of moisture saturation q if the moisture content and specific weight of the ground are known. This can be done in two ways:

a) calculate with the correlation

$$q = \frac{\gamma_{об} \cdot \gamma_{уд} \cdot w \cdot 0{,}01}{\gamma_{уд}(1 + w \cdot 0{,}01) - \gamma_{об}}; \qquad (6.7.20)$$

b) first, calculate $\gamma_{sk} = \gamma_{vol}/w$, and then find on the diagram of Figure 76 the value of the absolute moisture capacity w_a and calculate q:

$$q = \frac{w}{w_п}.$$

For example, we have loam with a moisture content of 20% by weight and a specific weight $\gamma_{vol} = 1.70$ g/cm^3. We find its degree of moisture saturation and then the thermophysical characteristics:

a) if the specific weight of the skeleton of loam $\gamma_{spec} = 2.71$ g/cm^3, then with (6.7.20) we find that the degree of moisture saturation is

$$q = \frac{1{,}7 \cdot 2{,}71 \cdot 0{,}01 \cdot 20}{2{,}71(1 + 0{,}2) - 1{,}70} = 0{,}6;$$

b) first we calculate $\gamma_{ск} = \frac{1{,}70}{1 + 0{,}2} = 1{,}40$ г/см³.

Then with Figure 76 we find that for $\gamma_{sk} = 1.40$ g/cm^3 the absolute moisture capacity $w_a = 0.33$. Consequently the degree of moisture saturation q = 20/33 = = 0.61.

Thus the values of the degree of moisture saturation, calculated in two ways, are very similar.

2. Determine the specific heat also in two ways:

a) calculate it with correlations (6.7.3) and (6.7.4);

b) find it on the diagram (Figure 63). However, in the frozen state the specific heat will be somewhat understated, since the diagrams were constructed without consideration of the amount of unfrozen water.

An example of calculation for the same loam at $w_{un} = 0$.

a) $$C_т = \frac{0{,}19 \cdot 100 - 20}{120} = 0{,}325 \text{ кал/г·град};$$

$$C_м = \frac{0{,}19 \cdot 100 + 0{,}5 \cdot 20}{120} = 0{,}235 \text{ кал/г·град};$$

b) from the diagram (see Figure 63) we have C_t = 0.33 cal/(g)(degree) and C_f = 0.24 cal/(g)(degree).

3. The coefficient of temperature conductivity (α) can be determined in two ways:

a) calculate for sands with formulas (6.7.9) and (6.7.8) and for loams and clays with formulas (6.7.5) and (6.7.6);
b) with the diagram (see Figure 74) we find the values.

For example, we find α for loam:

a) we obtain with the indicated formulas

$$a_т = 0{,}00187 \text{ м}^2/\text{час}; \quad a_м = 0{,}00201 \text{ м}^2/\text{час};$$

b) with the diagram (see Figure 74) we obtain

$$a_т = 0{,}00190 \text{ м}^2/\text{час}, \quad a_м = 0{,}00205 \text{ м}^2/\text{час}.$$

4. The coefficients of thermal conductivity can be obtained by any of three methods:

a) calculate through the degree of moisture saturation with formulas (6.7.14) and (6.7.15) for sands and formulas (6.7.12) and (6.7.13) for loams and clays;

b) find on the diagram (see Figure 74);

c) calculate with correlation (6.7.11) if α, C and γ are known. For example, we calculate λ for the above-indicated loam:

a) with the indicated formulas at q = 0.6

$$\lambda_т = 1{,}06 \text{ ккал/м·град·час}, \; \lambda_м = 0{,}894 \text{ ккал/м·град·час};$$

b) with the diagram (see Figure 74) we obtain

$$\lambda_т = 1{,}06 \text{ ккал/м·град·час}, \; \lambda_м = 0{,}90 \text{ ккал/м·град·час};$$

c) with formula (6.7.11)

$$\lambda_т = 0{,}00187 \cdot 0{,}325 \cdot 0{,}700 = 1{,}03 \text{ ккал/м·град·час},$$

$$\lambda_м = 0{,}00201 \cdot 0{,}240 \cdot 1700 = 0{,}819 \text{ ккал/м·град·час}.$$

It is evident from the presented examples that the coincidence of the thermophysical characteristics calculated by different methods is practically complete.

The somewhat lower value of λ_f obtained by the three methods is explained by the fact that the amount of unfrozen water was not taken into account in calculating C_f.

8. Regularities of the Change of Mechanical Properties of Frozen Dispersed Rocks

The distinctive features of the mechanical properties of frozen rocks and the difference of their transparency and deformational characteristics from unfrozen rocks are connected with the formation of ice and the cryogenic structure.

The formation in frozen soils of ice-cementation bonds leads to increase of their strength and reduction of their deformability and impermeability and contributes to an intensive development of rheological processes (creep, relaxation, and reduction of strength under the long effect of load).

The mechanical properties of frozen dispersed rocks are mainly determined by the character of the structural connections and the forces of interaction between structural elements (particles of the skeleton, aggregates of them, ice inclusions, etc) and also the strength and deformational properties of particles of ice and unfrozen water; the mechanical properties of the skeleton particles in most cases are of minor importance.

The structural connections in soils can have a chemical, molecular or ion-electrostatic nature and are formed as a result of physicomechanical and physicochemical processes developing in the course of the history of development of the rock. In that case, in frozen rocks a considerable role belongs to cryolithogenesis, connected with which is the formation of ice, of the cryogenic structure, and of the ice-cementation and other cryogenic bonds.

The character of the structural connections of frozen rocks is stipulated above all by their composition and structure. At the same time a great influence is exerted by the temperature, which is very variable and is controlled by the characteristics of the frozen rocks. In connection with that the forecasting of the mechanical properties of frozen rocks can be constructed on the basis of the dependence of those properties on the composition, structure and temperature.

The composition of frozen rocks on the whole is characterized by the composition of each of the main components (the skeleton, ice, unfrozen water and gases) and their quantitative relation.

Indicators of the latter can be the total moisture content, the ice content, the moisture content due to unfrozen water, the porosity, the specific gravity, the degree of moisture saturation and other characteristics. It must be noted that in forecasting the mechanical properties of frozen rocks those characteristics must be given in a definite complex which can assure determination of a definite correlation of all the principal components, and not only of some of them. Direct and more graphic indicators of that correlation are the relative content of each component -- the skeleton, ice, unfrozen water and gases.

In examining the dependence of the mechanical properties of frozen soils on their composition, structure and temperature, it is necessary to take their interconnection into consideration. In addition, it should be borne in mind that frozen dispersed rocks are characterized by intensive development of rheological processes. Therefore the influence of a given indicator (dispersion, moisture content, cryogenic texture, etc) as a function of the composition, structure and temperature of the frozen ground, and also of the time of effect of the load, can be different. The influence of the dispersion and the total moisture content is illuminated in very great detail by the available materials.

1. The Dependence of the Mechanical Properties of Frozen Grounds on Their Dispersion

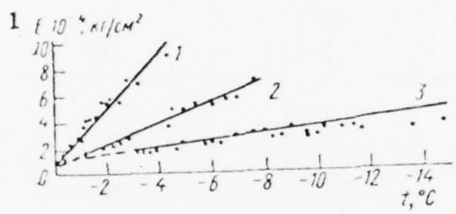


Figure 77. Dependence of the modulus of normal elasticity E of frozen rocks on the temperature t. Uniaxial compression, $\sigma = 2$ kg/cm^2; 1 - sand, w = 22%; 2 - sandy loam, w = 23%; 3 - clay, w = 29% (acc. to N. A. Tsytovich, 1937). 1 - E, 10^{-4}, kg/cm^2

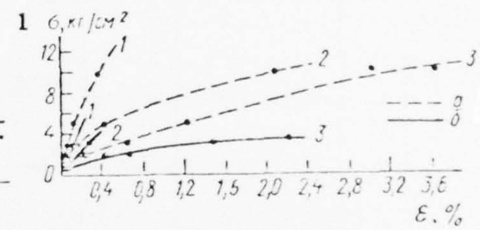


Figure 78. Dependence of the total deformation ε of frozen rocks on the load σ at different moments of time. Uniaxial compression: t = -5°, w = = 26%; 1 - sandy loam, 2 - loam, 3 - clay, a - τ = 240 hrs; b - τ = 1440 hrs. 1 - σ, kg/cm^2

The experimental data characterizing the behavior of frozen soils of different dispersion under the effect of external loads are rather numerous. However, far from all of them can be used to examine the influence of the granulometric composition, since other parameters connected with dispersion often intensify or weaken that influence. For example, in comparison with sands, in many cases clayey grounds have a high moisture content, with which various cryogenic textures of those rocks can also be connected.

The dispersion of frozen rocks has a substantial influence on the development of elastic and plastic deformations.

As follows from Figure 77, when the values of the moisture content are similar and the temperatures are equal the modulus of normal elasticity E diminishes noticeably in the transition from sand to clay. This indicates that with growth of dispersion the intensity of development of elastic deformations of frozen grounds increases, and the resistance to compression is correspondingly reduced under the "instantaneous" effect of the load. An analogous dependence is also noted for plastic deformations. The data presented on Figure 78 (constructed on the basis of results of creep tests) and in Table 39 show that in all cases, with growth of dispersion an increase in the intensity of

Table 39 The moduli of total deformation E_o of frozen soils of different dispersity (degree of moisture saturation, $q \simeq 1$; $t = -2.2^o$)

A Грунт	B Влажность, %	C Время действия нагрузки	D Диапазон нагрузки, $кг/см^2$	E E_0, $кг/см^2$
1 Песок	27	1 мин minute	0—5	3125
			5—10	2083
			10—20	385
		1 час hour	0—5	1020
			5—10	820
			10—17,5	64
		8 час hours	0—5	781
			5—10	232
			10—12,5	32
	22	1 мин	5—10	1000
			10—15	1000
		1 час	5—10	395
			10—15	294
			15—20	178
		8 час	5—10	294
			10—15	147
			15—20	53
2 Суглинок	26	1 мин	0—5	1000
			5—10	1000
			10—12,5	357
			12,5—15	132
		1 час	0—5	454
			5—10	128
			10—12,5	50
			12,5—15	30
		8 час	0—5	385
			5—10	78
			10—12,5	37
	22	1 мин	5—10	385
			10—15	217
			15—20	102
		1 час	5—10	156
			10—15	79
			15—20	57
		8 час	5—10	143
			10—15	59
			15—17	25

Key: A - Ground B - Moisture content, % C - Time of effect of load
D - Range of load, kg/cm^2 E - E_o, kg/cm^2 1 - Sand 2 - Sandy loam

the development of deformation (ε) is observed; correspondingly the modulus of total deformation, characterizing the resistance of frozen soils to the effect of load, diminishes.

The degree of influence of dispersity on the deformational properties of the frozen soils, the available materials show, depends on the temperature and is intensified when it is lowered. When there is a structural coupling in the frozen ground the influence of dispersity is manifested to a lesser degree and in a number of cases greater resistance can be noted in the case of more clayey soils. For example, comparison of the σ-ε curves for sandy loams and clay (when the moisture content values are similar) (see Figure 90) shows that at $t = -5^{\circ}$ clay, with structural coupling, exerts more resistance than sandy loam.

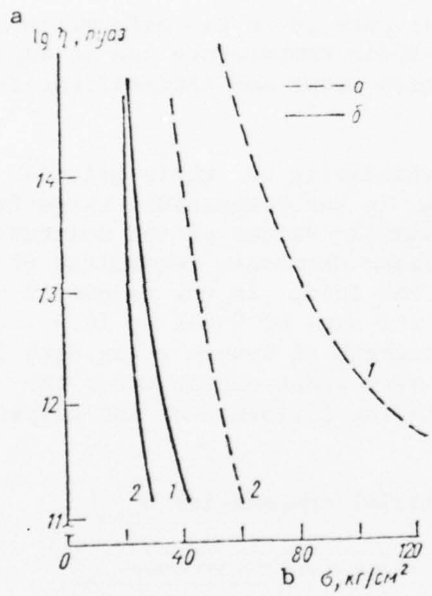


Figure 79. Influence of dispersity on the coefficient of viscosity of frozen rocks. Uniaxial compression: t = -20 (a) and -5° (b); 1 - sandy loam; 2 - clay
A - lg η, pause B - σ, kg/cm^2

The influence of dispersity on the deformational properties of frozen soils is also well illustrated by data on viscosity (Figure 79). As is evident from Figure 79, the coefficient of effective viscosity η of frozen rocks, which characterizes the resistance in the stage of steady creep (during flow at a constant rate), decreases with increase of dispersity. For example, at $t = -20^{\circ}$ and $\sigma = 55$ kg/cm^2 for clay, η is 1000 times smaller than for sandy loam. The noted difference depends on the temperature of the frozen rocks, decreasing when it rises.

The examined data, which testify to a reduction of the resistance of frozen rocks to loads with increase of dispersity, other conditions being equal, can be explained mainly by change of the ratio of the unfrozen water and ice. Side by side with that, the form of the elementary particles and their aggregates, and also distinctive features of the microstructure, have an influence.

As noted above, the mechanical properties of frozen rock are determined mainly by the forces of interaction between structural elements and also by the resistance of the ice itself and the unfrozen water. Particles of the skeleton of the ground interact with one another and with ice inclusions both directly and through films of unfrozen water which reduces the force of the structural bonds. In more disperse frozen rocks the film of water has a greater development (in length and thickness) and therefore the number and area of contacts between the structural elements with its participation increase, and the forces of interaction decrease with each contact; in addition, decrease of the ice content of the rock leads to reduction of the role of mechanical properties of the ice in the formation of resistance of the frozen rock as a whole.

The above-noted increase in the influence of dispersity on the deformational properties of frozen rocks with reduction of their temperature can be explained by reduction of the leveling role of the unfrozen water and intensification of the role of ice.

The strength of frozen rocks also depends substantially on their granulometric composition. As experimental data show, in the temperature range from 0 to -20^{0} with increase of the dispersity at similar values of the moisture content the strength of frozen soils in most cases decreases regardless of the type of deformation and the time of effect of the load. In the temperature range close to 0^{o} this is well illustrated by the data of Table 40 ($t = -2.2^{o}$), from which it follows that reduction of the strength of frozen soils with increase of their dispersity is manifested to a very great degree under the "instantaneous" effect of the load, and in time the influence of the dispersity diminishes.

Table 40 Strength of frozen soils during uniaxial compression σ_{com}

	A Грунт	w, %	B $\sigma_{сж}$, кг/см², при времени действия нагрузки						
			1 мин	10 мин	30 мин	1 час	8 час	24 час	50 лет
1	Песок	27	80,4	30,1	21,8	18,5	12,6	10,8	4,7
		22	88,2	37,6	27,8	23,8	16,5	14,2	6,3
2	Суглинок	26	38,8	23,6	19,1	17,0	12,6	11,1	5,7
		22	36,2	26,1	22,2	20,1	15,8	14,2	7,4

Key: A - Ground B - σ_{com}, kg/cm^2, during the time of effect of load of 1 min, 10 min, 30 min, 1 hr, 8 hrs, 24 hrs, 50 years 1 - Sand 2 - Loam

The influence of the granulometric composition on the long-term strength can be observed at any time of effect of the load, ending with the long-term strength. Thus, according to the data of S. S. Vyalov (1959) the long-term strength during pressing with a spherical stamp (equivalent to coupling) at $t = -3^{o}$ is characterized by the following values (kg/cm^2): 2.3 for sand, 1.0 for sandy loam, and 0.6 for loam.

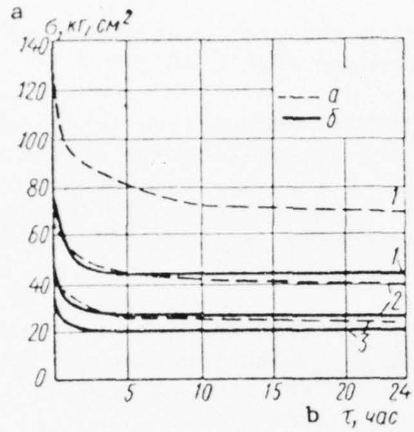


Figure 80. Curves of the long-term strength of frozen sandy loam (a) and clay (b) during uniaxial compression: 1 - $t = -20^o$; 2 - $t = -10^o$; 3 - $t = -5^o$. a - σ, kg/cm^2; b - τ, hrs

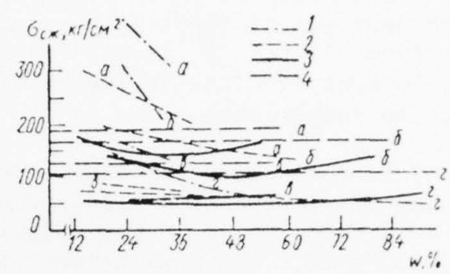


Figure 81. Dependence of the temporary strength of frozen rocks during uniaxial compression σ_{com} on the moisture content ($q = 0.8$-1.0): 1 - sand; 2 - loam; 3 - clay; 4 - ice; a - -55^o; b - -40^o; c - -20^o; d - -10^o. A - σ_{com}, kg/cm^2

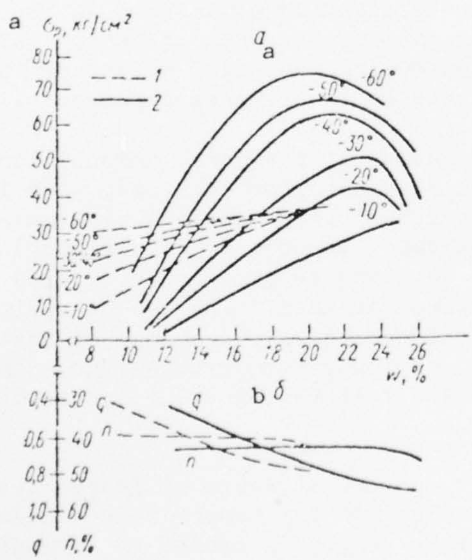


Figure 82. Dependence of the temporary tensile strength of frozen sand (1) and sandy loam (2) σ_t on the moisture content w and the degree of moisture saturation q on w (b). a - σ_t, kg/cm^2

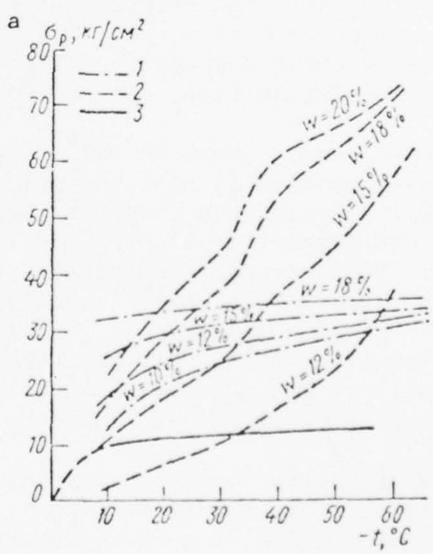


Figure 83. Dependence of the temporary tensile strength of frozen rocks σ_t on the temperature t at various moisture contents: 1 - sand, 2 - loam, 3 - ice. a - σ_t, kg/cm^2

It should be borne in mind, however, that during the prolonged effect of load the more dispersed frozen soils can become stronger (Table 40).

The above-noted reduction of resistance to uniaxial compression (σ_{com}) of frozen rocks with growth of their dispersity and intensification of that dependence with decrease of the length of effect of load in the range of low temperatures from -10 to -55° is manifested still more distinctly (Figures 80 and 81). In that case the influence of dispersity is intensified with reduction of the temperature.

The results of determinations of the temporary textile strength of frozen soils (Figures 82 and 83) show that a reduction of σ_{com} with increase of dispersity is noted in a narrower range of temperatures t (down to -25 to -35°), at still lower values of t the loam becomes stronger than sand.

The influence of the granulometric composition on the strength of frozen rocks, just as on their deformational properties, is caused to a considerable degree by change of the phase composition of the water. The reduction of strength of frozen rocks with increase of dispersity, noted other conditions being equal, and the intensification of the influence of dispersity with lowering of the temperature in the range from 0 to -20° can be explained in the same manner as similar dependences for deformational properties.

The reduction of the degree of influence of dispersity on the strength of frozen rocks during the long effect of load is connected with the rheological properties of the ice, the strengthening role of which diminishes in the course of time. In connection with that, the decrease of strength of frozen soils with increase of their dispersity, observed during the brief effect of load, gradually decreasing in time, can be replaced by the reverse dependence.

At lower temperatures, (down to -55°) the change of the phase composition of water in frozen grounds is also the principal reason for decrease, with increase of dispersity, of the temporary strength of those soils during uniaxial compression and intensification of the indicated dependence with reduction of temperature. The analogous dependence of the tensile strength of frozen soils on dispersity above -25 to -35° (sand is stronger than loam) can be explained mainly by the phase composition of water. The qualitative change of that dependence (sand becomes weaker than loam) at lower temperatures (down to -60°) evidently is connected with distinctive features of the microstructure of frozen rocks with different disperity.

The decrease of the influence of dispersity on the strength of frozen soils with increase of their moisture content, noted for the temporary resistance to uniaxial compression in a wide temperature range, is caused by intensification of the role of ice, which is reduced to weakening of the interaction between soil particles and to leveling of the wedging effect of films of unfrozen water.

2. Dependence of the Mechanical Properties of Frozen Soils on Their Moisture Content

The interconnection of the principal indicators of composition (dispersity, moisture content, the degree of moisture saturation, etc) and temperature can be clearly traced also in examining the dependence of the mechanical properties of frozen dispersed rocks on the total moisture content w.

The dependence of the strength of frozen soils on the moisture content at t down to -12^{o} was examined in very great detail in the works of N. A. Tsytovich (1937) and at t down to -60^{o} by Ye. P. Shusherina et al (1969, 1970).

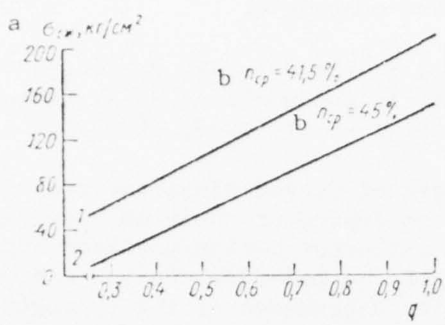
In studying the influence of w on the strength of frozen dispersed rocks it is necessary to take into account above all the degree of their moisture saturation and also the fact that the change of w can occur during constant and variable porosity. The conditions of change of the moisture content w have a substantial influence on the character of the dependence of the strength of the frozen rocks on it.

The increase of the total moisture content w of frozen rocks under conditions of practically constant porosity n and correspondingly increasing degree of moisture saturation q in all cases, regardless of the dispersity of the rock, temperature and type of deformation, causes increase of strength (Figures 82 and 84). This is connected mainly with intensification of the cementing effect of ice as a result of increase of the volume of the ice of conclusions, ans also increase of the area of contacts between particles of the skeleton and the ice.

During increase of the moisture content w of frozen soils under conditions of practically complete moisture saturation and correspondingly increasing porosity, in most cases a reduction of porosity is observed, but at sufficiently large values of w during compression there can also be an increase of strength (see Figure 81). The noted reduction of strength is caused primarily by weakening of the structural bonds between particles of the skeleton or aggregates of them as a result of shifting of the ice, which is only partially compensated by the cementing effect of the ice. However, the growth of strength during increase of w in the range of large values of the latter is connected with the developing localization of the unfrozen water (the ground framework is broken up by ice layers), and also by reduction of its quantity as a result of decrease of the surface of the skeleton and ice (per unit of volume of the ground).

The degree of manifestation of the indicated regularity depends substantially on the dispersity and temperature. For example, during increase of w under conditions of practically constant porosity with reduction of the temperature the dependence of the strength (tensile) on w for clayey frozen soils is intensified, and for sands weakened (see Figure 82). Under conditions of absolute moisture saturation and variable porosity the influence of increase of w on strength (compression) increases with decrease of the temperature and dispersity.

The variation of w of frozen soils occurs, as a rule, in far more complex conditions than those considered above. In different ranges of the moisture content both the porosity n and the degree of moisture saturation q can vary. Therefore the dependence of strength on the moisture content can have a rather complex character (can have a maximum, a minimum, a combination of them, etc), which, however, can be analyzed on the basis of the above-examined cases, as is illustrated by the schematic diagram presented on Figure 85.



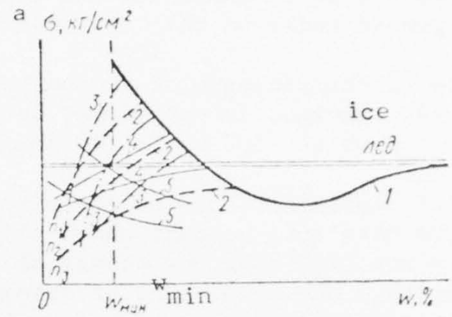


Figure 84. Dependence of the temporary uniaxial compressive strength σ_{com} of frozen rocks on the degree of moisture saturation q; $t = -40^{\circ}$; 1 - sand; 2 - loam. a - σ, kg/cm^2 b - η_{mean}

Figure 85. Dependence of the strength σ of frozen rocks on the moisture content w under various conditions of its variation (schematic diagram). Explanations are given in the text. a - σ, kg/cm^2

As is known, the interconnection between w, q and n can be represented in the form

$$w\gamma_{уд}(1-n) = 0,9qn + 0,1\gamma_{уд}w_н(1-n) \qquad (6.8.1)$$

or

$$w\gamma_{уд}(1-n) \cong 0,9qn,$$

where γ_{spec} is the specific gravity of the skeleton of the ground. On Figure 85 curve 1 depicts the case of increase of w at $q \cong 1$; $n \neq$ constant and increases in accordance with (6.8.1). Curves 2 characterize the case of increase of w at n = constant ($n_1 < n_2 < n_3$); $q \neq$ constant and increases in accordance with (6.8.1). Curves 3 relate to the case of decrease of n, and curves 4 to the case of its increase with increase of w; $q \neq$ constant and increases in accordance with (6.8.1). Curves 5 characterize the case of increase of w at q = constant < 1; $n \neq$ constant and increases in accordance with (6.8.1).

A combination of those cases can give a different character of the dependence of the strength of frozen rocks σ on their moisture content w. Thus, a combination of cases 2 and 1 gives a curve σ - w with two extremums (a maximum and a minimum). The same is obtained with a combination of 3 and 1 (but with a sharper maximum) and also 4 and 1 (the maximum is expressed more weakly). With a combination of cases 3, 5, 4 and 1 the curve q - w can have two maxima and two minima, etc.

The examined general form of the dependence of the strength of frozen rocks on their moisture content w applies to the case of brief effect of load. The dependence of the long-term strength of frozen soils on w with the same regularity will differ only quantitatively in connection with decrease of the cementing capacity of the ice under the long effect of load.

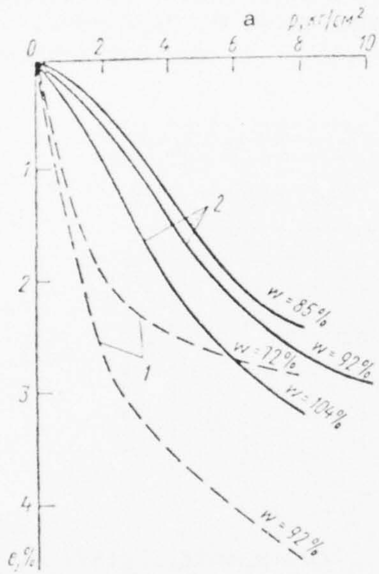


Figure 86. Dependence of the compressability e of frozen loams with a laminated cryogenic texture on the total moisture content w: 1 - at t = -0.4°; 2 - t = -3.6-4.0° (according to A. G. Brodskaya) a - p, kg/cm²

Also substantially dependent on the total moisture content w of frozen rocks is their compressability. According to the data of A. G. Brodskaya (1962), with increase of w the degree of shrinkage increases during compression (Tables 41 and 42 and Figure 86). This is noted both for frozen rocks with a cryogenic texture of a single type and for those with different. The obtained dependence is explained by the fact that the shrinkage of frozen rock under the effect of load is caused mainly by reduction of its ice content. The main mechanism of that process involves phase transformations of water (the liquid phase of water is squeezed out and continuously made up through the ice).

Table 41. The influence of the total moisture content (w) on the compressability (e) of frozen soils. Load p = 8 kg/cm² (according to A. G. Brodskaya, 1962)

	A Разновидность грунта	B Темпера-тура, °C	C Объемный вес, г/см³	w, %	D $w_н$, %	E Льдис-тость, %	e, %
1	Суглинок массивной текстуры	—4,2	1,83	36	12,3	23,7	1,8
2	То же	—4,2	1,84	33	12,3	20,7	1,4
	То же	—4,0	—	30	12,3	17,7	1,1
3	Песок массивной текстуры	—0,5	1,87	27	0,2	26,8	1,2
	То же	—0,5	1,98	21	0,2	20,8	0,5
4	Суглинок слоистой текстуры	—0,4	1,53	72	16,1	55,9	3,2
	То же	—0,4	1,43	92	16,1	75,9	5,2

Key: A - Type of ground B - Temperature, °C C - Specific gravity, g/cm³ D - w_{un}, % E - Ice content, %
1 - Loam of massive texture 2 - Ditto 3 - Sand of massive texture 4 - Loam of laminated texture

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FUNDAMENTALS OF FROST FORECASTING IN GEOLOGICAL ENGINEERING INV--ETC(U)
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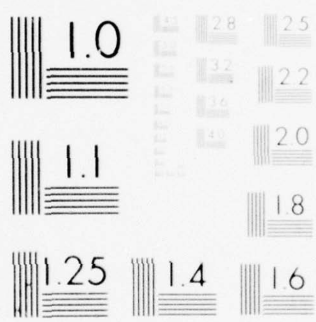


MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Table 42. Compressibility (e) of frozen loams with different total moisture content and cryogenic texture (according to A. G. Brodskaya, 1962). In the numerator, data at t = -4°, in the denominator, at t = 0.4°

A Криогенная текстура	w, %	B Льдистость, %	C Объемный вес, $г/см^3$	D e (%) при нагрузке $кг/см^2$				
				1	2	4	6	8
1 Массивная	30.0	17.7	1,82	0.1	0.2	0,4	0.8	1.0
1 Массивная	30,0	12,3	1,82	0,7	1,2	2,0	2,3	2,6
2 Сетчатая	41.9	29,6	1.71	0.1	0.2	0,5	1,4	2.0
3 Слоистая	71,8	55,7	1,53	1,8	2,4	2,8	3.0	3,2
3 Слоистая	103.6	92,0	1.36	0.5	1.1	2,3	3.1	3.8
3 Слоистая	91.5	75,4	1.43	1,9	3,3	4,1	—	5,1

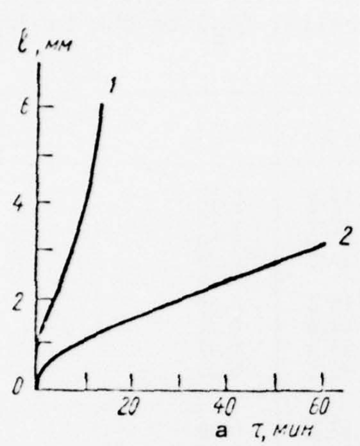
Key: A - Cryogenic texture B - Ice content, % C - Specific gravity, g/cm^3 D - e (%) at a load in kg/cm^2
1 - Massive 2 - Latticed 3 - Laminated

3. The Dependence of the Mechanical Properties of Frozen Soils on Their Structure

In examining the dependence of the mechanical properties of frozen rocks on their structure one should take into consideration its distinctive features, formed both before the freezing of deposits and in the process of cryogenesis.

Thus, the lamination which formed in the rock before the freezing has an effect on its mechanical properties also in the frozen state. This is clearly illustrated by the results of shear tests of layered clay after it has been frozen at a temperature of -40°, which assures the formation of a massive cryogenic texture. It follows from a comparison of the curves of creep of frozen clay during shear that if the shear plane passes along the lamination the deformation e of the rock at any moment of time is greater than in the case where the shear plane is perpendicular to the layers (Figure 87). Comparison of the short-term and long-term strength of clay in the frozen state in the presence of shear with a different direction of the shear surface to the lamination (Table 43) shows that, regardless of the amount of normal pressure and time before destruction, in the case of a direction of the shear surface perpendicular to the lamination the strength of the rock always proves to be far greater than in the case where the shear surface passes along the lamination.

The presented data show that, in spite of intensification of the strength of bonds between particles of rock as a result of its cementation by ice the anisotropy of the mechanical properties of the layered soil is also preserved after its transition into the frozen state.



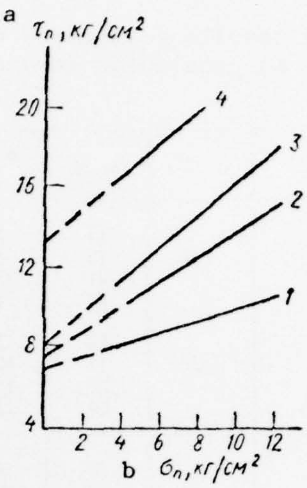


Figure 87. Curves of creep of frozen laminated clay in the presence of shear. The shear surface is parallel (1) and perpendicular (2) to the lamination; t = -10°, σ_n = 25 kg/cm², τ_n = 15 kg/cm². a - τ, minutes

Figure 88. Diagram of rapid shear of frozen clay at t = -2°: 1 - massive cryogenic texture, w = 30-31%; 2 - fine-lattice cryogenic texture, w = 29-31%; 3 - fine-lattice cryogenic texture, w = 32-34%; 4 - ice (according to N. K. Pekarskaya, 1963).

Table 43. Resistance to shear of frozen clay (kg/cm²) in the direction of the shear surface perpendicular ($\tau_\perp$) and parallel ($\tau_\parallel$) to the lamination. t = -10°C

A σ, кг/см²	B Время действия нагрузки											
	2 мин			10 мин			2 часа			24 часа		
	$\tau_\perp$	$\tau_\parallel$	$\tau_\perp/\tau_\parallel$	$\tau_\perp$	$\tau_\parallel$	$\tau_\perp/\tau_\parallel$	$\tau_\perp$	$\tau_\parallel$	$\tau_\perp/\tau_\parallel$	$\tau_\perp$	$\tau_\parallel$	$\tau_\perp/\tau_\parallel$
10	23,0	17,0	1,3	14,2	9,3	1,5	12,0	7,5	1,6	12,0	7,5	1,6
25	26,0	17,0	1,5	17,8	15,0	1,2	15,0	9,5	1,6	15,0	9,5	1,6
50	29,0	23,0	1,3	22,4	17,0	1,3	18,0	13,0	1,4	18,0	12,7	1,4

Key: A - σ, kg/cm² B - Time of effect of load (2 minutes, 10 minutes, 2 hours and 24 hours)

The mentioned anisotropy depends on the dispersity, moisture content and temperature of the frozen rock, which can be clearly traced on the basis of the data of rapid tensile tests of layered sands and loams frozen at -40° and having a massive cryogenic texture (Table 44).

The cryogenic structure also has a substantial influence on the mechanical properties of frozen rocks (Pekarskaya, 1963). Thus, in the region of intensive phase transitions, when the values of the total moisture content w

Table 44. Temporary tensile resistance of frozen soils in the direction of stress perpendicular ($\sigma_{\perp}$) and parallel ($\sigma_{\parallel}$) to the lamination

A Грунт	t, °C	w, %	$\sigma_{\perp}$, *кг/см²	$\sigma_{\parallel}$, *кг/см²	$\sigma_{\perp}/\sigma_{\parallel}$
1 Суглинок	−10	15	2,0	11,0	0,18
		18	8,5	18,0	0,47
		21	15,0	25,0	0,60
		24	21,0	30,0	0,70
	−20	15	5,5	18,5	0,30
		18	12,0	29,0	0,41
		21	18,5	39,0	0,47
		24	25,5	40,5	0,63
2 Песок	−10	10	4,0	13,5	0,30
		12	10,5	19,0	0,55
		15	21,5	26,5	0,81
		18	31,5	32,5	0,97
	−20	10	10,0	21,0	0,48
		12	15,5	25,0	0,62
		15	24,0	29,5	0,81
		18	32,5	39,0	0,98

Key: A - Ground * - kg/cm^2 1 - Loam 2 - Sand

are close the frozen clay in the case of massive cryogenic textures is characterized by smaller values of the resistance to rapid shear, coupling and friction than in the case of a latticed texture, for which with growth of the ice schlieren the strength of the rock increases (Figure 88). Comparison of the results of rapid shear tests of frozen clay with massive and laminated textures also testifies to a smaller strength of ground without ice schlieren. In that case, in samples of clay with a laminated texture, differing also in a larger value of w, the shear surface was directed practically perpendicularly to the lamination. Characterized by less strength are samples of frozen rock with a massive texture in comparison with samples containing ice schlieren and during long shear and uniaxial compression tests.

The obtained regularities can be explained by starting from an estimate of the strength of ice and the connection of different contacts. At high temperatures and with a brief effect of load the contacts of soil particles with one another and with the ice as a result of the presence of unfrozen water are weakened sections in comparison with ice. Therefore the appearance of ice schlieren and their development leads to strengthening of the frozen rock (in cases where the surface of destruction passes through ice inclusions).

When the temperature is lowered the correlation of strength of the indicated contacts and ice can change, as can be testified to by the results of compression tests of frozen soils in the range of temperatures from -10 to -55^{o}, according to which under certain conditions ice becomes weaker than frozen rock (See Figure 81). The correlation of the strength of ice and contacts between the structural elements of frozen ground also depends on the length of the effect of load, as the strength of ice diminishes in time practically to zero.

Also dependent on the cryogenic texture of frozen rocks is their compressibility (Brodskaya, 1962) which, as the experimental data show, increases with the development of the surface of resistance of ice with soil particles. Therefore the formation and growth of ice schlieren and the increase of their quantity and thickness, characterizing the amount of the schlieren ice content, lead to increase of the compressibility of frozen soils (see Table 42).

4. The Dependence of the Mechanical Properties of Frozen Soils on Their Temperature

Data on the dependence of the mechanical properties of frozen rocks on temperature are numerous, and in comparison with the factors touched upon above the influence of temperature can be considered in a "purer" form. The change of the deformational properties of frozen soils as a function of temperature can be traced on the basis of the moduli of elasticity and general deformation, the curves of creep, the coefficients of viscosity and the compression curves.

The modulus of elasticity of frozen soils depends essentially on the temperature t, increasing with its decrease (see Figure 77), which characterizes the the increase of the resistance of those soils to the effect of load in the region of elastic deformations. In that case the influence of t diminishes with increase of dispersity.

A similar influence of temperature on the resistance of frozen rocks in the region of plastic deformations is shown by comparison of the curves of the dependence of the total deformation ε on the load σ at different moments of time at different temperatures (Figure 89, a and b and Figure 90, a and b).

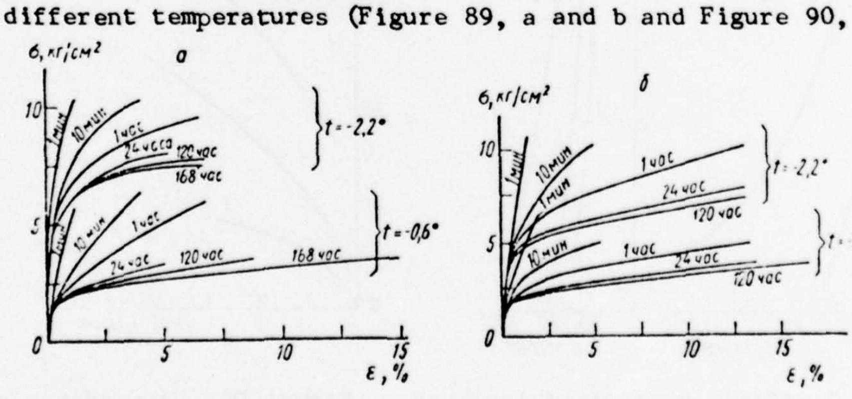


Figure 89. Dependence of the total deformation of frozen rocks ε on the load σ at different moments of time. Uniaxial compression: a - sand, b - loam. 1 - σ, kg/cm^2

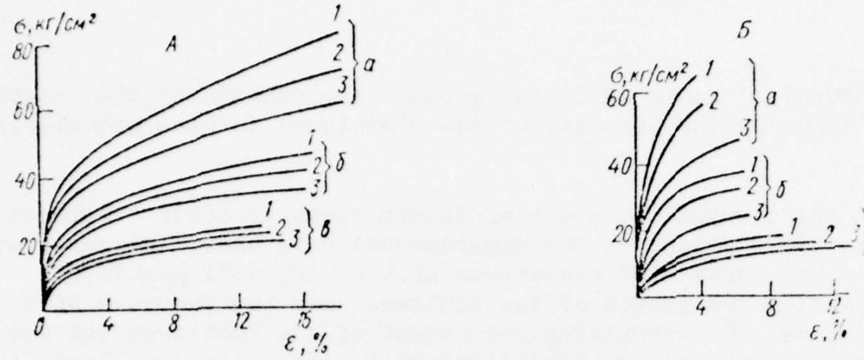


Figure 90. Dependence of the total deformation of frozen rocks ε on the load σ at different moments of time: 1 - 1; 2 - 3; 3 - 12 hours. Uniaxial compression: A - sandy loam; B - clay; a - at t = -20^{o}; b - at t = -10^{o}; c - at t = -5^{o} (according to S. E. Gorodetskiy, 1962).

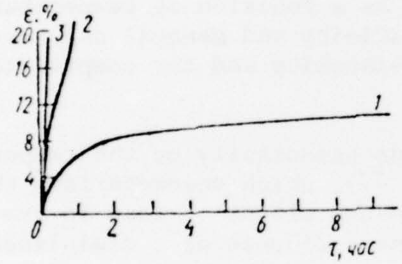


Figure 91. Curves of creep of frozen sandy loam at temperatures: 1 - -20^{o}; 2 - -10^{o}; 3 - -5^{o}. Uniaxial compression. 1 - τ, hrs

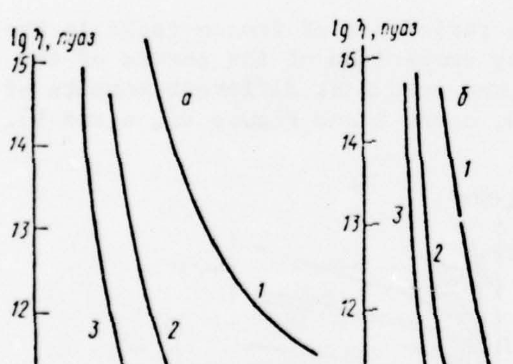


Figure 92. Coefficient of viscosity η of frozen sandy loam at temperatures: 1 - -20^{o}; 2 - -10^{o}; 3 - -5^{o}. Uniaxial compression: a - sandy loam; b - loam. 1 - lg η, pause 2 - σ, kg/cm^{2}

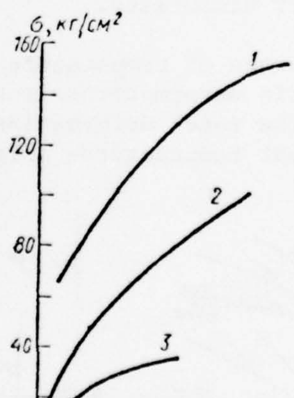


Figure 93. Dependence of the temporary resistance to compression of frozen rocks to temperature: 1 - sand, w = 16-17%; 2 - sandy loam, w = 11-12%; 3 - clay, w = 52-61% (according to N. A. Tsytovich, 1958).

The presented data show that the modulus of total deformation E_0, which is a variable and depends on the time τ and effective stress σ, increases when the temperature is lowered, that is, the resistance of the frozen soils is intensified with lowering of the temperature also in the region of plastic deformations.

Testifying to the same thing also is comparison at different temperatures of the curves of creep of frozen rocks (Figure 91) and the coefficients of viscosity η of the rocks, characterizing the resistance in the state of steady flow (Figure 92).

According to S. S. Vyalov the dependence between the stress σ and strain E for frozen rocks can be described with the expression:

$$\sigma = A(\tau) E^m. \qquad (6.8.2)$$

The influence of the temperature t on the value of E is manifested through the parameter A, the dependence of which on time is described by the equation $A(\tau) = \chi\tau^{-\lambda}$. The parameter λ is practically independent of t. However, the dependence of χ on t, according to S. E. Gorodetskiy (1962), in separate ranges of t can be represented in the form

$$\chi(t) = \omega(t + 1)^k. \qquad (6.8.3)$$

Through the modulus of total deformation $E_0 = \sigma/E$ the coefficient A can be expressed with the dependence

$$A(\tau) = E(\tau) E^{1-m} = E^m(t) \sigma^{1-m}. \qquad (6.8.4)$$

As follows from the presented materials, reduction of the temperature causes the same effect as the reduction of dispersity -- increase of the resistance of frozen soils to the effect of loads. This results because both those indicators are connected with the phase composition of the water in the frozen rocks, which is one of the important factors on which the mechanical properties of those rocks depend. However, when the temperature is lowered, not only does a change of the quantitative ratio of ice and the unfrozen water occur, but also other processes. As is known, during the cooling of dispersed rocks during the transition through 0° there is a sudden increase of their strength, connected with the transition of water into ice. Further lowering of the temperature to -20° according to numerous experimental data is characterized by a smooth increase of strength, occurring, regardless of the granulometric composition, with damping intensity (Figures 93 and 94), which is reflected by equations of the type

$$\sigma = A + Bt + Ct^n, \quad \sigma = A + Bt^n. \qquad (6.8.5)$$

The growth of the strength of frozen soils, like the above-noted dependence of their deformational properties on temperature, is caused mainly by increase of the strength of the ice and the viscosity of the unfrozen water and the further crystallization of water and structural transformations connected with that (including the dehydration of aggregates, processes of coagulation, and

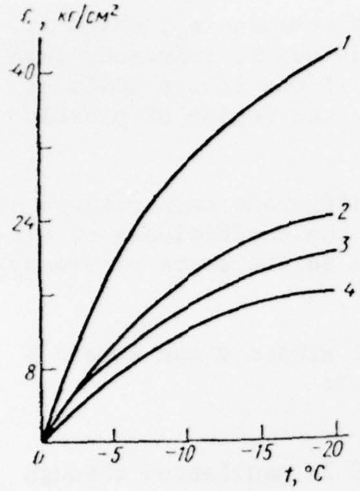


Figure 94. Dependence of the coupling forces of frozen sandy loam (C) on the temperature (t) at different times of the effect of load: 1 - 10 sec; 2 - 10 min; 3 - 1 hr; 4 - 24 hrs.
a - C, kg/cm^2 b - τ, ^oC

the migration of moisture and skeleton particles). However, the weakening of the dependence of the resistance of frozen soils on the temperature t when it is lowered is connected with the nature of the influence of t on the mechanical properties of ice and unfrozen water, and also with damping of those processes.

Investigations in the area of low negative temperatures (down to -60^o) show that below -20^o a smooth increase of strength σ with damping intensity can be observed only for frozen sand (see Figure 83). However, for frozen clayey soils (sandy loam, loam and clay) at a temperature below -20 to -30^o, however, there is a far more intensive increase of σ than in the region from -10 to -20^o (the σ - t curve has a point of discontinuity) (Figure 83). Evidently this can be linked with the freezing of new amounts of water in finer pores (for example, in aggregates, which are not present in sand), causing further structural transformations, which is completely linked with the results of investigations of temperature deformations of frozen soils during their cooling in the range of low temperatures. For frozen clayey ground the strains of contraction during the lowering of their temperature differs only down to -20^o, and expansion is observed in the range from -20 to -60^o. For frozen sand, however, down to -160^o there is only a smooth development of strains of contraction.

During the long effect of load the dependence of the strength of frozen soils on temperature weakens somewhat (Figure 94); this is mainly connected with the rheological properties of the ice.

According to S. S. Vyalov the reduction of the strength of frozen rocks in time can be described by an expression of the type of

$$\sigma(\tau) = \frac{\beta}{\ln \frac{t+1}{B}}. \qquad (6.8.6)$$

The dependence of the short-term and long-term strength of frozen rock on the temperature t in the range of down to -20^{o} is expressed through the parameter β in the form

$$\beta = \omega (t + 1)^{\alpha}. \qquad (6.8.7)$$

9. Forecasting Thermal Subsidences of Soils During Thawing

During construction it is important to take into consideration whether existing permafrozen rock masses will thaw or new formations of frozen rock masses will form within the limits of taliks, as in those cases there are very substantial changes of the frost and geological engineering conditions. For the forecasting of those processes it is necessary to know the composition, moisture content, the cryogenic textures of frozen rocks, their thermophysical characteristics, the possibility and probability of their thawing and the character of the frost and geological processes. On sections where the new formation of frost is possible as a result of economic activity, the regularities of its formation and development must be established both in accordance with existing natural conditions of the given region and with consideration of future changes.

During the thawing of permafrozen rock masses of great importance are the composition and ice content (cryogenic textures) of the rocks. It is precisely they which determine the degree of change of the properties of rocks during thawing, and also the degree and intensity of development of cryogenic processes. Each geological genetic complex of rocks, depending on the character of the freezing (the correlation of sedimentation and freezing), can be classed as syngenetic, epigenetic or polygenetic frozen rock masses. The genesis of loose thick rock masses is determined to a considerable degree by their stratigraphy. Deposits formed in pre-Quaternary time belong mainly to the epigenetic. In northern and arctic regions, syngenetic frozen rock masses could have formed during the entire Quaternary period. Near the contemporary southern boundary of frost propagation, syngenetic frozen rock masses formed in the Upper Pleistocene and mainly in the Holocene. Polygenetic frozen rock masses occur in all the frost-temperature zones and are determined by the conditions of the accumulation of contemporary sediments and their bedding in relation to permafrozen rocks.

The thawing of frozen rock masses and the character of the processes accompanying them, as has already been pointed out, depend to a great degree on the cryogenic structure of the rocks. For syngenetic rock masses, in compiling a frost forecast it is advisable to distinguish three principal types of ice content in rocks: 1) reopened-vein ices; 2) schlieren cryogenic textures; 3) monolithic cryogenic textures.

In the presence of ice veins the process of thawing of permafrozen rock masses proceeds slowly because of large phase transitions on account of the thawing of ice. This is clearly visible in outcrops with reopened-vein ices, where the thawing proceeds irregularly -- more slowly above veins and more rapidly above enclosing rocks. During the thawing of syngenetic rock masses of that kind very large thermal subsidences are noted which can reach 30-50% or more of the thickness of the thawing rock mass. Depending on the composition of the rocks as a result of the thawing of reopened-vein ices (on lowland sections) or baydzherakhi [hillocks remaining after glaciation] (on slopes, see Figure 123). The total depth of the thermal subsidence will be determined by the depth of the thawing basin with consideration of the weighted average ice content of the thawing rock mass.

During the thawing of syngenetic rock masses with a laminated and laminated-lattice texture the subsidences can also achieve large values -- 10-20% of the total thickness of the thawing rocks. As a result, alassy [depressions in the pergelisol] basins or thermokarst lakes form. During the thawing of syngenetic rock masses with a monolithic cryogenic texture the thermal subsidences usually are less considerable. Therefore a quantitative estimate of the thermal subsidences during the thawing of frozen rock masses must be given for each geological genetic complex of rocks with consideration of facial changes of the deposits and distinctive features of their moisture content and cryogenic textures. Thus, for example, syngenetic deluvial-solifluctional deposits are characterized by the presence of underground ices of both segregative and solifluctional origin and buried firn basins.

Segregational ices form in the process of accumulation of deluvial deposits with their annual seasonal freezing and thawing. The latter leads to frost differentiation of the material in the layer of seasonal thawing and the evacuation of fine-grained filler and as a result of that to the specific structure of that layer and the upper part of the profile of permafrozen rock masses (Figure 95). Frequently in that case under the layer ξ forms a layer of almost pure (bald peak) ice with a thickness of 0.3-0.5 m or more. From the surface usually lies large-fragment or block material, at times almost without filler. As a result of that an impression can form of the presence of a rocky base under them and the geological engineering conditions of that section can erroneously be considered favorable. Actually, on the given section the thermal subsidences can reach 1 or 2 meters or more.

The ice distribution in fork formations is extremely irregular and separate exploring holes may be unable to reveal their beddings. To reveal them and contour them for geological engineering purposes, geophysical surveying methods should be used. A thermal subsidence on such slopes causes slides, overflows and avalanches. As a rule, in that case the thickness of buried ices and firn fields is kept within the limits of tens of cm and rarely reaches 2 or 3 meters. A characteristic feature of those ices is their wide distribution in the form of separate small islands, which give considerable thermal subsidences.

Calculation of the Amount of Thermal Subsidence During the Thawing of Perma-Frozen Rocks (Example 30)

Determine the thermal subsidence of ground, for example, at the base of a highway being planned in a permafrost region. The starting data are: deposits to a depth of 18-20 meters consist of loams laQ_{3IV}, which in the layer of seasonal thawing have the values: γ_{sk} = 1100 kg/m^3; γ_{spec} = 2.7 g/cm^3; w = 38%; λ_t = 1.0; λ_f = 1.3 kcal/(m)(degree)(hr); C_{spec} = 0.19 kcal/(kg)(degree). Data on the amount of unfrozen water in frozen loam are presented on Figure 96.

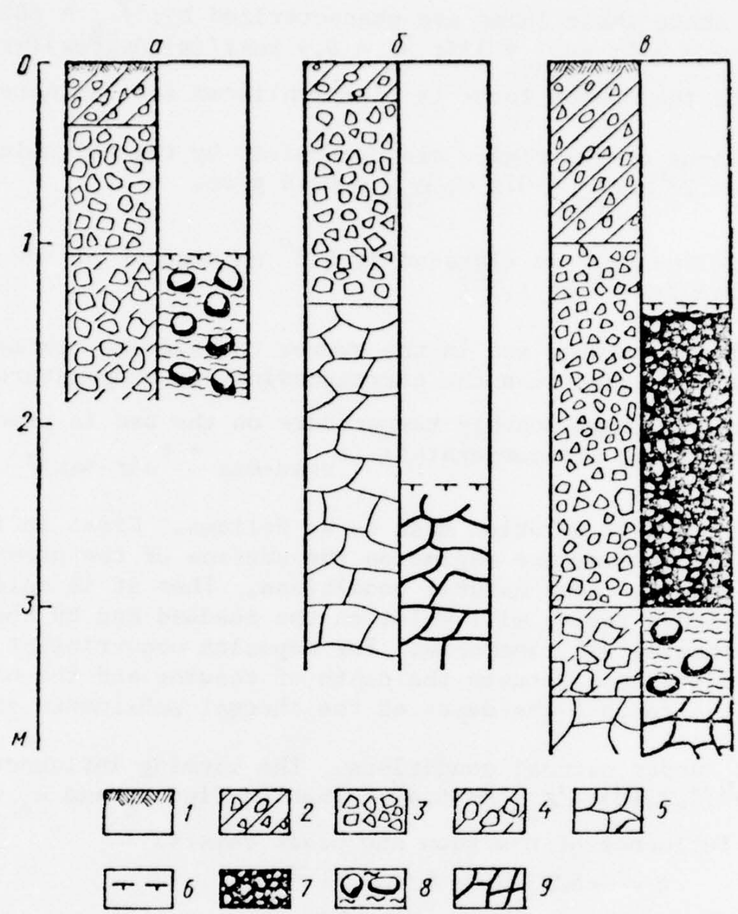


Figure 95. Examples of the correlation of the depth of thawing and of occurrence of deposits on steep slopes (according to T. N. Kaplina, 1970): a - depth of thawing coincides, b - considerably exceeds, c - does not reach the base of the layer of crushed stone without filler. In the left column is the lithology and in the right the cryogenic structure of the rocks: 1 - humified layer; 2 - sandy loam or loam with crushed stone; 3 - crushed stone; 4 - crushed stone with sandy loam and loam filler; 5 - crushed stone; 6 - depth of seasonal thawing; 7 - basal cryogenic texture; 8 - combination of crustal and fine-schlieren texture; 9 - fractured cryogenic texture.

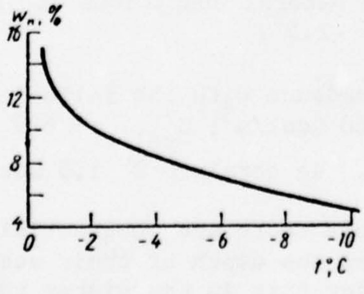


Figure 96. Graph of the dependence of w_{un} in loam on temperature.

In the permafrozen state those loams are characterized by: $\gamma_{sk} = 1000$ kg/m^3; $\gamma_{spec} = 2.7$ g/cm^3; w = 57%; w_{min} = 33%; $\lambda_t = 0.9$ kcal/(m)(degree)(hr). The cryogenic texture of the frozen loams is fine schlieren and laminated.

The climatic conditions of the region are determined by the following data: $t_{air} = -8.2^{\circ}$; $A_{air} = 23^{\circ}$; $z_{sn} = 0.5$ m; $\rho_{sn} = 0.19$ g/cc.

Under natural conditions t_{air} is elevated by 0.3° on account of the plant cover and A_{air} is reduced by 1.5°.

Snow is cleared off the roadbed and in the summer the average maximal temperature ($t_{road\text{-}max}$) is 8° higher than the corresponding air temperature ($t_{air\text{-}max}$); in winter the maximal average monthly temperature on the bed is practically equal to the corresponding air temperature ($t_{road\text{-}max} = t_{air\text{-}max}$).

Solution. The procedure in solution must be as follows. First it is necessary to determine the temperature regime on the surface of the ground and the depth of seasonal thawing under natural conditions. Then it is calculated what sort of temperature regime will exist on the roadbed and by how much the depth of seasonal thawing will increase. For deposits occurring at a depth determined by the difference between the depth of thawing and the natural conditions and under the roadbed the depth of the thermal subsidence is calculated.

1. We determine t_o under natural conditions. The warming influence of the snow is found with (5.3.10): $\Delta t_{sn} \approx 6.4^{\circ}$. Then we find t_o and A_o with consideration of the influence of the snow and plant covers:

$$t_0 = -8{,}2 + 0{,}3 + 6{,}4 = -1{,}5^{\circ},$$

$$A_0 = 23 - 6{,}4 - 1{,}5 = 15{,}1^{\circ},$$

$$t_{0,\text{зим}} = \frac{t_{\text{мин}}}{2} = \frac{-16{,}6}{2} = -8{,}3^{\circ}.$$

2. We calculate $c_{vol\text{-}t}$ and Q_{ϕ} with (4.1.7) and (4.1.8):

$$C_{\text{об.т}} = 0{,}19 \cdot 1100 + \frac{38 \cdot 1100}{100} = 627 \text{ ккал/м}^3\cdot\text{град}.$$

In calculating Q_{ϕ} we take w_{un} into account, assuming that the average monthly temperature in the freezing layer is close to $1/2\, t_{min} = -8.3^{\circ}$, at which the amount of w_{un} on the average for the layer will be ~ 6% (see Figure 96):

$$Q_{\Phi} = 80 \frac{(38-6)\,1100}{100} = 28\,160 \text{ ккал/м}^3.$$

3. We calculate ξ under natural conditions. To do that we find Δt_{λ} in the layer ξ with the nomogram (see Figure 33). Under natural conditions it is equal to ~ 1.0°. Consequently, $t_{\xi} = -1.5 - 1.0 = -2.5^{\circ}$.

With a nomogram (see Figure 15) we find ξ in accordance with the following starting data: $A_o = 15.1^{\circ}$; $t_{\xi} = -2.5^{\circ}$; $Q_{\phi} = 28{,}160$ kcal/m^3; $C_{vol\text{-}t} = 627$ kcal/(m^3)(degree); $\lambda_t = 1.0$ kcal/(m)(hr)(degree). We obtain $\xi \cong 1.5$ meters.

4. After construction of the road, along its route there are substantial changes of the temperature regime of the soils and the depth of their seasonal thawing. These changes are connected with the fact that in the winter the soils are greatly cooled, since snow is regularly removed from their surface.

and in the summer they are fairly well warmed as a result of the influence of the road covering (the albedo of asphalt is low, and the heat loss due to evaporation from the surface becomes in practice a very small amount). As a result the annual amplitude of temperature fluctuations on the surface of the roadbed (A_{road}) increases substantially, and the average annual temperature (t_{road}) is reduced slightly.

In accordance with the conditions of the problem, in the presence of a steady temperature regime we will have

$$A_{д} = 23 + 4 = 27° \left(A_{д} = A_{з} + \frac{1}{2} \Delta t_{макс};\ \Delta t_{макс} = t_{д.макс} - t_{в.макс} \right),$$

$$t_{д} = -8,2 + 4 = -4,2° \left(t_{д} = t_{в} + \frac{1}{2} \Delta t_{макс} \right).$$

That temperature regime determines the average established depth of seasonal thawing and the temperature of the ground at that depth. Upon attainment of a steady regime the character of the ground in the upper part of the layer of permafrozen rocks also changes. It is obvious that after the lapse of some time (3-5 years) the permafrozen soils which have gone into the seasonally thawed state will in their composition and properties approach soils of the layer of seasonal thawing under natural conditions. With the nomogram (Figure 15) we find that the depth of seasonal thawing of the ground corresponding to the steady regime is 1.85 meters (the initial data are as follows: A_{road} = 27^{o}; $t_{\xi} \approx -5.0^{o}$, with consideration of the temperature shift; Q_{g} = 28,160 kcal/m^3; C_{vol-t} = 627 kcal/(m^3)(degree); λ_t = 1.0 kcal/(m)(degree)(hr).

However, for calculation of the thermal subsidence the depth of ξ must be determined, not according to the average perennial values of A_{road} and t_{road}, but according to their values in the warmest year, the recurrence of which amounts to at least once in 10 years. Since in a separate warm year on the investigated section t_{air} increases by 2^{o} (Δt_{air}) and the amplitude increases by 4^{o} (ΔA_{air}) in relation to the average perennial values, then it is obvious that the depth of ξ that year will be deeper than 1.85 m.

5. We determine the depth ξ of the ground in a warm year. In that case we leave t_{ξ} equal to -5.0^{o} and we increase the amplitude of temperature fluctutions on the surface of the road by the sum of Δt_{air} and ΔA_{air}, as is done in calculations of the potential freezing or thawing (example 5), that is, A_{road} = 27 + 4 + 2 = 33^{o}.

With the nomogram (see Figure 15) we find that ξ = 2.0 m.

6. We calculate the size of the thermal subsidence during thawing of the icy layer of permafrozen rocks as a result of increase of the depth of seasonal thawing under the roadbed.

The thickness of the icy layer giving the thermal subsidence is

$$h = 2,0 - 1,5 = 0,5\ м.$$

We calculate the size of the subsidence (S) with the formula of A. M. Pchelintsev (1964)

$$S = \frac{\gamma_{уд} (\omega - \omega_{мин})}{1 + \gamma_{уд}} h, \qquad (6.9.1)$$

$$S = \frac{2,7 (0,57 - 0,33)}{1 + 2,7} \cdot 0,5 = 0,09 \text{ м}.$$

As a result of the calculation we find that during construction of a road on the section on which the given type of permafrost is widespread the thermal subsidence as a result of increase of the depth of seasonal thawing is 9 cm.

To verify the correctness of the determination of the maximal thickness of the layer of icy permafrozen ground giving a thermal subsidence as a result of increase of the depth of seasonal thawing, one should also calculate the depth of thawing forming in the first year of construction, when the temperature regime of the ground still differs insignificantly from that existing under natural conditions (t_ξ = 02.8°) and the profile of seasonally thawing rocks already has a two-layer structure. It is obvious that the temperature conditions on the surface of the roadbed in that year depend on weather conditions. For calculations it is advisable to take data characterizing a very warm year, that is, A_{road} = 33°. The two-layer structure can be taken into consideration by determining the values of C_{vol-t}, λ_t and Q_ϕ by the method of weighted-average values, assuming provisionally that the depth of thawing increases by approximately 30% as compared with natural conditions. The first layer represents soils which thaw seasonally annually. Their thermophysical properties were presented above. The second layer represents the upper part of the permafrozen rock mass. Their thermophysical properties are as follows:

$$C_{об.т} = 0,19 \cdot 1000 + 1,0 \cdot 570 = 760 \text{ ккал/м}^3\cdot\text{град}.$$

The amount of unfrozen water for that layer can be determined at a temperature equal to the average annual (-2.8°), at which w_{un} = 9%. Consequently:

$$Q_\phi = 80 \cdot 480 = 38\,400 \text{ ккал/м}^3.$$

Then the weighted average values of the characteristics will be:

$$C_{об.т} = \frac{627 \cdot 1,5 + 760 \cdot 10,5}{2,0} = 670 \text{ ккал/м}^3 \text{град},$$

$$Q_\phi = \frac{28\,160 \cdot 1,5 + 38\,400 \cdot 0,5}{2,0} = 30\,720 \text{ ккал/м}^3,$$

$$\lambda_т = \frac{1,0 \cdot 1,5 + 0,9 \cdot 0,5}{2,0} = 0,97 \text{ ккал/м}\cdot\text{град}\cdot\text{час}.$$

In accordance with the obtained data we determine the maximal depth of thawing under the roadbed in the first year after construction. With the nomogram (see Figure 15) we find that $\xi \cong$ 2.1 meters, that is, exceeds by 0.1 m the depth of thawing adopted in the calculation of the thermal subsidence. For a definitive estimate of the size of the subsidence it obviously is necessary to take a larger depth of thawing. In the given case it is preferable to assume that it is equal to 2.1 meters. But that will give an increase of the thermal subsidence totalling 1 cm, which is not of great importance. In other cases that difference can be greater, and then it should be taken into consideration.

In conclusion it must be noted that not under all conditions will the depth of thawing of ground observed in the first year be greater than the depth of thawing corresponding to the new temperature regime of the ground. Therefore in solving each concrete problem one should analyze the regularities in the formation of the depths of seasonal thawing of the ground, as was shown in example 30.

In the case of an unfavorable forecast, when the thermal subsidences reach large values, it is necessary to provide for measures to eliminate them and control the frost process. The latter can be achieved by construction according to the principle of preservation of permafrozen rock masses in the base of the construction or according to the principle of their preliminary thawing.

In the case where a large territory with widespread reopened-vein ice or permafrozen rock masses with schlieren cryogenic textures is to be opened up, when it becomes practically impossible to apply both those principles, it is necessary to plan the filling in of large-skeleton soils, the thickness of which must be especially calculated.

Calculation of the Height of an Embankment Constructed to Prevent Thermal Subsidences of Soils (Example 31)

Calculate the height to which an enbankment should be erected on a section of the route of a highway in order to exclude the thawing of strongly iced frozen soils which give a large thermal subsidence. As a result of a frost survey it has been established that the upper part of the profile of the frozen rock mass consists of strongly iced peatized loams, characterized by: γ_{sk} = 870 kg/m^3; γ_{spec} = 2.7 g/cm^3; w = 67%, w_{min} = 33%; λ_t = 0.78 kcal/(m)(degree)(hr).

In the layer of seasonal thawing lie silty loams with the characteristics: γ_{sk} = 1000 kg/cm^3; w = 36%, λ_t = 1.0 kcal/(m)(degree)(hr); C_{vol-t} = 540 kcal/(m^3)(degree); Q_ϕ = 24,000 kcal/m^3; w_{un} = 6%.

The temperature regime of the soils is characterized by the following data: A_o = 10^o; t_ξ = -1.7°.

During construction the temperature conditions varied along the path of the highway: t_ξ increased to -1.0^o and A_o increased to 25^o in warm years.

Solution. 1. We determine that under natural conditions the depth ξ amounts to 1.2 meter. It was calculated with the nomogram (see Figure 16) at the following initial data:

$$A_0 = 10^\circ,\ t_\xi = -1{,}7^\circ,\ C_{об.т} = 540\ ккал/м^3\,град,\ Q_\Phi = 24000\ ккал/м^3,$$
$$\lambda_т = 1{,}0\ ккал/м \cdot град \cdot час.$$

2. After construction, as a result of change of the temperature conditions, that is, at $A_o = 25^o$ and $t_\xi = -1.0^o$, the depth of the seasonal thawing in the same grounds increases to 2.2 m. Since under natural conditions from a depth of 1.2 m lie icy frozen loams, the newly formed depth in the first years will be smaller than 2.2 m. In subsequent years, when the temperature regime has become steady, the depth ξ obviously will attain that value if the loams which have thawed, which went over into the seasonally thawed state, will have properties characterized by values of C_{vol}, Q_ϕ and λ close to those cited above.

3. We determine the thermal subsidence S as a result of thawing of icy loams which before construction were in a permafrozen state.

Upon increase of the depth of thawing by 1.0 m (h = 2.2 - 1.2), in accordance with (6.9.1) the subsidence is equal to

$$S = \frac{2.7 \cdot (0.67 - 0.33)}{1 + 2.7} \cdot 1.0 \cong 0.25 \text{ м}.$$

When such a subsidence proves to be unallowable it is necessary to erect an embankment with a height of at least 1.0 m in order to protect the permafrozen rock mass against thawing. However, in planning the embankment one should calculate the depth of potential seasonal freezing in order to correctly determine the allowable height of the embankment at which the possibility of formation of non-convergent frost. Under the given conditions ($A_o = 25^o$; $t_\xi = -1.0^o$; $C_{vol-t} = 540$ kcal/(m^3)(degree); $Q_\phi = 24{,}000$ kcal/m^3; $\lambda_t^o = 1.0$ kcal/(m)(degree)(hr)) the depth of potential freezing is $\xi_p \approx 2.5$ m. Consequently, if the embankment is erected at the end of the summer, when seasonal thawing of the ground to a depth of 1.2 m has occurred, the height of the embankment must be at least 1.0 m but also not more than 1.3 m. It is obvious that the embankment height in case of need can be made larger if it is erected in winter or at the start of spring, but even in that case it cannot exceed 2.5 m.

In compiling a forecast of the change of the frost engineering geological conditions on sections where syngenetic permafrozen rock masses are widespread it also is necessary to take into consideration their distinctive features in connection with the latitudinal zonation and height zonation. This applies especially to the distribution of different types of syngenetic permafrozen rock masses containing reopened-vein ice. Within the limits of the first two frost-temperature zones reopened-vein ice, as a rule, either is absent or is developed locally and is of small thickness. In the three remaining northern zones they are widespread and their distribution increases from south to north, as a rule. Their thickness also increases in the same direction. In the fifth zone and in the northern part of the fourth their thickness attains 10-20 meters, in places 30-50 m. The width of the ice wedges in that case often reaches 3-5 m at dimensions of the ice lattice of 7-9 m, more rarely 12 m. At the same time the depth of bedding of reopened-vein ices from the surface decreases from south to north in accordance with the decrease of thickness of the layer of seasonal thawing. This has the result that the probability of thawing increases during the economic opening up of territory from south to north. In northern arctic regions even a slight damage of the plant cover can lead to the development of thermokarst (examples 14, 15 and 16).

Syngenetic rock masses with schlieren cryogenic textures are encountered locally within the limits of the first two frost-temperature zones. This is connected with the fact that freezing from below in the layer of summer thawing is extremely small in those zones. In more northerly zones freezing from below reaches considerable values (in zones IV and V up to 30-40% of the entire thickness of the layer of seasonal thawing). Because of this, permafrozen rock masses with thick, medium and thin schlieren laminated cryogenic textures in the upper levels are widespread in the north. The thickness of the latter can reach 10-20 m or more.

Buried and solifluctional ices and firn basins are widespread, as a rule, in frost-temperature zones III, IV and V, they are not encountered at all in zone I, and are encountered locally in zone II. They are most widespread in the northern parts of zone IV and in frost-temperature zone V. They also have maximal thickness there. Most often they are widespread on steep bare slopes composed of large-fragment material.

Thermal subsidences during the thawing of epigenetic frozen rock masses, just like syngenetic, are determined by the ice content and distinctive features of the cryogenic structure of the rocks. Epigenetic rock masses are characteristic of pre-Quaternary deposits and younger ones if their cryogenic age is determined by shorter time segments than the time of their formation. The cryogenic textures of epigenetic frozen rock masses are characterized by great variety and depend on the composition and genesis of the deposits, on their moisture content and the regime of the subsurface waters, and also on the conditions of freezing.

In compiling a forecast of the change of frost conditions, and in particular a forecast of thermal subsidences during thawing of epigenetic permafrozen rocks, it is necessary to use as a basis analysis of the types of frozen rock masses. In that case one should above all distinguish permafrozen rock masses of river valleys (and their slopes) and reservoirs. Within the limits of river valleys epigenetic frozen rock masses are characteristic of denudation terraces and slopes of valleys, including those deposited in solid rocks. In rock formations the ice inclusions are concentrated in the weathering joints and zones of tectonic faults. In that case, in the process of thawing the thermal subsidences usually are not observed, but during thawing the coefficient of filtration of rocks increases sharply.

Epigenetic rock masses in river valleys composed of loose Quaternary deposits have a different ice content in different latitudinal zones. In the first two frost-temperature zones the thicknesses of permafrozen rock masses usually form within the range of from several meters to 100-150 meters. Their ice content forms as a function of the composition of the rocks and the drainage conditions. In sandy-pebbled deposits in the presence of water-bearing horizons mainly massive, massive-porous, at times basal cryogenic textures are noted. Layers and lenses of ice are encountered rarely and are usually connected with injection ice. The thermal subsidences in that cases most often do not go beyond the range of 3-4 cm/m. When sandy loam and loam lenses and layers representing facial changes of alluvial deposits are encountered in sand and gravel deposits, the cryogenic textures in them can be laminated, lens-shaped and porphyraceous. During the thawing of those lenses the subsidence can attain considerable dimensions, of the order of 10-12 or more cm/m.

In sandy-pebbled alluvia, in the absence of subsurface waters the ice content is extremely small and no thermal subsidences are observed, as a rule, during their thawing.

In deposits of flood-plain, riverbed and oxbow lake deposits, consisting of sandy loam and loam, lens-shaped, laminated, lattice or cellular or grid cryogenic textures are encountered. To determine the thermal subsidence in that case it is necessary to have data on the moisture content and ice content of the frozen rocks by facies. In that cast it must be borne in mind that in the first two frost-temperature zones the maximal ice content of epigenetically frozen deposits is concentrated in the upper 10-20 m. Deeper, the ice content diminishes, as a rule. Therefore in compiling a forecast of thermal subsidences it is necessary to take into consideration the dimensions of the basins (aureoles) of thawing under the structure.

On water divides and their slopes a difference of epigenetic rock masses from those in valleys is noted especially with respect to the composition and genesis of deposits. In solid rocks the ice content of the deposits is connected with weathering joints and tectonic fissures. The thawing of those rock masses often leads to slides and avalanches on steep slopes. The thawing of such frozen rock masses at reservoirs and in valley floors often is connected with the formation of local fissure waters and the formation of perforated and unperforated taliks.

During the freezing of deluvial-eluvial deposits composed of wood and gravel materials, massive-porous, basal and, when there is a large accumulation of ice, ataxitic cryogenic textures are usually encountered. In sandy loam and loam material containing crushed stone, laminated, lens-shaped and crustal textures are encountered. In that case the character of the textures is determined by the hydrogeological conditions and, in particular, by the character of the subsurface waters and the moisture content of the rocks.

In conclusion it must be noted that in forecasting thermal subsidences it is necessary to take into consideration the possibility of the formation of accompanying cryogenic processes and effects (landslides, overflows, thermokarst, swamping, etc) which have a great influence on the stability of structures.

10. Forecasting Changes of the Geological Engineering Properties of Frozen, Freezing and Thawing Rocks

Frozen soils consist of a four-component system (mineral particles, ice, unfrozen water and air), the properties of which are determined by the percentage ratio of the content of each of the components and the structural features of the rock. The presence in frozen soils of a rock-forming mineral -- ice -- is a characteristic of them. The mineralogical, granulometric and saline compositions of frozen rocks are determined by their geological genetic features. The influence of cryogenic processes on the granulometric composition of rocks is expressed mainly in the formation of silty fractions. The ice content in soils is determined by the lithological facies to which they belong, the moisture content and hydrogeological conditions before the start of freezing, and the conditions of freezing and of moisture migration toward the front of freezing. As a result of that various cryogenic textures of frozen soils form, characterized by the presence of different ice inclusions (schlieren, patches, crusts, etc). In mineral layers of the soil a monolithic cryogenic texture with ice cement is observed. In those layers is the principal part of the unfrozen water of the frozen ground. Its quantity depends on the dispersity of the soil, its mineralogical and saline composition, its content of organic matter and its temperature. The character of the change of quantity of unfrozen water as a function of temperature for concrete soils is shown on Figures 12, 13, 14, 27, 31, etc.

The regularities of the change of the geological engineering properties of rocks as a function of composition, structure and temperature can be illustrated by the data on Figures 73-94. It is evident from the presented graphs that soils of sandy and gravel-pebble composition differ sharply in their properties from dispersed varieties. The unfrozen water contained in them exerts an especially great influence on the properties of frozen soils. The dependence of their transparency, rheological and thermophysical properties on the content of ice and unfrozen water is shown on Figures 76, 78, 79, 82, 83, etc.

Thus in the economic opening up of territory which leads to change of the temperature regime of frozen rocks and to change of their thermal state on the whole (frozen or thawed) there is a substantial change of the properties of the soils, in connection with which a need arises to forecast those changes. The forecasting of change of the geological engineering properties of rocks should be subdivided into two parts: 1) forecast of the change of properties of frozen rocks in connection with their thawing and of thawed rocks during their freezing; 2) forecast of the change of properties of frozen rocks in connection with change of their temperature regime in the range of negative values.

During the thawing of frozen soils, besides a sharp change of the composition, structure and properties of the soils one should take into consideration subsidence during thawing, which is a special characteristic of thawing grounds. During the freezing of thawed grounds it is necessary above all to take into account the change of their composition (the appearance of ice), the formation of cryogenic textures and heaving, as a result of which all the properties of the soils change. Consequently, in compiling a forecast of the change of frost conditions, sections are distinguished in which, as a result of the productive activity of man, the following are expected: a) thawing of permafrozen rock masses and the formation of taliks; b) formation of permafrozen rock masses again within the limits of a thawed mass of rocks and c) change of the temperature regime of rocks while preserving them in the frozen state.

In the case of thawing of permafrozen rock masses the principal forecasting characteristics are the dynamics of thawing in time, the character of the thermal subsidence and the expected consistency of the thawed soils and their properties. For the forecast it is necessary to have a complete characterization of the physical properties of the frozen soils, namely: the specific gravity of the skeleton of the rock, the specific gravity in the frozen and thawed states, the absolute moisture capacity, the natural moisture content, the limits of plasticity, the filtration coefficient and the thermophysical properties. The dynamics of thawing of frozen rock masses can be determined in accordance with the above-indicated procedure, analysed in examples 1, 3, 4, 5, 13, 21 and 22. A procedure for determination of subsidences during thawing is presented in example 30, from which it is evident that the thermal subsidence is calculated for the entire expected layer of thawing. If thawing of frozen rock masses is expected on large areas, the thermal subsidence is calculated for the entire thickness of the permafrozen rocks. In that case, when the process of thawing is limited to a basin, the subsidence is calculated only within the limits of its contour.

In relation to the expected properties of thawed rocks it should be said that the initial moisture content and ice content of soils are decisive aspects. When the total moisture content of the thawing frozen rocks does not exceed the critical moisture content (w_{cr}) the properties of those soils will correspond to the thawed state at the given moisture content. For thawing soils with a total moisture content greater than the critical the properties will be connected with the dynamics of thawing and the rate of separation of the excessive moisture accumulating on the surface. In the case of thawing of finely dispersed rocks, often a portion of the excessive moisture is entrapped in the ground and masses of moisture-saturated, often silty soils form which have a zero carrying capacity and are completely unsuitable as the base of any structure. Such soils require serious improvement measures, and later filling with large clump soils. A calculation of the amount of such filling is presented in example 31.

In the case of new formations of permafrozen rocks it is necessary to forecast their distribution over an area, the conditions of their bedding (in the sense of determination of the depth of permafrost) and the temperature regime. Methods of making that forecast have already been examined in examples 4, 5, 25 and 26. In forecasting the properties of freezing rocks the main attention ought to be given to the formation of cryogenic textures for each geological genetic type and lithological variety of rocks. A forecast of the formation of cryogenic textures can be obtained with the procedure presented in section 3 of Chapter 6. The data of section 7 can be used to determine the thermophysical properties of rocks. The procedure of their calculation is presented in example 28.

The change of the geological engineering properties of frozen rocks connected with change of the temperature of frozen rocks is very widespread. Noted in Chapter 4 were the role and importance of separate factors of the natural environment in the formation of the temperature regime of rocks. The procedure for taking into consideration the influence of the principal components of the natural environment on the formation of the temperature regime of rocks and the depths of seasonal freezing and seasonal thawing is presented in examples 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20. The procedure for calculating the summary influence of all the principal factors is shown in example 22.

In accordance with the results of the forecast and as a function of the technical conditions of the opening up of the territory, methods of controlling the frost process for productive purposes can be worked out. By regulating and controlling the temperature regime of rocks it is possible to control the process of freezing and thawing of soils, and also directively change the properties of soils in the frozen state. Thus, for example, in the process of formation of permafrozen rocks it is possible to observe the formation of cryogenic textures with a large quantity of ice schlieren, which in turn is accompanied by intensive heaving, as was shown in section 3 of Chapter .6 and section 3 of Chapter 9. Heaving of the ground can be prevented or substantially reduced by draining the area of their propagation and regulating the conditions and rate of freezing by means of snow, blackening the surface, shading it and other measures indicated in section 5 of Chapter 6.

In working up a forecast it is necessary to prepare a list of measures to control the frost process in order to obtain optimal conditions for the operation of structures and optimize the economics of construction. Depending on the type of construction and distinctive features of the geological engineering and frost conditions the specific content of the forecast of change of frost conditions and the properties of frozen rocks can vary, but the general basis will be preserved.

Everything said above indicates that a forecast of change of the geological engineering properties of rocks and the development of principles and methods of their control can be accomplished only on the basis of a frost survey, in the process of which the general regularities in the formation and development of the geological engineering conditions in a region of permafrost are studied.

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Chapter 7. Regularities in the Formation of Taliks and Forecasting Their Formation

1. Talik Classification by Reasons for Their Formation

In the area of permafrozen rocks taliks (tabetisols) are of great importance in the solution of theoretical and practical questions of geocryology, especially in the geological engineering evaluation of territory and the study of subsurface waters.

It is very important to explain the reasons for the formation of taliks and the conditions of their existence, which makes it possible to give not only a forecast of their development in the economic opening up of territory but also a forecast in the direction of searches for talik zones.

The formation of taliks is determined by the character of the heat exchange on the surface of the ground and in the rock mass as a function of the specific geological structure and geographical situation, and also other elements of the natural environment.

The reasons for the formation and existence of taliks ought to express both qualitative and quantitative interconnections of the temperature regime of the rocks and the radiation heat balance of the surface and thermal cycles in the soils. Such interconnnections are the basis of the genetic classification of taliks, on which their study must be based.

The classification developed by N. N. Romanovskiy (1970, 1972) is such a genetic classification, constructed on qualitative interconnections of definite types of taliks and the conditions in which they form and exist. A new genetic classification of taliks (Table 45) has been compiled for the main taxonomic subdivisions of the classification of N. N. Romanovskiy by establishing bilateral quantitative dependences in the heat exchange between taliks and the principal geological, geographic and thermophysical conditions characteristic of frost-temperature zones.

The determination of the main reasons for the formation and existence of taliks and the character of the connection of those causes with heat exchange are a distinctive feature of the given classification.

Calculating methods based on use of approximate formulas for solving the Stefan problem (Chapters 4, 5 and 6) can be used on the basis of the correlations, indicated in the classification, of types of taliks with the reasons for their formation in the study and characterization of taliks for the purpose of forecasting.

2. Distinctive Features and the Character of the Influence of Natural Factors on the Formation of Taliks

1. The Radiation Heat Balance of the Surface and Its Role in Talik Formation

Type of taliks -- radiation-heat, subtype -- radiation. Taliks belonging to the radiation subtype can form on account of change of one or a group of components of the radiation balance of the surface. Such taliks can above all be connected with increase of the incident solar radiation on slopes with a southern exposure and a definite steepness, and also during reduction of the albedo of the surface.

The maximal amount of solar radiation impinging on the surface is observed on slopes with a southern exposure and a steepness corresponding to perpendicular impingement of rays on the surface. For each specific region, depend-on the latitude of the place, that steepness and the relative amount of radiation can be determined from Tables 12 and 13. In field investigations on different types of landscapes the actual amount of direct and scattered radiation can be measured. For the winter period the amount of impinging solar radiation on slopes of different steepness and exposure can be assumed to be identical. The different amount of absorbed radiation in the summer leads to a difference in the temperature regime of the rocks on slopes in the annual cycle. In that case, when the absorbed radiation increases so much that the average annual temperature of the surface of the soil reaches 0° and goes over into the region of positive values, taliks of a radiation type form. The possibility of the ex.stence of those taliks in different regions can be determined by means of the following calculating scheme.

Calculation of the Possibility of Existence of a Radiation Talik on Slopes With a Southern Exposure (Example 32)

It has been established by field actinometric observations that on slopes with a southern exposure in the summer period about 55% more solar radiation arrives than on a horizontal surface. The effective radiation on slopes at that time increases by not more than 5%. The radiation balance of a horizontal surface and the air temperature above it in the course of a year are presented in Table 46.

On slopes and horizontal sections, deposits of detritus and wood lie on the surface. The plant cover consists of lichens growing in small patches. On the slopes the snow reaches a height of 0.2 meter at a density of 0.19 g/cc, and on horizontal sections the snow height increases to 0.3 meter at a density of 0.22 g/cc. It is necessary to determine the temperature regime on the surface of the rocks and clarify the possibility of existence of a talik on slopes with a southern exposure.

Table 45. Classification of taliks by main causes of their formation and existence

<table>
<tr><th rowspan="3">A
Тип</th><th rowspan="3">B
Подтип</th><th colspan="5">C Основные причины образования и существования таликов</th></tr>
<tr><th colspan="5">a Температурные зоны</th></tr>
<tr><th>I</th><th>II</th><th>III</th><th>IV</th><th>V</th></tr>
<tr><td rowspan="8">1. Радиационно-тепловой</td><td rowspan="2">a Радиационный</td><td>1. Южная экспозиция (класс талика — безводный, подкласс — термальный, вид — сквозной)</td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>2. Уменьшение альбедо в результате лесных пожаров (класс — безводный, подкласс — термальный, вид — несквозной и сквозной)</td></tr>
<tr><td rowspan="6">b Тепловой</td><td>1. Малое испарение с поверхности в связи с хорошими условиями дренажа пород на оголенных от растительности участках (класс — безводный, подкласс — термальный, вид — сквозной)</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">2. Положительная температурная сдвижка
до 1° до 2—3°
(класс — безводный, подкласс — термальный, вид — сквозной)</td><td></td><td></td><td></td></tr>
<tr><td colspan="3">3. Снежный покров по зонам высотой
0,4—0,7 м 0,7—1,0 м 1,0—2,0 м
(класс — безводный, подкласс — термальный, вид — сквозной и несквозной)</td><td></td><td></td></tr>
<tr><td colspan="4">4. Мощные снежники и ледники
(класс — безводный, подкласс — термальный, вид — сквозной и несквозной)</td><td></td></tr>
<tr><td colspan="3">5. Наличие плотного кустарничка и травяного покрова, обусловливающего рыхлость снега, неплотное его прилегание к почве с образованием пустот (класс — безводный, подкласс — термальный, вид — сквозной и несквозной)</td><td></td><td></td></tr>
<tr><td colspan="3">6. Заболоченность участков при наличии снежного покрова высотой 0,7—1,0 м (класс — безводный, подкласс — термальный, вид — сквозной)</td><td></td><td></td></tr>
</table>

		1
	Радиационно-инфильтрационный	Отепляющее влияние грунтовых вод и инфильтрующихся теплых атмосферных осадков на участках, сложенных с поверхности крупнодисперсными породами (класс — инфильтрационный, подкласс — термальный, вид — сквозной и несквозной)
II. Подводно-тепловой (гидрогенный)	a Шельфовый	1. Отепляющее влияние теплых морских течений при $t_{ср}$ воды в придонных слоях выше 0° (класс — безводный, грунтово-фильтрационный и инфильтрационный, подкласс — термальный, вид — сквозной)
		2. Соленость воды при $t_{ср}$ воды в придонных слоях ниже 0° (класс — инфильтрационный и напорно-фильтрационный, подкласс — криогидрогалинный, вид — сквозной)
	b Подозерный	1. Отепляющее влияние озер глубиной более той, где среднегодовая температура равна нулю и в которых донные отложения представлены нефильтрующими или слабофильтрующими породами (класс — безводный, подкласс — термальный, вид — сквозной, если диаметр озер превышает мощность многолетнемерзлой толщи; несквозной, если диаметр озер меньше мощности мерзлой толщи)
		2. Отепляющее влияние озер, существующих на хорошо фильтрующих породах при грунтово-фильтрационном и инфильтрационном питании (класс — грунтово-фильтрационный и инфильтрационный и застойный, подкласс — термальный, вид — в I и II температурных зонах сквозной, в остальных — сквозной и несквозной в зависимости от глубины, размеров озер и длительности их существования)
	c Подрусловой	1. Отепляющее влияние водотока на нефильтрующих и слабофильтрующих породах (класс — безводный, подкласс — термальный, вид — сквозной и несквозной в зависимости от глубины и режима водотока)
		2. Отепляющее влияние водотока на фильтрующих породах (класс — грунтово-фильтрационный и инфильтрационный, подкласс — термальный, вид — в I и II температурных зонах сквозной, в остальных—сквозной и несквозной в зависимости от режима водотока)
III. Водно-тепловой (гидрогеогенный)		1. Отепляющее влияние грунтовых вод (класс — грунтово-фильтрационный, инфильтрационный и напорно-фильтрационный; подкласс — термальный, вид — сквозной в I и II температурных зонах, сквозной и несквозной в III, IV и V в зависимости от режима водоносного горизонта)
		2. Отепляющее влияние напорных подмерзлотных вод (класс — напорно-фильтрационный, подкласс — термальный, вид — сквозной)
		3. Отепляющее влияние пластовых, пластово-трещинных и трещинных подмерзлотных вод (класс — инфильтрационный и напорно-фильтрационный, подкласс — термальный, вид — сквозной)

Table 45 (Continued) Key

A - Type B - Subtype C - Principal causes of the formation and existence of taliks a - Temperature zones

I - Radiation-thermal a - Radiation 1 - Southern exposure (class of talik -- anhydrous, subclass -- thermal, type -- permeating); 2 - Decrease of albedo as a result of summer fires (class -- anhydrous, subclass - thermal, type -- nonpermeating and permeating) b - Thermal 1 - Little evaporation from the surface in connection with good conditions of drainage of rocks on sections bare of vegetation (class - anhydrous, subclass -- thermal, type -- permeating) 2 - Positive temperature shift up to 1° up to $2\text{-}3^{\circ}$ (class, etc, as for "1") 3 - Snow cover by zones of height 0.4-0.7 m 0.7-1.0 m 1.0-2.0 m (class -- anhydrous, subclass -- thermal, type -- permeating and nonpermeating) 4 - Thick firn basins and glaciers (class, etc, as for "3") 5 - The presence of dense underbrush and grassy cover, causing looseness of snow, not packed close to the soil, with the formation of cavities (class, etc, as for "3") 6 - Swampiness of sections in the presence of a snow cover 0.7-1.0 m high (class, etc, as for "1") c - Radiation-infiltration Warming influence of ground waters and infiltrating warm atmospheric precipitations on sections composed from the surface of coarsely dispersed rocks (class -- infiltration, subclass -- thermal, type -- permeating and nonpermeating).

II - Underwater-thermal (hydrogenous) a - Shelf 1 - Warming influence of warm sea currents at t_m of water in bottom layers above 0° (class -- anhydrous, ground-filtration and infiltration, subclass -- thermal, type -- permeating) 2 - Water salinity at t_m of water in bottom layers below 0° (class -- infiltration and pressurized-filtration, subclass - cryohydrohalinic, type -- permeating) b - Sub-lake 1. The warming influence of lakes with a depth greater than that where the average annual temperature is equal to zero and in which the bottom deposits are composed of non-filtering or slightly filtering rocks (class -- anhydrous, subclass -- thermal, type -- permeating, if the diameter of the lakes exceeds the thickness of the permafrozen rock masses; nonpermeating if the diameter of the lakes is smaller than the thickness of the frozen rock mass) 2. The warming influence of lakes existing on well filtering rocks when there is ground-filtration and infiltration feeding (class -- ground-filtration and infiltration and stagnant, subclass -- thermal, type -- in temperature zones I and II, permeating, in the rest, permeating and nonpermeating, depending on depth, the dimensions of the lakes and the length of their existence) c - Sub-riverbed 1. The warming influence of current on nonfiltering and weakly filtering rocks (class -- anhydrous, subclass -- thermal, type -- permeating and nonpermeating, depending on depth and the regime of the current). 2. The warming influence of current on filtering rocks (class -- ground-filtration and infiltration, subclass -- thermal, type -- in temperature zones I and II, permeating, in the rest, permeating and nonpermeating, dpeending on the regime of the current.

III - Hydrogeogenic 1. The warming influence of ground waters (class -- ground-filtration, infiltration and pressurized-filtration, subclass -- thermal, type -- permeating in temperature zones I and II, permeating and nonpermeating in zones III, IV and V, depending on the regime of the water-bearing horizon). 2. The warming influence of pressurized subfrostal waters (class -- pressurized-filtration, subclass -- thermal, type --

permeating). 3. The warming influence of stratal, stratal-fissure and fissure subfrostal waters (class -- infiltration and pressurized-infiltration, subclass -- thermal, type -- permeating)

Table 46. Data of radiation balance and air temperatures on a horizontal surface

A Составляющие радиационного баланса	I	II	III	IV	V	VI	VII
1 $Q_п$, ккал/см² мес	0,3	0,60	1,6	2,9	8,0	13,0	10,8
2 I, ккал/см² мес	1,5	1,5	1,8	1,7	2,1	4,2	3,0
3 R, ккал/см² мес	—1,2	—0,9	—0,2	1,2	5,9	8,8	7,8
4 $t_в$, °C	—28,1	—25,6	—17	—6,2	3,3	13,2	16,7

Продолжение табл. 46

A Составляющие радиационного баланса	VIII	IX	X	XI	XII	Год
1 $Q_п$, ккал/см² мес	8,2	5,1	1,8	0,3	0,2	—
2 I, ккал/см² мес	3,1	2,4	2,3	1,4	1,4	—
3 R, ккал/см² мес	5,1	2,7	—0,5	—1,1	—1,2	—
4 $t_в$, °C	13,3	4,5	—6,5	—19,2	—25,9	—6,4

Key: A - Components of radiation balance 1 - $Q_п$, kcal/(cm^2)(month)
2 - I, kcal/(cm^2)(month 3 - R, kcal/(cm^2)(month) 4 - t_{air}, ^oC

Table 47 Data of radiation balance on slopes with southern exposure

A Составляющие радиационного баланса	I	II	III	IV	V	VI
1 $Q_п$, ккал/см²мес	0,30	0,60	1,60	3,48	9,6	15,6
2 I, ккал/см²мес	1,50	1,50	1,80	1,78	2,2	4,4
3 R, ккал/см²мес	—1,20	—0,9	—0,2	1,7	7,4	11,2

Продолжение табл. 47

A Составляющие радиационного баланса	VII	VIII	IX	X	XI	XII
1 $Q_п$, ккал/см²мес	12,9	9,48	6,12	1,8	0,30	0,20
2 I, ккал/см²мес	3,15	3,15	2,52	2,40	1,40	1,40
3 R, ккал/см²мес	9,71	6,69	3,60	—0,5	—1,1	—1,2

Key: As for Table 46

Table 48 Data of the radiation balance and calculation of the average annual temperature of rocks and amplitude on the surface of a horizontal area and a slope with a southern exposure

	A Составляющие радиац. баланса	I	II	III	IV	V	VI
1	R, ккал/м²час	−16,44	−12,33	−2,74	16,44	80,83	120,56
2	$\Delta t_R = \frac{R}{\alpha}$, °C	−0,8	−0,6	−0,1	0,8	4,0	6,0
3	$t_в$, °C	−28,1	−25,6	−17,0	−6,2	3,3	13,2
4	$t_в(R)$, °C	−28,9	−26,2	−17,1	−5,4	7,3	19,2
5	$Q_п$, ккал/см²мес	0,3	0,6	1,6	4,5	12,4	20,2
6	I, ккал/см²мес	1,5	1,5	1,8	1,7	2,1	4,2
7	R, ккал/см²мес	−1,2	−0,9	−0,2	2,8	10,4	16,0
8	R, ккал/м²час	−16,4	−12,3	−2,74	38,4	142,5	219,2
9	Δt_R, °C	−0,8	−0,6	−0,1	1,9	7,1	11,0
10	$t_{в(R)}$, °C	−28,9	−26,2	−17,1	−4,3	10,4	24,2

Продолжение табл. 48

	A Составляющие радиац. баланса	VII	VIII	IX	X	XI	XII
1	R, ккал/м²час	106,86	69,87	36,99	−6,85	−15,07	−16,44
2	$\Delta t_R = \frac{R}{\alpha}$, °C	5,3	3,5	1,8	−0,3	−0,8	−0,8
3	$t_в$, °C	16,7	13,3	4,5	−6,5	−19,2	−25,9
4	$t_{в(R)}$, °C	22,0	16,8	6,3	−6,8	−20,0	−26,7
5	$Q_п$, ккал/см²мес	19,7	12,7	7,9	1,8	0,3	0,2
6	I, ккал/см²мес	3,0	3,1	2,4	2,3	1,4	1,4
7	R, ккал/см²мес	16,7	9,6	−5,5	−0,5	−1,1	−1,2
8	R, ккал/м²час	228,8	131,5	75,4	−6,8	−15,1	−16,4
9	Δt_R, °C	11,4	6,6	3,8	−0,3	−0,8	−0,8
10	$t_{в(R)}$, °C	28,1	19,9	8,3	−6,8	−20,0	−26,7

Key: A - Components of radiation balance 1 - R, kcal/(m^2)(hr) 2 - t_R = = R/α, °C 3 - t_{air}, °C 4 - $t_{air(R)}$, °C 5 - Q_t, kcal/(cm^2)(month) 6 - I, kcal/(cm)(month) 7 - R, kcal/(cm^2)(month) 8 - R, kcal/(m^2)(hr) 9 - Δt_R, °C 10 - $t_{air(R)}$, °C

Solution. 1. We determine the radiation balance on rocks with a southern exposure with consideration of the regularity established for the region that on the slopes from April to September the monthly totals of absorbed radiation are larger than the corresponding sums on a horizontal surface by 55%, and the effective radiation by 5% (Table 47).

2. We determine the radiation correction for the temperature of the surface of the deposits on horizontal sections and on slopes with a southern exposure, assuming that the coefficient of heat transfer (α) from the surface in the

the course of a year varies little and is 20 kcal/(m^2)(hr)(degree). The calculated data are presented in Table 48. Thus with consideration of the radiation correction on the horizontal sections $t_{air} = -5^{\circ}$, $A_{air} = 25.5^{\circ}$ and on slopes with a southern exposure $t_{air} = -3.3^{\circ}$ and $A_{air} = 28.5^{\circ}$.

3. We find the value of the warming influence of snow on slopes and horizontal sections with formula (5.3.10). In accordance with the change of the snow height and temperature regime of the surface (with consideration of the radiation regime) we obtain:

1) on horizontal sections $\Delta t_{сн} = 25{,}5 \cdot 0{,}164 = 4{,}2^{\circ}$,

2) on southern slopes $\Delta t_{сн} = 28{,}5 \cdot 0{,}123 = 3{,}5^{\circ}$.

4. We determine the temperature regime on the surface of the soil on the slopes and the horizontal surface with consideration of the radiation correction and the warming influence of snow:

1) on a horizontal surface $t_0 = -5 + 4{,}2 = -0{,}8^{\circ}$, $A_0 = 25{,}5 - 4{,}2 = 21{,}3^{\circ}$,

2) on slopes with a southern exposure $t_0 = -3{,}3 + 3{,}5 = +0{,}2^{\circ}$, $A_0 = 28{,}5 - 3{,}5 = 25^{\circ}$.

Thus on slopes with a southern exposure there are conditions favorable for the existence of taliks on account of increase of absorbed radiation as compared with horizontal sections and slopes with other exposures.

The formation of permeating and nonpermeating taliks of the radiation type in frost-temperature zones I and II often involves an increase of the albedo of the surface, as a reduction of it by 5-10% leads to elevation of the average annual temperature of the surface of the soil by 1-2°. The conditions of the formation of taliks during change of the albedo of the surface can be determined by means of a calculation similar to that presented in example 32.

The formation of the given type of talik can also be connected with the character of the effective radiation. Very often nonpermeating taliks and non-converging frost form on account of that factor. As is known, the effective radiation is determined by the temperature of the surface and the bottom layer of air, the atmospheric humidity and the cloudiness. Increase of the moisture content of the air and cloudiness in the autumn and winter, usually accompanied by elevation of the air temperature, can lead in separate years to a sharp reduction of the effective radiation of the surface, which involves elevation of the average annual temperature of the soil and reduction of the depth of freezing of the rocks. The process occurs especially frequently in regions where cyclones are observed in winter.

The change of the effective radiation in different types of landscapes can be connected with different radiative capacity of the surface. Therefore that characteristic must be determined directly in field conditions.

Calculation of the Influence of the Effective Radiation on the Formation of a Radiation Talik (Example 33)

Determine how the temperature regime of the surface of the soil varies in years when in the region frequent cyclones are observed in the autumn-winter period, which lead to elevation of the air temperature and reduction of the heat loss due to effective radiation. The characteristics of the average perennial climatic conditions are presented in Table 49. Thus, at t_{air} = $= -6.5^{o}$ with consideration of Δt_R it is found that $t_{air(R)} = -5.0^{o}$ and $A_{air(R)} = 25.4^{o}$. In addition, in Table 50 are presented data obtained in years with frequent cyclones in the autumn-winter time. The height of the snow cover on the investigated section in winters with a cyclonic type of weather varies very insignificantly in comparison with the average perennial data as a result of wind transport. If the average annual height of snow is 30 cm in winters with cyclones, it increases by 5-7 cm (at a density of the snow of $\sim$ 0.25 g/cc).

Table 49 Average annual data of the radiation balance and air temperatures

	A Составл. радиац. баланса	I	II	III	IV	V	VI
1	R, ккал/см²час	—1,2	—0,9	—0,2	1,2	5,9	8,8
2	$t_в$, °C	—28,1	—25,6	—17,0	—6,2	3,3	13,2
3	$t_{в(R)}$, °C	—28,9	—26,2	—17,1	—5,4	7,3	19,2

Продолжение табл. 49

	A Составл. радиац. баланса	VII	VIII	IX	X	XI	XII
1	R, ккал/см²час	7,8	5,1	2,7	—0,5	—1,1	—1,2
2	$t_в$, °C	16,7	13,3	4,5	—6,5	—19,2	—25,9
3	$t_{в(R)}$, °C	22,0	16,8	6,3	—6,8	—20,0	—26,7

Key: A - Components of the radiation balance 1 - R, kcal/(cm^2)(hr)
2 - t_{air}, ^{o}C 3 - $t_{air(r)}$, ^{o}C

Solution. 1. We calculate the temperature regime of the surface (of the soil in summer and the snow in winter) in years with a cyclonic type of weather and increased cloudiness (Table 51), according to the data of which at $t_{air} = -4.9^{o}$ with consideration of Δt_R it was found that $t_{air(R)} = -3.3^{o}$ and $A_{air(R)} = 24.4^{o}$.

Table 50 Data of the radiation balance and air temperatures in years with frequent cyclones

A Составл. радиац. баланса	I	II	III	IV	V	VI
1 $Q_п$, ккал/см²мес	0,3	0,6	1,6	2,9	8,0	13,0
2 I, ккал/см²мес	1,2	1,3	1,5	1,7	2,1	4,2
3 R, ккал/см²мес	— 0,9	— 0,7	0,1	1,2	5,9	8,8
4 $t_в$, °C	—26,2	—21,4	—15,0	—4,2	3,3	13,2

Продолжение табл. 50

A Составл. радиац. баланса	VII	VIII	IX	X	XI	XII
1 $Q_п$, ккал/см²мес	10,8	8,2	5,1	1,8	0,3	0,2
2 I, ккал/см²мес	3,0	3,1	2,2	1,8	1,2	1,1
3 R, ккал/см²мес	7,8	5,1	2,9	0	— 0,9	—0,9
4 $t_в$, °C	16,7	13,3	4,5	—2,5	—17,5	—22,9

Key: A - Components of the radiation balance 1 - Q_t, kcal/(cm^2)(month) 2 - I, kcal/(cm^2)(month) 3 - R, kcal/(cm^2)(month) 4 - t_{air}, °C

Table 51

A Климатические характеристики	I	II	III	IV	V	VI
1 R, ккал/см²мес	— 0,9	— 0,7	0,1	1,2	5,9	8,8
2 R, ккал/м²час	—12,4	— 9,6	— 1,4	16,4	80,8	120,6
$\Delta t_R = \frac{R}{\alpha}$, °C	— 0,6	— 0,5	— 0,1	0,8	4,0	6,0
3 $t_в$, °C	—26,2	—21,4	—14,0	—4,2	3,3	13,2
4 $t_{в(R)}$, °C	—26,8	—21,9	—15,1	—3,4	7,3	19,2

Продолжение табл. 51

A Климатические характеристики	VII	VIII	IX	X	XI	XII
1 R, ккал/см²мес	7,8	5,1	2,9	0	— 0,9	— 0,9
2 R, ккал/м²час	106,9	69,9	39,7	0	—12,3	—12,3
$\Delta t_R = \frac{R}{\alpha}$, °C	5,3	3,5	2,0	0	— 0,6	— 0,6
3 $t_в$, °C	16,7	13,3	4,5	—2,5	—17,5	—22,9
4 $t_{в(R)}$, °C	22,0	16,8	6,5	—2,5	—18,1	—23,5

Key: A - Climatic characteristics 1 - R, kcal/(cm^2)(month) 2 - R, kcal/(m^2)(hr) 3 - t_{air}, °C 4 - $t_{air(R)}$, °C

2. The height of the snow on the section is 0.4 m. With formula (5.3.10) we find Δt_{sn} and t_o for the conditions of the cyclonic type of weather:

$$\Delta t_{св} = 24,4 \cdot 0,175 = 4,3°,$$
$$t_0 = -3,3 + 4,3 = 1,0°.$$

3. In years with a number of cyclones in the winter time close to the average annual value the temperature conditions on the surface of the soil with consideration of the radiation correction and the warming influence of snow in accordance with the data of Table 49 obtained are as follows:

$$\Delta t_{сн} = 25,4 \cdot 0,153 = 3,9°,$$
$$t_0 = -5,0 + 3,9 = -1,1°.$$

Thus in years with frequent winter cyclones and increased cloudiness in the autumn-winter period the temperature of the surface of the soil can increase by 2.1^{o} (from -1.1 to $+1.0^{o}$) on account of reduction of the effective radiation. Under the conditions of the investigated region that change will lead to separation of the seasonally frozen layer from the permafrozen rock mass.

/The radiation-thermal type of taliks, the thermal subtype/. Regularities in the formation of taliks relating to the thermal subtype should be regarded as a result of the complex interaction of a number of factors, each of which in itself determines the conditions of formation of the temperature regime of rocks, their seasonal freezing and thawing and the annual thermal cycles. The formation of taliks of the given subtype is connected with processes and phenomena which lead to change of the thermal balance of the surface. In this respect one should above all point out the change of the amount of evaporation from the surface of the ground and the formation of a positive temperature shift as a function of different geological and geographical conditions.

2. The Role of Evaporation and the Positive Temperature Shift in the Formation of Taliks of the Thermal Subtype

A substantial influence is exerted on the amount of evaporation by the plant cover, the conditions of runoff and drainage of the surface, the composition and moisture content of the rocks, and also the climatic conditions. Change of the amount of evaporation can lead to change of the average annual temperature of the rocks within the range of several degrees and, consequently, evaporation is a very strongly influencing factor in the formation of the temperatures of rocks. Of great importance in that case are the composition of the rocks and their moisture content, which affect the formation of the temperature regime through the annual thermal cycles which pass through in them, through the thermophysical properties of the rocks in the thawed and frozen states, and also through the phase transitions of water during freezing and thawing*. The quantitative aspect of that influence is determined through

*The influence of infiltration of atmospheric precipitations and subsurface waters on the temperature regime of rocks is omitted here and is examined in point 6 of the present section and section 8 of Chapter 5.

the temperature shift. The formation of taliks is connected with a positive shift of the average annual temperatures. This is noted for soils which in the frozen state have a thermal conductivity smaller than in the thawed. The amount of that shift usually is kept within the range of up to 1, more rarely 2°. Therefore the formation of taliks, connected with a positive temperature shift, is concentrated in frost-temperature zone I and more rarely zone II. In view of the fact that the amount of the temperature shift depends substantially on the composition and moisture content of the soils, the average annual temperature on the surface of the soil and the continental character of the climate, taliks whose formation is caused by a shift have a selective and limited distribution. They are confined mainly to sections of the propagation of soils with a lower moisture content mainly under the conditions of a sharply continental climate.

3. The Role of the Snow Cover in the Formation of Taliks of the Thermal Subtype

A very powerful factor leading to the formation of thermal taliks is the snow cover. The snow cover is one of the most strongly influencing factors increasing the average annual temperature of rocks. The amount of influence of snow depends on its height and density, the time of establishment and disappearance, the climatic conditions, the character of the plant cover and also the composition and moisture content of the soils. Of great importance in that is the character and depth of the seasonal freezing and thawing, and also the annual thermal cycles in the soils. Taliks form, as a rule, in all cases where the warming effect of the snow cover exceeds the value of the average annual air temperature in the region under consideration. Thus, within temperature zone I taliks can form at a height of the snow cover of 0.4-0.7 meter, at a height of 0.7-1.2 meter in II, at 1.0-1.5 m in III and at more than 1.5-2.0 meters in IV (under the conditions of their thawing).

In a detailed consideration of this question it is necessary to take into consideration regularities in the formation of the snow cover in the course of the winter as a function of the presence of thawings, compaction and shifting of snow by the stormy wind. To estimate the influence of snow on the formation of taliks it is advisable to determine the warming effect of the snow, the total for the entire winter, while calculating the number of degrees by which the average annual temperature of the rocks is increased.

At a small height of the snow cover (from 2 to 5-10 cm) in the southern regions (south of latitude 50°) it is necessary to take into account the influence of the snow on change of the albedo of the surface and components of the radiation balance. Frequently in that case for separate regions (the Northern Caucasus), on account of increase of the albedo in a thin snow cover (2-3 cm) its summary effect leads to a reduction of the average annual temperature of the soils of up to 1° as compared with sections free of snow.

At larger thicknesses (> 1.5-2.0 meters) in northern regions the disappearance of snow is delayed and the summer warming of the soils is postponed. This leads to some cooling of the soils and reduction of their average annual temperature. The amount of cooling can vary substantially as a function of the length of delay of the disappearance of snow. The summer temperatures of the soils cannot exceed 0^o maximally, when the snow remains lying the entire summer until the new snow falls.

At ordinary thicknesses of the snow (from 0.1-0.2 to 1.0 meter) the main influence on the temperature of the underlying rocks is connected with its effect as a heat insulator with a definite thermal resistance. Calculations taking into consideration the warming influence of snow give an idea of the possibility of the formation of taliks.

Calculation of the Critical Height of Snow at Which Taliks of the Thermal Subtype Can Form and Exist (Example 34)

Determine the possibility of existence of taliks on account of the warming influence of the snow cover in a region where the average annual air temperature is -8.3^o and the amplitude of the annual temperature fluctuations is 44^o. The ground conditions are characterized by the distribution of sands in a layer with annual fluctuations of temperature. The properties of the sands are: γ_{sk} = 1300 kg/m^3; w = 15%; C_{spec} = 0.19 kcal/(kg)(degree); λ_f = 1.5 kcal/(m)(hr)(degree). The height of the snow cover under natural conditions varies as a function of the microrelief and vegetation from 0.5 to 0.7 meter, the density of the snow is 0.22 and 0.3 g/cc respectively, and τ = 3240 hours (the time from the moment of establishment of the snow to the moment of the autumnal inversion of the sign of the heat cycle through the surface).

The solution of the problem is reduced to finding the critical height of the snow at which the average annual temperature of the surface of the soil under the conditions of the investigated region increases by more than 8^o. Having determined that height it is possible to say whether the formation and existence of taliks on account of the warming influence of the snow are possible. To do that we find the warming influence of snow with a height of 0.5 and 0.7 meter, using equation (5.3.5). The procedure of the calculations is presented in example 10. On the basis of the results of the calculations we construct a diagram of the variation of the warming influence of snow as a function of its height. The starting data for the calculations were: C_{vol-f} = 440 kcal/(m^3)(degree); Q_ϕ = 15,600 kcal/(m^3); λ_f = 1.5 kcal/(m)(hr)(degree).

1. We find the warming influence of snow with a height of 0.5 meter, a density of 0.22 g/cc and λ = 0.25 kcal/(m)(hr)(degree). For the construction of a diagram to solve transcendental equation (5.3.5) we are given the following values of Δt_{sn}: 3, 5 and 8^o. In accordance with those values we obtain all the calculation data (Table 52). When we have constructed the diagram with the obtained data (Figure 97) we find that the warming influence of the snow with a height of 0.5 meter is 6.7^o.

2. We find the warming influence of snow with a height of 0.7 meter at ρ_{sn} = 0.3 g/cc and λ_{sn} = 0.3 kcal/(m)(hr)(degree). We are given the following values of Δt_{sn}: 8 and 10^o. In accordance with those values we obtain new calculation data (Table 53). When we have constructed the diagram

(Figure 98) we find that the warming influence of snow with a height of 0.7 m reaches 9.3° under the conditions of the region.

Table 52 Calculating data for finding on a diagram the value of the warming influence of snow Δt_{sn} with a height of 0.5 m

A $\Delta t_{сн}$, °C	t_0, °C	A_0, °C	B $A_{ср}$, °C	C ξ_{2c}, м	ξ, м	D $Q_{гр}$, ккал/м²	E $t_{сн}$, °C	F $t_{о. зим}$, °C	G $Q_{сн}$, ккал/м²
3,0	−5,3	19,0	11,6	1,75	2,1	38 740	22,0—20,0	−16,2	11 970
5,0	−3,3	17,0	9,6	1,5	2,3	40 517	22,0—20,2	−13,5	33 660
8,0	−0,3	14,0	6,5	1,4	2,7	44 683	22,0—20,2	−9,5	49 500

Table 53 Calculating data for finding on a diagram the value of the warming influence of snow Δt_{sn} with a height of 0.7 m

A $\Delta t_{сн}$, °C	t_0, °C	A_0, °C	B $A_{ср}$, °C	C ξ_{2c}, м	ξ, м	D $Q_{гр}$, ккал/м²	E $t_{сн}$, °C	F $t_{о. зим}$, °C	G $Q_{сн}$, ккал/м²
8	−0,3	14	6,5	1,4	2,7	44 683	22,0	−9,5	29 700
10	1,7	12	6,9	1,3	2,1	30 168	22,0	−6,9	35 640

Key (for both tables): A - Δt_{sn}, $^{\circ}$C B - A_m, $^{\circ}$C C - ξ_{2c}, m D - Q_{gr}, kcal/m^2 E - t_{sn}, $^{\circ}$C F - $t_{0\text{-}wtr}$, $^{\circ}$C G - Q_{sn}, kcal/m^2

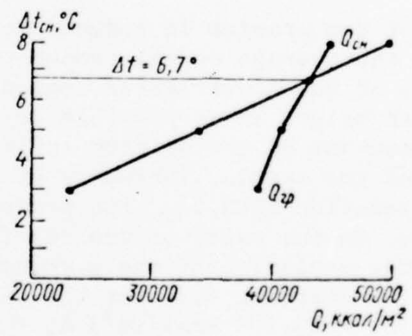

Figure 97. Diagram for finding t_{sn}.

a - Δt_{sn}, $^{\circ}$C b - Q, kcal/m^2

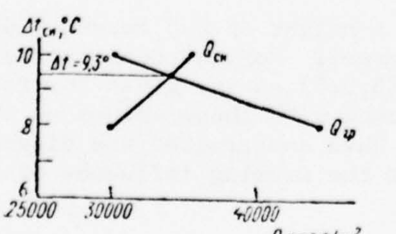

Figure 98. Diagram for finding the warming influence of the snow (Δt_{sn}).

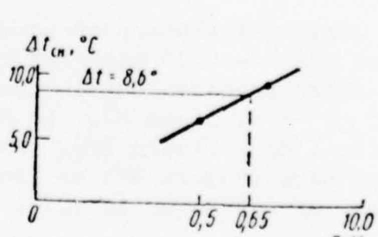

Figure 99. Diagram of change of the warming influence of the snow (Δt_{sn}).

3. We construct a diagram of the change of the warming influence of snow Δt_{sn} on its height (Figure 99) and find that at a height of the snow of 0.65 m the temperature on the surface of the ground is equal to zero, since Δt_{sn} in that case is equal to 8.6°. Consequently, under the conditions of the region taliks can form and exist on sections where the height of the snow is equal to or exceeds 0.65 meter.

On sections with a small hummock microrelief or covered with a grassy or underbrush cover a loose snow cover forms which adheres loosely (with the formation of cavities) to the soil. The specific conditions of the bedding of the snow and the presence of cavities have the result that its warming effect, even with a small height, reaches such large values that under its influence taliks form in temperature zones II and III, where the average annual temperature reaches -3 to -5°.

The formation of cavities under the snow has a great effect on the process of heat exchange on the surface of soil under snow. Heat fluxes from the soil and underlying rocks (negative heat cycles) arriving per unit of surface are carried off into the atmosphere only through that part of it on which the snow adheres closely to the soil, since molecular heat transfer is negligibly small through air layers. In that case, on sections with closely adherent snow the specific heat cycles passing through per unit of surface increase sharply, and that leads to increase of the warming effect of the snow.

The intensity of heat fluxes with and without consideration of cavities will be similar to the flow of filtering liquid through the ground (porous media) related to the effective and actual coefficients of filtration. By virtue of that fact the problem of heat transfer through snow with consideration of cavities can be considered a linear problem by reducing the intensity of the heat fluxes toward sections with a snow cover closely adherent to the surface of the soil.

To calculate the warming influence of the snow cover on the temperature of the surface in the presence of air layers under it, it is necessary to study in the field the conditions of bedding of the snow in order to obtain a coefficient expressing the percentage of cavities per unit of surface. In accordance with that coefficient the intensity of heat exchange through the surface of the soil which enters the calculating formula should be increased.

A general regularity of the phenomenon under consideration consists in the fact that the warming influence of the cavities forming under snow is directly proportional to the heat cycles passing from the soil through the snow and the latter in turn are proportional to the area (as %) with a closely adherent snow cover per unit of surface ($1\ m^2$). Thus, if on an area of $1\ m^2$ the low places between hummocks or mounds where the snow adheres closely to the surface of the soil occupies 20%, the warming effect of the snow (Δt_{sn}) must also be approximately 20% greater than in sections where other conditions being equal the snow adheres closely to the ground on the entire area. In regions where the warming influence of the snow cover is great (in the region of Igarka $\Delta t_{sn} \approx 7^{\circ}$) an increase of Δt_{sn} by 20% will give -1.4°, and in Zabaykal'ye, where $\Delta t_{sn} \approx 2^{\circ}$ it will be increased by a total of 0.4°.

Thus the absolute value of the increase of the warming influence of snow as a function of the area of the forming cavities under the snow will be different in different regions as a function of concrete geological and geographical conditions, in spite of the fact that their relative value (as %) can remain constant. For more precise determination of this effect it is necessary to take into account also the fact that with increase on account of cavities of the warming effect of the snow the average annual temperature of the ground changes and, consequently, the annual heat cycles. The change of the latter will be different in the region of seasonal freezing and in the region of permafrozen rocks. In the region of seasonal freezing, during elevation of the average annual temperatures of the ground the annual heat cycles decrease, and in accordance with that the warming effect of the snow also decreases.

In the region of seasonal thawing the reverse dependence is observed: with increase of the average annual temperature the annual heat cycles and the warming effect of the snow increase. As is known, the amount of change of heat cycles as a function of the change of the annual average temperatures of the ground can be calculated (see section 1, Chapter 4). The amount of that change will be different for different average annual temperatures and is subject to latitudinal geographic zonation. Near the southern boundary, in the region of seasonal freezing, that change will be maximal (during change of t_m from 0 to $+1^{\circ}$) and further south it will steadily diminish.

It follows from what has been said that the warming influence of cavities under the snow consists of two components. The first is connected with the fact that the presence of cavities leads to increase of the warming influence of the snow on account of reduction of the area through which the heat cycles pass, and the second with change in the number of heat cycles on account of change of the average annual temperatures of the ground. In the region of seasonal freezing the influence of those two components is different, and so the general effect of cavities under the snow will be somewhat smaller. In the region of permafrozen rocks the warming influence of the cavities under the snow increases sharply in comparison with the same effect (all other conditions being equal) in the region of seasonal freezing of the soils.

The maximal value of the warming influence of cavities under the snow must be noted near the southern boundary of the region in which permafrozen rocks are widespread. To the north and especially to the south of that boundary the thermal effect will diminish. Manifested in that is the latitudinal zonation of the phenomenon under consideration. The same should also be noted as a function of the continental climate. Under the conditions of a sharply continental climate (Central Yakutiya and Eastern Siberia) the thermal effect of cavities will attain maximal values, and under the conditions of a maritime climate will decrease to a minimum. The warming effect of snow containing cavities must also change in accordance with the height zonation. The maximal changes will be observed in that case in the region of the southern boundary of permafrozen rocks (distinguished by latitudinal zonation). The dependence of the warming influence of the snow on the annual heat cycles in the soil has the result that on swampy sections the thermal influence of cavities is considerably greater than on drained sections. It should also be pointed out that of very great importance in the question under consideration is the influence of the plant cover, as distinctive features and the character of the latter determine the conditions of formation of cavities under the snow and their warming effect on the soil.

Comparison of the Warming Influence of Snow on Sections With Its Close and Loose Adherence (Example 35)

Calculate how the warming influence of the snow cover with a height of 0.2 m and a density of 0.29 g/cc ($\lambda_{sn} \cong$ 0.2 kcal/(m)(hr)(degree) varies, other conditions of its adherence to the surface of the soil being equal. It is known that on sections with a sparse grassy cover the snow adheres closely and uniformly to the soil on that surface, and on sections with a dense underbrush under the snow, cavities form which occupy 0.3 m^2 per m^2 of surface. The average annual air temperature is -10.5° and the annual amplitude of air temperatures is 44°. The soils in the layer of annual temperature fluctuations are composed of sandy loams with thin, sparse layers of sand, characterized by the following thermophysical data: $C_{\gamma \theta 1-t}$ = 550 kcal/(m^3)(degree), λ_t = = 1.0 kcal/(m)(hr)(degree) and Q_ϕ = 18,600 kcal/m^3.

The time from the moment of stable transition of the temperature through 0° (coincides with the moment of establishment of snow) to the moment of autumn inversion of sign of the heat cycle through the surface of the snow τ is 4600 hours.

Solution. 1. We determine the warming effect of the snow cover with a height of 0.2 m on sections with snow closely adherent to the surface of the soil. We use for that the procedure presented in example 10. The results of the calculations are presented in Table 54. Having constructed a diagram (Figure 100), we find that Δt_{sn} = 2.8°.

Table 54 Calculating data for determination of Δt_{sn} on sections with closely adhering snow

A $\Delta t_{сн}$, °C	t_0, °C	A_0, °C	B $A_{ср}$, °C	C ξ_{2c}, м	ξ, м	D $Q_{гр}$, ккал/м²	E $Q_{сн}$, ккал/м²
2	−8,5	20	14	0,84	1,25	28 220	19 000
4	−6,5	18	11,7	0,9	1,4	30 476	44 364

Key: A - Δt_{sn}, °C B - A_m, °C C - ξ_{2c}, m D - Q_{gr}, kcal/m^2
E - Q_{sn}, kcal/m^2

2. When the snow does not adhere closely its warming influence increases in proportion to the area occupied by the cavities. To calculate Δt_{sn} for that purpose it is necessary to increase Q_{gr} by 30%. In that case the data for construction of the diagram will be those in Table 55. It is evident from the diagram that in that case the warming influence of the snow cover is 3.6°, that is, when the adherence was not close it increased by 0.8°.

Table 55 Calculating data for determination of Δt_{sn} on sections with not close adherence of the snow

A $\Delta t_{сн}$, °C	B $Q_{гр}$, ккал/м²	C $Q_{сн}$, ккал/м²
2	36 686	19 000
4	36 619	44 364

Key: A - Δt_{sn}, ^{o}C B - Q_{gr}, kcal/m^2 C - Q_{sn}, kcal/m^2

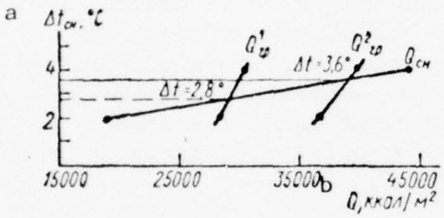

Figure 100. Diagram for finding the warming influence of snow (Δt_{sn}). a - Δt_{sn}, ^{o}C b - Q, kcal/m^2

4. Influence of Firn Basins and Glaciers on the Formation of Taliks of the Thermal Subtype

The influence of thick firn basins and glaciers on the temperature field of underlying rocks is a process still not much studied. The temperatures in the base of a glacier are determined by the average annual temperatures in the ice mass, their amplitudes on its surface, the value of the temperature gradient in the body of the glacier and its thickness.

The average annual temperature is determined by the radiation thermal balance of the surface of the glacier and the character of its feeding. Thus the average annual temperature of a glacier with ice feeding is approximately equal to the average annual air temperature, and on sections with firn feeding is considerably higher. In the first case the average annual temperature of the ice of glaciers of Franz Jozef Land was -10 and -11°, and under the firn caps increased to -3°. The lowest annual average temperatures of glaciers of as low as -30 and -50° are noted in Greenland and Antarctica.

The amount and character of change of the geothermal gradient depends on the values of the thermal conductivity of the ice, the history of development of the glacier and its dynamics, and the heat fluxes from the depths of the Earth. In Antarctica, according to the data of temperature measurements in a drill-hole near the Vostok station the temperature gradient in the half-kilometer ice mass increased with depth from 0.6 to 0.82°/100 m. In the simplest case (a stationary glacier with steady temperature regime) the temperature gradient in the ice is $g_i = q/\lambda_i$, where q is the heat flux from the depths of the Earth, characteristic of the given geological structure, in kcal/(m^2)(hr); λ_i is the thermal conductivity of the ice, assumed to be constant and equal to 1.56 kcal/(m)(hr)(degree).

The motion of a glacier contributes to an elevation of temperatures at its base as a result of friction against the surface of the rocks to 0°, which makes possible the existence of thawed tocks under them.

The thickness of glacier covers reaches 4300 meters (Antarctica), 3400 meters (Greenland), on islands of the Arctic Ocean it usually does not exceed 500-700 meters, on a continent they reach 1000 meters (the Pamirs), but usually they are measured in tens and the first hundreds of meters. Earlier it was considered that permafrozen rocks could not exist under thick glaciers. According to recent data, permafrozen rocks have been discovered under the Greenland glacier cap. The temperature at the base of the glacier at a depth of 1400 meters proved to be -13°. The presence of permafrozen rocks has also been noted in the edges of small glaciers of North America.

The temperature at the base of a glacier, upon the assumption of a steady or almost steady temperature regime, can be calculated in first approximation with the formula

$$t_{z.л} = t_{ср.л} - z \cdot \frac{q}{\lambda_{л}},$$

where t_{z-i} is the temperature at the base of the glacier, t_{av-i} is the average annual temperature of the ice, and z is the thickness of the glacier, in meters.

5. The Influence of the Flooding and Swampiness of Sections on the Formation of Taliks of the Thermal Subtype

It has long been established that on the northern part of the Western Siberian lowland, on swampy sections, as a rule, the average annual temperatures of rocks are higher than on dry sections. It also is widely known that in the region of Zabaykal'ye and the Far East, on swampy sections, the average annual temperatures are considerably lower than on drained sections. Near the southern boundary of the permafrost region, within the limits of the first frost-temperature zone, in the Western Siberian lowland, taliks are concentrated in swampy sections, and within the limits of Zabaykal'ye and the Far East on swampy sections permafrozen rocks usually are noted.

To calculate the thermal influence of swampiness in individual cases where on swampy sections a layer of water with an open mirror with a total area of more than half the surface is permanently present, formula (5.5.2) can be used, which was proposed for calculation of the thermal influence of small drainless bodies of water (section 5, Chapter 5). In that case it is necessary to take into consideration the difference between the temperature of the water surface and the air, since the albedo on swampy sections is small in the summer time and the water cover is warmed more than the air.

In the absence of a constant layer of water or in the presence of it in narrow interblock spaces the influence of swampiness on the temperature of rocks is expressed above all through change of the value of the warming influence of the snow cover and change of the temperature shift into the layer of seasonal freezing (thawing). Since swampiness usually leads to increase of the annual heat cycles of the soil and increase of the difference of the values of the coefficient of thermal conductivity of rocks in the frozen and thawed states, the warming effect of the snow and the amount of the temperature shift on swampy sections are larger than on drained sections.

As calculations have shown, swampiness can under different conditions be either a cooling or a warming factor. In cases where on swampy sections in the layer of seasonal thawing (frezing) of rocks a large temperature shift forms, severe frost conditions usually are connected with swampiness. Thus, for example, in the Far East the height of the snow cover is small (0.1-0.2 m) and the continental character of the climate is great (the amplitude of annual air temperature fluctuations is more than 40^{o}). Under those conditions the warming effect of snow on swampy sections does not exceed $1\text{-}2^{o}$ and is somewhat smaller on drained sections. The amplitude of temperature fluctuations on the surface of the soil is reduced very insignificantly and therefore the amount of the annual heat cycles in the soil remains large, which involves the formation of a temperature shift which attains values of $2\text{-}3^{o}$. On drained sections the temperature shift rarely exceeds 1^{o}. As a result, on swampy sections the average annual temperature of the rocks is 1 or 2^{o} lower than on dry sections.

In the Western Siberian lowland the thickness of the snow reaches 0.8-1 m. On swampy sections the heat cycles are large and the warming influence of the snow reaches $7\text{-}10^{o}$. On dry sections, in connection with decrease of the heat cycles, the snow cover warms the surface by not more than $5\text{-}6^{o}$. The temperature shift on swampy sections does not go beyond the limits of 1^{o}, since under snow the amplitude of the temperature fluctuations is sharply reduced. As a result, on those sections the average annual temperature of the rocks is always considerably (by $2\text{-}3^{o}$) higher than on dry sections, which causes the formation of taliks from the surface at a different depth, depending on the lower boundary conditions. Thus the calculations explain why, when there is a thick snow cover (0.7-1 meter) swampiness leads an increase of the average annual temperatures, whereas when there is a small snow cover, on the contrary, it leads to their sharp reduction. The composition and moisture content of the rocks is very essential in this question. A maximal difference of the thermal conductivity of thawed and frozen rocks is noted for sands and sandy loams when they are completely saturated with water. By virtue of this the maximal effect of the phenomenon under consideration is noted precisely for those soils.

It is interesting to note that a maximal influence of swampiness is manifested in the conditions of a sharply continental climate (Siberia and the Far East). Under the conditions of a maritime climate that influence is almost unnoticeable and is connected with the fact that the annual heat cycles of the soil are small there. The dependence of the thermal influence of swampiness on the annual heat cycles determines its geographical zonation and height zonation. A maximal manifestation of the influence of swampiness on the temperature regime of the soils and the formation of taliks is noted where the thermal cycles are maximal, that is, near the southern boundary of the region of permafrozen rocks. To the south and north the thermal cycles decrease and in accordance with that there is a decrease in the influence of swampiness and also of the formation of taliks of the thermal subtype. In the Far North, where the frost conditions are extremely severe and the average annual temperatures of the rocks reach values of -7 to -12°, on dry and swampy sections no great difference in the temperature regime is observed. A similar picture is also observed for the height zonation.

Calculation of the Warming Influence of Swampiness Leading to the Formation of Taliks of the Thermal Subtype (Example 36)

Calculate what influence is exerted by swampiness on the temperature of rocks under the conditions of a moderate maritime climate and a sharply continental climate. One of the sections is located in a region where $t_{air} = -7.0^{\circ}$ and $A_{air} = 22^{\circ}$ and the other in a region where $t_{air} = -6.8^{\circ}$ and $A_{air} = 23^{\circ}$. In both cases the snow reaches a height of 0.5 m and has a density of 0.28 g/cc, $\lambda_{sn} = 0.23$ kcal/(m)(hr)(degree). The time from the moment of establishment of snow to the moment of the autumn inversion of sign of the heat cycle through the surface τ under the conditions of a sharply continental climate on swampy sections is 4000 hours, on drained sections is 4300 hours, and under conditions of a moderate maritime climate, on the same sections, is 4300 hours.

The soils on swampy sections in both regions were composed of loams with small peat inclusions. Their properties are characterized by: $\gamma_{sk} = 630$ kg/m^3; $w_{vol} = 65\%$; $w_{un} = 15\%$; $C_{vol-t} = 770$ kcal/(m^3)(degree); $Q_{\phi} \cong 40{,}000$ kcal/m^3; $\lambda_t = 0.6$, $\lambda_f = 1.1$ kcal/(m)(hr)(degree). On drained sections in the regions under consideration are loams with $C_{vol-t} = 550$ kcal/(m^3)(degree); $Q_{\phi} = 15{,}000$ kcal/m^3; $\lambda_f = 1.2$, $\lambda_t = 1.0$ kcal/(m)(hr)(degree).

It also has been established as a result of frost investigations that on swampy sections in a region with a sharply continental climate the radiation corrections for the temperature regime of the surface are: $\Delta t_R = 0.7^{\circ}$ and $\Delta A_R = 1.5$; in a region with a moderately maritime climate they are much smaller and are: $\Delta t_R = 0.3^{\circ}$ and $\Delta A_R = 0.8^{\circ}$. On drained sections those corrections are $\Delta t_R = 0.3^{\circ}$ and $\Delta A_R = 1^{\circ}$ in the first case and $\Delta t_R = 0.2^{\circ}$ and $\Delta A_R = 0.5^{\circ}$ in the second.

The plant cover exerts on all sections a warming effect on the rock temperature. Under the conditions of a sharply continental climate on swampy sections: $\Delta t_{plant} = 0.3^{\circ}$ and $\Delta A_{plant} = 2.5^{\circ}$; on drained sections: $\Delta t_{plant} = 0.2^{\circ}$ and $\Delta A_{plant} = 1^{\circ}$. Under the conditions of a moderately maritime climate on swampy sections: $\Delta t_{plant} = 0.3^{\circ}$ and $\Delta A_{plant} = 1.8^{\circ}$; on drained sections: $\Delta t_{plant} = 0.1^{\circ}$ and $\Delta A_{plant} = 0.7^{\circ}$.

Solution. 1. We calculate the warming influence of snow similarly to example 10 and the amount of the temperature shift similarly to example 9 on swampy sections in a region with a sharply continental climate. The average annual temperature (t') and the annual amplitude of temperatures on the surface of the soil (A')° with consideration of Δt_R and Δt_{plant} but without consideration of the influence of the snow are:

$$\Delta t_0' = -6{,}8 + 0{,}7 + 0{,}3 = -5{,}8°,$$

$$\Delta A_0' = 23 + 1{,}5 - 2{,}5 = 22°.$$

Being given in accordance with the procedure in example 10 the values Δt_{sn} = = 7 and 9^{o} at C_{vol-t} = 770 kcal/(m^3)(degree), Q_ϕ = 40,000 kcal/m^3; λ_t = 0.6, λ_f = 1; λ_{sn} = 0.25 kcal/(m)(hr)(degree); z_{sn} = 0.5 m; τ = 4000 hrs we obtain the following calculation values (Table 56).

Table 56 Calculation data for determining Δt_{sn} on swampy sections

A $\Delta t_{сн}$, °C	t_0, °C	A_0, °C	ξ, м	B $A_{ср}$, °C	C ξ_{2c}, м	D $Q_{гр}$, ккал/м²	E $t_{сн}$, °C	F $t_{о.зим}$, °C	G $Q_{сн}$, ккал/м²
7	+1,2	15	1,40	6,7	0,74	57 820	−21,0	−9,2	47 200
9	+3,2	13	1,05	8,0	0,51	44 000	−21,0	−6,5	58 000

Key: A - Δt_{sn}, ^{o}C B - A_m, ^{o}C C - ξ_{2c}, m D - Q_{gr}, kcal/m^2
E - t_{sn}, ^{o}C F - t_{o-wtr}, ^{o}C G - Q_{sn}, kcal/m^2

With the data of Table 56 we construct a diagram (Figure 101) from which we find that Δt_{sn} = 8.0^{o}. Consequently, the temperature regime on the surface of the soil under a snow cover is characterized by

$$t_0 = -5{,}8 + 8{,}0 = 2{,}2°$$

$$A_0 = 22 - 8 = 14°.$$

For the starting data: t_o = 2.2^{o}; A_o = 14.0^{o}; λ_t = 0.6, λ_f = 1.0, λ_{lim} = = 0.82 kcal/(m)(hr)(degree); τ = 4000 hours, we find the temperature shift. Being given in accordance with the procedure in example 9 the values of Δt_λ, equal of 1, 2 and 3^{o}, we obtain data (Table 57) for the contruction of a diagram (Figure 102), from which we find that Δt_λ = 1.8^{o}. Consequently, t_ξ = 2.2 - 1.8 = 0.4^{o}.

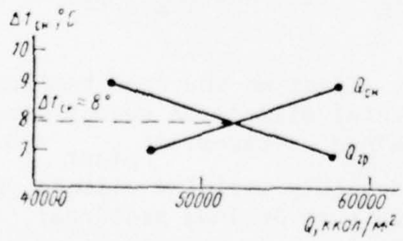

Figure 101. Diagram for finding Δt_{sn}. a - Δt_{sn}, ^{o}C b - Q, kcal/m^2

Table 57 Calculation data for determining Δt_λ on swampy sections

Δt_λ, °C	t_ξ, °C	ξ, м	$A_{ср}$, °C	$\frac{\xi^2 (Q_\phi + A_{ср} C)}{\tau} \cdot \frac{\sqrt{\lambda_м} - \sqrt{\lambda_т}}{\lambda_{пр}}$
1,0	1,2	1,25	7,4	2,2
3,0	−0,8	1,0	7,2	1,4

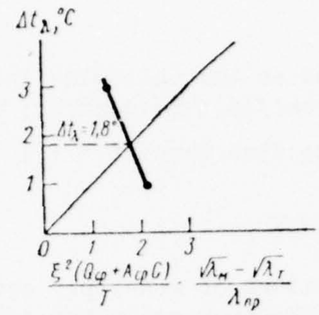

Figure 102. Diagram for finding Δt_λ

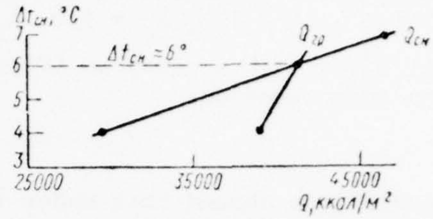

Figure 103. Diagram for finding Δt_{sn}. a - Δt_{sn}, °C b - q, kcal/m²

2. We calculate the warming influence of snow and the amount of the temperature shift on drained sections in a region with a sharply continental climate. Without consideration of the influence of the snow but with consideration of the influence of Δt_R and Δt_{plant}, t_o and A_o are equal to

$$t_0' = -6{,}8 + 0{,}3 + 0{,}2 = 6{,}3°,$$

$$A_0' = 22 + 1{,}0 = 23°.$$

Being given Δt_{sn} = 4, 6 and 7°, we obtain at C_t = 550 kcal/(m³)(degree), Q_ϕ = = 15,000 kcal/m³; λ_t = 1.0, λ_{sn} = 0.25 kcal/(m)(hr)(degree) and τ = 4300 hrs the following data (Table 58). Having constructed on the basis of the data of Table 58 a diagram (Figure 103), we find that $\Delta t_{sn} \cong 6°$. Consequently, t_o = -6.3 + 6 = -0.3° and A_o = 23 - 6 = 17°.

Table 58 Calculating data for determining Δt_{sn} on drained sections

A $\Delta t_{сн}$, °C	t_0, °C	A_0, °C	ξ, м	B $A_{ср}$, °C	C ξ_{2C}, м	D $Q_{гр}$, ккал/м²	E $t_{сн}$, °C	F $t_{0.\ зим}$, °C	G $Q_{сн}$, ккал/м²
4	−2,3	19	2,20	9,7	1,60	38 939	−21	−14,2	29 240
6	−0,3	17	2,45	7,6	1,77	41 285	−21	−11,5	40 850
7	+0,7	16	2,30	7,4	1,79	40 805	−21	−10,2	46 440

Key: As for Table 56

Table 59 Calculating data for determining Δt_{sn} on swampy sections

A $\Delta t_{сн}$, °C	t_0, °C	A_0, °C	ξ, м	B $A_{ср}$, °C	C ξ_{2C}, м	D $Q_{гр}$, ккал/м²	E $t_{сн}$, °C	F $t_{0.\ зим}$, °C	G $Q_{сн}$, ккал/м²
1	−5,4	9,0	0,3	7,3	0,16	13 610	−12,3	29,6	11 610
2	−4,4	8,0	0,4	6,1	0,17	17 451	−12,3	−8,3	17 200
4	−2,4	6,0	0,46	4,2	0,18	19 309	−12,3	−5,6	28 810

Key: As for Table 56

The temperature shift on drained sections is found at the following initial data: t_o = -0.3°, A_o = 17°, λ_t = 1.0, λ_f = 1.2 kcal/(m)(hr)(degree); Q_φ = = 15,000 kcal/m³. With a nomogram (Figure 33) we find that $\Delta t \approx 1°$. Consequently, on those sections

$$t_\xi = -0,3 - 1,0 = -1,3°.$$

Thus the calculations showed that under the conditions of a sharply continental climate at a sufficiently large height of the snow cover the swampiness exerts a substantial warming influence in comparison with drained sections (t_ξ = -1.3°) and can lead to the formation and existence of taliks.

3. We calculate Δt_{sn} and Δt_λ on swampy sections in a region with a moderately maritime climate. Without consideration of the influence of snow:

$$t_0' = -7,0 + 0,3 + 0,3 = -6,4°,$$

$$A_0' = 11 + 0,8 - 1,8 = 10°.$$

Being given in accordance with the procedure of example 10 the values Δt_{sn} = = 1.2 and 4° at C_{vol-t} = 770 kcal/(m³)(degree), Q_ϕ = 40,000 kcal/m³; $\lambda_t \cong$ 0.25 kcal/(m)(hr)(degree) we obtain the following calculating data (Table 59); z_{sn} = 0.5 m; τ = 4300 hours.

Having constructed a diagram in accordance with the data of Table 59 we find that Δt_{sn} = 2.1° (Figure 104). Consequently t_o = -4.3°; A_o = 7.9°; λ_t = 0.6, λ_f = 1.1, λ_f = 1.0 kcal/(m)(hr)(degree); Q_ϕ = 40,000 kcal/m³ we find the temperature shift. Being given in accordance with the procedure of example 9 the values of Δt_λ = 0.2, 0.5 and 1°, we obtain the calculating data (Table 60), after plotting which on the diagram we find that $\Delta t_\lambda \approx 0.3°$. Consequently t_ξ = -4.3 + (-0.3) = -4.6°.

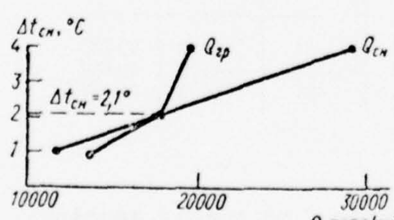

Figure 104. Diagram for finding Δt_{sn}. a - Δt_{sn}, °C b - Q, kcal/m²

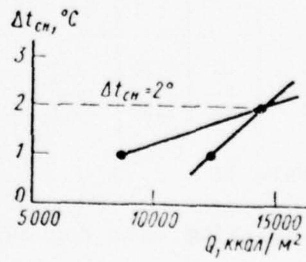

Figure 105. Diagram for finding Δt_{sn}. a & b as for Figure 104.

4. We calculate Δt_{sn} and Δt_λ on drained sections in a region with a moderately maritime climate. Without consideration of the influence of snow

$$t_0' = -7{,}0 + 0{,}2 + 0{,}1 = -6{,}7°,$$

$$A_0' = 11 + 0{,}5 - 0{,}7 = 10{,}8°.$$

Being given the values of Δt_{sn} of 1^o and 2^o at the initial data C_{vol-t} = 550 kcal/(m^3)(degree); Q_ϕ = 15,000 kcal/m^3; λ_t = 1.0, λ_{sn} = 0.25 kcal/(m)(hr)(degree); τ = 4300 hours, we obtain data for the construction of a diagram (Table 61).

Table 60 Calculating data for determining Δt_λ on swampy sections

Δt_λ, °C	t_ξ, °C	ξ, м	A_{cp}, °C	$\frac{\xi^2 (Q_\Phi + A_{cp}C)}{\tau} \cdot \frac{\sqrt{\lambda_м} - \sqrt{\lambda_т}}{\lambda_{пр}}$
0,2	−3,2	0,5	5,5	0,35
0,5	−3,5	0,46	5,7	0,3

Table 61 Calculating data for determining Δt_{sn} on drained sections

A B C D E F G

$\Delta t_{сн}$, °C	t_0, °C	A_0, °C	ξ, м	A_{cp}, °C	ξ_{2c}, м	$Q_{гр}$, ккал/м³	$t_{сн}$, °C	$t_{о,\ зим}$, °C	Q_{cd}, ккал/м³
1	−5,7	9,8	0.75	6.8	0,45	12 315	−12,3	−10,3	8 600
2	−4,7	8,8	0,85	6,7	0,45	14 352	−12,3	−9,0	14 190

Key: As for Table 56

Having constructed a diagram (Figure 105) we find that $\Delta t_{sn} = 2^o$ and then

$$t_0 = -6{,}7 + 2 = -4{,}7°,$$

$$A_0 = 10{,}8 - 2 = 8{,}8°.$$

We find Δt_λ corresponding to the following data: $t_o = -4.7^o$, $A_o = 8.8^o$; λ_t = 1.0; λ_f = 1.1 kcal/(m)(hr)(degree); Q_ϕ = 15,000 kcal/m^3. On the basis of a nomogram (Figure 33) we obtain $\Delta t_\lambda \cong 0.18^o$. Therefore

$$t_\xi = -4{,}7 + (-0{,}18) = -4{,}9°.$$

Consequently, under the conditions of a moderately maritime climate on both drained and swampy sections the warming influence of snow and the temperature shift are similar in value. As a result of their summary influence the average annual temperature of the rocks on the two sections remains negative and substantially lower in value than on the corresponding sections with a sharply continental climate.

6. The Influence of Infiltration of Precipitations on the Formation of Taliks of the Radiation-Infiltration Subtype

The type of talik is radiation-thermal and the subtype is radiation-infiltration. A great influence on the formation of such taliks is exerted by infiltrating atmospheric precipitations.

On sections composed of coarse well-filtering rocks, in the summer atmospheric precipitations, filtering, carry additional heat into the soils. In the region of propagation of permafrozen rocks in the layer of summer thawing the entire store of heat in the infiltrating precipitations goes to elevate the temperature of the layer of thawing and for phase transitions of the additional depths of thawing. In first approximation this can be quantitatively considered reduced by the above formula (5.8.3). For practical calculations with that formula it is necessary to take from meteorological handbooks the monthly or 10-day sums of the precipitations falling in summer and the average monthly and average 10-day air temperatures respectively. It is assumed that the precipitations enter the soil with a temperature equal to the air temperature. Therefore the summary heat flux ($v \times t_{prec} \times C_{air}$) entering the soil with precipitations can be determined as the sum of the product of the monthly (10-day) sums of the precipitations times the average monthly (10-day) air temperature during the entire summer period.

Calculation of the Possibility of Existence of Taliks on Account of Infiltration of Atmospheric Precipitations (Example 37)

In the investigated region on sections of terrace composed from the surface to a depth of at least 10 meters of sands, taliks are developed. In the sands there is a water-bearing horizon, the level of which fluctuates in the range of 3 meters, and the water temperature in the course of the summer varies insignificantly and on the average is $+0.5^{\circ}$. On sections of the same terrace, where the sand deposits are covered from the surface with loams 1 or 2 meters thick, permafrozen rocks are distributed. The conditions on the surface of the sections differ slightly: on thawed sections the plant cover consists of sparse pine forest with an admixture of birch, with sparse underbrush and herbage. On sections with permafrozen rocks, green mosses are encountered in patches, not more than 2-3 cm in height. The snow height on both those sections reaches 0.3 m and its density is 0.17 g/cc on the average. The climatic conditions are characterized by $t_{air} = -1.2^{\circ}$ and $A_{air} = 4.4^{\circ}$, and the regime of fall of summer precipitations is determined by the data in Table 62.

Table 62 Characteristics of precipitation in the summer period and calculating data for determination of Δt_{prec}

A Показатели	V	VI	VII	VIII	IX	B Σv_{oc} V–IX
B v_{oc}, мм	40	30	70	60	80	2761
C $t_в$, °C	+3,8	10,7	13,5	15,0	7,3	
BC $\Sigma v_{oc} (t_в - 0,5)$	140	306	901	870	514	

Key: A - Indicators B - oc = precip C - B = air

On sections composed from the surface of sands the precipitations completely infiltrate into the deposits. On sections composed of loams in the layer of seasonal thawing, the precipitations go out into the surface runoff.

The plant cover has a cooling effect on the rock temperatures: $\Delta t_{plant} = -0.2^{\circ}$ and $\Delta A_{plant} = 0.8^{\circ}$.

The soils on the sections are characterized by the following properties: the sands have $\gamma_{sk} = 1250$ kg/m^3; w = 12%; $\lambda_t \cong \lambda_f = 1.5$ kcal/(m)(degree)(hr); $C_{vol-t} = 370$ and $C_{vol-f} = 310$ kcal/(m^3)(degree); $Q_{\phi} = 9600$ kcal/m^3; the loams have $\gamma_{sk} = 1150$ kg/m^3; w = 20%; $\lambda_t = 1.0$ and $\lambda_f = 1.2$ kcal/(m)(degree)(hr); $C_{vol-t} \cong 400$ kcal/(m^3)(degree); $Q_{\phi} = 16{,}000$ kcal/m^3. It is required to determine the conditions of existence of a talik on sections composed from the surface of sands.

Solution. 1. We determine the temperature regime on the surface of soil on both sections with consideration of the plant and snow covers (with 5.3.10): $t_{sn} = 22 \times 0.188 = 4.1^{\circ}$. Consequently

$$t_0 = -4,2 + 4,1 - 0,2 = -0,3^{\circ};$$
$$A_0 = 22 - 4,1 - 0,8 = 17,1^{\circ}.$$

2. On sections composed from the surface to a depth of 1-2 m of loams, in the layer of seasonal thawing a temperature shift forms. We find it with a nomogram (Figure 33) for the following starting data: $t_0 = -0.3^{\circ}$, $A_0 = 17.1^{\circ}$; $\lambda_t = 1.0$ and $\lambda_f = 1.2$ kcal/(m)(degree)(hr); $Q_{\phi} = 16{,}000$ kcal/m^3. In that case $\Delta t_{\lambda} = 0.8^{\circ}$. Consequently, $t_{\xi} = -0.3 - -0.8 = -1.1^{\circ}$.

3. On sections composed from the surface of sands there is infiltration of atmospheric precipitations into the deposits. As a result of convective heat exchange the temperature of the rocks rises. We find the warming influence of the infiltrating precipitations with formula (5.8.3), taking into account that in the process of infiltration the water is cooled to $+0.5^{\circ}$, reaching the level of the ground waters at a depth of 30 m:

$$\Delta t_{oc} = \frac{c_{в} \cdot \Sigma (v_{oc} \cdot t) \xi}{\lambda \cdot T} = \frac{1 \cdot 2761 \cdot 3}{1,5 \cdot 8760} = 0,6^{\circ}.$$

4. We find the average annual temperature of the rocks on sections where the infiltration of atmospheric precipitations occurs freely:

$$t_{\xi} = t_0 + \Delta t_{oc} = -0,3 + 0,6 = 0,3^{\circ}.$$

Under those conditions the depth of the seasonal thawing will be 3 m.

Thus the infiltration of atmospheric precipitations on sections composed from the surface of sands assures the conditions of existence of a talik.

Besides the considered approximate method of calculating the thermal influence on rocks of the infiltrating atmospheric precipitations, in section 4 of Chapter 9 was presented the solution of a unidimensional problem of the thawing of coarsely dispersed soils with consideration of the infiltration of summer precipitations on digital computers.

7. The Influence of Water Covers and the Composition of the Bottomset Beds on the Formation of Taliks of the Underwater Type

Type of taliks -- underwater-thermal (hydrogenous), subtype -- shelf.

Taliks of the underwater type are encountered in bottomset beds of the sea shelf, under lakes and under riverbeds. The thickness and composition of the layer of water in water bodies to a considerable degree determines the structure of the radiation-thermal balance of the surface and the thermal regime of the bottomset beds.

The thermal state of the bottomset beds within the limits of the shelf is determined by the temperature of the bottom layers of water in the course of the year. In that case, depending on the depth and salinity of the water, there can be two cases: the first, when the temperature in the bottom layer remains above zero during the entire year, and the second, when a positive temperature is observed only in the summer period. In the latter case the bottomset beds freeze seasonally and therefore in the hydrogeological sense as a feeding region they are switched off in the winter period.

When there is a negative average annual temperature in the bottom layers of the water on sections of the shelf there will be permafrost of the bottomset beds. Most often in that case the temperature of the deposits at great depths of the body of water is kept at about -1.9° (the freezing point of sea water). Under those conditions the taliks in the bottomset beds belong to the class of infiltration and pressurized-filtration and to the subclass of cryohydrohalinic. The temperature of the rocks in those taliks usually is below 0°, but the rocks contain no ice. The upper part of the talik within the range of negative temperatures is in essence a zone of frozen rock masses. The thickness of the latter can be calculated in accordance with the geothermal gradient and the temperature regime on the surface of the bottomset beds. The indicated taliks are a zone of active water exchange during the entire year.

Type of taliks -- underwater-thermal, subtype -- underlake. In fresh bodies of water the layer of water has a warming influence on the temperature of the rocks, protecting them against severe winter cooling. At a depth of bodies of water surpassing the depth of their freezing, the bottomset beds remain in the thawed state the year round and have a positive average annual temperature. At a depth of the body of water below the depth of freezing the bottomset beds freeze with the formation of either seasonally frozen or permafrozen rocks. The formation of taliks in bottomset beds under the influence of lakes must be regarded in connection with the composition of the deposits. If they are composed of non-filtering or poorly filtering rocks, the taliks could exist on account of conductive heat transfer. But if they are composed of rocks which filter well, their existence involves convective heat transfer.

At a depth of the body of water greater than the depth of its freezing the tempe ture of the bottomset beds is kept above 0° during the entire year, and this leads to the formation of a talik. The thickness of the talik depends on the width of the body of water and the thickness of the permafrozen rock masses in the coastal massif, and also the time of existence of the lake. If the thickness of the frost (ξ_{per}) exceeds the width of the lake (B), then in the case of absence of convective heat exchange in the bottomset beds under the lake a non-permeating talik will occur. The time of formation of that talik and its configuration can be determined with the following calculating procedure.

Calculation of the Time of Formation of a Talik and Its Configuration (With the Method of D. V. Redozubov) Under a Thermokarst Lake (Example 38)

The lake is situated on a lake-alluvial plain composed of a mass of silty light and medium sandy loams similar in composition. In the frozen state the specific gravity of the rock skeleton γ_{sk} is 1000 kg/m^3 on the average, w_{vol} = = 40%; λ_t = 1.3 kcal/(m)(hr)(degree); Q_{ϕ} = 32,000 kcal/m^3. The average annual temperature of the permafrozen lake-alluvial deposits surrounding the talik is -7°, and the geothermal gradient in the frozen rock mass (g) is close to 0°. The dimensions of the lake, which is oval, are 100 x 70 m, and its depth is 1.8 m. The average annual air temperature in the region is -13.5° and the average amplitude of temperatures is 44°. The thickness of the snow on the lake reaches 0.3 m and its density is 0.25 g/cc. The thickness of the ice (H_i) in stagnant bodies of water of that region reaches 2.2 m.

Solution. 1. To determine the configuration of the talik under the lake, and later the time of its formation, it is necessary to determine the temperature regime of the bottomset beds at a depth of 1.8 m. In accordance with the procedure of example 17 we find successively:

$$\Delta t_{сн} = 22 \cdot 0{,}153 = 3{,}4^{\circ} \text{ (по 5.3.10)},$$

$$t_0 = -13{,}5 + 3{,}4 = -10{,}1^{\circ},$$

$$A_0 = 22 - 3{,}4 = 18{,}6^{\circ},$$

$$t_{макс} = 8{,}5^{\circ},\ t_{мин} = -28{,}7^{\circ},$$

$$t_{h=1,8} = \frac{\frac{2{,}2-1{,}8}{2{,}2}\cdot(-28{,}7)+8{,}5}{2} \cong +1{,}7^{\circ} \text{ (по 5.5.1)}.$$

2. With formula (5.5.2) we find the depth of the lake at which the isotherm of the zero annual average temperature passes, below which on bank slopes the talik starts:

$$h_{t=0} = H_л\left(1 + \frac{t_{макс}}{t_{мин}}\right) = 2{,}2\left(1 - \frac{8{,}5}{28{,}7}\right) = 1{,}54 \text{ м}.$$

3. We determine the configuration of the talik under the lake at the moment of the steady temperature regime with the method of D. V. Redozubov (see section 6, Chapter 3). For the given case, assuming the origin of the coordinates to be on the left bank of the lake (Figure 106), we obtain an expression for the temperature of the bottomset beds at any point (x, z) of the region:

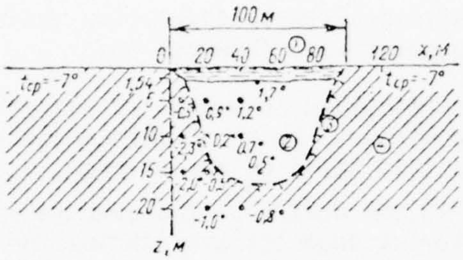

Figure 106. Configuration of talik under lake at the moment of steady temperature regime: 1 - lake; 2 - talik under lake; 3 - boundary of frozen and thawed rocks; 4 - frozen rock mass.

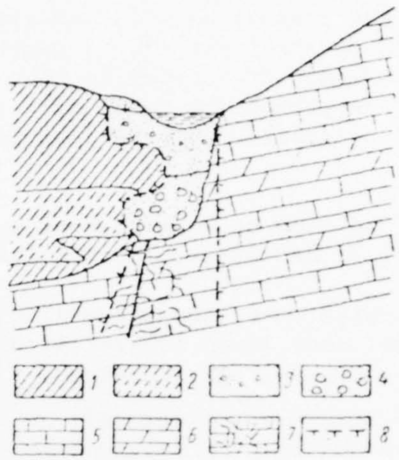

Figure 107. Layered frozen rock mass forming as a result of the riverbed dynamics: 1 - loam; 2 - sandy loam; 3 - sand containing gravel; 4 - coarse gravel; 5 - limestone; 6 - marl; 7 - fault with fractured zone; 8 - boundary of permafrozen rocks.

$$t(x, z) = \frac{z}{\pi}\int_{-\infty}^{+\infty}\frac{\varphi(S)\,dS}{(S-x)^2+z^2} + gz = 0 + \frac{1}{\pi}\left[-7\left(\frac{\pi}{2} - \operatorname{arctg}\frac{x}{z}\right) + \right.$$
$$\left. + 1{,}7\left(\operatorname{arctg}\frac{100-x}{z} + \operatorname{arctg}\frac{x}{z}\right) - 7\left(\frac{\pi}{2} - \operatorname{arctg}\frac{100-x}{z}\right)\right]. \quad (7.1.1)$$

The results of calculations for 10 points at arbitrary values of **x** and **z** are presented in Table 63. On the basis of those data, on Figure 106 is shown the configuration of the talik, the depth of which reaches 15.2 meters.

Table 63 Calculation data for determining the configuration of a talik under a lake by the method of D. V. Rodozubov

x, м	5	5	5	20	40	20	40	20	50	20
z, м	5	10	15	5	5	10	10	15	15	20
t, °C	−0,5	−2,3	−2,0	0,9	1,2	0,2	0,7	−0,45	0,5	−1,0

4. We find the time of formation of the talik with the approximate Stefan formula (3.7.7) at the starting data λ_t = 1.3 kcal/(m)(hr)(degree), Q_ϕ = = 32,000 kcal/m^3, t = 1.7^o and $H_t \approx$ 15.2 meters:

$$\tau = \frac{H^2 Q_\Phi}{\lambda t} = \frac{15{,}2^2 \cdot 32\,000}{1{,}3 \cdot 1{,}7} = 3\,345\,376 \text{ час} \approx 382 \text{ года}.$$

Thus under the lake a talik with a depth of 15.2 meters could have formed in 382 years.

/Type of taliks -- underwater-thermal, subtype -- under-riverbed/. Taliks under riverbeds, just like those under lakes, are connected with the warming influence of the layer of water covering the bottomset beds. Therefore the above-considered regularities of the formation of taliks under lakes remain valid also for those under riverbeds. At the same time, there are differences between them which relate above all to distinctive features of the temperature regime of the layer of water in the summer and winter periods, and also distinctive features of the formation of the ice cover. The temperature regime of the water in the rivers differs above all on account of the flow of the water. In the summer period this leads to a certain reduction of temperature of the water in the river, and in winter to a reduction of the depth of freezing of rivers as compared with lakes. The latter has the result that the formation of taliks under riverbeds usually involves smaller depths of rivers than of lakes. When the bottomset beds are composed of non-filtering or poorly filtering rocks, the taliks under riverbeds belong to the class of anhydrous, subclass thermal. The type of talik -- permeating or nonpermeating -- is determined by the depth, width and regime, and also by the dynamics of the riverbed and history of development of the valley. Often when the riverbed is wide enough and the river deep enough nonpermeating taliks of small thickness are noted where according to the temperature conditions of the river current they ought to be permeating. Those taliks are connected with an intensive shift of the riverbed under conditions of continuous permafrozen rocks within the limits of the valley profile. On stable sections of the riverbed, where a rather thick layer of water exists a long time, taliks form with an almost stationary temperature field. By virtue of that, when the talik is wide enough it usually is permeating. In the presence of intense dynamics (shifting) of the riverbed taliks can form which are underlain by continuous permafrozen rock masses, that is, interlayered by thawed and frozen horizons (Figure 107).

Buried taliks of the sub-bed type can often be encountered in the valleys of large rivers. Their formation is connected with shift of the riverbed and the formation in its place of sand islands, on which from the surface there is perennial freezing of deposits. With time the buried talik under the riverbed can freeze completely. On the basis of the character of the bedding of the permafrozen rock mass and its temperature regime on those sections it is possible to determine by calculation the time of existence of the talik under the riverbed and its freezing rate.

The probability of the formation of permeating taliks as a function of the temperature regime and the thickness of the permafrozen rock masses on river banks and the temperature regime of the river flow can be calculated as follows.

Determination of the Character of the Talik Forming in a Weakly Filtering Alluvium Under a Riverbed (Example 39)

The bottomset beds under a riverbed are composed of poorly filtering loams. The conditions on the investigated section are as follows: on the left of the riverbed extends a narrow flood plain composed of permafrozen sandy loams and loams, the average annual temperature of which is -0.5°. On the right the river undercuts a steep scarp of the first terrace above the flood plain, composed of permafrozen sands with an average annual temperature of -2.5°. The temperature gradient in the frozen rock mass is 0.02 deg/m on the average. The river bed is 60 m wide and the average annual temperature of the water in the bottom layers is +1.5°.

Solution. We find the configuration of the talik under the riverbed at the moment of the established temperature regime with the method of D. V. Redozubov. For the given case, assuming the origin of the coordinates to be on the left bank of the lake (Figure 108), we have an expression for the temperature at any given point (with respect to x and z) in the region of the investigations:

$$t(x, z) = gz + \frac{1}{\pi}\left[-0{,}5\left(\frac{\pi}{2} - \operatorname{arctg}\frac{x}{z}\right) + 1{,}5\left(\operatorname{arctg}\frac{60-x}{z} + \operatorname{arctg}\frac{x}{z}\right) - 2{,}5\left(\frac{\pi}{2} - \operatorname{arctg}\frac{60-x}{z}\right)\right]. \quad (7.1.2)$$

The results of the calculation for 17 points at arbitrary given values of x and z are presented in Table 64 and on Figure 108. Those data testify that a permeating talik forms under the riverbed.

Table 64 Calculation data for determination of the configuration of a talik under a riverbed by the method of D. V. Redozubov

x, м	z, м	t, °C	x, м	z, м	t, °C
60	5	−0,4	−60	25	−0.01
−10	5	−0,5	80	30	−0,77
0	10	0,4	−20	30	0,24
20	10	1,0	60	35	−0.14
40	15	0,5	−30	40	0,37
−20	15	0,03	−20	50	0,59
−5	20	0,1	120	50	−0,87
60	20	−0,6	80	50	−0,35
			60	55	0,12

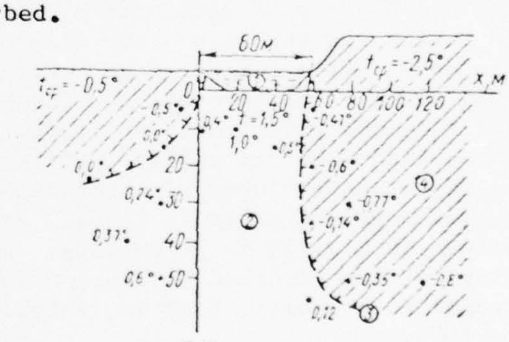

Figure 108. Configuration of a talik under a riverbed at the moment of the established temperature regime: 1 - river; 2 - talik under river; 3 - boundary of frozen and thawed rocks; 4 - frozen rock mass.

Taliks of the sub-riverbed subtype which form in the presence of well-filtering bottomset beds belong to the class of ground-filtering and infiltration and the thermal subclass. The warming influence of the water flow, the depth of which is greater than the depth of freezing, besides a thermal insulation effect which determines the conductive heat exchange in the bottomset beds, is accomplished through additional convective heat transfer. Where the bottomset beds are composed of sandy and gravel-pebble rocks the waters of the flow under the riverbed, circulating, carry with them a large quantity of additional heat, thaw permafrozen rock masses and sharply elevate the temperature of the rocks. In that case permeating taliks form, more rarely non-permeating.

At a small depth of the riverbed and its complete freezing in winter, in the summer the taliks under the riverbed are completely restored and the water in them circulates until they are completely or partially frozen in winter. In that case, when the river depth remains rather large even in the winter, those taliks exist the year round.

In the valleys of large rivers the sandy-pebble deposits of the riverbed facies often are widespread even within the limits of a narrow flood plain. Usually confined to those deposits is an alluvial flow, closely connected with the riverbed in its conditions. Through movement of the alluvial flow form taliks of the water-thermal type. Genetically they are closely connected with the sub-riverbed subtype of underwater taliks.

8. The Influence of Subsurface Waters on the Formation of Taliks of the Water-Thermal Type

Taliks of the water-thermal (hydrogeogenic) type owe their existence to the presence of additional convective heat exchange on account of the movement of subsurface waters. Depending on the type of subsurface waters (descending infiltration, ground pressurized and unpressurized or ascending subfrostal), taliks of three subtypes are distinguished (see Table 45).

/The subtype of taliks forming on account of the warming influence of ground waters/. Taliks of this subtype are widespread in valleys and on slopes. The warming influence of ground waters is concentrated in the regions in which they are fed by the infiltration of warm summer precipitations, and also on the path of their movement in valleys and down along slopes in well-filtering sandy, gravel-pebble and crushed rock-gruss deposits. A substantial warming influence on rocks can be exerted by ground waters at the places of their discharge (emergences of sources).

Determination of the Possibility of Forming an Infiltration Talik Through the Runoff of Surface Waters in a Fractured Zone (Example 40)

The bottom of a valley to a depth of 2 m is composed of sandy-loam and pebble alluvium (alQ_{IV}), more deeply -- of strongly fractured granites and granitized gneisses of the Archean. The thickness of the permafrozen rock mass is 200 m on the average, $t_m = -2^o$, $A_o = 15^o$, $\xi_{seas} = 2$ m; λ_1 of alluvial deposits and λ_2 of bedrocks are 1 and 2 kcal/(m)(hr)(degree) respectively. The thermal conductivity (α) of the bedrocks is 40×10^{-4} m^2/hr and Q in layer ξ is 17,000 kcal/m^3. The average annual temperature of the waters running off along the slope and infiltrating along a fault in the valley bottom is 4°. The radius of the water-absorbing zone (r) is 10 meters. The infiltration rate (v) is 0.02 m/hr. The length of the infiltration period (τ_{inf}) is 2400 hours.

Solution. During the heat exchange of infiltrating waters moving along a talik, with frozen rocks a portion of the heat of those waters is expended, as a result of which their temperature declines. Some heat is expended in that case in the layer of annual heat cycles on thawing the layer of winter freezing and forming positive average annual temperatures of the rocks. In that case there is a change of the average annual temperature from negative on the surface of the soil to positive on the base of the layer of seasonal freezing. A temperature shift forms on account of the influence of the rising flow of water.

If the temperature on the surface of the soil is t_o^*, then for the average annual temperature to be not below 0° in the base of the layer of freezing with a thickness ξ_f (a condition necessary for the existence of a talik), one requires a heat flux of

$$q = C_в \cdot v \cdot \Delta t_1 \cdot \tau_{ин} = Q_ф (\xi_{п.пр} - \xi) + \frac{\lambda_{пр} |t_{ср}|}{\xi} T, \qquad (7.1.3)$$

where C_w is the heat capacity of water, kcal/(m^3)(degree); v is the specific flow rate of water, m^3/hr; Δt is the reduction of the water temperature in the infiltrating flow, °C; ξ_{p-f} is the depth of potential freezing of rocks in the zone of a talik at an average annual temperature on the surface of the soil of t_m; λ_f is the reduced thermal conductivity of the rocks in the layer of seasonal freezing, kcal/(m)(hr)(degree).

From equation (7.1.3) it is possible to determine the reduction of the temperature of water necessary for the formation of a zero average annual temperature in the base of the layer of seasonal freezing, and then it remains to determine the reduction of the water temperature as a result of heat exchange with the frozen rocks surrounding the talik zone. In accordance with what has been said the problem is calculated as follows:

1. We determine the heat flux necessary to increase the average annual temperature from -2 to 0°

$$q_1 = Q_ф (\xi_{п.пр} - \xi) + \frac{\lambda_{пр} |t_{ср}| T}{\xi};$$

$$q_1 = 17\,000 \cdot 0{,}8 + \frac{1 \cdot 2 \cdot 8760}{2} = 22\,360 \text{ ккал/м}^3.$$

2. We determine the reduction of temperature of infiltrating water (Δt_1) caused by the expenditure of heat on elevation of the temperature of the rocks to 0° from the left side of equation (7.1.3):

$$\Delta t_1 = \frac{22\,360}{2400 \cdot 1000 \cdot 0{,}02} = 0{,}5^{\circ}.$$

---

*Without taking into consideration that g $t_o = t_{\xi} = t_m$.

3. We find the amount of heat absorption by the frozen walls of the thawed zone (β, kcal/(m)(hr). To do that we construct a diagram (Figure 109) of the change of the value of β with time with the equation of Carslow and Yeager:

$$\beta = \frac{\lambda t}{r}\left\{(\pi T')^{-\frac{1}{2}} + \frac{1}{2} - \frac{1}{4}\left(\frac{T}{\pi}\right)^{\frac{1}{2}} + \frac{1}{8}T\right\}. \qquad (7.1.4)$$

where $t = t_m/2$ and $T' = \alpha\tau/r^2$ (τ is the time in hours).

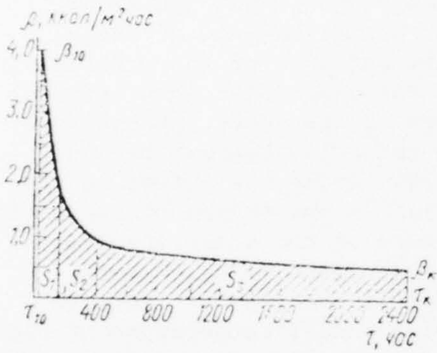

Figure 109. Diagram based on the Carslow and Yeager equation (7.1.4), where S_1, S_2 and S_3 are parts of the cross-sectional area of the talik.
a - β, kcal/(m^2)(hr) b - τ, hrs

4. We determine the amount of the heat flux through the lateral surface of the talik during the entire period of infiltration (q_2). To do that, with Figure 109 we calculate the area of the figure $\tau_{10}\beta_{10}\beta_k\tau_k$, equal to S mm^2 (1 mm^2 on the diagram corresponds to 1 kcal/m^2):

$$S_1 = 8 \cdot \frac{32+80}{2} = 448\ мм^2,\quad S_2 = 12 \cdot \frac{32+20}{2} = 312\ мм^2,$$

$$S_3 = 100\ \frac{20+10}{2} = 1500\ мм^2,\quad S = S_1 + S_2 + S_3 = 2260\ мм^2.$$

$$q_2 = S \cdot 1 = 2260\ ккал/м^2.$$

5. We find the reduction of temperature of the infiltrating water (Δt_2) from the formula of Kh. R. Khakimov (1962):

$$\Delta t_2 = \frac{q_2 S_б}{\tau_{нн} C_в v S}, \qquad (7.1.5)$$

where S_{lat} is the lateral surface of the talik, m^2; $S_{lat} = 2\pi rH$; S is the cross-sectional area of the talik, m^2; $S = \pi r^2$:

$$\Delta t_2 = \frac{2260 \cdot 2 \cdot 3{,}14 \cdot 10 \cdot 200}{2400 \cdot 1000 \cdot 0{,}02 \cdot 3{,}14 \cdot 100} = 1{,}9°.$$

6. We find the total reduction of the water temperature on the outlet of the thawed zone (Δt):

$$\Delta t = \Delta t_1 + \Delta t_2 = 0{,}5 + 1{,}9 = 2{,}4°.$$

Thus during the infiltration of a subsurface flow with given characteristics the existence of a talik is possible upon the condition that the average annual water temperature is not below 2.4°.

It is evident from the cited example that the warming influence of subsurface waters is a very complex process and quantitatively should be regarded complexly in a close connection with the radiation-thermal balance of the surface, the composition, the moisture content and the properties of the rocks, and also in connection with the latitudinal zonation and height zonation. It is generally known that the components of the thermal balance of the ground surface are essentially determined by the temperature of the underlying rocks. Because of that the warming influence of the convective heat exchange through ground waters, by changing the temperature of the rocks, at the same time changes the amount of the components of the radiation-thermal balance. In particular, E, P and B increase (see Chapter 2). Under the effect of the ground waters the heat cycles change in the soils. In the region of seasonal freezing, increase of the average annual temperature of the soils involves decrease of the thermal cycles of the soil. In the region in which permafrozen rocks are widespread the reverse dependence is observed -- increase of the thermal cycles with elevation of the average annual temperature of the rocks. In that case, when elevation of the temperature on account of convective heat exchange leads to change of the sign of the average annual temperature (negative values of ξ become positive) both increase and decrease of the annual heat cycles can occur. Thus the change of heat cycles in the soil under the effect of ground waters occurs in accordance with a complex law. In addition, the warming influence of the waters under consideration also leads to change of the temperature shift in the layer ξ. All that has been said testifies to a complex interaction of convective and conductive heat exchange in soils and rocks.

However, on sections on which ground waters are widespread, the influence of the latter does not always lead to the formation of taliks. At times that influence is manifested in the form of elevation of the temperatures of permafrozen rock masses within the range of negative values and increase of the depth of the seasonal thawing of rocks. In that case the taliks can form only during the superposition of an additional warming influence of other natural factors. Thus, for example, the warming influence of ground waters in combination with the influence of a thick snow cover can lead to the formation of both nonpermeating and permeating taliks. In such cases taliks will have a complex genesis, and this must be taken into consideration in considering the conditions of their formation.

/The subtype of taliks forming on account of ascending flows of subfrostal waters/. The reason for the existence of taliks of the given subtype, like the preceding, is the warming influence of subsurface waters on account of convective heat exchange. The difference between them consists in the fact that the temperature regime of pressurized artesian waters has its own laws of formation. The warming influence of those waters should be examined as a function of concrete characteristics of the hydrogeological structures.

The regions of feeding of artesian basins within the limits of permafrost represent ground-filtration and infiltration taliks. The infiltration of warm summer precipitations and surface waters leads to a relatively large elevation of the temperature of the rocks and talik zones as compared with the temperature of the surrounding masses. The area of propagation and the conditions of occurrence of taliks are linked with the propagation and occurrence of well-filtering water-bearing complexes and their outcrops on the surface. The warming influence of subsurface waters in the given case can be calculated with the procedure presented in examples 20 and 37.

The climatic conditions in the foci of discharge of subsurface waters in accordance with the long-period fluctuations of temperature on the surface of the ground also have an influence, reducing or enlarging the area of the talik around the sources. In separate periods on account of change of the hydrogeological and surface conditions the taliks can freeze from the surface and the subsurface waters with a deep circulation can be discharged at a new place. Connected with that is shift of taliks along and across valleys laid on faults on fractured zones and usually covered with thin alluvium. A procedure for calculating the influence of long-period fluctuations on the conditions of existence of taliks in foci of discharge is presented in the following example.

Calculation of the Influence of Long-Period Fluctuations of Temperature on the Surface on the Conditions of Existence of a Talik in Foci of Discharge of Subsurface Waters (Example 41)

It must be determined whether there is freezing of the focus of discharge of pressurized subsurface waters on account of periodic fluctuations of temperature on the surface with a period T = 10,000 years and an amplitude $A = 4^{\circ}$ at an average temperature of the surface of the ground $t_{o} = 0^{\circ}$ during the period.

The geological profile is composed (from the bottom upward) of dolomites and marls of the Cambrian with pressurized bed-karst fresh waters, covered by a water-resistant rock mass of clayey deposits of the Ordovician. The thickness of the water-resistant rock mass is 80 m. The angle of incidence of the fault along which the subsurface waters are discharged is 45°. The rate of filtration of the subsurface waters over the fractured zone is 0.005 m/hr and the average bed temperature $t = 2^{\circ}$.

The thermophysical properties of the loose formations are: the thermal conductivity $\lambda_{f} = 1.5$ kcal/(m)(hr)(degree), the temperature of phase transitions $Q_{\phi} = 10{,}000$ kcal/m^3 and $C_{vol-f} = 350$ kcal/(m^3)(degree). The geothermal gradient g = 0.01 degree/m. The maximal depth of the seasonal thawing of rocks during the period T is 2 m.

Solution. 1. We determine the effective value of the geothermal gradient (g_{eff}) with consideration of the convective component of the heat flux (Glusov, 1970):

$$g_{эф} = g - \frac{C_0 v t}{\lambda} \cdot \cos\alpha, \qquad (7.1.6)$$

$$g_{эф} = 0{,}01 + \frac{1000 \cdot 0{,}005 \cdot 2}{1{,}5} 0{,}71 = 0{,}95 \text{ град/м},$$

where C_0 is the heat capacity of water, 1000 kcal/(m^3)(degree); α is the angle between the direction of freezing and the flow of subsurface waters; $\alpha = 90 - 45 = 45^\circ$.

2. We determine with formula (4.2.1) the depth of permafrost at $g = g_{eff} = 0.95$ degree/m. Solving the equation by trial and error, we find that $\xi_{p-f} = 1.5$ m.

Since the depth of seasonal thawing under the conditions of the given region is 2 m, it is obvious that the 10,000-year fluctuations of temperature on the surface of the soil cannot lead to permafrost at the focus of discharge of pressurized subsurface waters.

The configuration of talik zones at the places of emergence of pressurized subfrostal waters on the surface is linked with the propagation and conditions of occurrence of well-filtering rocks, the temperature of the water and the intensity of the sources. Connected with the emergences of subfrostal waters of deep circulation are zones of the propagation of large and gigantic ice bodies -- ice sills. The latter often protect the underlying rocks against freezing and contribute to the preservation of thick talik zones.

Within the limits of the central part of artesian basins, on the path of movement of waters their warming influence is determined by the convective heat transfer and also by the thermal interaction of the subsurface waters and permafrozen rocks, which depends on the spatial interrelationship of the latter. Here one should distinguish the case where subfrostal waters contact the lower surface of permafrozen rock masses and the case where such contact is absent and the subsurface waters have been separated from the frozen rock mass by a lithological confining bed of different thickness. It is obvious that the warming influence of waters on frozen rocks will be maximal in the case of their contact. When pressurized subsurface waters are shallow in regions where there are thin frozen rock masses their warming influence can lead to the formation of permeating talik zones. In regions where there are thick frozen rock masses, in the central part of artesian basins taliks can form and exist only on zones of faults and tectonic dislocations.

/The subtype of taliks forming from stratal-fracture and fracture waters/. The warming influence of the indicated type of subsurface waters is determined by the character of their occurrence, propagation and the conditions of motion, discharge and feeding. The formation of permeating taliks and talik zones usually is confined to the southern regions of a permafrost area. In regions of thick permafrozen rock masses, permeating taliks can form and exist in the places of discharge and feeding of stratal-fracture and fracture waters concentrated in zones of tectonic dislocations. Nonpermeating taliks can form on the path of movement of those waters.

The procedure for calculating the age of a pressurized-filtration talik through the warming influence of stratal-fracture and fracture waters can be shown on the following example.

Calculation of the Minimal Age of a Pressurized-Filtration Talik (Example 42)

We will assume that a talik of the indicated subtype is confined to the fractured zone in dolomites of Devonian age. It is known that the talik radius (r) is 20 m; the thickness of the permafrozen rock mass (H) reaches 400 m and its average annual temperature (t_m) is -6°; the thermal conductivity (λ_f) and temperature conductivity (α) of the rocks are 2 kcal/(m)(hr)(degree) and 4 x 10^{-3} m^2/hr respectively; the filtration rate of the waters within the talik is 0.008 m/hr; the difference in the water temperature at the outlet into the thawed zone and at the base of the layer of annual heat cycles is 0.5°.

Solution. 1. We determine the change of the amount of heat absorption by the frozen talik walls (β, kcal/(m^2)(hr)) in time, using the Carslow and Yeager formula (7.1.4);

$$\beta = \frac{2\lambda T'}{r}\left\{\frac{1}{\ln(4T) - 1{,}154} - \frac{0{,}577}{[\ln(4T') - 1{,}154]^2}\right\}, \qquad (7.1.7)$$

where $T' = \alpha T/r^2$ for T equal to 1000, 5000 and 10,000 years. The calculated data are presented on the diagram (Figure 110).

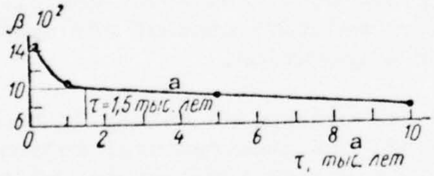

Figure 110. Diagram for determination of the minimal age of a talik. a - 1000 years

2. We find the value of β at the moment of observation with the formula of Kh. R. Khakimov (7.1.5):

$$\beta = \frac{C_в \cdot S}{S_б} \cdot v \cdot \Delta t, \qquad (7.1.8)$$

$$\beta = \frac{1000 \cdot 3{,}14 \cdot 400}{2 \cdot 3{,}14 \cdot 20 \cdot 400} \cdot 0{,}008 \cdot 0{,}5 = 0{,}1 \text{ ккал/м}^2 \text{ час}.$$

3. With the diagram (Figure 110) we determine the amount of T corresponding to the value β = 0.1 kcal/(m^2)(hr); T = 1500 years. That value also is the minimal age of the given talik.

3. Distinctive Features of the Propagation of Taliks With Consideration of the Latitudinal Zonation and Geostructural Conditions

Latitudinal zonation in the region of propagation of permafrozen rocks is expressed in a regular reduction of the average annual temperatures of the rocks, increase of the continuity of the propagation of frost over the area and increase in the thickness of permafrozen rock masses in the direction from south to north. In accordance with that a general regularity in the propagation of taliks in a permafrost region is expressed in a regular decrease of their area from south to north. Then, in connection with a gradual reduction of the average annual temperature of the rocks the propagation of taliks from south to north is linked with more and more strongly acting factors.

/The propagation of taliks of the radiation-thermal type/. Taliks of the radiation subtype (see Table 45), connected with a southern exposure of the terrain and a small value of the albedo of its surface, are encountered in all geostructural regions mainly in the first frost-temperature zone. A very substantial change of the radiation-thermal balance for slopes with a southern exposure is noted within the limits of mountain folded regions, where the steepness of slopes reaches 10-20^{o}. On sections of such slopes which are free of plant cover the albedo of the surface is relatively low (see Table 8). A large difference is noted on slopes with a northern and southern exposure in the moisture content of soils of the surface layer, which is reflected in the amount of evaporation. For the indicated reasons the components of the radiation-thermal balance on slopes with a southern exposure vary in such a way that this leads to a substantial increase of the average annual temperature of the soil and a wide distribution of taliks of the radiation subtype.

Within the limits of regions of the platform type on slopes with a southern exposure taliks of the radiation subtype also are noted, but far more rarely than in mountainous folded regions. This is connected with the smaller steepness of the slopes, the distribution of the plant cover and shade and also the poorer drainage of the soils, their greater moisture content and swampiness and, consequently, the greater amount of evaporation.

Taliks of the thermal subtype (see Table 45) are widespread mainly in the first two zones, more rarely in the third in all the geostructural regions. Taliks of the thermal subtype connected with a positive temperature shift from 1 to 2-3^{o} are widespread in platform and plains regions composed from the surface of a layer of Quaternary deposits of a sandy-loam-loam composition at a moisture content close to the absolute moisture capacity. In a mountainous folded region such taliks can be encountered only on swampy plateaus and highlands or in the bottoms of valleys and on accumulative river terraces composed of fine-grained material.

Taliks of the thermal subtype connected with the influence of the snow cover can be widely encountered within the limits of both platform and of mountainous folded regions. In platform regions their distribution is more restrained and more often connected with the micro- or mesorelief and the character of the vegetation. In a tundra zone such taliks are connected with the transport of snow which accumulates under the steep sections of leeward slopes.

In mountainous folded regions taliks of that subtype are confined also to leeward steep and gentle slopes, where snow accumulates in a layer of up to 2 m or more, and the forming snow slides often lead to a sharp warming of the underlying rocks. In mountainous folded regions the formation of thermal taliks is connected with the presence of perennial firn basins and glaciers. In narrow gorges and valleys when the area of firn basins and glaciers is relatively small, water flows form under the latter which exclude the freezing of the underlying rocks and are a reason for the formation of taliks of the thermal and often of the radiation-infiltration type.

Taliks of the thermal subtype often are encountered in all the geostructural regions on account of the formation of a looser snow cover and its not close adherence to the soil with the formation of cavities within the sections of underbrush and grassy vegetation which have grown during the summer. The taliks are mainly non-permeating and are characteristic mainly of leeward snow-drifted slopes. In the first two frost-temperature zones thermal taliks connected with a thick snow cover 0.7-1.0 meter high are encountered in all the geostructural regions.

Taliks of the radiation-infiltration subtype (see Table 45) are encountered within the limits of the first four zones and, as a rare exception, in the fifth. Taliks of that subtype are very widespread within the limits of old crystalline rock masses deprived of a thick mantle of loose Quaternary deposits. Often they are confined to flat water divides composed of fissured crystalline rocks covered by a layer of detrital eluvium. The presence of radiation-infiltration taliks in that case is linked with the warming influence of the ground waters and the infiltration of warm atmospheric precipitations (see examples 20 and 37). The latter is noted mainly is regions with large summer precipitations.

In mountainous folded regions such taliks are encountered on sections composed of intrusive and effusive bodies of fractured crystalline rocks, and also within the limits of the outcropping on the surface of solid and semisolid rocks of the Paleozoic and Mesozoic. In that case the taliks are linked with the warming influence of fissure and fissure-stratal waters. In the case where covering Quaternary deposits are represented by well-filtering rocks the formation of taliks of that subtype is linked also with the infiltration of warm atmospheric precipitations. In mountainous folded regions such taliks are often encountered in the regions of feeding of subsurface waters. Of great importance in that are regions of tectonic dislocations and zones of fracture.

Taliks of the radiation-infiltration subtype are encountered in platform regions within the limits of the mantle of Quaternary deposits composed of coarse, well-filtering formations. Often they are confined to river valleys and accumulative river terraces within the limits of mainly a riverbed facies.

/The propagation of taliks of the underwater-thermal type/. Taliks of this type are widespread in all geostructural regions and in all frost-temperature zones (see Table 45).

Taliks of the shelf subtype are encountered on the coasts of northern seas within the limits of propagation of warm sea currents, at a water temperature in the bottom layers of above 0^{o} (class -- anhydrous, ground-filtration and infiltration, subclass thermal, type permeating). Very often this subtype of talik is caused by the salinity of sea water at a water temperature below 0^{o} (class -- infiltration and pressurized-filtration, subclass -- cryohydrohalinic, type -- permeating).

Within the limits of a shelf, especially in its coastal part, where the depth of the sea does not exceed the first 10 meters, at an average annual temperature above 0° seasonal freezing of the bottom deposits is often encountered. This is connected with lowering the temperature of the saline water to -2° in the winter period. As a result of that the bottom deposits also acquire a negative temperature and often freeze at a depth of several tens of cm. In the warm period of the year that layer thaws. Usually on those areas of the shelf a talik of the underwater type, the shelf subtype, is observed.

At average annual temperatures above 0° such a talik can exist because of the high mineralization of the water in bottomset beds. In that case, in the warm period of the year, in the upper horizon of those deposits forms a layer with positive temperatures, corresponding to the layer of seasonal thawing. The absence of ice in the bottomset beds having a temperature below zero, when they have a corresponding composition, leads to the presence of a water-bearing horizon in the bottomset beds, which creates conditions favorable both for the feeding and for the discharge of the subsurface waters.

Under the conditions of regions of the platform type on seacoasts, within the limits of the coastal part of the shelf, the mass of bottomset beds is composed of loose moraine-like sediments representing confining strata or formations filtering extremely poorly. In that case the taliks do not have decisive importance in the feeding and discharge of subsurface waters. If the bottomset beds consist of well-filtering sediments of the type of near-mouth gravel-pebble-sand alluvium with a large thickness (for example, buried mouths of large rivers: the Lena, Yenisey, Ob', etc), such shelf taliks are usually classed as ground-filtration and pressurized filtration and, as a rule, are regions of discharge of fresh subsurface waters, and therefore have great hydrogeological importance.

Under the conditions of a mountainous folded region, when the coast and coastal part of the shelf are composed of pre-Quaternary solid rocks, the distribution of the shelf taliks, as a rule, is connected with the circulation of fissure and stratum-fissure waters. Such taliks can be regions of the discharge of subsurface waters for the coastal mass of rocks and regions of feeding during the infiltration of sea waters into the coastal mass of solid fractured rocks. In the latter case the taliks usually are cryohydrohalinic and the rocks composing them often have a relatively low temperature (below 0°) to a great depth. An example of this is the region of Amderma, where on account of the infiltration of saline sea water into lower-lying horizons the temperature of the masses of cooled rocks, equal to -5°, extends to 200-300 meters.

All coastal shelf taliks of the cryohydrohalinic subclass are cryopegs. Taliks of the shelf subtype at an average annual temperature of the rocks above 0° are encountered, as a rule, in the first three frost-temperature zones. At a temperature below zero taliks of the cryohalinic subclass (cryopegs) are confined mainly to frost-temperature zones IV and V.

Taliks of the sub-lake subtype are connected with the warming influence of the layer of water and therefore are determined by the depth and width of the body of water and the length of its existence. By virtue of that the character of the talik, and also the conditions of its formation and the length of its existence, are connected primarily with the genesis of lakes and the geological conditions of the region itself. Under the conditions of platform regions of a plains character, water divide and valley sub-lake taliks should be distinguished. In the first case there can be sub-lake taliks, confined to closed negative forms of the relief of glacier origin. The bottomset beds of those lakes can be represented as moraine, silty, non-filtering sediments and as sandy, rubble-gravel material. In the former case the taliks, as a rule, are thermal underwater taliks. Their hydrogeological importance is extremely insignificant. In the latter case the taliks, as a rule, are ground-filtration and infiltration taliks, often thermal, and have great hydrogeological importance in relation to both the feeding and discharge of subsurface waters. That subtype of taliks is azonal and can be encountered in any frost-temperature zone.

Watershed sub-lake taliks in regions of the platform type, especially on lake-alluvial watershed plains, can form on account of the thawing of syngenetic reopened-vein ices. In that case the existence of permeating taliks under lakes is connected with the dynamics of the development of those lakes. During rapid displacement of those lakes there can be non-permeating taliks and new formations of frozen rock masses on dried sections. By virtue of that, taliks are encountered not only under the lakes themselves but in the coastal mass in the form of interfrost talik zones. Such taliks can serve as temporary sources for the feeding of subsurface waters. As a rule, they are widespread in frost-temperature zones IV and V, where reopened-vein ices are widely encountered.

Widespread in river valleys and on river terraces are lakes of alluvial origin (ox-bow lakes) and thermokarst lakes resulting from the thawing of reopened-vein ices. The former are similar in their character to watershed glacial lakes and are characterized mainly by the character of the bottomset beds and their genesis. The latter also are very similar to the above-indicated thermokarst lakes. Valley sub-lake taliks are often regions of the feeding and discharge of subsurface waters of a complex of alluvial deposits composing river valleys. They can be connected with the regime of rivers and alluvial flows. Such taliks are of especially great hydrogeological importance in the first two frost-temperature zones. There the thermokarst lakes are often connected with the thawing, not of reopened-vein, but of epigenetic stratal and schlieren ices. In that case the lakes are characterized by muddy sediments and the sub-lake taliks have no substantial hydrogeological importance.

In mountainous folded regions, lakes are connected with negative forms of the relief and are characterized by the length of their existence. Their bottoms usually are composed either of well-filtering loose formations or of solid fractured rocks. Therefore they belong to the class of infiltrating taliks and are of great hydrogeological importance.

Sub-riverbed taliks are connected with the warming effect of riverbed waters and therefore are encountered only where there are sufficient depth and width of the channel (see section 5, Chapter 5). During the long existence of a riverbed in a single place, permeating taliks form. During frequent shift of the riverbed, non-permeating taliks form. In this way the occurrence and propagation of taliks in the bottom of a valley depends on the history of the development of the river. In regions of negative neotectonic movements, where there is a process of sedimentation and wide river valleys of large rivers are noted, shifting of the riverbed, the formation of meanders and reprocessing of the river alluvium are traced. Taliks of the sub-riverbed subtype in that case have a complex bedding, are rarely covered by new formations of permafrozen rock masses, are connected with alluvial flow and are regions of subaqueous feeding and discharge of subsurface waters. These taliks are azonal and are encountered in all five frost-temperature zones. The indicated sub-riverbed taliks usually are connected with large and medium-sized rivers mainly in regions of a platform type.

In mountainous folded regions, especially where deep erosion is noted in connection with a bulging up of the territory, accumulative deposits, as a rule, are absent and sub-riverbed taliks form in solid fractured rocks. The river valleys in that case often coincide with the zones of fractures, characterized by well-developed fissures and shatter zones which assure the infiltration of surface waters and ascending flows of subsurface waters. Such taliks are regions of feeding and discharge and are of great hydrogeological importance.

The propagation of taliks of the water-thermal type. In platform regions water-thermal taliks (see Table 45) are abundant in river valleys in sand-pebble deposits mainly within the limits of the first three frost-temperature zones. On watersheds such taliks are connected with the warming influence of pressurized subfrostal waters in zones of tectonic jointing.

Within the limits of the spread of crystalline rocks taliks of that type are connected with fissure and stratal-fissure waters. They are least abundant in the first three zones.

In mountainous folded regions water-thermal taliks are connected with all varieties of subsurface waters, both ground and pressurized stratal and stratal-fissure, and especially with ascending thermal waters. In each concrete case the class and subclass of a talik is determined by the concrete frost and hydrogeological situation.

In any frost-temperature zones and geostructural regions the taliks of the water-thermal type under consideration are connected with concrete heat exchange through the warming influence of subsurface waters. By virtue of that the regime of a talik is essentially determined by the thermal and hydrodynamic regime of the subsurface waters. Therefore taliks connected with the warming influence of subsurface waters undergo very great changes in the course of a year. Taliks connected with pressurized subfrostal stratal-fissure and stratal-pore waters are stable. Taliks connected with fissure and stratal-fissure waters have within the limits of crystalline masses a different character. When they are covered by a confining stratum they give rise to a regime of taliks similar to taliks of artesian basins. When they are connected with fissure and stratal-fissure waters not covered by a confining stratum, they are similar to taliks resulting from the warming influence of ground waters.

All that has been said testifies that during the examination of regularities in the formation of taliks the influence of very varied factors should be examined complexly, as a function of distinctive features of each landscape type of the locality. Hence one cannot give a single standard procedure for calculating the conditions of origination and existence of taliks. In each specific case it is necessary to study in detail the influence of the various factors and their interaction and on the basis of that compile a scheme of calculations after determining the type of talik and the principal factors participating in its formation.

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Chapter 8. Regularities in the Formation and Prediction of the Development of Geocryological Processes and Phenomena

Geocryological (cryogenic) phenomena are connected with distinctive features of the accumulation of Quaternary sediments, distinctive features of seasonal and perennial freezing and thawing of rocks, the change of their moisture and temperature regimes annually and perennially, and also the regime of surface and subsurface waters acting both round the year and seasonally. Therefore in analyzing the regularities in the development of geocryological (cryogenic) processes and phenomena it is necessary to use as a basis the above-considered regularities in the formation of seasonally frozen and permafrozen rock masses.

1. Regularities in the Development and the Prediction of Heaving of Soils

/Connection of the process of the heaving of soils with the main characteristics of natural conditions/. In general the heaving of soils depends on their composition and structure, aqueous properties, the moisture and temperature regimes and also the conditions of freezing. By the latter is understood the system in which the freezing proceeds (open or closed), the front of freezing and the rate, the velocity of freezing. In an open system the freezing of the ground is accompanied by an inflow of moisture toward the front of freezing from lower-lying thawed or unfrozen layers; in a closed system, freezing proceeds without an inflow of moisture from outside toward the front of freezing.

The process of heaving in sandy and loamy soils proceeds under quite different conditions. In an open system sands do not heave; in a closed, they heave if their moisture content is equal or close to the absolute moisture capacity. The amount of heaving of sands is determined by the thickness of the freezing layer of water-saturated sand. The dependence of the amount of heaving on the rate of freezing is determined by the influence of the rate of freezing on the depth of freezing.

In loamy soils during their freezing in an open system the heaving reaches a maximum under corresponding conditions. In a closed system, only loams with a small amount of swelling heave if their moisture content is greater than that of the threshold of heaving. The moisture content of the threshold of heaving is determined by the minimum of ice crystals forming on the front at the given rate of freezing which is necessary for disruption of the initial density of the skeleton of the ground. Therefore it is determined by the aqueous properties of the rocks and the temperature conditions on the surface of the soil. V. A. Kudryavtsev proposed the following formula for determining the moisture content of the threshold of heaving w_{t-h} (1971):

$$w_{\text{п.п}} = w_{\text{н}} + n\,(w_{\text{кр}} - w_{\text{н}}), \qquad (8.1.1)$$

where $w_{cr} = 0.91\ w_a$; w_a is the absolute water capacity; n is a coefficient which depends on the continental character of the climate. The values of n are presented in Table 65 and are selected as a function of the value of the amplitude of fluctuation of temperatures on the surface of the ground A_o.

Table 65 Values of the coefficient n in formula (8.1.1)

A_0, °C	<14	14–19	>19
n	0,5	0,6	0,7

The moisture content of the threshold of heaving differs greatly from the absolute moisture capacity of the ground, and the difference between them is all the greater the lower the rate of freezing. The dependence of the moisture content of the threshold of heaving on the rate of freezing has the result that loamy soils with different moisture contents can have an identical relative amount of heaving if their rates of freezing are different. Therefore under the conditions of a maritime climate, at a relatively low moisture content of the ground, the relative amount of heaving reaches large values. Under continental conditions such an amount of heaving is observed when there is considerably greater moisture (soils of identical composition are considered).

A characteristic regularity of the process of heaving of loamy soils also is the fact that, other conditions being equal, the amount of heaving depends on the rate of freezing, determined by the average annual temperature t_o and the amplitude of annual fluctuations of temperature A_o on the surface of the soil. The influence of different natural factors on heaving can be estimated by determining their influence on t_o, A_o and w. Maximal heaving is noted at values of t_o close to 0^o, and at maximal depths of seasonal freezing (thawing). In that case the heat cycles in the soils reach maximal values but the rates of freezing still remain low. At average annual temperature above and below zero the amount of heaving decreases, other conditions being equal.

During change of the continental character of the climate the following regularity is noted in heaving. With increase of A_o, that is, with increase of the continental character, the heat cycles and depth of ξ_t and ξ_f of the rocks increase. Simultaneously with that the rate of freezing increases. Therefore in loamy soils, in which the heaving is inversely proportional to the rate of freezing, during increase of A_o the heaving decreases in spite of increase of ξ_f. In this the snow cover is of great importance. Increase of the height of the snow leads to increase of t_o and decrease of A_o (on the surface of the soil), decrease of the depths of seasonal freezing of the soils and increase of the depth of their seasonal thawing (see section 3, Chapter 5). In the case of seasonal freezing the rate of increase of its depth in the winter period under snow will be considerably smaller than on snowless sections. This leads to the possibility of a greater inflow of moisture toward the front of freezing on sections bare of snow, toward the formation of layers of ice before heaving of the soil. In the presence of a water-bearing horizon lying in the range of 1 or 2 meters below the base of the layer of seasonal freezing, the inflow of moisture toward the front increases considerably and the heaving can attain a greater value. Irregularity of freezing, often linked with a height of the snow cover irregular over the area, leads to the formation of hummocks of heaving.

In examining the influence of the snow cover on the heaving of ground it is necessary to take into consideration that increase of the snow height leads not only to decrease of the rates of freezing but also to a reduction of the annual heat cycles in the soils. The latter is expressed in a certain reduction of the depth of freezing and, consequently, a reduction in the total amount of heaving in comparison with sections with a small snow cover.

In a region of seasonal thawing the winter freezing of dispersed soils is often accompanied by heaving, as thanks to the presence of a frozen confining bed their natural moisture content usually exceeds that of the threshold of heaving and approaches the absolute moisture capacity. During freezing of the layer of summer thawing the regime of the waters above the frost changes substantially. During irregular freezing on separate sections the layer of waters above the frost can freeze completely, on account of which downward along the movement of the flow from the place of freezing the intensity of the heaving processes diminishes sharply or they are completely absent.

In sandy soils in the layer of seasonal thawing, during the freezing of waters above the frost ice lenses form through the inflow of water along the slope and their head during irregular freezing. Those lenses form underground ice coatings, which are examined in the second part of this section.

It is obvious that with decrease of the thickness of the layer of summer thawing the amount of heaving of rocks decreases. Therefore in frost-temperature zones IV and V the amount of heaving is extremely small even in water-saturated soils.

The influence of the plant cover on heaving should be examined in connection with the influence of that factor on the temperature regime of the rocks (see section 4 of Chapter 5) and their moisture content. As indicated above, the plant cover leads to reduction of the amplitudes of the annual fluctuations of temperature on the surface of the ground and to reduction of the depth and rates of seasonal freezing and thawing. As a result of that influence, more favorable conditions are created for the development of the process of heaving, which is also intensified because the plant cover protects the soil against desiccation. The moisture content of rocks below a plant layer, as a rule, is higher than on sections bare of vegetation. Similarly to the snow and plant covers, all the remaining factors of the natural environment (the radiation-thermal balance, steepness and exposure of the slopes, etc) have an influence on the development of the process of heaving.

It follows from all that has been said that in a frost survey within the limits of the distinguished microregions, homogeneous in geological and genetic complexes and types of deposits, it is necessary to distinguish sections, each of which is characterized by homogeneity of the composition of the rocks in the layer of seasonal thawing and freezing (including the layer of seasonal thawing and freezing under a pipeline, road or other communications), definite intervals of change of their aqueous properties (w, w_{un}, w_{t-h} and pre-winter natural moisture content, the temperature regime on the surface of the soil and definite regimes and conditions of occurrence of ground waters. With formula (8.1.1) we determine the moisture content of the threshold of heaving (w_{t-h}) and in a comparison with the natural (pre-winter) moisture content of the soils and with consideration of the depth of occurrence of the ground waters, sections of the development of heaving ($w > w_{t-h}$) and non-heaving ($w < w_{t-h}$) soils are distinguished (the system of freezing also is taken into account in that case). The value of w_{t-h} is calculated on the basis of the average annual values of t_o and A_o. For grounds classed as non-heaving it is necessary to make an additional calculation for the climatically most unfavorable years -- a warm winter, a minimal amplitude of the air temperatures and a large snow cover.

When a territory is opened up there are changes in the natural situation which involve changes of the temperature and moisture regimes of deposits. Therefore in forecasting the calculations of the heaving of soils must be made with consideration of the variable temperature regime and moisture content. On a map and profiles one must note the sections of development of heaving soils, sections on which the heaving of soils is possible in separate unfavorable years, and sections on which after development of the territory the soils will go over into the category of heaving soils.

The amount of heaving should be calculated with the method presented in section 12, Chapter 3. The starting parameters must consider the influence of all natural factors in their interaction. A list of those parameters and conditions includes: the aqueous properties of the soils -- the initial moisture content over the profile, the absolute moisture capacity, the limits of plasticity, the content of unfrozen water in the range of temperatures, the coefficient of thermal and moisture conductivity, the temperature regime of the deposits, the coefficient of thermal conductivity and the specific heat of the ground in the thawed and frozen states and of its skeleton, the depth of the seasonal thawing, the possibilty of disconnecting the layer of winter freezing from the surface of permafrozen rocks, the depth of occurrence of the water-bearing horizon, the time of its existence and the rate of its exhaustion. Some examples of the calculations are presented in section 3, Chapter 9. In addition, the probability of development and an approximate estimate of the amount of ground heaving can be determined with the following calculating procedure.

Determination of the Probability of Development of Heaving and Its Possible Amount (Example 43)

For example, in the process of a frost survey the following data were obtained. The investigated section with a high flood plain is composed of alluvial loams

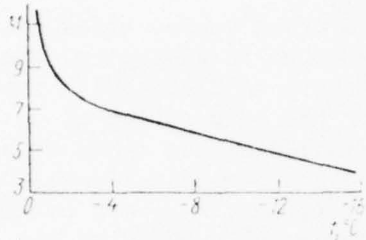

Figure 111. Diagram of w_{un} as a function of temperature for loam.

(alQ_{IV}) with a density of 1.1 g/cm^3, the natural moisture content of which in the layer of seasonal freezing in the autumn (before the start of freezing) varies from 18 to 35%; ground waters are absent. The temperature regime on the surface of the soil is characterized by: $t_o = 0.5^o$ and $A_o = 14^o$, and the average temperature on the surface of the ground during the winter period (t_{o-wtr}) is -8^o. The data of laboratory determinations of the properties of loam are as follows: w_{un} = 39% and w_b = 20% (w_b is the moisture content on the boundary of rolling); at w = 20-25%, $\lambda_f = 1.2$ and at w = 35%, λ_f = 1.3 kcal/(m)(degree)(hr); the amount of unfrozen water in loam as a function of the negative temperature varies according to the diagram (Figure 111); C_t = 0.18 kcal/(m^3)(degree). It is required to determine the sections with the greatest probability of development of ground heaving and calculate the amount of heaving.

Solution. 1. To contour the sections in which the process of heaving develops during freezing we calculate the moisture content of the threshold of heaving of loam (w_{t-h}) with formula (8.1.1) in accordance with the rate of freezing characteristic of the given temperature regime of the surface of the soil. According to Table 65 the coefficient n is equal to 0.5; at $t_{o-wtr} = -8^o$, w_{un} = 6.0%

$$w_{кр} = 0{,}91 \cdot w_{н} = 0{,}91 \cdot 39 = 35{,}5\%;$$
$$w_{н.п} = 6 + 0{,}5\,(35{,}5 - 6) \approx 21\%.$$

Consequently, within the limits of the investigated section under natural conditions heaving will occur everywhere where the moisture content of the loam exceeds 21%.

2. To distinguish within the section areas with a different degree of manifestation of heaving we calculate its value in loams with moisture contents of 25 and 35%. To do that we use the formula of V. O. Orlov (1970):

$$h_{п} = \frac{\gamma_{ск} \cdot h}{\gamma_{в}} \left(0{,}09A_1 + \eta BC \sqrt{\frac{A_1}{A_2 + \eta C}}\right); \qquad (8.1.2)$$

$$A_1 = w - [K_{н}(t_{в})]\, w_{p}; \; A_2 = w - [K_{н}(t_{э})]\, w_{p}; \qquad (8.1.3)$$

$$B = 1{,}09 \sqrt{\frac{t_{э}}{t_0}}; \qquad (8.1.4)$$

$$C = \frac{(w - w_{p})^2}{w_{p}}, \qquad (8.1.5)$$

where γ_{sk} is the specific gravity of the skeleton of the thawed ground (g//cm^3); γ_w is the specific gravity of water (g/cc); h is the depth of the seasonal freezing of the ground (cm); w is the autumn moisture content of the thawed ground within the limits of the freezing layer, in fractions of unity (with consideration of variation in the process of use); w_b is the moisture content on the boundary of rolling (in fractions of unity); $K_c(t_{air})$ and $K_c(t_g)$ are correction factors taken in accordance with Table 66 as a function of the average winter air temperature (t_{air}) (equated to the temperature of freezing of the ground) and the temperature (t_g) at which the movement of moisture in the ground ceases; η is a correction factor taken in accordance with Table 66 as a function of the type of ground and t_g; w_{pl} is the plasticity number.

3. We determine the depth of the seasonal freezing of loam with moisture contents of 25 and 35% at $t_\xi = t_o = 0.5^o$; $A_o = 14^o$.

a. For loam with w = 25% in accordance with (4.1.8) and (4.1.6) Q_ϕ = 16,720 kcal/m^3; C_f = 370 kcal/(m)(degree), λ_f = 1.2 kcal/(m)(degree)(hr). According to the nomograms (Figures 17 and 19) we find that $\xi_{w=25\%} = 2.1\sqrt{1.2} = 2.4$ m.

b. For loam with w = 35% correspondingly Q_ϕ = 25,520 kcal/m^3; C_f = 424 kcal/(m)(degree) and λ_f = 1.3 kcal/(m)(degree)(hr). With the same nomograms we find that $\xi_{w=35\%} = 1.7\sqrt{1.3} \cong 2.0$ m.

4. We find the amount of heaving during freezing of loam with moisture contents of 25 and 35%. In that case we assume that for the given ground t_g = $= -2.5^o$; from the condition of the problem $t_o = -8^o$. From Table 66 we find that η = 5; with (8.1.4) we find that

$$B = 1{,}09\sqrt{\frac{2{,}5}{8}} = 0{,}61.$$

a. For loam with w = 25% we find with (8.1.5) and (8.1.3)

$$C = \frac{(25-20)^2}{20} = 1{,}25,\quad A_1 = 25 - 0{,}45\cdot 20 = 16,$$

$$A_2 = 25 - 0{,}57\cdot 20 = 13{,}6.$$

In accordance with the obtained initial data with (8.1.2) we find that

$$h_n = \frac{1{,}1\cdot 2{,}4}{1{,}0}\left(0{,}09\cdot 16 + 5\cdot 0{,}61\cdot 1{,}25\sqrt{\frac{16}{13{,}6+5\cdot 1{,}25}}\right) = 21{,}6 \text{ см}.$$

b. For loam with w = 35% we find accordingly that

$$C = \frac{(35-20)^2}{20} = 11{,}2;\ A_1 = 35 - 0{,}45\cdot 20 = 26 \text{ и } A_2 = 35 - 0{,}57\cdot 20 = 23{,}6;$$

$$h_n = \frac{1{,}1\cdot 2{,}0}{1{,}0}\left(0{,}09\cdot 26 + 5\cdot 0{,}61\cdot 11{,}2\sqrt{\frac{26}{23{,}6+5\cdot 11{,}2}}\right) = 52{,}3 \text{ см}.$$

As so we obtain the answer that the amount of heaving of loams with a moisture content of 25% at the given temperature regime on the surface and depth of seasonal freezing (2.4 meters) is 21.6 meters, and the heaving of loams with

Table 66 Values of indices and coefficients in formula (8.1.2) according to V. O. Orlov (1970)

Наименование грунта	Число пластичности $w_{пл}$	Температура $t_з$, °C	Значение η	Значение $K_н(t_в)$ и $K_н(t_з)$					
				−0,3	−0,5	−1	−2	−4	−10
Супесь легкая Супесь пылеватая	$2 < w_{пл} \leq 7$	−1,5— −2,0	3,55— 5,00	0,6	0,5	0,4	0,35	0,3	0,25
Суглинок легкий Суглинок легкий пылеватый	$7 < w_{пл} < 13$	−2,0— −2,5	4,25 5,00	0,7	0,65	0,6	0,5	0,45	0,4
Суглинок тяжелый Суглинок тяжелый пылеватый	$13 < w_{пл} \leq 17$	−2,5— −3,0	3,80 5,35	—	0,75	0,65	0,55	0,5	0,45
Глина	$w_{пл} > 17$	−4,0	2,5	—	0,95	0,9	0,65	0,6	0,55

Key: A - Type of ground B - Plasticity number w_{pl} C - Temperature t_g, °C
D - Value of E - Values of $K_c(t_{air})$ and $K_c(t_g)$

1 - Light sandy loam 2 - Silty sandy loam 3 - Light loam
4 - Light silty loam 5 - Heavy loam 6 - Heavy silty loam
7 - Clay

Note 1. For intermediate values of the temperature the value of the indicators is obtained by interpolation.

Note 2. The value of the coefficients K_c is determined at temperatures of 0.5 t_g and 0.5 t_o.

a moisture content of 35% at the same temperature regime and a corresponding depth of freezing (2 m) reaches 52.3 cm. The relative amount of heaving in the second case is larger than in the first by 30.7 cm.

/Regularities of the formation and predicting the development of hummocks of heaving/. Under the natural conditions of the region of propagation of permafrozen rocks and deep seasonal freezing the processes of heaving can lead to the formation of hummocks of heaving.

The formation of hummocks of heaving involves processes of water migration during freezing and the accumulation of ice. The local accumulation of ice can occur: 1) by the formation and accumulation of segregation ice as a result of moisture migration under the influence of the temperature gradient and moisture content and 2) by the movement of water under the effect of the hydrostatic pressure developing in closed systems during their freezing. Thus form hummocks on the basis of the accumulation of injection ice.

We will examine first, on the basis of the first of those processes, the formation of peat hummocks which are widespread in a region of permafrost.

It is known that peat and moss contain a large amount of moisture and so the thermal conductivity of frozen peat is considerably greater than that of thawed peat and it is more strongly cooled in winter than it is warmed in summer. As a result of that, in it forms a temperature shift which often reaches 2° or more. A large amount of solar radiation on swampy sections is expended on the evaporation of moisture from the surface of moss and peat, which also leads to a reduction of the temperature of the latter as compared with sections composed from the surface of mineral soil. What has been said above is shown schematically on Figure 112.

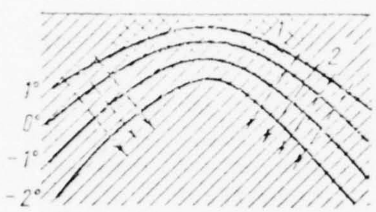

Figure 112. Course of the isotherms and the direction of water migration in the summer under the area of moist peat or moss (1) and on loamy soil (2).

Moisture present in the ground will as a result of thermal diffusion migrate in the direction of the heat flux and accumulate mainly in a frozen rock mass extended above the surface, determined by the isotherm 0°.

If a hummock or system of hummocks has heaved slightly, then in subsequent years their further growth is facilitated and intensified because the snow is blown off them and accumulates in the lows between the hummocks. This assures more intensive cooling of the apices of the hummocks and weaker cooling of the lows between them. In the summer the lows are warmed more strongly than the apices, since water which accumulates substantial amounts of heat is accumulated in them. As a result of that the isotherms under the hummocks bend more sharply, the migration of moisture under them is intensified and their growth is accelerated. The course of the isotherms and heat fluxes under a system of hummocks of migration heaving is depicted on Figure 113.

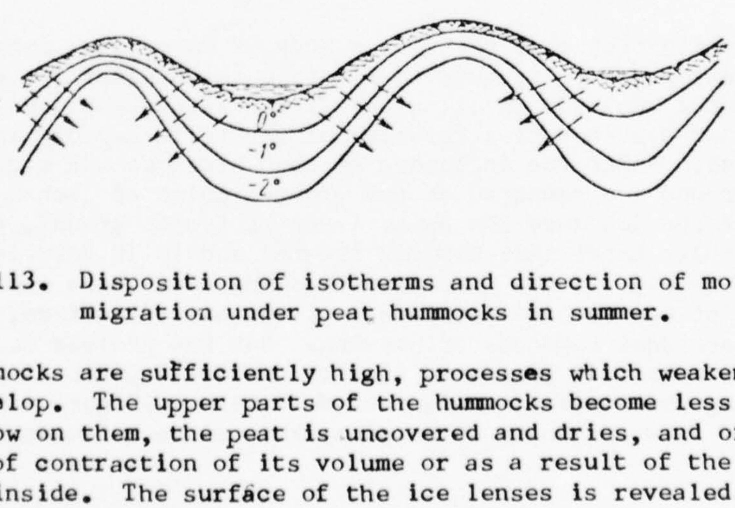

Figure 113. Disposition of isotherms and direction of moisture migration under peat hummocks in summer.

When the hummocks are sufficiently high, processes which weaken their growth start to develop. The upper parts of the hummocks become less moist, moss ceases to grow on them, the peat is uncovered and dries, and often cracks as a result of contraction of its volume or as a result of the bulging out by ice from inside. The surface of the ice lenses is revealed, they thaw out and the hummock subsides. At times that bulging out of the rock by a forming lens of ice attains great force and can lead to destructive consequences.

Hummocks of migration heaving usually achieve a height of 1.5-2 meters and often go beyond the range of 4 meters. Their diameters are very varied. On peat bogs complex systems of hummocks form, separated by channels. Such formations usually are called hummocky peat bogs.

According to the observations of A. I. Popov in Western Siberia (Popov, 1957) the natural moisture content of loam under a peat bog reached 35-80%, whereas beyond its limits on the periphery it did not exceed 15-20%. This fact confirms the presence of the above-described process of moisture migration under peat bogs. Peat bogs with hummocks of migration heaving are encountered more often in high-temperature peripheral zones of regions of permafrost south of the zone of underground vein ices. They are very greatly developed in the European North of the USSR and in Western Siberia, and less developed in the eastern regions of Siberia. North of that zone the development of hummocky peat bogs is complicated by the formation in them of system of vein ices and accompanying processes. The formation of migration hummocks of heaving often is observed also in mineral soils. Usually they are confined to the periphery of the bands of runoff and the marginal part of lake basins and swamps. The main conditions of their formation are irregularity of freezing and the presence of thawed masses of water-saturated rocks, from which moisture migration toward the front of freezing proceeds. It is precisely for this reason that mineral hummocks of heaving develop on the marginal part of taliks, on sections with nonconverging frost.

The second type of hummocks of heaving consists of hummocks forming as a result of the accumulation of injection ice under the conditions of freezing of large closed systems. The mechanism of formation of such hummocks, called "bulgunnyakhi" in the USSR (Yakutiya) and hydrolaccoliths (at the suggestion of L'vov, 1916 and Tolstikhin, 1932), and in America "pingo", can be described as follows.

If above a permafrozen rock mass there is a body of water with a relatively small basin of thawing under it, then when such a talik freezes a mass of thawed soil and water enclosed on all sides (a closed system) usually forms. The pressure in that system during freezing of the talik rapidly increases to very high values. Under the influence of that pressure the water and the water-saturated ground are squeezed at the weakest point of such a system, where they elevate the ice into the upper layer of frozen ground, forming a hummock (Figure 114). Later that hummock freezes and in it form lenses or layers of injection ice. If the process of heaving ends there and in summer there are thawing of the ice and subsidence of the heaved surface, such hummocks are called seasonal hummocks of heaving. But the process can recur in the course of a number of years and lead to the development of perennial hummocks of heaving which reach a height of 8-12 meters (Figure 115). The maximal height and rate of growth of bulgunnyakhi can be calculated as follows.

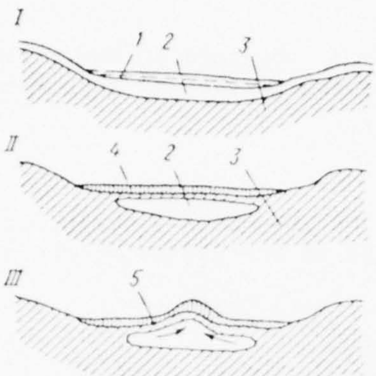

Figure 114. Schematic diagram of the formation of bulgunnyakhi (according to B. N. Dostovalov, 1967): 1 - water; 2 - thawed ground; 3 - frozen rock mass; 4 - ice; 5 - thawed and frozen ground which formed a bulgunnyakh, squeezed upward. I - initial stage, summer thawing; II - freezing of water and ground on the bottom and formation of a closed system; III - increase of pressure in the system during freezing and squeezing out of thawed and frozen ground upward at a weak point, leading to the formation of a bulgunnyakh.

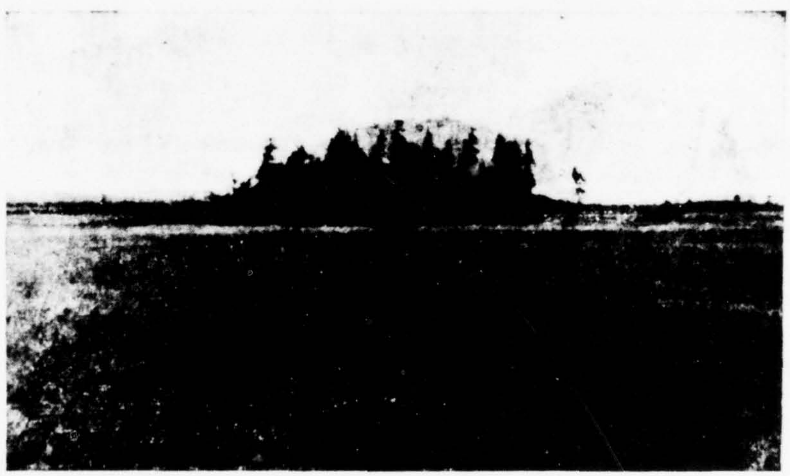

Figure 115. Hummock of heaving.

Determination of the Dimensions of a Perennial Hummock of Heaving and the Dynamics of Its Growth (Example 44)

In the investigated region, in connection with the drying of thermokarst lakes and the new formation of permafrozen rocks, it is necessary to determine the possible maximal volume of a perennial hummock of heaving and the dynamics of its growth. The frost conditions are characterized by the following data: $t_{air} = -11^{o}$; $A_{air} = 23^{o}$; $z_{sn} = 0.4$ m; $\rho_{sn} = 0.22$ g/cc. The deposits within the limite of the sub-lake talik consist of a water-saturated sandy loam with $\lambda_{sk} = 1450$ kg/cm^3, w = 30%, the porosity n = 0.46; $\lambda_f = 1.3$ kcal/(m)(degree)(hr); $Q_{\phi} = 35,000$ kcal/m^3. The area of the sub-lake talik is 1500 m^2 and the depth of the basin of thawing at the moment of drying up of the lake and the start of freezing of the rocks attains 20 meters.

Solution. 1. We calculate the volume of water (V_w) contained in the talik. Since the volume of the ground in the basin of thawing is 1500 m^2 x 20 m = = 30,000 m^3 and the porosity is 0.46, then

$$V_в = 30\,000 \cdot 0{,}46 = 13\,800\ м^3.$$

2. We calculate the total increase of volume of the ground (ΔV) during the complete freezing of a talik on account of increase of the volume of the water which has become ice:

$$V_л = 13\,800 \cdot 1{,}09 = 15\,042\ м^3,$$
$$\Delta V_л = V_л - V_в = 15\,042 - 13\,800 = 1242\ м^3.$$

3. It has been established by observations that during the freezing of a talik, side by side with the formation of a hummock of heaving there is a swelling of the surface (hydrothermal movement) rather uniform in area. The height of the uplift of the surface is 7 cm on the average. We calculate the increase in volume of the ground corresponding to it:

$$V_{поз} = 1500 \cdot 0{,}07 = 105\ м^3.$$

4. We find the maximally possible volume of the hummock (V) during complete freezing of tocks in the basin of thawing:

$$V = \Delta V_л - V_{поз} = 1242 - 105 = 1137\ м^3.$$

If the area of the base is 120 m^2 then, obviously, the height of the hummock of heaving will be at least 9.4 meters.

5. For an approximate estimation of the dynamics of growth of a perennial hummock of heaving it is possible to use the approximate Stefan formula (3.7.7). Since in accordance with that formula the depth of freezing is proportional to $\sqrt{\tau}$, then, consequently, the increase of volume of the frozen ground will also be proportional to $\sqrt{\tau}$, and, correspondingly, the growth of the hummock will follow that regularity.

We find the depth of freezing of the talik after 10, 50 and 100 years. The average annual temperature on the surface of the deposits in the period of freezing is: $\Delta t_{sn} = 23 \times 0.213 = 5^o$, $t_o = -11 + 5 = -6^o$. Then in accordance with (3.7.7):

$$H_{\tau=10} = \sqrt{\frac{1{,}3 \cdot 6 \cdot 10 \cdot 8760}{35\,000}} = 4{,}4\ м;$$
$$H_{\tau=50} = 4{,}4 \cdot \sqrt{5} = 9{,}9\ м;$$
$$H_{\tau=100} = 4{,}4 \cdot \sqrt{10} = 13{,}9\ м.$$

In accordance with the depth of freezing the increase of volume of the ground will occur; in the first 10 years the increase will be:

$$V_п = 1500 \cdot 4{,}4 \cdot 0{,}46 = 2640\ м^3;$$
$$\Delta V = 2640 \cdot 0{,}09 = 237{,}6\ м^3.$$

If of that volume about 7% of the total increase of volume will be used for swelling of the surface over the entire area of the talik, the following volume will be required for growth of the hummock:

$$V = 237{,}6 - (0{,}07 \cdot 237{,}6) = 221\ м^3.$$

At an area of the base of the hummock of 120 m^2, in 10 years its height will reach 1.8 meters. In 50 years it will grow to be 2.25 times as large, and in 100 years 3.15 times.

The proposed method of estimating the dynamics of growth of a perennial hummock is approximate, as in the calculations substantial assumptions are made, in particular, moisture migration toward the front of freezing is not taken into consideration. However, that method can be used in a frost survey for an approximate prediction.

Bulgunnyakhi are widespread in Central Yakutiya, in "alasy" -- depressions which form during the thawing of thick reopened-vein ices. They are also widespread in the northern part of the European part of the USSR and Western Siberia, in the Northern Urals, Zabaykal'ye, the northeastern part of the USSR and North America.

Injection one-year hummocks of heaving are formed by waters on top of frost. A necessary condition of their formation is irregularity of the freezing of a water-bearing seasonally thawing layer, as a result of which in the forming closed space the waters on top of the frost create a head during further freezing of the layer. As a result of the introduction of water under pressure hummocks form with an ice lens consisting of underground ice. The possibility of the formation and the amount of the underground ice are determined, besides the indicated reasons, by the composition of the soils in the layer of seasonal freezing and thawing, their filtration properties, and the regime and dynamics of the waters on top of the frost.

Heaving which appears sometimes on roads, air strips and construction sites represents the formation for the most part of seasonal hummocks of migration and injection origin. They are observed both in the zone of permafrost and in regions of deep seasonal freezing. Preventing and combatting them are important problems of engineering geocryology.

/Processes of heaving in different frost-temperature zones and geostructural regions/. For latitudinal zones the maximal depths of seasonal freezing and thawing are noted near the southern boundary of permafrost. South and north of it those depths diminish, and the thermal cycles in the layer of seasonal freezing and thawing behave in a corresponding manner. The minimal rates of freezing (thawing) are noted near the southern boundary of the permafrost region. South and north of it those rates increase sharply. All this testifies to more favorable conditions for the development of heaving, all other conditions being equal, near the southern boundary of permafrost in both the region of seasonal freezing and the region of seasonal thawing.

In transitional and semi-transitional types of seasonal freezing and thawing (t_{ξ} from 0 to $\pm 2^{\circ}$) the maximal amounts of heaving are noted, which reach 10-20 and even 50 cm. A characteristic feature under those conditions is the development of processes of heaving during the entire winter. Often the intensity of heaving does not only not decrease but it even increases in the spring. The maximal total amount of heaving, as a rule, is confined to the upper third of the layer of seasonal freezing (thawing). Within the limits of the first two frost-temperature zones, sharp changes of the air temperature and especially warmings are of very great importance. The latter lead to substantial decrease of the temperature gradients in the layer of seasonal freezing and to a sharp decrease of the freezing rates. As a result, favorable conditions are created for the formation of ice schlieren and the growth of heaving.

It is obvious that during seasonal freezing that effect is intensified from north to south and weakens within the limits of permafrost from south to north. Within the limits of frost-temperature zones IV and V it is almost completely excluded. Within the limits of all frost-temperature zones of a different kind of cover (plant and snow), as a rule, they contribute to the formation of schlieren ices during the freezing of the ground and the development of heaving. This factor is of especially great importance within the limits of spread of transitional and semi-transitional types of seasonal freezing and thawing of rocks.

The formation of ice schlieren and the differentiation of moisture in the layer of seasonal freezing and thawing lead to the structurizing of the soils and their dispersion.

A substantial difference in the heaving of soils is noted in different climatic zones. The maximal rates of freezing are characteristic of regions with a sharply continental climate, and minimal for maritime coasts, where the continental character of the climate is minimal. Because of this, in spite of the deeper freezing in regions with a sharply continental climate and small depths in regions with a maritime climate, the amount of heaving has a reverse dependence in relation to depth. This is connected with the more frequent recurrence of warmings and thawings under the conditions of a maritime climate than of a sharply continental.

Of great importance in the formation of heavings is the composition of the soils and their moisture regime. With respect to the first factor it should be said that it mainly is azonal and connected with distinctive features of the geological and genetic complexes and types of rocks. Within the range of spread of thick rock masses of Cenozoic deposits, consisting of coarse-grained material (gravel-pebble deposits and sands of different particle sizes) of alluvial, lake-alluvial, fluvoglacial and marine genesis heavings, as a rule, are absent. Within the limits of development of finely dispersed rocks of alluvial, deluvial, eluvial and glacier genesis heavings can be encountered everywhere in the presence of a corresponding moisture regime.

Side by side with geological and genetic features, the composition of rocks also has a certain dependence on latitudinal zonation. It is generally known that silty soils are specific for a permafrost region and also are prevalent in northern regions of the region of seasonal freezing. This is connected with distinctive features of processes of weathering during multiple freezing and thawing of soils. Silty soils, as a rule, are characterized by a large coefficient of thermal and moisture conductivity and therefore, all other conditions being equal, have more heaving. In relation to the moisture regime it should be said that strongly moistened soils are very widespread on the plains territories of platform regions. Therefore here the process of heaving can be spread over great areas. Within the limits of rugged terrain in mountainous folded regions the higher moisture content of rocks is encountered locally where drainage of the soils is difficult. Within the limits of the spread of old crystalline masses where, as a rule, coarse-grained eluvium and deluvium is developed, heaving is almost not encountered at all. On sections covered with finely dispersed Quaternary deposits of glacial or any other genesis, the development of heaving will be determined by the moisture regime of those deposits.

2. Regularities in the Formation and Prediction of the Development of Ice Bodies

Ice bodies is the name usually given to horizontal layers of ice formed during the freezing of water which has flowed out under pressure on the surface of river ice and the adjacent part of a valley, as a result of freezing of the riverbed, and also the introduction under pressure and the freezing of water between layers of rock (underground ice bodies). Ice bodies form both in the region of development of permafrost and beyond its limits under the conditions of a continental climate. Ice bodies are characterized by different form and dimensions.

On the basis of genesis ice bodies are distinguished which are formed:

a) from the surface waters of rivers, brooks and lakes;

b) from underground waters -- waters of the layer of seasonal thawing and the ground waters of thawed zones, the waters of alluvial currents under channels (permeating and non-permeating taliks) and waters of deep subfrostal and interfrostal circulation;

c) from mixed feeding sources (surface and subsurface waters). Such ice bodies are very widespread.

Ice bodies of surface waters form as a result of winter freezing of the cross section of the current and, in connection with that, increase of the hydrodynamic pressure of the water, which leads to disruption of the continuity of the ice cover and the outflow of water on the surface of the ice.

Ice bodies of subsurface waters are formed: 1) at the places of emergence of steady subaerial sources; 2) at the places of emergence of steady subaqueous sources. Above and below the sources often there are spaces of open water which do not freeze during the entire or a large part of the winter; 3) during the compression of the cross section of the flow of ground waters (including the sub-channel) as a result of the winter freezing of rocks, the acquisition of pressure by the waters, their bursting through the seasonally frozen layer and their flow out on the surface. The compression of the flow of ground waters can be caused by either natural factors or the intervention of man.

On the basis of position on the relief ice bodies are subdivided into watershed, permeating the base of slopes, terrace, detrital cone, of ravines, flood-plain, riverbed, scarps and artificial workings.

On the basis of their bedding ice bodies are divided into those on the ground and buried (subsurface), and of the time of formation into contemporary and old (mineral).

The mechanism of formation of underground ice bodies has great similarity with that of injection hummocks of heaving -- bulgunnyakhi or hydrolaccoliths.

On the basis of length of existence ice bodies are divided into seasonal (forming in winter and existing part of the summer), summering (forming in winter and lasting to the end of summer) and perennial ("taryny"). These are gigantic ice bodies encountered in the northeastern part of Yakutiya and forming in winter due to emergences on the surface of large-discharge sources under the conditions of an especially sharp continental climate. In summer they thaw only partially.

Ice bodies have different periods of formation. Some ice bodies are formed completely at the start of winter, and others are formed in the middle and even at the end of winter, in the period of maximally deep winter freezing. A third are formed in the course of the entire winter.

Depending on the discharge of the sources, the morphological structure of the ice section and the frost conditions on the adjacent territory of the area occupied by the ice bodies, the volumes of ice of the ice bodies (Table 67) are also different.

Table 67 Classification of ice bodies by area and by volume of the ice

	A Наименование	B Площадь наледи ($м^2$) по В. Г. Петрову	C Объем льда наледи ($м^3$) по А. И. Калабину	D Объем льда наледи ($м^3$) по А. С. Симакову
1	Очень малые . . . . . a	менее 10^2	—	до 10^4
2	Малые . . . . b	от 10^2 до 10^3	до 10^4	от 10^4 до 10^5
3	Средние	от 10^3 до 10^4	от 10^4 до 10^5	от 10^5 до 10^6
4	Большие	от 10^4 до 10^5	от 10^5 до $5 \cdot 10^5$	от 10^6 до 10^7
5	Очень большие	от 10^5 до 10^6	от $5 \cdot 10^5$ до 10^6	от 10^7 до 10^8
6	Гигантские c	более 10^6	более 10^6	более 10^8

Key: A - Class B - Area of ice body (m^2) according to V. G. Petrov
C - Volume of ice of ice body (m^3) according to A. I. Kalabin
D - Same according to A. S. Simakov 1 - Very small 2 - Small
3 - Medium-sized 4 - Large 5 - Very large 6 - Gigantic
a - smaller than 10^2 b - from 10^2 to 10^3 c - larger than 10^6

The length of existence of ice bodies is determined by the regime of the springs forming those ice bodies. The increase of size of the ice body is determined by the discharge of the source in the winter, and the destruction by the intensity of thawing of the ice in the spring and summer period. The process of thawing of ice is determined by the amount of absorbed solar radiation and the structure of the radiation-thermal balance of the surface. Under the conditions of the Far North very large and gigantic ice bodies do not succeed in completely thawing during the short summer period, as a result of which perennial ice bodies form. They can be developing, stable and degrading. The conditions of their development can be recorded in the following manner:

1) $Q_{input} > Q_{output}$ -- developing,
2) $Q_{input} = Q_{output}$ -- stable,
3) $Q_{input} < Q_{output}$ -- degrading.

The general procedure for calculating the regime and dynamics of an ice body can be illustrated as follows.

Estimation of the Dynamics of Growth of an Ice Body and Its Thawing (Example 45)

In the investigated region in the valleys of small rivers annual and perennial ice bodies are observed which form at the places of discharge of subsurface waters. In one such valley the regime has been studied of a source situated in the lower part of a slope and confined to the fault zone along which the discharge of subfrostal stratal-fissure waters of a Cambrian complex of deposits proceeds. The discharge of the source in the period of formation (or enlargement) of the ice body varies in the following manner (Table 68).

Table 68 Discharge of the source (m^3/day) forming an ice body

A Дата замера						
1965 г.			1966 г.			
1/X	1/XI	4/XII	2/I	1/II	1/III	5/IV
31	27,5	20,0	13,5	10,0	6,5	4,0

Key: A - Date of measurement

The height of the ice body at the end of winter reaches 1.9 meters on the average, occupying an area of about 1500 m^2. The growth of the ice body starts about 10-15 November.

The climatic conditions of the region are characterized by: $t_{air} = -9.5^{\circ}$; $A_{air} = 26.4^{\circ}$, and the average annual air temperature ($t_{air-ann}$) is 9.7°. The coefficient of heat transfer on the surface of the ice body in the summer is 11 kcal/(m^2)(degree)(hr). It must be determined whether that ice body is annual or perennial and the share of the surface waters and atmospheric precipitations in the formation of the ice body.

Solution. 1. We determine the volume of water of the source (V_w) used to form the ice body. In accordance with the length of growth of the ice body (from 15 November to 1 April) and change of the flow of the source in that time (see Table 68) with the average monthly values of the flow (Figure 116) we find that $V_w = 22.5 \times 15 + 17.2 \times 31 + 11.7 \times 31 + 8.3 \times 28 + 5.2 \times 31 = 1627\ m^3$.

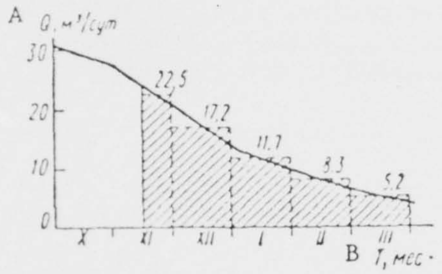

Figure 116. Flow of the source forming an ice body; the hatched area shows the flow of water in the period of enlargement of the ice body. a - Q, m^3/day b - τ, months

Consequently the volume of the ice body due to water of the source will be:

$$V_л = 1627 \cdot 1{,}09 = 1773{,}4\ м^3.$$

2. We determine the average height of the ice body forming through the source (h') if it is known that the area of the ice body (S) before the start of thawing is 1500 m^2:

$$h' = \frac{V_п}{S} = \frac{1773{,}4}{1500} \approx 1{,}2\ м.$$

3. We determine the share of the surface waters and atmospheric precipitations (V_w') in the formation of an ice body if the average height of the ice body at the end of winter is 1.9 m, and through waters of the source it can reach 1,2 m:

$$V_в' = (1{,}9 - 1{,}2) \cdot 1500 \cdot 0{,}99 = 1039{,}5\ м^3.$$

4. We find the maximally possible thickness of thawing of the ice body (h_t) through the heat of solar radiation:

$$h_{от} = \frac{\alpha t_{в.лет}\, \tau}{Q_{ф.л}}; \qquad (8.2.1)$$

$$h_{от} = \frac{11 \cdot 9{,}7 \cdot 3240}{80\,000} \approx 4{,}3\ м.$$

Consequently, an ice body with a thickness of 1.9 m is an annual one. Under the conditions of the given region it thaws in the first half of the summer, all the more so since the waters of atmospheric precipitations and surface runoff will exert an additional warming influence on the thawing.

The presence of an ice body affects the temperature regime of the basement rocks. As a rule, perennial ice bodies lead to a sharp reduction of temperatures in the basement rocks, which is connected with the following circumstances. In the summer period the presence of a layer of ice excludes the warming of the soil under it, the soil temperature does not exceed zero degrees and seasonal thawing under the ice body is completely absent, that is, the ground beneath ice bodies is in a permafrozen state for a number of years. This results in a sharp reduction of the average annual temperatures. In addition to that, the year round the ice cover, by reflecting the arriving solar radiation, leads to a sharp reduction of the temperature of the surface of the ice as compared with the air temperature and, consequently, to a reduction of the temperature of the basement rocks.

The procedure for calculating the influence of the ice body on the temperature regime of basement rocks can be illustrated as follows.

Calculation of the Influence of an Ice Body on the Temperature Regime of Basement Rocks (Example 46)

Calculate the temperature regime of alluvial deposits on a section of a flood plain where in the winter period an annual ice body forms through surface and subsurface waters. In frost investigations it was established that the growth of an ice body starts on 1-5 October (from the moment of stable transition of the air temperature through 0°) and continues to the end of February. The height of the ice body in that period is 2.8 meters on the average. In the first days of May the ice body starts to thaw, and it disappears by the end of July. The length of the winter period is 7 months. Table 69 presents data characterizing the annual course of components of the radiation balance and the average monthly air temperatures. It has been established that after the end of growth of the ice body, in March and April, its surface does not have a permanent snow cover, as it is blown away by winds whose velocity is great in those months (13-15 days of the month the wind velocity exceeds 6 m/sec).

The coefficient of heat transfer on the surface of an ice body is 10 kcal/(m^2)(hr)(degree), and on the surface of the soil -- 20 kcal/(m^2)(hr)(degree).

Solution. To determine the average annual temperature of the rocks on the section of development of an ice body it is necessary to determine the course of temperature on the surface of the ice body in the period of its existence and on the surface of the soil after its disappearance. It is obvious that during the growth of the ice body, that is, from October to February inclusively, the temperature of its surface will be considerably above the air temperature, as in that period water is crystallized and the released heat prevents lowering of the surface temperature. The reverse picture will occur in the period of thawing of the ice body, from May to July, when the phase transition of the ice into water creates a "zero screen" on the surface. In accordance with what has been said we determine the course of the average monthly temperatures on the surface of the ice body and the soil.

1. We find the average monthly temperature of the surface of the ice body in March and April, in the period when the ice body was formed, its further growth does not occur and thawing has not yet started. From the conditions of the problem it follows that the temperature of the surface of the ice body at that time will differ from the air temperature only by the amount of the radiation correction. In those months the radiation balance is 0.47 kcal/cm^2 in March and 2.46 kcal/cm^2 in April. Correspondingly the radiation correction will be:

$$\Delta t_{R,III} = \frac{0,47 \cdot 13,7}{10} \approx 0,5°; \quad \Delta t_{R,IV} = \frac{2,46 \cdot 13,7}{10} \approx 3,0°.$$

Table 69 The annual course of change of the components of the radiation balance and air temperature on the section of development of an ice body

		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Год
1	Суммарная радиация на горизонтальную поверхность $Q_{сум}$, ккал/см²мес	1,1	2,9	7,1	13,0	14,8	15,6	13,2	10,1	6,9	4,0	1,6	0,8	
2	Альбедо поверхности α, %	65	65	68	68	50	45	41	18	26	40	70	70	
3	Поглощенная радиация $Q_п$, ккал/см²мес	0,38	1,0	2,27	4,16	7,4	8,58	7,8	6,56	5,51	2,4	0,48	0,2	
4	Эффективное излучение земной поверхности I, ккал/см²мес	1,5	1,5	1,8	1,7	2,1	4,2	3,0	3,1	2,4	2,3	1,4	1,4	
5	Радиационный баланс R, ккал/см²мес	−1,1	−0,5	0,47	2,46	5,3	4,38	4,8	3,46	3,11	0,1	−0,92	−1,16	
6	Среднемесячная температура воздуха, $t_в$, °C	−35,9	−29,0	−21,1	−7,6	3,6	12,5	15,7	12,2	4,5	−7,3	−20,9	−35,6	$t_в = -9,2$; $A_в = 25,3$

Key: 1 - Summary radiation on the horizontal surface Q_{sum}, kcal/(cm^2)(month) 2 - Albedo of the surface α, % 3 - Absorbed radiation Q_{ab}, kcal/(cm^2)(month) 4 - Effective radiation of the surface of the ground I, kcal/(cm^2)(month) 5 - Radiation balance R, kcal/(cm^2)(month) 6 - Average monthly air temperature, t_{air}, °C

Consequently the average monthly temperatures of the surface of the ice body are $t_{III} = -21 + 0.5 = -20.6°$ and $t_{IV} = -7.6 + 3.0 = -4.6°$.

2. From May to July, in the period of thawing of the ice body, the temperature on its surface is 0°. If we assume that the temperature regime in the ice body (in its depth) is at that time isothermic as a result of filtration of thawed and surface waters through the ice body, we find that the temperature on the surface of the soil (below the ice body) at that time also is 0°.

3. In August and September the surface of the soil is freed of the ice body and is covered by sparse grassy vegetation. The albedo of the surface changes substantially and in August is 18% and in September 26%. With consideration of the radiation correction the average monthly temperature of the surface of the soil is 14.5° in August and 6.6° in September.

4. From October to February inclusively, that is, in the course of 5 months, the formation of the ice body occurs. It is obvious that the heat released during the crystallization of water in the process of the growth of the ice body raises the temperature on the surface of the ice body in proportion to the amount of water arriving and freezing per unit of surface.

If the growth of the ice body were not limited by the stream discharge, by the end of the winter period the ice body would have achieved the maximal thickness, which can be calculated with (8.2.1) at an average winter air temperature of -22.6°:

$$h = \frac{10 \cdot 3500 \cdot 22,6}{80\,000} = 10,2 \text{ м}.$$

In that case the temperature on the surface of the ice body in the period of its formation, that is, in the course of τ_{wtr}, would have been 0°.

Under real conditions the feeding of the ice body is limited and its thickness reaches only 2.8 m. Consequently, the elevation of the temperature on the surface of the ice body as compared with the air temperatures will occur at a certain value of Δt_{i-b} which can be determined with (8.2.1):

$$\Delta t_{\text{нал}} = \frac{h_{\text{нал}} \cdot Q_{\text{ф.л}}}{\alpha \cdot \tau_{\text{зим}}} = \frac{2,8 \cdot 80\,000}{10 \cdot 3500} = 6,2^{\circ}.$$

If the growth of the ice body in time occurs uniformly it can be assumed that the average monthly temperatures on the surface of the ice body in the period from October to February is 6.2° higher than the average monthly air temperatures.

Table 70 Annual course of the average monthly temperatures on the surface of an ice body in the period of its formation and on the surface of the soil after its disappearance

A Месяц	B Температура, °C
I	−30,7
II	−22,8
III	−20,6
IV	−4,6
V	0
VI	0
VII	0
VIII	14,5
IX	6,6
X	−1,1
XI	−14,7
XII	−29,4
C Год	−8,6

Key: A - Month B - Temperature, °C
C - Year

The values of the average monthly temperatures on the surface of the ice body in the period of its existence and on the surface of the soil after its disappearance are presented in Table 70. When the growth is irregular one can calculate the corrections for the average monthly temperatures in proportion to the change of the stream discharge, using the same equation.

5. In order to determine the average annual temperature on the surface of the soil, obviously it is necessary to determine the influence of the ice body as a thermal insulation cover. This can be done either by using a Fourier equation or with the method of V. A. Kudryavtsev, using

an equation for determination of the influence of the snow cover through the thermal cycles (example 14).

We will examine the solution of the problem on the basis of a Fourier equation.

The influence of the ice body on the temperature of the rocks in the period of existence of negative air temperatures is determinable by calculating the minimal temperature of the surface of the soil with equation (3.3.4), in which we make the following substitutions: we assume that A_o is equal to the minimal average monthly temperature of the surface of the ice body (-30.7^o); K is the coefficient of thermal conductivity of ice, equal to 4.5×10^{-3} m^2/hr; T is the length of the nominal period of fluctuations of temperature, equal to $2\tau_{wtr}$; h is the average thickness of the ice body during the period τ_{wtr}. The value of h is determined by the method of weighted-average values, in which from October to February the average thickness of the ice body is assumed to be 1.4 meters, since its growth occurred uniformly, and in the course of March and April it was invariable and amounted to 2.8 meters, that is,

$$h_{cp.} = \frac{1,4\cdot 5 + 2,8\cdot 2}{7} = 1,8 \text{ м}.$$

With the indicated initial data, using formula (3.3.4), we find that

$$t_{0,\text{мин}} = -30,7\cdot e^{-1,8\sqrt{\frac{3,14}{4,5\cdot 10^{-3}\cdot 10,2\cdot 10^{3}}}} = -19,1^\circ.$$

If we assume that the average winter temperature of the surface of the soil is approximately equal to 2/3 $t_{o\text{-}min}$, we find that $t_{o\text{-}min} = -12.7^o$. We find the average annual temperature of the surface of the soil in accordance with the data of Table 70:

$$t_{лет} = \frac{0 + 0 + 0 + 14,5 + 6,6}{5} = 4,2^\circ.$$

Then the average annual temperature of the surface of the soil under the ice will be equal to:

$$t_0 = \frac{-12.7\cdot 7 + 4,2\cdot 5}{12} = -5,6^\circ.$$

6. We determine the influence of the ice body on the temperature of the surface in the winter period through the heat cycles by the method of V. A. Kudryavtsev, as in (5.3.5). Being given certain values of Δt_{i-b}, as was done in the calculation of Δt_{sn} we determine the heat cycles through the surface of the soil and the surface of the ice body (see Table 71). Then by trial and error we determine the sought value of Δt_{i-b}. The initial data for calculation of the heat cycles in accordance with the conditions of the problem were assumed to be the following: γ_{sk} = 1350 kg/m^3; $\lambda_f = \lambda_t$ = 1.5 kcal/(m)(hr)(degree); w = 23%; w_{un} = 0%; $C_{vol\text{-}f}$ = 432 kcal/(m^3)(degree); Q_ϕ = 24,840 kcal/m^3; λ_i = 2 kcal/(m)(hr)(degree). Assuming successively the values of 8.6, 7.0 and 6.0^o for Δt_{i-b}, we obtain calculation data (see Table 71) on the basis of which, having constructed a diagram, we find that the sought value of Δt_{i-b} is 7.2^o.

Table 71 Calculation data for determination of Δt_{i-b}

A $\Delta t_{нал}$, °C	B $Q_{гр}$, ккал/м²	C $Q_{нал}$, ккал/м²
8,6	$54,68 \cdot 10^3$	$64,21 \cdot 10^3$
7 0	$53,35 \cdot 10^3$	$52,28 \cdot 10^3$
6,0	$51,22 \cdot 10^3$	$44,8 \cdot 10^3$

Key: A - Δt_{i-b} B - Q_{gr}, kcal/m^2
C - Q_{i-b}, kcal/m^2

With consideration of Δt_{i-b} we determine the average winter temperature on the surface of the soil under the ice body:

$$t_{0,зим} = t_{нал,зим} + \Delta t_{нал} = 2/3 t_{нал,мин} + \Delta t_{нал};$$
$$t_{0,зим} = 2/3(-30,7) + 7,2 = -13,2°.$$

Whence the average annual temperature of the surface of the soil will be:

$$t_0 = \frac{-13,2 \cdot 7 + 4,2 \cdot 5}{12} = -6,1°.$$

The values of t_o obtained by the two indicated methods have satisfactory convergence.

The calculations showed that the formation of an annual ice body with a height of 2.8 meters under the conditions of the investigated region leads to elevation of the average annual temperature of the surface of the soil under the ice body by 3.1-3.6° in comparison with the average annual air temperature of -9.2°. The same influence is exerted in the region by a snow cover with a height of 0.2-0.25 meter at a density of 0.19-0.2 g/cc.

Calculation of the Influence of Freezing of the Surface of an Ice Body on Its Thawing (Example 47)

In the region of investigation in a frost survey it was established that in river valleys where subsurface waters are discharged ice bodies form on a large area with a height of 3.5-4.0 meters. The ice bodies thaw by the end of summer. To work placer deposits it is necessary to accelerate the thawing of the ice body by 2 months. For that purpose its surface is covered with coal dust at the start of spring (April or May).

Table 72 presents data characterizing the radiation balance and temperature regime of the surface of an ice body in the period of its thawing, obtained by calculation with the use of meteorological data with consideration of the albedo (A) of the blackened surface.

It is necessary to determine the time of complete thawing of the ice body.

Solution. 1. We determine what total of the degree-hours is necessary to thaw an ice body with a height of 4.0 meters, using equation (8.2.1);

$$\Sigma\tau \cdot t_{сут} = \frac{80\,000 \cdot 4,0}{14} = 22\,857 \text{ град·час}.$$

Table 72 For calculation of the temperature of the blackened surface of an ice body in the spring and summer

	A Составляющие радиационного баланса, температура воздуха и поверхности наледи	B Месяцы V	VI	VII	VIII	IX
1	$Q_{сум}$, ккал/см²·мес	14,8	15,6	13,2	6,9	4,0
	A, %	32	15	15	15	20
2	$Q_п$, ккал/см²·мес	10,06	13,26	11,22	8,58	5,52
3	I, ккал/см²·мес	2,1	4,2	3,0	3,1	2,4
4	R, ккал/см²·мес	7,96	9,06	8,22	5,48	3,12
	$\Delta t_R = \frac{R}{\alpha}$, °C;					
5	$\alpha = 14$ ккал/м²·час·град	~7,8	8,8	8,0	5,4	3,0
6	$t_в$, °C	3,6	12,5	15,9	12,2	4,5
7	$t_{нал} = t_в + \Delta t_R$, °C	11,4	21,3	23,9	17,6	7,5

Key: A - Components of the radiation balance, air temperature and temperature of the surface of the ice body B - Months
1 - Q_{sum}, kcal/(cm^2)(month) 2 - Q_a, kcal/(cm^2)(month)
3 - I, kcal/(cm^2)(month) 4 - R, kcal/(cm^2)(month)
5 - α = 14 kcal/(m)(hr)(degree) 7 - $t_{i-b} = t_w + t_R$, ^oC

2. We determine at which average daily temperature the ice body can thaw in 2 months (1440 hours):

$$t_{сут} = \frac{\Sigma\tau}{1440} = \frac{22\,857}{1440} = 15{,}9°.$$

It is evident from Table 72 that the average daily temperature of the surface of the ice body during May and June is $\sim 16.3^o$ on the average (11.4 + + 21.3)/2 $\approx$ 16.3^o)). Consequently, by the end of June the ice body has thawed completely, which satisfies the conditions of the posed problem.

The effect of ice bodies on engineering structures is an undesirable phenomenon, and so in the construction of any objects it is combatted. Ice bodies often form where before construction and, consequently, disturbance of the natural conditions their formation was not observed. Therefore in an engineering forecast special attention must be given to the question of ice bodies. Many methods have been proposed for combatting ice bodies: explosive work, artificial thawing of ice, the construction of barriers to the flow of the water forming the ice body and the application of water-removing and drying drainage. For example, anti-ice-body belts are used as measures preventing the formation of underground ice bodies. The creation of belts consists in arranging, on the slope above the section, where an ice body forms under natural conditions, a ditch with an embankment laid out in the direction of the used area. In that case the ditch serves as a drain for the water on top of the frost, and the embankment as a dam which does not allow water to pass down the slope. The latter is achieved if the upper boundary of the permafrozen rock mass under the embankment is raised to the level of the day surface of the adjacent sections. An example of the calculation of such belts is given below.

Calculation of the Height of the Embankment of an Anti-Ice-Body Belt (Example 48)

In a frost survey it was established that, on long (> 500 meters) gentle slopes of a lake-glacier plain with a curved longitudinal profile, ground ice bodies form through the runoff of surface waters and waters on top of frost. In the construction of sections of a slope it is necessary to guard against the formation of ice bodies, which can be accomplished by creating an anti-ice-body belt. It is necessary to calculate the height of an enbankment which assures elevation of the upper boundary of frost to the level of the day surface.

Soils on the investigated section consisted of sandy loams with γ_{sk} = 1250 kg/m^3; w_{vol} = 27%; λ_t = 1.1 kcal/(m)(hr)(degree). The pouring off of the embankment is accomplished from the same soil (sandy loam) but its moisture content in the embankment is 18%, and γ_{sk} = 1200 kg/m^3 and λ_t = 1.2 kcal/(m)(degree)(hr).

The climate of the region is characterized by the following data: t_{air} = = -10.4°, A_{air} = 22°; z_{sn} = 0.6 m; ρ_{sn} = 0.19 g/cc.

When a "belt" is created the temperature regime of the ground in the groove and on the embankment changes substantially in comparison with natural conditions, as a redistribution of the snow cover will occur. The height of the snow in a ditch with a depth of 0.5-0.6 meter will increase to 1 meter at a density of 0.25 g/cc, and on the embankment will decrease to 0.3 m at a density of 0.22 g/cc.

Solution. 1. We determine the temperature regime of the surface of the soil and the depth of seasonal thawing under natural conditions. According to formula (5.3.10) Δt_{sn} = 7.1°, and then t_o = -3.3° and A_o = 14.9°.

In accordance with the moisture content and temperature regime of the soils we find with (4.1.7) and (4.1.8) that C_{vol-t} = 500 kcal/(m^3)(degree) and Q_ϕ = 21,600 kcal/m^3. With a nomogram (see Figure 17) we find that ξ = = $1.6 \times \sqrt{1.1}$ = 1.7 m.

2. We determine the temperature regime of the surface of the ground and the depth of seasonal thawing on the embankment at Δt_{sn} = 3.8°, t_o = -6.6° and A_o = 18.2°.

In accordance with the temperature regime and moisture content of the ground we find that C_{col-t} = 400 kcal/(m^3)(degree) and Q_ϕ = 14,400 kcal/m^3. With the nomogram we determine that $\xi = 1.45 \times \sqrt{1.2}$ = 1.6 m.

Therefore the height of the embankment must not less than 1.6 m for the soils under the embankment on the level of the day surface of the surrounding sections not to undergo seasonal thawing.

3. We determine the temperature regime and depth of seasonal thawing of the ground in the ditch at Δt_{sn} = 9.3°, t_o = -1.1° and A_o = 12.7°.

In accordance with the temperature regime of the ground in the ditch we find that Q_{ϕ} = 20,000 kcal/m^3 and with the nomogram (Figure 17) we determine that $\xi = 1.73 \times \sqrt{1.1} = 1.8$ m.

Comparison of the depth of thawing of the ground in the ditch and on the slope under natural conditions shows that even a shallow (0.5 meter) ditch will serve as a good drain for waters on top of frost. However, it should be taken into consideration that in constructing a ditch an undercutting of the slope occurs which can lead to the activation of solifluction and thermal erosion. Therefore the calculation of anti-ice-body belts must be accompanied by a forecast of the development of other frost processes.

3. Processes of Frost Cleavage of Rocks and Polygonal Formations

Polygonal-vein formations are developed not only in regions of contemporary prevalence of permafrozen rocks and seasonal freezing, but also far beyond their limits and in mountains. A common feature of those formations is their polygonal (often tetragonal) form, created by a network of fissures or ditch-like depressions which bound the polygons or more often quadrangles. At times the network of such ditches is combined with a system of ridges on the periphery of the polygons, forming a polygonal-ridge microrelief. In the case of a small hummock or large-hummock microrelief the network of ditchlike depressions bounds systems of small or large hummocks arranged in a checkerboard manner. Polygonal formations include various polygonal systems of ice and soil veins and also medallion spots, stone wreaths, "cryoturbations" and "boiling kettles" and other structural polygonal forms. The dimensions of the polygonal forms vary from several centimeters to tens or more meters.

Grid polygonal systems of ice veins are widespread in the Arctic and Subarctic and are developed mainly in finely dispersed and peaty soils. They at times amount to more than 50% of the volume of the enclosing rock and often create the specific appearance of the landscape and have complex forms and a structure reflecting the influence of the physicomechanical, thermal and facial conditions of their development.

Shown on Figure 117 is an outcrop of old ices of Mus-Khay on the Yana River. The mineral rocks, both finely dispersed and sandy-pebbled, enclosing the systems of ice veins have a laminated structure, as is clearly visible on Figure 118.

Contemporary polygonal systems of reopened-vein ices form on the surface a specific polygonal-ridge microrelief composed of regular polygons of tetragonal form on rocks uniform in their lithological composition. On lithologically different rocks polygons with an irregular form develop. Polygonal-veined formations form as a result of processes of frost cleavage.

In accordance with concepts of the physical essence of the process of frost cleavage presented in the works of B. N. Dostovalov (1952, 1959 and 1967) and developed in the works of N. N. Romanovskiy (1970, 1971 and 1972), the formation of fissures is connected with the following conditions. Above all the phenomenon can occur only in a solid frozen mass of rocks. The condition

Figure 117. Outcrop of vein ice of Mus-Khay on the Yana River.

is fulfilled in the establishment of low negative temperatures on the surface of the soil and at small depths of seasonally thawed (seasonally frozen) layers, which is determined by the latitudinal zonation and the continental character of the climate.

It follows from the first condition that frost cleavage is connected with the distribution of definite geological genetic complexes and types of rocks. By virtue of that the phenomenon under consideration is widespread on watersheds within the limits of occurrence of covering loams, moraine clayey deposits and also silty sandy-sandy loam-loamy deposits of alluvial and lake-alluvial plains. Frost cleavage is also widespread in river valleys.

Very favorable conditions for the development of the given process are noted in the region of permafrozen rock masses, where at the moment of complete freezing of the seasonally thawed layer occurs a solid mass of frozen rocks from the surface to a great depth. In that case the tensile stresses forming in the frozen seasonally thawed layer can appear at temperatures lower than the temperature of the main phase transitions of water for the given type of deposits. The intensity of frost cleavage for that type of deposit increases in proportion to the lowering of the negative temperature on the surface from t_m to the minimal (t_{min}). At a constant value of t_m ($t_o = t_\xi$) the intensity of the cleavage increases with increase of A_o. In both cases the increase of intensity of cleavage leads to reduction of the distance between fissures

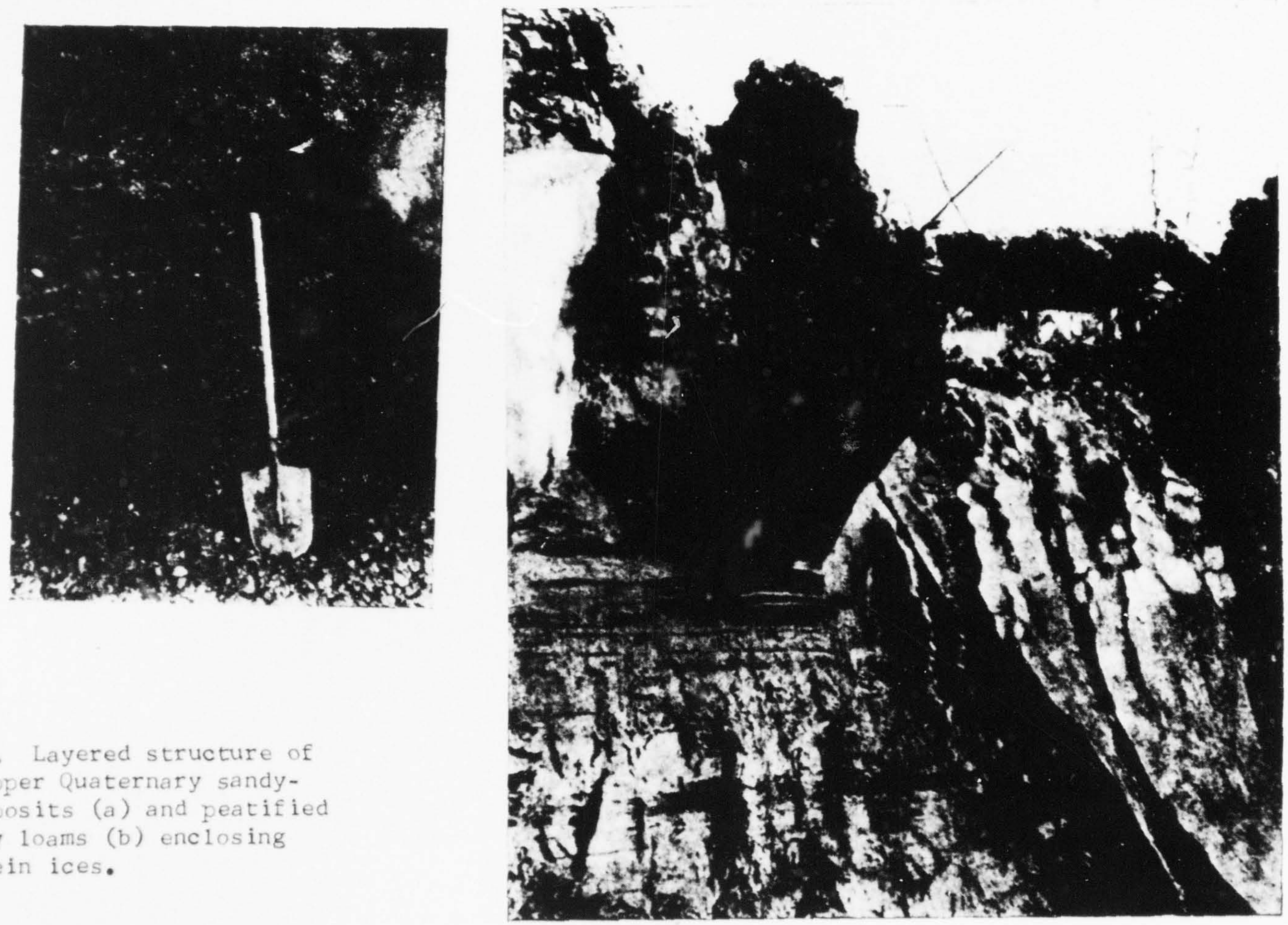

Figure 118. Layered structure of alluvial Upper Quaternary sandy-pebbled deposits (a) and peatified silty sandy loams (b) enclosing reopened-vein ices.

and to increase of the depths of penetration of fissures (a single order of generation). The temperature gradients in the seasonally thawed (seasonally frozen) layer are determined by the difference of temperatures on its surface and base and depends on the layer thickness and its lithological and moisture characteristics and consequently are connected with a definite type of seasonal thawing (seasonal freezing) according to V. A. Kudryavtsev.

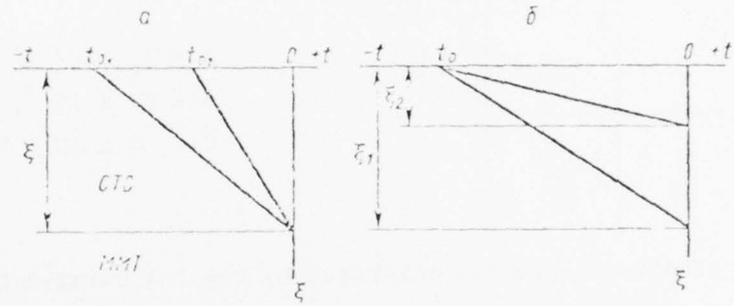

Figure 119. Change of the temperature gradient as a function of temperature on the surface of the ground (a) and on the thickness of the seasonally thawed layer (b): CTS -- seasonally thawed layer; MMT -- permafrozen rock mass.

What has been said can be illustrated by a schematic diagram, from which it is evident that at one and the same thickness of the seasonally thawed layer (ξ) different temperatures on the surface (t_{o1} and t_{o2}) cause different temperature gradients in that layer (Figure 119a). The same thing is noted at one value of t_o but at different thicknesses of the layer ξ (Figure 119b). The probability of frost cleavage is largest on sections with maximal temperature gradients in the layer ξ in the winter. Therefore within the limits of the first frost-temperature zone the process of fissure formation can occur only under the conditions of a sharply continental climate, in regions with little snow, on sections with a sparse plant cover. In northern regions, in frost-temperature zones IV and V frost cleavage fissures can be encountered even in conditions close to a maritime climate. The probability of frost cleavage of soils can be calculated for each concrete region as follows.

Determination of the Conditions of Formation and the Distances Between Frost Cleavage Fissures (Example 49)

In conducting a frost survey it is very important to determine the frequency of formation of frost cleavage fissures in soils. In the region of the investigation the first terrace above the flood plain is widespread. Its surface is level and covered with moss-lichen cover and sparse, low (15-20 cm) underbrush.

In the process of the survey it was established that the terrace in the upper part of the profile to a depth of 17-20 cm is composed of light silty loams with a moisture content of 27% at γ_{sk} = 1240 kg/m^3. The degree of moisture saturation of loam is equal to unity. Laboratory investigations of samples of the soil showed that $\lambda_f = 1.5$ and $\lambda_t = 1.3$ kcal/(m)(hr)(degree), $C_{vol-f} = 422$ and $C_{vol-t} = 560$ kcal/(m^3)(degree), $Q_\phi = 21{,}824$ kcal/m^3, and the coefficient of temperature conductivity $\alpha_f \approx 0.0035$ m^2/hr. The dependence of the tensile strength of the frozen ground (σ_t), the coefficient of linear expansion (α) and the modulus of deformation (g) on temperature is presented in Table 73.

Table 73 Mechanical properties of loam

	A	B	C
t, °C	σ_p, кг/см²	$\alpha \cdot 10^{-6}$, 1/град	$G \cdot 10^4$, кг/см²
—5	24	130	2,6
—10	24	130	3,5
—20	29	130	4,6

A - σ_t, kg/cm^2
B - $\alpha \times 10^{-6}$, 1/degree
C - $G \times 10^4$, kg/cm^2

The climatic conditions are characterized by the following data: $t_{air} = -10.4^\circ$, $A_{air} = 25.5^\circ$, $z_{sn} = 0.3$ m, and $\rho_{sn} = 0.22$ g/cm^3. The plant cover in the region exerts a warming influence on the temperature regime of the soil: $\Delta t_{plant} = 0.1\text{-}0.2^\circ$; $\Delta A_{plant} = 0.3\text{-}0.5^\circ$.

Determine the distance between the frost cleavage fissures forming in the layer ξ during its complete freezing, and also between fissures penetrating the permafrozen rock mass.

Solution. 1. We calculate the temperature regime on the surface of the soil and the depth of the seasonal thawing of rocks:

$$t_0 = t_в + \Delta t_{сн} + \Delta t_{раст}; \qquad \Delta t_{сн} = 25{,}5 \cdot 0{,}164 \approx 4{,}2^\circ;$$
$$t_0 = -10{,}4 + 4{,}2 + 0{,}2 = -6{,}0^\circ;$$
$$A_0 = A_в - \Delta A_{сн} - \Delta A_{раст}; \quad A_0 = 25{,}5 - 4{,}2 - 0{,}3 = 21^\circ.$$

With a nomogram (Figure 33) we find the amount of the temperature shift in the layer ξ : $\Delta t_\lambda \approx 0.8^\circ$. Consequently, $t_\xi = -6.0 - 0.8 = -6.8^\circ$.

At the initial values $t_\xi = -6.8^\circ$; $A = 21^\circ$; $\lambda_t = 1.3$ kcal/(m)(hr)(degree); $C_{vol-t} \approx 560$ kcal/(m^3)(degree); $Q_\phi = 21{,}824$ kcal/m^3, and with nomograms (Figures 15 and 17) we find the depth of the seasonal thawing $\xi = \sqrt{1.3} \times 1.45 \approx 1.65$ m.

2. We determine the temperature gradient in layer ξ at the moment of its joining during freezing with the permafrozen rock mass. To do that, on a nomogram (Figure 61) we find the temperature on the surface of the soil at the moment of joining (see examples 27 and 28).

Under the given conditions ($t_o = -6.0^\circ$; $A_o = 21^\circ$; $Q_\phi = 21{,}824$ kcal/m^3) we find that $t_j = -22.6^\circ$. Consequently, the gradient in layer ξ at the moment τ_j will be

$$\text{grad}_\xi t = \frac{t_{\text{см}}}{\xi} = \frac{22,6}{1,65} \approx 13,6 \text{ град/м}.$$

3. We determine the distance between fissures (x) in layer ξ which form at the moment τ_j, using the formula of B. N. Dostovalov (1967):

$$x = \frac{2\sigma_p}{\alpha \cdot G \cdot \text{grad}_\xi t}; \qquad x = \frac{2 \cdot 24}{130 \cdot 10^{-6} \cdot 4,6 \cdot 10^{3} \cdot 13,6} \approx 0,6 \text{ м}.$$

4. We determine the layer thickness h and the minimal temperature on its surface with consideration of the asymmetry of the temperature envelopes (see example 6). In accordance with the conditions of the problem the initial data in the calculation are assumed to be as follows: $t_\xi = -6.8^{\circ}$; $A_o = 21^{\circ}$; $\alpha_f = 0.0035$ degree/m^2; $\xi = 1.65$ m; $\xi_{t-f} = 3.19$ m. We find A_ξ' with (4.4.1) and A_ξ with (4.4.2):

$$A'_\xi = 21,0 \cdot e^{-1,65\sqrt{\frac{\pi}{0,0035 \cdot 8760}}} = 12,38^{\circ};$$

then

$$A_\xi = \sqrt{1 - \left[\frac{1,65}{3,19}\left(1 - \frac{6,8}{6,8+21}\right)\right]^2} = 11,4^{\circ};$$

$$h = \sqrt{\frac{KT}{\pi}} \ln \frac{A_\xi}{0,1} = \sqrt{\frac{0,0035 \cdot 8760}{3,14}} \ln \frac{11,4}{0,1} = 14,8 \text{ м}.$$

When the values of t_ξ and A_ξ are known we readily find the minimal temperature at the depth ξ:

$$t_{\xi,\text{мин}} = t_\xi - A_\xi = -6,8 - 11,4 = -18,2^{\circ}.$$

5. We determine the temperature gradient in the layer h at the moment of establishment of the minimal temperature at the base ξ and the distance between the forst cleavage fissures forming at that time in the permafrozen rock mass:

$$\text{grad}_h t = \frac{|t_{\xi,\text{мин}}|}{h} = \frac{18,2}{14,8} = 1,23 \text{ град/м}.$$

Then

$$x = \frac{2 \cdot 24}{130 \cdot 10^{-6} \cdot 3,5 \cdot 10^{3} \cdot 1,23} \approx 8,6 \text{ м}.$$

Thus under the conditions of the investigated section of the first flood-plain terrace in the winter period frost cleavage fissures form. The distance between the fissures in the layer ξ at the moment when it joins the permafrozen rock mass during freezing is 0.6 m, and the distance between fissure penetrating the layer h and forming in the period of maximal gradients in that layer reaches 8.6 m.

It is obvious that on small areas within the limits of the placement of very important structures the question can be posed of purposeful change of frost conditions in order to prevent the possibility of fissure formation. Improvement measures must be selected with consideration of concrete [illegible] conditions, as only in that case can the necessary results be achieved.

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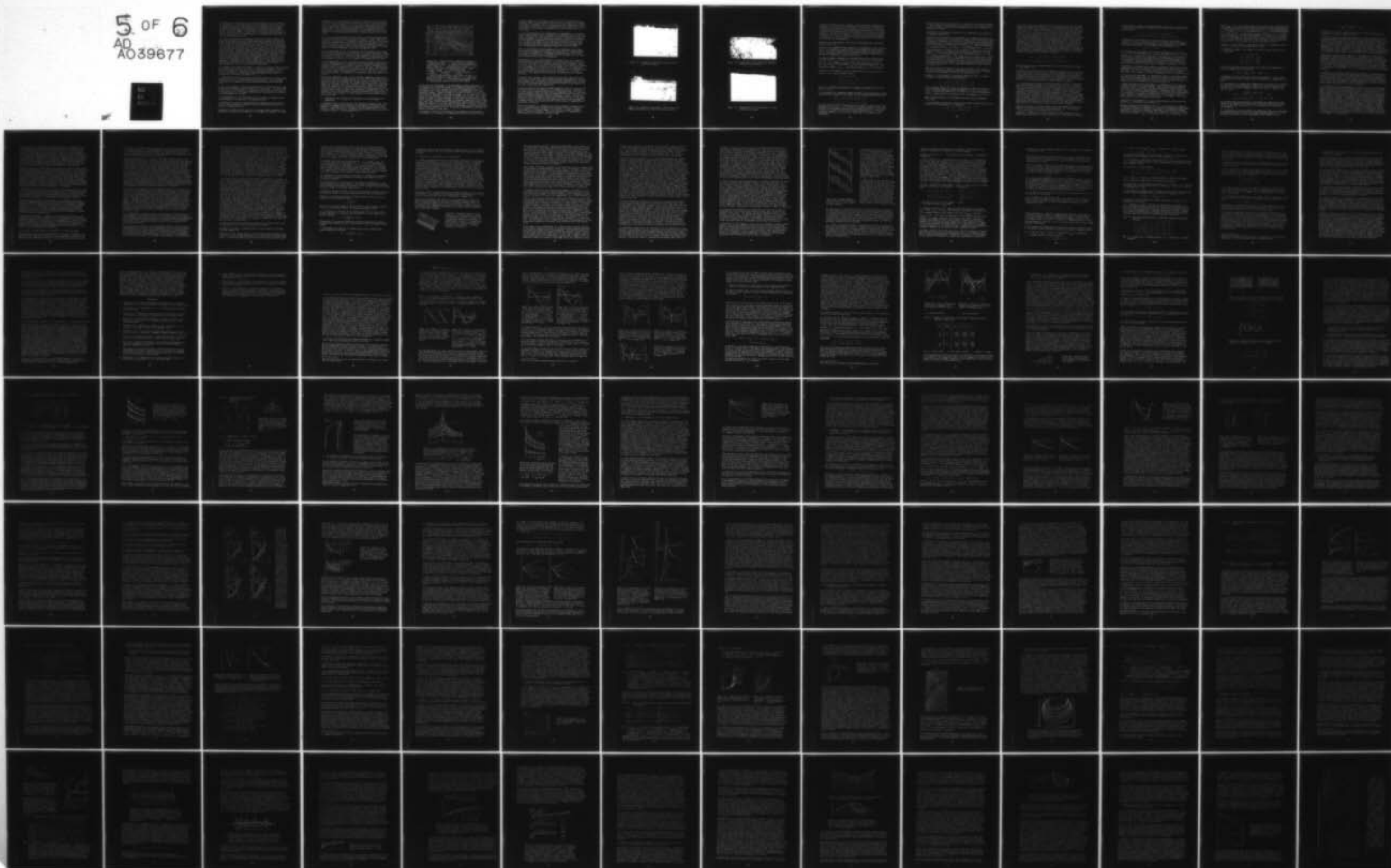

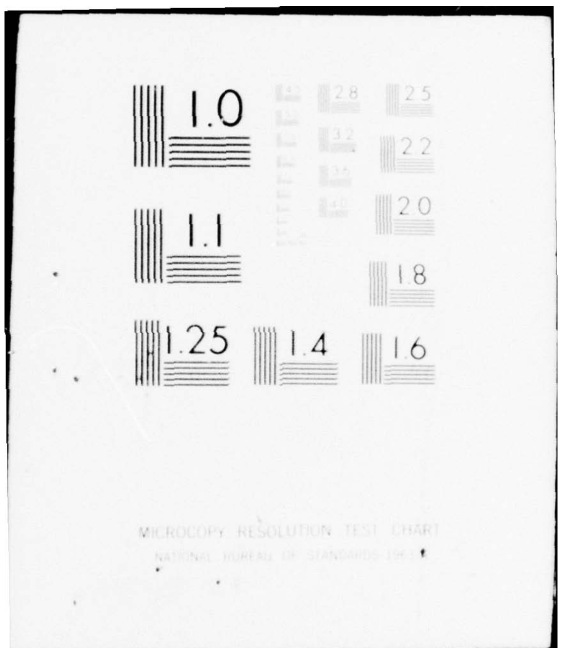
1.0
2.8
2.5
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MICROCOPY RESOLUTION TEST CHART

In different frost-temperature zones and under different ground conditions the effectiveness of the same methods of controlling the frost process will be different. Thus, for example, measures to accumulate snow will give a far greater effect under the conditions of a sharply continental climate than of a maritime climate. The same measures in zones I and II lead to a greater effect than in more northern zones. A corresponding difference will be noted with respect to the plant cover, soil drainage, the installation of various artificial covers, etc.

In a region of prevalence of permafrozen rocks the fissures forming in the winter in the surface layer of seasonal thawing can penetrate to different depths in different frost-temperature zones. Thus frost cleavage fissures in the rocks of a seasonally thawed layer extend to its base at temperatures of -2 and -3^{o} (in sandy loams and peats) or to -4 and -6^{o} (in sands, pebbles and rock debris-gruss formations). At lower values of t_m they penetrate the frozen rock mass. During the spring flood and melting of snow the temperature of the walls of gaping fissures in the frozen rock mass remains sufficiently low and the water falling in them freezes rapidly, cementing the fissures and transforming the frozen rock mass again into a solid mass. The repeated alternating frost cleavage and cementation of the fissures by ice leads to the development of polygonal systems of ice veins representing an aggregate of elementary veins. The vertical extent of such ice veins sometimes reaches 40 meters at a transverse thickness of up to 6-8 meters, at average dimensions of 12-24 and 4-6 meters respectively.

The polygonal vein formations forming as a result of processes of frost cleavage are represented by ice and soil veins. In that case, depending on the conditions of formation, on the basis of the correlation of the ice and soil parts of the veins N. N. Romanovskiy (1972) distinguished four types of polygonal-vein formations (see Figure 52).

1. Primordially soil veins forming in seasonally thawed and seasonally frozen layers as a result of periodically recurrent processes -- frost cleavage, the formation of elementary ice veins, and their thawing and filling with rock.

2. Ice veins forming in a frozen rock mass, below the seasonally thawed zone. Polygonal-vein formations with vein ice are two-stage forms: the upper stage is of soil and has the features of primordially soil veins, and the lower is of reopened-vein ice. That type includes ice-soil veins which are variations of the development of ice veins.

3. Pseudomorphoses on ice veins which arise as a result of thawing of vein ice and filling of the space with overlapping deposits.

4. Primarily soil veins of sandy composition, forming in zones of wind activity in winter during the pouring of sandy and fine gravel material into open frost cleavage fissures.

/Latitudinal zonation of processes of frost cleavage fissure formation/. The dependence of polygonal-vein formations on the temperature regime of the rocks determines their zonal character, the presence of transitional forms from the first to the second type often within the limits of a single polygonal system or systems arranged in a row. For various lithological-facial varieties of rocks that transitions is accomplished in different temperature regimes (Figure 120). Simultaneously with that on latitudinal zonation regularities is superposed the influence of a continental character of the climate and structural geological features having a regional, azonal character.

In the first frost-temperature zone the process of fissure formation during favorable conditions is developed mainly on sections on which permafrozen rock masses are prevalent. They usually are absent on taliks. The necessary temperature gradients for the formation of frost cleavage fissures are observed in that zone only in regions with a sharply continental climate. The process of fissure formation here is connected with the formation of primordially soil veins and extremely rarely with reopened-vein ices.

In frost-temperature zone II the processes of frost cleavage fissure formation also are developed under the conditions of a sharply continental climate, are concentrated mainly in sections in which permafrozen rock masses are prevalent and, as a rule, do not lead to the formation of reopened-vein ices. Also widespread in that zone is the formation of primordially soil veins.

In frost-temperature zone III frost cleavage fissure formation is widely developed, and depending of the combination of the temperature regime and the continental character of the climate on separate sections predominates the formation either of primordially soil veins or of ice veins. Characteristic of that zone is the formation of two-stage veins, the lower part of which is of ice and the upper, often predominant, is of soil. The size of the polygonal lattice varies as a function of the conditions from a few meters to 10 or more.

Within frost temperature zones IV and V favorable temperature gradients for the formation of frost cleavage fissures are noted almost everywhere with the exception of regions with thick snow and moss covers. Cleavage occurs most intensively under the conditions of a sharply continental climate, when the dimensions of polygons reach tens of meters. Under conditions of a maritime climate the dimensions of a polygonal grid increase. Within the limits of flood plains, and also of coastal-lake and coastal maritime sections, frost cleavage fissure formation leads to the formation of reopened-vein ices. The thickness of those ices during the intensive accumulation of deposits can reach several tens of meters. On high elements of the relief mixed veins of ice and soil form.

4. Regularities of the Formation and Prediction of the Development of a Thermokarst

A thermokarst forms in connection with the thawing of subsurface ices. This phenomenon is accompanied by subsidence of the surface of the ground, the formation of negative forms of the relief and their being swamped. In the absence of runoff of the water, thermokarst lakes form in lows, and when there is intensive runoff -- a dry thermokarst low.

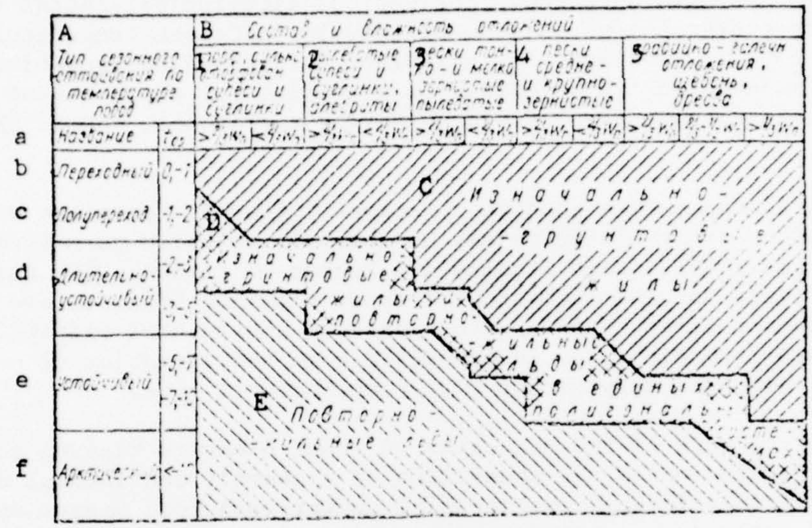

Figure 120. Zonation of vein formations according to N. N. Romanovskiy (1972). A - Type of seasonal thawing by temperature of rocks B - Composition and moisture content of deposits C - Primordially soil veins D - Primordially soil veins and reopened-vein ices in single polygonal systems E - Reopened-vein ices
a - Name b - Transitional c - Semi-transitional d - Long-stable e - Stable f - Arctic
1 - peat, strongly peatified, sandy loams and loams 2 - silty sandy loams and loams, aleurites 3 - sands, fine-grained and silty 4 - medium and coarse-grained sands 5 - gravel-pebble deposits, crushed stone, gruss.

In both cases the development of the thermokarst proceeds differently. In the formation of a lake there always is a progressive development of the thermokarst either to the complete thawing of the permafrozen rocks or to the formation of a stable thawing basin and the establishment of a steady temperature regime. Complete thawing of the rocks is noted in that case if the transverse dimensions of the lake are larger or of the same order of magnitude with the thickness of the frozen rock mass and the depth of the lake exceeds the depth of winter freezing. In that case, if the thickness of the frozen rock mass considerably exceeds the transverse dimensions of the thermokarst lake, under it forms a stable thawing basin. Before establishment of the regime the dimensions of the lake and the talik under it can have a different dependence. Then, if we know the depth of occurrence of the frozen rock mass, with the Stefan formula (3.7.7) it is possible to approximately determine the length of existence of that lake.

Two principal conditions are necessary for the formation and development of a thermokarst: 1) the presence of underground ices and 2) the depth of the seasonal thawing of the rocks must exceed the depth of occurrence of the subsurface ices. The development of the process of thawing of the basement rocks under a thermokarst lake is observed apart from a dependence on their ice content. In the case of a dry thermokarst low the process often is suspended even in the presence of subsurface ices and is renewed only in isolated years.

A reason for difference in the development of a thermokarst is a covering layer of water, which leads to warming of the underlying bottomset beds (see section 5, Chapter 5). With increase of the depth of the body of water its warming influence on the bottomset beds increases. Therefore the progressive development of thermokarst lakes under any conditions, even very dry ones, is easily explained. Having once formed, in each subsequent summer the thermokarst will develop more intensively than in the preceding one until the subsurface ice is completely thawed and a lake is formed.

Since in the formation of dry thermokarst lows the process can be damped, there follows the important practical conclusion that the struggle against the harmful aftereffects of a thermokarst assumes above all the drying of the surface of sections where subsurface ices and strongly icy soils lying near the base of the layer ξ are widespread.

Reopened-vein ices are widely developed in northern parts of the region in which permafrozen rocks are prevalent. On sections where the depth of ices is close to the depth of the seasonal thawing of rocks, thermokarst starts to develop when there are relatively small changes of the external conditions. Thus, for example, the clearing of a forest (Figure 121), the construction of a highway (Figure 122), etc, lead to the development of a thermokarst even when the average annual temperatures of the rocks are low. The form of thermokarst lakes depends on the conditions of occurrence of the reopened-vein ices.

On sections with intensive runoff baydzherakhi (hillocks remaining after deglaciation) of conic shape form, usually arranged in the manner of a checkerboard (Figure 123). On sections of watersheds with hindered runoff of water the forms of the baydzherakhi are flat and their height is considerably less than that of the conic ones (Figure 124).

In southern areas of the region in which permafrozen rocks are prevalent this variety of ice is rarely encountered or is completely absent. Subsurface ice bodies in those regions consist mainly of segregation ices of a schlieren lenslike bedding. Those ice bodies are epigenetic and their formation is connected with moisture migration during the freezing of rocks. The depth of occurrence of those ice bodies is most often greater than that of the summer thawing of rocks. Therefore in such regions the start of the development of a thermokarst often is connected with the overall process of degradation of permafrozen rock masses.

Thus the development of a thermokarst can be both regional, arising as a result of planetary secular changes of heat exchange, and local, caused by local changes of the thermal regime of the rocks as a result of limited changes of natural conditions. Such local phenomena of a thermokarst are often caused by the productive activity of man.

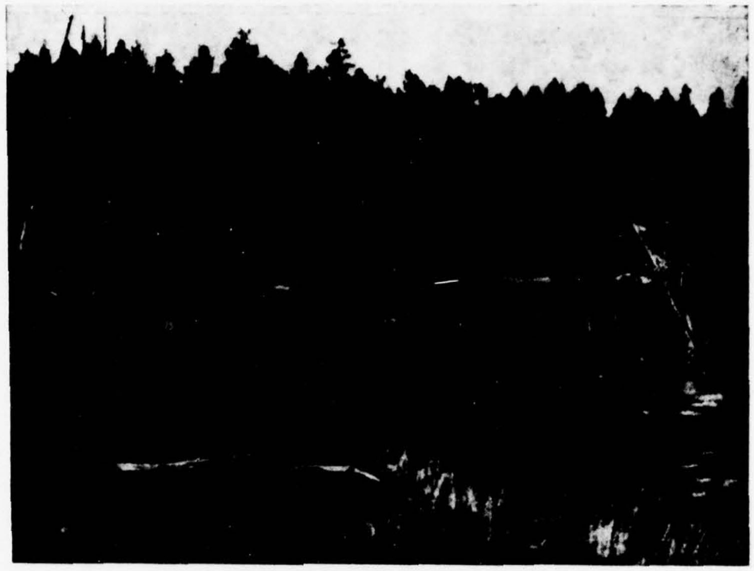

Figure 121. Thermokarst which began to develop after a forest was cleared.

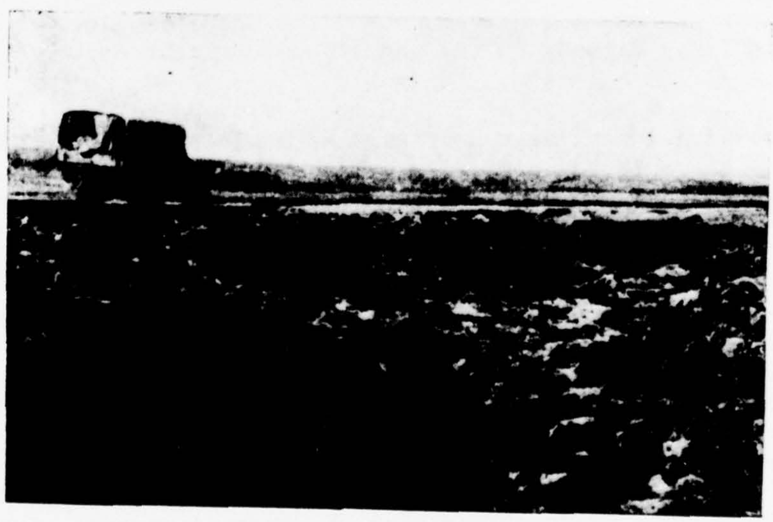

Figure 122. Thermokarst which began to develop after the highway from Kular to the bank of the Yana was built.

Figure 123. Baydzherakhi on the slope of the Mus-Khay outcrop on the Yana River.

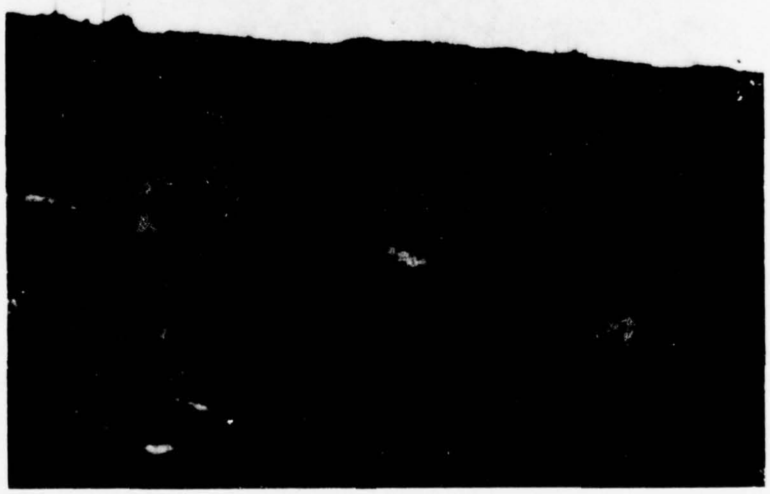

Figure 124. Baydzherakhi on the flat surface of a lake-alluvial plain (Q_{III}).

It follows from what has been said that in frost geological engineering investigations for the purpose of compiling a forecast of the formation of a thermokarst in connection with the opening up of territory it is necessary to study in detail the frost characteristics and correlations of the depth of occurrence of underground ice and the thickness of the layer of seasonal thawing as a function of a complex of components of the natural environment (see Chapter 5). All those changes of the natural conditions which cause an increase of the depth of seasonal thawing of rocks lead to the formation and development of a thermokarst.

Estimation of the Possibility of the Start of Development of a Thermokarst Process (Example 50)

Determine the possibility of the formation of a thermokarst in the following cases: a) elevation in separate years of the average annual air temperature by 1.5° and b) increase of the amplitude (physical) of fluctuations of the air temperature by 5° if it is known that soils with a large amount of schlieren ice occur from a depth of 1.6 meters.

The natural climatic conditions are characterized by the following average perennial data: $t_{air} = -8.7^{o}$, $A_{air} = 21^{o}$, $z_{sn} = 0.5$ m, $\rho_{sn} = 0.22$ g/cc. The soils in the layer of seasonal thawing are composed of loams with w = 30%, $w_{un} = 7\%$, $\gamma_{sk} = 1200$ kg/cm^3; $C_{spec} = 0.19$ kcal/(m^3)(degree), $\lambda_t = 1.0$ kcal/(m)(hr)(degree), $C_{vol-t} = 588$ kcal/(m^3)(degree), $Q_{\phi} = 22{,}080$ kcal/m^3.

Solution. 1. We determine t_o and ξ_t during elevation of t_{air} by 1.5° in relation to its average perennial value:

$$\Delta t_{\text{сн}} = 21 \cdot 0{,}259 = 5{,}4^{\circ},$$

$$t_0 = -8{,}7 + 1{,}5 + 5{,}4 = -1{,}8^{\circ},$$

$$A_0 = 21 - 5{,}4 = 15{,}6^{\circ}.$$

Under those conditions ξ according to a nomogram (see Figures 15 and 17) is equal to 1.55 m.

2. We determine the temperature regime and depth of seasonal thawing of rocks during increase of A_{air} by 5° in relation to the average perennial amplitude:

$$A_{\text{в}} = 21 + 5 = 26^{\circ}, \quad \Delta t_{\text{сн}} = 26 \cdot 0{,}259 = 6{,}7^{\circ},$$

$$t_0 = -8{,}7 + 6{,}7 = -2^{\circ}, \quad A_0 = 26 - 6{,}7 = 19{,}3^{\circ}.$$

The depth of the thawing of rocks under those conditions reaches 1.8 m. Thus during elevation of t_{air} by 1.5° the depth of seasonal thawing does not reach the depth of occurrence of icy grounds, and during increase of A_{air} by 5° at a value of t_{air} equal or close to its average perennial value (-8.7°) the thawing embraces the icy horizon, which leads to the formation of a thermokarst.

Measures necessary for the prevention of a thermokarst are reduced primarily to various kinds of fillings which are carried out so that the depth of the seasonal thawing does not reach the underground ices. Such fillings can be calculated as follows.

Calculation of the Height of Fillings of Soil to Prevent the Development of a Thermokarst (Example 51)

Determine the possibility of the development of a thermokarst during the removal of the plant cover and calculate the height of the filling of the ground to prevent it if during a frost survey the following data were obtained. The construction site is situated on a lake-alluvial plain composed from the surface of loams with γ_{sk} = 1000 kg/m^3, w = 35%, C_{vol-t} = 530 kcal/(m^3)(degree), Q_ϕ = 20,000 kcal/m^3, λ_t = 1.0 and λ_f = 1.2 kcal/(m)(degree)(hr). The permafrozen rock mass in the upper part of the profile is composed of loams with frequent schlieren of ice with a total moisture content of 60%, γ_{sk} = 800 kg/m^3, C_{vol-t} = 760 kcal/(m^3)(degree), Q_ϕ = 40,000 kcal/m^3, λ_t = 0.7 and λ_f = 1.0 kcal/(m)(degree)(hr). The average annual air temperature is -10.6°, the annual amplitude of the air temperature is 21.5°, and the height of the snow reaches 0.5 m at a density of 0.22 g/cc. The plant cover, consisting of moss-underbrush cover, increases the average annual temperature by 0.3° and reduces the amplitude of the temperature fluctuations on the surface of the soil by 2.2°.

When the plant cover is removed, on account of reduction of the albedo of the denuded surface the annual average temperature of the soil is elevated by 0.9°, and the amplitude in that case is increased by 2.5°.

Solution. 1. We determine the temperature regime and depth of seasonal thawing of the ground under natural conditions:

$$\Delta t_{сн} = 21,5 \cdot 0,259 = 5,6^\circ;$$

$$t_0 = t_в + \Delta t_{сн} + \Delta t_р = -10,6 + 5,6 + 0,3 = -4,7^\circ;$$

$$A_0 = A_в - \Delta t_{сн} - \Delta A_р = 21,5 - 5,6 - 2,2 = 13,7^\circ.$$

With a nomogram (Figure 33) we find the temperature shift in the layer of seasonal thawing. With the starting parameters t_o = -4.7°, A_o = 13.7°, C_{vol-t} = 530 kcal/(m^3)(degree), λ_t = 1.0 and λ_f = 1.2 kcal/(m)(degree)(hr), and Q_ϕ = 20,000 kcal/m^3 we obtain $\Delta t_\lambda \approx 0.4^\circ$. Consequently, t_ξ = -5.1°.

With a nomogram (Figure 17) we determine that under natural conditions $\xi \cong 1.0$ m.

2. We determine t_ξ and ξ with the plant cover removed to make clear the possibility of the development of a thermokarst:

$$t_0 = t_в + \Delta t_{сн} + \Delta t_R = -10,6 + 5,6 + 0,9 = -4,1^\circ;$$

$$A_0 = 21,5 - 5,6 + 2,5 = 18,4^\circ.$$

In the first year, after removal of the plant cover, in the layer of annual temperature fluctuations there will be an unsteady regime of the temperature field: the average annual temperature on the surface of the soil will be 0.6° higher (4.7 - 4.1) and at the depth of the seasonal thawing will remain equal to -5.1°. The influence of the elevation of the annual average temperature of the surface on the depth of the seasonal thawing in that case can be calculated similarly to what is done in the calculation of the potential seasonal freezing, that is, the difference Δt_o of 0.6° should be added to the value of the amplitude. Correspondingly the depth of the seasonal thawing of the ground in the first year after removal of the cover must be calculated at $t_\xi = -5.1^{\circ}$ and $A_o = 19^{\circ}$. Since the depth of thawing in that case is increased and embraces the upper horizon of the permafrozen rock mass, it is necessary to take into account the change of the ice content and the properties of the ground in determining the starting data for finding ξ. With the method of weighted averages, assuming that the depth of thawing increases by not more than 0.5 meter, we find that

$$C_{\text{т}}^{\text{ср.вз}} = \frac{530 \cdot 1{,}0 + 760 \cdot 0{,}5}{1{,}5} = 600 \text{ ккал/м}^3\text{град},$$

$$\lambda_{\text{т}}^{\text{ср.вз}} = 0{,}9 \text{ ккал/м}\cdot\text{град}\cdot\text{час}, \quad Q_{\text{ф}}^{\text{ср.вз}} = 26\,000 \text{ ккал/м}^3.$$

With those data, with a nomogram (Figure 17) we find the depth of thawing: $\xi = 1.44 \sqrt{0.9} = 1.4$ m.

Consequently, when the plant cover is removed the depth of the seasonal thawing of the ground in the very first season will increase by 40% in relation to the depth under natural conditions. When the drainage of the surface is difficult this leads to the development of a thermokarst, since during the thawing of strongly iced grounds of a frozen rock mass in the range of depths of 1.0 - 1.4 m a subsidence of 0.1 m occurs during the thawing.

3. In determining the height of filling necessary to prevent a thermokarst one should determine the depth of the seasonal thawing of the soil forming under new and changed conditions in very warm years. If in the analysis of the climatic data (for the last decade or a longer period of years) it turns out that a very warm year from the point of view of a high average annual temperature is characterized by a small amplitude, then it is preferable for calculations to select a year characterized by a very long and hot summer and a very large amplitude of fluctuations (even if the average annual temperature remains on the level of the average perennial). In selecting a warm year one can be guided by the fact that the elevation of the average annual temperature of the ground by 1° and an increase of the amplitude by 2° brings each separately to an identical increase of the depth of thawing.

Under the conditions of the given region the steady temperature field after removal of the plant cover will be characterized by: $A_o = 18.4^{\circ}$ and $t_\xi = -4.8^{\circ}$ (since under the new conditions $\Delta t_\lambda \approx 0.7^{\circ}$).

According to climatic data in separate warm years (which recur twice in a decade) t_{air} increases by 2° and A_o increases by 6°. The height of the snow in those years is close to the average perennial norm, that is, about 0.5 m. In that case the temperature conditions on the surface of the soil will be characterized by:

$$t_{\text{п}} = -8{,}6^\circ;\quad A_{\text{п}} = 24{,}5^\circ;\quad \Delta t_{\text{сн}} = 24{,}5 \cdot 0{,}259 = 6{,}3^\circ;$$

$$t_0 = -8{,}6 + 6{,}3 + 0{,}9 = -1{,}4^\circ;\quad A_0 = 24{,}5 + 2{,}5 - 6{,}3 = 20{,}7^\circ.$$

In calculations of the depth of the seasonal thawing in warm years t_ξ is assumed to be -4.8° in accordance with the new steady temperature regime of the rocks. The elevation of the average annual temperature on the surface of the soil (from -4.1 to -1.4°) is taken into consideration in A_0:

$$A_0 = 20{,}7 + 2{,}7 = 23{,}4^\circ.$$

The properties of the grounds in the layer of thawing are assumed to be: $C_{vol-t} = 530$ kcal/(m^3)(degree), $\lambda_t = 1.0$ kcal/(M)(degree)(hr), $Q_\phi = 20{,}000$ kcal/m^3 (it is assumed that the grounds of the upper part of the profile of the rock mass, in density, moisture content and thermophysical properties after repeated thawing and freezing, in the period of establishment of the new temperature regime become similar to grounds of the layer ξ formed before removal of the plant cover). With a nomogram we find that $\xi = 1.9$ m.

Consequently, to exclude the possibility of thawing the upper horizon of icy permafrozen rock mass the height of the filling of the soil must be not less than $(1.9 - 1.0) + 0.2 = 1.1$ m (0.2 is given in reserve, as the fillings with time will be lowered as a result of packing of the soil, denudation, etc).

In the process of development of a thermokarst, bodies of water with permeating and non-permeating taliks can form. The forecast of the formation of those taliks and their development in time, and also the change of their configuration, can be determined as was shown in Chapter 7.

Of great importance for the development of a thermokarst and the formation of bodies of water is the water balance. When the runoff and evaporation exceed the arrival of moisture (through precipitations, surface runoff and water from the thawing of underground ice), a thermokarst body of water dries up. In that case the temperature regime of the bottomset beds changes sharply in the direction of lowering of temperatures. When the body of water has dried completely, permafrozen rock masses start to form again. The new formation of frost can be calculated as follows.

Calculation of the Thickness of a Layer of New Formation of Frost After the Drying Up of a Thermokarst Lake (Example 52)

During the drying up of thermokarst lakes on a lake-alluvial plain composed of loam-sandy loam frozen rock masses of deposits, non-permeating taliks under a lake start to freeze. Determine at what depth of the lake the bottomset beds start to freeze and at what depth they freeze after 25 years if the following data are known: $t_{air} = -10.6^\circ$, $A_{air} = 23.5^\circ$, $z_{sn} = 0.3$ m, $\rho_{sn} = 0.22$

g/cm^3, H_{ice} = 2 m. The bottomset beds consist of silty loams; during freezing they acquire a fine-schlieren multilayered cryogenic texture characterized by a total moisture content of 55%. Frozen loams have the following properties: γ_{sk} = 800 kg/m^3, λ_f = 1.2 kcal/(m)(degree)(hr), C_{vol-f} = 400 kcal/(m^3)(degree), Q_ϕ = 36,000 kcal/m^3. The temperature regime on the surface of the bottomset beds is determined by the following conditions: in the course of the first 5 years from the moment of start of freezing a layer of water with a thickness of 0.3 m on the average is preserved in the lake basin. After complete drying the height of the snow cover in the basin increases to 0.5 m.

Solution. 1. We calculate at what critical depth of a thermokarst lake the perennial freezing of the bottomset beds can start.

The temperature regime on the surface of the body of water under the snow is determined:

$$\Delta t_{сн} = 23{,}5 \cdot 0{,}164 = 3{,}8°;$$

$$t_{0,\text{вод}} = -10{,}6 - 3{,}8 = -6{,}8°;$$

$$A_{0,\text{вод}} = 23{,}5 - 3{,}8 = 19{,}7°;$$

$$t_{\text{макс}} = 12{,}9°; \quad t_{\text{мин}} = -26{,}5°.$$

The depth of the body of water at which the average annual temperature is zero is determinable from the equation

$$h_{t=0} = H_л \left(1 + \frac{t_{\text{макс}}}{t_{\text{мин}}}\right) = 2{,}0 \left(1 - \frac{12{,}9}{26{,}5}\right) \approx 1{,}0 \text{ м}.$$

Consequently when the water level in the lake is reduced to 1.0 m or more the bottomset beds start to freeze, with the formation of a permafrozen rock mass.

2. We determine the average annual temperature on the surface of the bottomset beds which a) is established at a depth of the body of water of 0.3 m and also b) in the case of complete drying of the lake at a height of the snow of 0.5 m:

$$\text{a)}\ t_{h=0.3} = \frac{\frac{2{,}0 - 0{,}3}{2{,}0}(-26{,}5) + 12{,}9}{2} = -4{,}8°,$$

$$\text{б)}\ \Delta t_{сн} = 23.5 \cdot 0{,}259 \approx 6{,}0°; \quad t = -10{,}6 + 6{,}0 = -4{,}6°.$$

As is evident from the calculation, the average annual temperature of the surface of the deposits will change in the course of 25 years from -4.8 to -4.6°. We will take the value t_o = -4.6° for calculation of the depth of the permafrost.

3. The thickness of the new formation of permafrozen rocks can be calculated by the method of exact solution of the Stefan problem (Chapter 3). For approximate calculation we use the approximate Stefan formula (3.7.7);

$$\xi_{мн} = \sqrt{\frac{1{,}1 \cdot 4{,}6 \cdot 25 \cdot 8760}{35\,000}} \approx 5{,}5 \text{ м.}$$

Consequently, a layer of permafrozen loams with a high ice content and a thickness of 5.5 meters forms in 25 years.

During the formation of permafrozen rock masses the water-bearing horizon in a non-permeating talik becomes interfrostal and acquires a large head. Irregularity of the drying of the lake and subsequent freezing of the rocks has the result that in places not yet dried or where the thickness of the rock masses is minimal from the surface the ground bulges and large hummocks -- bulgunnyakhi -- form. The hummock size reaches 30-50 meters in height and 200-300 meters in cross section with ice lenses in the hummock of 30-40 meters. The possibility of the formation of such bulgunnyakhi can be calculated and an approximate estimate made of their dimensions as was shown in example 42.

/Distinctive features of the formation of a thermokarst in different latitudinal zones and geostructural regions/. The character of a thermokarst is determined primarily by the genesis and conditions of occurrence of the underground ices and the latitudinal zonation of their prevalence. The development of a thermokarst also depends substantially on the ratio of the depths of occurrence of the underground ices and the depths of the seasonal thawing of the rocks. That dependence has the result that the latitudinal zonation of a thermokarst is determined also by the latitudinal zonation of the seasonal thawing.

As is known, the latitudinal zonation of the prevalence of underground ices is characterized by the fact that during movement from south to north in the upper part of the profile the epigenetic ices are replaced by syngenetic. In frost-temperature zones I and II epigenetic ices are prevalent everywhere in the form of both ice-cement and separate schlieren and lenses of ice. As a rule, those ices are confined to the uppermost horizon of permafrozen rock masses (within the range of 1 or 2 tens of meters).

In the indicated zones the thermokarst is connected with the thawing of epigenetic ices in finely dispersed epigenetic frozen loose deposits. Usually they are deluvial and eluvial deposits, alluvial flood-plain and ox-bow facies, alluvial-lake and coastal-lake deposits of watershed plains, glacier, mainly moraine, deposits, and also marine deposits consisting of clayey and loamy varieties. The form of a thermokarst is determined by the conditions of occurrence of the ices. The bedding of the ices in the form of separate lenses causes the formation of small basin-like lows and small lakes (with a diameter of tens of meters) with a small depth. A characteristic feature of a thermokarst of that type is the group arrangement of lakes on relatively large areas, measured in tens and hundreds of square kilometers. In that case the separate lakes are divided by isthmuses, relatively dry, composed of deposits containing little ice. The formation of a thermokarst in zones I and II is very often connected with the thawing of permafrozen rock masses. The dynamics of the layer of seasonal thawing of rocks lead much more rarely to the formation of a thermokarst.

In zones IV and V a thermokarst is connected with the thawing mainly of reopened-vein ices in syngenetically frozen rock masses. A specific feature of the bedding of those ices is their direct location at the base of the seasonally thawed layer, in connection with which a very slight increase of the depth of the summer thawing of rocks, as a rule, leads to the formation of a thermokarst. Epigenetic ices in frost-temperature zones IV and V usually are at great depths (20-30 meters or more) and therefore cannot be of great importance in the formation of a thermokarst. Syngenetic ices are confined to the flood-plain facies of alluvial deposits, the coastal-lake facies of lake and the coastal-marine facies of marine deposits. Reopened-vein ices are prevalent mainly in finely dispersed varieties of those deposits.

Within the limits of the zones under consideration the depth of the seasonal thawing is very small, and so the reopened-vein ices are at a depth of several tens of centimeters from the surface. A very slight change on the surface of the soil (disturbance of the plant cover, change of the snow height, change of the moisture content of the soils in the seasonally thawed layer, etc) can lead to the start of formation of a thermokarst. The thawing of ices, subsidence of the surface and the formation of water bodies lead to a warming effect of the layer of water, as a result of which the average annual temperature of the rocks is elevated and the depth of the seasonal thawing under the water body increases. Thus the formation of a thermokarst lake creates favorable conditions for the further development of the process.

A thermokarst in frost-temperature zones IV and V can also form during irregular seasonal thawing of strongly iced syngenetic frozen solifluctional deposits and strongly iced coastal-lake and alluvial deposits with layered cryogenic textures. The forms of the thermokarst in that case are very different and often are determined by the character of the production activity.

In frost-temperature zone III the formation of a thermokarst through the thawing of epigenetic ices weakens and almost ceases completely. Simultaneously with that a thermokarst forming on account of the thawing of reopened-vein ices also is not widespread, as those ices are encountered extremely rarely in the zone. In zone III a thermokarst occurs mainly on sections anomalous for that zone. On sections with a temperature regime of rocks characteristic of zones I and II the development of a thermokarst is linked with the thawing of epigenetic ices.

The spread and character of a thermokarst are essentially different in different geological structural regions. It is most widespread on lowland-plain territories composed of a thick mass of loose sediments. In regions of solid rocks a thermokarst is not widespread and is confined mainly to alluvial terraces, and more rarely to upland terraces. In that case the origin of a thermokarst is connected with the thawing of buried and solifluctional ices, and also with the thawing of buried glaciers.

5. Processes of Thermal Abrasion and Prediction of Their Development

Thermal abrasion is a process of destruction of the coasts of northern seas, lakes and rivers composed of permafrozen rocks, occurring as a result of the mechanical and thermal action of water, and also under the effect of the heat of air masses. Thermal abrasion is noted on the coasts of northern seas, on lowlands near the sea and in intermountain areas composed of icy deposits of a different genesis. Thermal abrasion is a specific phenomenon for a region of permafrozen rocks and is linked with regions where underground ices mainly of the reopened-vein type are prevalent.

During thermal abrasion, as a result of the summary thermal effect of the main factors destroying coasts, thawing of rocks on steep, often perpendicular, slopes occurs. Then forms a layer of thawed strongly moistened rocks which either slide to the base of the slope on the surface of a frozen rock mass under the effect of the force of gravity or are washed away by the surf and draining streams of mud. More often a combined effect is noted, when the lower part of a slope is washed away by waves and deep niches are washed out in a coastal scarp. This causes caving of masses of thawed ground making up the upper part of the slope. In contrast with ordinary abrasion of coasts, during thermal abrasion the caving of coastal masses of rock leads to denudation of the frozen ground and its subsequent intensive thawing. The intensity of the processes of thermal abrasion is determined by the conditions of bedding of the permafrozen rock masses, their ice content and cryogenic textures, the character and regime of the water bodies and climatic characteristics (the direction of the prevailing winds, the character of summer precipitations and snow accumulation and the continental character of the climate).

Besides the above indicated factors, of great importance are the dimensions of ice veins and the dimensions of the lattice of those ices. When the dimensions of the polygons are large (20-30 m) during the thawing of relatively small ice veins perpendicular "towers" form which when the niches are washed out are destroyed and washed away by the water. When the upper and lower parts of the scarp have unequal ice contents, at the bottom gently sloping scarps -- "thermal terraces" -- can form, and if the activity of the wave-cut factor is not intensive enough here the process of thermal abrasion can be damped. When the ice content of the soils is relatively small, between the vein ices, as a result of their thawing, form baydzherakhi which, upon being unloaded, assume the shape of cones (see Figure 123). If vein ice lying parallel to the scarp is uncovered by thermal erosion, it has the form of a solid wall of ice.

In estimating the stability of coasts and the intensity of the processes of thermal erosion it is necessary above all to determine the geomorphological and stratigraphic-genetic classification of the rocks making up the permafrozen rock coastal masses, the presence, character and genesis of the underground ices, their extent, bedding and dimensions, and also the intensity of the coastal marine abrasion.

The thermal abrasion of the shores of thermokarst lakes, in contrast with the thermal abrasion of seacoasts, is characterized by the fact that the effect of the surf factor is sharply reduced and the transport of thawed material into the depth of the lakes on account of sliding (floating) of loose material along the surface of the gentle slopes of the lake bottom predominates, and also the work of currents of rainwater and floodwaters. A characteristic feature of the development of thermokarst lakes in the northern and northeastern parts of the country is the simultaneous thawing and formation again of syngenetic reopened-vein ices. Thermokarst lakes form in the process of the thawing of those ices. Near the shores of those lakes, through the sliding of soils which have thawed on the coastal slopes, form shallows which during subsequent drying of the lake emerge on the surface and begin to freeze. In the period of the flood-plain regime of those lakes in the shallows sediments are deposited which are subjected to syngenetic freezing, frost cleavage and the formation of ice veins in them. Thus in one part of the alasy lakes an intensive process of thermal abrasion and expansion of the lake is observed, and in another, usually opposite, proceeds the process of sedimentation and the formation of syngenetic permafrozen rock masses. Thermal abrasion is usually noted on coasts with a southern exposure and on windward slopes, where the effect of the surf factor is manifested very intensively. The formation of permafrozen rock masses and encroachment of the coast are noted on the opposite side. As a result of the noted processes a migration of alasy [depressions in the pergelisol] thermokarst lakes is observed.

It is obvious that the effect is very important in the general evaluation of cryogeological engineering conditions and, in particular, in compiling a forecast of the development of thermal erosion in regions of the production opening up of sections and the placement of structures. It is natural that in that case the basis for such a forecast is study of the distribution of genetic geological complexes of rocks and types of frozen rock masses, their temperature regime, the ice content and conditions of bedding or reopened-vein ices. Of no little importance in the study of the dynamics of thermokarst lakes and the processes of thermal abrasion of their shores is the question of the use of such lakes for agricultural purposes. In connection with the fact that within the limits of taliks on the bottom of alasy basins there forms a temperature regime of the soils which is essentially different from the coastal mass and in proportion to the emergence of the lake bottom on the surface the temperatures of the soils on those sections are held for a definite time in the region of positive values. On account of that, in the vegetation period the temperatures of the soils are so high that the soil climate proves to be favorable for the growing of agricultural crops even under the conditions of the Extreme North and the Arctic. Therefore in solving problems of the national economy it is necessary to compile a forecast of the destruction of the coasts, the dynamics of the drying up of swamps and lakes and the change of the moisture and temperature regimes of the bottomset beds, and on the basis of that to develop methods of monitoring and control of the frost process for purposes of regulation of the soil climate for needs of agriculture.

The general procedure for calculation of processes of thermal abrasion and the destruction of coasts can be illustrated as follows.

Estimation of the Rate of Thermal Abrasion of Coasts Composed of Strongly Icy Rocks (Example 53)

Observations of the thermal abrasion of coasts were conducted during a frost survey on a section of undermined cliff of an outlier of an old lake-alluvial plain. In the outlier with a height of more than 20 meters above the water line of a river, reopened-vein ices are exposed, with enclosing deposits consisting of horizontal layered silty sandy loams, peatified, with thin layers of fine-grained silty sand. The icy rock mass is covered from above with a layer of solifaction-deluvial silty sandy loams with a thickness of 2.0-2.5 meters. As a result of intensive lateral thawing there is encroachment of the cliff of the outlier and the formation of baydzherakhi (see Figure 123) on sections with both a southern and a northern exposure.

The climatic conditions are characterized by: $t_{air} = -14.0^{\circ}$, $A_{air} = 21^{\circ}$, the average annual air temperature ($t_{air-ann} \sim 5.6^{\circ}$ (according to average perennial data). The length of the warm period is 3 months ($\tau_{year} = 2160$ hours). On precipitous slopes (more than 50°) of an outlier with a southern exposure the average annual air temperature on the surface is 1° higher than on horizontal sections and slopes with a northern exposure on account of increase of the inflow of direct solar radiation.

The coefficient of heat transfer on the surface of thawing ices, calculated with the data of field microclimatic observations (see example 18) is 11 kcal/(m^2)(hr)(degree).

The observations also established that the surface of thawing ices is covered periodically for a short time by a thin layer of floating soliflunctional deluvial deposits thawing from the surface. On the average the ices are closed 20% of the length of the entire summer period.

Determine the distance which the scarps of an outlier with northern and southern exposures retreat in 10 years, counting from the time of observations.

Solution. To calculate the amount of thermal abrasion of the coast in the summer period we use equation (8.2.1)

$$h = n \frac{\alpha \cdot t_{\text{в.лет}} \cdot \tau_{\text{лет}}}{Q_{\phi,\text{л}}},$$

where n is the relative length of the period with an open surface of the ice on slopes, in fractions of unity. For the given problem n = 0.8.

1. We determine the thickness of the thawing layer of ice on slopes with a northern exposure. In accordance with the conditions of the region, in one year the thawing is

$$h = \frac{0{,}8 \cdot 11 \cdot 5{,}6 \cdot 2160}{80\,000} = 1{,}33 \text{ м}.$$

Consequently, in 10 years the cliff with a northern exposure of an outlier at places where reopened-vein ices are uncovered, retreats 13.3 meters on the average.

2. We determine the thickness of a thawed layer of ice on slopes with a southern exposure in one year:

$$h = \frac{0{,}8 \cdot 11 \cdot 6{,}6 \cdot 2160}{80\,000} = 1{,}57 \text{ м}.$$

Consequently, in ten years the cliff with a southern exposure of the outlier retreats 15.7 meters, that is, 2.4 meters further than slopes with a northern exposure.

6. Solifluction and Prediction of Its Development

In a region of permafrozen rocks the processes of solifluction are widespread and are connected with the presence of the permafrozen rocks. A necessary condition of the development of processes of solifluction is the specific composition of the soils, their moisture content, usually equal to the absolute moisture capacity (or close to it), the presence of slip planes of the ground and its creeping. In a region of permafrost a characteristic feature of covering deposits is great siltiness of the soils. As is known, that is connected mainly with physical weathering of minerals and processes of coagulation and aggregation of fine clayey fractions. When the thickness of the seasonally thawed layer is small and the upper boundary of the permafrozen rocks in northern regions has a close bedding, the conditions are created for strong moistening of the soils of the seasonally thawed layer. When they have an exceptionally silty composition, such moistening leads to the formation of soils with a fluid or similar consistency. In connection with that the presence of very small inclinations of the upper surface of the permafrozen rocks contributes to the soliflucional flow of soils and the thawed layer of soil.

Depending on small or large change of the strength properties of the soil and the velocity, the processes of solifluction can be subdivided into: 1) slow creep, 2) plastic-viscous flow and 3) progressive or liquid flow.

Slow creep of dispersed soil along a slope is accomplished as a result of multiple heavings during the freezing of subsequent sediments during thawing. A schematic diagram of such motion is presented on Figure 125. It is evident on that diagram that as a result of recurrent multiple freezings with heaving and thawing with precipitation the surface layer of dispersed soil slowly creeps downward.

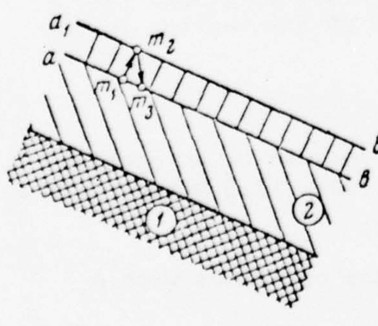

Figure 125. Schematic diagram of creep along the slope of dispersed soil during its multiple heaving and subsidences during recurrent freezings and thawings: 1 - permafrozen rock mass; 2 - thawing layer; m_1, m_2 and m_3 - successive positions of the point m at the start of freezing, heaving and subsidence.

There are relatively few data available on measurements of the actual rates of solifluction. According to measurements made by the Department of Geocryology of Moscow State University in the northeastern part of Western Siberia, and also according to data of the Institute of Geocryology of the Siberian Department, USSR Academy of Sciences and foreign investigators, the solifluction rate on the average is 8-10 cm/year and only in exceptional cases reaches 30 cm/year on slopes with a steepness of more than 10°. The movement of soils is completed very rapidly on the surface and usually halts at a depth of 50-60 cm. In cases where slow plastico-viscous flow changes into liquid flow, the rate of movement of the ground increases sharply and solifluction changes into mud flows and mud avalanches.

Under natural conditions, processes of the flow of the soil are a rather powerful geological and geomorphological factor, under the influence of which slope deposits form and the relief of the terrain changes. All this can be shown graphically on the example of an investigation of solifluction in the Yenisey River valley in the region of Dudinka, where it is developed on slopes of the river valley, gorges and lake basins composed of Sanchugovka glacial-maritime and Zyryanka water-glacial silty sandy loams and loams. In the layer of seasonal thawing those deposits have an indicator of consistency larger than unity, that is, their moisture content under natural conditions of 30-50% is always higher than that of the lower level of plasticity. In the spring and after summer rains the soils as a rule acquire a liquid consistency. Therefore even on sections with slopes of $5\text{-}7^{\circ}$ the flow of soils is observed.

A favorable condition for the intensive development of solifluction is also the formation of an increased moisture content of rocks in the base of the layer of seasonal thawing. The wide distribution of low-temperature frozen rock mass (t_m below -4°) and low air temperatures ($t_{air} = -10.3^{\circ}$) within the limits of a lake-glacial plain cause freezing of the seasonally thawed layer in the autumn and winter from both above and below. Moisture migration during freezing from below leads to the formation of a large quantity of horizontal ice schlieren with a thickness of fractions of a cm to 2-5 cm. The moisture content by weight of the frozen rocks reaches 80-100% during thawing. The dynamics of the depths of the layer of seasonal thawing under those conditions leads to the formation of heavily iced horizons of permafrozen rock masses on the boundary with the seasonally thawed layer, which has a thickness of 1 m on the average. On sections of the accumulation of solifluctional deposits the icy horizons of the lower part of the layer ξ change into the permafrozen state as a result of their burial. The thickness of the ice layer of the upper part of the frozen rock mass increases to 3-4 cm in that case. The icy horizon is also an additional source of moistening of the soils during their thawing and the surface along which they flow in the diluted state.

In the region of the investigation, in connection with distinctive features of the moisture regime of the layer of seasonal thawing, sections with plastico-viscous and liquid flow are distinguished. Slow plastico-viscous flow of soils occurs during the thawing of rocks, the moisture content of which does not exceed that of the yield point. In that case, on gentle slopes of a lake-glacial plain form solifluctional terraces with a length of up to 10 meters and a width of 1-3 meters. The height of the front benches of those terraces varies from 0.1 to 0.4 meter. On such sections the flow of soils is often accompanied by rupture of the sod cover and the formation on the foot of front benches of ditchlike depressions with a width of 0.4-0.5 meter and a depth of about 0.3 meter. In those depressions, under a layer of peatified soil with a thickness of 0.1-0.15 meter, lies a lens of ice or heavily iced soil with a thickness of 0.1-0.2 meter (up to 0.6 meter).

During the thawing of icy soils of a seasonally thawed layer, the moisture content of which considerably exceeds their absolute moisture capacity, liquid or progressive flow develops. This process develops especially actively on extensive swampy solifluctional slopes cut through by deep ravines. In that case overflows are observed, characterized by the movement of large volumes of rock, which in isolated cases leads to the burial of firn basins. Thus, for example, in the course of 3 years in the given region a process of liquid flow of soil and the burial of the flowing mass of rock of firn basins on the edge of a deep ravine were observed. That ravine was cut into deposits of Sanchugovka age to a depth of up to 30 meters. It has a curved longitudinal profile and a V-shaped cross-section, and the steepness of the slopes is $40-50^{\circ}$. Along the bottom of the ravine passes a brook which flows during the entire warm period of the year. The water flow is due to runoff from a lake lying in a basin in the upper part of the ravine. Firn fields are preserved in the upper part of the ravine until the end of summer. Some of them are partially or completely covered with soil flowing from a higher gentle slope. The liquid mud flows with the exception of boulder and pebble material move the stream in separate sections toward the right edge of the ravine. The thickness of the mud flows in the bottom of the ravine reaches 0.5-0.8 m. The burial of firn basins by soliflctional flows also occurred in the past. Thus, one of the holes drilled on a fold of a gentle slope of a ravine revealed a firn basin with a thickness of about 5 meters under a 4-meter layer of soliflctional deposits.

Closely connected with distinctive features of the formation and freezing of deluvial-soliflctional deposits are thermal erosion processes. They proceed especially intensively in ravines cut in silty sandy loams and loams of Sanchugovka age at different depths. In the upper reaches of all the ravines are strongly swamped lowlands or lakes, the runoff from which is the main source of feeding the currents. If the runoff from lakes into adjacent ravines is considerable and occurs during the entire warm period, the deluvial-soliflctional material arriving from the slopes does not form thick accumulations but is carried off into the valleys of larger currents. In ravines, the constant connection of which with lakes has been halted as a result of lowering of the water level of the lakes and their enlargement, the transport of deluvial-soliflctional material is slowed down and the accumulation of that material exceeds its outflow. In that case masses of soliflctional ravine deposits form, the thickness of which reaches 5-10 m.

In conducting drilling work it was established that the composition of those deposits is characterized by great variability in the plane and cross-section: horizons enriched with boulder and pebble material alternate with horizons of sandy loams and loams and with horizons enriched with peat-plant material. The ice content of the soils and the cryogenic structure vary sharply in the deposits. The massive texture is replaced by lens-shaped, grid, basal and ataxitic. Horizons of ice with inclusions of boulder-pebble material are encountered. Variegation in the distribution of ice in frozen rocks is evidently caused not only by heterogeneity of the composition of the deposits but also by the character of the influx of material (slow creep or progressive flow), the conditions of its drainage, the character of the freezing and the presence of buried firn basins. Horizons with a large ice content either are formed during the freezing of a greatly flooded layer, the runoff from which down along the slope was hindered, or are connected with the burial of firn basins or, finally, are a result of freezing of the layer from below and its burial. The presence of those greatly iced horizons predetermines the development of thermal erosion in the ravines. When the depth of seasonal thawing under temporary currents reaches the icy horizon, intensive thawing of the ice occurs. Sinks and niches form at the bottom of ravines. Often the thawing proceeds along a heavily iced horizon lying below a less icy frozen layer, as a result of which underground traverses form. On sections where the seasonal thawing does not reach the icy horizons or the latter are absent, the temporary currents do not have a deep cut and a constant channel. In that case the surface waters run off without disturbing the sod-plant cover.

It follows from the presented material that in studying solifluction in order to issue a forecast of its probable formation it is necessary above all to turn attention toward the conditions of development of the process. First it is necessary to map the composition of the soils of the seasonally thawed layer on sections of gentle slopes and slopes of moderate steepness. The contouring of sections of the propagation of silty dispersed rocks determines the probable area of distribution of solifluction. The moisture regime of the soils and its variation in connection with the productive activity of man and sharp moistening or drying will testify to the probability of the development or discontinuance of the process of solifluction on a given section. Of great importance in that is the character of the relief of the surface of the permafrozen rocks and its probable change in connection with the productive activity of man. In that case a decisive factor will be the change of the temperature regime of the soils and the change of the depth of their seasonal thawing. A procedure for forecasting the latter is given in Chapter 5.

The formation of solifluctional and upland terraces. The rate of creep of dispersed rock along a slope, other conditions being equal, depends on the steepness of the slope. On sections with great steepness the rate of solifluction is greater than on sections with little steepness. This leads to an accumulation of flowing matter on more gently sloping sections and to change of the relief of the slope. Thus change of the steepness of the slope is the first main factor in the formation of solifluctional terraces.

A second factor is the tendency of the process of solifluction toward a steady regime of flow in which identical volumes of matter (V) must flow through areas (S) of different cross-sections of flow rock in a unit of time. Therefore, where the rate of flow (u) is small, S must be large, and the reverse, that is, the condition $V = u_1S_1 = u_2S_2$, which leads to the accumulation of matter and a thickening of the flowing layer on sections of the slope with relatively little curvature and rate of flow, is fulfilled.

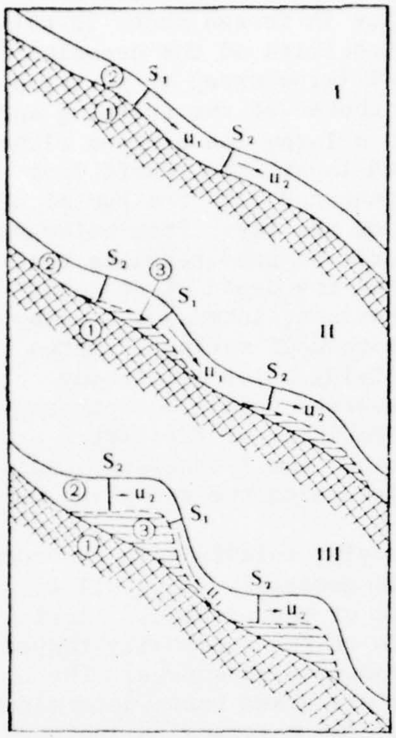

Figure 126. Schematic diagram of the formation of solifluctional terraces: I, II & III -- stages of their formation.

A third factor influencing the formation of solifluctional terraces on slopes is the relative constancy of the average depth of seasonal thawing. As a result of that the lower parts of the thickenings of the flowing layer of soil go over into the permafrozen state, increasing the sharpness of the relief of the upper surface of the permafrozen rock mass and intensifying the difference of the rates of flow of the ground on sections of the slope with a different curvature. The combined effect of those factors forming solifluctional terraces is shown schematically on Figure 126.

The final stage in the development of solifluctional terraces (III) is reached when the gently sloping sections of a slope (that is, the terraces which have formed) become horizontal and the steep slopes approach the angle of rest of thawed rock. In that case solifluction ceases or becomes very slow and steady. In the latter case the solifluctional terraces must slowly move upward over the slope as a result of more intensive drainage of the soils with steep sections of slope and growth of the horizontal sections of the "terraces" upward.

When the process of solifluction reaches the final stage and in the case of the outcropping of bedrocks on the slopes the retreat of terraces upward on the slope can continue, since the steep slopes of the benches are destroyed by weathering processes and the forming silt is distributed over the surface of the terraces. Such terraces are called "upland." The difference between solifluctional and upland terraces is that the formation of the former is directly connected with frost processes, and upland terraces are classed as geological phenomena properly speaking.

As a result of distinctive features of the relief of the slope and the conditions of moistening the ground on separate sections can be liquified and flow under the plant cover, forming "tongues" of soil extended over the slope. The transport of silt on those sections causes a collapse of the belts of vegetation, and in places where the moss and sod layer is broken and in sections of discontinuities the liquid ground flow out on the surface, forming mud flows and debris cones.

A general procedure for calculating the probability of development of the process of solifluction is shown in the following example.

Evaluation of the Conditions of the Formation and Development of Solifluction (Example 54)

In a frost survey it was established that in the region of an investigation solifluction develops on gentle slopes of a lake-alluvial plain in the summer. To determine the conditions of its development and characterize that process, slopes with different steepness and exposure were studied and it was established that solifluction is confined very often to slopes with steepnesses of 8-10 and 15-17^{o}. The soils in the layer of seasonal thawing are composed of light silty loams, the moisture content of which varies from 40 to 45% and the specific gravity of the skeleton of which is 1.2 g/cm^3. The thermophysical properties of the soil are characterized by $\lambda_f = \lambda_t = 1.0$ kcal/(m)(degree)(hr) and $C_{vol-t} = 500$ kcal/(m^3)(degree). In a test of the soils for long shearing strength the following data (Table 74) were obtained. The slopes are grassed under natural conditions. The plant cover is composed of mosses and low bushes. The tensile strength of the sod (σ_t) is ~ 22 g/cm^2.

Table 74 Long-term shearing strength of loams (according to L. A. Zhigarev, 1967)

Влажность w, % к сухой навеске	30	35	40	45
Сопротивление сдвигу $\tau_{дл}$, $г/см^2$	94	51	17	7.5

a - Moisture content w, % of dry weight
b - Shearing resistance σ_{long}, g/cm^2

Table 75 Rates of seasonal thawing of the ground

Месяц	V	VI	VII	VIII	IX
ξ, %	10	55	82	95	100

a - Month

The climatic conditions in the region are characterized by the following: $t_{air} = -10.4^{o}$, $A_{air} = 21^{o}$, $z_{sn} = 0.5$ m and $\rho_{sn} = 0.22$ g/cm^3. The plant cover has a warming influence on the surface of the soil: $\Delta t_{plant} = 0.5^{o}$ and $\Delta A_{plant} = 1.0^{o}$. As a result of observation of the radiation balance of the surface in the summer period data were obtained which made it possible to determine (see example 18) that on slopes with a southern exposure and a steepness of 15^{o} the average annual temperature of the surface is 1^{o} higher and the annual amplitude of temperatures is 2.5^{o} larger than on slopes with a northern exposure and a horizontal surface. On gentler slopes practically no influence of exposure on the arrival of solar radiation is found.

According to weather station data the rates of seasonal thawing of the ground were obtained for the end of each month (Table 75). It is necessary on the basis of the presented materials to determine the conditions of the formation and the duration of solifluction on slopes with steepnesses of 10 and 17^{o}.

Solution. As is known (Zhigarev, 1967), it is a condition of the formation of solifluction that

$$\tau > \tau_{дл} + \sigma_{дл},$$

where τ_l is the long-term shearing strength of the ground, g/cm^2; σ_{long} is the long-term tensile strength of the sod cover, g/cm^2; τ is the tangential stress in the ground, causing the plastico-viscous deformation of the ground on the slope.

Since $\tau = \gamma \xi' \sin\alpha$, where γ is the specific gravity of the ground, kg/cm^3; ξ' is the thickness of the thawing layer at the moment of time under consideration, m; α is the steepness of the slope, then obviously it is possible to readily determine the minimal thickness of the thawing layer of ground on the slope at which solifluction starts:

$$\xi_{мин} = \frac{\tau_{дл} + \sigma_{дл}}{\gamma \sin \alpha}. \qquad (8.6.1)$$

In a frost survey determinations are made for different sections of slopes of the temperature regime (A and t) and the depth of the seasonal thawing of the ground (ξ), and physical (γ and w) and mechanical (τ_{long}) properties of the thawing ground and the sod cover (σ_{long}). Then the course of the seasonal thawing of the ground in time is constructed (by the Tumel' method) and the moment of time when the depth of thawing reaches ξ_{min} is determined. It is obvious that from that time solifluction starts on the slope.

For the given concrete conditions the solution of the problem in such a formulation will be as follows.

1. We determine the temperature regime on the surface of the soil and the depth of seasonal thawing on slopes with a steepness of 10° at a moisture content of the loams of 40 and 45%. According to (5.3.10) $\Delta t_{sn} = 21 \times 0.259 \approx 5.4°$; then

$$t_0 = t_л + \Delta t_{сн} + \Delta t_{расч} = -10,4 + 5,4 + 0,5 = -4,5°,$$
$$A_0 = A_л - \Delta t_{сн} - \Delta A_{расч} = 21 - 5,4 - 1,0 = 14,6°.$$

If we assume that w_{un} = 9% then in accordance with (4.1.8) we find that at a moisture content of 40% Q_ϕ = 30,000 kcal/m^3, and at a moisture content of 45% $Q_\phi \approx$ 35,000 kcal/m^3. With nomograms (see Figure 17) we find that at the initial data t_o = -4.5°, A_o = 14.6°, Q_ϕ = 30,000 kcal/m^3 and C_t = 500 kcal/(m^3)(degree) we obtain ξ = 1.1 m; at the same values but with Q_ϕ = 35,000 kcal/m^3, ξ = 0.9 meter.

2. We calculate the minimal depth of thawing of loams with moisture contents of 40 and 45% at which solifluction starts on slopes with a steepness of 10°.

a) w = 40%, $\gamma = \gamma_{sk}(1 + 0.01\,w)$ = 1.68 g/cm^3, τ_{long} = 1.7 g/cm^2, σ = = 22 g/cm^2, whence according to (8.6.1)

$$\xi_{мин} = \frac{17 + 22}{1,68 \cdot \sin 10°} = 133 \text{ см.}$$

b) w = 45%, γ = 1.74 g/cm^3, τ_{long} = 7.5 g/cm^2, σ = 22 g/cm^2, whence:

$$\xi_{мин} = \frac{7,5 + 22}{1,74 \cdot 0,174} = 97 \text{ см.}$$

The obtained data indicate that under natural conditions on slopes with a steepness of 10° solifluction will not occur, since $\xi_{min} > \xi$.

3. We determine the temperature regime and depth of seasonal thawing of loams with a moisture content of 45% on slopes with southern and northern exposures and a steepness of 17°.

On slopes with a southern exposure:

$$t_0 = t_в + \Delta t_{сн} + \Delta t_{раст} + \Delta t_R = -10,4 + 5,4 + 0,5 + 1 = -3,5^\circ,$$
$$A_0 = A_в - \Delta A_{сн} - \Delta A_{раст} + \Delta A_R = 21 - 5,4 - 1 + 2,5 = 17,1^\circ.$$

At t_o = -3.5°, A_o = 17.1°, Q_ϕ = 35,000 kcal/m^3, C_t = 500 kcal/(m^3)(degree) with a nomogram (see Figure 17) we find that $\xi \approx$ 1.3 meter.

On slopes with a northern exposure the conditions are similar to those observed on gentler slopes and horizontal sections, that is, t_o = -4.5°, A_o = 14.6° and ξ = 0.9 meter at w = 45%.

4. We calculate the minimal depth of thawing of loams with a moisture content of 45% at which solifluction starts on slopes with a steepness of 17°:

$$\xi_{мин} = \frac{7,5 + 22}{1,74 \cdot \sin 17^\circ} = \frac{29,5}{1,74 \cdot 0,292} \approx 58 \text{ см.}$$

Consequently, on both northern and southern slopes with a steepness of 17° solifluction will develop.

5. To determine the time of the start of solifluction, we determine the course of thawing of loams in time by the Tumel' method, having the relative rate of thawing of the ground on the basis of the data of weather stations. If we assume the maximal depth of thawing, calculated at the end of summer, to be 100%, we obtain the depth of thawing of loams with a moisture content of 45% on slopes with northern and southern exposures (Table 76).

Table 76 Depths of thawing (meters) on slopes with southern and northern exposures

	V	VI	VII	VIII	IX
a На склонах южной экспозиции	0,13	0,71	1,1	1,23	1,3
b На склонах северной экспозиции	0,09	0,50	0,[illegible]6	0,85	0,9

Key: a - on slopes with a southern exposure; b - on slopes with a northern exposure

Since the minimal depth of thawing of loams with a moisture content of 45% on slopes with a steepness of 17° is 0.58 meter, it is obvious that on slopes with a southern exposure solifluction starts at the end of the second 10-day period of June, and on the northern in the middle of July, that is, more than a month later.

In conclusion it can be determined whether solifluction starts on slopes with a steepness of 10° if the sod and plant cover is removed in the process of economic opening up.

6. We determine how the temperature regime on the surface of the soil changes if upon removal of the plant cover the height of the snow on the slopes decreases to 0.3 meter (ρ = 0.22 g/cm^3) and the radiation corrections increase and are equal* to: $\Delta t_R = 0.8^{\circ}$ and $A_R = 2^{\circ}$

$$\Delta t_{\text{сн}} = 21 \cdot 0{,}164 = 3{,}4^{\circ},$$
$$t_0 = -10{,}4 + 3{,}4 + 0{,}8 = -6{,}2^{\circ},$$
$$A_0 = 21 - 3{,}4 + 2 = 19{,}6^{\circ}.$$

7. We find the depth of seasonal thawing of loams with a moisture content of 45% (Q_{ϕ} = 35,000 kcal/m^3, C_{vol-t} = 500 kcal/(m^3)(degree) on sections of slopes with the plant cover removed. With a nomogram (Figure 17) we find that ξ = 1.1 meters.

8. We determine the minimal depth of thawing of loams on slopes with a steepness of 10° with the sod and plant cover removed at which solifluction can start:

$$\xi_{\text{мин}} = \frac{7{,}5}{1{,}74 \cdot 0{,}174} = 25 \text{ см}.$$

Consequently, on slopes with a steepness of 10° after removal of the plant cover solifluction starts to develop already from the first half of June, whereas under natural conditions it was not observed at all.

The considered regularities testify that very favorable conditions for the development of solifluction are noted in the Extreme North in frost-temperature zones IV and V. Noted there is a minimal thickness of the seasonally thawing layer, a mainly silty composition of the surface deposits and a high moisture content of the soils, often equal to the absolute moisture capacity. In frost-temperature zone III those conditions are less favorable and the processes of solifluction are confined to limited sections. In zones I and II solifluction phenomena are encountered in the form of upland solifluctional terraces and are linked with the flow of soils on sloping sections and steep slopes.

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*Under natural conditions in the presence of a plant cover on slopes with a steepness of 10° those corrections will be negligibly small.

7. Regularities of the Formation of Landslides, Runoffs, Mud Flows, Mud Streams and Avalanches

Moistening of the soil on the surface of frozen rock masses on slopes can reduce the strength of adhesion of the soil layer to the frozen rock mass and cause slides of extensive sections of the layer of summer thawing. At times sliding of such layers, bearing a forest cover, is observed (Tyrtikov, 1963). When the sliding layer has great fluidity, solifluctional runoffs form with an irregular movement of the mass of the soil.

At a still greater fluidity of the soil, usually in narrow valleys of mountainous rivers and small streams, large mud flows can form. Such mud flows are especially large in the valleys of mountain rivers flowing from under glaciers, and at times have catastrophic consequences. Masses of immobile ice or slides of rocks can dam up the bottom and contribute to an accumulation of liquefied soil. Upon the melting of the ice or bursting of the blocking obstacle the liquefied soil and water carrying a mass of boulders, rock fragments and broken trees rush down, washing away everything in their path. Such catastrophic mud flows are called mud streams.

In the mechanism of their development and manifestation some snow avalanches are similar to mud streams. They are divided into avalanches of clean dry snow and wet avalanches, carrying large masses of silt, boulders and tree fragments. The latter type of snow avalanches are of great geomorphological importance, as they move considerable masses of material.

The physical conditions of the formation of landslides, runoffs and slides are determined by the angle of inclination of the slip planes and the value of the coefficient of friction or angle of internal friction of the soils. Those two conditions are determined for frozen masses by the composition of the rocks, their moisture content and cryogenic textures, and also the conditions of bedding of the frozen rocks (their stratification, connected with different lithology and moisture content) and the character of the subsurface waters.

Landslides, runoffs and slides in the area of permafrozen rocks are mainly confined to the contact of thawed masses of soil with frozen ones. The slip plane is usually the icy boundary of the frozen rock mass. The development of processes is noted during increase of the thickness of the seasonally thawed layer on steeper sections of a slope and during irregular thawing of frozen rock masses on steep sloping sections, when the thickness of thawing is greater upwards on the slope than downwards on the slope. Especially favorable conditions are created when during the thawing of permafrozen rock masses slip planes form according to the descent of the layers. In addition, in the region of permafrozen rock masses the landslides, runoffs and slides are connected with distinctive features of the moisture regime on the contact of the frozen and thawed rocks. The underlying mass of frozen rocks is a confining bed and therefore in the process of summer or perennial thawing the soils of the thawed layer on that contact are greatly moistened.

Within the limits of frost-temperature zone I the landslides, runoffs and other slides are linked for the most part with perennial thawing of frozen rock masses, and so the processes spread to a greater depth and often embrace larger masses of soil.

In more northern zones the development of landslides, runoffs and other slides is connected mainly with the seasonal thawing of rocks. In frost-temperature zones IV and V those processes proceed in connection with thermal abrasion, thermal erosion and the thawing of thick ice bodies in uncovered scarps.

The probability of the formation of runoffs, landslides and other slides is determined by the configuration of planes of contacts of the thawed part of a mass with the frozen part and is connected with the character of the soils and their properties on that contact.

The configuration of the plane of contact depends on the dynamics of change of frost conditions. Especially sharp changes can exist in different kinds of construction. The probability of the formation and development of those processes must be predicted on the basis of a general forecast of change of the frost conditions with consideration of the character of the productive opening up of the territory. If calculations have established the probability of the formation of a slide or runoff of ground on the slip plane, it is advisable to make provision for measures to preserve the rocks in the frozen state and at the same time prevent the development of the process. Measures of improvement can be calculated by determining the influence of various factors on the temperature regime and depth of thawing of the frozen rock masses (see Chapters 4 and 5).

8. The Development of the Process of Differentiation of Large-Fragment Material

During the freezing of the layer of summer thawing, in the region of permafrozen rocks there is a redistribution of the moisture in that layer on account of the formation of ice schlieren of segregation ice. The heterogeneity of the soils in the layer of summer thawing leads to difference in the intensity of its freezing. In the presence of boulder-pebble and block material in alluvial, proluvial and deluvial deposits under boulders, blocks and rock debris the freezing sets in earlier than on neighboring sections, and therefore large ice lenses form under them. This in turn causes bulging of the rock material by the amount of the ice schlieren. In the summer period the ice lenses and schlieren thaw, but the elevated bounders and rock debris do not return to their previous position when they settle, but remain elevated by several millimeters as compared with the position of the previous year as a result of the flow of rock particles with the water into the formed space. During freezing in the course of a number of years the rock material of the layer of seasonal thawing bulged completely out on the surface. The length of that process can be estimated as several tens (or even hundreds) of years, taking into consideration that the yearly bulging can reach several millimeters, and sometimes centimeters.

The process of differentiation of large fragment material can proceed in parallel with the process of formation of slope deposits. In that case frost differentiation of the deposits on slopes proceeds syngenetically.

Near the southern boundary of the region in which permafrozen rocks are prevalent the thickness of the layer of summer thawing reaches very large values. The freezing of the layer of thawing in that case proceeds only upwards and therefore ice lenses form under the lower surface of the boulders and block material, which leads to their intense bulging. The thickness of the ice schlieren and lenses forming as a result of moisture migration toward the front of freezing in the presence of water above the frost increases with depth. This is connected with deceleration of advance of the front of freezing deep into the freezing layer. By virtue of that the process of bulging of rock material in southern regions embraces the entire thickness of the layer of summer thawing. In northern regions where the thickness of that layer is extremely small and where 30-50% of the thickness of the thawing layer freezes from the bottom, the intensity of bulging of the rock material becomes far smaller and embraces mainly the upper part of the layer.

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Chapter 9. Solution of Some Geocryological Problems of Heat and Mass Transfer

The theoretical principles of heat and mass transfer in frozen rocks, presented in Chapter 3, make it possible to pose and solve a number of special questions of great practical importance. In particular, in examining the formation of seasonally and perennial frozen rocks, of basic importance is the examination of periodic temperature fluctuations on the surface, the character of the change of the temperature field and the dynamics of the processes of freezing with consideration of phase transitions under the conditions of a multifront problem. Section 1 of this chapter is devoted to an examination of a complex of those questions. Evaluation of the influence of phase transformations in the range of negative temperatures is of great importance in studying the process of freezing of rocks in both the seasonal and the perennial cycles. Section 2 is devoted to this. The application of calculating methods in considering moisture migration in the process of freezing is examined in section 3. These questions qunatitatively determine not only the course of the process of freezing but also the formation of cryogenic textures of seasonally and perennially frozen rocks, and also their heaving. Given in section 4 is a formulation of the problem of thawing of coarsely dispersed rocks with consideration of the infiltration of summer precipitations and its digital computer solution under concrete conditions. The concluding section of this chapter consists of examples of the solution of concrete problems connected with the freezing and thawing of rocks which are of great importance in the solution of some practical questions of frost prediction.

1. Solution of the Problem of the Temperature Regime in a Homogeneous Medium With a Periodically Varying Phase State of the Water

One of the most important practical applications of the problem of determining the temperature field and the rate of advance of the interfaces in a multizone medium* (the Stefan problem) is the case of periodic change of the phase state of the water in separate layers of rock, connected with change of the boundary conditions. As a rule, a triple-layer medium forms in that case. Typical examples are the seasonal freezing and subsequent thawing of moist ground, the formation and development of permafrozen rocks, etc. The scheme

*Here and henceforth by zones are understood layers of ground in the thawed and the frozen states.

of the process under consideration in a very simple case is as follows: At the initial moment a homogeneous medium is in a single-zone thawed (frozen) state. From the moment of sign inversion of the temperature on the surface a new zone forms which develops until the moment of the subsequent sign inversion on the surface (a triple-layer medium). When the two mobile interfaces merge, a single-layer medium again forms, etc. A solution of such a multifront problem is readily found by reducing the Stefan problem to a system of ordinary differential equations (Chapter 3, section 13). Presented as an example on Figure 127 are the results of calculation of the temperature regime of a layer of ground with a depth of up to 12 meters in the course of a year at the surface temperature:

$$\Phi_0(\tau) = t_0 - 24 \sin \frac{2\pi}{T} \tau,$$

where $t_0 = \pm 3$ and $\pm 12^{\circ}$, $T = 8760$ hours. In that case it is assumed that $\lambda_t = \lambda_f = 1.07$ kcal/(m)(degree)(hr), $Q_\phi = 23{,}680$ kcal/m^3 and $\alpha_t^2 = \alpha_f^2 = 2.14 \times 10^{-3}$ m^2/hr. Let us note that, as is evident on Figure 127, thawing from below becomes noticeable in practice at ground temperatures not lower than $+5^{\circ}$.

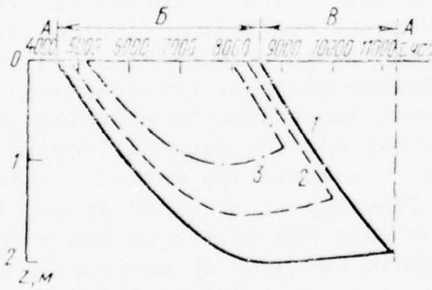

Figure 127. Dynamics of the fronts of freezing and thawing in time at different average surface temperatures: 1 - ± 3, 2 - ± 7, 3 - $\pm 12^{\circ}$C; A, B and C are the single-, double- and triple-layered media respectively.

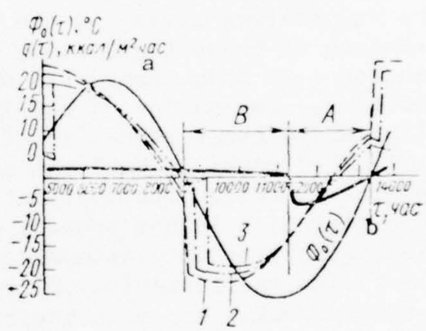

Figure 128. Course of the variation of heat flows at different depths in the course of the annual cycle at $t_m = 3^{\circ}$, $A_0 = 24^{\circ}$: 1 - z = 0 m, Q = 62,300 kcal/m^2; 2 - z = 0.2 m, Q = 57,800 kcal/m^2; 3 - z = 0.5 m, Q = 48,600 kcal/m^2; 4 - (heavy line) - z = 2 m, Q = 8700 kcal/m^2; A and B are the single and triple-layered media respectively. a - $q(\tau)$, kcal/(m^2)(hr) b - τ, hrs

The indicated method of solving a multifront Stefan problem permits simultaneously investigating heat flows in the ground and also calculating the heat cycles of the soil during a given course of temperature variation in time on its surface during any time interval. In that case the input and output parts of the heat cycles are calculated separately (that is, the summary value of the heat cycles with one and the same sign) during an entire cycle or part of it. These characteristics can be calculated both for the surface of the soil and for any given depth. The results of calculation of the curves of variation of heat fluxes q in the time τ and the corresponding values of the heat cycles in the annual cycle Q at different depths are presented in Figures 128-130. During transition of the zone interfaces through the point where the heat fluxes are measured the latter undergo sharp changes*. The maximal error in the calculation of heat cycles does not exceed 1%.

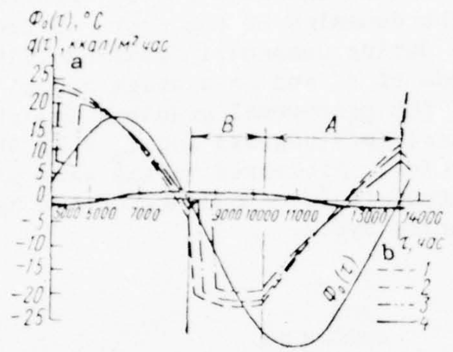

Figure 129. Course of the variation of heat fluxes at different depths in the course of the annual cycle at $t_m = -7^o$ and $A_0 = 24^o$: 1 - z = 0 m, Q = 61,200 kcal/m^2; 2 - z = 0.2 m, Q = 55,600 kcal/m^2; 3 - z = 0.5 m, Q = 46,200 kcal/m^2; 4 - z = 5 m, Q = 3800 kcal/m^2; A and B are the single- and triple-layered media. a - q(τ), kcal/(m^2)(hr) b - τ, hrs

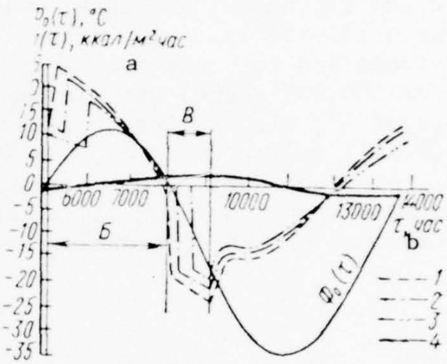

Figure 130. Course of the variation of heat fluxes at different depths in the course of the annual cycle at $t_m = -12^o$ and $A = 24^o$: 1 - z = 0 m, Q = 56,500 kcal/m^2; 2 - z = 0.2 m, Q = 51,000 kcal/m^2; 3 - z = 0.5 m, Q = 41,700 kcal/m^2; 4 - z = 5m, Q = 4700 kcal/m^2; B and C are the single- and triple-layered media. a and b as for Figure 129.

At different values of t_o the rate of change of $\phi_o(\tau)$ and the value of the heat flow to the surface at the moment of sign inversion $\phi_o(\tau)$ are different, and this leads to a different value of the jumps. The formation of a triple-layered medium leads to a discontinuity of the curves of the heat fluxes at the points of the internal zone at which the temperature starts to decrease, approaching a zero gradientless distribution.

As in the absence of phase transitions, in the case under consideration of a multizone medium at different thermophysical characteristics in the frozen and thawed states, the heat input into any section during a cycle in a periodically established regime is equal to the output.

The proposed numerical method can also be used in the case where, besides the temperature variation on the surface of the soil, in the course of the year there is a variation in the average annual temperatures of the ground from year to year. In that case the input part of the heat balance of the soil will not be equal to the output part. Their difference gives the amount of

*This is very clearly manifested during formation of a new zone.

heat in calories accumulated by the rock mass if it is heated in a perennial cycle or lost by it in a cooling cycle. Thus it is possible to calculate in calories the heat balance of the soil both during degradation of permafrozen rock masses and during their aggradation. The problem of perennial freezing (thawing) is calculated similarly. It is characteristic here that on the lower boundary of the region of investigations in that case it is necessary to give the geothermal gradient, which plays an enormous role. Presented on Figures 131-133 are calculations of the dynamics of the depth of freezing of the rocks and the heat cycles in time during perennial freezing (with a period of 100,000 years) with an amplitude of 6^{o} and an average annual temperature of 4^{o}, with different values of the geothermal gradient (g) at λ = 0.69 kcal/(m)(hr)(degree), C = 500 kcal/(m^3)(degree) and Q_ϕ = 23,680 kcal/m^3, obtained during two complete cycles. Presented in the same places are the values of the input (Σ_q^-) and output (Σ_q^+) components of the heat cycles through the surface during the entire cycle.

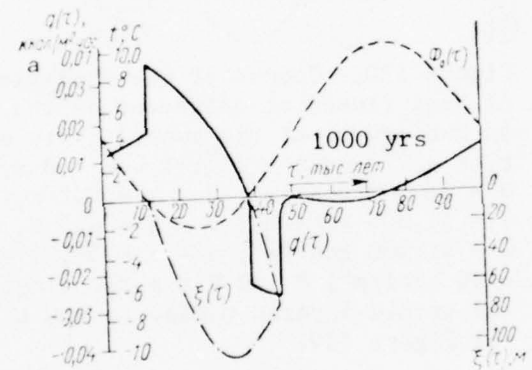

Figure 131. Dynamics of the depth of freezing (thawing) $\xi(\tau)$ and the heat fluxes on the surface of the ground during perennial freezing (g = 0.01 degree/m; $\Sigma_q^- \approx \Sigma_q^+ \approx$ 4,840,000 kcal/m^2) a as on Figure 129

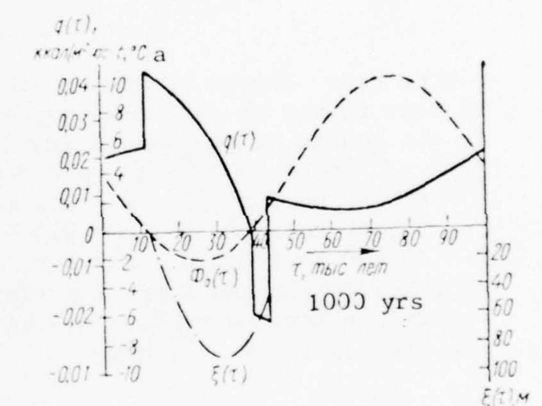

Figure 132. Dynamics of the depth of freezing (thawing) $\xi(\tau)$ and the heat fluxes on the surface of the ground during perennial freezing (g = 0.02 degree/m, $\Sigma_q^- \approx \Sigma_q^+ \approx$ 4,287,000 kcal/m^2) a as on Figure 129

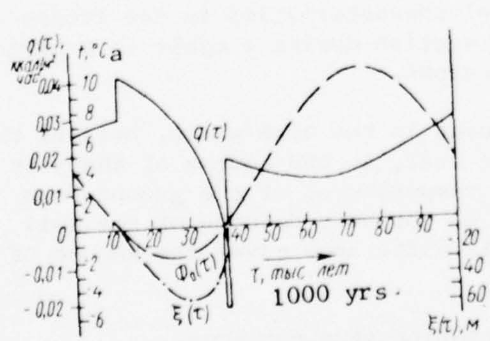

Figure 133. Dynamics of the depth of freezing (thawing) $\xi(\tau)$ and the heat fluxes on the surface of the ground (g = 0.03 degree/m, Σ_q^- = 3,846,500 and Σ_q^+ = 3,096,100 kcal/m^2) a as on Figure 129

It is evident from the data of Figures 131-133 that at small values of g the heat balance after two cycles is fulfilled within the limits of the precision of the calculations. The difference between the input and output components of the heat cycles during considerable geothermal gradients is connected with the fact that the time of achievement of a periodically steady regime is in that case rather large.

1. Numerical Calculation of Heat Cycles in Soils With Phase Transitions of Water as a Criterion for Determination of the Temperature Shift

The input and output parts of the heat balance during a complete cycle T at an arbitrary depth z_k within the layer of annual fluctuations are calculated in the form

$$\int_{\tau_1}^{\tau_2} q(z_k, \tau)\, d\tau \text{ и } \int_{\tau_2}^{\tau_1+T} q(z_k, \tau)\, d\tau,$$

where τ_1 and τ_2 are the roots of the equation $q(z_k, \tau) = 0$. It is obvious that in the case of a periodically steady temperature regime the output part of the heat cycles must be equal to the input part, and the heat balance in any section must be equal to zero. Both theoretically and under natural conditions, in cases where the thermophysical characteristics of the medium in the presence of phase transformations of water are practically unchanged, such a situation is encountered only in the presence of equality of the average annual temperature on the surface of the soil (t_o) and at the depth of the base of the layer of annual temperature fluctuations (t_H)*.

In the presence of inequality of the thermophysical characteristics during phase transitions, and primarily at $\lambda_t \neq \lambda_f$, the average annual temperature on the surface of the soil t_o in a periodically steady regime will be different from the average annual temperature of the ground at the base of the layer of seasonal freezing (thawing), and in the absence of a gradient in the layer of annual temperature fluctuations (H) -- at the base of that layer by the value of the so-called temperature shift Δt_λ ($\Delta t_\lambda = t_o - t$) (section 2, Chapter 5). The sign of the shift is determined by the value of the ratio λ_t/λ_f, namely

$$\operatorname{sgn} \Delta t_\lambda = \operatorname{sgn}\left(\frac{\lambda_т}{\lambda_м} - 1\right), \text{ т. е. } \Delta t_\lambda > 0 \text{ при } \frac{\lambda_т}{\lambda_м} > 1,$$

$$\Delta t_\lambda < 0 \text{ при } \frac{\lambda_т}{\lambda_м} < 1.$$

In addition, in the case of freezing of the ground ($t_o > 0$) the shift at $\lambda_t/\lambda_f > 1$ is greater than in the similar case during thawing. This divergence in shifts, as well as the value of the shifts themselves, increases with increase of the heat expended on the phase transformations of water. Therefore in calculations of heat cycles in the ground it is necessary to take into consideration the amount of the shift in designating the lower boundary condition.

*In concrete calculations the latter, as a rule, is taken to be the lower boundary condition ($t_o = t_H$ = constant).

Finding the amount of the shift in a problem with mobile interfaces analytically encounters a number of mathematical difficulties and in practice has not been worked out. There is only an approximate formula proposed by V. A. Kudryavtsev (see section 1, Chapter 4, and section 2, Chapter 5). At the same time the calculation of heat fluxes in different sections in the upper layers of the lithosphere in the course of a complete cycle makes it possible to determine with adequate precision the value of Δt_λ. This is achieved by the fact that when the value of Δt_λ has been incorrectly selected the input and output parts of the heat cycles in any section during a year (positive and negative components) will not coincide. When the value of Δt_λ is reduced (the soil temperature at the depth of the annual zero amplitudes has been overstated) the heat input in sections within the layer of freezing (thawing) is greater than the output, and below the active layer is smaller. But if Δt_λ has been overstated, the reverse regularity will be noted. On the basis of that position it is possible to propose a procedure for calculating the temperature shift by successive approximations. Having obtained the heat cycles at an arbitrarily taken value of t_H, it is necessary, depending on the value and sign of the inbalance, in various sections to vary t_H before establishing (with the prescribed precision) equality of input and output of heat in them*.

The difference between t_o and t_H in that case will be equal to the temperature shift in the given concrete case.

In accordance with the indicated procedure a number of calculations have been made of heat cycles in a layer of ground with the depth 1 = 12.05 meters in the sections z = 0, 0.2, 0.5 and 5.0, as a result of compilation of which the amount of the shift was determined under the following conditions: $\alpha_t^2 = 2.14 \times 10^{-3}$, $\alpha_f^2 = 1.38 \times 10^{-3}$ m^2/hr, $\lambda_f = 1.07$, $\lambda_t = 0.69$ kcal/(m)(hr)(degree), $Q_\phi = 23{,}680$ kcal/m^3, $\phi_o(\tau) = t_o - 7.5 \sin \frac{2\pi}{T}\tau$ and T = 8760 hours. Presented on Figures 134 and 135 are curves of the change of heat fluxes in the course of time in the indicated sections, the course of the interfaces $\xi_1(\tau)$ and $\xi_2(\tau)$ (the number of zones varies from 1 to 3) and graphs of $\phi_o(\tau)$ at $t_o = \pm 3^\circ$. As a result of the calculations the following values of Δt_λ were determined:

$$\text{при } t_0 = 3^\circ \; \Delta t_\lambda = 0{,}75^\circ \; (t_H = 2{,}25^\circ),$$
$$\text{при } t_0 = -3^\circ \; \Delta t_\lambda = 0{,}45^\circ \; (t_H = -3{,}45^\circ).$$

The values of the input and output parts of the heat cycles in the sections under consideration obtained at the found values of Δt_λ are presented in Table 77. Presented there are the results of calculation of the heat cycles at an incorrectly selected value of Δt_λ, when at $t_o = 3^\circ$ Δt_λ was originally assumed to be equal to the shift at $t_o = -3^\circ$, that is, was understated (0.45 instead of 0.75).

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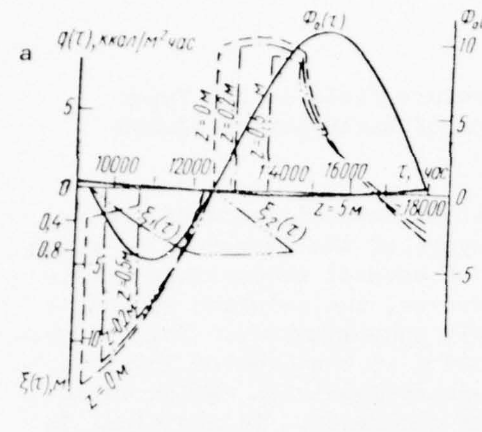
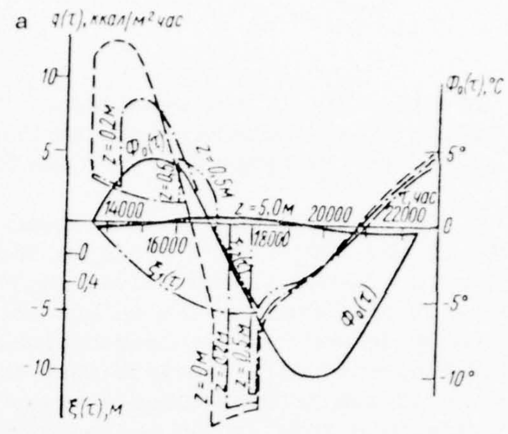
*The course of the interfaces changes insignificantly in that case.



$$\Phi_0(\tau) = 3 + 7{,}5 \sin \frac{2\pi}{8760}\tau$$



$$\Phi_0(\tau) = -3 + 7{,}5 \sin \frac{2\pi}{8760}\tau$$

Figure 134. Course of freezing and thawing and the dynamics of the heat fluxes at different depths at

a - $q(\tau)$, kcal/(m^2)(hr)

Figure 135. Course of freezing and thawing and the dynamics of the heat fluxes at different depths at

a - $q(\tau)$, kcal/(m^2)(hr)

Table 77 Results of calculations of heat cycles at different depths at different values of Δt_λ

A Глубина, м	B Теплообороты, ккал/м²	$t_0=-3°$, $\Delta t_\lambda=0{,}45°$	$t_0=3°$, $\Delta t_\lambda=0{,}45°$	$t_0=3°$, $\Delta t_\lambda=0{,}75°$
0	a расход	−27 464	−27 542	−26 726
0,2		−19 964	−22 601	−21 396
0,5		−10 962	−14 079	−13 279
5,0		−128	−784	−591
0	b приход	27 507	30 1,53	26 908
0,2		20 090	24 725	21 504
0,5		10 914	16 305	13 190
5,0		130	667	605

Key: A - Depth, meters B - Heat cycles, kcal/m^2 a - output b - input

It follows from Table 77 that the balance of heat during a cycle depends sharply on the amount of the temperature shift. Quite analogously the value of Δt_λ is found in the case of boundary conditions of the second kind on any of the boundaries of the layer of annual fluctuations.

2. Investigation of the Dynamics of the Temperature Field in the Upper Layers of the Lithosphere With Consideration of Early-period Fluctuations of the Temperature of the Surface

One of the cardinal questions of geocryology is the investigation of the dynamics of the temperature field in the upper layers of the lithosphere, which forms as a result of the mutual superposition of several temperature oscillations of different period on the surface. However, the solution of that problem up to now has been accomplished extremely approximately. This is primarily connected with the fact that the examination of the problem involves solution of the problem under a superficial boundary condition which varies according to a complex law in the course of long intervals. In addition, in the given case the problem is known to be a multifront one, as the number of mobile boundaries within the limits of the largest of the periods of temperature oscillation on the surface varies from 0 to 3 or more. Since the difference between the periods of the oscillations is rather large, in the course of the process there systematically occurs the formation of a zone with a surface and the degeneration of one or several zones into a point. Finally, in the case under consideration, considering the length of the process, the region of investigation also must be rather large (of the order of $\sum_{s=1}^{N} l_s$, where l_s is the depth of propagation of the s-th oscillation of temperature on the surface, which is found with consideration of the corresponding amplitude and period of oscillations, analogously to the depth of the "zero" annual amplitudes with a reserve from the well-known Fourier solution (3.3.4), and N is the number of vibrations). Therefore on the lower boundary of the region of investigation it is necessary to give a boundary condition of the second type, determined by the value of the geothermal gradient.

Everything said above has the result that the use of known difference methods of solving Stefan problems in the investigation of the problem under consideration is made difficult.

At the same time, for numerous cases where the heterogeneity of the medium within the limits of the region of investigation can be neglected, the solution of the problem of the dynamics of the temperature field in the upper layers of the lithosphere can be rather simply and effectively found by the method of reducing the Stefan problem to a system of ordinary differential equations. Let at the initial moment ($\tau = 0$) the medium be in the thawed (frozen) state, that is, single-phase. Then the change of the phase state can be presented schematically in the following form (for simplicity we will limit ourselves to a 5-front problem) (Figure 136). There the figures indicate the number of zones, and cases the probability of the formation of which is low are marked with an asterisk.

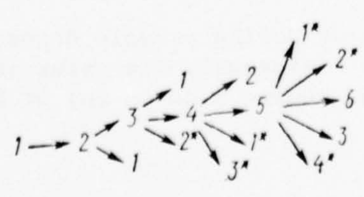


Figure 136. Schematic diagram of the change of the number of zones during perennial freezing and thawing.

As an illustration we will examine the dynamics of the temperature field at

$$\Phi_0(\tau) = 2 + 20 \sin \frac{2\pi}{T} \tau \pm 2 \sin \frac{2\pi}{11T} \tau \pm 2 \sin \frac{2\pi}{40T} \tau,$$

where T is the period of annual oscillations (8760 hours). The selection of the indicated harmonics (annual, 11- and 40-year oscillations) and their amplitudes was determined with the data of regime observations in the Skovorodino borehole. On the lower boundary of the region of investigation (l = 200 meters) is given the heat flux corresponding to different values of the geothermal gradient g.

Adopted in the calculations were the following values of the thermophysical characteristics: $\lambda_f = \lambda_t = 1$ kcal/(m)(hr)(degree), $C_f = C_t = 500$ kcal/(m^3)(degree) and $Q_\phi = 25{,}000$ kcal/m^3.

The results of numerical integration of the problem under consideration within the limits of half the largest of the periods (40 years) are presented in Figures 137 and 138. If the oscillations coincide in phase (in that case the change of the average annual temperature on the surface $t^*(\tau)$ in time is written in the form:

$$t^*(\tau) = 2 + 2 \sin \frac{2\pi}{11T} \tau + 2 \sin \frac{2\pi}{40T} \tau),$$

the calculations were made at a geothermal gradient of 0.03 degree/m. However, in the case where the perennial oscillations are in counterphase with the annual, that is,

$$t^*(\tau) = 2 - 2 \sin \frac{2\pi}{11T} \tau - 2 \sin \frac{2\pi}{40T} \tau,$$

the examination was conducted at different values of the geothermal gradient (g = 0.01 and 0.03 degree/m).

Figure 138 presents the corresponding diagrams of $t^*(\tau)$ within the limits of the time interval under consideration. As follows from an examination of Figure 138, when the oscillations coincide in phase, within the next 20 years $t^*(\tau)$, while varying on account of perennial oscillations, remains positive. In that case it was assumed that $\lambda_t = \lambda_f$ and, consequently, the number of zones does not exceed 3. The dependence of the maximal depth of freezing on time reflects completely the dynamics of change of $t^*(\tau)$. As follows from Figure 138, under the given conditions the formation of a frozen layer (intergelisol) will set in not earlier than the 28th year. It is obvious that if a temperature shift occurs, then by at least the 20th year the formation of an intergelisol of small thickness will occur, the time of existence of which will be short.

However, if the perennial oscillations are in counterphase with the annual, $t^*(\tau)$ within the limits of the time interval under consideration undergoes sign inversion 4 times. However, the formation and length of existence of the intergelisols in that case depends sharply on the value of the geothermal gradient. Thus, at a maximal value of g (0.03 degree/m), in spite of the negative sign of $t^*(\tau)$, in the interval from 1.5 to 5 years the formation of permafrost does not set in. In that case the depth of freezing reaches 2.15

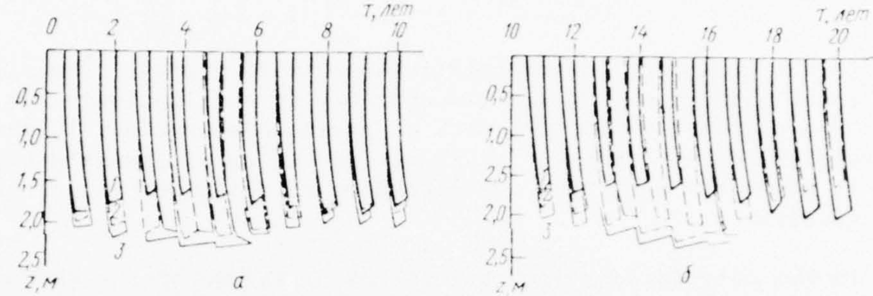


Figure 137. Change of the depth of freezing (thawing) in time: a - in a period of 1-10 years; b - 11-20 years 1 - τ, years

1 — $g = 0{,}03$ град/м,

$$t^*(\tau) = 2 + 2\sin\frac{2\pi}{11T}\tau + 2\sin\frac{2\pi}{40T}\tau;$$

2 — $g = 0{,}03$ град/м,

$$t^*(\tau) = 2 - 2\sin\frac{2\pi}{11T}\tau - 2\sin\frac{2\pi}{40T}\tau;$$

3 — $g = 0{,}01$ град/м,

$$t^*(\tau) = 2 - 2\sin\frac{2\pi}{11T}\tau - 2\sin\frac{2\pi}{40T}\tau.$$

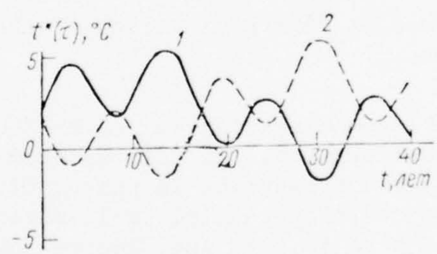


Figure 138. Change of the average annual temperature of the surface of the ground in time: 1 - τ, years

$$1 - t^*(\tau) = 2 + 2\sin\frac{2\pi}{11T}\tau + 2\sin\frac{2\pi}{40T}\tau;$$

$$2 - t^*(\tau) = 2 - 2\sin\frac{2\pi}{11T}\tau - 2\sin\frac{2\pi}{40T}\tau$$

m. The seasonal freezing is replaced by seasonal thawing only during the subsequent establishment of negative values of $t^*(\tau)$ (11-15.8 years); an intergelisol with a thickness of 12 cm forms in the 14th year and disappears after 2.5 years. The total thickness of the frozen ground in the given cases varies from 1.68 (20th year) to 2.29 m (15th year). However, if the geothermal gradient is 0.01 degree/m, the formation of intergelisols occurs during both the first and third sign inversions of $t^*(\tau)$. The values of the intergelisols corresponding to that reach 10 and 22 cm, and the thickness of the frozen ground is 2.2 and 2.38 m. The delay in the formation and disappearance of intergelisols in comparison with the corresponding moment of inversion of $t^*(\tau)$ reaches 1.5-2 years, that is, during decrease of the value of g the length of existence of the permafrozen layer increases by 1.5 years. The minimal depth of freezing in the first 17 years reaches 1.94 m in the 8th year at g = 0.01 degree/m.

Thus, in spite of the fact that, during change of the geothermal gradient in rather broad limits the total depth of the ground frozen in the course of 20 years differs insignificantly, from the qualitative point of view the picture changes sharply. Increase of g leads to a considerable decrease of the length of existence of intergelisols. Finally, if at small values of the geothermal gradient negative temperatures on the surface, as a rule, lead to the formation of intergelisols, at a normal geothermal gradient the formation of permafrozen ground occurs only in cases where the sum of the frost-degree-hours [determined on the basis of $t^*(\tau)$] is at least 30-35 x 10^3 degree-hours. It is obvious that in the presence of a temperature shift everything said above must be corrected in an appropriate manner.

2. Investigation of the Processes of Freezing and Thawing of Rocks With Consideration of Phase Transitions of Unfrozen Water By Means of a Self-modeling Solution

1. Freezing (Thawing) of Porous Bodies With Consideration of the Curve of Unfrozen Water in a Self-modeling Case (the Classical Stefan Problem)

At the present time a solution of the problem of the freezing of porous bodies in the range of negative temperature during arbitrary boundary conditions is possible only by one of the difference methods of solving a quasi-linear Stefan problem. Of great interest in that connection is the determination by means of a self-modeling solution of some general regularities of the influence of the ice-content curve on the process of freezing in order to use them subsequently in correcting the results of corresponding calculations in the generally useful classical Stefan formulation (section 7, Chapter 3.

For a quantitative estimate of the influence of phase transitions of water on the course of freezing and thawing a number of calculations of the freezing of real soils -- loam and sand (their thermophysical properties in the thawed and frozen states, and also the effective heat capacity C_{eff}, are presented in Table 78) were made by means of a self-modeling solution on a continuously acting IPT-5 computer installed in the Department of Geocryology of MSU and also a "Strela" digital computer of the MSU Computer Center.

Table 78 Thermophysical characteristics of loam and sand adopted in calculations

A Обозначение	B Размерность	C Суглинок	D Песок
λ_0 2	ккал/м·час·град	0,86	1,04
$\lambda(0-0)$	»	1,13	1,34
Q 3	ккал/м^3	9 669,6	16 054,53
C_0 4	ккал/м^{3}град	678,49	551,35
1 $C_{эф}(0-0°)$	»	10 330,55	12 450,94
$C_{эф}(-0.5°)$	»	5 261,41	6 621,50
$C_{эф}(-1°)$	»	3 641,82	1 612,89
$C_{эф}(-2,5°)$	»	2 058,32	588,72
$C_{эф}(-5°)$	»	1 425,06	526,57
$C_{эф}(-10°)$	»	1 311,96	455,25

Key: A - Symbol B - Dimensions C - Loam D - Sand
1 - C_{eff} 2 - kcal/(m)(hr)(degree) 3 - kcal/m^3 4 - kcal/(m^3)(deg)

The calculations were made at different surface temperatures T_o and initial temperatures of the medium T_1 both with consideration of the ice-formation curve and in the Stefan formulation. In the latter case it was assumed that if the temperature $T_o > 0^o$ is given on the surface then the quantity of heat of phase transitions at any values $t < 0^o$ corresponds to the ice-formation curve at the temperature $t = T_o$.

The influence of the curve of ice formation on the depth of freezing can be estimated by comparing the values of α, which determine the position of the zero isotherm in time (section 8, Chapter 3), obtained in the two cases under consideration.

The results of calculation of those values of α as a function of T_1 at different values of T_o are represented by the solid and broken lines respectively on Figure 139 for the case of loam (heavy line) and sand (light line). As follows from an examination of them, in the case of calculation of the curve of ice formation the process of freezing proceeds more intensively than in the ordinary Stefan formulation. Maximal divergence occurs at the zero initial temperature of the medium and gradually decreases during its increase.

As follows from Figure 139, at $T_o = -5^o$ the maximal divergence for loam is 15.5%, whereas it is only 3.1% for sand. In that case the reduction of the temperature on the surface from -0.5 to -5^o affects greatly the increase of divergence, as it continues to increase at $T_o < -5^o$, whereas in the case of sand the maximum of divergence is weakly expressed and at $T_o < -1^o$ (outside the region of substantial phase transitions) the divergence gradually decreases.

Thus, as follows from the results of calculations, for sandy soils in practice one can neglect calculation of the formation of the freezing zone. However, when the phase transitions of water occur in the range of temperatures, that is, the effective heat capacity decreases gradually during a temperature reduction (finely dispersed soils), during calculations in the Stefan formulation the depth of freezing will be considerably understated, especially at average annual temperatures of the medium near 0^o.

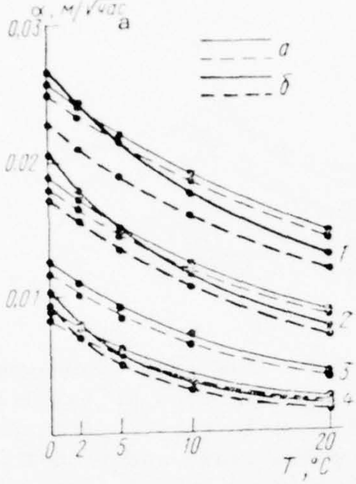



Figure 139. Results of calculation of the value of α as a function of the ground temperature T at different temperatures on the surface T_o in calculation of the zone of freezing (solid line) and front formation (broken line) for (a) sand and (b) loam: 1 -- $T_o = -5^o$, 2 -- $T_o = -2.5^o$, 3 -- $T_o = -1^o$, 4 -- $T_o = -0.5^o$. a -- hrs

However, also in that case at small values of T_o (above -2^o) and at rather high values of T_1 (above $5\text{-}10^o$) the influence of the zone of freezing will be small -- less than 5%.

2. The Influence of Consideration of the Ice Formation Curve on the Thawing of the Ground

Investigations of the phase transitions of water during the thawing of rocks are conducted analogously.

Figure 140 presents the results of calculations of the values of α determining the course of thawing under different boundary conditions (T_o and T_1) for two types of ground -- loam and sand. Presented in the same place are values of α corresponding to the ordinary formulation of the Stefan problem. The thermophysical data are presented in Table 79.

As follows from the presented results, in contrast with the freezing, the thawing of porous bodies with consideration of the curve of the ice content proceeds at different values of $T_1 < 0^o$ less intensively than in the Stefan formulation.

At $T_1 = 0^o$ divergence obviously is not present, as no zone of thawing is present. With increase of $|T_1|$ the divergence grows, but at sufficiently large values of $|T_1|$ (since the curve of ice formation has a horizontal asymptote) it decreases. In connection with that, for each value of T_o a value $T_1 < 0^o$ is found at which the divergence reaches a maximum, its position then being dependent on the ice content curve. This is very graphically evident in the case of loam on the curve $T_o = 0.5^o$ (at values $T_o > 0.5^o$ approximation of the curve starts at $T_1 < -10^o$). In the case of sand the maximum divergence sets in practically independently of the value of T_o at $T \approx -2^o$.

Such a picture is explained by the fact that thawing in the temperature range leads to phase transitions of water anywhere at $t < 0^o$, that is, in the entire

Table 79 Thermophysical characteristics of loam and sand adopted in calculations

A Температура, °C	B Суглинок	C Песок
	a $W_н$, %	a $W_н$, %
0	24.1*	21,7*
0—0	16.6	7,25
—0.5	14.0	3.0
—1,0	12,0	1,75
—2,75	9,7	1,12
—10,15	5,7	0,65
1 $\lambda_м(0)$, ккал/м·час·град	1,13	1,46
2 $\lambda_т$ »	0,86	1,04
3 $\gamma_{об\cdot ск}$, кг/м³	1611,6	1388,8

Key: A - Temperature; B - Loam; C - Sand;
a - W_{un}, %
1 - $\lambda_f(0)$, kcal/(m)(hr)(degree)
2 - λ_t 3 - γ_{vol-sk}, kg/m^3
*Natural moisture content

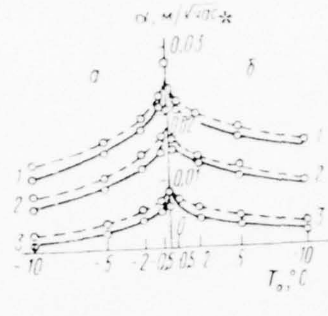


Figure 140. The rate of thawing of soils of the type of (a) loam and (b) sand in the temperature range (solid line) and with the formation of an interface (broken line): 1 -- 5°; 2 -- 2.5°; 3 -- 0.5°. * - hrs

frozen rock mass, as the temperature curve at $t > 0^o$ will be above the initial distribution of temperatures $t = T_1$. At the same time, in the Stefan formulation the phase transitions occur only in the interval $[0, \xi]$, in connection with which the thawing in that case increases. For grounds of the type of sand the principal phase transitions occur in a narrow temperature range at $t > -1^o$, since at $T_1 < -2^o$ the elevation of temperatures in the main part of the frozen mass, connected with increase of T_o, has an insignificant influence on the process of thawing. In the case of loam, however, increase of T_o has the result that even a slight elevation of the temperatures at great depths, where the temperatures are close to T_1, can substantially increase the total amount of heat of phase transitions. Therefore in the case of loam the influence of the curve of ice formation will decreaase at considerably larger values of $|T_1|$ than for sand.

As follows from the results of the calculation, the influence of the curve of ice formation on the depth of thawing can be considerable even for sandy soils. Thus, at $T_o = 0.5^o$ consideration of the phase transitions in the temperature range for sand at $T_1 = -2^o$ leads to a reduction of thawing by 19%, and in the case of loam by 26%. For sand, however, elevation of T_o or $|T_1|$ leads to a sharp decrease of the influence of the curve of ice formation, whereas in the case of loam the maximal divergence in the rate of thawing is only shifted in the direction of decrease of T_1. Besides that, in contrast with freezing, during thawing the consideration of the curve of ice content has a substantial influence on the temperature distribution by depth. As an illustration, the results of calculation of the temperature distribution during thawing, both with consideration of the ice content curve and in the Stefan formulation are presented on Figure 141, according to the data of which it follows that temperatures by depth when the curve of the ice content is taken into consideration are considerably lower than in the Stefan formulation. In the case of sand at $T_o = 5^o$ and $T_1 = -2^o$ the divergence reaches 0.5^o, but in the case of loam at $T_o = 5^o$ and $T_1 = -2^o$ the divergence exceeds 2^o.

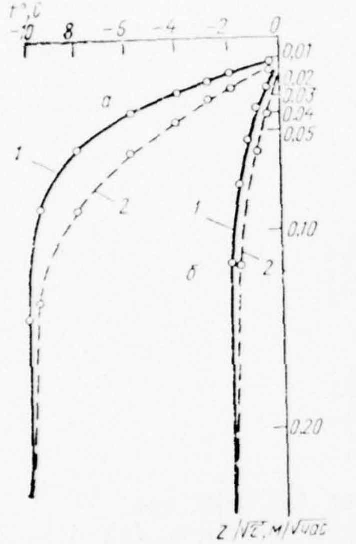


Figure 141. Temperature distribution by depth in frozen soils of the type of loam (a) and sand (b) at $T_o = 5^o$: 1 -- in the case of thawing in the temperature range; 2 - with formation of an interface

Thus calculations of thawing of soils by the usual procedure without consideration of phase transitions in the temperature range lead for finely dispersed soils (with smooth change of the ice content curve) to considerable distortion of the temperatures and decrease of the depths of thawing by up to 20%. In the case of sandy soils the indicated effect occurs only when there are low temperatures on the surface.

3. The Influence of Consideration of the Ice Content Curve on the Process of Freezing and Thawing of Soils With Different Natural Moisture

We will examine the influence of change of the amount of free water (or the natural moisture content of the soil) on divergence between solutions of the problem of freezing with consideration of the curve of unfrozen water and the classical Stefan problem under analogous conditions.

During both freezing and thawing with consideration of the ice formation curve change of the amount of free water (w_{nat}), that is, the natural moisture content of the soil, does not change the non-linear differential equation to which the problem under consideration is reduced in the self-modeling case. This is connected with the fact that the ice content curve for the given soil in practice* is a physical characteristic independent of the moisture content in the natural state.

*Cases where the natural moisture content is smaller than the maximal molecular do not play an important role.

Figure 142 presents the results of calculation of the values of α determining the dynamics of freezing during consideration of the ice formation curve as a function of w_{nat} for the values of the initial temperature $T_1 = 0.2$ and 10^o and the temperature of the surface $T_o = -0.5$, -2.5 and -5^o. Presented at the same place are similar calculations of the problem in the classical Stefan formulation. The calculations were made for two essentially different ice formations in the range of negative temperatures of the types of soil: loam and sand.

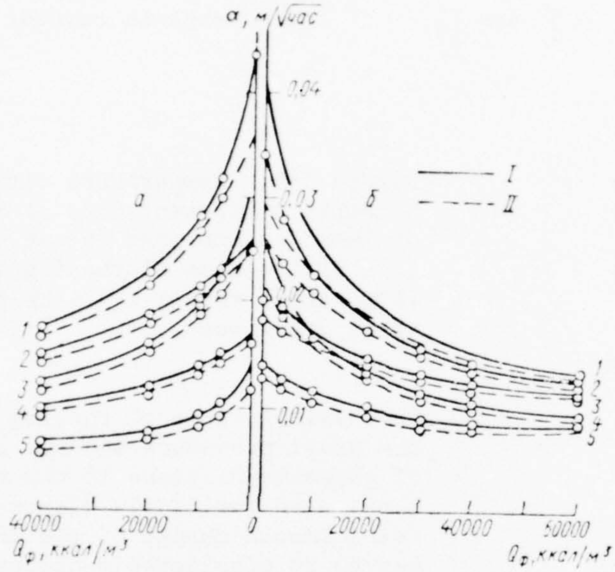


Figure 142. Freezing of soils of the type of (a) sand and (b) loam during consideration of the ice formation curve (I) and in the Stefan formulation (II) (as a function of the amount of free water): 1 -- $T_o = -5^o$, $T_1 = 0^o$; 2 -- $T_o = -5^o$, $T_1 = 2^o$; 3 -- $T_o = -5^o$, $T_1 = 10^o$; 4 -- $T_o = -2.5^o$, $T_1 = 2^o$; 5 -- $T_o = -0.5^o$, $T_1 = 2^o$. a - α, m/$\sqrt{\text{hr}}$ b - Q_ϕ, kcal/m^3

As follows from the obtained results, for a soil of the type of loam change of w_{nat} plays a substantial role from the point of view of the influence of the curve of ice formation on the course of freezing in comparison with the Stefan formulation. An especially sharp difference occurs at a natural moisture content close to the maximal molecular (Q_ϕ is small) and the difference achieves a maximum at $Q_\phi = 0$ kcal/m^3. At a fixed value of T_o the largest divergence occurs at $Q_\phi = 0$ kcal/m^3 and $T_1 = 0^o$. For example, at $Q_\phi = 0$ kcal/m^3 for the case $T_o = -5^o$ and $T_1 = 0^o$ the increase of the depth of freezing during consideration of the ice formation curve reaches 26.2% in comparison with the corresponding Stefan problem and can be still larger at $T_o < -5^o$. With increase of Q_ϕ the rate of freezing during consideration of the ice formation curve falls considerably more sharply than in the Stefan formulation (while still remaining always larger than the latter). In connection with that, at large enough values of Q_ϕ the depths of freezing calculated by the two methods converge.

For soils of the type of sand the difference in the course of freezing during change of w_{nat} has a substantial influence only at small values of $|T_o|$ of the order of 0.5^o (for example, at $T_o = 0.5^o$, $T_1 = 2^o$ and $Q_\phi = 0$ kcal/m^3 that difference reaches 33%). However, during decrease of T_o or increase of T_1 the depths of freezing obtained at different values of Q_ϕ in the Stefan formulation and with consideration of the ice formation curve converge far more rapidly than for loam. In addition, during increase of w_{nat} the influence of the ice formation curve also diminishes far more intensively. Thus, already at $Q_\phi = 5000$ kcal/m^3 (that is, at $w_{nat} - w_{un}$ of the order of 5%) the divergence of the freezing depths at $T_o = -0.5^o$ and $T_1 = 2^o$ drops to 13%, and at larger values of $|T_o|$ and T_1 -- less than 4%.

The results of similar calculations in the case of thawing of soils of the type of loam and sand are presented on Figure 143. The calculations were made at $T_1 = -2^o$ at different values of T_o as a function of Q_ϕ. As follows from an examination of Figure 143, calculations of the Stefan problem give substantially overstated values of the thawing depths, and upon increase of T_o for both loam and sand a convergence of ξ in the Stefan formulation and with consideration of the ice formation curve is characteristic. The maximal divergence, as during freezing, is noted during the absence of free water, but in both cases only at small values of T_o (at $T_o = 0.5^o$, 40.4% in the case of loam and 19.4% in the case of sand).

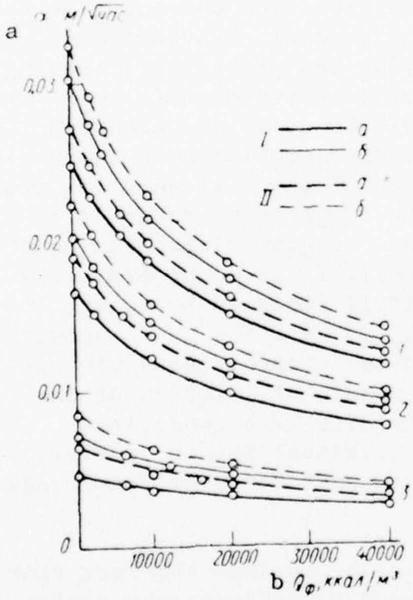


Figure 143. Thawing of soil of the type of (a) loam and (b) sand during consideration of the ice formation curve (I) and in the Stefan formulation (II) (as a function of the amount of free water: 1 -- $T_o = 5^o$, $T_1 = -2^o$; 2 -- $T_o = 2.5^o$, $T_1 = -2^o$; 3 -- $T_o = 0.5^o$, $T_1 = -2^o$.
a - κ, m/$\sqrt{hr}$ b - Q_ϕ, kcal/m^3

In addition, upon increase of Q_ϕ the convergence of the corresponding curves occurs far more slowly than during freezing, and even at $Q_\phi > 40{,}000$ kcal/m^3 the divergence of thawing depths is considerable (at $T_o = 2.5$ and $T_1 = -2^o$ for soil of the type of loam above 12%, and 4% for sand). In that case it must be borne in mind that for soil of the type of sand at values of T_1 different from $T_1 = -2^o$ the divergences will be smaller, while for loam at $T_o > 0.5^o$ and $T_1 < -2^o$ the divergences can be greater.

Thus change of the amount of free water has a substantial effect in calculations of freezing or thawing not only of finely but also of coarsely dispersed soils in comparison with the corresponding calculations in the Stefan formulation. In calculations of the Stefan problem for coarsely dispersed soils during both freezing and thawing a correction must be introduced only at small values of $w_{nat} = w_{un}$ (up to 5%) and at small absolute values of the surface temperatures. In the case of finely dispersed soils, however, during thawing a correction must be introduced at any values of w_{nat}, whereas during freezing only at $w_{nat} - w_{un} \leq 20\%$. Since in comparison with the Stefan problem ice formation in the temperature range increases the freezing depth and reduces the thawing depth, then it is necessary to introduce a positive correction during freezing and a negative one during thawing.

4. Determination of the Effectiveness of the Ice Formation Curve

The above-examined comparison of solutions of "linear" and quasi-linear problems of the Stefan type was made for a definite curve of unfrozen water, obtained experimentally for a specific soil. At the same time the curve of unfrozen water also influences the divergence of the results of solutions of those problems. All this leads to a need in solving the problem of freezing and thawing to constantly use a laboratory-determined curve of unfrozen water for each specific soil, and this makes it difficult to solve and excludes the application of known solutions of the classical Stefan problem with a freezing boundary. However, by using the indicated solution of the self-modeling problem with consideration of phase transitions in the temperature range it is possible to propose a method which permits solving the general problem under consideration by means of calculations of the classical Stefan problem with a precision adequate for practical purposes. This is achieved by using, instead of the natural curve of phase transitions $Q_\phi(t)$ at $t \leq 0^o$, a certain effective curve $\bar{Q}_\phi(t)$. The latter is determined by integral quadratic approximation of the freezing depths obtained as a result of solution of the self-modeling problem in the temperature range for different conditions $y = y_1(T_o, T_1)$ corresponding to solutions of the classical Stefan problem. The values of Q_ϕ as a function of T_o found in that case for a certain range of temperatures T_1 determine the curve $\bar{Q}(t)$.

The thus-constructed effective curve of ice formation, besides the fact that it permits making qualitative estimates, can be used to estimate the depths of seasonal freezing (thawing) in a given soil by means of an ordinary Stefan problem. To do that it is sufficient, under certain initial and boundary conditions on the surface of the ground, to determine with the curve $\bar{Q}(t)$ the corresponding quantity of the heat of phase transitions, the release of which is assumed at the freezing (thawing) temperature $t = 0^o$. Solution of the Stefan problem at that value of $\bar{Q}$ also will give the sought value of the freezing (thawing) depth for the given soil with consideration of the curve of phase transitions at practically any average annual ground temperature.

A solution of that variational problem in the case of freezing for four standard types of soil with different curves of unfrozen water (curves b, Figure 144), other conditions being equal, was obtained on the "Strela" computer of the MSU Computer Center (Melamed, 1967). As a result of the solution, corresponding effective curves of the phase transitions were obtained (curves a, Figure 144).

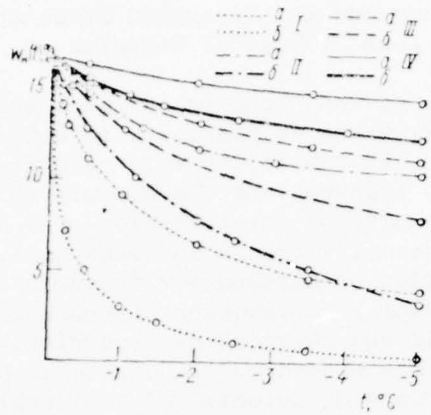


Figure 144. Dependence of the effective (a) and natural (b) curves of ice formation in the temperature range of 0 to -5° for four types of soils of different dispersion: from sand (I) to loam (IV), $w_{nat} = 24.1\%$, $w_{un(0)} = 16.6\%$

As follows from an examination of the results, in the case of freezing the effective curves of phase transitions lie above the natural. It is essential that their form differ insignificantly, and the effective curves emerge to the asymptote somewhat more rapidly.

The degree of divergence of the effective and natural curves of ice formation depends essentially on their inclinations on the section near $\pi/4$ (curves 2-3). In that case the divergence, increasing with lowering of the temperature, at temperatures below -5° exceeds a moisture content of 7%. Even in the case of a curve of ice formation typical of sands, where consideration of the phase transitions curve is considered correct, the divergence at -2° reaches 4% (curve IV). And even in cases where the phase transitions at $t < 0^{\circ}$ are small (curve 1), calculations of the Stefan problem with the usual procedure do not lead to substantial divergences.

Thus calculations of the problem of freezing of moist ground in the case where phase transitions occur in the temperature range must be corrected by means of the effective curve of phase transitions containing a form similar to the natural and corresponding to a larger quantity of unfrozen water. In that case the degree of correction depends on the inclination in the region of the main phase transitions. If high precision of calculations of the freezing depth is required, however, then to solve the Stefan problem by means of the proposed variational method it is necessary to preliminarily find the indicated effective curve with a digital computer.

As calculations have shown, the effective curve is practically independent of the amount ot the natural moisture content, which is an essential circumstance from the point of view of its use.

Thus, for calculations of the Stefan problem in the case of phase transitions in the temperature range for a specific medium it is sufficient at an arbitrary value of the natural moisture content to find the effective curve of phase transitions $\overline{Q}(t)$ and use it henceforth for any calculations with the ordinary method.

3. Investigation of the Dynamics of Freezing and the Cryogenic Structure of Finely Dispersed Rocks in the General Case By Digital Computer

The investigation, discussed in Chapter 3, of the processes of heat and mass transfer in freezing finely dispersed rocks makes it possible, in a formulation sufficiently close to real conditions, to reveal the main quantitative regularities of the formation of cryogenic textures and the amounts of heaving. Presented in this section are the results of investigation of a number of typical problems of the freezing of seasonally frozen and seasonally thawed soils which permit determining the connection of distinctive features of the cryogenic structure and the amount of heaving as a function of concrete climatic data in the presence of the hydraulic and physical characteristics of the rocks and the conditions of heat and mass exchange. In that case the corresponding problems are examined both by means of solution of the problem of freezing with moisture in a general formulation (under variable boundary conditions with consideration of the possibility of change of the freezing regime in the course of the process) and in the self-modeling case.

1. Investigation of the Dynamics of the Temperature and Moisture Fields and the Amount of Heaving During Seasonal Freezing in Moisture-saturated Finely Dispersed Soils

In solving questions of the prediction of change of frost conditions, of considerable interest are calculations of heat and mass exchange in the layer of annual temperature fluctuations during the winter freezing of the rocks. These questions are examined in a homogeneous medium under the conditions of a closed system (there are no inflows of moisture toward immobile boundaries of the region). Given as the temperature boundary condition on the surface ($z = 0$) is change of temperature according to the harmonic law $t(0, \tau) = t_m + A_o \sin 2\pi/T\ \Pi$, where t_m is the average annual temperature of the surface, A_o is the amplitude of oscillations, $t_m > 0^o$, $A_o > 0^o$ and T is the period of oscillations (8760 hours). At the depth of the zero annual amplitudes, as usual, the condition of temperature constancy is assumed. Since the time interval during which a solution is sought is limited to the winter freezing, consideration of the influx of geothermal heat, which can readily be accomplished with the given program, is not necessary.

The problem is set as follows. Taken as the initial moment of time is $\tau = 0$, when the temperature of the surface is t_m and the ground is in the thawed state. The initial temperature distribution by depth is taken in the form

$$t(z, 0) = t_{cp} + (t(h, 0) - t_{cp}) \cdot \operatorname{erf} mz, \quad 0 \leqslant z \leqslant h,$$

where h is the depth of the zero annual amplitudes, and m is a constant > 0. For simplicity, before the start of freezing moisture exchange in the layer of annual fluctuations is excluded from consideration, and at $\tau = 0$ the moisture in depth is assumed to be constant (w_1). Thus, until the next sign inversion of $t(0, \tau)$ the problem consists in determining the dynamics of the temperature field in the layer of annual fluctuations, which is described by an ordinary equation of thermal conductivity. The purpose of solution of the problem in this state is to find the temperature distribution in depth natural for the given region before the start of freezing. Later a solution is found for the problem of heat and mass exchange during freezing, one sought until the moment of the subsequent temperature inversion (when thawing from the surface has to start and the problems becomes a two-front one) or, if it occurs earlier, until the start of thawing from below.

As a result of the solution of that problem the dynamics of movement of the interface and of the heaving in time are determined, and also the cryogenic structure in the layer of seasonal freezing and the moisture distribution in the thawed ground at the indicated moment of time. It is obvious that at t_m values close to zero the freezing of the ground can occur also in the course of some time interval after the moment of sign inversion of $t(0, \tau)$. It must be borne in mind, however, that migration occurs far more slowly in a frozen than in a thawed zone and, in addition, the rate of freezing after the start of thawing from the surface approaches zero. Therefore the cryogenic structure of the frozen layer obtained as a result of calculations in accordance with what was said above within the framework of a single-front problem, in practice can be considered definitive. Let us note that insufficient consideration of the freezing possible after sign inversion of $t(0, \tau)$ leads to a somewhat understated value of the ice content near the base of the frozen layer.

The algorithm for solution of the problem of freezing with moisture migration in a general formulation with consideration of heaving and the formation of ice interlayers which was examined in Chapter 3 was implemented in the Department of Geocryology of MSU by means of a program for a model M-20 digital computer. The counting time of a single variant was about 1 hour. In calculations with a BESM-4 provision was made for the use of an alphanumeric printer. The maximal number of junction points in depth is 240. At a rapid rate of freezing, and also during entry into the layer and departure from it, a need arises to subdivide the step in time. The minimal value of a step was taken to be 6.25 hours. Since the coefficient of potential conductivity varies by several orders of magnitude during change of the moisture content from w_o to w_{abs}, in finding the moisture content on a mobile boundary in the process of formation of an ice layer iteration is used.

The calculations were made at the following values of the coefficients of heat and mass transfer: $K(w) = 0.3786 \times 10^{-5} \times \exp\{16.4646w\}$, m^2/hr (taken according to the data of V. R. Gardner, 1958), the thermal conductivity of the thawed ground was $\lambda_t = 0.67$ kcal/(m)(hr)(degree), that of the frozen ground λ_f was calculated in accordance with the dependence $\lambda_f = \lambda_t\,(1 + 2\,\frac{w_t}{100})$ (in that case it is assumed that $\lambda_f = 2$ kcal/(m)(hr)(degree) within the ice layer, when $w_t = 100\%$), the heat capacity of the thawed ground $C_t = 600$ kcal/(m^3)(deg), and the effective heat capacity is calculated with the formula of mixing with consideration of the curve $w_{un}(t)$ in the form

$$C(t, w_c) = 500\,[w_c - K_w w_н(t)] + [1000\, w_н(t) + \gamma_{ск} C_{ск} + 80\,000\, w'_н(t)]\, K_w,$$

where $K_w = 1$, $w_t \leqslant w_{abs}$; $1 - w_t/w_{abs}$; $w_t > w_{abs}$; $\gamma_{sk} = 1800$ kg/m^3; $C_{sk} = 0.2$ kcal/(kg)(degree); w_t is the total moisture content by volume. The curve of unfrozen water was given in the form

$$w_u(t) = \frac{a_0}{b_0 - t}, \quad t < 0.$$

where a_o and b_o were determined from the conditions: $w_{un}(0^o) = 15\%$, $w_{un}(-10^o) = 2\%$. The moisture content on the interface in the absence of ice schlieren (corresponds to the moisture content close to rolling in a cord) was given as 20% and the absolute moisture capacity 40%. The temperature at the depth of the zero annual amplitudes (12 meters) was assumed to be 3^o and the average annual temperature on the surface of the soil $t_o = 1^o$.

The calculations of freezing with consideration of moisture migration were made at different values of the natural moisture content of the ground w_1 (within the range from the moisture of rolling to the absolute moisture capacity with a step of 5%) and amplitudes of fluctuations of the temperature of the surface A_o (A_o = 7, 10, 20 and 30^o). The initial step in time was assumed to be 50 hours, and the step on the coordinate was 0.05 meter.

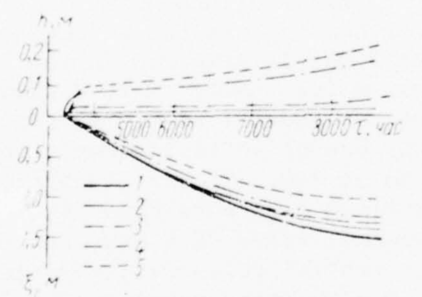


Figure 145. Dynamics of the depth of freezing and heaving in time at different values of A_o and w_1:

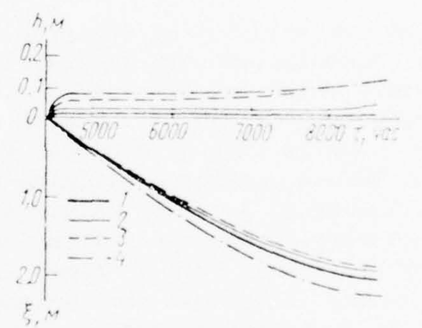


Figure 146. Dynamics of the depth of freezing and heaving in time at different values of A_o and w_1:

Presented on Figures 145-146 are the dynamics of the freezing and heaving in time at different values of w_1 at $A_o = 10^o$ and 20^o respectively. In addition, presented on Figure 146 are data on freezing and heaving at $A_o = 7^o$ and $w_1 = w_{abs}$. As was to be expected, reduction of A_o, other conditions being equal, leads to greater ice-saturation and increase of heaving. This is manifested very rarely at small values of A_o. The dependence of the amount of heaving on the amplitude at $w_1 = w_{abs}$, and also on the initial moisture content at different values of A_o, is illustrated on Figure 147. It must be stressed that the dependence of the amount of total heaving H on w_1 is essentially non-linear. Actually, the heaving at values of w_1 close to w_{abs} is negligibly small and increases slowly at $w_o < w_1 < 1/2(w_o + w_{abs})$. At the same

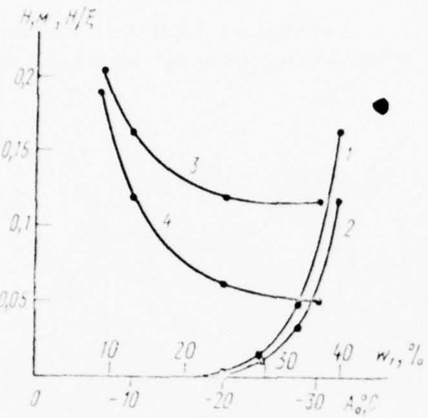


Figure 147. Dependence of the total heaving (H) on the initial moisture content (w_1) at $A_o = 10^o$ (1) and $A_o = 20^o$ (2) and amplitude of temperatures on the surface at $w_1 = w_{abs}$ (3), and the dependence of the relative heaving (H/ξ) on the amplitude of temperatures on the surface at $w_1 = w_{abs}$ (4).

time at $w > 1/2\ (w_o + w_{abs})$ and especially at values of w_1 close to w_{abs} the value of H increases sharply with increase of w_1. A qualitatively analogous picture occurs also in the self-modeling problem.

Figures 148 and 149 present the cryogenic structure over the profile of a frozen layer with consideration of heaving at $A_o = 10^o$ and 20^o at different values of w_1. The diagrams of the cryogenic structure characterize the distribution of the ice content by depth only at the moment of conclusion of freezing. Therefore, taking into account that in different cases the heaving is different, for their uniformity one must introduce a marker of the surface of the ground minimal in the year and common to all (that is, the surface of the ground before measurement), and over it, the axis of the hydrothermal movements of the surface v generated by the heaving. The connection between any point on the diagram of ice content and the moment of time when the front of freezing reached the corresponding depth and the indicated ice content formed is accomplished with the curve $\xi(\tau)$. In that case, it is necessary to determine for the selected point on the curve of the ice content its position in relation to the maximum of the hydrothermal movement [that is, the value of $\xi(\tau)$], after which, on the diagram where the dynamics of the process of freezing in time are presented, on the curve $\xi(\tau)$ the sought moment of time and the corresponding temperature of the surface are determined. Let us note that the correspondence between the indicated diagrams will be mutually identical only if in the course of the winter period $\xi'(\tau) > 0$. If in some time intervals during the winter freezing ceases, ($\xi'(\tau) = 0$), however, then as the sought moment of time it is necessary to take the largest time to which the given value of ξ corresponds on the diagram $\xi = \xi(\tau)$.

In examining the diagrams of the cryogenic structure it also is necessary to bear in mind that the ice content obtained by calculation is the mean-integral on the step of integration on the coordinate. Therefore solid ice schlieren with a thickness of not less than a step correspond to the points where the curve of the ice content in depth on the diagram reaches the value $w_t = 1$. But if the values of the ice content are in the interval between $w_t = w_{abs}$

and $w_t = 1$, then at the corresponding depth a laminated texture forms, the total thickness of the ice schlieren on a step being proportional to the value $w_t - w_{abs}$.

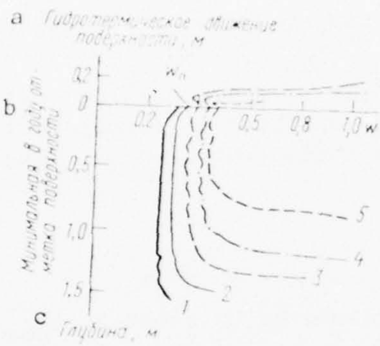


Figure 148. Distribution of ice content by depth of the frozen layer at different values of A_o and w_1. Symbols the same as on Figure 145.
a - Hydrometric motion of surface, m
b - Minimal marking of surface in yr
c - Depth, m

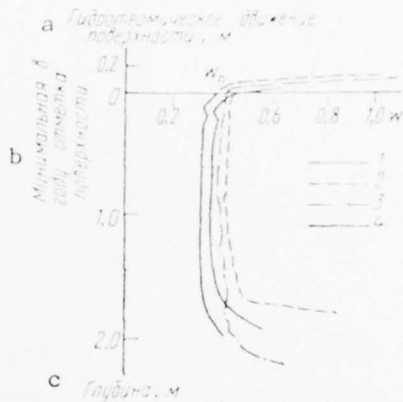


Figure 149. Distribution of ice content by depth of the frozen layer at different values of A_o and w_1. Symbols the same as on Figure 145.
a, b and c as on Figure 148

It follows from an examination of Figures 148 and 149 that, during freezing under the conditions of a closed system (in the absence of external sources of moisture) in the case of harmonic temperature oscillations on the surface, ice formation occurs irregularly over the profile of the seasonally frozen layer. At the same time it is characteristic that intensive ice formation occurs only near the surface and base of the freezing layer. In an overwhelming majority of it its total moisture content (with consideration of the volume expansion during the transition of water into ice) insignificantly exceeds the initial. This is manifested very rarely under the conditions of a sharply continental climate, when inside the freezing layer the moisture is actually fixed in place.

However, it must be borne in mind that intensive ice formation near the surface occurs primarily in connection with the fact that the pre-winter moisture content of the ground by a convention is assumed to be constant over its profile. It is obvious that if the ground near the surface has been dried by that moment, then heaving which occurs at the start of freezing under the conditions assumed in the calculations will be absent. Therefore the heaving in the course of time can be regarded as a linear function only in cases where at the pre-winter moment the ground near the surface is sufficiently dried and also if the start of freezing is accompanied by sharp cooling.

Since in geocryology calculations of the depths of freezing, as a rule, are made without consideration of mass transfer, for comparison solutions of analogous problems were obtained in the Stefan formulation, but with consideration of phase transitions in the range of negative temperatures. In that case, in contrast with the above-considered problem, on the mobile interface occur phase transitions of all the "free" moisture (that is, from w_1 to $w_{un}(0)$).

It must be stressed that, as follows from the presented calculations, the summary (with consideration of heaving) moisture content of the frozen ground in the problem with consideration of migration in the conditions under consideration proves in practice to be close to the maximal depth of freezing in the corresponding Stefan problem. This is connected to a considerable degree with the above-indicated irregularity in ice formation over the profile of the freezing layer. In addition, a definite role in that is also played by the dependence of the thermal conductivity of the ground on the total moisture content. Nevertheless the indicated circumstance will be violated inevitably when the values of A_o are sufficiently small, when intensive heaving occurs.

Thus calculations of the depth of freezing of moisture-saturated finely dispersed rocks in a closed system during harmonic oscillations of the temperature of the surface under the conditions of a continental climate can, with an accuracy sufficient for practical purposes, be made without considering migration. At the same time, under the conditions of a maritime climate, by virtue of substantial change of the cryogenic structure in the depth of the layer of freezing and considerable heaving, investigation of freezing within the framework of the Stefan problem leads to distortion of the picture. To a still greater degree, independently of climatic conditions, this applies to freezing under the conditions of an open system, particularly if there are water-bearing horizons situated within the range of 2-3 meters from the surface, and also during the freezing of a seasonally thawed layer in the presence of waters above the frost.

2. Investigation of the Dependence of the Cryogenic Structure and the Dynamics of Heaving on the Character of the Temperature Conditions on the Surface During the Freezing of Moisture-saturated Finely Dispersed Soils

Investigation of freezing in the presence of moisture migration toward the front of freezing by virtue of the interconnection between the cryogenic structure, heaving and the rate of freezing is sharply complicated, as in that case it is necessary to determine the dynamics of freezing in time. Thus, for moisture-saturated finely dispersed rocks, in contrast with the generally accepted method of calculating seasonal feeezing, it is necessary to consider in detail the change of temperature on the surface of the ground in the course of time.

As follows from part 1 of the present section, irregular ice formation within the layer of winter freezing can be connected only with essentially nonmonotonic change of the temperature of the surface of the ground in the process of freezing. From that point of view very great interest is presented by the case where on the surface during the winter season there is a successive change of rather long periods of warming and cooling, which are often widespread under natural conditions.

For a quantitative estimation of the influence of the non-monotonic nature of the change of $\Phi(\tau)$ in the winter time on freezing, calculations were made of heat and mass transfer during cooling at a temperature of the surface which attains several extremums in the course of the winter period. For that purpose within the limits of the winter period (at $\Phi(\tau) = t^{o}$, where t^{o} is the temperature of the start of freezing) selected as the temperature of the surface were functions of the type

$$\Phi_n(\tau) = A_n[\mu \cos \alpha_n(\tau - \beta_n) + \cos n\alpha_n(\tau - \beta_n) - M_n] + B_n.$$

Here μ assumes the values of 0 or 1, n are whole numbers, $n \geqslant 1$, M_n, $n > 1$ is the value of the largest of the internal maximums of the function $f(t) = \mu \cos t + \cos nt$ in the interval $0 < z < 2\pi$.

Since $\Phi_n(\tau)$ is determined only at $\Phi \leqslant 0$, then the function $\Phi(\tau)$, generally speaking, is compound.

It is obvious that the problem under consideration at n = 1 can be reduced to the above-considered case when $\Phi(\tau)$ is described by a simple harmonic with a period equal to a year.

Common for the functions $\Phi(\tau)$ and $n > 1$ describing the temperature of the surface in the course of the winter at $\mu = 0$ and $\mu = 1$ is the fact that the number of maximums internal for the inverval of freezing is equal to n - 1 (n minimums), and with increase of n the length of each oscillation diminishes. At the same time, as a function of μ the functions $\Phi_n(\tau)$ differ substantially at $n > 1$. Actually, at $\mu = 1$ the function $\Phi(\tau)$ is a result of the superposition of two harmonic functions with periods differing by n times and represents a series of oscillations in relation to a mean which varies in time. In that case the amplitude of such oscillations diminishes with increase of n.

In contrast with that, at $\mu = 0$ the function $Q_n(\tau)$ represents a number of identical oscillations (with one and the same amplitude and average annual temperature).

Thus the investigation of the dynamics of freezing of moisture-saturated finely dispersed soils at the indicated values of $\Phi(\tau)$ in the cases $\mu = 0$ and $\mu = 1$ permits quantitatively estimating the influence of the non-monotonic character of the change of temperature of the surface in the winter time during coolings and warmings of different types and number.

We will examine the solution of a number of problems with consideration of moisture migration for different values of $\Phi_n(\tau)$ at $n > 1$ and compare (other conditions being equal) the results of those calculations with the solution of an analogous problem where the temperature of the surface in the course of a year is described by $\Phi_1(\tau)$. To make the results of those calculations comparable at different values of n the problem is posed in the following manner:

1) the time interval in the course of which freezing occurs is in all cases identical and is determined by the moments τ_1 and τ_2 of sign inversion of $\phi_1(\tau)$ and $0 < \tau_1 < \tau_2 < T$. The points of adhesion $\phi_1(\tau)$ and $\phi_n(\tau)$, $n > 1$ correspond to the two successive nearest to zero positive roots of the equation $f(z) = M_n$;

2) in the course of the summer period the course of the temperature of the surface at any values of n given is one and the same (corresponding to $\phi_1(\tau)$;

3) the sum of the frost-degree-hours at any values of n is invariable and corresponds to the case where $\phi_1(\tau)$ occurs on the surface.

It can readily be seen that the cited conditions at any given $\phi_1(\tau)$ permit unequivocally determining all the parameters $\phi_n(\tau)$ at $n > 1$.

In addition, it follows from condition 2 that in all cases, regardless of n, the temperature field by the moment of start of freezing is identical.

The region of investigation (within the limits of the layer of annual oscillations), the boundary conditions and the coefficients of heat and moisture transfer are given in the same manner as in the preceding part of the present section. Let us only note that the full water capacity (by volume) $w_{abs} = 40\%$, and the initial distribution of moisture (as has been pointed out, it coincides with the pre-winter) is assumed to be constant in depth and equal to w_{abs}.

The results of calculations of the dynamics in time of the depth of the layer of freezing $\xi(\tau)$ and the amount of heaving $h(\tau)$, and also the cryogenic structure of the layer of seasonal freezing at different values of $\phi_n(\tau)$ in the case $\mu = 1$ are presented on Figure 150. Presented in the same place are diagrams of $\phi_n(\tau)$, and also the position of the mobile boundary of the thawed zone in relation to the marker of the surface of the ground minimal in the year $y(\tau)$. It is obvious that in the process of formation of ice layers the value of $y(\tau)$ remains invariable in time, whereas $\xi(\tau)$ by virtue of heaving increases. In all cases the function $1 + 10 \sin 2\pi/T \, \tau$ was taken as $\phi_1(\tau)$.

As follows from the obtained results, upon fulfilment of the above-indicated conditions the change of the course of the temperature of the surface has a practically insignificant effect on the total depth of freezing and the amount of heaving in a season. At the same time, the cryogenic structure in the freezing layer changes substantially during change of n. There is a very sharp interconnection between the change of warmings and coolings in the winter time and irregularity in the cryogenic structure of the freezing layer manifested at small values of n, when the length and intensity of those oscillations are rather large.

With increase in the number of oscillations the heterogeneity in the frozen layer increases substantially. Thus, at $n = 2$, when in the middle of the winter there is a single warming of considerable length, inside the freezing layer forms one layer with a thickness of more than 15 cm. In the rest of the frozen layer the ice content is close to the full moisture capacity. At the

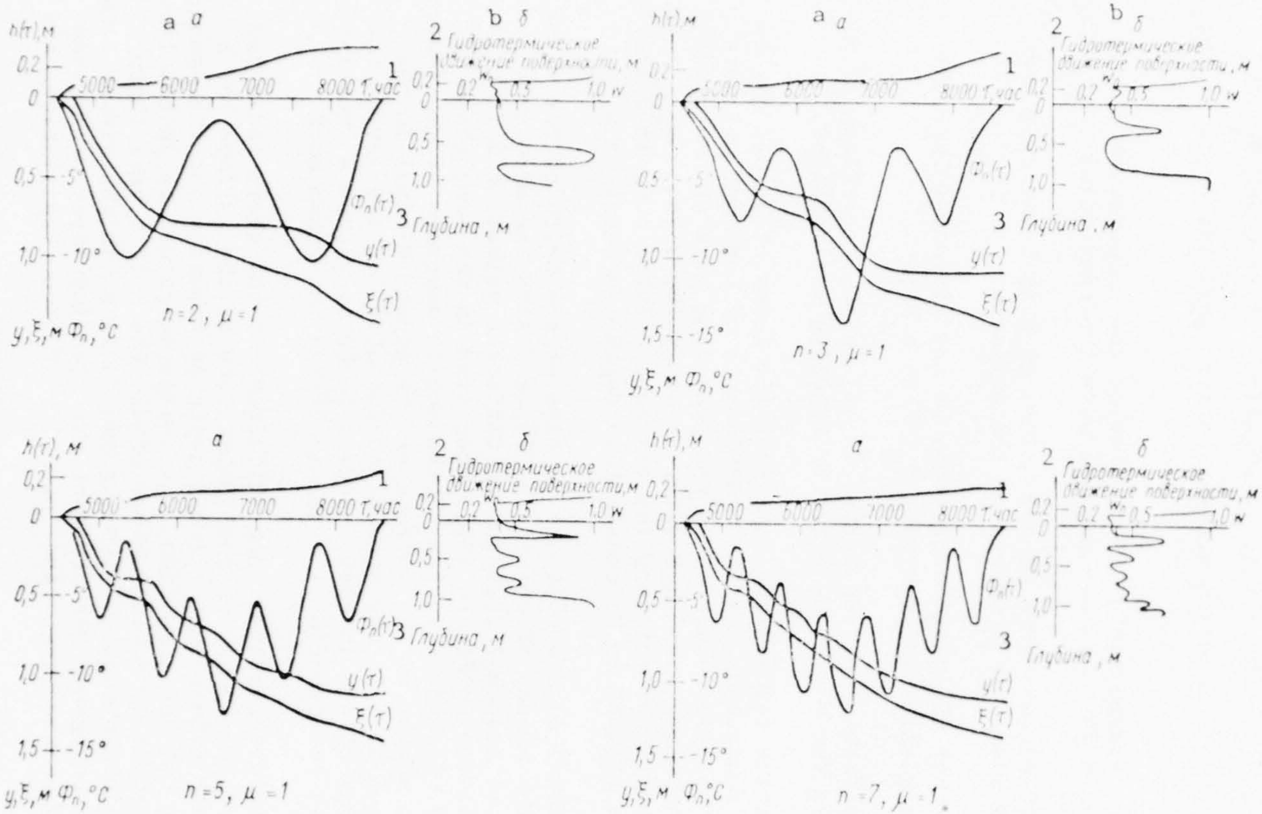


Figure 150. Dynamics of the depth of freezing and heaving in time, and also the cryogenic structure of the layer of freezing at different values of Φ_n for $\mu = 1$: a - results of calculations of the dynamics of the depth of freezing in time $y(\tau)$ in relation to the minimal marking of the level of the surface (0), the thickness of the frozen zone $\xi(\tau)$ and the heaving $h(\tau)$, and also the change of the temperature of the surface $\Phi_n(\tau)$; b - distribution of the total moisture by depth of the frozen zone by the moment of start of thawing of the surface. 1 - τ, hrs; 2 - Hydrometric movement of surface, m; 3 - Depth, m

same time at $n > 2$ isolated ice layers with a thickness of not less than 5 cm do not form. In that case during freezing an interstratification of sections with different ice-saturation occurs, and on sections with a high ice content schlieren form, the thickness of which is known to be smaller than the step on the coordinate. Of very great importance from the point of view of the formation of ice schlieren are warmings at the start of winter, when the thickness of the freezing layer still is not great. This statement remains valid at any values of $n > 1$ in spite of the fact that with growth of n both the amplitude and the length of the oscillations diminish.

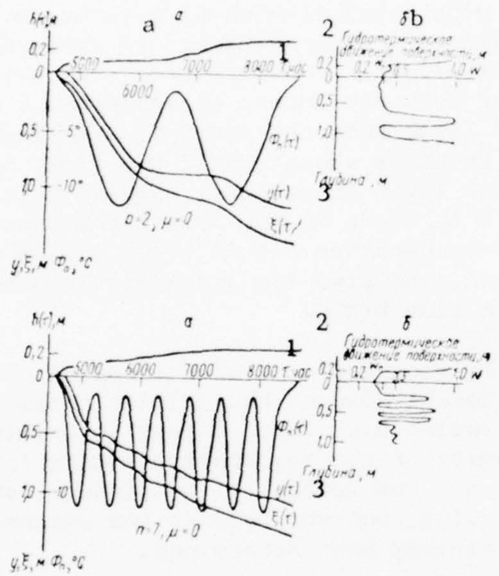


Figure 151. Dynamics of the depth of freezing and heaving in time, and also the cryogenic structure of the freezing layer at different values of ϕ_n for $\mu = 0$. Designations are the same as on Figure 150.

On the presented diagrams of the cryogenic structure this is manifested in the fact that at $n > 2$ the number of layers with a different ice content in depth is smaller than the number of warmings, which is $n - 1$. At $\mu = 0$ (Figure 151) the picture is similar on the whole. However, since at $\mu = 0$ all the oscillations in the winter period have an identical length, amplitude and average annual temperature, in the given case the connection between the non-monotonic character of the temperature of the surface and the formation of ice schlieren is sharper. Thus, for example, at $\mu = 0$ the thick ice layers (not less than 5 cm) correspond to at least the first three fluctuations of temperature, whereas at $\mu = 1$ they correspond to only the first warming.

Thus the heterogeneous character of ice formation during freezing can be quantitatively connected with the non-monotonic character of the change of climatic conditions in the winter time, expressed in an alternation of periods of warming and cooling.

The possibility of establishing a quantitative interconnection between the climatic conditions and the cryogenic structure opens up broad possibilities in the solution of very complex problems of frost prediction and historical geocryology.

3. Calculation of the Dynamics of the Temperature and Moisture Fields and the Amount of Heaving During Freezing of the Seasonally Thawed Layer

The mutual dependence of the rates of freezing and moisture migration has the result that the cryogenic structure of the seasonally thawing layer is a result of a complex process which depends on many factors. Very important in that case is the regime of waters on top of frost, characterized by different feeding conditions and time of existence of the water-bearing horizon.

We will examine the results of a number of calculations of the dynamics of freezing of the seasonally thawing layer with consideration of moisture migration in the case of harmonic oscillations of temperature on the surface of the ground. The calculations were made within the framework of a single-front problem, that is, under the condition that from the moment of achievement of the maximum of thawing (which sets in not later than the start of freezing) and advance of the upper boundary of the frost is absent until the frost has washed away. At first the thickness of the layer of seasonal thawing ξ is determined, and also the temperature field in it up to the moment the thawing ceases τ_0. At $\tau > \tau_0$ the dynamics of the temperature and moisture fields in the seasonally thawing layer are determined, and also its cryogenic structure and the change of the amount of heaving in time $h(\tau)$.

It must be noted that in connection with heaving the size of the region of investigation in that last case can considerably exceed the initial thickness of the seasonally thawing layer. In connection with that, below, in examining the cryogenic structure within the limits of the seasonally thawing layer as the start of the readoff we will take, not the surface of the ground, which in time moves the amount of the sought heaving, but the base of the seasonally thawing layer, the position of which has already been determined.

To obtain very great heaving, in all the calculations it was assumed that the pre-winter moisture is equal to the full moisture capacity. In addition, to consider the change of density of the thawed ground before and after the completion of thawing the value of λ_t at $\tau < \tau_0$ and $\tau > \tau_0$ was assumed to be different.

The influence of change of the regime of feeding the waters above the frost on the formation of its cryogenic structure was examined by variation of the corresponding boundary condition in the equation of moisture transfer. In the presence of constantly existing non-freezing waters above the frost, a constant moisture is fed on the base of the seasonally thawing layer, moisture equal to the full moisture capacity. In the case of absence of waters above the frost a condition of isolation was given, that is, the flow of moisture on the base of the seasonally thawing layer $q(\xi, \tau)$ was assumed to equal zero.

Finally, in the intermediate case the freezing of the seasonally thawing layer is examined under the condition that in the course of the winter period the waters above the frost are used up. For generality it is assumed that the consumption starts after a certain time $\Delta\tau_1$ from the moment of the start of freezing, after which in the course of a certain time $\Delta\tau_2$ the level of the waters above the frost drops to zero.

An estimate of the influence of the conditions of moisture exchange on the base of the seasonally thawing layer was made at different amplitudes A_0 of fluctuations of the temperature of the ground and the same hydro- and thermo-physical characteristics of the medium as in part 1 of the present section. The difference consists only in the fact that in accordance with the above it is assumed that

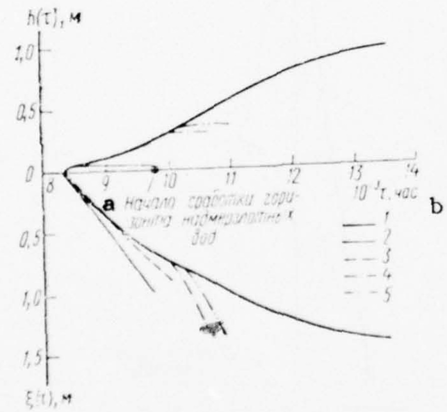
$$\lambda_т = 1{,}0\ ккал/м \cdot час \cdot град \quad \lambda_м = \lambda_т\left(1 + \frac{w_c}{100}\right) \text{ при } \tau < \tau_0;$$

$$\lambda_т = 0{,}6\ ккал/м \cdot час \cdot град \quad \lambda_м = \lambda_т\left(1 + 2\frac{w_c}{100}\right) \text{ при } \tau > \tau_0.$$

The temperature on the surface was given in the form

$$\Phi(\tau) = -3 - A_0 \sin\frac{2\pi}{8760}\tau, \quad A_0 = 10,\ 20^\circ C.$$

In addition, in a problem with a continuous nature of the feeding of the waters above the frost $\Delta\tau_1$ = 1450 hours and $\Delta\tau_2$ = 1000 hours. Finally, for comparison the cases $\Delta\tau_1$ = 2030 hours, and also during a gradual cessation of feeding of the waters $\Delta\tau_2$ = 3000 hours were examined at $A_0 = 10^\circ$.



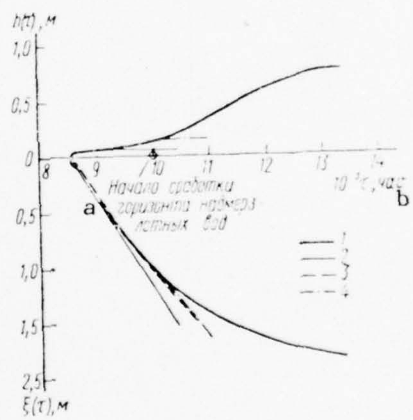


Figure 152. Dynamics of freezing of the ground (ξ) and heaving (h) in time (τ) under different conditions of moisture exchange on the base of the seasonally thawing layer in the case $A_0 = 10^\circ C$: 1 - with a constantly existing horizon of waters above the frost; 2 - in the absence of waters above the frost; 3 - dynamics of freezing without consideration of moisture migration (according to Stefan); 4 - during consumption of the horizon of waters above the frost in the course of 1000 hours; 5 - the same in the course of 3000 hours. a - Start of consumption of horizon of waters above the frost b - τ, hrs.

Figure 153. Dynamics of freezing of the ground (ξ) and heaving (h) in time (τ) under different conditions of moisture exchange on the base of the seasonally thawing layer in the case $A_0 = 20^\circ C$. Designations are the same as on Figure 152.

Presented on Figures 152 and 153 are the dynamics of the freezing of a seasonally thawed layer and the amount of heaving of the ground under different conditions of moisture exchange at the base of a seasonally thawed layer at A_0 = = 10 and 20° respectively.

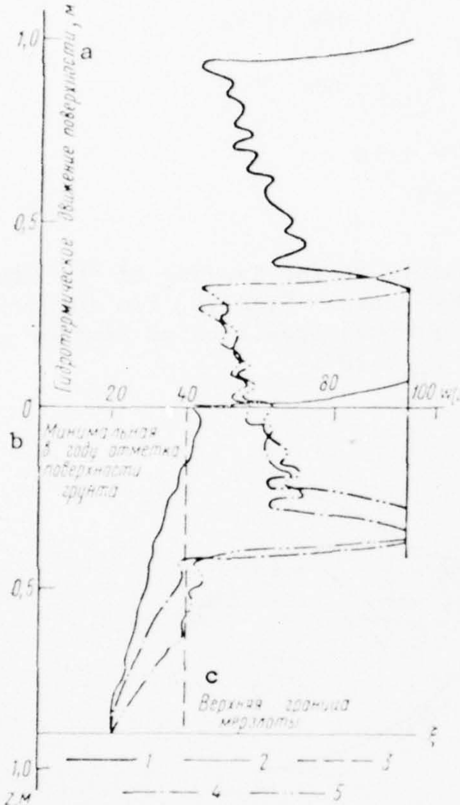


Figure 154. Distribution of the total moisture within the limits of the seasonally thawed layer under different conditions of moisture exchange on its base at $A_o = 10^oC$. Designations are the same as on Figure 152. a - Hydrometric movement of surface, m; b - Marking of surface of the ground minimal in the year; c - Upper boundary of frost.

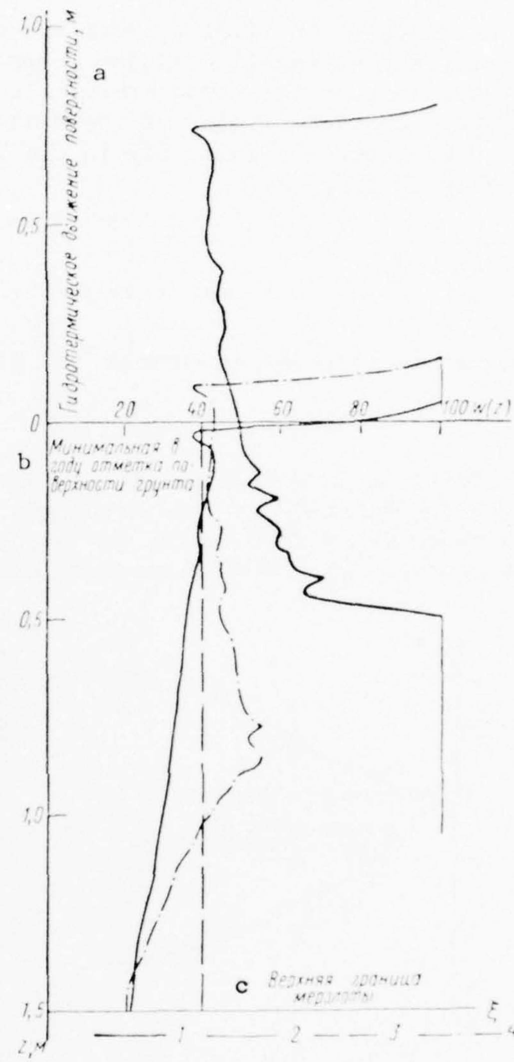


Figure 155. Distribution of the total moisture within the limits of the seasonally thawed layer under different conditions of moisture exchange on its base at $A_o = 20^o$. a, b & c as on Figure 154; other designations as on Figure 152.

Shown on Figures 154 and 155 are the curves of the distribution of the total moisture over the profile of a seasonally thawed layer, reduced to the base of the layer, that is, to the depth ξ, obtained under the same conditions.

In cases where complete freezing of the seasonally thawing layer occurs, the curves presented on Figures 154 and 155 correspond to the moment the seasonally thawing layer is washed away from the frozen rock mass, and in the contrary case to the moment of start of thawing of the latter. Presented for comparison on Figures 152-155 are the results of solution of corresponding problems without migration, obtained within the framework of the Stefan problem. In that case the curves of the distribution of moisture over the depth of the seasonally thawing layer represent the straight lines $w = w_{abs}$.

As follows from the examination of Figures 152 and 153, under fixed climatic conditions the heaving of the ground increases sharply with increase of the inflow of water above the frost. Thus, in the absence of an inflow of moisture a seasonally thawing layer freezes completely. In that case, in connection with the gradual cooling of the ground in the presence of the harmonic $\phi(\tau)$ and considerable initial moisture near the surface, an ice layer with a thickness of 2-4 cm forms. In the process of further freezing the heaving practically ceases and, as follows from Figures 154 and 155, soil with little ice on the whole forms. However, in the case of contantly acting waters on top of frost the seasonally thawing layer freezes only partially and the heaving occurs extremely intensively. It is essential that under the given conditions the heaving is mainly connected with the formation inside the seasonally thawing layer, at the base of the frozen layer, of a thick ice layer. It must be noted that under the conditions adopted in the calculations the formation of the indicated ice layer occurs independently of the amplitude of oscillations of the air temperature at a distance of about 0.5 meter from the water-bearing horizon.

The freezing of a seasonally thawing layer in the presence of a continuous character of feeding of waters on top of the frost depends on the values of $\Delta\tau_1$ and $\Delta\tau_2$. It is obvious that before the level starts to fall the process of freezing coincides completely with the case just considered. It is evident from Figures 151 and 152 that at $\Delta\tau_1$ = 1450 hours the level of waters on top of the frost starts to fall after the ice layer forms inside the seasonally thawing layer. However, later, since in the selected regime of waters on top of the frost the lowering of their level substantially outruns the rate of freezing, an intensive drying of the rest of the thawed part of the seasonally thawed layer occurs. As a result of that the seasonally thawed layer freezes completely and the ground near the base of that layer proves to contain little ice.

Thus by using such calculations when the values of $\Delta\tau_1$ and $\Delta\tau_2$ vary it is possible to quantitatively estimate the change of the cryogenic structure and heaving on different sections depending on the regime of waters on top of the frost, which are determined by the relief and the time of freezing of feeding regions. Comparison of the results of calculation at different values of A_o (Figures 153 and 154) make it possible to find the dependence of the distribution of the ice content over the section of the seasonally thawing layer on the value of A_o. It must be noted that in contrast with seasonal freezing, when, other conditions being equal, a widely known regularity of decrease of ice content with increase of the freezing rates is noted, during the freezing of seasonally thawing layers the picture is complicated. This is illustrated very graphically by calculations in the absence of waters on top of the frost. Actually, at the start of freezing, when the difference in the dimensions of seasonally thawing layers does not start to have an influence, the ice content obtained in calculations with different values of A_o is completely subject to the indicated law. However, during further freezing, because the thickness of a seasonally thawing layer decreases with decrease of A_o, the dependence of ice formation on A_o becomes the reverse -- at the same depth the ice content is lower at smaller values of A_o. Actually, increase of the intensity of ice formation at the start of freezing with decrease of A_o leads, if the dependence of the thickness of the seasonally thawing layer on A_o is taken into consideration, to a more rapid drying of the underlying part of that layer. In the same cases, when at the base of the seasonally thawing layer there are waters on top of the frost, the dependence of ice formation in the depth of a seasonally thawing layer on A_o is complicated in connection with the formation of an internal ice layer. In that case, however, since at small values of A_o (and in the presence of small seasonally thawing layers) the formation of an ice layer occurs closer to the surface, on the whole the generally accepted dependence of the intensity of ice formation on the value of A_o is preserved. As follows from Figures 152 and 153, in the presence of nonfreezing waters on top of the frost at the base of the seasonally thawing layer the total amount of heaving at $A_o = 10^o$ is 20 cm larger than at $A_o = 20^o$, whereas a surface ice layer differs insignificantly at the indicated values of A_o.

Thus the solution of the problem of the freezing of seasonally thawed layers with consideration of moisture migration with a computer makes it possible to investigate rather completely a number of extremely complex phenomena of practical importance connected with change of the cryogenic structure in freezing ground. The solution of such problems in series with different regimes of the waters on top of the frost permits making a frost forecast for rather large areas, bypassing the unwieldy calculations of multidimensional problems difficult in practice at the present time.

4. Investigation of the Freezing of Soil With Moisture Migration by Means of a Self-modeling Solution

It is advisable to solve a problem of the freezing of soil with moisture migration in the general case, because of unwieldiness, selectively in order to obtain reference data, and only in cases where the conditions of self-modeling are known not to occur. Typical examples of such problems are investigations of heat and mass transfer during freezing of a seasonally thawing layer, during freezing from below, under the conditions of a multifront problem, etc. At the same time, in a considerable number of cases in solving problems of frost prediction, to obtain estimates it is possible to limit onself to solving a self-modeling problem.

To apply a self-modeling solution in concrete cases it is necessary, while preserving all the above-indicated main properties of soils (the presence of unfrozen water and variability of the heat and mass exchange characteristics), to only average the boundary conditions in the equations of thermal and moisture conductivity. If it is taken into consideration that ice accumulation is inversely proportional to the rate of movement of the front, it is possible to take as the boundary temperature a certain overstated value, obtaining in that case the ice content with a certain reserve.

We will examine the application of a self-modeling solution of the problem of freezing with moisture migration in both a very simple formulation (without consideration of heaving) and in a complete formulation for quantitative investigation of some regularities of the process.

/Determination of the value of the moisture content of the threshold of heaving by means of a very simple self-modeling solution of a non-linear problem of freezing with moisture migration/. The exclusion of heaving from the formulation of the problem in a very simple self-modeling solution substantially limits the area of its application. At the same time, even by means of a very simple self-modeling solution it is possible to solve a number of important questions connected with predicting heaving. In particular, by means of such calculations it is possible to quantitatively determine the connection between the boundary conditions in temperature and moisture problems and the heaving hazard of soils.

Under constant boundary conditions for each value of the initial moisture content it is possible to indicate a "critical" temperature on the surface such that at all higher (but negative) boundary temperatures freezing will be accompanied by heaving, and at lower temperatures heaving will be absent. The concept of the heaving threshold will be used below precisely in that sense, which is convenient from the point of view of heaving prediction. A criterion of the fact that under the given concrete conditions freezing will be accompanied by heaving is the exceeding of the total moisture capacity by the total moisture content obtained as a result of solution of the self-modeling problem. Let us note once more that, if the indicated exceeding is large, the obtained solution (since heaving is not considered in its formulation) is purely formal and has no practical value (see section 11, Chapter 3).

The simplicity of solution of the self-modeling problem permits making serial calculations of the freezing of moist finely dispersed soils under different boundary conditions and for real heat- and mass-transfer characteristics of the medium. As an illustration let us examine the results of calculations of the cryogenic structure as a function of the initial moisture content and climatic conditions. The calculations were made with the same values of the heat- and mass-transfer characteristics as above (section 1). The initial temperature of the ground over all its depth is assumed to be $1^{\circ}C$.

The moisture content of the heaving threshold w_{h-t} at each given value of the total moisture capacity w_t is found in the following manner. First, from the obtained results of calculation of the non-linear self-modeling problem is found the dependence of the total moisture content in the frozen rock mass w_t as a function of the natural moisture content. Then on a curve corresponding to the given value of T_0 a point is sought at which $w_t = w_{abs}$, after which the value of the natural moisture content, which also is the sought w_{h-t}, is determined directly.

The thus-obtained values of the moisture content of the heaving threshold as a function of the continental character at those values of the total moisture capacity are presented on Figure 156. It is evident from it that during increase of the average annual temperature the value of the heaving threshold increases by more than 10%, and with increase of the total moisture capacity of the ground the value of w_{h-t} also increases. A very sharp change of the heaving threshold as a function of climatic conditions occurs at small amplitudes of the temperature of the surface and is practically stabilized at amplitudes higher than 20^{o} (T_{o} = -12.13^{o}). As calculations have shown, the change of the initial ground temperature in rather broad limits ($1-2^{o}$) has an insignificant effect on either the depth of freezing or the amount of the ice content in the frozen zone. At the same time the change of the coefficients of potential and thermal conductivity of the frozen ground has a sharp influence on the results of the solution, primarily on the ice saturation of the frozen ground and, consequently, the moisture content of the heaving threshold. That circumstance is of great importance from the point of view of the applicability of various approximate solutions of the problem of freezing with moisture migration, since in them the indicated parameters are assumed to be constant (in that case the problem is linearized).

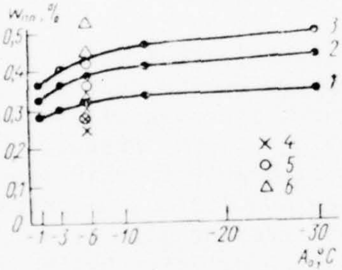


Figure 156. Dependence of the value of the moisture content of the heaving threshold w_{h-t} on the continental character of the climate with different values of the total moisture capacity in non-linear (1-3) and linear (4-6) formulations: 1 -- w_{t} = 40%; 2 -- w_{t} = 60%; 3 -- w_{t} = 80%; 4 -- K(w) = K(40%); 5 -- K(w) = K(30%); 6 -- K(w) = K(20%).

In connection with that, for quantitative estimation of the possibility of using such approximate solutions in finding the heaving threshold, corresponding calculations were made of a linearized self-modeling problem. The latter is obtained from the preceding (non-linear) problem during replacement of the parameters dependent on temperature and moisture [K(w), (t, w_{t}) $w_{un}(t)$] by certain averaged values.

The results of calculations of the value of the heaving threshold obtained in solving the problem in a linear formulation at T_{o} = -5.76^{o} (which corresponds to an amplitude of temperature oscillations on the surface of the ground of 10^{o}) are also presented on Figure 156. In that case the coefficient of thermal conductivity of frozen ground was assumed to be 1 kcal/(m)(hr)(degree) and the phase transitions of water in the range of negative temperatures were eliminated from consideration. The examination was conducted at different values of the coefficient of potential conductivity. The remaining parameters were assumed to be the same as in the above-considered nonlinear formulation. For convenience the values of the potential conductivity were selected by starting from the exponential dependence of K(w) at different fixed values of w (w = 20, 30 and 40%), indicated in section 3 of this chapter. Since K(w) is a sharply increasing function of w, and the latter in the process of freezing is known to be below the limit of rolling (20%) and not more than

w_{nat}, K(20%) and K(40%) correspond to the smallest and largest (at w_t = 40%) possible values of the potential conductivity. As follows from the presented results, the process of freezing and ice formation is extremely sensitive to the value of the potential conductivity of the ground. In that case reduction of the coefficient of potential conductivity, other conditions being equal, leads to a sharp increase in the value of the heaving threshold, and the reverse. In the nonlinear problem, just as in the linear, increase of w_t leads to a reduction of the value of w_{h-t}, but during linearization that dependence increases sharply in proportion to reduction of the potential conductivity.

At the same time the selection of the coefficient of thermal conductivity of the frozen ground plays a considerable role in freezing with migration. In that case, reduction of the coefficient of thermal conductivity (in relation to the thermal conductivity of frozen ground, which actually forms as a result of freezing) leads to increase of the ice content and, consequently, to a reduction of the heaving threshold, and the reverse. It must be stressed that change of those coefficients of heat and mass transfer in one direction leads to a certain self-compensation. Very great error corresponds to the case where the taken value of one of the coefficients is understated and that of the other is overstated.

Thus the use of approximate methods of calculating a linearized problem of freezing with migration (at constant parameters) can lead to considerable errors and in essence is impossible without preliminary determination of the values of those coefficients optimal for those conditions.

/Solution of the self-modeling problem of freezing with moisture migration with consideration of heaving and the formation of ice interlayers/. This problem was solved for three types of finely dispersed soils with different moisture-retaining capacity and substantially different coefficients of potential conductivity. In all cases the dependence of the curve of unfrozen water on the temperature was taken in the form $w_{un}(t) = a_o(b_o - t)^{-1}$, where a_o and b_o are constants determined with experimental data. Similar is the dependence of λ_f on the total moisture content w_t in the form $\lambda_f = \lambda_t (a_1 + b_1 w_t)$, where a_1 and b_1 are constants defined by the SNIP (Construction Norms and Regulations), and also from the condition that at w_t = 100% $\lambda_f(1) = \lambda_t$ = 2 kcal/(m)(degree)(hr). According to V. R. Gardner (1958) the function K(w) was approximated for all grounds in the form of $a_2 \exp\{b_2 w\}$, where a_2 and b_2 are constants. The initial temperature of the medium was assumed to be 3°C in all cases. Data characterizing the composition of the soils adopted in the calculations and also the thermophysical characteristics are presented in Table 80. The temperatures adopted for the start of freezing of light loam, light clay and medium loam were -0.2, -0.2 and 0°C respectively.

The results of the calculations of the self-modeling problem of freezing with consideration of moisture migration and heaving as a function of the conditions of freezing and the initial moisture content (other conditions being equal) for light and medium loam are presented on Figures 157 and 158*. In finding

*For light clay with a low moisture-retaining capacity, the influence of migration at $T_o \leq -1^o$ is relatively small.

Table 80 Thermophysical characteristics of loams and clay adopted in calculations

A Грунт	w_0	a $w_п$	b $\lambda_т$	$C_0\gamma_0$	c $C_т$	a_1	b_1
1 Суглинок легкий	0.36	0.54	0.80	209	750	1,25	1,25
2 Глина легкая	0.40	0,50	1.05	283	758	0.405	1,50
3 Суглинок средний (по В. Р. Гарднеру, 1958) . .	0,20	0,40	0.(6)	400	600	1,0	2.0

Продолжение табл. 80

A Грунт	$a_2 \cdot 10^5$	b_2	α	β	a_0	b_0
1 Суглинок легкий	$6,336 \cdot 10^{-3}$	18,090	3,4632	16,108	3,463	15,908
2 Глина легкая	$1,28 \cdot 10^{-8}$	45,649	1,1464	3,4636	1,146	3,264
3 Суглинок средний (по В. Р. Гарднеру, 1958) . .	3,786	16,446	0,2308	1,5385	0,231	1,538

Key: A - Soil a - w_t b - λ_t c - C_t 1 - Light loam 2 - Light clay 3 - Medium loam (according to V. R. Gardner, 1958)

the dependence of the ice content w_t and also the parameters α and β characterizing the thickness of the frozen ground and the amount of heaving on the temperature on the surface T_o (Figure 157), the initial moisture content was assumed to be 0.9 w_f. For the soils under consideration this is 51.3, 45 and 36% respectively. In addition, for a quantitative estimate of the influence of consideration of heaving on the solution of the self-modeling problem, on Figure 157 also are plotted the ice content values obtained for light loam as a result of solution of the corresponding simplified problem. It is obvious that the exclusion of heaving from the formulation of the problem leads to overstated values of the ice content forming in the freezing layer. In particular, with reference to medium loam, solution of the corresponding problem without consideration of heaving under the same freezing conditions leads to absurd values of the total moisture content by volume, considerably larger than unity.

As is evident from Figures 157 and 158, the heaving which arises during the freezing of medium loam is considerably greater than under analogous conditions for light loam. In addition, at w_1 = 0.9 w_f for soil of light loam the heaving ceases at $T_o \approx -8^o$, whereas for medium loam it is known to occur even at $T_o < -10^o$. This is mainly connected with the fact that, as follows from Table 80, the value of the coefficient of potential conductivity for light loam on the whole is far smaller than for medium loam. At the same time, as was to be expected, upon decrease of T_o or w_1 the results of solution of the problem of freezing with moisture migration both with consideration of heaving and in a simplified formulation converge.

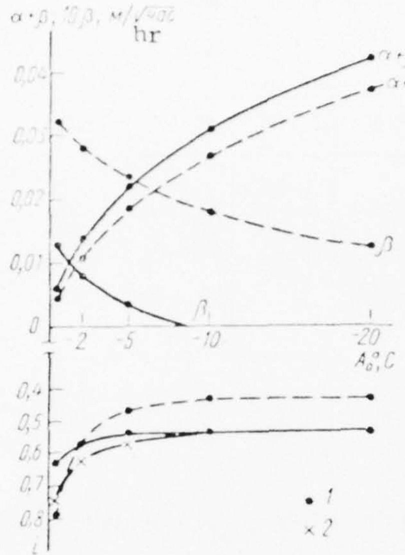


Figure 157. Dependence of the parameters of heaving (β), the thickness of the frozen zone (α + β) and the ice content (k) on the temperature of the surface T_0 for light and medium loam (broken): Results of calculation of the problem with (1) and without (2) consideration of heaving.

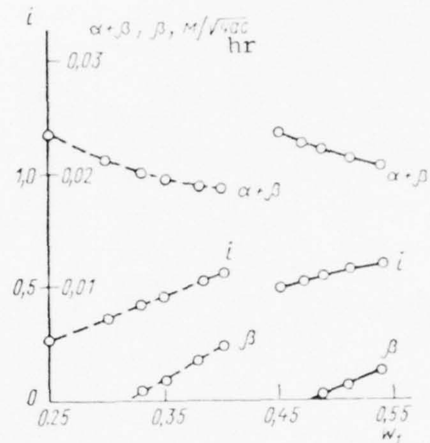


Figure 158. Results of self-modeling problem with consideration of heaving for light and medium (broken line) loam at different moisture contents for $T_0 = -5^\circ C$. Designations as on Figure 157.

Thus the cryogenic structure and heaving during freezing in self-modeling conditions depend substantially on the heat- and mass-transfer characteristics of the medium. It is obvious that this presents definite requirements for precision in finding those parameters. However, the finding of $K = K(w)$ involves serious difficulties and at values of w_1 close to w_f the experimental error increases sharply. In connection with that it is of interest by means of a self-modeling solution to quantitatively estimate with what precision it is necessary to determine the coefficient of potential conductivity in laboratory conditions. For that purpose, for the above-indicated soils a comparison was made of the results of calculations of the problem of freezing with moisture migration with consideration of heaving at different values of $K(w)$. All the other parameters of the process of heat and mass exchange are assumed to be constants. For simplicity the solution was made for $K(w)$ obtained from corresponding experimental curves of $K_0(w)$ (Table 81) by multiplication by a certain number.

The results of solution of such a problem for $jK_0(w)$, $j = 1$, 2 and 5 under different conditions of freezing ($T_0 = -0.5$ and -5°) are presented in Table 81 for all the soils under consideration. It was assumed that $w_1 = 0.95\ w_f$.

Table 81 Results of solution of a self-modeling problem

A Грунт		$T_0 = -0.5°C$			$T_0 = -5°C$		
		$K_0(w)$	$2K_0(w)$	$5K_0(w)$	$K_0(w)$	$2K_0(w)$	$5K_0(w)$
1 Суглинок легкий	$10^2\alpha$	0,3428	0,2171	0	1,9837	1,8492	1,5740
	$10^2\beta$	0,20	0,2944	0,4713	0,1449	0,2311	0,4142
	w_c	0,7095	0,8048	1,0	0,5713	0,5911	0,6358
2 Глина легкая	$10^2\alpha$	0,5636	0,5219	0,4402			
	$10^2\beta$	0,0408	0,0662	0,1172			
	w_c	0,5337	0,5563	0,6051			
3 Суглинок средний	$10^2\alpha$	0,0796	0	0	1,5399	1,2956	0,8388
	$10^2\beta$	0,4094	0,4780	0,4770	0,3420	0,5216	0,8855
	w_c	0,9023	1,0	1,0	0,5030	0,5722	0,7081

Key: A - Ground 1 - Light loam 2 - Light clay 3 - Medium loam

It follows directly from the obtained results that for soils in which $w_c - w_0 \geqslant 15\%$ the requirements for precision of determination of $K_0(w)$ must be rather high. At the same time the indicated circumstance depends to a certain degree on the freezing conditions. Thus, for medium loam even at low temperatures of the surface ($T_0 < -10°$) very small changes of $K_0(w)$ lead to a sharp change of the cryogenic structure. For light loam, however, such a picture is noted only at T_0 not below -2 to $-3°$, whereas at $T_0 < -5°$ the difference (absolute) in the ice content upon increase of $K_0(w)$ by 100% does not exceed 2%. At the same time, for silty clay (with a low moisture-retaining capacity) a noticeable difference in ice-saturation during such a change of $K(w)$ occurs only during slow ($T_0 > -1°$) freezing. Along with that, as is evident from Table 81, imprecisions in the calculation of $K_0(w)$ have a still greater effect on heaving (if it occurs) than on the ice content.

It must be noted, however, that everything said above is valid only in cases where freezing is not accompanied by the formation of an ice interlayer. As calculations have shown, if for any value of $K_0(w)$ in a certain range of A an interlayer forms on the surface, then at all values of $K(w) = nK_0(w)$, where n is a constant > 1, other conditions being equal an interlayer also forms. In that case, in accordance with section 12 of Chapter 3, the increase of $K(w)$ leads only to a corresponding increase in the moisture content of the soil. A quantitative illustration of that is the freezing of medium loam at $A = 0.5°$ and $K(w) = jK_0(w)$, j = 2 and 5, when an ice interlayer forms on the surface. Let us note that for the indicated cases the moisture content on the surface of the ground is 26.7 and 32.2% respectively (at $w_0 = 20\%$).

Thus the presented calculations show that when heaving is taken into consideration in a self-modeling solution the latter permits investigating the process of freezing of finely dispersed soils under any conditions of freezing. At the same time the results of such calculations convincingly affirm the need to conduct extensive experimental work to determine the coefficients of potential conductivity with sufficiently high precision. If the complexity of such experiments is taken into consideration, self-modeling solution can be successfully used to plan them.

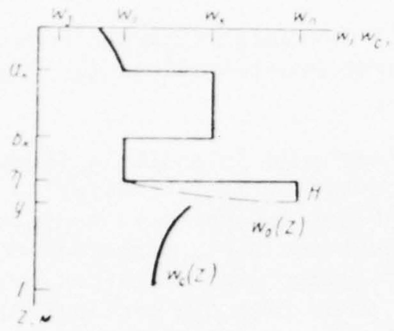
4. Solution of the Problem of the Thawing of Coarsely Dispersed Rocks With Consideration of the Infiltration of Summer Precipitations (by Digital Computer)

One of the urgent problems of heat and mass exchange in soils is study of the dynamics of thawing with consideration of convective heat transfer in the thawed zone. The influence of filtrational flows is expressed especially strongly on the rate of thawing of well-permeable coarsely dispersed rocks. Under certain conditions the infiltration of summer precipitations can play a decisive role in the dynamics of thawing. That fact, in particular, is widely used in the practice of working of placer deposits and the construction of industrial and civil objects on the territory of Siberia and the Extreme North and Northeast of the USSR.

There are a number of approximate solutions of the problem under consideration which make it possible to find the maximal value of the depth of thawing of soil during given courses of the air temperature and quantities of infiltrating precipitations (section 8, Chapter 5). It is a shortcoming of such solutions that in them the process of infiltration is linearized and in essence is considered independently of heat exchange, as a result of which the problem is solved by superposition. At the same time, such an approach, with all its simplicity, can be successfully applied for preliminary estimates in a frost survey. The influence of infiltration on the course of thawing was also investigated by G. Z. Perl'shteyn (1968) and V. T. Balobayev (1964).

Since atmospheric precipitations are intermittent, a region of seepage with mobile boundaries periodically forms and disappears in a thawed layer. For convenience it is assumed that in the ground not more than one region of seepage can simultaneously exist in the ground, that is, the next rain starts after the preceding precipitations have been "absorbed" by a water-bearing horizon. If the considerable rate of seepage is taken into consideration, that assumption in the problem of thawing of soils in the region in which permafrozen rocks are prevalent is natural. Fluctuations of the level of ground waters are determined by the ratio of infiltrational feed and excessive "lateral" runoff, which is given as a function of time. Generally accepted assumptions have also been made that the temperature of the infiltrating water and of the skeleton of the ground are equal and that the precipitations start to fall after the thawing has reached a certain finite depth. In addition, for simplicity the capillary rim is replaced by a layer of water-saturated soil with a thickness H, equivalent in the quantity of water. With consideration of what was said above, the scheme of the distribution of moisture by depth in the thawed zone will assume the form depicted on Figure 159.

A mathematical formulation and algorithm for the numerical integration of a single-front problem of thawing with consideration of infiltration upon the indicated assumptions for an arbitrary number of rains in a summer period ($\tau_0 \leq \tau < \tau_{sum}$) were presented in the work of V. G. Melamed and G. Z. Perl'shteyn, 1971). To estimate the influence of infiltration on the dynamics of



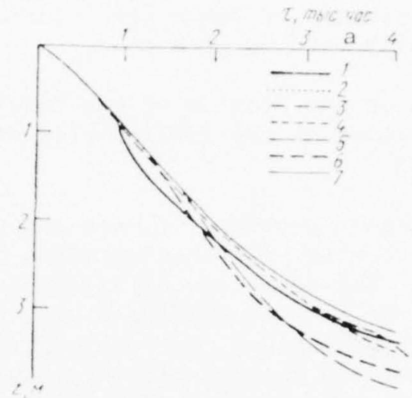


Figure 159. Schematic diagram of the distribution of moisture by depth in the presence of a seepage zone.

Figure 160. Dynamics of thawing in time under different conditions of infiltration (explanations in text). a -- τ, 1000 hrs

seasonal thawing in coarsely dispersed soils we will examine some results of solution of the corresponding problem without consideration of evaporation. The problem was solved under the following boundary conditions and with the following characteristics of the medium: the air temperature varies in accordance with the sinusoidal law:

$$t_{a}(\tau) = -3 + 20 \sin \frac{2\pi}{T} \tau, \quad T = 8760 \text{ час};$$

$$\tau_0 = 500 \text{ час} \leqslant \tau \leqslant \tau_{\text{лет}} = 4130 \text{ час}; \; \xi(\tau_0) = 0{,}4 \text{ м}; \; \eta(\tau_0) = 0{,}3 \text{ м};$$

$$l = 10 \text{ м}; \quad w_{п} = 250 \text{ кг/м}^3;$$

$$t(z, \tau_0) = \frac{\xi(\tau_0) - z}{\xi(\tau_0)} t(0, \tau), \; 0 < z < \xi(\tau_0); \; K_{ф} = 0{,}2 \text{ м/час};$$

$$t(z, \tau_0) = -2 \operatorname{erf} \cdot 2[z - \xi(\tau_0)], \quad \xi(\tau_0) < z < l;$$

$$w(z, \tau_0) = w_0 = 25 \text{ кг/м}^3, \quad 0 < z < \eta(\tau_0), \quad w_{н}(0) = 5 \text{ кг/м}^3;$$

$$w_c(z, \tau_0) = 67{,}5 \text{ кг/м}^3, \quad \xi(\tau_0) < z < l; \quad H = 0{,}1 \text{ м};$$

$$\lambda_{м}(t, w_c) = 2{,}2 \text{ ккал/м} \cdot \text{час} \cdot \text{град}, \quad C_{м}(t, w_c) = 405 \text{ ккал/м}^3 \cdot \text{град};$$

$$\underset{(\text{ккал/м}^3\text{град})}{C_{т}(z, w)} = \begin{cases} 280, & 0 < z < a_k(\tau), \; b_k(\tau) < z < \eta(\tau), \\ 320, & a_k(\tau) < z < b_k(\tau), \\ 460, & \eta(\tau) < z < \xi(\tau); \end{cases}$$

$$\underset{(\text{ккал/м} \cdot \text{час} \cdot \text{град})}{\lambda_{т}(z, w)} = \begin{cases} 0{,}9, & 0 < z < a_k(\tau), \; b_k(\tau) < z < \eta(\tau), \\ 1{,}2, & a_k(\tau) < z < b_k(\tau), \\ 1{,}5, & \eta(\tau) < z < \xi(\tau). \end{cases}$$

Here $a_k(\tau)$, $b_k(\tau)$ and τ_k (see Figure 159) are the upper and lower boundaries of the seepage zone and the time of start of fall of the k-th portion of the precipitations respectively ($k \geqslant 1$), $\eta(\tau)$ is the upper boundary of the water-saturated layer, and K_f is the filtration coefficient.

The number of nodal points was assumed to be 100 (the step in depth was 0.1 m). The step in time h was 5 hours in the presence of a zone of seepage and 20 hours without it.

To clarify the quantitative influence of the length of precipitations and aspects of their fall on the temperature regime and rate of thawing, the following variants were examined:

1) all precipitations, concentrated in the form of a single rain lasting 50 hours and with the intensity $\omega_1 = 0.004$ m/hr, fall in the first half of the summer period ($\tau_1 = 1000$ hours);

2) the same rain falls in the second half of the summer ($\tau_1 = 3300$ hours);

3) the same thing occurs at the end of the thawing ($\tau_1 = 3800$ hours);

4) the start of the fall of precipitations $\tau_1 = 1600$ hours, its length is 1000 hours and its intensity $\omega_1 = 0.0002$ m/hr;

5) $\tau_1 = 1200$ hours, the length is 2000 hours and $\omega_1 = 0.0001$ m/hr;

6) the precipitations fall in the second half of the summer ($\tau_1 = 2800$ hours) in the form of five rains lasting 10 hours each and having the intensity of 0.004 m/hr each; the interval between the rains is 250 hours and the conclusion of the last rain is 3810 hours*.

Thus the sum of the summer atmospheric precipitations is invariable in all the variants and is 200 mm. For comparison, a seventh variant was also calculated in the absence of infiltration. In that case it was assumed that the water-bearing horizon was absent $[\eta(\tau) \equiv \xi(\tau)]$.

The calculations were made with a BESM-4 computer of the MSU Computer Center. The results of the calculations of the dynamics of the depth of thawing in time in the cited seven cases are presented on Figure 160, on which the curve numbers correspond to the variants. As follows from its consideration, the relatively small infiltration assumed in the calculations has a substantially different influence on the course of thawing, depending on the conditions of the precipitation. Thus, precipitation in the form of a single brief rain (curves 1-3) leads to increase of the depth of seasonal thawing in comparison with the case without infiltration (curve 7) by 10, 24 and 16 cm respectively. In that case, in the first case (infiltration in the first half of the summer),

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*In the fourth and fifth variants the long rains fall in the period of highest summer temperatures.

immediately after the precipitation falls the depth of thawing increases by almost 20 cm as compared with the case without infiltration, but by the end of the summer period the difference in depths of thawing decreases. A somewhat greater effect is achieved during brief infiltration in the second half of the summer, practically independently of the precipitation conditions (curves 2 and 6). It must be noted that the falling of precipitations, even at the end of the thawing (curve 3), leads to a greater increase of the depth of thawing than infiltration in the first half of the summer at a higher air temperature.

Along with that the lengthy precipitation with a low intensity (variants 4 and 5) gives a considerable increase of depth of thawing -- by 41 and 60 cm respectively. In that case it is essential that in the latter cases the temperature on the base of the layer of seasonal thawing (corresponding to the case without infiltration) increases considerably by the end of the summer period and reaches 0.87 and 1.18° respectively, whereas, for example, in the former case the indicated temperature does not exceed 0.23°.

Thus when the sum of the summer precipitations is invariable the conditions of their fall play an important role from the point of view of a warming influence on the seasonally thawed layer. Under certain conditions (in particular, when the precipitations occur in short periods and especially at the start of the summer period) in coarsely dispersed soils the infiltration has little practical influence on either the depth of thawing or the temperature regime. By the same token, in the indicated cases the use of generally accepted methods of solving the Stefan problem for calculations of the seasonal thawing is completely justified. In the same cases, when the precipitations are lengthy, disregard of infiltration leads to a considerable understatement of the depth of thawing and the average annual temperature of the ground.

5. Examples of the Solution of Problems of Frost Prediction in Connection With the Construction of Structures of Various Types (on an Analog Computer)

The solution of the problem of frost prediction in connection with the economic opening up of territory often leads to a need to investigate a multidimensional and multifront problem of the Stefan type in regions of a complex type. Typical examples of problems of that kind are the questions, considered below, of calculation of the basin of thawing under buildings erected in a region in which premafrost is prevalent, aureoles of thawing and freezing around buried gas and petroleum pipelines, etc, and also in finding the configuration of the boundaries of frost under natural conditions in regions with a complex relief. It is obvious that the investigation of questions of this type in general presents considerable difficulties and is possible only with the use of computers.

In the formulation of such problems the question arises first of the selection of a calculating procedure, in particular of designating the area of investigation, the boundary conditions, etc. Of course, those questions must be solved specially in each concrete case. However, as practice in calculating multidimensional Stefan problems has shown, there are some adequate conditions which permit simplifying the calculating procedure. In particular, if the source of disturbance of the temperature field appears in the plane of a rectangle with a ratio of the sides of not less than 3:1, then the heat fluxes along the building are negligibly small (except the ends, the influence of which is small) in comparison with the heat fluxes in transverse direction. The situation is similar with cylindrical sources of disturbance of considerable extent, etc. What has been said above is of great importance, as when those conditions are fulfilled the problem can be reduced to a two-dimensional one, which considerably simplifies the calculations. In addition, in a considerable number of cases the temperature field in the base of a structure has symmetry in a given direction. Under those conditions the area of investigation can be reduced by half, which is important from the point of view of increasing the precision of calculations. In that case on the axis of symmetry conditions of isolation can be given, that is, equality of the flux to zero. On the bounding surface (far from sources of disturbance), depending on the type of problem either the characteristic ground temperature (with consideration of the temperature shift) or the geothermal gradient or the conditions of isolation are given. As an illustration we will examine the solution of some problems of forecasting typical of geocryology in connection with the change of frost conditions.

1. Solution of the Problem of the Dynamics of the Thawing Basin in the Foundation of Buildings Erected on Permafrozen Soils

We will examine the solution of a problem of the formation of a thawing basin under a building 32 meters long by 8 meters wide in the region of Yakutsk. It is assumed that the temperature in the building is constant during the year and equal to 12°C, the thermal resistance of the field is 0.428 hour-degree/kcal, the ground temperature at the depth of zero annual amplitudes is 4.5°, and the initial (15 April) temperature distribution by depth for the entire region of investigation was taken on the basis of weather data for 15 October. The thermophysical characteristics of the ground (silty loam) are presented in Table 82. The problem is solved as two-dimensional (in the plane perpendicular to the longitudinal axis of the building), with consideration of symmetry. A rectangle with a cross-section 29 x 37 m^2 in size is taken as the area of investigation.

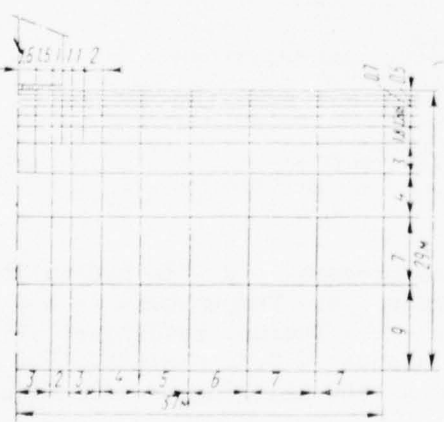

Figure 161. Subdivision of the area of investigation into blocks in the problem of a thawing basin under structures.

Table 82 Thermophysical characteristics of soil adopted in calculations

	A Наименование	B Принятое обозначение	C Числовая величина		D Размерность
1	Объемный вес скелета грунта	$\gamma_{ск}$	1 600	a	$кг/м^3$
2	Влажность грунта в % от веса скелета	w	20		%
3	Температура промерзания грунта	$t°$	0		°C
4	Льдистость грунта	i	0.9		—
5	Скрытая теплота плавления льда в единице объема грунта	$Q_ф$	23 000	b	$ккал/м^3$
6	Объемная теплоемкость талого грунта	$C_т$	640	c	$ккал/м^3град$
7	То же мерзлого грунта	$C_м$	496		$ккал/м^3град$
8	Коэффициент теплопроводности талого грунта .	$\lambda_т$	1.8	d	ккал/мград·час
9	То же мерзлого грунта	$\lambda_м$	2.4		ккал/м·град·час
10	Коэффициент теплоотдачи с поверхности . . .	α	20	e	$ккал/м^2·град·час$

Key: A - Characteristic B - Symbol C - Numerical value D - Dimensions 1 - Specific gravity of skeleton of the ground a - kg/m^3 2 - Moisture content of soil as % of weight of skeleton 3 - Freezing temperature of the soil 4 - Ice content of the soil 5 - Latent heat of fusion of ice per unit of volume of the soil b - $kcal/m^3$ 6 - Volume heat capacity of thawed soil c - kcal/(m)(degree 7 - The same of frozen soil 8 - Coefficient of thermal conductivity of thawed soil 9 - The same of frozen soil d - kcal/(m)(degree)(hr) 10 - Coefficient of heat transfer from the surface e - kcal/(m^2)(degree)(hr)

We present a solution of the indicated problem obtained on the hydraulic integrator of V. S. Luk'yanov*. The division of the region of investigation into blocks is presented on Figure 161. The scale ratios between the thermal system and its hydraulic model adopted in the calculations are presented in Table 83.

Table 83 Scale ratios between thermal system and its hydraulic model adopted in calculations

	A Наименование		B В исследуемом тепловом процессе		C В воспроизводящем его гидравлическом процессе
1	Масштаб высот	a	Температура — 1°C	1)	Напор — 1 см
2	Масштаб сопротивлений	b	Термическое сопротивление — 1 град·час/ккал	2)	Гидравлическое сопротивление — 3,34 $мин/см^2$
3	Масштаб емкостей	c	Теплоемкость C — 100 ккал/град	3)	Гидравлическая емкость (площадь поперечного сечения сосуда) — 0,312 $см^2$
4	Масштаб времени	d	Время — 1 год	4)	Время — 91,86 мин
5	Масштаб количества воды	e	Количество тепла — 1000 ккал	5)	Количество воды — 3,125 $см^3$

Key: A - Scale B - In investigated thermal process C - In hydraulic process reproducing it 1 - Height scale a - Temperature -- 1°C 1) Head -- 1 cm 2 - Resistance scale b - Thermal resistance -- 1 (degree)(hr)/kcal 2) Hydraulic resistance -- 3.34 min/cm^2 3 - Capacity scale c - Thermal capacity C -- 100 kcal/degree (Continued)

*A solution of this problem was described in detail by M. D. Golovko (1958). On the method of hydraulic analogies see the work of V. S. Luk'yanov and M. D. Golovko (1957).

Table 83 Key (Continued)

3) Hydraulic capacity (cross-sectional area of vessel) -- 0.312 cm^2
4 - Time scale d - Time -- 1 year 4) Time -- 91.86 minutes
5 - Scale of quantity of water e - Quantity of heat -- 1000 kcal
5) Quantity of water -- 3.125 cc

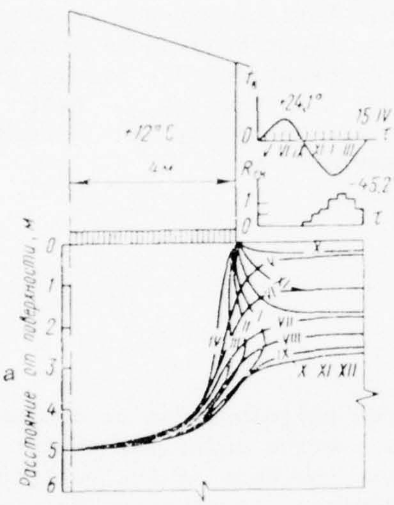

Figure 162. Diagram of zero isolines during a year in periodically established regime. a - Distance from surface, m

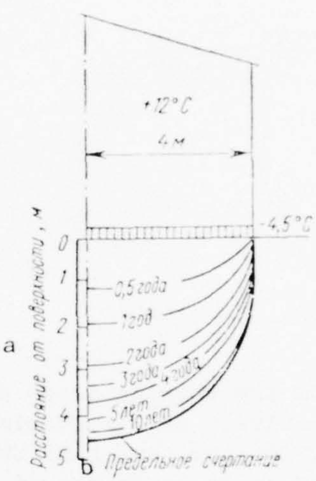

Figure 163. Dynamics of change of a thawing basin in the course of operation. a - Distance from surface, m b - Limiting outline

As a result of solution of the forumulated problem on Figure 162 are presented diagrams of the zero isolines in the course of a year during a periodically established thermal regime in the foundation of a building. In that case it is necessary to stress that, as was to be expected, change of the temperature at the surface of the ground near the building has practically no effect on the contour of the thawing basin under its central part, in contrast with the ends. Presented on Figure 163 are the dynamics of change of the thawing basin in the course of time up to the limiting state, obtained under averaged conditions on the surface of the ground. As is evident from consideration of it, the formation of the basin occurs rather rapidly -- already after 5 years its depth amounts to about 85% of the limiting, and after 10 years reaches 95%.

Of considerable practical interest is study of the dependence of the thawing basin on the surface conditions near the building. To illustrat this, on Figure 164 are presented the results of calculations of the building considered above, when on both sides of the building there is an identical snow accumulation (in that case symmetry is preserved in transverse direction). The snow accumulation is taken into consideration by proportional variation of the thickness of the natural snow cover by k times. Shown on Figure 164 is the outline of the thawing basin in the first two years of operation of the building obtained as a result. As follows from examination of it, the presence of snow drifts near the building sharply influences the outline of the thawing basin, this being reflected both in expansion of the basin and in increase (true, to a lesser degree and with some delay) of its depth.

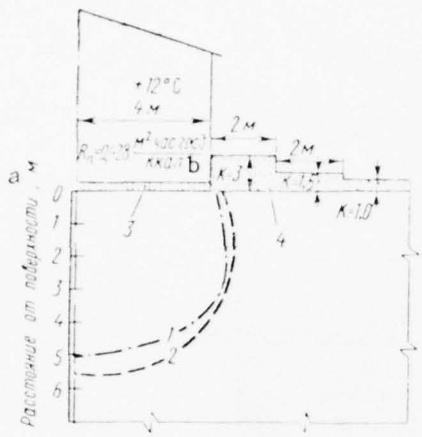

Figure 164. Outlines of the thawing basin after 1 (curve 1) and 2 (curve 2) years of operation: 3 - insulation in building; 4 - snow. R_f -- heat insulation of the field. a - Distance from the surface, m

Thus the depth of the thawing basin of fairly large buildings can be characterized by a relatively stable thawing basin in a section under the center of the building. At the same time the thawing near the ends of the building, depending substantially on change of natural conditions, fluctuates sharply within the limits of a year. Since the depth under the center of the building, as follows from the presented results, vary rapidly approaches the limiting value obtained from a solution of a considerably simpler stationary problem, the question arises of approximate estimation of the time when the limiting outline of the basin is reached. From the engineering point of view it often proves sufficient to estimate the time during which the thawing basin reaches 80-90% of the maximum. It follows from an examination of the criteria of similarity of the unsteady temperature fields that the time necessary for formation of the thawing basin with change of the dimensions of the building varies approximately in proportion to the square of the ratio of the widths of the buildings. This permits on the basis of the results of a single calculation obtaining in first approximation in the given region a time of formation of thawing basins for different structures with a similar temperature and heat insulation in them. Let us note that the dependence of the time necessary for the formation of a thawing basin on the width of the building is more precise the smaller the heat resistance of the insulating coverings on building. As has already been pointed out, the finding of the limiting thawing basin under a building in the presence of different insulations inside and outside the building but under averaged surface conditions is accomplished fairly easily with either a digital computer or a model EI-12 or EGDA differential analyzer.

Thus, as a result of serial calculations with a differential analyzer a nomogram was obtained for calculation of the limiting configurations of a thawing basin for buildings of any width under the conditions of symmetry under arbitrary (constant) boundary conditions, insulation inside the building and any correlations of the coefficients of thermal conductivity of the ground in the thawed and frozen states. The nomogram (Figure 165) consists of a diagram linking the ratio of the greatest depth of thawing ξ to the building width a with the ratio of thickness of the thawed ground equivalent in resistance to the thermal resistance of the insulation in the building s to the building width for different values of the coefficient $\delta = t_{bldg} - t'/t_{bldg} - t_o$. Here t_{bldg} and t_o are the temperatures in the building and on the surface of the ground respectively and t' is the temperature of the start of freezing.

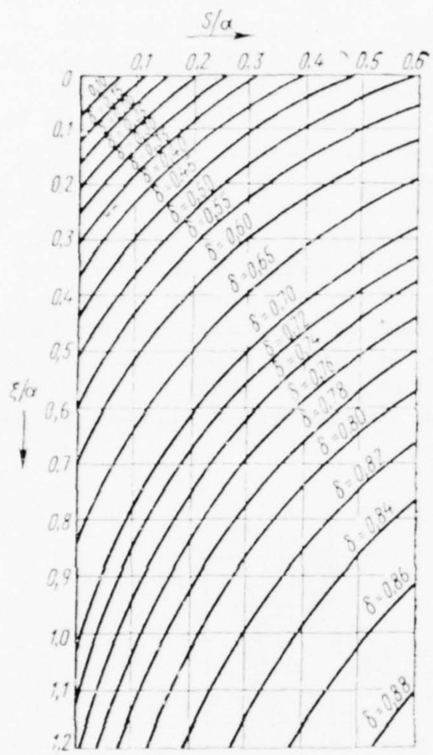

Figure 165. Nomogram for calculation of the thawing basin.

The greatest depth of thawing is calculated with the nomogram as follows. The calculating temperature in the building t_{bldg} is replaced by the reduced with the formula $t'_{bldg} = t_{bldg} (\lambda_t/\lambda_f)$. Then the difference of temperatures inside the building and in the ground at the depth of constant temperatures $t_{bldg} = t_o$ is taken to be 100% and the coefficient $\delta = t'_{bldg} - t'/t'_{bldg} - t_o$, which determines the position of the zero isotherm under the center of the building, is calculated.

If we calculate with the thermal resistance of the field R_f the value of $s = {}_tR_f$, on the basis of s/a and δ we find on the nomogram the ratio of the depth of thawing to the building width ξ/a and from that the depth of maximum thawing ξ_m.

2. Calculation of the Thawing Basin in the Construction of Large Structures With Irregular Heat Release

As an example of a more complex problem of the thawing basin we will examine the dynamics of the temperature fields under a building of large dimensions with irregular heat release. Such a problem is typical in the case of large industrial complexes consisting of structures essentially different in temperature conditions. The results of the calculations of the two-dimensional problem of the rate of thawing of rocks in the base of a building 58 meters wide, consisting of two sections, in the course of 70 years of operation are presented on Figure 166 (according to the work of S. A. Zamolotchikova and V. G. Melamed, 1972). In the first section, with a width of 18 meters, a constant temperature of $+60^{\circ}$ is maintained, and in the second, 40 meters wide, of $+20^{\circ}$. It is obvious that the indicated circumstance excludes symmetry of the temperature field in the ground. Taken as a region of investigation is a rectangle 400 x 160 meters in size, and near the surface a strongly fractured layer with a thickness of 3 meters is separated, the thermophysical characteristics of which differ from those in the basement rocks. To estimate the dependence of the rate of thawing on the quantity of phase transitions of water, two variants of calculations with a different ratio of the moisture contents in the indicated layers were examined, other conditions being equal. The properties of deposits adopted in calculations of the rate of thawing of rocks are presented in Table 84. A temperature of 0° was given on the lower boundary of the region of investigations, which coincides with the base of the permafrozen rocks. The distribution of the initial temperatures by depth is presented in Table 85.

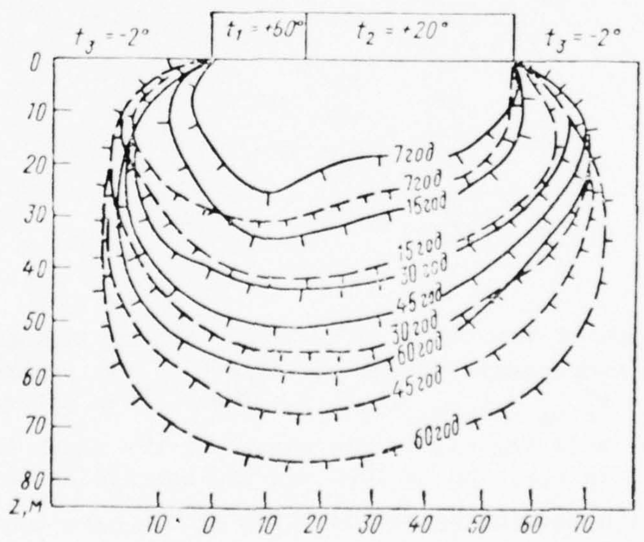

Figure 166. Boundaries of the thawing of rocks in time under a section with positive t_1 and t_2 at a moisture content of the rocks of 6% in the first layer and 3% in the second (solid line) and at a moisture content of the rocks of 18 and 9% (broken line).

Table 84 Properties of rocks adopted in calculations

	A Наименование характеристик грунта	B Размерность	C Вариант 1		D Вариант 2	
			a 1 слой	b 2 слой	a 1 слой	b 2 слой
1	Объемный вес скелета	кг/м³	2 290	2470	1 895	2 110
2	Весовая влажность	%	6,5	3,0	18,0	9,0
3	Относительная льдистость		1,0	1,0	1,0	1,0
4	Теплота фазовых превращений	ккал/м³	11 908	5928	27 288	15 192
5	Объемная теплоемкость мерзлого грунта	ккал/м³·град	510	506	510	506
6	Объемная теплоемкость талого грунта . .	то же	584	543	584	543
7	Коэффициент теплопроводности (талого и мерзлого) грунта	ккал/м× ×град·час	3,0	3,0	3,0	3,0

Key: A - Characteristics of the ground B - Dimensions C - Variant 1
D - Variant 2 a - first layer b - second layer
1 - Specific weight of skeleton, kg/m^3 2 - Moisture content by weight, % 3 - Relative ice content 4 - Heat of phase transformations, kcal/m^3 5 - Volume specific heat of frozen ground, kcal/(m^3)(degree) 6 - Volume specific heat of thawed ground, kcal/(m^3)(degree) 7 - Coefficient of thermal conductivity (of thawed and frozen ground), kcal/(m)(degree)(hr)

Table 85 Distribution of the initial temperatures of rocks by depth

A Глубина, м	10	20	40	60	80	100	120	140	160
B Температура, °C	−4	−4	−3,4	−2,5	−2,2	−1,5	−1	−0,5	0

Key: A - Depth, m B - Temperature, °C

It is natural that in the first years of operation a thawing basin grows especially rapidly under a building with a temperature of +60°, under which by the 7th year its depth reaches 30 meters (Figure 166). Under the building with a temperature of +20° the depth of the thawing basin by the 7th year changes from several to 20 meters, increasing toward the central part of the warm section. Later the maximal thickness of the thawing basin gradually moves out from under the building with a temperature of +60° toward the center of the entire warm section, where by the 60th year it reaches 70-77.5 m.

The above-presented data permit saying that a permafrozen rock mass with a thickness of 160 meters under a warm section completely thaws in not less than 300 years.

The moisture content of the rocks has a substantial influence on the thawing rate. At a moisture content of the rocks in the upper layer of 18% and in the lower of 6% in the first year under the section with a temperature of +60° the rate of thawing reaches 9 meters/year, and under the section with a temperature of +20° it varies from 5 to 2 meters/year. From the 5th to 11th years the rate of thawing of the rocks becomes 1 or 2 meters/year, and by the 60th year it decreases to 0.4-0.5 meter/year.

The divergence in the depths of thawing in the 60th year exceeds 20 meters and still continues to grow.

Comparison of the results on the thawing of rocks for variants with different average temperatures on the surface of -2, -4 and -7° showed that for buildings with such intensive heat release those temperatures have little influence on the formation of a thawing basin in the course of 60 years.

Thus the question of the formation of outlines of a thawing basin under the central part of a building can be solved rather simply in each concrete case. Making such calculations is an important part of frost forecasting in the development of territory in a region where permafrozen rocks are prevalent, as it permits solving the question of the optimal selection of a foundation. At the same time, of enormous importance from the point of view of the stability of buildings is the dynamics of thawing of the ground on the perimeter of a structure. The latter, as indicated above, depends substantially on the character of the frost conditions outside the building.

The solution of that general problem of frost forecasting in connection with the construction of structures leads to a need to examine an extremely complex multidimensional multifrontal non-stationary problem. At the same time its solution permits investigating the question of the selection of optimal conditions near the building (the creation of lawns, snow retention, etc). In addition, making such calculations gives a considerable saving, as it permits solving problems of frost forecasting with the use of local materials and makes it possible to dispense with costly hauling of ground from special quarries.

3. Investigation of the Thermal Regime of the Body and Base of an Earthen Dam During Construction and Operation

We will examine a number of specific problems relating to frost which arise if the base of a dam and all other structures is a thick mass of loose deposits, in the frozen state to a considerable degree. These questions have been studied on the example of the Western Siberian lowlands.

The most important of those problems is that of the state and stability of an earthen dam. It was proposed to erect the dam by building it up hydromechanically with local silty loams, sandy loams and sands in the course of a number of years. It was planned to build it up only in the summer period. Naturally the question arises of the freezing of the embankment both in the period of the buildup and later, in the period of operation of the structure.

Irregular freezing and possible thawing, and also the formation of frozen interlayers in the body of an earthen dam in the process of its erection, can lead to the formation of undesirable and dangerous glide planes and destruction of the dam. In connection with that the problem arises of determining optimal working conditions which would assure an absence of dangerous frozen interlayers in the body of a dam in the construction period.

In addition it is necessary to determine the thermal state of an earthen dam at the end of construction and its change later, in the operating period and, in particular, determine the probability of the formation of permafrost of the body of the earthen dam.

A very essential aspect is solution of the problem of the probable thawing of a basal, well-filtering coarse sand and pebble horizon lying at the base of a dam under sandy alluvium at a depth of 25-30 meters.

No less important, finally, is the solution of the problem of the behavior of slopes in places where an earthen dam is in contact with the sides of a valley. In that case it is necessary to determine the course of thawing of the frozen rock masses and the steepness of the planes of possible sliding of the thawed earthen masses over the upper surface of permafrozen rock masses. Those questions could be solved only with consideration of the entire complex of the concrete natural situation of the region of construction.

/The problem of the temperature regime of the embankment in the process of construction/ because of the considerable dimensions of a dam can be reduced to a unidimensional multifront problem of the Stefan type. Its solution with a hydraulic integrator presents no substantial difficulties even during discharge.

The data of a frost survey of the lithological composition of the soil indicate substantial heterogeneity of the soil at the base of the embankment: sandy loam and loam soils lying from the surface to a depth of 5.3 meters are underlain by fine sand to a depth of 25 meters. It is proposed to pour the embankment off the silty sand. The thermophysical characteristics of the soils in the body and base of the dam are presented in Table 86. As a result of calculations of the thermal regime of the body and base of the dam* it has been found that under average perennial conditions the reserve of heat in the embankment accumulated in the process of buildup is eliminated in the course of the first 10 years of operation (Figure 167). But if the buildup is carried out in cold years with little snow then, as is evident from Figure 168, in the body of the buildup form layers of frozen ground of considerable thickness.

If we compare the results of all the calculations made we can conclude that during the construction of an embankment it is necessary to construct snow retention on its surface in order to prevent the formation of permafrozen layers of ground in it. The optimal amount of thermal insulation assuring the preservation of the body of the embankment in a thawed state is roughly equivalent to 1.5 times the height of the average snow cover observable in the region of construction through the years. This conclusion remains valid for sections of the region where the average annual temperatures of the ground are not lower than -1°. On sections with lower annual average temperatures of the ground a double thickness of the snow cover should be assured.

/Thawing of the frozen rock mass under a reservoir and dam/ on account of motion of water in a filtrating layer. To judge the stability of a structure

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*The course of the conducting of the indicated calculations and also their results are described in the work of V. A. Kudryavtsev, V. G. Melamed, M. D. Golovko and N. I. Trush (1961).

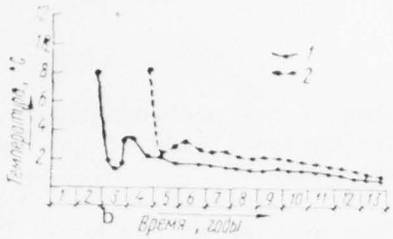

Figure 167. Change of the temperature of the ground in the body of an embankment in the course of 8 years of construction and 5 years of operation: 1 - temperature at a height of 9.6 meters; 2 - " " " " " 21.6 " . a - Temperature, °C b - Time, years

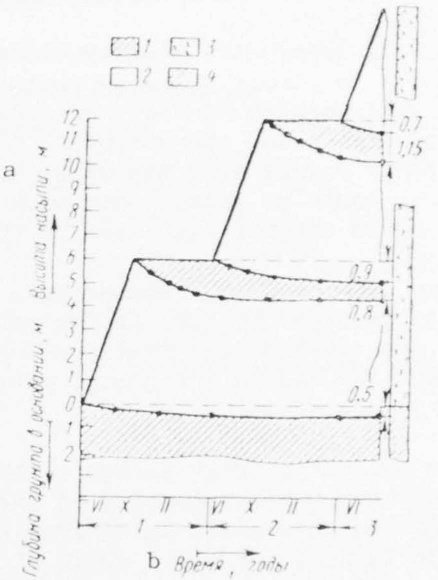

Figure 168. Change in time of the boundary of freezing and thawing of the ground in the body and base of an embankment in the course of 2 "cold" years with little snow: 1 - frozen ground; 2 - thawed ground; 3 - silty loam; 4 - light loam. a - height of embankment; b - Depth of ground at the base, meters; c - Time, years

Table 86 Thermophysical characteristics of ground adopted in calculations

	A Наименование	B Размерность	C Суглинок	D Мелкозернистый песок	E Пылеватый песок
1	Объемный вес грунта	$кг/м^3$	1 850	1 840	1 920
2	Влажность грунта в % (ко всему весу грунта)	%	34	21,0	20
3	Температура промерзания грунта	°C		0	0
4	Скрытая теплота плавления льда в единице объема грунта	$ккал/м^3$	49 000	30 920	30 800
5	Объемная теплоемкость талого грунта . .	$ккал/м^3 \cdot град$	500	536	500
6	Объемная теплоемкость мерзлого грунта	$ккал/м^3 \cdot град$	500	536	500
7	Коэффициент теплопроводности талого грунта	$ккал/м \cdot град \cdot час$	0,69	1,59	1,6
8	Коэффициент теплопроводности мерзлого грунта	$ккал/м \cdot град \cdot час$	1,07	2,12	2,12
9	Коэффициент теплоотдачи с поверхности	$ккал/м^2 \cdot град \cdot час$			20

Key: A - Characteristic B - Dimensions C - Loam D - Fine sand E - Silty sand 1 - Specific weight of ground, kg/m^3; 2 - Moisture content of ground as % (of total weight of the ground); 3 - Freezing point of the ground; 4 - Latent heat of fusion of ice per unit of volume of the ground, $kcal/m^3$; 5 - Volume specific heat of thawed ground, kcal/(m^3)(degree); 6 - Volume specific heat, kcal/(m^3)(degree); 7 - Coefficient of thermal conductivity of thawed ground, kcal/(m)(degree)(hr); 8 - Coefficient of thermal conductivity of frozen ground, kcal/(m)(degree)(hr); 9 - Coefficient of heat transfer from the surface, kcal/(m^2)(degree)(hr) in subsequent years of its operation it is necessary to calculate the thawing of those frozen rocks in the base during a long period of operation of a reservoir, taken to be 50 years. In connection with that, calculations were made of the thawing of a permafrozen rock mass from above (through heat arriving from the reservoir) and below (through heat transported by filtration of water in the layer underlying the mass of frozen soils). The former calculation is extremely simple and is not illuminated here, but the second required a special approach.

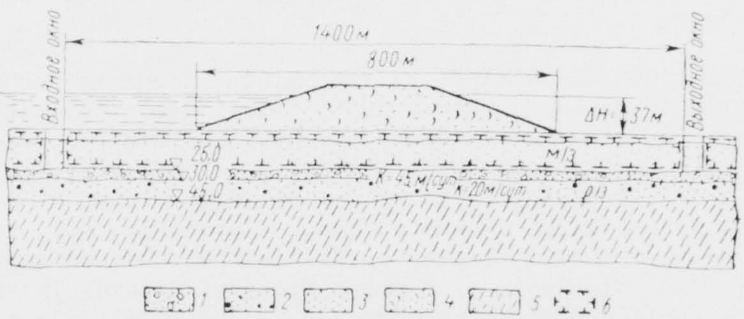

Figure 169. Geological and lithological conditions of the formulation of the problem of thawing of frozen rock under a dam through the influence of the water flow in the filtering horizon: 1 - gravel-pebble-boulder horizon; 2 - sand of various particle sizes; 3 - fine sand; 4 - silty sand; 5 - loam; 6 - boundary of frozen ground. a - Input window; b - Output window.

Figure 169 presents a geological scheme of the formation of the problem under consideration. In the calculations it was assumed that the filtration of water from a filled reservoir under a dam occurred mainly through thawed "windows." In connection with movement of the filtration flow in the basal horizon and in the horizon of filtering various-sized sand underlying it the frozen rock mass both under the reservoir and under the dam will gradually thaw. Since there are fairly many input and output windows, it was assumed that the movement of ground waters along the filtering layer* in the direction perpendicular to the motion of flow will be identical. The length of the filtration path in that case was assumed to be L = 1400 meters. The flow rate of the water filtering from the upper into the lower waters of the reservoir is determined with the formula**

$$V_1 = K_{cp} \frac{\Delta H}{L} b, \text{ м}^3/\text{сут (на 1 пог. м ширины потока)}.$$

*That is, the basal horizon and the layer of sand underlying it.

**The hydraulic resistance of the input and output in the filtering horizon can be neglected.

Here K_m = 26.2 m/day is the weighted average value of the filtration coefficient of the soils of the filtering horizon; b = 20 m is the thickness of the filtering horizon; ΔH = 37 m is the difference of heads of the water in the upper and lower waters of the reservoir.

In connection with the fact that the filtering horizon is covered by poorly filtering sands, the movement of water in thawing higher-lying sands was disregarded in the calculations. Below the filtering horizon lies loam which is practically water-impermeable. Therefore the conditions of water filtration are assumed to be invariable in time and unconnected with changes of the thermal regime of the base.

The thickness of the filtering horizon is very small in comparison with the length of the filtration path and therefore in calculations the difference in water temperatures in the cross-section of the filtering flow can be neglected. To simplify calculation of the thermal interaction of the filtering flow with higher- and lower-lying rocks it can be assumed that it is composed of homogeneous soil, the filtration coefficient of which is equal to the weighted average value of the filtration coefficient of the two layers of that horizon. Therefore the calculations of the thawing of grounds lying above and below the filtering horizon under consideration can be made separately, taking into consideration in each of them only half the flow rate of the moving water $V = V_1/2$. Presented below are calculations of the thawing of the grounds lying only above the filtering horizon (Figure 170).

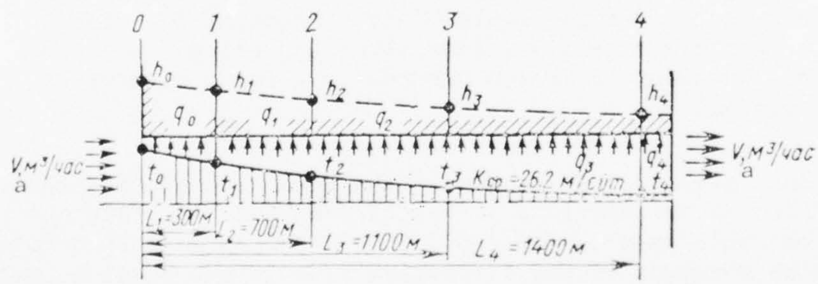

Figure 170. Schematic diagram of the conditions adopted in calculating the thawing of the frozen rock mass from below through movement of water in the filtering horizon. The depth of thawing is h_i, meters; the heat flow from the subsurface flow into the roof q_i, kcal/(m^2)(hr) in each i-th section, i is 0, 1, 2, 3, 4. a -- V, m^3/hr

During the movement of water along a filtering horizon there is a gradual reduction of its temperature, in connection with which the intensity of thawing of higher-lying frozen soils gradually decreases with removal from the start of the filtration path.

The two-dimensional problem of the thawing of frozen rocks by the filtration flow was solved approximately by conducting a series of unidimensional calculations of thawing at different points along the length of the filtration path.

To obtain the course of the change of water temperature at those points in time (used later in the thawing problem as boundary conditions of the first kind), an approximate calculation was preliminarily made of the water temperature drop during its movement along the filtering horizon on account of upward heat transfer.

The proposed method of calculating the depths of thawing on account of motion of water into the filtering layer represents to a certain degree the method of successive approximations. Taken as the first approximation of the depths of thawing of the roof h_i along the length of the filtration path were depths calculated with the simplified Stefan formula (3.7.7) at different water temperatures (for example, +4, +3, +2 and +1°). Obtained from that was the distance at which under those conditions by the moment $\Delta\tau$ the water temperature will be t, and also data on the change of the water temperature in time for different points on the filtration path. Taking them as boundary conditions of the first kind for solving the problem of determining the course of thawing, a second approximation of the thawing depths was obtained with the hydraulic integrator.

Then with the obtained data on the thawing of the frozen rock mass covering the filtering horizon it is possible to refine the equation of the heat balance for different sections of the filtering flow and by the same token more precisely calculate the variation in time of the water temperature at various points of the flow. Repeating then the calculation of the thawing depth with the hydraulic integrator, one can find the subsequent approximation of the thawing depth, etc. Since the starting data on the regime of motion of the subsurface waters from the upper and lower waters of the reservoir through the filtering horizon were known approximately and are very simplified in the adopted calculating procedure, it was decided in calculating the thermal regime of the ground interacting with the filtering flow to limit ourselves to the second approximation.

The results of calculation of the temperature change in the roof of the filtering horizon in the various sections -- at the beginning, middle and end of the dam and in the outlet window (see Figure 169) in the course of 50 years are presented on Figure 171. Those curves were also taken as boundary conditions of the first kind in the refined calculation of the thawing of the frozen rock mass covering the filtering layer with the hydraulic integrator.

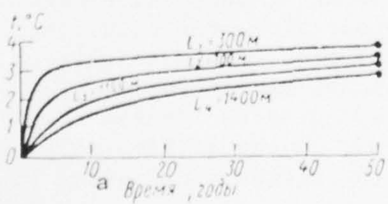

Figure 171. Change of the water temperature in time in various sections of the filtering horizon. a - Time, years

The second boundary of the region where a constant ground temperature of -0.5° is preserved was taken at a distance of about 10 meters from the roof of the filtering horizon. The initial temperature distribution of the investigated region was also assumed to be constant ($t = -0.5^{\circ}$).

The results of calculation of the thawing of the frozen rock mass on account of motion of the water in the filtering horizon, obtained as a second approximation of the depths of thawing of the ground, are presented in the form of diagrams of the variation of those depths in time in various sections in the course of 50 years of operation, and also (in its upper part) of the curve of variation of the depth of thawing from above under the reservoir and dam during the same period (Figure 172). On the basis of those diagrams the configuration of the bedding of permafrozen rocks under the reservoir and dam at different moments of time is readily determined. It is obvious that even after 50 years of operation of hydrosystems a considerable portion of the frozen rock mass still will remain in the frozen state.

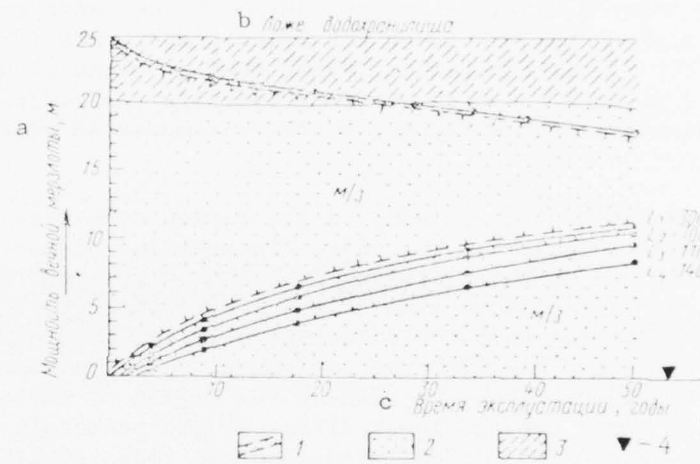

Figure 172. Change of the depth of thawing in time in the base of the dam and along the length of the filtering horizon in different sections: 1 - boundary of the frozen soil; 2 - fine sand; 3 - loam; 4 - roof of the filtering horizon; a - Thickness of permafrost, meters; b - Bed of reservoir; c - Time of operation, years.

The following conclusions can be drawn as a result of the calculations. The thawing of permafrozen rock masses from below on account of convective heat exchange between the filtered waters from the reservoir along the filtering horizon will occur slowly. Complete thawing of the frozen rock mass will occur after several hundred years. Therefore one cannot expect rapid catastrophic sediments of an earth dam on account of thawing of the frozen base from below. The quantity of those sediments and their course in time will be determined only by the course of the thawing of the frozen rock masses in the base from above.

The calculation testifies to the possibility of formulating and solving with an accuracy sufficient for practice problems of convective heat exchange with a hydraulic analyzer for such complex conditions as were assumed in the problem under consideration above.

/Investigation of the dynamics of thawing of the ground in bank slopes of a reservoir/. After a reservoir has been filled, thawing of the rock masses starts in its underwater part. In the bank slopes of the reservoir, especially near the waterline, where the layer of the underwater talik thins out, such thawing leads to a sharp change of the relief of the upper surface of the permafrozen rock masses. In the presence of considerable slopes on the upper surface of the frozen rock masses, runoffs and slides of thawed rocks are possible. In connection with that the problem was set of determining the possible very large slopes of the upper surface of a frozen rock mass which arise as a result of thawing under a reservoir, which can lead to the formation of landslides, runoffs and other slides of coasts.

The problem was solved in a simplified manner. The sought outline of the boundary of thawing of rocks in the slope of the left bank of the reservoir was obtained as a result of the solution of several unidimensional problems, that is, the heat fluxes in horizontal direction were not taken into consideration in the calculations. In such an approach the difference in the thermal regime of the different sections of the slope was determined mainly by the time of start of warming of those sections. The program of filling of the reservoir is shown on Figure 173a in the form of a diagram of the change of the water horizon in it in the course of time.

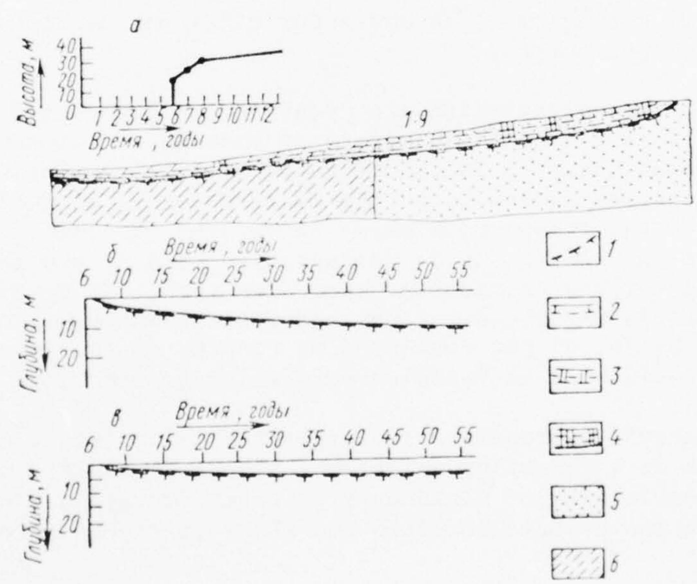

Figure 173. Change of the depth of thawing of the ground in slopes of the left bank of a reservoir: a - diagram of the change of the water horizon in time; b & c - diagrams of the change in time of the depths of thawing of the ground, preliminary and final: 1 - boundary of frozen ground; 2 - thawing of ground in the 7th year of filling of the reservoir; 3 - thawing of ground in the 30th year of filling of the reservoir; 4 - thawing of ground in the 50th year of filling of the reservoir; 5 - sand; 6 - loam. a - Height, m b - Depth, m c - Time, years

Also of definite importance were the composition of the soils lying on different sections of the slope and the difference of their thermophysical characteristics. Neglecting several insignificant heterogeneities in the structure of the slope of the left bank, on it we distinguished two sections with essentially different soils. In the bed of the reservoir and on the half of the adjacent slope up to a height of 15 meters lies loam, and in the higher-lying half of the slope, fine sand (Figure 173). With such heterogeneity of the soils lying on the slope taken into account the course of thawing was calculated for 50 years on a hydraulic integrator. In that case it was assumed that from the moment of warming on the surface of the ground is established a temperature of 4^{o}, equal to the average annual temperature of the layers of water near the bottom in the reservoir. The results of the described preliminary calculations are presented on Figure 173b in the form of diagrams of the change of the depth of thawing of the ground in the course of time. Using those diagrams with consideration of the program for filling the reservoir, we determine the sought outlines of the boundary of thawing of the ground in the slope of the left bank. To do that the depths of thawing of the ground, determined for identical moments of time, which were obtained from diagrams on Figures 173b and c for different sections of the slope, were connected by smooth curves.

The final results of the calculation are presented on Figure 173 in the form of diagrams of the position of the boundary of thawing of the ground in the slope of the left bank after 7, 30 and 50 years from the moment the reservoir is filled. The following proved to be the largest angles of inclination of the upper surface of the frozen rock mass: 13^{o} at the 7th year, 23^{o} at the 30th and 27^{o} at the 50th year. As is evident, in spite of the fact that the intensity of thawing of the ground decreases with time, the probability of formation of a runoff in the slopes of the reservoir increases with the course of time. The probability of the formation of runoffs on the interface of the frozen and thawed soils must be verified by special geotechnical calculations.

In conclusion, it should be stressed that specific calculations of the change of frost conditions as a result of construction became possible only on the basis of a close combination of field work on frost surveying with laboratory and office work, in the process of which the above-considered problems were simulated.

4. Forecast of the Temperature Regime of the Body and Base of a Dam in Rocky Soils, Established in the Process of Operations

In the process of planning an earthen water-pressure dam of a hydroelectric power station with its base and ends fastened in rock a number of questions connected with the behavior of the dam in the process of long-term operation arise. Among the most important of them is determination of the degree of filtration of water from the upper into the lower waters through the body of the dam. Therefore special attention is turned toward the erection of an economically advantageous anti-filtration screen capable of restraining to some degree the movement of water through the dam. These questions have been examined on the example of solution of corresponding problems for the region of the main distribution of permafrost. In our example, a rockfill dam with a gravel-loam screen must be laid on the thawed rocky base of a riverbed. The severe climatic conditions in the region of construction can lead in the course of time to the formation of permafrozen rocks in the body of the dam. If the lower surface of the permafrozen rocks descends below the base of the dam, the formed frozen core will fill the role of a natural anti-filtration screen. In connection with that, in planning the dam the problem arises of determining the possible established configuration of the frozen core forming in the body of the dam in the process of operation.

Examined below is the prediction of a steady temperature regime in the body and rocky base of a dam of a hydroelectric power station as a function of various conditions on the surface. The corresponding calculations were made with an EI-12 differential analyzer installed in the Department of Geocryology of Moscow University.

Preliminarily, the problem of determining the steady temperature regime at the dam site under natural conditions was solved in order to verify the correctness of the given temperature conditions on the boundaries of the investigated region (mainly on the lower boundary -- about 300 meters) and compare the results obtained with the EI-12 with available full-scale temperature determinations. After that, under reliable boundary conditions, the problem of forecasting the steady regime which is established in the body of a dam and in lower-lying rocks after construction of the hydro station is completed was solved. In that case different variants of the temperature conditions on the open surface of the dam were examined.

/Determination of the stationary temperature regime of a dam and rocks along the line of a section under natural conditions/. This problem was solved in the plane along the line of the section, the region investigated was limited from above by the day surface, and the lateral boundaries were selected at such a distance that the influence of the river was practically not expressed (250-300 meters from the river). In connection with that the problem was solved on a fairly large scale (a step of 50 meters on the x and y axes), the upper layer of loam (1-2 meters) was not considered, and the mass of diabases in the base was assumed to be homogeneous in the entire investigated depth. The average annual temperature on the bottom of the river was assumed to be 4°, on the slopes of the valley the temperature was given in accordance with the change of height, steepness, exposure of the slopes and character of the covers and basement rocks, and at a depth of 350 meters below the level of the river (the lower boundary of the investigated region) was $+1^{\circ}$ (Figure 174). Presented on the same profile is the established temperature distribution over the section of the planned dam, obtained as a result of solution of the problem under consideration. Under the riverbed a permeating talik is well traced, and the thickness of the permafrozen rocks under the sides of the valley reaches 300-350 meters. Comparison of the temperatures at different depths obtained with the differential analyzer with the data of temperature measurements in drillholes showed good agreement of the results.

/Determination of the stationary temperature regime of the rocks of a solid rock base under different conditions on the surface of a dam/. In the process

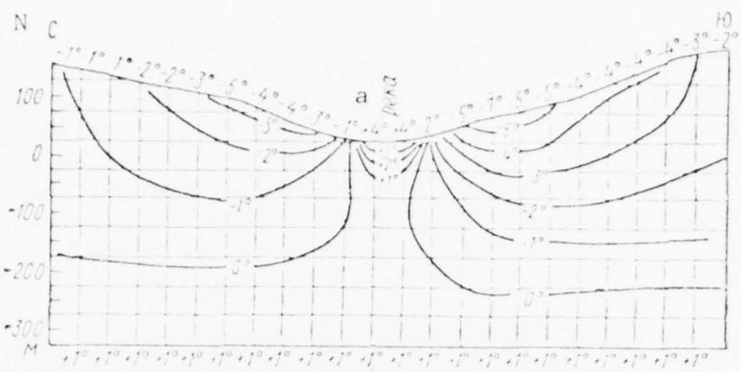

Figure 174. Temperature field in the plane along the line of a section under natural conditions. a - River

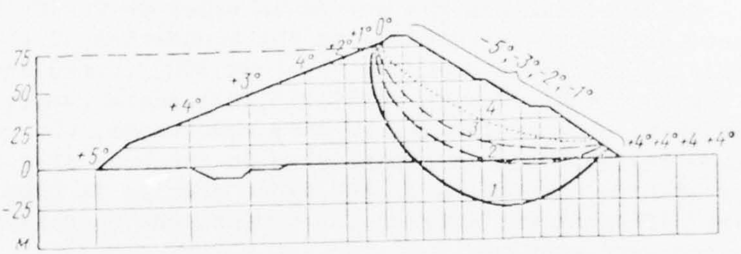

Figure 175. Temperature field in the body and base of a dam in the presence of a layer of water at the foot of a dam in the lower race. Scale on the vertical 1:50: 1-4 -- zero isolines at temperatures on the surface of -5, -3, -2 and -1°C respectively.

of operation of a dam the temperature regime of the rocks forming the base varies qualitatively. The erection of a dam, the opening up of the territory adjacent to it and also the accumulation of a large amount of water in the upper waters considerably change the temperature distribution in the embankment and rocky base.

The problem under consideration of the stationary temperature regime of the dam and sections adjacent to it was solved in the plane perpendicular to the line of the section along the river over the center of the channel. On the side of the upper and lower waters the boundary of the investigated region passed at a distance of 150 meters from the dam. The depth of the investigated region was assumed to be 300 meters. To reduce the error during the discretization of the investigated region the step selected along the x and y axes was variable, while the step selected in the body of the dam was minimal and equal to 10 meters (Figure 175).

The coefficient of thermal conductivity of the diabasic base, as in the preliminary problem, was 1.8 kcal/(m)(hr)(degree). The dam is heterogeneous in the composition of the soil and is composed of rock diabasic fill and rock debris-gruss-loamy soil of the screen. The rock fill of blocks of diabases, thanks to the presence of a large number of cavities, will have a coefficient of heat transfer of about 0.9 kcal/(m)(hr)(degree), and the rock debris-gruss-loamy soil according to the results of laboratory determinations also has a coefficient of thermal conductivity of the same value. On the first boundary (under the water), depending on the depth of the water, a temperature of 0 to +5 was given.

On the open surface of the dam different temperatures were used for different calculations. The average annual air temperature in the investigated region varies around -9° and on the surface of the dam, with consideration of the warming influence of snow, it reaches about -5°. However, thanks to infiltration of warm summer precipitations from above the body of the dam is intensively warmed and the average annual temperature on the surface can rise by $2\text{-}3^{\circ}$. Some warming of the dam can occur also during the filtration of water from the upper waters through the screen. In connection with what was said above, besides the calculation in which those warming factors were not taken into consideration, calculations also were made under the condition that the average annual temperatures on the surface of the dam are -3, -2 and -1° respectively.

In addition, variants of the thermal regime of the dam were investigated as a function of the conditions at the foot of the dam in the lower waters. The first series of calculations was made with consideration of the fact that at the foot of the dam there is a layer of water of 2-3 meters, that is, the temperature was assumed to be the same as on the surface of the dam. In that case, in the body of the dam more severe temperature conditions will form and, consequently, a thicker frozen core. As in the preliminary problem, in the lower boundary (at a depth of 300 meters) a temperature of $+1^{\circ}$ was given. However, in contrast with natural conditions, to which those temperatures correspond, in the given case the $+1^{\circ}$ isoline will pass somewhat below 350 m. Consequently, such a lower boundary condition in the given case will lead to the formation of permafrozen rocks with a somewhat smaller thickness and by the same token will create a certain reserve factor.

The results of the first series of calculations are presented on Figure 175. As follows from their examination, in the case where the average annual temperature on the surface of a dam is assumed to be -5° the zero isoline (Figure 175, curve 1) penetrates to a depth of more than 25 meters below the base of the dam. The forming frozen core with considerable width and cross-section of the dam is a reliable natural screen.

However, at a temperature on the surface of -3° or higher the zero isoline (Figure 175, curves 3-4) remain within the limits of the body of the dam and frozen rocks obviously will not exist permanently there, but will depend completely on seasonal fluctuations of temperature and the infiltration of warm summer precipitations from above.

The second series of calculations was made on the assumption that at a distance of 5 meters from the dam a small levee with a height of up to 5 meters

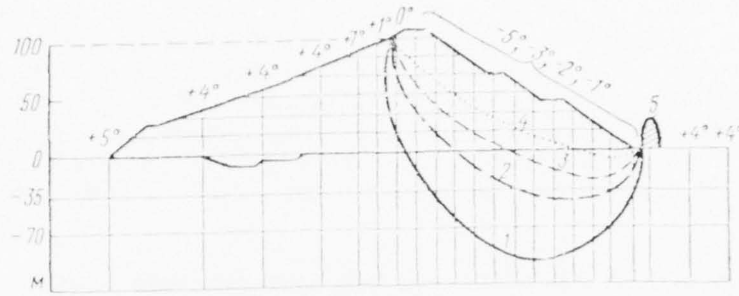

Figure 176. Temperature field in the body and base of a dam in the absence of water at its foot. Vertical scale, 1:25: 1 - 4 -- zero isolines at temperatures on the surface of -5, -3, -2 and -1°C respectively; 5 - levee.

will be created, one which blocks the access of water to the foot of the dam (Figure 176). In that case, conditions at the foot of the dam in the lower waters more severe than in the first series of calculations will lead to a substantial increase of the thickness of the permafrozen rock masses.

However, here too at a temperature of -1° the zero isoline reaches the base of the dam and the presence of permafrozen rocks on the entire height of the dam cannot be guaranteed.

It follows from the results of the above-described calculations that if the infiltration of warm summer precipitations and the warming of the dam connected with that are absent, then regardless of the presence of water at the foot in its body in the process of long operation a thick frozen core forms which embraces the base of the dam and serves as a wholly reliable natural anti-filtration screen. Under the condition that the surface of the dam and sections near it are warmed by summer precipitations, that is, the temperature increases to -3°, the formation of a frozen core on the entire height of the dam cannot be guaranteed under all conditions. However, if in that case a small levee prevents the access of water to the foot of the dam in the lower waters, a 25-30-meter core forms (below the base of the dam) which will exist permanently. At a temperature of -2° on the surface a frozen core forms only in the absence of water at the foot of the embankment and under certain conditions can, evidently, serve as an anti-filtration screen. Finally, in that case, if the warming with summer precipitations is extremely intensive, and also some quantity of heat will penetrate from the upper waters with the filtered water (in that case the temperature on the surface of the dam was assumed to be -1°), the rocks will not freeze at all.

Thus, starting from the point of view that the formation of a frozen core is desirable for increase of filtration through the body of the dam, one can propose a number of measures which will contribute to a very rapid formation of frost on the entire height of the dam: 1) measures to lower the average annual temperature on the surface of the open part of the dam are needed.

To do that in the winter time it is desirable to remove from the surface the snow cover preventing intensive winter cooling; 2) it would be desirable to isolate somehow the surface of the dam from the penetration of precipitations inside the dam; 3) it is desirable to maintain very cold conditions at the foot of the dam in the lower waters. To do that it is necessary to provide for the construction of a low levee which prevents the access of water to the foot of the dam.

At the same time the second variant, in which the body of the dam will be in the thawed state, should not be ignored. In that case, on the contrary, it is necessary to adopt all measures to elevate the temperature in the body of the dam. In the winter it is desirable to maximally increase the thickness of the snow cover. In the lower waters it is necessary to create a constant layer of water with a depth of not less than 3-4 meters. In addition, it is necessary to exclude any sort of water-insulating coating on the surface of the dam in order to have the infiltration of thawed precipitations in summer be maximal. The selection of a given variant must be made only after detailed calculation and comparison of the two variants.

5. Prediction of Change of Frost Conditions Under Embankments as a Function of the Upper Boundary Temperatures

Up to now predictions of change of permafrozen rocks under embankments and in ditches have been compiled by calculations extremely rarely, because in each case it is necessary to solve a laborious two- or three-dimensional problem of the freezing and thawing of rocks. A second factor which has long held back solution of questions of prediction of the change of frost conditions under embankments is difficulties in determining the values of the surface temperature which are an upper boundary condition.

An earthen bed, and in particular an embankment, has no heat sources and is poured of soils. Therefore the formation of frost conditions within its limits occurs, as under natural conditons, under the influence of an entire complex of natural factors. Consequently the available procedure for determining the influence of geological and geographic factors on the temperature regime of soils can be used to calculate the boundary temperatures in the region of an earthen bed and, in particular, on an embankment.

A prediction of change of frost conditions under embankments can be compiled on the basis of the results of a frost survey. In the compilation of such a prediction the principle of construction of the embankment must be determined first of all. At times it is sufficient for that to determine the direction of the frost process, which is characterized by the conditions on the surface, and in a number of cases, to determine the configuration of the frozen and thawed rocks in the region of the embankment in the regime established after construction. In the second stage of compilation of the forecast it is necessary to determine the configuration of the frozen and thawed rocks with consideration of the planned measures, the rate of freezing and thawing, and also the temperature regime of the rocks in the region of the embankment. For example, we will examine the case where the temperature on the surface is positive only on the slopes of the embankment.

The dependence of the limiting boundaries of permafrozen rocks in the body and base of an embankment on the surface temperature conditions was found by modeling a series of problems on a differential analyzer. The thermophysical characteristics of the rocks in the base of the embankment and on adjacent sections, and also their thickness, are presented in Table 87.

In the solution the following values of the average annual temperatures were adopted: on a roadbed -2.2°, on slopes $+0.8^{\circ}$ and on sections adjacent to an embankment, -0.5, -1, -2 and -3°. The lower boundary conditions were given at a depth of 150 meters in accordance with the temperature gradients existing in the region. In calculations the height of the embankment was assumed to be 12 meters, and the steepness of the slopes 1:2. The gentle slopes and large height of the embankment were taken because the greater the height of the embankment and the more gentle its slopes, the greater the probability of thawing the heavily iced rocks in the base of the embankment.

The results of solution of that series of problems are presented on Figure 177. Analysis of the results shows that the possibility of thawing rocks in the base of an embankment depends on the ratio of the boundary temperatures on elements of the embankment and the sections adjacent to it and also the dimensions of the regions where those temperatures prevail.

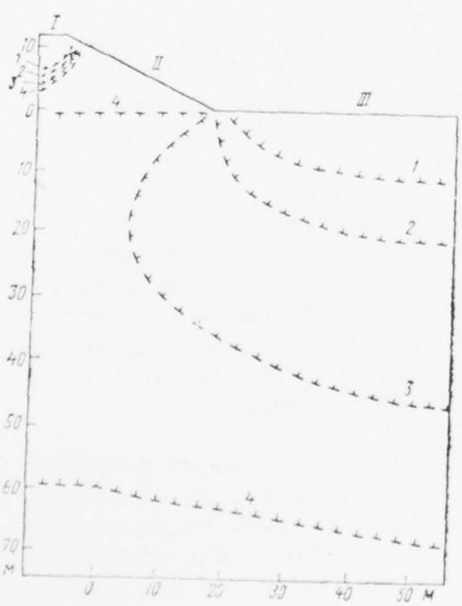

Figure 177. Boundaries of the freezing of rocks in the body and base of an embankment at a constant t_s of the embankment: I -- -2.2°; II -- $+0.8^{\circ}$; at different values of t of the adjacent section III: 1 -- 0.5°; 2 -- -1.0°; 3 -- -2°; 4 - -3.0°.

As the calculations showed, under the conditions under consideration a thawing basin always forms under the slope of an embankment. Depending on the correlation of the boundary temperatures, the thawing basin can be small and be situated above the base of the embankment. At higher temperatures a permeating and more rarely a non-permeating talik forms under the embankment. In those cases a core of permafrozen rocks forms in the body of the embankment.

Table 87 Thermophysical characteristics of rocks taken in the calculations

	A Наименование	B Размерность	C Наименование пород и их мощность в основании насыпи и участках, к ней прилегающих					D Породы в насыпи — суглинки со щебнем
			a торф, 1 м	b суглинок тяжелый, 3 м	c супесь, 2,5 м	d гравий и галька с супесчаным заполнителем, 2,5 м	e аргиллиты с прослоями мергелей, 200 м	
1	Суммарный объемный вес скелета грунта	кг/м³	59,0	889,0	1280,0	1230,0	1700,0	1360,0
2	Суммарный объемный вес оттаявшего грунта	кг/м³	590	1034,0	1350,0	1230,0	1700,0	1360,0
3	Суммарная влажность мерзлого грунта	% к сухой навеске	1315,0	80,0	39,0	29,0	15,0	25,0
4	Суммарная влажность талого грунта	% к сухой навеске	1315,0	45,0	30,0	29,0	15,0	25,0
5	Относительная льдистость грунта		1,0	0,85	1,0	1,0	0,9	0;85
6	Температура фазовых переходов воды в лед	°C	0	0	0	0	0	0
7	Скрытая теплота плавления льда в единице объема грунта	ккал/м³	62068,0	48361,6	39936,0	28536,0	18360,0	21760,0
8	Объемная теплоемкость мерзлого грунта	ккал/м³·град	402,0	497,9	499,2	428,8	501,3	432,0
9	Объемная теплоемкость талого грунта	ккал/м³·град	790,0	672,1	640,0	611,7	578,0	592,0
10	Коэффициент теплопроводности мерзлого грунта	ккал/м·час·град	0,36	1,09	1,68	1,9	1,25	1,31
11	Коэффициент теплопроводности талого грунта	ккал/м·час·град	0,2	0,81	1,2	1,38	1,3	1,1

Key: A - Characteristic B - Dimensions C - Kind of rocks and their thickness in the base of the embankment and sections adjacent to them D - Rocks in the embankment -- loam containing rock debris 1 -- Summary volume weight of the skeleton of the soil, kg/m^3 2 - Ditto of the thawed soil, kg/m^3 3 - Summary of the moisture content of the frozen ground, % of dry weighed portion 4 - Ditto of the thawed ground 5 - Relative ice content of soil 6 - Temperature of phase transitions of water into ice 7 - Latent heat of fusion of ice per unit of volume of the soil 8 - Volume heat capacity of frozen ground, kcal/(m^3)(degree) 9 - Ditto of thawed ground 10 - Coefficient of thermal conductivity of frozen ground, kcal/(m)(hr)(degree) 11 -- Ditto of thawed ground

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FUNDAMENTALS OF FROST FORECASTING IN GEOLOGICAL ENGINEERING INV--ETC(U)
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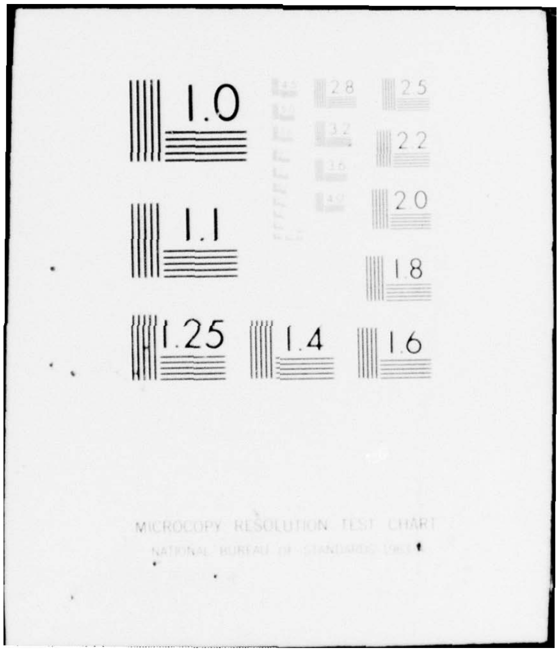
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MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS

Analysis of the obtained modeling data shows that there are critical ratios of the average annual temperatures of rocks in elements of the embankment and adjacent sections at which thawing of the permafrozen rocks in the base of the embankment starts. At temperatures close to the critical ratio a small additional elevation of temperatures can lead to thawing of rocks in the base of the embankment. For one embankment with a height of 12 meters a critical ratio was found in which on a roadbed the average annual temperatures of the rocks are -2.2°, on the slope $+0.8^{\circ}$ and on sections adjacent to the embankment, $-3^{\circ}C$.

For the variant in which the average annual temperatures are positive only on slopes, finding the critical ratios of temperatures at which thawing of the rocks in the base of the embankment starts makes it possible to substantiatedly approach the selection of the principle of construction of an embankment of a certain height. If thawing of permafrozen rocks is observed in the base of an embankment, it should be erected with consideration of the thawing of rocks, and if thawing ought not occur in the base of the embankment, it should be constructed on the principle of preservation of the permafrozen rocks.

Determination of the configuration of the frozen and thawed rocks in the body of the embankment was obtained as a result of solution of a series of problems at different ratios of the average annual temperatures on elements of the embankment and adjacent sections. In that case it was found that at average annual temperatures higher than the critical in the body of an embankment (under a slope) a thawing basin forms. In a number of cases a thawing basin forms with such a form at which intensive creep of the slopes of the embankment must occur. The same thawing basins probably also form under the slopes of ditches. If the form of the thawing basin is regulated by conducting land improvement measures, it is possible, evidently, to achieve a reduction of the intensity of creep of the slopes.

To estimate the stability of an embankment it is very important to know the rate of thawing of the rocks. High thawing rates are observed, as a rule, on sections where the warming influence of the surface and subsurface waters is great. On such sections the subsidence of the embankment due to the thawing of heavily iced rocks in the base can reach 3 meters in 4 years of operation of a road. If the influence of surface and subsurface waters is excluded, the rate of thawing of rocks will be considerably lower.

In the calculations of the thawing and freezing of rocks under consideration it was assumed that the embankments were constructed on loams. The question of with what it is advisable to pour the embankment is a complex one. As construction experience shows, one of the best material for pouring embankments is gravel-pebble and stone debris soils, as in that case the processes of heaving of rocks and creep of slopes are not strongly manifested. However, under embankments constructed of rocks filtering water well, the thawing and subsidence of rocks in the base will occur more intensively as a result of the warming influence of the infiltration of atmospheric precipitations and the possible appearance of subsurface waters. In particular, during the construction of embankments of pebbles or rock debris the changes of the permafrozen rocks will be substantially different from those obtained in the calculations. For example, in the body of an embankment the core of frozen rocks can not form and in the base of the embankment a perennial thawing of rocks will be observed almost always. In the case of the pouring of an embankment with loams and clays a reduction of the rate of thawing of rocks in the base of the embankment will be observed, and in a number of cases even a preservation of permafrozen rocks under embankments, but at the same time the loams are inclined to heave and creep.

Of great interest are the designs of embankments proposed by N. A. Peretrukhin (1958), who recommends on permafrozen rocks pouring embankments from above with gravel-pebble or rock debris soils with a thickness equal to the layer of seasonal freezing or thawing, and below, in the core of the embankment, local sandy loams and clays containing rock debris. Such a design excludes to a considerable degree the heaving of rocks and creep of slopes and at the same time contributes to preservation of permafrozen rocks or reduction of the rate of thawing of rocks in the base of the embankment.

6. Solution of the Problem of Protection of Fluvial Deposits Against Freezing by Means of the Creation of an Artificial Ice Body

A considerable portion of placer gold deposits is located in regions where permafrozen rocks and deep seasonal freezing are widespread. In connection with that, enormously important in the application of the dredge method of working placers are questions of the thawing of frozen and the protection of thawed rocks against deep winter freezing. At the present time the methods of artificial thawing of frozen soils (sprinkling, filtration-drainage, water-needle, steam-needle, artificial thawing with removal in layers, etc) have been worked out relatively well and are being widely used in practice. However, in some cases it is economically advantageous to create artificial ice bodies to protect fluvial deposits against winter freezing. Calculations of the thickness of such an ice body and of the regimes of flooding with water have been made for the rivers of the Vitimo-Patomskoye highlands.

The industrial part of the placer is almost completely situated within the limits of the riverbed and shore. The alluvial deposits represent well-rolled boulder and pebble material with sand and gravel filler and fine silty sand.

Pebbles during thawing have high filtration capacity. Their filtration coefficient varies from 100 to several hundred meters per day. In sands the filtration coefficient is small and amounts to 1 - 10 meters per day.

On all river sections where the depth of the water by the start of freezing does not exceed 0.5-1 meter all the alluvium and the upper part of the bedrocks are in the frozen state. The total thickness of the permafrozen rock mass in the channel evidently reaches 10-12 meters.

Let us note that the application of hydraulic methods of thawing (needle and filtration-drainage) is impossible under the given conditions because of the rapid current of the river and high floods, the considerable number of boulders in alluvial deposits in the upper parts of the profile and the low filtration properties of the rocks in the parts of placers near dams.

Under those conditions it is advisable to preserve against winter freezing the alluvial deposits in the channel and on the river banks by creating an artificial ice body which must be frozen by the feeding of layers of water of considerable thickness. As a result of preliminary calculations and taking account of the technical possibilities in the given conditions a thickness of the indicated layers of water of 1 meter was adopted. With such a method, in winter only the water of "floods" must freeze, and the bottom deposits remain thawed with a temperature close to 0°.

To determine the necessary thickness of the ice body and select the most effective regime of floods of water and its freezing, with an IG-1 hydraulic integrator an investigations was conducted of the dynamics of freezing of the floodable layers of water with a thickness of 1 meter under different flooding conditions.

The time of investigation is taken from the moment of the first flood, which occurs simultaneously with complete freezing (about 15 October), to the onset of positive average daily air temperatures (about 1 May). The water temperature in that time interval can be assumed to be practically equal to 0°.

Starting from practical goals requiring the obtaining of results with a definite guarantee, in conducting the investigation under consideration it is advisable to adopt some assumptions leading to a very slight increase of the thickness of the ice body and at the same time sharply simplifying the calculations. In connection with that the following are excluded from consideration:

1) the summer store of heat available in the underlying ground, which permits regarding only the flooding water as the area of investigation;
2) convective heat exchange in the water layer in the process of freezing;
3) the presence of a snow cover, since its thickness in the region of investigation is not great and, in addition, it is blown away to a considerable degree.

The investigation was conducted for two methods of freezing an ice body under the following flooding conditions:

1) the flooding occurs after complete freezing of the flooded layer; 2) the flooding occurs after freezing of the upper 0.5 meter of water of the preceding flood.

The results of calculations of the dynamics of freezing of flooded layers of water in time for the two cases of flooding are presented on Figure 178 (curves 1 and 2 respectively). In the case where the following flood occurs after complete freezing of the preceding (curve 1), by the end of March an ice body with a thickness of 4 meters forms, and, consequently, to preserve the underlying ground in the thawed state it is necessary to freeze a little more than 4 meters of ice.

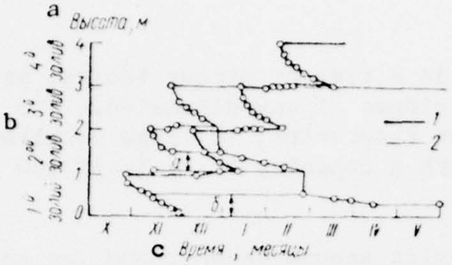

Figure 178. Dynamics of the freezing of the ice-body water: 1 - in the first flooding condition; 2 - in the second flooding regime. a - Height, m b - 1st, 2nd, 3rd and 4th floods c - Time, months.

In the second case (curve 2), when the following flood occurs after freezing of only 0.5 m of water, for the same purpose 3 1-meter floods are known to be adequate, and by the moment of complete freezing an unfrozen layer of water with a thickness of about 0.3 meter remains in the lower part of the ice body.

The gain in total thickness of the ice body obtained in the second case can be explained as follows.

In the first case the freezing of each alternate portion of water occurs intensively because of its closeness to the day surface. In the second case, however, the layers of water a and b (Figure 178) start to freeze in the presence above them of ice cushions with thicknesses of 1.5 and 2.5 meters respectively. As a result of that the rate of freezing of layers a and b diminishes considerably, which also leads to a reduction of the total thickness of the ice body. It is obvious that a further reduction of the total thickness of the ice body also is possible through reduction of the thickness of the frozen layers. Under the conditions for which the problem was solved an ice body does not have a cooling effect in summer, as it will have been carried away in the time of powerful spring floods.

Technically an ice body can be created by pouring water into "baths" bounded on all sides by water-impermeable ridges of ice or soil. Either river ice or water-impermeable frozen soil must serve as their bottom. Their form and dimensions must be determined by the contour of the section of alluvial deposit requiring protection against freezing, the microrelief of the section (the slope of the bottom) and the technical possibilities of feeding large portions of water into the "baths" in a short time. The one-meter layer of water must be poured in a few hours to avoid its freezing in the process of being poured.

Thus it is possible to protect against freezing both sections where by the moment of complete freezing some layer of river water is preserved and sections completely drying up by the moment when the winter freezing starts.

Some difficulty is presented by the retention of the water of the first flood on dry sections of the river and its banks. One should make transverse and longitudinal levees here of local soils. From the start of frosts the bottom of the section being protected and the wall of the levee, to avoid filtration losses, should be encrusted with ice with a thickness of 5-10 cm. After that the first flood occurs. On underwater sections of a polygon the first flood must start after complete freezing.

Since the second and subsequent floods occur in a time of strong frosts, at that time it is not difficult to freeze side ridges of any dimension. The water must be fed rapidly into the created ice reservoirs, in large portions. This can readily be done by means of pumps with a capacity of at least 100 liters per second.

7. Calculation of Established Temperature Fields Around Underground Ice and Soil Reservoirs

In connection with the construction of underground containers of ice and soil in regions where permafrozen rocks are widespread, of great importance is calculation of the temperature fields in the ground surrounding the container. Together with such characteristics of frozen rocks as the granulometric composition, moisture content, etc, the temperature factor is one of the main factors affecting the strength properties of those rocks. In connection with the fact that cooling of the product before the container is filled greatly increases the cost of the ice and soil containers, the question arises of the possibility of flooding the reservoir with petroleum product with a positive temperature. Therefore in planning an underground reservoir under the given concrete conditions the main task is assuring a low temperature (optimally -2 or -3°) of the ice coating of the container and the surrounding ground.

The problem of the temperature regime of permafrozen rocks surrounding an ice and ground reservoir, during the multiple flooding of liquid into it, is in principle a non-stationary problem of thermal conductivity. However, the question of the possibility of operating a reservoir buried beneath the layer of annual fluctuations can be solved by determining the temperature field established around the reservoir in the case where the temperature of the liquid at the center of the cross-section of the reservoir remains unchanged.

It is natural that the formulation of such a question is possible only for regions with a ground temperature not higher than the limiting (-2°). In regions with a higher ground temperature it is necessary to accomplish special measures to increase the strength properties of the soils (for example, systematic purging of the reservoir with frosty air in the winter time, etc). These questions can be investigated only when there is a solution of the non-stationary problem of thermal conductivity and are not examined here.

In connection with the great length of a reservoir it is advisable to regard the given problem as two-dimensional, neglecting the heat fluxes along the structure. With the exception of the ends of the chamber where the temperature distribution will be more favorable, the solution of the two-dimensional problem will practically not differ from the three-dimensional.

Everything said above has the result that investigation of the temperature fields of the rock mass surrounding the underground ice and ground reservoir can be conducted with a model EI-12 differential analyzer with some approximation. If it is taken into account that the liquid, ice and soil are homogeneous, the problem under consideration is described by a system of three equation of an elliptical type with different (piecewise constant) conductivities.

In connection with axial symmetry (the curvature of the vault can be neglected) it is advisable in solving with the instrument to take into consideration only a fourth of the cross-section of the reservoir and the ground surrounding it. It is natural that on the axes of symmetry equality of the heat fluxes to zero is given. On the boundaries of the investigated region, which must be selected sufficiently far from the reservoir, it is possible to assume the condition of constancy of the ground temperature. Given as a boundary condition at the center of the section is a constant temperature of the liquid equal to its temperature at the moment of pouring. If the fact that the liquid is poured into the container many times is taken into consideration, that requirement will sufficiently reflect the real picture and the solution in that case will contain a certain reserve. There will be a still greater reserve in the case of solution of the problem of fuel conservation. In that case the increase of cross-sectional dimensions of the ice and soil container brings the calculations close to the natural conditions.

With the proposed procedure one can readily calculate the stationary temperature fields in the reservoir and the ground surrounding it in the presence of different cross sections of the container and temperatures of the liquid. However, since ice and soil reservoirs will satisfy all the requirements during operation if the temperature of the ice coating does not exceed -2°, it is more advisable to pose the reverse problem. In that case, being given the temperature of the liquid at the moment of pouring, it is necessary to find the average ground temperature at which the coating will have an optimal temperature. On the basis of calculations made at different average annual temperatures of the ground it can be concluded that at a temperature of the poured liquid of +2 and $+10^{\circ}$ ice and soil containers can be constructed in regions with temperatures not higher than -2.5° and -4° respectively. Increase of the dimensions of the cross-section of the reservoir leads to considerable warming of the mass of rocks and has practically no influence on the solution of the question.

8. Prediction of Change of Frost Conditions Around Buried Pipelines With Consideration of Changes of the Surface Temperature

Quantitative investigation of the dynamics of the temperature field in permafrozen ground around a pipe with variable heat release, laid at a small depth from the surface, presents considerable difficulties. Even under certain natural assumptions relating to the temperature field in a pipe (in particular, that the temperature on the pipe wall is a function only of time, varying similarly to the air temperature) the problem is reduced to a two-dimensional multifront Stefan problem for a complex region. It is obvious that in selecting the region of investigation it is advisable to take into consideration the symmetric character of the problem in relation to a vertical line passing through the center of the circle. At the same time the presence of the horizontal surface of the ground, and also the horizontally laminated heterogeneity of the ground make it extremely difficult to introduce cylindrical coordinates in solving the problem. The application of known substantiated algorithms for the solution of such Stefan problems requires large expenditures of both programmer and machine time. In addition, to assure finding a solution with sufficient precision it is necessary to take a variable step on the coordinates, making it smaller near the pipe, whereas the entire region of investigations, the dimensions of which are determined by the zone of disturbance in connection with the gas pipeline and surface conditions, must be sufficiently large. In connection with that an important role is played in the investigation of the problem under consideration by analog computers which make it possible to obtain a solution of a multidimensional Stefan problem. Presented below is an example of the solution of a concrete problem of the dynamics of thawing of grounds around the pipe, and also the course of freezing of the formed aureole of thawing in order to solve questions of the heaving and subsidence of ground under a pipe.

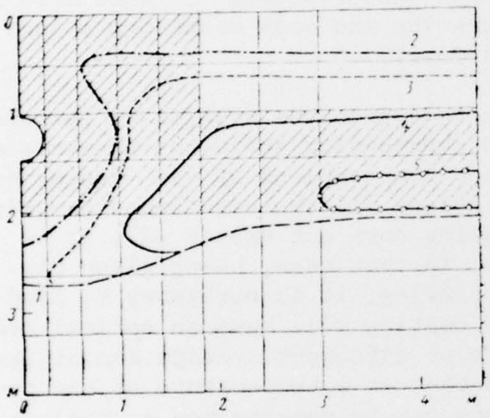

Figure 179. Results of calculations of the dynamics of thawing and freezing around a gas pipeline: 1 - maximal aureole of thawing (on 10 November); 2 to 4 - position of the zero isoline during freezing of the aureole of thawing on 1 December, 24 December and 1 February respectively; 5 - maximum of freezing (on 3 June). Thawed ground at the moment of start of operation is shown by hatching.

The pipe was laid in permafrozen rocks with an average annual temperature of close to -0.4°. In its profile the ground consists of a laminated heterogeneous medium made up of three layers with a substantially different moisture contant and thermophysical characteristics. A condition of the third kind is specified on the surface, and of the first kind on the pipe walls. It is assumed that the zone of influence of the pipe does not exceed 20 d. The breakdown of the region adopted in the calculation is shown on Figure 179. In the course of the winter period a change of thickness of the snow cover characteristic of a construction process (0.2 meter) was assumed. The gas pipeline was laid in the winter period inside a talik with a thickness of 1 meter. Since for the regions under consideration the heat output in the annual cycle greatly exceeds the input, the warming influence of the gas pipeline will be greatest precisely in the first year of operation. Therefore in making the calculations we will limit ourselves only to the time interval including one interval of positive air temperatures.

To obtain maximal thawing around the pipe the gas temperature is assumed to be equal to the air temperature with a correction for solar radiation. The depth of the pipes was assumed to be 1 meter. The results of calculations of the dynamics of the temperature field around a pipe with a diameter of 0.5 meter are presented on Figure 179, from which it is evident that the aureole of thawing toward the middle of October (the maximum of thawing) embraces a considerable area around the gas pipeline. During subsequent freezing of the summer aureole of thawing, first occurs a joining of the fronts of freezing above the pipe (the layer of seasonal freezing from the surface is joined with freezing from the pipe). Then occurs freezing of the aureole of thawing directly under the pipe, and lastly the thawed rocks on the side sections freeze. The reduction of the snow cover during construction leads to reduction of the thickness of the talik far from the gas pipeline. If at the place where the pipe is laid there is a talik, the aureole of thawing increases sharply through lateral expansion.

On the whole it can be concluded from the calculations under consideration that regardless of the temperature regime of the soils, during the laying of pipe at a depth of 1 meter under the given conditions the depth of thawing on the axis of symmetry is practically determined by the gas pipeline itself. In addition, the laying of pipe in winter reduces somewhat the thawing near the pipe (up to 30 cm).

6. Approximate Method of Calculating the Thickness of the Layer of Seasonal Thawing (Freezing) of Soils Around Buried Pipelines

In connection with the intensive construction of pipelines in northern regions much importance has been acquired by questions of the interaction of a given kind of structure with permafrozen soils. One of the main questions of that problem is their thermal interaction. In that case of considerable interest is the case where the temperature regime of the transported product is determined by the seasonal air temperature fluctuations. Then in the soils surrounding the pipeline forms a layer of seasonal thawing (freezing) which is of great importance for the stability of the structure.

As a result of thermal interaction of the pipeline with soils its temperature regime varies along the length of the pipe. The depth of thawing (freezing) under it changes correspondingly. The problem of variation of the temperature regime along the length of a pipe laid in frozen ground and, consequently, the finding of the size of the thawed zone around the pipe with consideration of the influence of the surface is a complex three-dimensional problem of mathematical physics, the solution of which in complete volume is difficult at the present time. Therefore, taking into account that the heat flux in the ground in the direction of the pipe axis is many times smaller than that in radial direction, that problem can be broken down into two with a precision adequate for practical purposes:

a) determination of the aureole of thawing around the pipe at the given temperature regime of the heat transfer agent for any section of the pipe and
b) determination of the variation of the temperature regime of the heat transfer agent along the pipe length.

/Determination of the depth of thawing under the pipe as a function of the temperature regime of the heat transfer agent/. When a pipeline is laid below the layer of seasonal thawing (freezing) under natural conditions the influence of the surface of the ground on the temperature regime of the soils under the pipe depends essentially on its diameter. As calculations with a hydraulic integrator of the system of V. S. Luk'yanov have shown, at a diameter of the pipeline of at least 0.4 meter the influence of the surface of the ground becomes negligibly small. If the pipeline is laid in sufficiently homogeneous ground, then with a precision high for practical purposes the problem can be assumed to be axisymmetric and, consequently, finding the depth of thawing (freezing) of the ground under the pipe is reduced to solving a unidimensional Stefan problem. However, as is well-known, even in that case its solution under variable boundary conditions can be obtained only with a computer. The approximate formulas used in practice for calculating the zone of thawing (freezing) around pipelines are based on assumption of constancy of the temperature of the heat transfer agent, which sharply limits the possibilities of their application under natural conditions. In connection with that it is necessary to create approximate formulas which will make it possible with precision adequate for practice to calculate the aureoles of thawing (freezing) around pipelines with consideration of unsteadiness of their temperature regime.

We will examine the formation of the aureole of thawing around a pipeline with a radius r in homogeneous ground with an average annual temperature i upon the condition that the temperature on the pipe wall varies according to a sinusoidal law with an amplitude of oscillations A and a period T equal to 1 year. The depth of thawing of the ground under those conditions in the formulation corresponding to the plane problem can be calculated with formula (4.1.4) of V. A. Kudryavtsev.

The difference of the axisymmetric problem of thawing of the ground around the pipeline in comparison with the problem of thawing of the plane surface of the ground in essence consists in the fact that the volume of the thawing ground is determined by the volume of a hollow cylinder, and not a parallelopiped. Therefore the heat cycles during thawing of the ground around a pipeline can be expressed through the reduced values of the heat capacity and phase transitions for the corresponding plane problem.

Thus, if the influence of the form of the limiting surface is taken into consideration, it is possible to obtain an expression which with adequate precision connects the aureole of thawing around the pipe (h) with the depth of thawing of the surface of the ground (ξ).

As was shown in the work of V. A. Kudryavtsev, V. G. Kondrat'yev and V. G. Melamed (1971), to calculate h it is sufficient to substitute in formula (4.1.4) the values of C and Q_ϕ with the coefficient h + d/d. Then the maximal aureole of thawing in the ground around the pipeline in a periodically established regime is determined from the equation

$$h = \xi \sqrt{\frac{d}{d + h}}. \qquad (9.6.1)$$

In the indicated work a nomogram was compiled with that formula which makes it possible to determine the depth of thawing under a pipe h as a function of the pipe diameter d and the parameter ξ with a precision of within 10-15%.

/Calculation of the depth of seasonal thawing (freezing) around a pipeline with consideration of change of its temperature regime along the pipe length/. The change of the temperature regime of the heat transfer agent along the pipe length L can be calculated in the following manner. We will break the length of the underground section of a pipeline into several elementary sections. On the basis of the given amplitude of oscillation of the temperature of the transported product at the start of the i-th elementary section (A_i), the known temperature of the soils (t) and their thermophysical properties (λ and C) the depth of thawing (freezing) h_i was calculated with formula (9.6.1). In that case at any point of the pipeline the annual heat cycles in the ground surrounding the pipe per running meter of its length amounts to (section 1, Chapter 4):

$$q_k = \pi (h_k + d) \left\{ (2A_{\text{cp}_k} C + Q_\Phi) h_\kappa \sqrt{\frac{h_k + d}{d}} + \right.$$
$$\left. + [2t - (2 - \sqrt{2}) A_k] \sqrt{\frac{\lambda TC}{\pi}} \right\}$$

It is obvious that q_k at k = k, i + 1 represents the heat cycles in the ground at the start and end of the i-th elementary section. The total heat cycles in the ground around the pipeline in the i-th section with the length L_i is

$$Q_i = \frac{q_i + q_{i+1}}{2} L_i. \qquad (9.6.2)$$

On the other hand, since the temperature within the pipeline at any point of it varies according to a harmonic law, in half a year the pipeline on that section loses half the heat, equal to

$$Q_i = VC_{\text{np}} (A_i - A_{i+1}) \frac{T}{\pi}. \qquad (9.6.3)$$

where VC_{prod} is the flow rate of the transported product, kg/hr and C_{prod} is its heat capacity, kcal/(kg)(degree). Having equated (9.6.2) and (9.6.3), we obtain

$$A_{i+1} = A_i - \frac{(q_i + q_{i+1}) L_i \pi}{2VC_{\text{np}} T}. \qquad (9.6.4)$$

The thus-obtained temperature regime of the heat transfer agent at the end of the i-th section will be the initial one for the following (i + 1)-th section on the basis of it the temperature regime at the start of the (i + 2)-th section is determined, etc.

Equation (9.6.4) at the given value of L_i is transcendental with respect to A_{i+1}, as it contains the unknowns H_{i+1} and $A_{m_{i+1}}$, which in turn depend non-linearly on A_{i+1}. Therefore the solution is found by trial and error. The unwieldiness of the calculations hinders use of the obtained expression in calculations of the change of the temperature regime of the transported product along the pipe length. At the same time that calculation can be made far more simply if one is given the drop of amplitude of the transported product ΔA and the corresponding value of L_i is sought. In that case on the

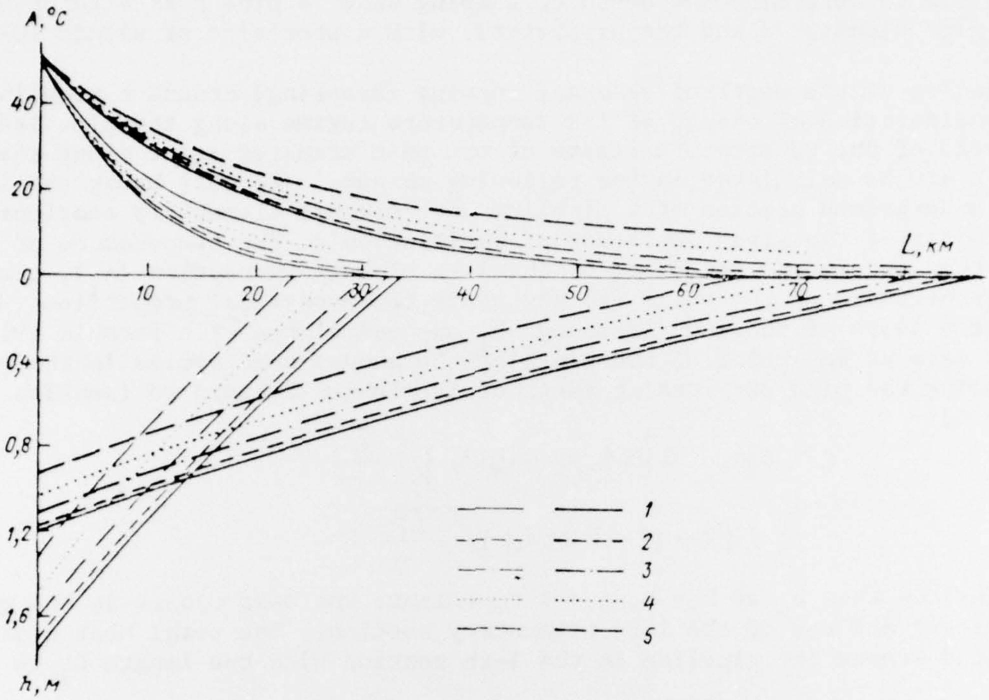

Figure 180. Nomogram for calculating the depth of seasonal thawing (freezing) of soils around a buried pipeline with consideration of the variation of its temperature regime along the pipe length: 1 - 5 -- t = 10, 6, 3, 1, 0°C respectively; $Q_ф$ = 40,000 kcal/m^3; thick lines d = 0.325 m, thin d = 1.42 m.

basis of the known amplitude at the start of the section (A_i) and at its end ($A_i - \Delta A_i$) h_i and H_{i+1} and q_i and q_{i+1} respectively are calculated, where $h_{i+1} = h(A_i - \Delta A_i)$ and $q_{i+1} = q(A_i - \Delta A_i)$. After that the sought value of L_i is found directly from the correlation flowing from (9.6.4):

$$L_i = \frac{2\Delta A_i VC}{\pi (q_i + q_{i+1})}. \qquad (9.6.5)$$

Since in the compilation of the heat balance the heat cycles in the ground around the gas pipeline in section L_i are averaged (see formula 9.6.2), the precision of calculation of L_i increases with decrease of ΔA_i.

According to the indicated scheme at $\Delta A_i \equiv \Delta A$ = constant > 0 calculations of the values of h_i and L_i were made on a BESM-6 computer

$$i = 1, 2 \ldots \left[\frac{A-t}{\Delta A}\right] \text{ при}^1 \ \Delta A = 1°C,$$

*The sign [s] designates the integral part of s.

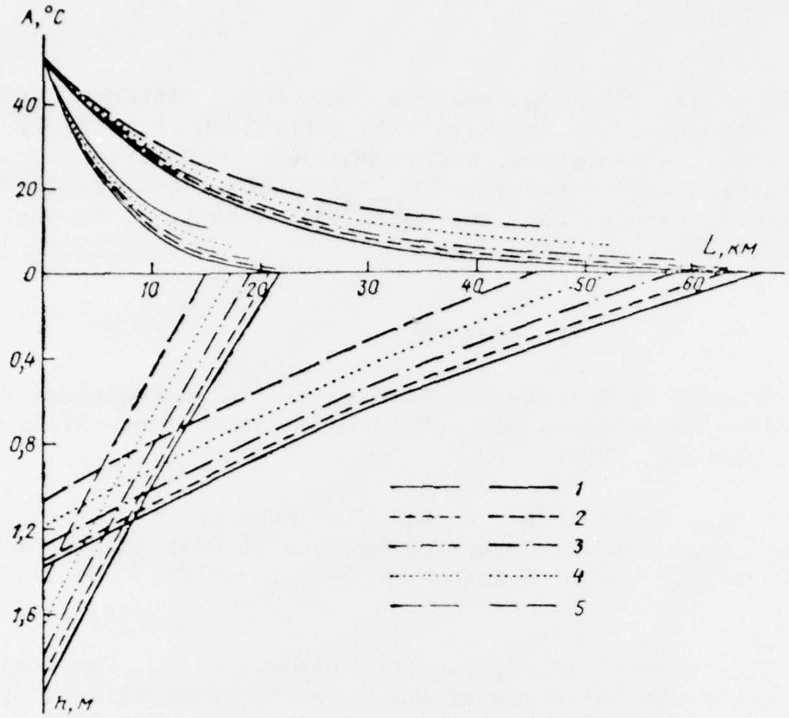

Figure 181. The same as on Figure 180; thick lines d = 0.529 m, thin -- d = 2.50 m.

A = 50°, t = 0, 1, 3, 6 and 10°, Q_ϕ = 40,000 kcal/m³; d_3 = 0.325, 0.529, 1.42 and 2.5 m; λ = 1 kcal/(m)(degree)(hr); C = 800 kcal/m³; V = 100,000 kg/hr; C_{prod} = 0.99 kcal/(kg)(degree). The results of calculation, presented in the form of a series of curves h = h(L) and A = A(L) at all the values of the input data under consideration, are presented on Figures 180 and 181. The obtained nomograms make it possible to determine the amplitude of oscillation of the temperature of the heat transfer agent and the depth of seasonal thawing (freezing) of soils under pipelines of different diameter at any distance from the section of pipeline with a known temperature regime.

As follows from the nomograms and formula (9.6.4), more rapid reduction of the amplitude of oscillation of the temperature of the heat transfer agent along the pipe length and, consequently, the depth of the seasonal thawing (freezing) of soils under it, occurs during increase of the pipe diameter and the heat capacity and thermal conductivity of the ground and reduction of the flow rate and heat capacity of the gas, and also of the absolute value of the average annual temperature of the ground. The above adopted set of values of pipeline diameters for which the present nomograms were constructed is connected with the fact that, as the calculations showed, the influence of change of diameters on the dependence A(L) and h(L) weakens with increase of d.

During use of the presented nomograms it must be borne in mind that they were compiled for fixed values of V and C_{prod}, and also for λ = 1 kcal/(m)(hr)(degree). It can readily be seen, however, that the obtained nomograms can be used directly for any values of the parameters V and C_{prod} different from those adopted in the nomogram. To do that it is sufficient to multiply the value of L obtained with the nomograms, corresponding to the given drop in amplitude, by a correction factor equal to $VC_{prod}/99,000$. However, if $\lambda \neq = 1$ kcal/(m)(hr)(degree), then to find the dependence A(L) it is necessary to create nomograms similar to those presented.

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Chapter 10. Basic Principles of Frost Surveying and Mapping for Purposes of a Geological Engineering Frost Forecast

As has already been pointed out, the forecasting of change of frost conditions in connection with construction is possible only on the basis of comprehensive study of the regularities in the formation and development of seasonally and permanently frozen rocks as a function of a complex of natural conditions and the productive activity of man. In other words, the forecasting of frost conditions in the process of production activity obligatorily assumes the study and understanding of regularities in the formation of frost conditions of the territory under consideration on the regional and zonal levels, that is, study of the regularities in the formation and change of frost conditions over the territory in connection with existing and changing natural conditions. Those regularities are studied in the process of conducting a frost survey. Because of that a forecast of change of frost conditions can be compiled only on the basis of frost survey data and cannot be reduced to thermophysical calculations. The compilation of a frost forecast in connection with construction must be the concluding stage of a frost survey.

Consequently it is evident that the examination of questions of frost surveying in the given work must conclude with the basic principles of a frost survey as the principles of a frost forecast.

The results of a frost survey are presented in the form of reports and geological engineering frost maps. The data of reports containing actual and calculated materials are the starting data for the compilation of a frost forecast in an investigated region and determination of its distinctive features for different landscape types as a function of the specifics of the economic development. The extent of the frost and geological engineering characteristics of rocks in a regular connection with complexes of natural conditions, stipulated in the scale of the survey, is taken from geological engineering frost maps. In connection with that, in the present section of the work it is necessary to determine the principles of frost mapping and the use of frost maps for frost forecasting.

The principles of frost surveying have been published in a number of works (Poltev, 1963; Baranov, 1965; "Polevyye geokriologicheskiye issledovaniya" [Field Geocryological Investigations], 1961, etc) and very completely described in "Metodika kompleksnoy merzlotno-gidrogeologicheskoy i inzhenerno-geologicheskoy s"yemki masshtabov 1:200 000 and 1: 500 000" [Procedure of a Complex Hydrogeological Frost and Geological Engineering Survey With Scales of 1:200,000 and 1:500,000] (1971). In the first chapter of the present work, in examining the principles of frost prediction, the basic methodological principles of frost investigations in general and of frost surveying in particular are given. Therefore only a short list of the basic principles of frost surveying is presented here.

/Short list of the basic principles of frost survying/. The purpose of a frost survey is to study the regularities in the formation and development of seasonally and permanently frozen rocks in connection with distinctive features of an investigated territory. As a result of that, studies must be made of regularities in the prevalence, conditions of occurrence, composition of cryogenic textures and properties of frozen rocks, their temperature regime, and also cryogenic and other geological phenomena accompanying them. All this must be examined in the process of development as one of the pages of geological history in the Quaternary period of the investigated region as a function of the general course of development of the entire geological and geographic situation. The productive activity of man substantially changes the geological and geographic conditions of the region, and by the same token its frost characteristics.

The geological engineering frost conditions characterizing the conditions of construction and operation of any structures within the limits of the selected construction sites are determined by the composition, cryogenic structure and thermal state of the rocks. Because of that, in compiling a frost survey the main attention must be turned toward these characteristics of frozen rocks. The main geological and genetic complexes and types of rocks must be above all studied in the investigated region. Their distribution and character usually are linked with geomorphological peculiarities of the region. Therefore in a frost survey within the limits of geological structural regions a microzoning of the territory is made on the basis of a geomorphological sign, as a result of which sections are distinguished which are uniform in a geomorphological respect and similar in geological structure. The landscape types of a locality distinguished on that basis are characterized by uniformity also in hydrogeological conditions, swampiness, the plant cover, microclimatic peculiarities, etc, that is, within the limits of each landscape a completely definite composition of rocks, character of composition, extent, properties and moisture regime of deposits is noted. In accordance with a complex of natural conditions within the limits of each landscape forms its own radiation-heat balance of the surface and temperature regime of the soils. Thus in the process of a frost survey a zoning and microzoning of the territory is made for each distinguished type of landscape, their specific features of composition are determined, as well as their cryogenic textures, occurrence, extent, the temperature regime of the frozen rock masses, the depths of the seasonal freezing and thawing and accompanying cryogenic and other geological processes and effects. This determines the content and sequence in making a frost survey.

The influence of the productive activity of man on change of the geological engineering frost situation is examined simultaneously. In a frost survey all these questions of the interconnection of elements of natural conditions with the characteristics of the seasonally and perennial frozen rock masses must obligatorily be characterized not only qualitatively but quantitatively, by making corresponding calculations in the field by means of existing rapid methods of calculation. On that basis a forecast is compiled of the change of frost conditions in connection with the productive opening up of the territory for each landscape type of the locality separately. Mapping of the geomorphological characteristics of the region and the geological and genetic complexes and types of rocks corresponding to them, and also a characterization of the remaining components of the natural complex, are achieved by reflection of the distribution of landscape types of the locality on the maps. On the basis of data of investigations on key sections within the limits of each landscape type are shown all the frost characteristics studied in the field and calculated for the moment of the survey with nomograms. A forecast of the change of frost conditions as a function of possible natural changes of the environment and as a function of the character of the productive opening up of the territory can be calculated with the same nomograms.

Besides the geological engineering frost maps corresponding to the conditions at the moment of the survey, forecast frost maps can be compiled for special purposes, for example, a map of the depths of foundations for cities and settlements, a map of a forecast of the thawing of frozen rocks under large reservoirs, a map of the thawing of frozen rock masses during the preparation of drainage polygons for development, etc.

Geological engineering frost mapping during the compilation of a frost forecast should be examined in two aspects. Geological engineering frost maps above all, being one of the forms of generalization of materials of a frost (and a geological engineering frost) survey, represent the basis for the compilation of a frost forecast. At the same time a geological engineering frost map can be regarded as one of the forms of a frost forecast. It is advisable to consider these aspects separately for small-scale maps and for medium and large-scale maps.

/Geological engineering frost maps as the basis for a frost forecast/. Specialized literature is devoted to the principles and methods of compilation of frost and geological engineering frost maps, and so they are not examined in the present work. It must be emphasized, however, that such maps must satisfy the requirements presented for both geological engineering and for frost maps. Their principal distinctive feature is that they must be compiled on the basis of study of particular, general and regional regularities in the formation and development of seasonal and permanently frozen rocks and cryogenic and other geological phenomena accompanying them as a function of a complex of natural conditions and their separate components.

Since the formation of the temperature regime of rocks, and in connection with that, of the cryogenic structure and thickness of frozen rock masses, occurs as a result of heat exchange on the surface and depends on the composition and moisture content of the rocks of the layer of seasonal freezing and thawing, it is advisable to show established particular regularities within the limits of each landscape type on a map of types of seasonal freezing and thawing of rocks. That map is used primarily for the compilation of a frost forecast.

On the basis of that map and the text of the report in which an estimate ought to be given of the influence of each factor of the geological and geographic medium on the formation of the temperature regime and the depths of seasonal freezing and thawing of soils, forecast data can be calculated with respect to the main characteristics of seasonally and permanently frozen rock masses in connection with construction and the productive opening up of a territory. This applies to both the temperature regime and the depths of seasonal thawing and freezing and to the determining of the bearing capacity of frozen rocks, the forces of freezing together, rheological properties, etc. In addition, a map of types of seasonal freezing and thawing of rocks is the basis for forecasting the development of cryogenic processes and phenomena (thermokarst, fissure formation, heaving, solifluction, etc). In that case, as the basis for compilation of a forecast one should use a geological engineering frost map characterizing the composition and cryogenic textures of the permafrozen rock masses, the moisture (ice) content and the possible amount of thermal subsidence.

In small-scale (1:500,000 and 1:200,000) frost (and geological engineering frost) survey maps are the basis for the compilation of a frost forecast in a general form characterizing the change of frost conditions in connection with natural change of separate elements and of the entire complex of natural conditions. Such a forecast can be made specific in larger-scale investigations, depending on the character of the productive opening up of territory for separate sites and specific objects of construction. As a result of such refinement a concrete forecast must be compiled, the basis of which ought to be medium- and large-scale surveys and maps. Such medium- and large-scale maps are usually compiled for industrial regions with intensive development, under definite hydraulic and thermal engineering objects, cities, living quarters and various linear structures. In those cases already during the survey more definite and specific tasks are set, the solution of which is closely connected with the forecast of frost conditions. Therefore such maps are special forecast maps. In that case data are drawn on the map regarding change of frost conditions under the effect of one or several production measures, such as the removal of the snow and plant covers, warming of the area, plowing, the working of minerals, the pouring of embankments, the drainage and flooding of land, the level ing of the terrain, construction, tree-planting and sodding, artificial coatings (asphalt and concrete), etc. Among specialized forecast maps are maps of the depths of foundations, water pipes, sewers and other pipelines.

For specialized forecast maps methods of controlling the frost process must be given, methods which can be developed only on the basis of the proposed methods of calculation. Indicated on those maps are measures which prevent the development of thermokarst, of processes of fissure formation, heaving, sagging, ice bodies and other phenomena. Measures to regulate the depths of seasonal freezing and thawing of rocks and their temperature regime also must be pointed out obligatorily.

The examples of calculations presented in chapters 4-9 testify that frost (and geological engineering frost) maps are a necessary basis for the compilation of a frost forecast and one of the most generalized forms of that forecast.

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