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SOLID EARTH TIDES IN THE EQUATIONS OF MOTION OF THE TERRA SYSTE--ETC(U)  
FEB 77 W J GROEGER, R B MANRIQUE  
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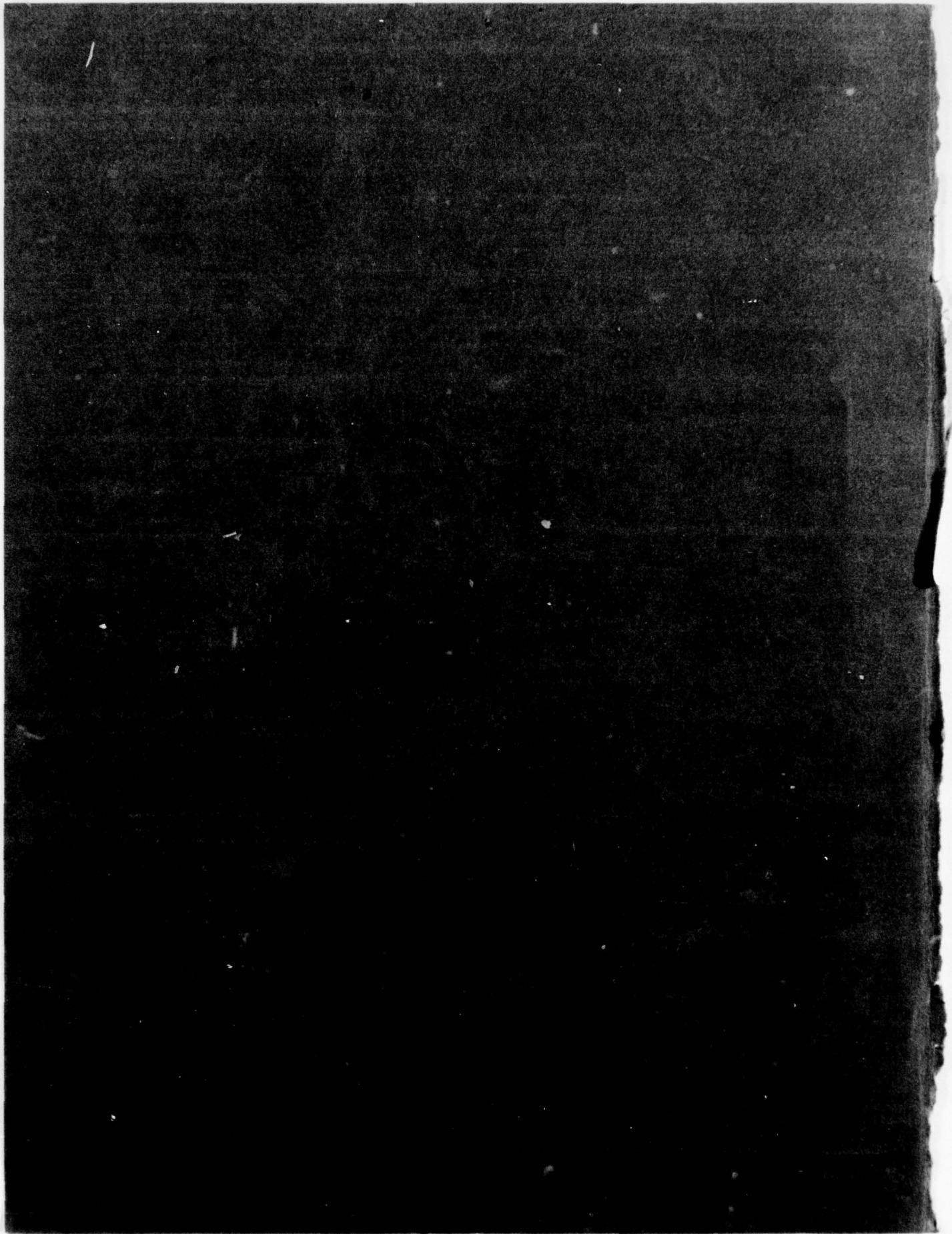
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to allow for tidal lag. The latter manifests itself as a time delay in the response of the tidal mass redistribution to the tidal stress field. Its origin are the elasticity and plasticity of the earth and the inertia of the masses involved.

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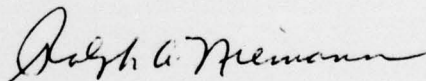
## FOREWORD

This report documents the last of four tidal perturbing terms appearing in the force equations for satellite motion in the TERRA system of satellite geodesy computer programs. An algorithm is presented for the gravitational action by which the tidal bulge of the solid earth perturbs satellite motion. Three earlier reports contain similar algorithms for the air tides caused by the sun and the moon, as well as for the ocean tides.

The problem analysis was rather tedious because it involved certain expressions which were unusually complicated algebraically. The authors are greatly obliged to Mr. Alfred Morris of the Computer Program Division (DK-70), who verified the results with the aid of his symbolic computer language FLAP.

The work was done in the Astronautics and Geodesy Division (DK-10) in support of astronautics computer program development. This report has been reviewed and approved by R. J. Anderle, Head, Astronautics and Geodesy Division.

Released by:



R. A. NIEMANN, Head  
Warfare Analysis Department

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## INTRODUCTION

All tides except one\* are caused by the fields of lunar and solar mass attraction within which the earth rotates. These fields impart to each terrestrial particle an acceleration which depends on the specific position of the particle within the field. Generally, this acceleration is different from that experienced by the earth as a whole. The relative acceleration of the particle with respect to the earth gives rise to certain geophysical forces which balance it, thus enabling the particle to remain attached to the earth. These forces constitute a very complicated stress field. It is customary to classify the various tides by decomposing the tidal stress field into a number of individual components according to the nature of the terrestrial constituents (solid earth, ocean, or atmosphere) and to the body (sun or moon) which generates the particular stress field component.

Each tide is essentially a redistribution of the earth's masses. As such, it will modify the terrestrial gravity field which in turn may be expected to perturb the satellite motion. For most purposes of satellite geodesy, tidal perturbation of the orbits was regarded inconsequential until quite recently. However, data acquisition has now become so accurate and the art of orbit computation has been refined to the point where it is generally agreed that to avoid tide related biases, the tides are required to appear amongst the perturbing terms in the force equations for satellite motion.

The necessary improvement in the force model for geodetic satellites was recently implemented at the Naval Surface Weapons Center. The additional force terms which have been developed for the new TERRA system of computer programs for satellite geodesy reflect the state of the art and account for tides due to the combined effects of solar gravitation and solar irradiation,<sup>1</sup> tides caused by the moon's gravity,<sup>2</sup> ocean tides,<sup>3</sup> and solar as well as lunar tides of the solid earth. For the two atmospheric tides, surface pressure functions corresponding to the atmospheric tide bulges were readily available in the geophysical literature. The disturbing acceleration was derived for each tide from surface pressure via Poisson integration followed by spatial differentiation. The algorithm for the ocean tides starts from a table for the global tide amplitudes and phase angles. It approximates the gravitational tide potential exterior to the earth by that of a system of point masses which are located on the ocean surface and which are chosen to resemble the masses of the tidal bulge.

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\* The solar air tide is partly a thermal effect related to the absorption of sunlight.



A suitable potential function for the solid earth tide was compiled from current literature.<sup>4</sup> This was, however, very complicated algebraically. The partial derivatives necessary for the perturbing acceleration were derived manually; and to assure correct results, this effort was duplicated on the computer with the help of the symbolic language FLAP.<sup>5,6</sup> In addition, it was difficult to verify by hand, in a reasonable time span, that the formula collection for the potential actually satisfied the Laplace differential equation. Consequently, verification was performed entirely by computer which speedily resolved the problem.

The potential function just mentioned is probably the most flexible expression now available for the Newtonian gravitation associated with the solid earth tide. However, it was not immediately applicable to our purpose because, as transcribed from Reference 4, it represents the mass attraction of the equilibrium tide.\* Our attention was directed to this defect by Reference 7 (page 2) which also indicates that the necessary correction can be accomplished by interpreting the just mentioned equilibrium tide potential as being produced by a fictitious moon or sun. A rather simple transformation from the lunar and solar ephemeris coordinates to those of the fictitious moon or sun will change the equilibrium potential into a potential function which contains the desired effects of tidal lag.

## THE TIDE POTENTIAL

The following is a summary of the tide potential associated with the tidal mass redistribution occurring within the solid earth. This was abstracted from Reference 4 (see especially pages 258 through 266). The formulas chosen were, however, not simply copied. When comparing the equations which appear on the next pages with their counterparts in the reference, one will notice that the unit vector components and, generally, the coordinates which indicate respectively the positions of sun and moon are distinguished below from those of the satellite by a star-shaped superscript, while in the reference, they are primed. In the reference as well as here, the prime denotes the ephemeris coordinates, while the star is attached to the positions of the fictitious sun and moon mentioned above in the Introduction and further elaborated in the next chapter.

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\*This is a stationary solution of the tidal problem. If taken for the tide proper, the unrealistic assumption would be implied of a tidal mass redistribution capable of instantaneously responding to the motion of the celestial body which causes the tide.

The potential is, of course, valid exterior to the earth. Observe that the properties of the tide are assumed to vary latitudinally; that is, they are expressed by including Love coefficients which depend on latitude. To be precise, the Love numbers are expansions in terms of zonal harmonics, latitude\* being the variable. Although the expansion coefficients for the Love numbers occur frequently in the tide potential and later in the disturbing acceleration, the explicit expressions for the Love numbers are not needed. Those interested in the geophysical background may wish to peruse pages 260 and 261 of Reference 4. An introduction to the basic Love number concept may be found in Reference 8.

In detail, the tide potential is:

$$U = \sum_{\ell=0}^4 C_{\ell} \frac{V_{\ell}}{r^{\ell+1}} \quad (1)$$

where

$$C_0 = \left( \frac{2}{3} \epsilon^2 k_{20} - \frac{2}{5} k_{22} \right) a K A'_0 \quad (2a)$$

$$C_1 = K \alpha k_{21} a^2 \quad (2b)$$

$$C_2 = K \alpha^2 a^3 \quad (2c)$$

$$C_3 = K \alpha^3 a^4 \quad (2d)$$

$$C_4 = K \alpha^4 a^5 \quad (2e)$$

and

$$K = \frac{m'}{M} n^2 a^2 \alpha'^3 \beta^{*3} \quad (3a)$$

$$a^3 n^2 = \mu = MG \quad (3b)$$

$$\alpha' = \frac{R}{a'} \quad (3c)$$

---

\*We intentionally avoid distinguishing between the various types of latitude because we expect that any inaccuracy arising from this simplification will be masked by the much larger uncertainties which otherwise prevail in current tide work.

$$\alpha = \frac{R}{a} \quad (3d)$$

$$\beta^* = \frac{a'}{r^*} \quad (3e)$$

and

$$V_0 \equiv 1 \quad (4a)$$

$$V_1 = \frac{1}{5} (A'_3 \lambda + A'_4 \mu - 4A'_0 \nu) \quad (4b)$$

$$V_2 = \sum_{n=0}^4 P'_n P_n \quad (4c)$$

$$V_3 = \sum_{n=1}^7 S'_n S_n \quad (4d)$$

$$V_4 = \sum_{n=1}^7 T'_n T_n \quad (4e)$$

and

$$A'_0 = \frac{1}{4} (1 - 3\nu^{*2}) \quad (5a)$$

$$A'_1 = \frac{3}{4} (\lambda^{*2} - \mu^{*2}) \quad (5b)$$

$$A'_2 = 3\lambda^* \mu^* \quad (5c)$$

$$A'_3 = 3\lambda^* \nu^* \quad (5d)$$

$$A'_4 = 3\mu^* \nu^* \quad (5e)$$

and

$$B'_1 = \frac{3}{8} \lambda^* (1 - 5\nu^{*2}) \quad (6a)$$

$$B'_2 = \frac{3}{8} \mu^* (1 - 5\nu^{*2}) \quad (6b)$$

$$B'_3 = \frac{1}{4} \nu^* (3 - 5\nu^{*2}) \quad (6c)$$

$$B'_4 = \frac{5}{8} \lambda^* (\lambda^{*2} - 3\mu^{*2}) \quad (6d)$$

$$B'_5 = \frac{5}{8} \mu^* (3\lambda^{*2} - \mu^{*2}) \quad (6e)$$

$$B'_6 = \frac{15}{4} \nu^* (\lambda^{*2} - \mu^{*2}) \quad (6f)$$

$$B'_7 = 15\lambda^* \mu^* \nu^* \quad (6g)$$

and

$$P'_0 = \left( k_{20} + \frac{2}{7} k_{22} \right) \left( 1 - \frac{55}{42} \epsilon^2 \right) A'_0 + \frac{3}{7} k_{31} \alpha' \beta^* B'_3 \quad (7a)$$

$$P_0 = 1 - 3\nu^2 \quad (7b)$$

$$P'_1 = \left( k_{20} - \frac{2}{7} k_{22} \right) \left( 1 - \frac{5}{14} \epsilon^2 \right) A'_1 + \frac{1}{7} k_{31} \alpha' \beta^* B'_6 \quad (7c)$$

$$P_1 = \lambda^2 - \mu^2 \quad (7d)$$

$$P'_2 = \left( k_{20} - \frac{2}{7} k_{22} \right) \left( 1 - \frac{5}{14} \epsilon^2 \right) A'_2 + \frac{1}{7} k_{31} \alpha' \beta^* B'_7 \quad (7e)$$

$$P_2 = \lambda\mu \quad (7f)$$

$$P'_3 = \left( k_{20} + \frac{1}{7} k_{22} \right) \left( 1 - \frac{15}{14} \epsilon^2 \right) A'_3 - \frac{8}{7} k_{31} \alpha' \beta^* B'_1 \quad (7g)$$

$$P_3 = \lambda\nu \quad (7h)$$

$$P'_4 = \left( k_{20} + \frac{1}{7} k_{22} \right) \left( 1 - \frac{15}{14} \epsilon^2 \right) A'_4 - \frac{8}{7} k_{31} \alpha' \beta^* B'_2 \quad (7i)$$

$$P_4 = \mu\nu \quad (7j)$$

and

$$S'_1 = -\frac{1}{5}k_{21}A'_3 + k_{30}\alpha'\beta*B'_1 \quad (8a)$$

$$S_1 = \lambda(1 - 5\nu^2) \quad (8b)$$

$$S'_2 = -\frac{1}{5}k_{21}A'_4 + k_{30}\alpha'\beta*B'_2 \quad (8c)$$

$$S_2 = \mu(1 - 5\nu^2) \quad (8d)$$

$$S'_3 = \frac{3}{5}k_{21}A'_0 + k_{30}\alpha'\beta*B'_3 \quad (8e)$$

$$S_3 = \nu(3 - 5\nu^2) \quad (8f)$$

$$S'_4 = k_{30}\alpha'\beta*B'_4 \quad (8g)$$

$$S_4 = \lambda(\lambda^2 - 3\mu^2) \quad (8h)$$

$$S'_5 = k_{30}\alpha'\beta*B'_5 \quad (8i)$$

$$S_5 = \mu(3\lambda^2 - \mu^2) \quad (8j)$$

$$S'_6 = k_{21}A'_1 + k_{30}\alpha'\beta*B'_6 \quad (8k)$$

$$S_6 = \nu(\lambda^2 - \mu^2) \quad (8l)$$

$$S'_7 = k_{21}A'_2 + k_{30}\alpha'\beta*B'_7 \quad (8m)$$

$$S_7 = \lambda\mu\nu \quad (8n)$$

and

$$T'_1 = \left\{ \frac{15}{14}\epsilon^2 \left( k_{20} + \frac{1}{7}k_{22} \right) - \frac{9}{14}k_{22} \right\} A'_3 + \frac{15}{7}k_{31}\alpha'\beta*B'_1 \quad (9a)$$

$$T_1 = \lambda\nu \left( 1 - \frac{7}{3}\nu^2 \right) \quad (9b)$$

$$T'_2 = \left\{ \frac{15}{14} \epsilon^2 \left( k_{20} + \frac{1}{7} k_{22} \right) - \frac{9}{14} k_{22} \right\} A'_4 + \frac{15}{7} k_{31} \alpha' \beta' B'_2 \quad (9c)$$

$$T_2 = \mu \nu \left( 1 - \frac{7}{3} \nu^2 \right) \quad (9d)$$

$$T'_3 = \left\{ \frac{3}{14} \left( k_{20} + \frac{2}{7} k_{22} \right) \epsilon^2 - \frac{9}{70} k_{22} \right\} A'_0 - \frac{1}{7} k_{31} \alpha' \beta' B'_3 \quad (9e)$$

$$T_3 = 3 - 30\nu^2 + 35\nu^4 \quad (9f)$$

$$T'_4 = k_{31} \alpha' \beta' B'_4 \quad (9g)$$

$$T_4 = \lambda \nu (\lambda^2 - 3\mu^2) \quad (9h)$$

$$T'_5 = k_{31} \alpha' \beta' B'_5 \quad (9i)$$

$$T_5 = \mu \nu (3\lambda^2 - \mu^2) \quad (9j)$$

$$T'_6 = \left\{ \frac{5}{14} \left( k_{20} - \frac{2}{7} k_{22} \right) \epsilon^2 - \frac{3}{4} k_{22} \right\} A'_1 - \frac{1}{7} k_{31} \alpha' \beta' B'_6 \quad (9k)$$

$$T_6 = (\lambda^2 - \mu^2)(1 - 7\nu^2) \quad (9l)$$

$$T'_7 = \left\{ \frac{5}{14} \epsilon^2 \left( k_{20} - \frac{2}{7} k_{22} \right) - \frac{3}{4} k_{22} \right\} A'_2 - \frac{1}{7} k_{31} \alpha' \beta' B'_7 \quad (9m)$$

$$T_7 = \lambda \mu (1 - 7\nu^2) \quad (9n)$$

The  $k_{ij}$  terms occurring in the formulas above are the expansion coefficients for the Love numbers. These are geophysical constants of the TERRA system. It is expected that the following values approximate the  $k_{ij}$  sufficiently; otherwise, they may be regarded to be initial values for a parameter improvement procedure.

$$\left. \begin{array}{ll} k_{20} = 0.3 & k_{21} = 0.01 \\ k_{30} = 0.1 & k_{31} = 0.01 \\ k_{22} = 0.1 & \end{array} \right\} \quad (10)$$

The necessary notations are compiled in Table 1.

**Table 1. Glossary for the Tide Potential Formulas**

M	Mass of the earth
G	Newton's constant
R	Equatorial radius of the earth (semimajor axis of the reference ellipsoid in use)
$\epsilon$	Eccentricity of earth's meridian (eccentricity of the reference ellipsoid in use)
$k_{ij}$	Expansion coefficients for the Love numbers
$m'$	Mass of the sun or moon
$a'$	Semimajor axis of the sun or moon orbit relative to the earth (Numerical values to be compatible with the ephemeris tapes used in TERRA for the sun and moon. May optionally be taken from a current edition of "American Ephemeris".)
$\lambda^*, \mu^*, \nu^*$	Equatorial components of the unit vector $\hat{r}^*$ pointing from the center of the earth to the fictitious sun or moon in a Cartesian coordinate system referred to mean equator and equinox of epoch
$r^*$	Geocentric distance of fictitious sun or fictitious moon
n	Mean motion of the satellite
a	Semimajor axis of the satellite orbit (This is an osculating orbit element which may be obtained from position and velocity of the satellite, for each time line in the orbit integration, via the energy integral.)
$\lambda, \mu, \nu$	Equatorial components of the unit vector $\hat{r}$ pointing from the center of the earth to the satellite in a Cartesian coordinate system referred to mean equator and equinox of epoch
r	Geocentric distance of the satellite

## TIDAL LAG

We mentioned earlier that the original equations for the tide potential (transcribed from Reference 4) are part of a stationary tide model. Essential to the latter is the assumption that the tidal mass redistribution occurs instantaneously or (which is the same) that the motions relative to the earth, of sun and moon, involve but vanishing velocity. Either one of these notions is unrealistic because they omit tidal lag. For TERRA, we desired to include the latter effect, hoping to be able to measure how it acts on the satellite orbits.

The tidal lag feature was readily introduced by the expedient of a fictitious sun and a fictitious moon. These are the ephemeris sun and the ephemeris moon; each, however, retarded in their geocentric motion with respect to the ephemeris positions and advanced in the direction of the earth's rotation. The retardation in the geocentric motion is expected to reflect how, at any given location on the globe, viscosity and inertia combine to delay the forming of the tidal bulge while the sun and moon traverse the sky. Simultaneously, the advance of either of the two fictitious bodies in the direction of the earth's rotation represents the tidal bulge being carried along by that rotation.

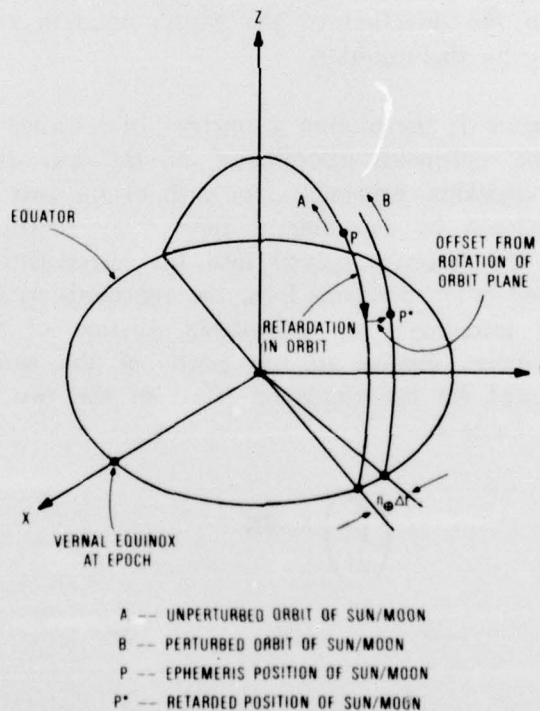
As illustrated in Figure 1, the motion geometry just specified was realized by a transformation from the ephemeris coordinates of the sun and moon to the corresponding fictitious positions. Separately, for each of the two bodies, the orbital retardation was accomplished by including a time delay (equal to the tidal lag parameter appearing in the formulas below) into the interpolations which produce the instantaneous sun and moon positions from the ephemeris data. By contrast, the rotational advance was introduced by a suitable rotation of the orbital planes associated with the motions, relative to the earth, of the sun and moon. The following algorithm resulted for the combined effect of the two motions; and, the notation is explained in Table 2.

$$\begin{pmatrix} x^* \\ y^* \\ z^* \end{pmatrix} = MN \quad (11)$$



**Table 2. Glossary for the Tidal Lag Equations**

$x_j, y_j, z_j$	Table values of the coordinates for the sun and moon from the ephemeris tapes
$\Delta T$	Time increment from the ephemeris tapes
$\Delta t$	Tidal lag parameter
$x', y', z'$	Position of the sun or moon at time $t$ as computed from the ephemeris tape by interpolation ("ephemeris coordinates")
$x^*, y^*, z^*$	Coordinates of the fictitious sun or moon at time $t$
$n_\oplus$	Earth's sidereal rate of rotation



**Figure 1. Simulating Tidal Lag by Orbital Retardation**

$$M = \begin{pmatrix} \cos n_{\oplus} \Delta t & -\sin n_{\oplus} \Delta t & 0 \\ \sin n_{\oplus} \Delta t & \cos n_{\oplus} \Delta t & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (12)$$

$$N = \begin{pmatrix} x' - (x_{i+1} - x_i) \frac{\Delta t}{\Delta T} \\ y' - (y_{i+1} - y_i) \frac{\Delta t}{\Delta T} \\ z' - (z_{i+1} - z_i) \frac{\Delta t}{\Delta T} \end{pmatrix} \quad (13)$$

$$\lambda^* = \frac{x^*}{r^*} \quad (14a)$$

$$\mu^* = \frac{y^*}{r^*} \quad (14b)$$

$$\nu^* = \frac{z^*}{r^*} \quad (14c)$$

$$r^* = \sqrt{x^{*2} + y^{*2} + z^{*2}} \quad (15)$$

### GRADIENT ALGORITHM

The following formulas represent the gradient of the tide potential. The notation is that indicated above. In addition,  $\bar{r}$  is the geocentric position vector of the satellite and  $\hat{x}$ ,  $\hat{y}$ ,  $\hat{z}$  (unit vectors) are the basis vectors for the Cartesian equatorial reference frame mentioned in Table 1.

$$\begin{aligned}
\bar{T} = + \text{grad } U = & \frac{-1}{r^3} \left\{ \left[ C_0 + \frac{3C_1 V_1}{r} + \frac{C_2}{r^2} (5V_2 - 2P'_0) + \frac{C_3}{r^3} (7V_3 - F) \right. \right. \\
& + \left. \frac{C_4}{r^4} (9V_4 - H) \right] \hat{r} - \left[ \frac{C_1}{5} A'_3 + \frac{C_2}{r} \rho_{11} + \frac{C_3}{r^2} \rho_{12} + \frac{C_4}{r^3} \rho_{13} \right] \hat{x} - \left[ \frac{C_1}{5} A'_4 \right. \\
& + \left. \frac{C_2}{r} \rho_{21} + \frac{C_3}{r^2} \rho_{22} + \frac{C_4}{r^3} \rho_{23} \right] \hat{y} - \left[ \frac{-4C_1}{5} A'_0 + \frac{C_2}{r} \rho_{31} \right. \\
& + \left. \frac{C_3}{r^2} \rho_{32} + \frac{C_4}{r^3} \rho_{33} \right] \hat{z} \left. \right\} \quad (16)
\end{aligned}$$

$$\rho_{11} = 2\lambda P'_1 + \mu P'_2 + \nu P'_3 \quad (17a)$$

$$\rho_{12} = (1 - 5\nu^2)S'_1 + 3(\lambda^2 - \mu^2)S'_4 + 2\lambda(3\mu S'_5 + \nu S'_6) + \mu\nu S'_7 \quad (17b)$$

$$\rho_{13} = \nu \left( 1 - \frac{7}{3}\nu^2 \right) T'_1 + 3\nu(\lambda^2 - \mu^2)T'_4 + 6\lambda\mu\nu T'_5 + (1 - 7\nu^2)(2\lambda T'_6 + \mu T'_7) \quad (17c)$$

$$\rho_{21} = -2\mu P'_1 + \lambda P'_2 + \nu P'_4 \quad (17d)$$

$$\rho_{22} = (1 - 5\nu^2)S'_2 - 6\lambda\mu S'_4 + 3(\lambda^2 - \mu^2)S'_5 + \nu(\lambda S'_7 - 2\mu S'_6) \quad (17e)$$

$$\rho_{23} = \nu \left( 1 - \frac{7}{3}\nu^2 \right) T'_2 - 6\lambda\mu\nu T'_4 + 3\nu(\lambda^2 - \mu^2)T'_5 + (1 - 7\nu^2)(\lambda T'_7 - 2\mu T'_6) \quad (17f)$$

$$\rho_{31} = -6\nu P'_0 + \lambda P'_3 + \mu P'_4 \quad (17g)$$

$$\rho_{32} = -10\nu(\lambda S'_1 + \mu S'_2) + (3 - 15\nu^2)S'_3 + (\lambda^2 - \mu^2)S'_6 + \lambda\mu S'_7 \quad (17h)$$

$$\begin{aligned}
\rho_{33} = & (1 - 7\nu^2)(\lambda T'_1 + \mu T'_2) - 20\nu(3 - 7\nu^2)T'_3 \\
& + \lambda(\lambda^2 - 3\mu^2)T'_4 + \mu(3\lambda^2 - \mu^2)T'_5 \\
& - 14\nu[(\lambda^2 - \mu^2)T'_6 + \lambda\mu T'_7] \quad (17i)
\end{aligned}$$

$$F = 2(\lambda S'_1 + \mu S'_2 + 3\nu S'_3) \quad (17j)$$

$$H = 2[\nu(\lambda T'_1 + \mu T'_2) + 6(1 - 5\nu^2)T'_3 + (\lambda^2 - \mu^2)T'_6 + \lambda\mu T'_7] \quad (17k)$$

## TIDES OF THE SOLID EARTH IN THE TERRA EQUATIONS OF MOTION

The desired perturbing term can now readily be formulated. Note that the vector  $\bar{T}$  just introduced (Equation 16) depends essentially on the coordinates of the fictitious sun or moon, on the mass and the semimajor axis of the sun or moon, and on the satellite coordinates:

$$\bar{T} = \bar{T}(x^*, y^*, z^*, m', a' ; x_j, y_j, z_j) \quad (18)$$

To produce the perturbing acceleration caused by the solar tide, substitute into the latter equation, separately for each step (time line) in the satellite orbit integration, the position coordinates of the fictitious sun, the satellite position, and the remaining two solar parameters ( $m'_S$  and  $a'_S$ ):

$$\bar{T}_S = \bar{T}_S(x_S^*, y_S^*, z_S^*, m'_S, a'_S ; x_j, y_j, z_j) \quad (19)$$

For the lunar tide component, enter the corresponding (lunar) data to obtain:

$$\bar{T}_L = \bar{T}_L(x_L^*, y_L^*, z_L^*, m'_L, a'_L ; x_j, y_j, z_j) \quad (20)$$

The tide potential, as compiled by Reference 4, is the sum of the potentials associated with the lunar and solar tide components. This implies that the disturbing acceleration due to the combined effect of the two tide components is, simply, the sum of the individual accelerations:

$$\bar{T}_{\text{SOLID EARTH TIDE}} = \bar{T}_S + \bar{T}_L \quad (21)$$

All the equations above are referred to an inertial reference frame. The latter is identical with the coordinate system in terms of which TERRA performs the satellite orbit integration. In particular, the perturbing acceleration specified by Equation 21 is compatible with all the other perturbing terms appearing in the TERRA equations of motion. No further coordinate transformation is needed. The

force equations for satellite motion in TERRA are a rather complicated collection of algorithms, each of which reflects one of the perturbing forces acting on earth satellites. The entire force model has been formulated and is available for implementation on the computer; and, it will be discussed in a future report.

It should finally be noted that the tide potential, as it appears in Reference 4, is a function of the ephemeris coordinates of moon and sun. To introduce tidal lag, we substituted for the ephemeris positions the fictitious positions of moon and sun. We are aware that this is not strictly correct. To insert the tidal lag feature in the right place, it would have been necessary to re-derive the tide potential. The positions of the fictitious moon and sun could then have been considered while dealing with the tidal surface deformation, in the earth-fixed reference frame, prior to calculating the tide potential and prior to transforming the latter back into the inertial frame. This would have accurately accounted for the effects of precession and nutation. But as we expect these to be negligible when compared with the lack of precision inherent in our tide lag model, we decided on the approach actually chosen.

The reader may now expect to be presented with a quantity of numerical information concerning items such as the average magnitude of the perturbing acceleration at typical orbital altitude, plots of the disturbing force along a typical satellite trajectory, and estimates of the tidal effect on the various orbital elements. However, no serious attempt was made to produce such data by manual calculation. Rather, the authors are planning to take advantage of the fact that the TERRA force equations are intended for use with a direct, numerical, orbit integration. It is expected that, upon completion of the program coding, all numerical questions will best be explored by trial runs of the relevant program segments.

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