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EFFECTIVE THICKNESS OF PAPER: APPRAISAL AND FURTHER DEVELOPMENT--ETC(U)
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Abstract

The rough surface and heterogeneous internal structure of paper complicate and restrict an analysis of paper by classical engineering mechanics. Problems in establishing physical properties stem from an uncertainty of exactly what is the "thickness" of a rough surfaced material. The concept of "effective thickness" obtained from a simultaneous solution of equations for flexural and extensional stiffness is proposed to mathematically transform the rough surfaced fibrous paper structure into a mechanically equivalent, smooth homogeneous sheet. The effects of density gradients within the sheet on effective thickness are examined. The U.S. Forest Products Laboratory (FPL) modified dial micrometer is shown to yield an expedient laboratory approximation to the effective thickness.



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EFFECTIVE THICKNESS OF PAPER:

APPRAISAL AND FURTHER DEVELOPMENT¹

By **MICHAEL R. ROSENTHAL**, Engineer

Forest Products Laboratory,² Forest Service
U.S. Department of Agriculture

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Introduction

Paper properties dependent on thickness are likely to be substantially in error when thickness values are obtained with standard micrometers. Surface roughness and the resiliency of paper's heterogeneous fibers cause thickness measurements obtained with a standard TAPPI micrometer to be always too high. For example, Setterholm found rough corrugating media where measurements were in error by as much as 80 percent.³ Measurements with such degrees of error may still be useful to obtain comparative values, such as for purposes of quality control. However, a more accurate means of obtaining paper thickness is needed for research and engineering purposes.

To use classical equations of solid mechanics in the study of paper physics, several basic physical parameters must be known. The most commonly known and used of these parameters is the elastic modulus (E). This is frequently obtained from extensional stiffness (the initial linear portion of a tensile load-strain curve) by dividing by specimen thickness and width. This is a trivial calculation for most materials, but paper with its rough surface and fibrous structural nature presents special problems. The problems arise when attempts are made to measure the thickness of paper. Due to the irregular surface contours and resilient fibrous structure of paper, the value obtained using a micrometer type instrument will be very sensitive to the area and shape of the micrometer pressure foot.

TAPPI has long recognized the problem in handsheet thickness determination by requiring a stack of five sheets to be measured with a standard micrometer, the thickness of a single sheet being the average of the measured value of the stack. The stacking process results in a nesting between individual sheets that tends to lower the average.

Setterholm³ proposed a new definition and method for determining thickness to alleviate the problems caused by surface roughness of paper sheets. The proposed definition of "effective thickness" is that value obtained from simultaneous solution of equations for extensional and bending stiffness. This definition can be viewed as a means of mathematically transforming paper, with its intrinsic surface roughness and heterogeneous fibrous nature, into a mechanically equivalent smooth homogeneous sheet.

¹Acknowledgment is made to Craig A. Jackson, Engineer, for design of the jig used for determining paper bending stiffness, and to John Wichmann, Technician, for collecting the experimental data.

²Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

³Setterholm, Vance C., 1973. A New Concept in Paper Thickness Measurement. Tappi 57(3) 164.

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Effective Thickness

To determine the effective thickness of a paper both its extensional and bending stiffness must be experimentally determined. It is then assumed that the material is homogeneous and linearly elastic for small strains and deflections. Material isotropy does not have to be assumed because the relations for tensile loading and pure bending are the same for both an anisotropic and isotropic material.⁴ This accommodates the current view that paper is an orthotropic material with principal inplane axes corresponding to the machine and cross-machine directions. The bending stiffness S_B and the extensional stiffness S_E are given by

$$S_B = EI = E \frac{1}{12} WT^3 \quad (1)$$

$$S_E = EA = EWT \quad (2)$$

The elastic modulus, E in these equations, is the one associated with the direction of the applied load in extension and the direction of the axis of the beam in pure bending. Solving these equations simultaneously for T yields:

$$T_{eff} = \sqrt{\frac{S_B}{12 S_E}} = \sqrt{\frac{EI}{12(EA)}} = \sqrt{\frac{EWT^3}{E12WT}} \quad (3)$$

where W is width,
 T is thickness,
 E is elastic modulus,
 S_B is bending stiffness,
 S_E is extensional stiffness,
 I is moment of inertia, and
 A is cross sectional area.

In practice, specimens of different widths may be used in the bending and tension tests. If this is the case, equation (3) is still valid if the stiffnesses are used on a per unit width basis. The third form of equation (3) shows the importance of the assumption that the material is homogeneous. Without this assumption the elastic modulus could be different in the

numerator and denominator (that is, different in bending than in tension). This difference, caused by a varying elastic modulus through the thickness, will result in a difference between T_{eff} and T (assuming that T is known).

This difference is expressed by

$$\frac{T_{eff}}{T} = \sqrt{\frac{E_B}{E_E}} \quad (4)$$

where E_B is elastic modulus in bending and E_E is elastic modulus in tension.

Effective thickness, once determined by equation (3), can be substituted into either equation (1) or (2) to obtain the appropriate elastic modulus, E . This modulus E is an equivalent modulus such that when coupled with the effective thickness it will reproduce the experimentally determined bending and extensional stiffnesses. This implicitly assumes that the material is linearly elastic and homogeneous. Hence the rough-surfaced and inhomogeneous paper structure, which may be nonlinearly elastic, is represented by a smooth homogeneous material having an effective thickness and an equivalent elastic modulus.

Instruments used in thickness measurement included a mercury pycnometer,³ the TAPPI automated micrometer, and a dial micrometer modified at the U.S. Forest Products Laboratory to provide a practical method of obtaining an expedient laboratory approximation to the actual effective thickness (fig. 1).³ Thickness of five types of paper was measured with these three instruments, and thickness values were also derived with the effective thickness formula (table 1). The five papers selected could all be used in structural applications where the elastic modulus is of quantitative importance. Mercury pycnometer, FPL micrometer, and effective thickness values are all in close agreement. However, the TAPPI micrometer measurements are larger than the effective thickness by 2 to 34 percent.

⁴Lekhnitshii, S. G., 1968. Anisotropic Plates. Gordon and Breach, New York, N. Y. p.58 - 60.

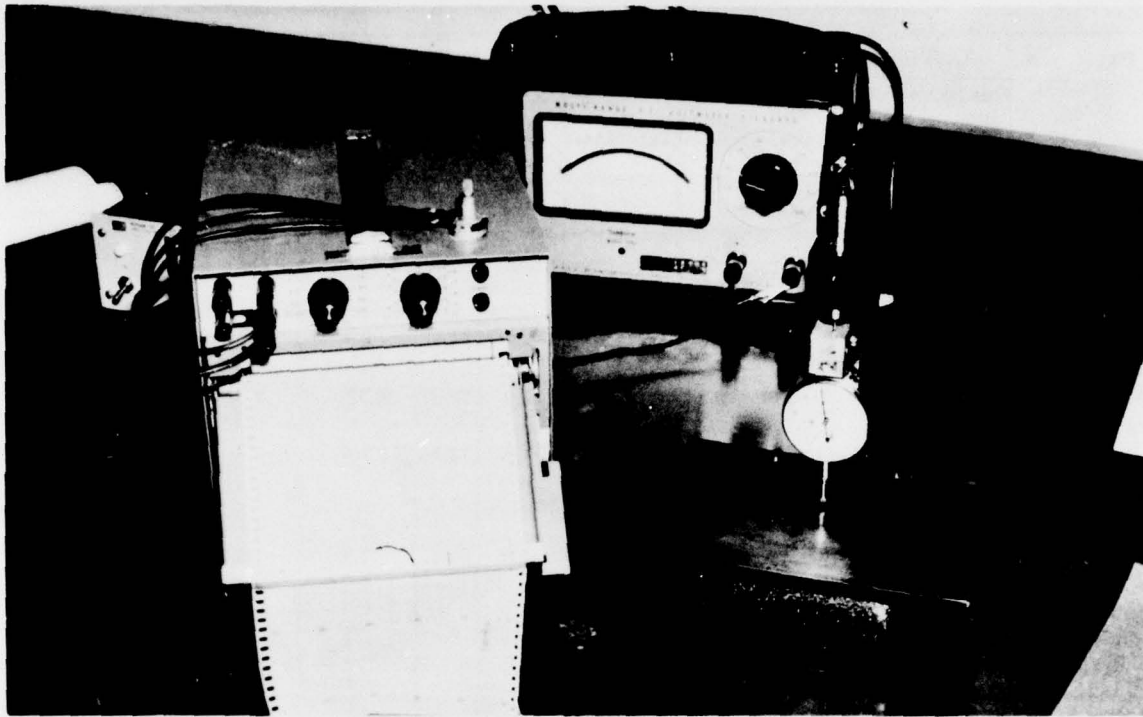


Figure 1. -- The FPL modified dial micrometer produces a graphic thickness contour as a paper sheet is drawn between the platens. A transducer and a recorder are shown attached to the micrometer.

(M 140 847)

The largest discrepancy occurred with a commercial corrugating medium which had the roughest surface of the five papers. It can be expected that the coarser the sheet, the larger this error will become. Setterholm³ reports errors as large as 80 percent for rough corrugating media.

The two values of effective thickness, one determined in the machine direction and the other in the cross-machine direction (table 2), are not significantly different at the 10 percent level (Welsh test) for each of the five papers. If the average of the two effective thicknesses is used with the appropriate elastic modulus, the experimentally determined extensional and bending stiffnesses can be reproduced in either machine or cross-machine directions with substantially less error than that due to the variation of the elastic modulus.

The advantage of the effective thickness definition is easily demonstrated. Suppose that the thickness of a paper is measured with a TAPPI micrometer with a 34 percent error (as the commercial corrugating medium in table 1) and the elastic modulus is calculated from a tensile test. If these values are then used to calculate the bending stiffness -- which is needed, for example, in bending and buckling relations -- the stiffness obtained will be in error by 80 percent, which is substantially greater than the variation in the elastic modulus. In general, if this process is followed and the error in thickness is P percent, the error in the resulting bending stiffness will be $(2P + P^2/100)$ percent. This error will propagate through any relation in which this erroneous bending stiffness is used.

Table 1. -- Thickness determinations of paper specimen materials by four methods

Paper type	TAPPI micrometer			FPL micrometer			Pycnometer			Effective thickness	
	Reading	Coefficient of variation	Deviation from effective thickness	Reading	Coefficient of variation	Deviation from effective thickness	Reading	Coefficient of variation	Deviation from effective thickness	Calculated value ¹	Coefficient of variation ¹
	Mil		Pct	Mil		Pct	Mil		Pct	Mil	
Commercial three-ply linerboard	15.93	3.2	8.1	14.36	2.6	-2.5	14.35	2.7	-2.6	14.73	5.6
FPL single-ply linerboard	11.03	2.0	3.6	10.34	2.0	-2.9	10.27	2.3	-3.6	10.61	4.5
FPL food board	14.85	1.4	3.7	14.10	1.9	-1.6	14.14	1.4	-1.3	14.32	4.1
Commercial three-ply cylinder board	25.87	3.0	2.0	25.03	2.2	-1.3	24.90	1.9	-1.8	25.37	3.7
Commercial corrugating medium	11.18	5.8	33.6	8.33	2.8	-0.5	8.41	3.3	.4	8.37	5.4
Commercial six-ply cardboard	25.71	.6	--	25.43	.9	--	25.09	2.1	--	32.43	3.3

¹Average of values for machine and cross-machine directions.

Table 2. -- Effective thickness and elastic modulus for machine and cross-machine axes of paper sheets

Paper type	Effective thickness				Elastic modulus			
	Machine direction		Cross-machine direction		Machine direction		Cross-machine direction	
	Derived value	Coefficient of variation	Derived value	Coefficient of variation	Derived value	Coefficient of variation	Derived value	Coefficient of variation
	Mil		Mil					
Commercial three-ply linerboard	14.66	5.9	14.79	5.3	526.0	11.0	213.1	11.8
FPL single-ply linerboard	10.70	6.1	10.59	2.9	985.7	7.5	450.9	6.1
FPL food board	14.39	4.2	14.25	4.0	910.8	8.4	306.9	8.9
Commercial three-ply cylinder board	25.37	4.0	25.37	3.4	656.6	9.1	113.9	9.5
Commercial corrugating medium	8.23	5.1	8.50	5.7	1,066.0	11.1	364.3	13.4
Commercial six-ply cardboard	31.55	2.5	33.30	4.1	413.4	7.1	146.3	10.9

Density Gradient

The definition of effective thickness implicitly assumes that the paper sheet is elastically homogeneous. Yet it is generally accepted that paper possesses a density gradient due to the nonuniform distribution of fines and their varying degrees of compaction through the sheet thickness. Changes in density have been shown to be reflected by changes in elastic modulus.⁵ Hence, the existence of a density gradient will in turn produce a variation in elastic modulus through the sheet thickness. The effect of elastic modulus variations on effective thickness values can be estimated in the following manner.

Assume for the purposes of illustration that the faces of the paper sheet are perfectly smooth, but that a linear density gradient exists through the thickness of the sheet. Then the only factor affecting the effective thickness is a variation in elastic modulus through the sheet thickness. The elastic modulus has been empirically shown to vary directly as the cube of the density for wet-pressed sheets.⁵ The error in effective thickness will arise due to the difference between the elastic modulus calculated from a bending test (E_B) and a tension test (E_E) by using the usual elementary relations, equations (1) and (2). Assuming for convenience that the sheet is of unit width with thickness, T , and the elastic modulus for a particular direction is a function of the thickness variable, y , alone, then E_E and E_B are given by

$$E_E = \int E(y) dy / T \quad (5)$$

$$E_B = \int E(y) y - \bar{y}^2 dy / (T^3/12) \quad (6)$$

where \bar{y} is $\int E(y)y dy / \int E(y)dy$ and

$$E(y) \text{ is } K\rho(y)^3$$

It is easily shown that if $E(y) = E$, equations (5) and (6) reduce to $E_E = E_B$. The error caused by a particular density gradient, $\rho(y)$, is found by evaluating equations (5) and (6) and substituting the results into equation (4).

Three forms of a density gradient are considered where the density varies linearly from ρ to $(1 + \alpha)\rho$, $0 < \alpha < 1$ (fig. 2). These three forms are meant to approximate or bound the presumed density gradient in a single ply sheet or two similar sheets wet pressed together. Using equations (4), (5), and (6) the errors for the three forms can be shown to be

$$\frac{T_{eff}}{T} = \left[\frac{1 + 3\alpha + 3.7\alpha^2 + 2.4\alpha^3 + 0.9\alpha^4 + 0.2\alpha^5 + 0.02\alpha^6}{1 + 3\alpha + 4.25\alpha^2 + 3.5\alpha^3 + 1.75\alpha^4 + 0.5\alpha^5 + 0.0625\alpha^6} \right]^{1/2} \quad (7)$$

$$\frac{T_{eff}}{T} = \left[\frac{1 + 0.75\alpha + 0.3\alpha^2 + 0.05\alpha^3}{1 + 1.5\alpha + \alpha^2 + 0.25\alpha^3} \right]^{1/2} \quad (8)$$

$$\frac{T_{eff}}{T} = \left[\frac{1 + 2.25\alpha + 1.8\alpha^2 + 0.5\alpha^3}{1 + 1.5\alpha + \alpha^2 + 0.25\alpha^3} \right]^{1/2} \quad (9)$$

Equations (7), (8), and (9) are graphed in figure 2. Note that approximately 15 percent density variation ($\alpha = 0.15$) will induce only a 5 percent error in effective thickness for any of the three forms considered. This degree of error is minor compared to the degree of error common for TAPPI micrometer measurements (table 1).

A common-type paper and paperboard in which a definite elastic modulus gradient exists is the multi-ply sheet formed by layering plies of different pulps together. Two types of construction are analyzed here, the two-ply sheet (fig. 3) and the symmetric three-ply sheet (fig. 4). The analysis assumes that the exact thickness and elastic modulus of each ply are known and also that the elastic modulus is constant within a ply but changes abruptly at ply interfaces. However, in the actual sheet formation process the fibers of two adjacent

⁵Setterholm, Vance C. and Warren A. Chilson. 1964. Effect of Restraint During Drying on the Tensile Properties of Handsheets. U.S. Forest Serv. Res. Pap. FPL 11, Forest Prod. Lab., Madison, Wis.

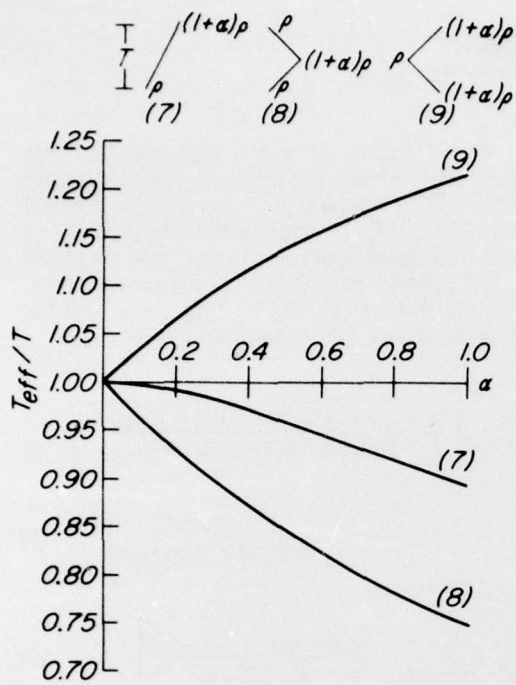


Figure 2. -- Effective thickness error in sheets containing three different linear density profiles. (M 144 478)

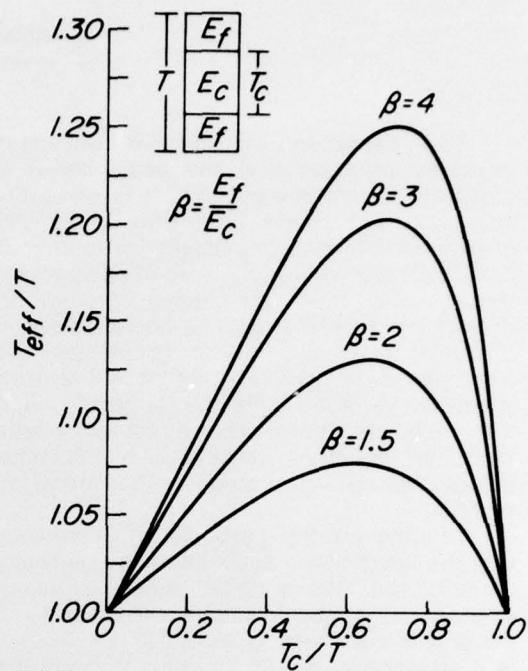


Figure 4. -- Effective thickness error in sheets of three-ply symmetrical construction (M 144 479)

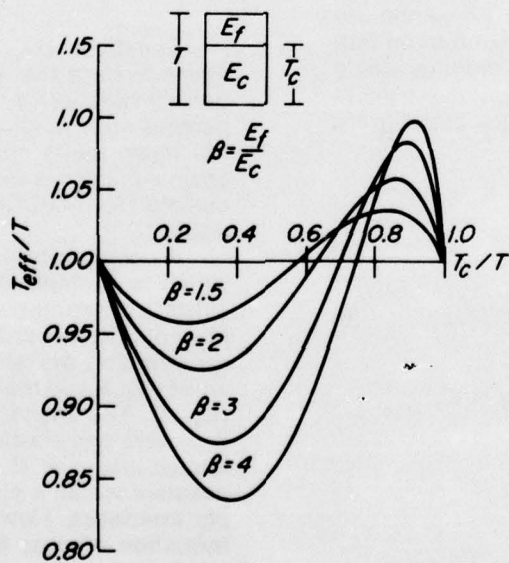


Figure 3. -- Effective thickness error in sheets of two-ply construction. (M 144 480)

plies intermingle at the interface. This mingling produces an irregular ply boundary layer with a varying elastic modulus between the two neighboring plies.

The degree to which two pulps intermingle at a ply boundary depends upon their moisture content when they come into contact; a sheet formed with a multiple headbox would have more diffuse ply boundaries than one formed on a cylinder paper machine. The two cases presented can be looked upon as approximations which bound the error in an actual layered paper. This error again is due solely to the presence of an elastic modulus gradient, as the surfaces are assumed to be smooth. The error in effective thickness for the two-ply sheet is given by equation (10) which is graphed in figure 3. The error for the three-ply symmetric sheet is given by equation (11), graphed in figure 4.

$$\frac{T_{eff}}{T} = \left[\frac{4(\beta(1-\gamma)^3 + \gamma^3)(\beta(1-\gamma) + \gamma) - 3(\beta(1-\gamma)^2 - \gamma^2)^2}{(\beta(1-\gamma) + \gamma)^2} \right]^{1/2} \quad (10)$$

where γ is $\frac{T_c}{T}$ and

$$\beta \text{ is } \frac{E_f}{E_c}$$

$$\frac{T_{eff}}{T} = \left[\frac{\beta + \gamma^3(1-\beta)}{\beta + \gamma(1-\beta)} \right]^{1/2} \quad (11)$$

where γ is $\frac{T_c}{T}$ and

$$\beta \text{ is } \frac{E_f}{E_c}$$

Figures 3 and 4 show that use of the effective thickness concept for the multi-ply type of sheet construction can lead to substantial errors. However, the smoothing of the elastic modulus gradients caused by the intermingling of fibers at the ply interfaces reduces this error. This effect can be demonstrated by comparing the maximum error (25 pct) of a

three-ply symmetric sheet (fig. 4) having an elastic modulus ratio of 4 to the maximum error of a sheet with an elastic modulus that varies linearly from E at the center to $4E$ at the edges. Using equations (4), (5), and (6), the error caused by the smooth linear modulus gradient is only 14 percent. Hence, it is probable, because an elastic modulus ratio of 4 is an extreme case, that no unfinished paper will have an effective thickness greater than its conventionally measured TAPPI thickness. That is, its increased bending stiffness will increase its effective thickness no more than its surface roughness will increase its TAPPI thickness relative to its volume displacement (mercury pycnometer) thickness. The effective thickness definition is just as valid for the multi-ply type sheet; in this case the effective thickness formula transforms a fibrous material with a rough surface, known to be inhomogeneously elastic, into the mechanistically equivalent, smooth, homogeneous sheet. That is, using the effective thickness and the calculated equivalent elastic modulus the experimentally determined extensional and bending stiffnesses are both reproducible.

Note that for some multi-ply paper materials the effective thickness definition is inappropriate. An example is the six-ply commercial cardboard noted at the bottom of table 1. This is a coated cardboard material, not unfinished paper for which effective thickness is intended. The coating minimizes the influence of surface roughness while at the same time drastically increasing the elastic modulus of the outer plies. The increase in the elastic modulus of the outer plies will in turn increase the ratio of the bending to the extensional stiffness. The increase in this ratio will significantly increase the difference between the effective thickness and the actual physical thickness as can be seen from equation (4). The significant differences between the TAPPI and FPL micrometer measurements, as well as between these two measurements and the pycnometer thickness, are due to the different stylus and pressure used in each method. Using the pycnometric thickness as the actual value and as a basis of comparison, the effective thickness is in error by 26 percent in the machine direction and 34 percent in the cross-machine direction.

The difference in error is due to different elastic modulus ratios between the facing and core materials in the two directions. It should be noted that the FPL micrometer method gives the best approximation to the pycnometric thickness.

Method of Measurement

The various thickness measurements were performed using three types of specimens randomly cut from sheets of each type of paper. A 1/2- by 4-inch specimen was used in the specially designed mercury pycnometer.³ The necked tension specimen was 1/2 by 6 inches and was tested in a table model Instron. The tension specimen was also used in the FPL modified dial micrometer³ which produces a graphical thickness contour through which an average line is fitted (fig. 1). This FPL modified micrometer used spherical platens with radii of 0.0938 inch ; the load on the dial stem was adjusted until readings approximated the thickness values derived with the effective thickness formula.

The bending specimen was 1 by 2 inches and was tested on a recently developed paper bending jig which fits in a table model

Instron. The jig employs a 1-inch span with quarter-point loading to obtain a pure bending mode in the center portion of the specimen. The bending specimen was also used with the TAPPI automated micrometer with the thickness being the average of four readings taken at different points on the specimen. Twenty-four specimens of each type of paper were used in the TAPPI automated micrometer and the FPL modified dial micrometer. Twelve specimens were used in the mercury pycnometer and in the tensile and bending tests in both the machine and cross-machine directions. The effective thickness in each direction was calculated from the 144 possible combinations of the 12 tension and bending tests. Table 2 documents the average values for each direction with corresponding coefficients of variation.

Summary

The concept of effective thickness transforms a rough surfaced, heterogeneous fibrous material into a mechanistically equivalent smooth homogeneous sheet. The effective thickness is in good agreement with a physical thickness determined from a volume displacement measured in a mercury pycnometer. It also agrees well with a dial micrometer which was modified to provide a practical method of obtaining an expedient laboratory approximation to the actual effective thickness. The automated TAPPI micrometer yields a value that is always greater than the effective thickness due to the rough surface of a paper sheet. The thickness obtained in this manner

leads to substantial errors if it is used as a factor in a mechanistic relation. The effect of a physically reasonable density gradient within a sheet has no significant effect on its effective thickness. The unfinished multi-ply sheet with a definite elastic modulus gradient may minimally increase the effective thickness but not to the extent that it becomes physically unacceptable. In spite of any sheet inhomogeneities in structure or material, the effective thickness concept produces a mechanistically equivalent section which is a good approximation for purposes of applying the classical equations of mechanics to paper.

U.S. Forest Products Laboratory.

Effective thickness of paper: Appraisal and further development, by Michael R. Rosenthal, Madison, Wis., 1977.

9 p. (USDA For. Serv. Res. Pap. FPL 287).

The concept of "effective thickness" is developed to mathematically transform rough surfaced paper sheets into the equivalent of smooth, homogeneous sheets. The concept is also applied to multi-layer paper materials.

KEYWORDS: Micrometer, paper, sheet thickness, thickness measurement.

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