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QUALITATIVE STABILITY CHARACTERISTICS OF A FLEXIBLE MISSILE SUB--ETC(U)

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QUALITATIVE STABILITY CHARACTERISTICS OF A FLEXIBLE
MISSILE SUBJECTED TO A CIRCULATORY THRUST

G. L. Anderson

February 1977



BENET WEAPONS LABORATORY
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WATERVLIET, N.Y. 12189

TECHNICAL REPORT

AMCMS No. 611102.11.35D00

DA Project No. 1F161102A35D

Pron No. EJ-5-Y0015

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER WVT-TR-77007	2. GOVT ACCESSION NO.	3. REPORT'S CATALOG NUMBER
4. TITLE (and Subtitle) Qualitative Stability Characteristics of a Flexible Missile Subjected to a Circulatory Thrust.	5. TYPE OF REPORT & PERIOD COVERED Technical Rept.	
7. AUTHOR(s) G. L. Anderson	8. CONTRACT OR GRANT NUMBER(s)	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Benet Weapons Laboratory Watervliet Arsenal, Watervliet, N.Y. 12189 SARWV-RT-TP	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AMCMS No. 61102.11.35000 DA Proj. No. 1F161102A35D Pron No. EJ-5-Y0015	
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Armament Command Rock Island, Illinois 61201	12. REPORT DATE Feb 1977	13. NUMBER OF PAGES 29
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) UNCLASSIFIED	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Circulatory Thrust Dynamics Flexible Missile Stability		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The stability characteristics of an elastic, flexible missile of constant mass, modelled mathematically as a system consisting of three rigid bars elastically hinged at the joints and subjected to a constant circulatory thrust, are examined. Plots of eigencurves are presented and used to identify the regions of stability, divergence, and flutter in related stability maps. For anti-tangential and sub-tangential thrusts, the system is found to be divergent for any sufficiently small, positive thrust. Moreover, flutter does not occur for anti-tangential and super-tangential thrusts.		

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1. INTRODUCTION

The problem of determining the state of stability of a slender, flexible missile, modeled mathematically as a free-free elastic beam subjected to a thrust, has been the subject of several recent investigations [1] - [9]. It is of particular interest that Wu [9] has determined that, in the absence of a feedback directional control, the mechanism of instability of the missile system described in references [2], [4], and [8] is not flutter but rather divergence. This discrepancy appears to result from the form of approximate solution assumed in those investigations. For example, to solve approximately the fundamental non-self-adjoint boundary value problem

$$y^{IV} + Q(xy')' - \omega^2 y = 0, \quad 0 < x < 1, \quad (1)$$

$$y'' = y''' = 0 \quad \text{at} \quad x = 0, 1, \quad (2)$$

where $y' = dy/dx$, etc., and Q and ω denote the dimensionless thrust and frequency parameters, respectively, Beal [2] employed the Galerkin procedure. He assumed a solution that included the rigid body motions of translation and rotation, and, as a result, always obtained two zero frequencies. However, it is very easy to verify that equations (1) and (2) admit a rigid body mode of rotation whenever $Q > 0$, i.e., there exists only a single zero frequency. Subsequently, Wu [9] reported that the loss of stability of the nature of a structural mode that degenerates to the rigid body mode of rotation when $Q \rightarrow 0$ rather than by the onset of flutter which ensues upon the coalescence of two non-null natural frequencies at a positive critical value of the thrust parameter Q .

The objective of the present investigation is to examine a related missile problem from a somewhat simpler viewpoint, so as to expose rather clearly through elementary mathematical relationships the qualitative stability characteristics of a flexible missile. The system will be modeled mathematically as a system comprising three rigid, weightless bars joined by elastic springs, carrying a concentrated mass at the center of each bar, and subjected to a circulatory thrust, whose direction may be adjusted through a variation in the value of a so-called tangency coefficient.

2. THE EQUATIONS OF MOTION

Consider the system depicted in Figure 1 that consists of three rigid, weightless bars of equal length ℓ with elastically hinged joints that exert the following linear restoring moments: $c(\phi_1 - \phi_2)$ and $c(\phi_3 - \phi_2)$, where the ϕ_j 's, $j = 1, 2, 3$, are the (assumed small) angles formed between the horizontal and each of the respective bars and c denotes a constant stiffness parameter. The neutral equilibrium configuration corresponds to $\phi_j = 0$. The concentrated masses m_j are affixed at the center of each bar, and x and y are the coordinates of the position of the central mass m_2 relative to the fixed xy -coordinate frame. A thrust of constant magnitude T_0 with its line of action specified by the tangency coefficient α , whose vector form \underline{F} is $\underline{F} = T_0 (\cos \phi_1 \underline{i} + \sin \alpha \phi_1 \underline{j})$, where \underline{i} and \underline{j} are unit vectors parallel to the fixed x - and y -axes, respectively, is applied at the left free end of the system. It is assumed that the system can translate in the vertical y - as well as the horizontal x - directions.

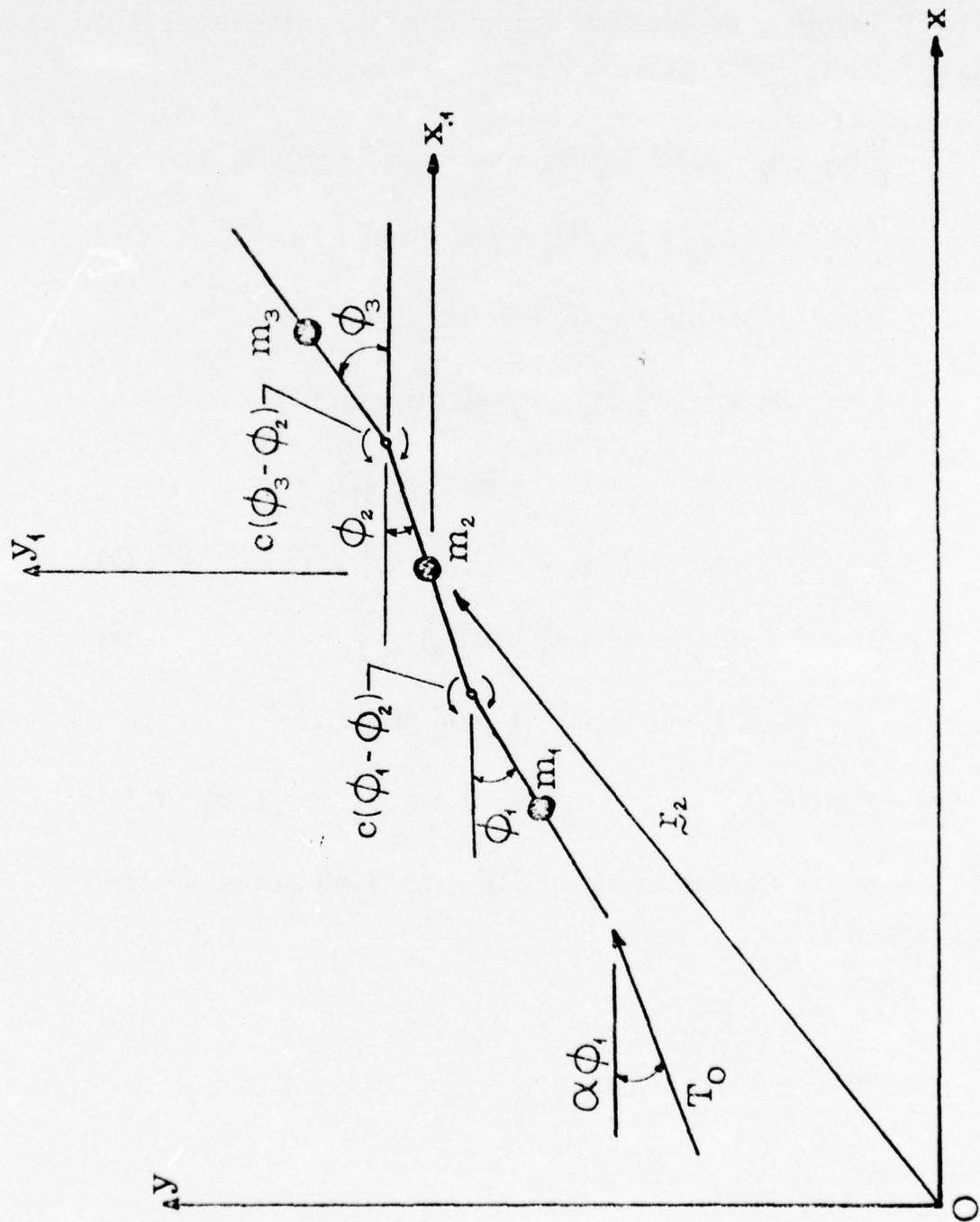


Figure 1. A simple model of a missile.

In the usual manner, one can derive the following expressions for the kinetic energy T , the potential energy V , and the generalized forces Q_x, \dots, Q_3 :

$$\begin{aligned}
 T = & \frac{1}{2} (m_1 + m_2 + m_3) \dot{x}^2 + \frac{1}{2} (m_1 + m_2 + m_3) \dot{y}^2 + \frac{1}{8} m_1 \ell^2 \dot{\phi}_1^2 + \\
 & + \frac{1}{8} (m_1 + m_3) \ell^2 \dot{\phi}_2^2 + \frac{1}{8} m_3 \ell^2 \dot{\phi}_3^2 + \frac{1}{2} m_1 \ell \ddot{x} \sin \phi_1 \dot{\phi}_1 + \\
 & + \frac{1}{2} (m_1 - m_3) \ell \dot{x} \sin \phi_2 \dot{\phi}_2 - \frac{1}{2} m_3 \ell \dot{x} \sin \phi_3 \dot{\phi}_3 - \\
 & - \frac{1}{2} m_1 \ell \dot{y} \cos \phi_1 \dot{\phi}_1 - \frac{1}{2} (m_1 - m_3) \ell \dot{y} \cos \phi_2 \dot{\phi}_2 + \\
 & + \frac{1}{2} m_3 \ell \dot{y} \cos \phi_3 \dot{\phi}_3 + \frac{1}{4} m_1 \ell^2 \cos (\phi_2 - \phi_1) \dot{\phi}_1 \dot{\phi}_2 + \\
 & + \frac{1}{4} m_3 \ell^2 \cos (\phi_2 - \phi_3) \dot{\phi}_2 \dot{\phi}_3, \quad (3)
 \end{aligned}$$

$$V = \frac{1}{2} c (\phi_1^2 - 2\phi_1\phi_2 + 2\phi_2^2 + \phi_3^2 - 2\phi_2\phi_3), \quad (4)$$

$$Q_x = T_o \cos \alpha \phi_1, \quad Q_y = T_o \sin \alpha \phi_1,$$

$$Q_1 = T_o \ell \sin (1 - \alpha) \phi_1, \quad Q_2 = \frac{1}{2} T_o \ell \sin (\phi_2 - \alpha \phi_1), \quad Q_3 = 0. \quad (5)$$

In view of the forms of equations (3) - (5), the Lagrange equations may be written as

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}} \right) = Q_x, \quad \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}} \right) = Q_y, \quad (6)$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\phi}_j} \right) - \frac{\partial T}{\partial \phi_j} + \frac{\partial V}{\partial \phi_j} = Q_j, \quad j = 1, 2, 3$$

Therefore, substituting equations (3) - (5) into equations (6), one finds that the linearized equations of motion for the given system are

$$(m_1 + m_2 + m_3)\ddot{x} = T_0 \quad (7)$$

$$(m_1 + m_2 + m_3)\ddot{y} - \frac{1}{2} m_1 \ell \ddot{\phi}_1 - \frac{1}{2} (m_1 - m_3) \ell \ddot{\phi}_2 + \frac{1}{2} m_3 \ell \ddot{\phi}_3 = \alpha T_0 \phi_1, \quad (8)$$

$$\frac{1}{4} m_1 \ell^2 \ddot{\phi}_1 + \frac{1}{2} m_1 \ell \ddot{x} \phi_1 - \frac{1}{2} m_1 \ell \ddot{y} + \frac{1}{4} m_1 \ell^2 \ddot{\phi}_2 + c (\phi_1 - \phi_2) = T_0 \ell (1 - \alpha) \phi_1,$$

$$\begin{aligned} & \frac{1}{4} (m_1 + m_3) \ell^2 \ddot{\phi}_2 + \frac{1}{2} (m_1 - m_3) \ell \ddot{x} \phi_2 - \frac{1}{2} (m_1 - m_3) \ell \ddot{y} + \frac{1}{4} m_1 \ell^2 \ddot{\phi}_1 + \\ & + \frac{1}{4} m_3 \ell^2 \ddot{\phi}_3 - c \phi_1 + 2c \phi_2 - c \phi_3 = \frac{1}{2} T_0 \ell (\phi_2 - \alpha \phi_1), \end{aligned} \quad (9)$$

$$\frac{1}{4} m_3 \ell^2 \ddot{\phi}_3 - \frac{1}{2} m_3 \ell \ddot{x} \phi_3 + \frac{1}{2} m_3 \ell \ddot{y} + \frac{1}{4} m_3 \ell^2 \ddot{\phi}_2 + c \phi_3 - c \phi_2 = 0,$$

where the familiar small angle approximations have been made and higher order terms have been neglected.

Solving equations (7) and (8) for \ddot{x} and \ddot{y} , one finds

$$\ddot{x} = \mu T_0 \quad (10)$$

and

$$\ddot{y} = \mu \left[\frac{1}{2} m_1 \ell \ddot{\phi}_1 + \frac{1}{2} (m_1 - m_3) \ell \ddot{\phi}_2 - \frac{1}{2} m_3 \ell \ddot{\phi}_3 + \alpha T_0 \phi_1 \right], \quad (11)$$

where $\mu = 1/(m_1 + m_2 + m_3)$. Insertion of equations (10) and (11) into equation (9) yields the following set of three equations of motion:

$$\frac{1}{4} \mu m_1 (m_2 + m_3) \ell^2 \ddot{\phi}_1 + \left[c - \frac{1}{2} \mu T_0 \ell (1 - \alpha) (m_1 + 2m_2 + 2m_3) \right] \phi_1 +$$

$$+ \frac{1}{4} \mu m_1 (m_2 + 2m_3) \ell^2 \ddot{\phi}_2 - c \phi_2 + \frac{1}{4} \mu m_1 m_3 \ell^2 \ddot{\phi}_3 = 0,$$

$$\frac{1}{4} \mu m_1 \ell^2 (m_2 + 2m_3) \ddot{\phi}_1 - \left[c - \frac{1}{2} \mu \alpha (m_2 + 2m_3) T_0 \ell \right] \phi_1 +$$

$$\begin{aligned}
& + \frac{1}{4} \mu \ell^2 (m_1 m_2 + 4m_1 m_3 + m_2 m_3) \ddot{\phi}_2 + [2c - \\
& - \frac{1}{2} \mu T_0 \ell (m_2 + 2m_3)] \dot{\phi}_2 + \frac{1}{4} \mu m_3 \ell^2 (2m_1 + m_2) \ddot{\phi}_3 - c \phi_3 = 0, \\
& \frac{1}{4} \mu m_1 m_3 \ell^2 \ddot{\phi}_1 + \frac{1}{2} \mu m_3 \alpha T_0 \ell \dot{\phi}_1 + \frac{1}{4} \mu m_3 (2m_1 + m_2) \ell^2 \ddot{\phi}_2 - d \dot{\phi}_2 + \\
& + \frac{1}{4} \mu m_3 (m_1 + m_2) \ell^2 \ddot{\phi}_3 + (c - \frac{1}{2} \mu m_3 T_0 \ell) \phi_3 = 0.
\end{aligned} \tag{12}$$

It is convenient to use a dimensionless form of equations (12). For this purpose, the following definitions may be made:

$$\begin{aligned}
m_1 &= \mu_1 m, & m_2 &= m, & m_3 &= \mu_3 m, \\
t &= \sigma \tau, & \sigma^2 &= m \ell^2 / 4c (1 + \mu_1 + \mu_3), \\
Q &= q / (1 + \mu_1 + \mu_3), & q &= T_0 \ell / 2c.
\end{aligned} \tag{13}$$

It is now a straightforward process to verify that the system of equations (12) assumes the form

$$\sum_{n=1}^3 [A_{mn} \ddot{\phi}_n + (C_{mn} + Q D_{mn}) \dot{\phi}_n] = 0, \quad m = 1, 2, 3 \tag{14}$$

where now $\ddot{\phi}_n = d^2 \phi_n / d\tau^2$, and

$$\begin{aligned}
A_{11} &= \mu_1 (1 + \mu_3), & A_{12} &= \mu_1 (1 + 2\mu_3), & A_{13} &= \mu_1 \mu_3, \\
A_{21} &= \mu_1 (1 + 2\mu_3), & A_{22} &= \mu_1 + 4\mu_1 \mu_3 + \mu_3, & A_{23} &= \mu_3 (1 + 2\mu_1), \\
A_{31} &= \mu_1 \mu_3, & A_{32} &= \mu_3 (1 + 2\mu_1), & A_{33} &= \mu_3 (1 + \mu_1), \\
C_{11} &= 1, & C_{12} &= -1, & C_{13} &= 0, \\
C_{21} &= -1, & C_{22} &= 2, & C_{23} &= -1, \\
C_{31} &= 0, & C_{32} &= -1, & C_{33} &= 1,
\end{aligned}$$

$$\begin{aligned}
D_{11} &= (\alpha - 1)(2 + \mu_1 + 2\mu_3), & D_{12} &= 0, & D_{13} &= 0, \\
D_{21} &= \alpha(1 + 2\mu_3), & D_{22} &= -(1+2\mu_3), & D_{23} &= 0, \\
D_{31} &= \alpha\mu_3 & D_{32} &= 0, & D_{33} &= -\mu_3.
\end{aligned} \tag{15}$$

3. THE FREQUENCY EQUATION

If a solution of the form

$$\phi_n(\tau) = X_n e^{i\omega\tau}, \quad n = 1, 2, 3 \tag{16}$$

where the X_n 's are constants, is substituted into equation (14), then a system of homogeneous algebraic equations in the X_n 's is obtained. This system has a non-trivial solution if and only if the determinant of the coefficient matrix vanishes. Expansion of this determinant yields

$$p_0 \omega^4 - p_2 \omega^2 + p_4 = 0, \tag{17}$$

where

$$\begin{aligned}
p_0 &= 8\mu_1\mu_3(1 + \mu_1 + \mu_3) - \mu_1\mu_3 Q[3 + 4(\mu_1 + 2\mu_3) + (\mu_1 + \mu_3)(\mu_1 + 5\mu_3) - \\
&\quad - \alpha(1 + \mu_1 + \mu_3)^2],
\end{aligned}$$

$$\begin{aligned}
p_2 &= 4(\mu_1 + 4\mu_1\mu_3 + \mu_3) - Q[3\mu_1 + 11\mu_3 + 36\mu_1\mu_3 + \mu_1^2 + 13\mu_3^2 + \\
&\quad + 34\mu_1\mu_3^2 + 10\mu_1^2\mu_3 - \alpha(\mu_1 + 11\mu_3 + \mu_1^2 + 22\mu_1\mu_3 + 11\mu_3^2 + \\
&\quad + 10\mu_1^2\mu_3 + 10\mu_1\mu_3^2)] + 2\mu_3 Q^2 [1 + 3\mu_1 + 4\mu_3 + 11\mu_1\mu_3 + \\
&\quad + \mu_1^2 + 3\mu_3^2 + 7\mu_1\mu_3^2 + 3\mu_1^2\mu_3 - \alpha(1 + 2\mu_1 + 4\mu_3 + \\
&\quad + 7\mu_1\mu_3 + \mu_1^2 + 3\mu_3^2 + 3\mu_1\mu_3^2 + 3\mu_1^2\mu_3)],
\end{aligned} \tag{18}$$

$$p_4 = -Q(1 - \alpha)[\mu_3(2 + \mu_1 + 6\mu_3 + 2\mu_1\mu_3 + 4\mu_3^2)Q^2 - (2 + \mu_1 + 11\mu_3 + 4\mu_1\mu_3 + 10\mu_3^2)Q + 3 + \mu_1 + 5\mu_3] .$$

A priori, one would have expected the frequency equation (17) to be a bi-cubic polynomial in ω^2 . However, one can easily verify that the inertia matrix $A = (A_{mn})$ is singular which, therefore, accounts for the fact that the frequency equation is a biquadratic. Consequently, the system under consideration is a quasi -dynamic system [10], which is characterized by the existence of an internal constraint. To determine this constraint, one merely adds the equations for $m = 1$ and 3 obtained from equation (14) and then subtracts the equation for $m = 2$ from the result to obtain

$$[2 - Q\alpha(1 + \mu_3) - Q(1 - \alpha)(2 + \mu_1 + 2\mu_3)]\phi_1 - [4 - Q(1 + 2\mu_3)]\phi_2 + (2 - \mu_3Q)\phi_3 = 0 . \quad (19)$$

It is obvious from equation (18) that $p_4 = 0$ when $Q = 0$. Thus, the frequency equation (14) leads to $\omega^2 = 0$, p_2^*/p_0^* , where

$$p_0^* = 8\mu_1\mu_3(1 + \mu_1 + \mu_3), \quad p_2^* = 4(\mu_1 + 4\mu_1\mu_3 + \mu_3) .$$

The value $\omega^2 = 0$ corresponds to a rigid body rotation of the system.

4. STABILITY CONSIDERATIONS

If Q has an arbitrarily small positive value, then $p_4 \neq 0$ and $\omega^2 = 0$ is no longer a root of the frequency equation (17). Thus, one must conclude that the system can no longer undergo a rigid body rotation. As the value of Q is increased above $Q = 0$, an eigencurve

emanating from the origin in the $Q\omega^2$ -plane must enter either the first or second quadrant, which implies that the system is either stable or unstable, respectively. Clearly, if one can establish a relationship between Q and ω^2 that is invalid in a neighborhood of the origin in the $Q\omega^2$ -plane, then some conclusions regarding the stability of the system can be drawn.

To accomplish this for small thrusts Q , the frequency parameter ω^2 is expanded as a power series in the perturbation parameter Q as follows:

$$\omega^2 = Q\alpha_1 + Q^2\alpha_2 + (Q^3), \quad (20)$$

where the coefficients $\alpha_1, \alpha_2, \dots$ are to be determined. Substituting equation (20) into the frequency equation (17) and following the steps of the classical perturbation process, one finds

$$\omega^2 = \frac{-Q(1 - \alpha)(3 + \mu_1 + 5\mu_3)}{4(\mu_1 + 4\mu_1\mu_3 + \mu_3)} + (Q^2), \quad (21)$$

provided that $0 \leq Q \ll 1$. From this equation it is evident that the sign of ω^2 is determined solely by the magnitude of the tangency coefficient α . Specifically, if $\alpha < 1$, then $\omega^2 < 0$ and the system is divergent for an arbitrarily small positive thrust. Hence, the value of the critical thrust of divergence is $Q_b = 0$ when $\alpha < 1$. If $\alpha > 1$ (a super-tangential thrust), then $\omega^2 > 0$ and the system is stable for sufficiently small, positive thrusts.

If the thrust is a tangential force ($\alpha = 1$), then, provided that $Q > 0$, p_4 again vanishes, as is evident from equation (18). In this

case, the frequency equation yields

$$\omega^2 = 0, \quad p_2/p_0, \quad (22)$$

where

$$\begin{aligned} p_0 &= 2\mu_1\mu_3[4(1 + \mu_1 + \mu_3) - Q(1 + \mu_1 + 3\mu_3 + 2\mu_1\mu_3 + 2\mu_3^2)], \\ p_2 &= 4(\mu_1 + 4\mu_1\mu_3 + \mu_3) - 2Q[\mu_1(1 + 7\mu_3) + \mu_3^2(1 + 12\mu_1)] + \\ &\quad + 2\mu_1\mu_3(1 + 2\mu_3)^2Q^2. \end{aligned} \quad (23)$$

A change in the sign of ω^2 , as given in equation (22), will be signalled by the vanishing of either p_2 or p_0 . Specifically, if $p_2 = 0$, then, according to equation (23) one has

$$\begin{aligned} \mu_1\mu_3(1 + 2\mu_3)^2Q_b^2 - [\mu_1(1 + 7\mu_3) + \mu_3^2(1 + 12\mu_1)Q_b + \\ + 2(\mu_1 + 4\mu_1\mu_3 + \mu_3)] = 0, \end{aligned} \quad (24)$$

whereas, if $p_0 = 0$ the critical thrust for divergence is given by

$$Q_b = \frac{4(1 + \mu_1 + \mu_3)}{1 + \mu_1 + 3\mu_3 + 2\mu_1\mu_3 + 2\mu_3^2}. \quad (25)$$

It must be noted that p_4 can vanish whenever

$$\begin{aligned} \mu_3(2 + \mu_1 + 6\mu_3 + 2\mu_1\mu_3 + 4\mu_3^2)Q_b^2 - (2 + \mu_1 + 11\mu_3 + 4\mu_1\mu_3 + \\ + 10\mu_3^2)Q_b + 3 + \mu_1 + 5\mu_3 = 0, \end{aligned} \quad (26)$$

which also provides a critical thrust for divergence. The values of Q_b obtained from equation (26) are completely independent of the tangency coefficient α .

The condition for the determination of the critical thrust for flutter, namely that of coalescence of a pair of roots of the frequency equation (17), is $p_2^2 - 4p_0 p_4 = 0$ which, by virtue of equation (18), can be expressed as the following quartic polynomial in Q :

$$\sum_{n=1}^5 r_n Q^{n-1} = 0, \quad (27)$$

where the r_n 's are lengthy polynomials in μ_1, μ_3 , and α which will not be recorded here.

5. SOME EIGENCURVES

To identify regions of stability, flutter, and divergence in the stability maps that will be presented later, it is first efficacious to solve the frequency equation (17) and then to examine some typical eigencurves, such as those shown in Figures 2 to 4. These figures were prepared for three values of the tangency coefficient: $\alpha = 3/10, 4/5$, and $3/2$, in the case of $\mu_1 = \mu_3 = 1$. In agreement with the conclusion drawn from equation (21), one sees from Figures 2 and 3 that the branch of the eigencurve emanating from the point $(\omega^2, Q) = (0, 0)$ extends immediately into the second quadrant. Thus, the system is unstable by divergence for arbitrarily small, positive values of Q . Indeed, in the case of $\alpha = 3/10$, the value of the critical thrust of divergence must be $Q_b = 0$. In the case of $\alpha = 4/5$ (Figure 3), one has $\omega^2 = 0$ at $Q = 0$, $\omega^2 < 0$ for $0 < Q < 0.4127$, and $\omega^2 > 0$ for $0.4127 < Q < 0.506$. The bounds in these inequalities are obtained from equation (26). Consequently, one may conclude that the system diverges whenever the

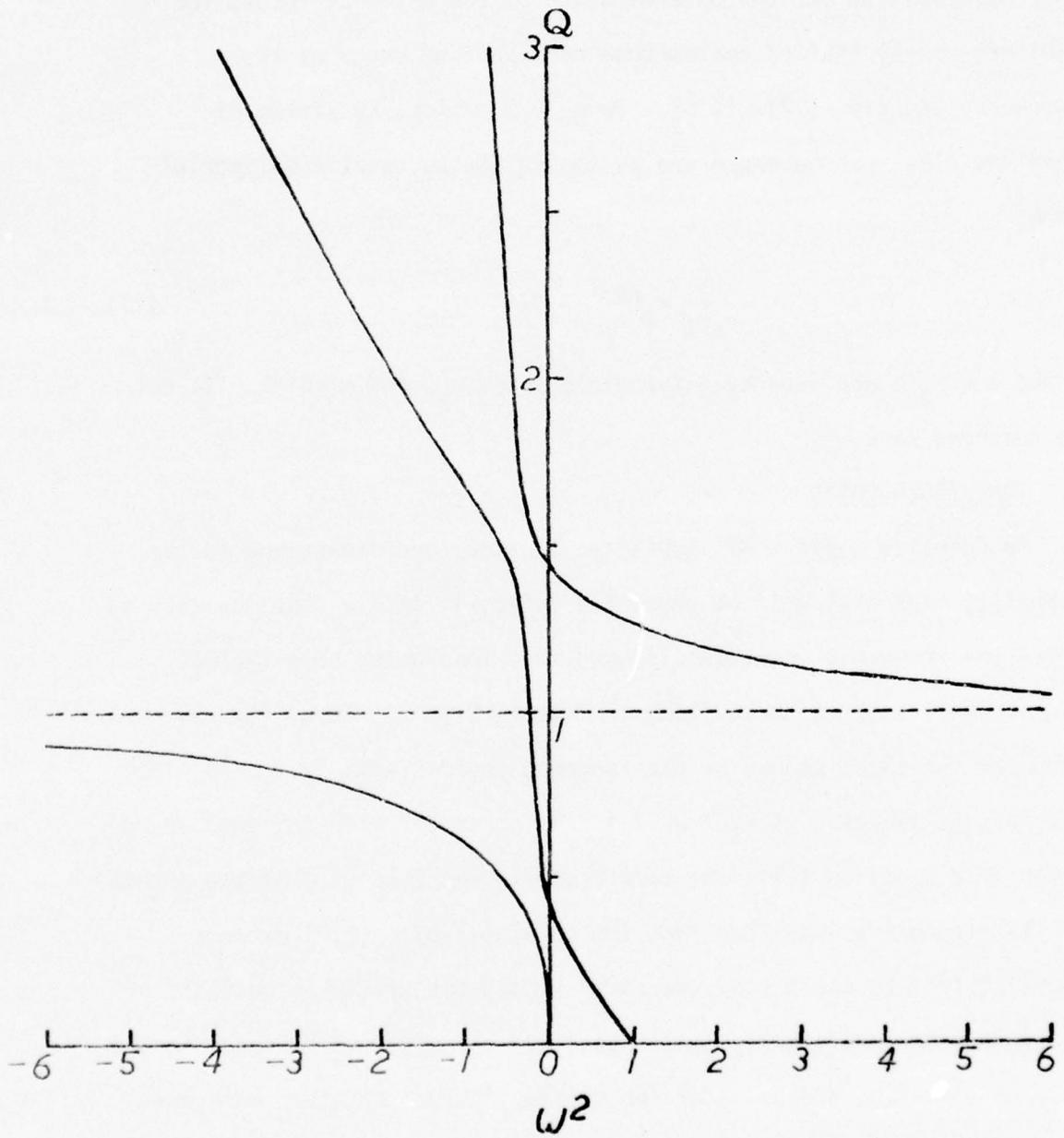


Figure 2. Eigencurve for $\alpha = 3/10, \mu_1 = \mu_3 = 1$.

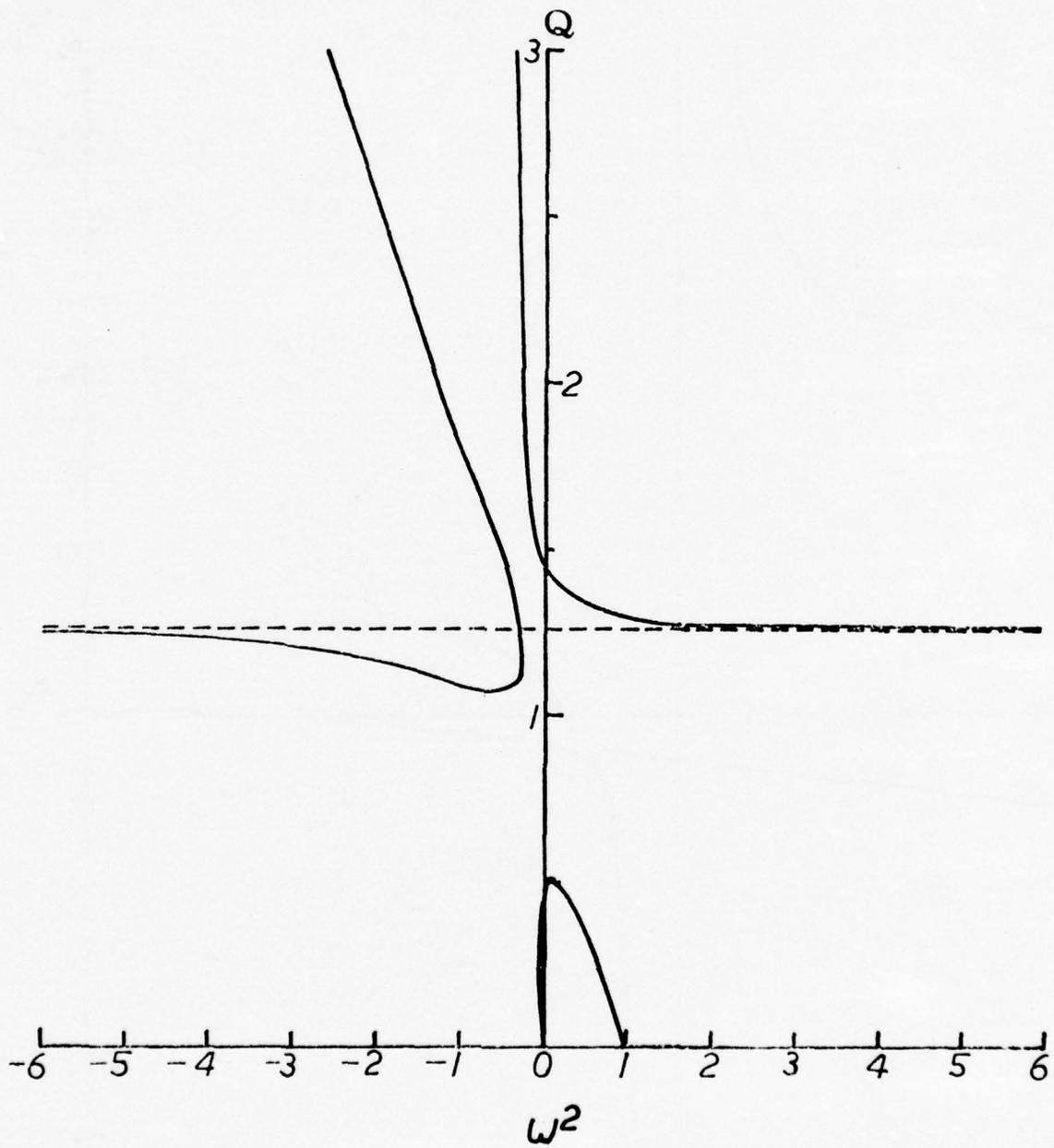


Figure 3. Eigencurve for $\alpha = 4/5$, $\mu_1 = \mu_3 = 1$.

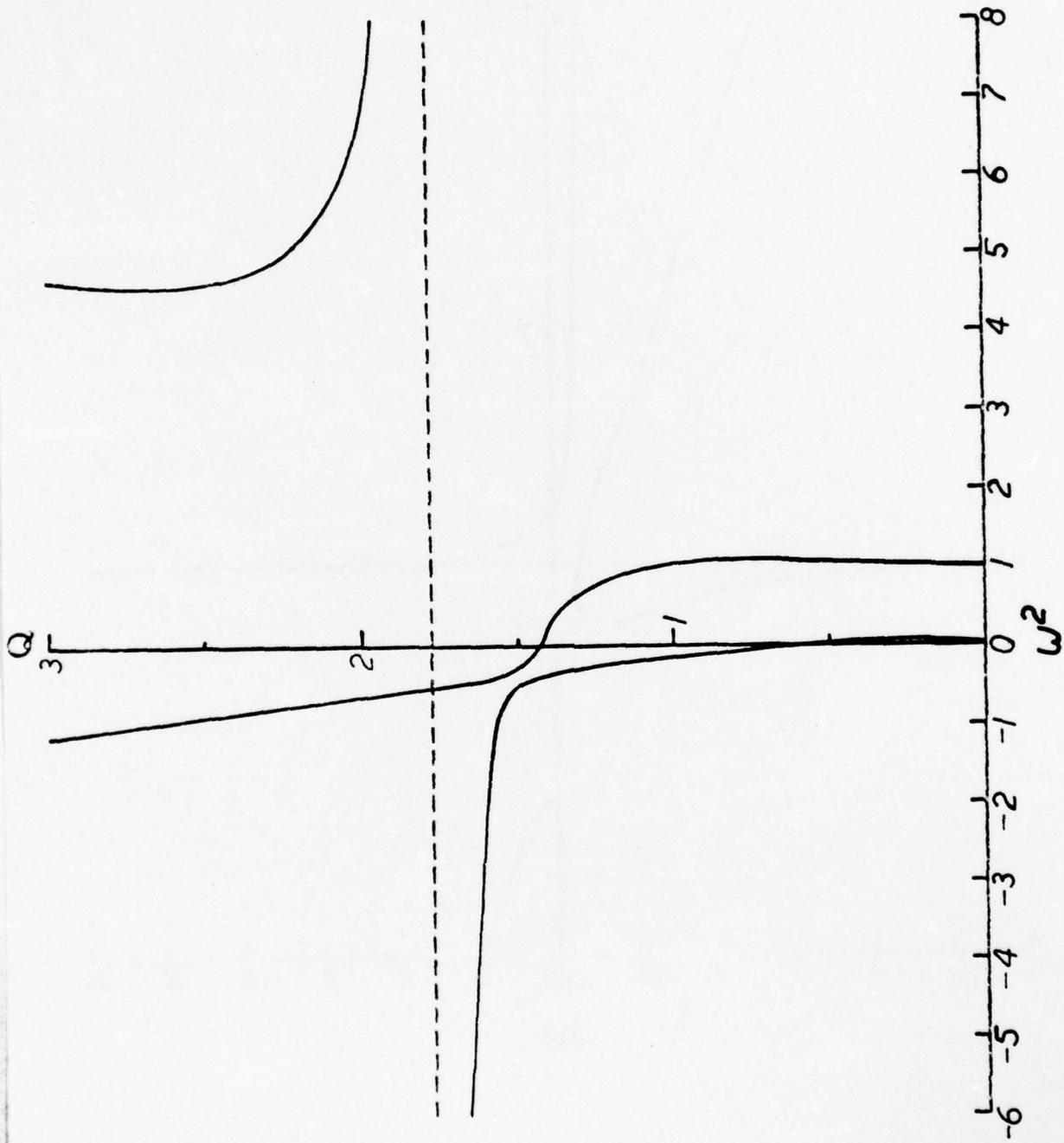


Figure 4. Eigencurve for $\alpha = 3/2$, $\mu_1 = \mu_3 = 1$.

thrust parameter Q is in the interval $0 < Q < 0.4127$ and that it is stable when Q is in the interval $0.4127 < Q < 0.506$. At $Q = 0.506$, the frequencies of the first and second modes merge, which signals the onset of instability by flutter. For $Q > 1.2$, flutter gives way to divergence.

In Figure 4 plotted for $\alpha = 3/2$, the branch of the eigencurve initiating at the origin enters the first quadrant, reaches a maximum value of ω^2 , and then decreases until it intersects the load axis at $Q_b = 0.4127$.

The dashed horizontal lines appearing in Figures 2 to 4 are horizontal asymptotes which represent the value of Q at which ω^2 becomes infinite. This value of Q , hereafter designated as Q_a , is determined from the condition $p_0 = 0$, which, by virtue of equation (18), leads to

$$Q_a = \frac{8(1 + \mu_1 + \mu_3)}{3 + 4(\mu_1 + 2\mu_3) + (\mu_1 + \mu_3)(\mu_1 + 5\mu_3) - \alpha(1 + \mu_1 + \mu_3)^2} .$$

6. STABILITY MAPS

Having in mind the observations made in the preceding section, one now can identify the regions of stability, divergence, and flutter in $q\alpha$ -plane.* In Figures 5 to 7, stability maps are shown in the $q\alpha$ -plane

*The thrust parameter q was defined in equation (13). It is more desirable to consider the $q\alpha$ -plane rather than the $Q\alpha$ -plane since the quantity Q tends to mask the influence of the mass distribution of the system on the value of the critical thrust.

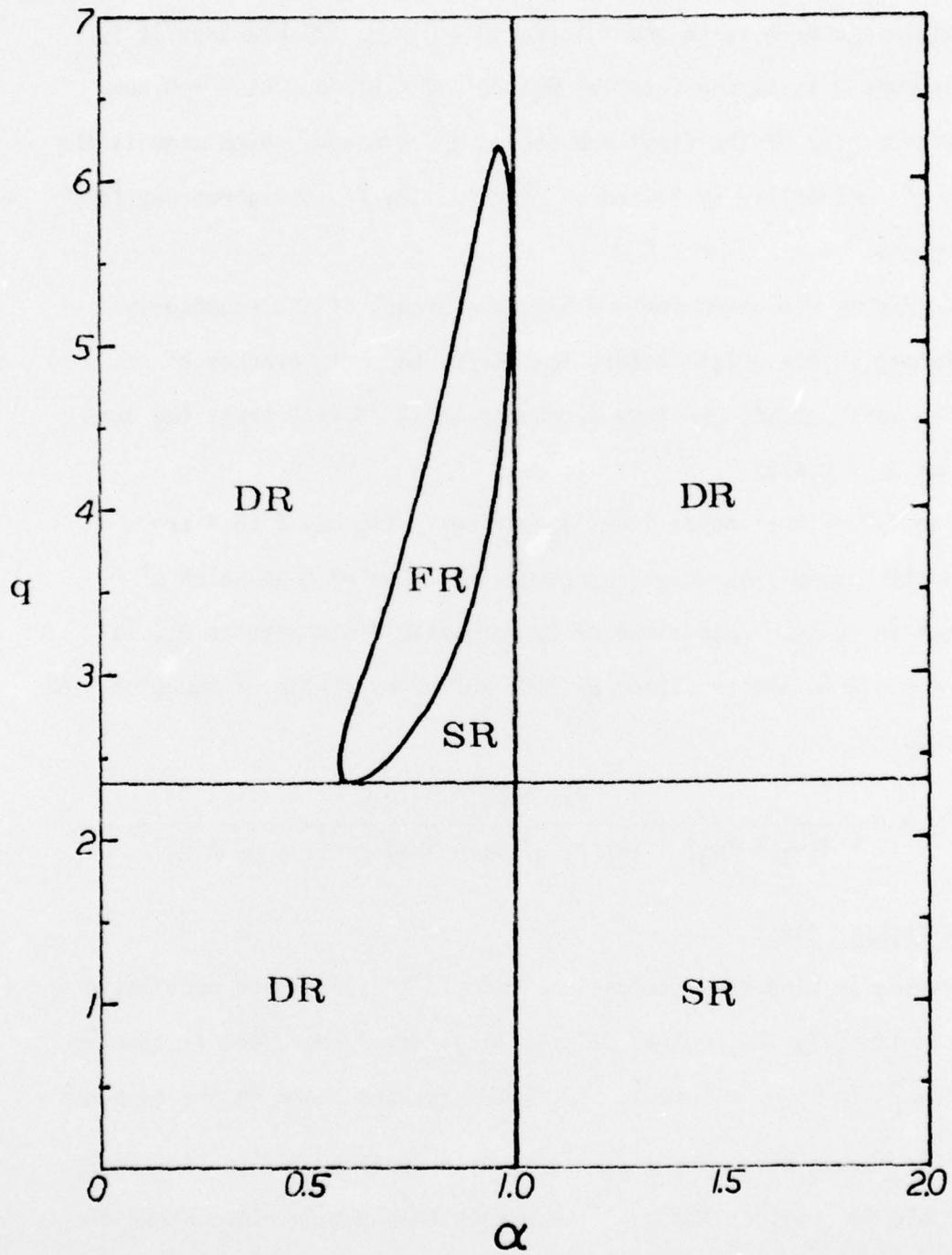


Figure 5. Stability map for $\mu_1 = 5$, $\mu_3 = 1$.

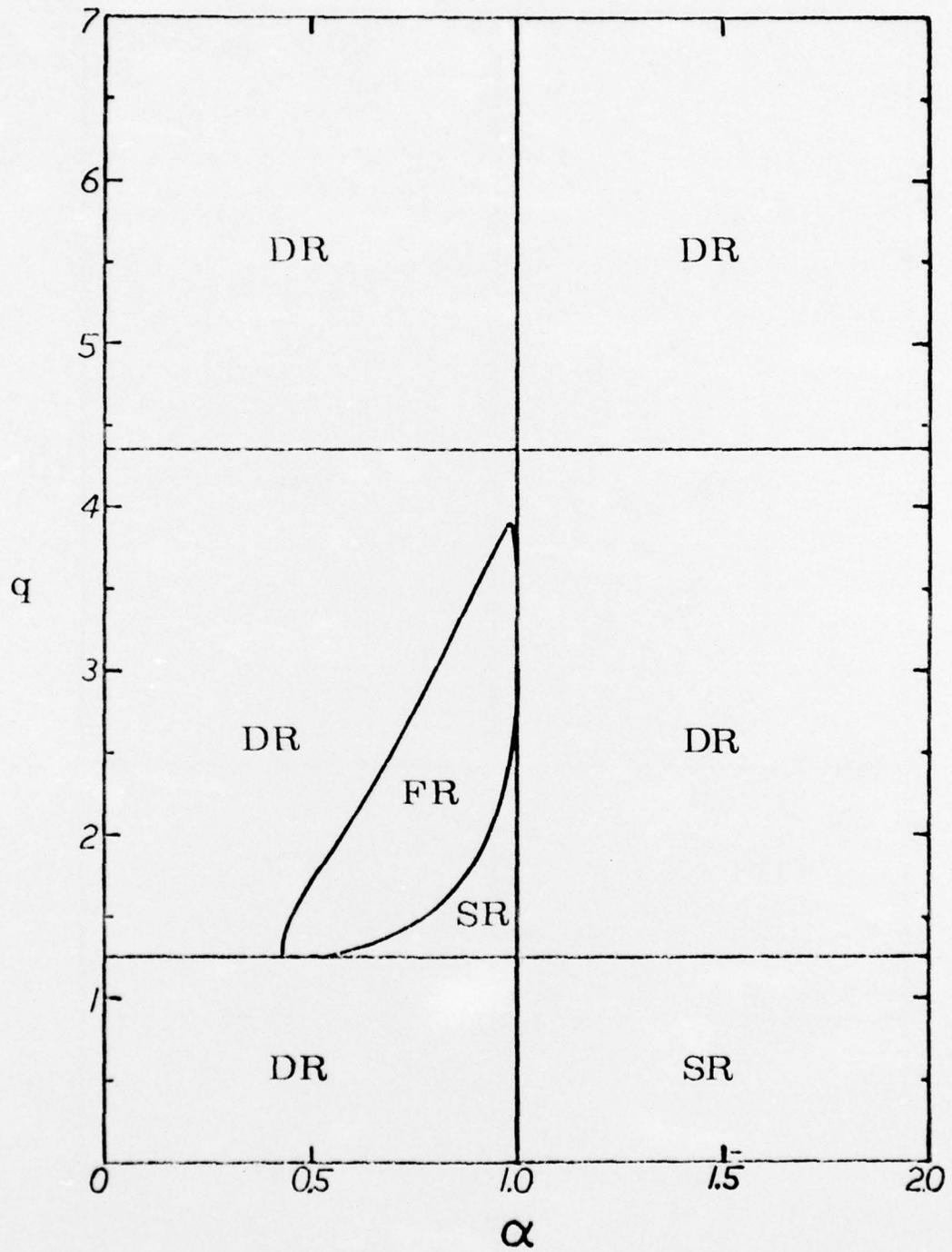


Figure 6. Stability map for $\mu_1 = \mu_3 = 1$.

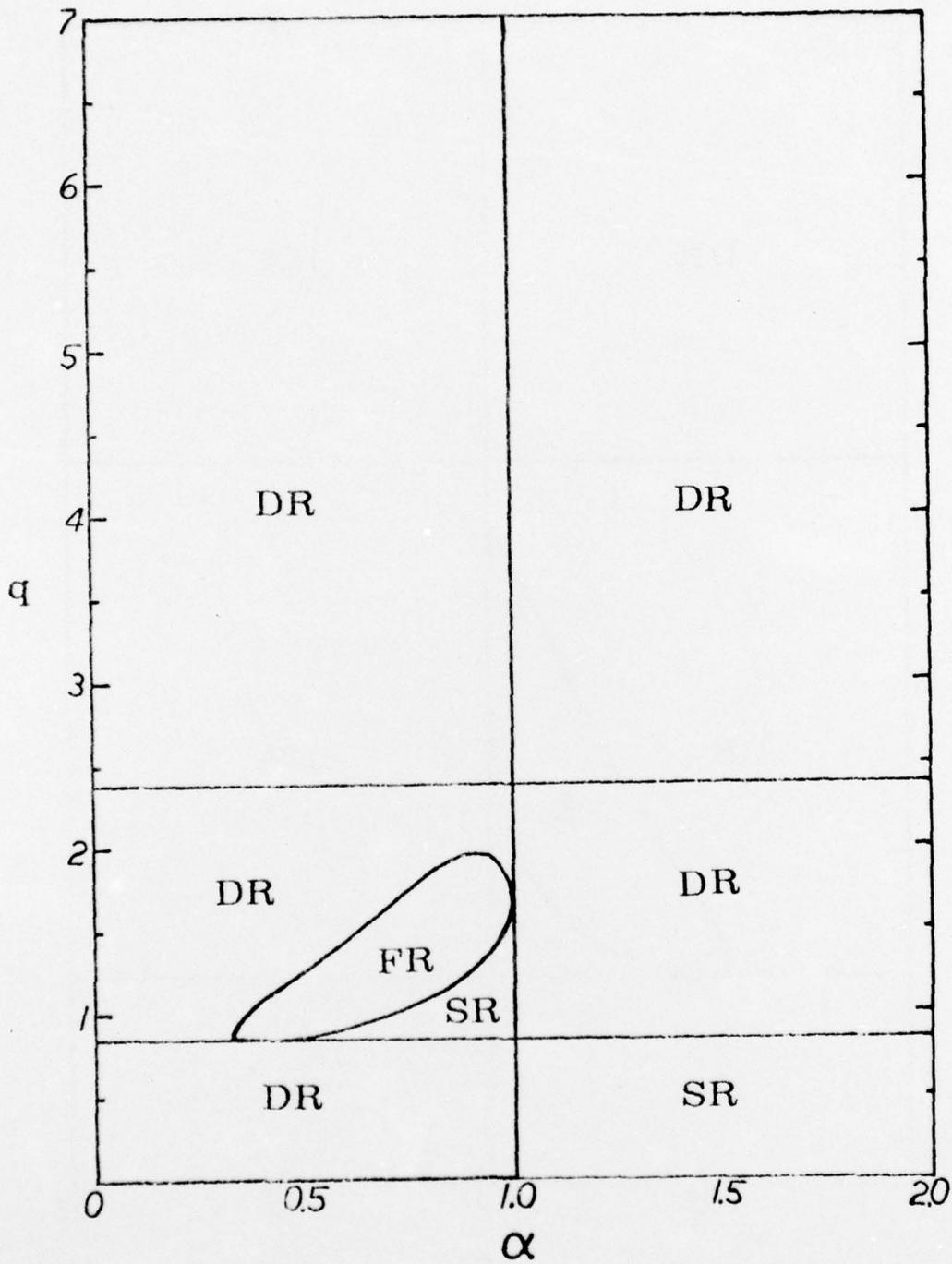


Figure 7. Stability map for $\mu_1 = 1, \mu_3 = 5$.

over the range $0 \leq \alpha \leq 2$ for (a) $\mu_1 = 5, \mu_3 = 1$, (b) $\mu_1 = \mu_3 = 1$, and (c) $\mu_1 = 1, \mu_3 = 5$, respectively. The regions of stability, flutter, and divergence are labelled with the symbols SR, FR, and DR.

It should be noted that, for $\alpha < 1$, the minimum critical thrust of divergence is $q_b = 0$, whereas, for $\alpha > 1$, its value is positive and is associated with the smallest of the two roots of equation (26). For anti-tangential ($\alpha < 0$) and super-tangential ($\alpha > 1$) thrusts, flutter does not occur.

From Figures 5 to 7, it is clear that there exists a value of α , say α_* , at which $q_b = q_e$. The value of α_* may be computed from the requirement that equation (27) and equation (26), written now in the form

$$b_1 + b_2 Q_b + b_3 Q_b^2 = 0,$$

where

$$b_1 = 3 + \mu_1 + 5\mu_3, \quad b_2 = -(2 + \mu_1 + 11\mu_3 + 4\mu_1\mu_3 + 10\mu_3^2),$$

$$b_3 = \mu_3(2 + \mu_1 + 6\mu_3 + 2\mu_1\mu_3 + 4\mu_3^2),$$

possess a common root in the thrust parameter. This requirement leads, according to Sylvester's dialytic method of elimination [11], to the condition

$$\begin{vmatrix}
 r_5 & r_4 & r_3 & r_2 & r_1 & 0 \\
 0 & r_5 & r_4 & r_3 & r_2 & r_1 \\
 b_3 & b_2 & b_1 & 0 & 0 & 0 \\
 0 & b_3 & b_2 & b_1 & 0 & 0 \\
 0 & 0 & b_3 & b_2 & b_1 & 0 \\
 0 & 0 & 0 & b_3 & b_2 & b_1
 \end{vmatrix} = 0 .$$

Expansion of this determinant produces a polynomial in α from which the value of α_* may be determined numerically for given values of μ_1 and μ_3 .

Consequently, one may now observe that for a sub-tangential thrust ($\alpha < 1$) for which $0 < \alpha < \alpha_*$, flutter cannot occur and the minimum critical thrust of divergence is $q_b = 0$. However, for a sub-tangential thrust for which $\alpha_* < \alpha < 1$, flutter can occur (e.g., see Figure 3). For α in this latter range, the system is first unstable by divergence for $0 < q < q_{b_1}$, where $q_{b_1} = Q_{b_1} (1 + \mu_1 + \mu_3)$, with Q_{b_1} denoting the smallest root of equation (26), and it becomes stable whenever $q_{b_1} < q < q_{e_1}$, where $q_{e_1} = Q_{e_1} (1 + \mu_1 + \mu_3)$, with Q_{e_1} denoting the smallest real root of equation (27). For $q_{e_1} < q < q_{e_2}$, the system is prone to flutter, and for $q_{e_2} < q$ it is divergent once again.

The stability maps in Figures 5 to 7 indicate that a stable flight at relatively low values of the thrust parameter q can be achieved provided that the direction of the thrust is controlled in such a way that $\alpha > 1$. The critical super-tangential thrust is, of course, q_{b_1} .

Suppose next that the value of α is only slightly greater than unity. If, as the value of q is increased to q_{b_1} , the value of α is changed to a value very slightly less than unity, then a stable motion with thrusts greater than q_{b_1} would be possible. In this second region of stability, the maximum permissible sub-tangential thrust would be $q < q_{e_1} < q_{b_1}^*$, where the values of q_{e_1} and $q_{b_1}^*$ can be obtained from equations (27) and (24), respectively. For α only slightly less than unity, $q_{e_1} \approx q_{b_1}^*$. For example, with $\mu_1 = \mu_3 = 1$, one finds $q_{b_1} = 1.2381$ and $q_{b_1}^* = 3$. Thus, a significant increase in the admissible magnitude of the thrust appears possible, provided, certainly, that the control system can accomplish the necessary reduction in the value of α in a period of time that is sufficiently brief so as to avoid large angular changes relative to the equilibrium configuration of the system.

Finally, it is of interest to consider the influence of the distribution of mass of the system on the value of the critical super-tangential thrust. To do this, one may first presume that the total mass of the system is assigned some value. Then, in terms of the dimensionless mass parameters, one may write $1 + \mu_1 + \mu_3 = \beta + 1$, where β is assigned a fixed value. Therefore, $\mu_1 = \beta - \mu_3$, where $0 \leq \mu_3 \leq \beta$. Substitution of $\mu_1 = \beta - \mu_3$ into equation (26) and introduction of $Q_b = q_b / (\beta + 1)$ lead to

$$\begin{aligned} \mu_3[2 + \beta + (5 + 2\beta)\mu_3 + 2\mu_3^2]q_b^2 - (\beta + 1)[2 + \beta + 2(5 + 2\beta)\mu_3 \\ + 6\mu_3^2]q_b + (\beta + 1)^2(3 + \beta + 4\mu_3) = 0 . \end{aligned} \quad (28)$$

In Figure 8, the variation of q_{b_1} , i.e., the lowest root of equation (28), versus μ_3 for $\beta = 1, 2, \dots, 5$ has been plotted. All the curves show essentially the same behavior, namely, the value of q_b decreases monotonically as μ_3 increases, with the difference $\Delta = q_b(\beta') - q_b(\beta'')$, where $\beta' < \beta''$ and μ_3 held fixed, increasing as the difference $\beta'' - \beta'$ is increased. Consequently, significantly greater critical thrusts result when the tail portion of the system is more massive than the head portion. The lowest critical thrusts are obtained when the head is more massive than the tail.

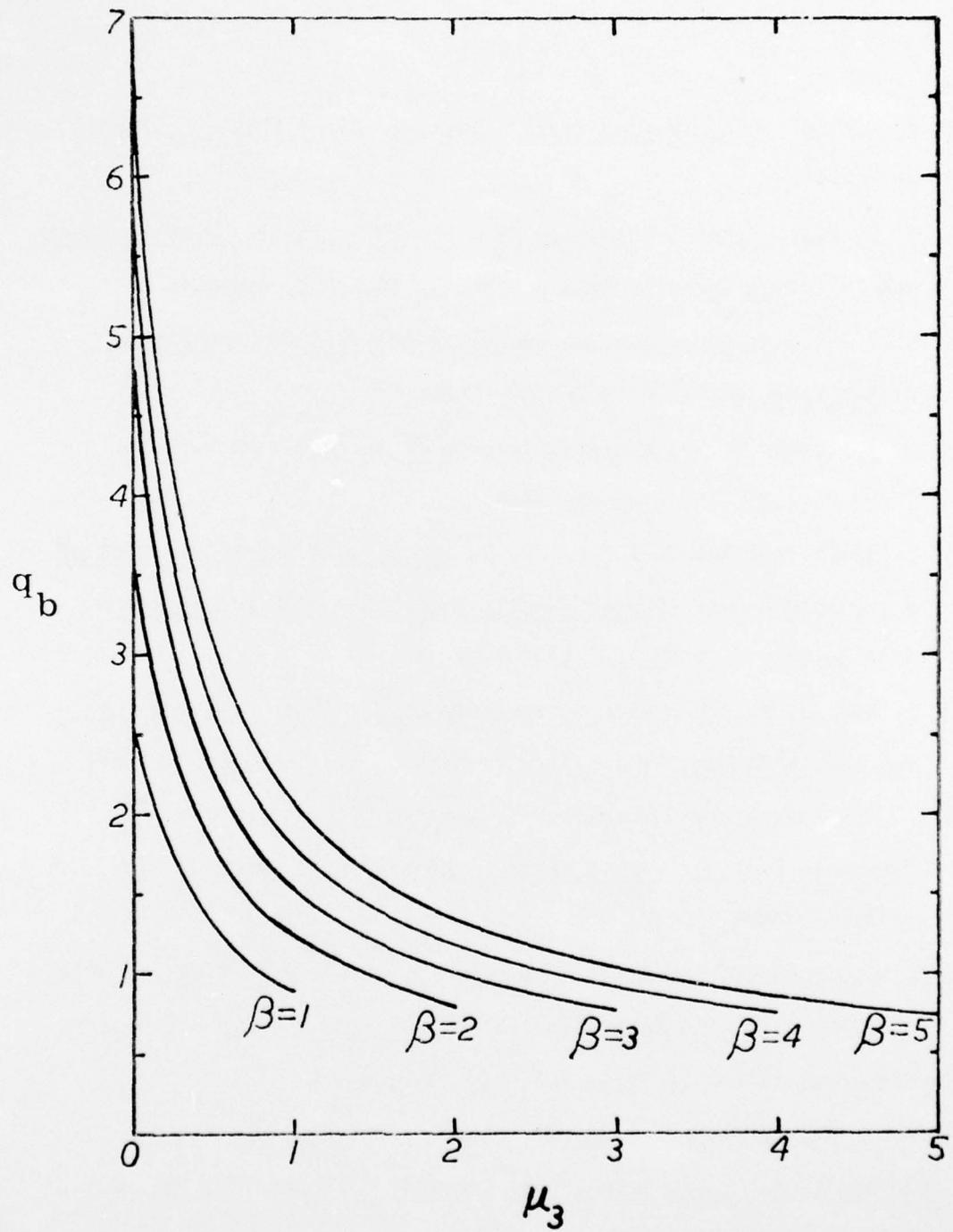


Figure 8. Variation of q_b with μ_3 for five values of β .

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