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March 1977

WALSH TRANSFORM PULSE DETECTION IN HIGH NOISE ENVIRONMENTS

Ronald Okupski

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ROME AIR DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
GRIFFISS AIR FORCE BASE, NEW YORK 13441

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The Walsh Hadamard Transformation is shown through computer generated graphics to be very useful for detecting rectangularly shaped information carrying pulses that exist in high noise environments. This transformation is used to process the time domain input signal and determine if one or more pulses are present in the noise. A Fast Walsh Transform (FWT) algorithm is used to generate the sequency domain coefficients. It is shown that by low pass filtering of the Walsh Spectrum coefficients, the pulse can be detected since the higher order coefficient contribute only to the noise.

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A combination of low pass filtering and amplitude thresholding of the Walsh spectrum is shown to provide an optimal signal detection capability.

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PREFACE

The author of this report is grateful to the following project engineers at RADC (IRAP), Griffiss AFB NY 13441 for their advice and assistance in generating this report: David Clark, Albert Proctor, Walter Czyzycki, and Bruno Beek.

Several government agencies (Naval Electronic Laboratory Center) and private corporations (American Electronic Laboratories) are known to be using Walsh Transform processing. However, up until now, very little information has been published describing the technology involved.

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SECTION I

Introduction

In the area of digital signal processing, the representation of time domain data by the superposition of simple functions is an important analysis technique. This is often done in the frequency domain through the use of the Fourier transform. Once the transform of the input data is obtained, the Fourier coefficients which contribute to the information part of the signal can be retained, while those that contribute only to the noise can be dropped.

In the Fourier transform, sine and cosine terms for the set of basis functions which are used to selectively reconstruct the information contained in the input data. Most naturally occurring disturbances are sinusoidal in nature, example: all forms of electromagnetic radiation, human speech, seismic events, etc., and thus the Fourier transform has proven to be useful in the analysis of these signals. Many forms of information collection and transmission systems built by man, however, do not consist of sinusoidally varying values, but of rectangularly shaped on-off pulses. Some examples are digital communications channels, radar pulse envelopes, and telemetry data. The analysis of these signals through the utilization of a transform that is based on sine and cosine functions can be cumbersome.

Walsh functions form a complete orthonormal set of rectangular waves. The first eight are shown in figure one.[4] There are several ways to generate (mathematically) a Walsh function series. Each has certain advantages:

1. By means of a difference equation
2. From products of Rademacher functions
3. Through the Hadamard matrix
4. By the use of Boolean Synthesis.

The derivation of Walsh functions is not the subject of this study and more information in this area can be obtained from reference 1 at the end of this report.

Walsh functions take on two possible values, +1 or -1. The Walsh transform can be used to process time domain signals of any form, but are particularly useful for signals which are rectangularly shaped.

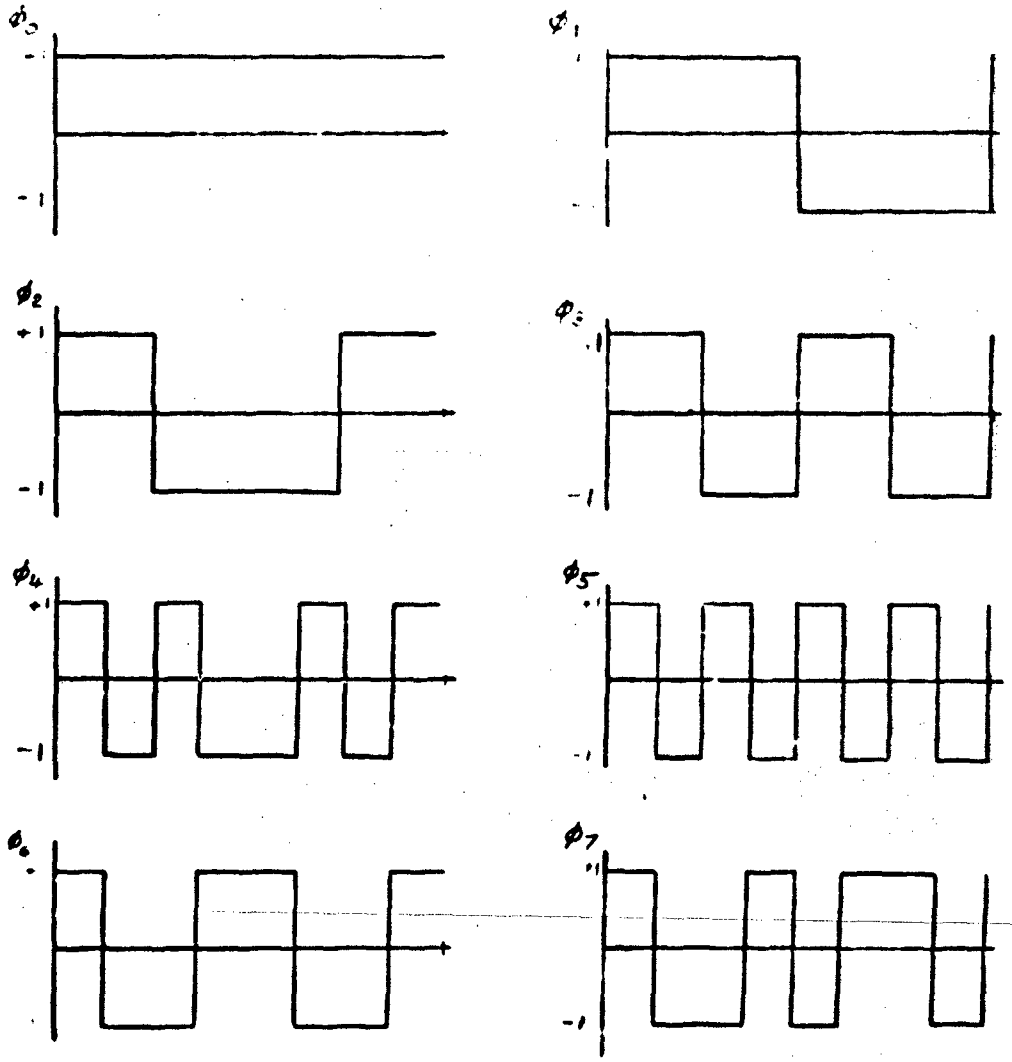


FIGURE 1 First 8 Walsh Functions

SECTION II

Comparison of Walsh and Fourier Spectrum Processing

The decision of whether to use Fourier or Walsh domain processing techniques is dependent upon the characteristics of the signal. A smoothly varying continuous signal favors the Fourier transform whereas a rectangular waveform is reconstructed with fewer terms by using the Walsh transform. (See figure 2)

The Fourier transform exhibits the Gibb's phenomena (divergence at the zero crossing) and this necessitates the use of a large number of high frequency terms in the Fourier transformation to retain the square corner characteristics of a pulse. The Walsh transform is uniquely adapted to pulsed signals and does not exhibit the Gibb's phenomena.

The Walsh transformation is a real one, while the Fourier transformation is complex. In digital signal processing, the Fast Fourier Transform (FFT) requires a total of $N \, LOG_2 N$ complex multiplications and additions. The Fast Walsh Transform (FWT) requires $N \, LOG_2 N$ real additions or subtractions. This makes the FWT much faster than the FFT. Also, sine and cosine functions cannot be represented exactly by a finite number of bits; thus a source of truncation noise is introduced by the discrete Fourier transform which involves repeated multiplication by a complex number. The FWT involves only additions and subtractions of real numbers and precise representation are possible. Thus, the FWT is not a source of noise.

The amplitude coefficients of the discrete Fourier transform are not affected by the phase of the input signal. This causes a given signal to have the same complex amplitude spectral coefficients regardless of its position

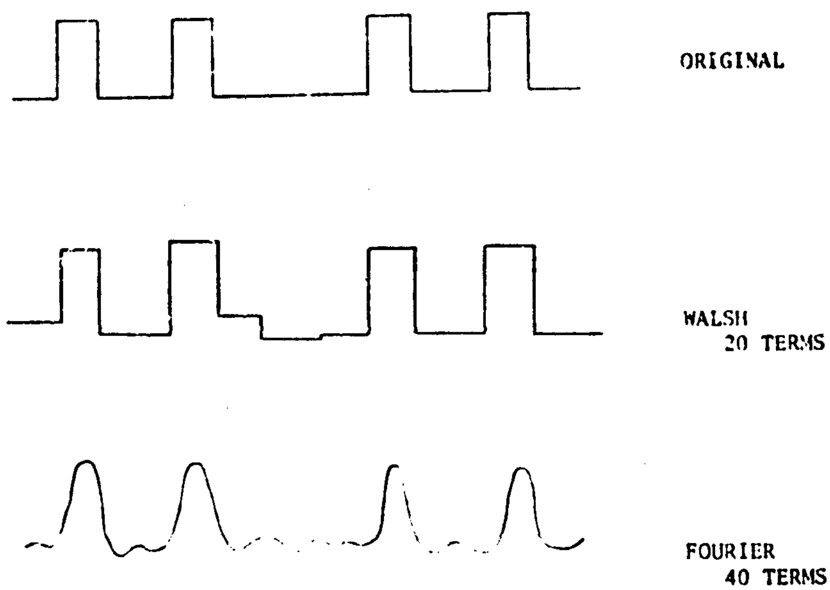
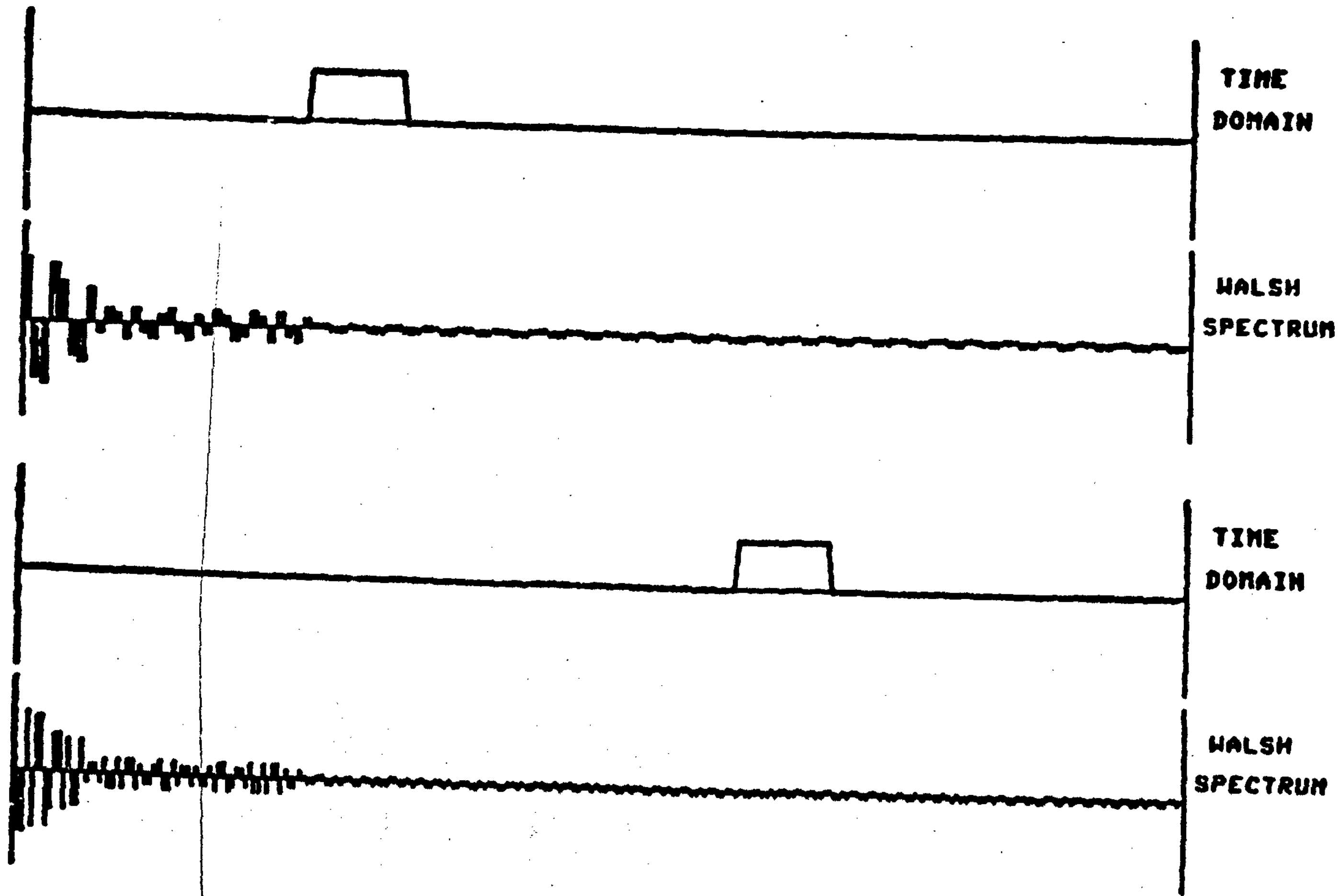


FIGURE 2 Synthesis of Pulsed Signals

within the sampling window. The Walsh transform will exhibit a different spectrum as the signal is shifted within the window. (See figure 3)

FIGURE 3 Effect of Phase Shift on Walsh Transform



SECTION III

Data 1 Description

In each of the data plots that follow a 256 point window is used. Present in this window are one or more pulsed signals plus added Gaussian noise. Pulses of different widths and signal-to-noise ratios are examined.

Data Table 1 lists all the parameters which pertain to data plots 1-1 through 1-13. Only low pass filtering of the Walsh spectrum is performed.

Line 1 of each data page shows the input time domain pulse plus added Gaussian noise.

Line 2 is the Walsh transform spectrum of the line 1 input data.

Line 3 shows the coefficients which were used to reconstruct the time domain input data. This is, in effect, sequency domain low pass filtering. Only the terms that are plotted in this line are used in the reconstruction; all others are set equal to zero.

Line 4 is the reconstructed time domain data. It was obtained by taking the inverse FWT of line 3.

Line 5 is the time domain amplitude threshold of line 4. All data below the given threshold value are not output or plotted.

Clarification of the Data Table 1 entries is given:

START OF PULSE - This gives the number of the sample in the 256 point window where the pulse begins.

END OF PULSE - This is the number of the sample where the pulse ends. This information is given because it will not always be obvious in low signal to noise ratio data. The pulse position is also outlined by dashed lines.

SEQUENCY LOW PASS # OF TERMS - The Walsh coefficients are plotted in ascending sequency order. The number of low order terms used when the inverse FWT is calculated is given here. All other coefficients in the array are set equal to zero. This is the line 3 data.

INPUT DATA S/N (db) - The signal to noise ratio of the line 1 input data is given here. There are many methods of calculating S/N ratios. Here the input signal is assumed to be a voltage and the formula used is:

$$S/N(db) = 20 \text{ LOG } \frac{\text{(average value of signal)}}{\text{(average value of noise)}}$$

TIME THRESHOLD - The last line (line 5) of the data plots shows how all of the reconstructed data below the listed value of amplitude threshold in the time domain is truncated. Only the samples which have values greater than this threshold value are plotted on the screen. This cleans up the pulse further.

PULSE WIDTH ERROR LEADING EDGE - A low S/N ratio pulse will often exhibit some error as to the exact point where the pulse begins. The number of samples that the leading edge of the reconstructed pulse is in error as compared to its position in the line 1 input data is given here.

PULSE WIDTH ERROR TRAILING EDGE - The number of samples that the reconstructed pulse's trailing edge is in error as compared to line 1 is given here.

DATA TABLE 1

Data Number	Start of Pulse	End of Pulse	Sequency Low Pass # of Terms	Input Data S/N db	Time Threshold	PW Error Leading edge (#of samples)	PW Error Trailing Edge (#of samples)
1-1	64	128	30	11	.3	0	0
1-2	64	128	8	11	.3	0	0
1-3	20	50	14	11	.3	5	0
1-4	100 190	130 205	20	11	.3	8 4	5 2
1-5	200	240	20	4	.2	3	0
1-6	100	130	8	4	.1	8	8
1-7	100	128	16	0	.15	7	0
1-8	10	40	14	-1	.15	6	8
1-9	160	200	10	-2	.14	0	8
1-10	180	200	10	-2	.15	30	10
1-11	200	250	16	-4	.1	10	10
1-12	64 84 104	74 94 114	20	2	.2	*	*
1-13	64 84 104	74 94 114	30	2	.2	0 5 0	3 1 1

*See Notes on Data 1

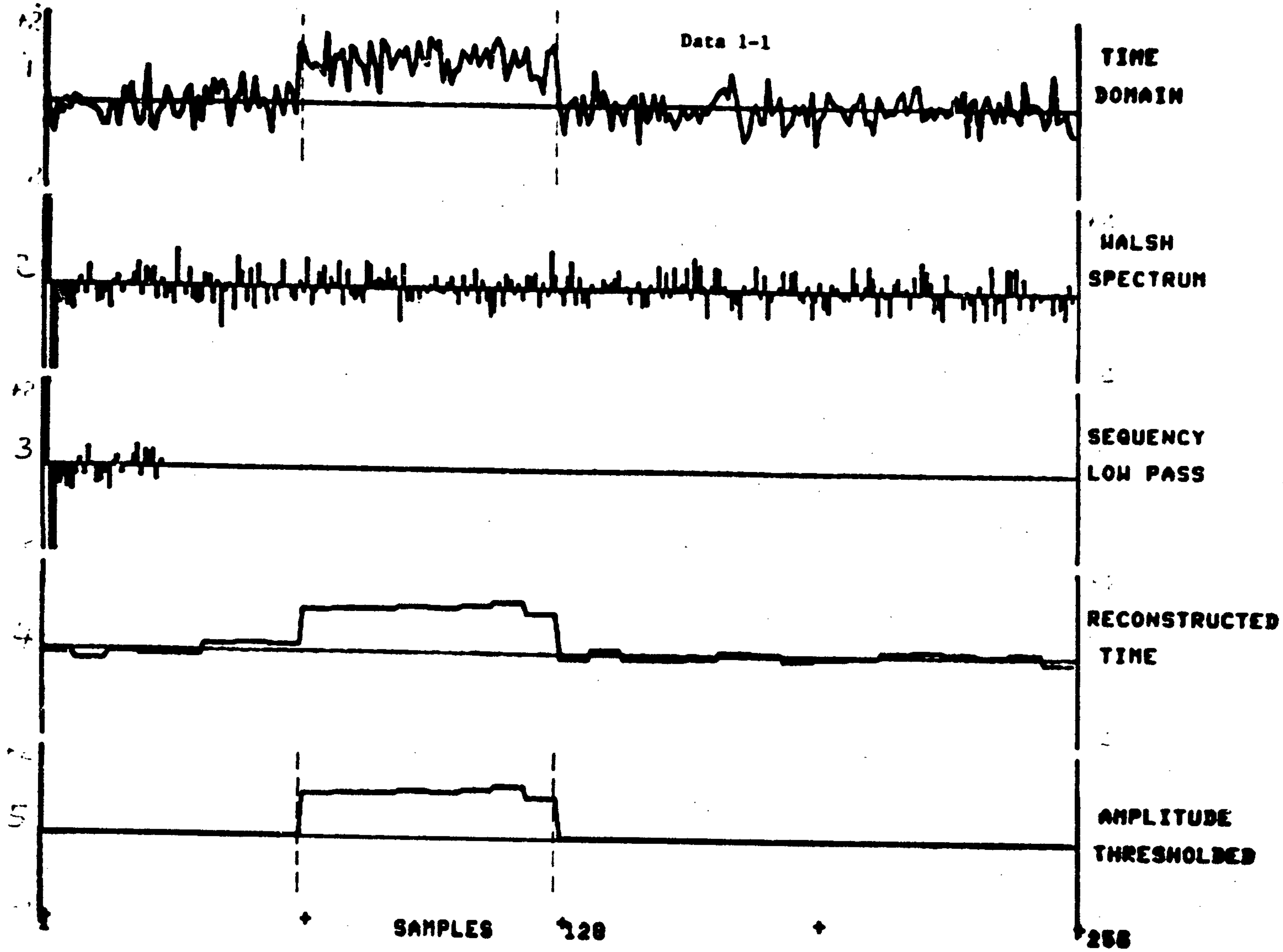
Notes on Data Table 1

Data 1-1 and 1-2 - These plots have identical input data, but 1-1 processes 30 Walsh coefficients in order to obtain the reconstructed time domain pulse while 1-2 processes only 8. In this case, 8 terms reconstruct the pulse with the same accuracy of pulse width and amplitude as do 30 terms. The first 4 sequency terms dominate because of their large amplitudes.

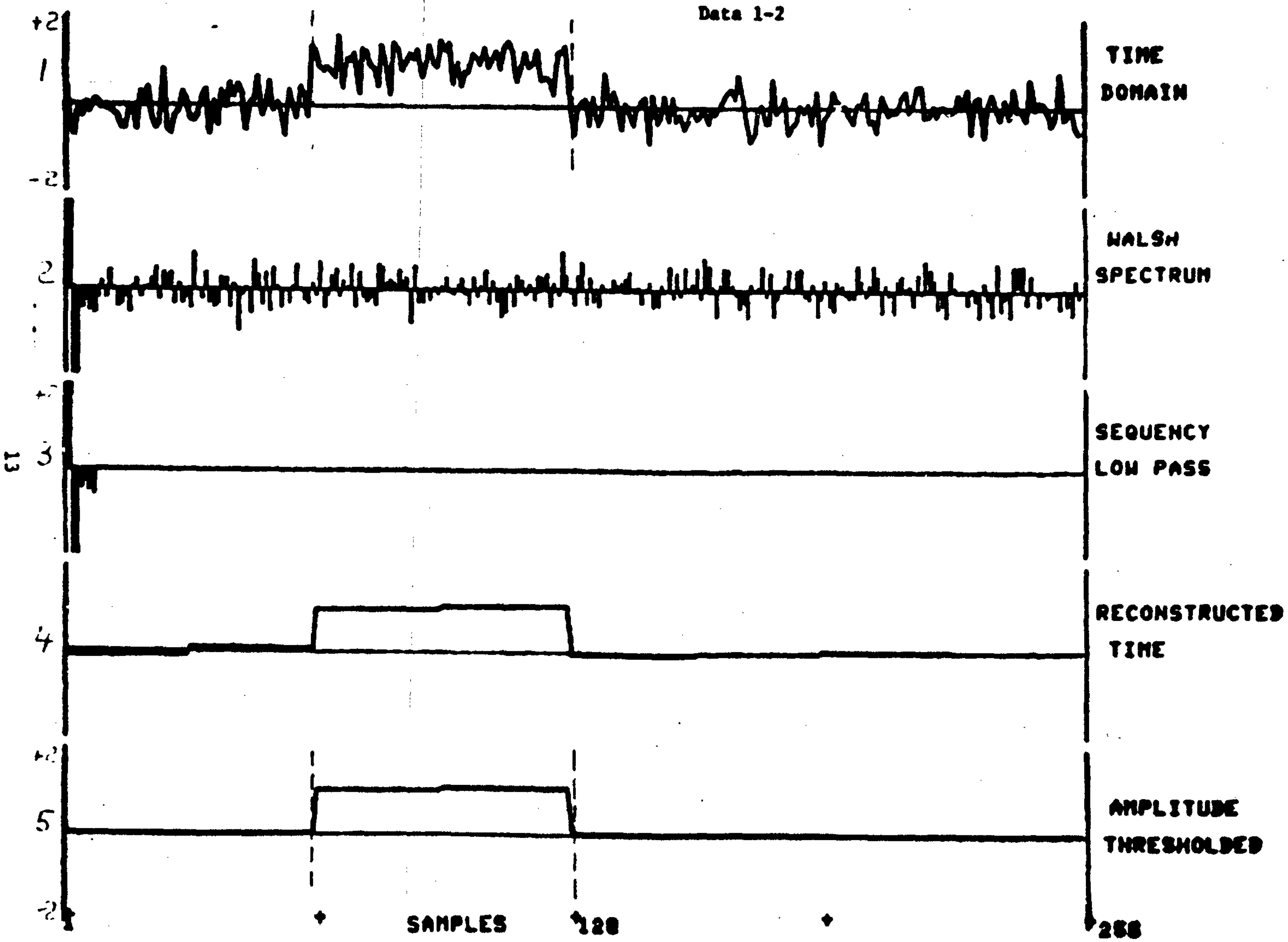
Data 1-4 - Two pulses of differing pulse widths exist in the window. The first pulse has a width of 30 samples. The second pulse has a width of 15 samples.

Data 1-11 - At this low S/N ratio (-4 db) only one pulse actually exists in the noise. Its position is indicated by the dashed lines. The other pulse that shows up in line 5 is a false indication. A close look at the input data reveals that the noise does take on slightly more positive values where the pulse is indicated.

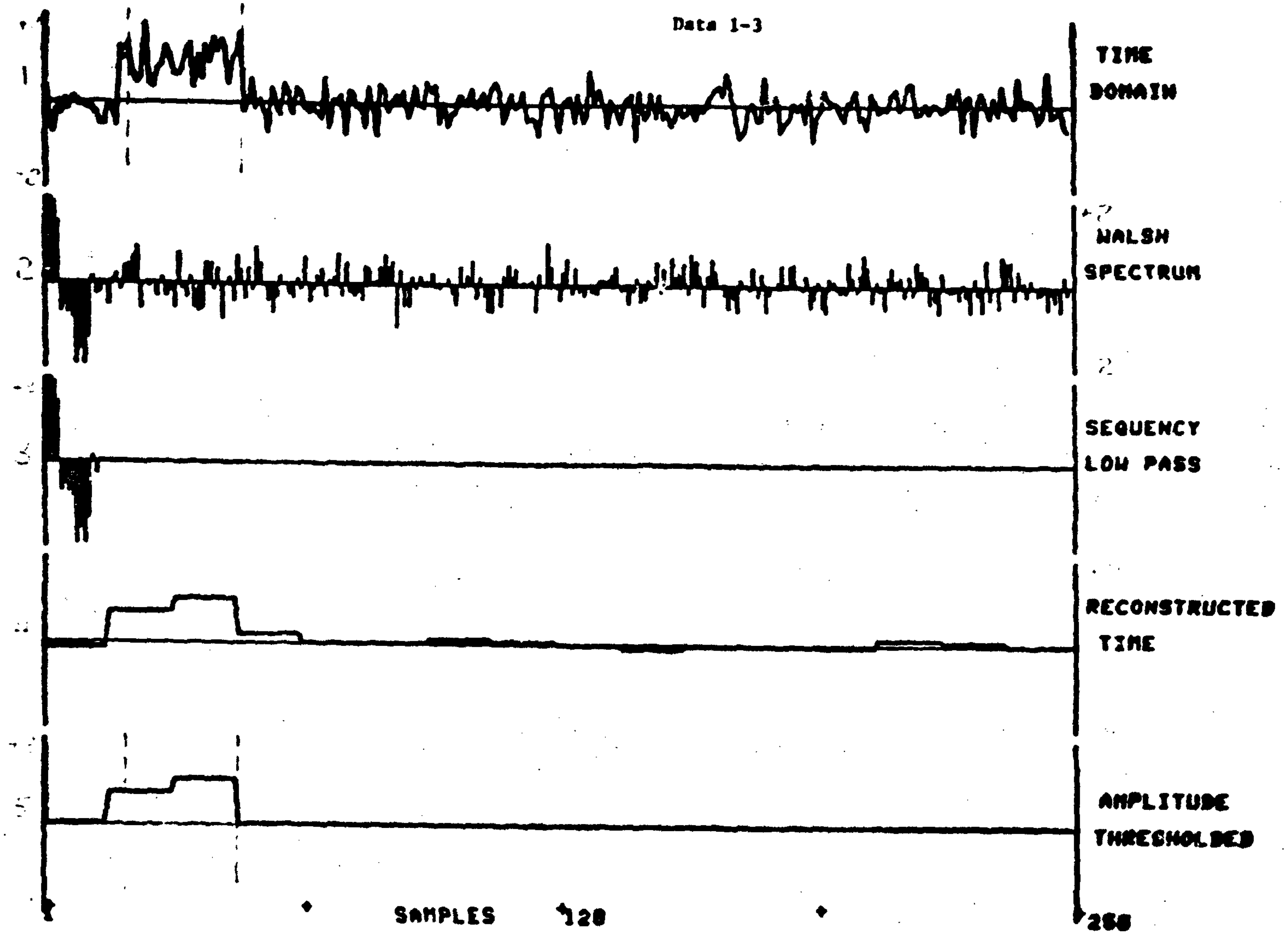
Data 1-12 and 1-13 - These plots have identical line 1 input data. It consists of 3 pulses, 10 samples wide, with 10 samples separation. Data 1-12, line 5 incorrectly shows only one pulse present in the noise. Data 1-13 shows all 3 pulses because a sufficient number of terms was used in the reconstruction of the input data. If an insufficient number of terms is processed, several closely spaced pulses may be misintrepreted and combined into one pulse, particularly at low S/N ratios.


Data 1-1
TIME DOMAIN
WALSH SPECTRUM
SEQUENCY LOW PASS
RECONSTRUCTED TIME
AMPLITUDE THRESHOLDED
SAMPLES
128
256
1
2
3
4
5

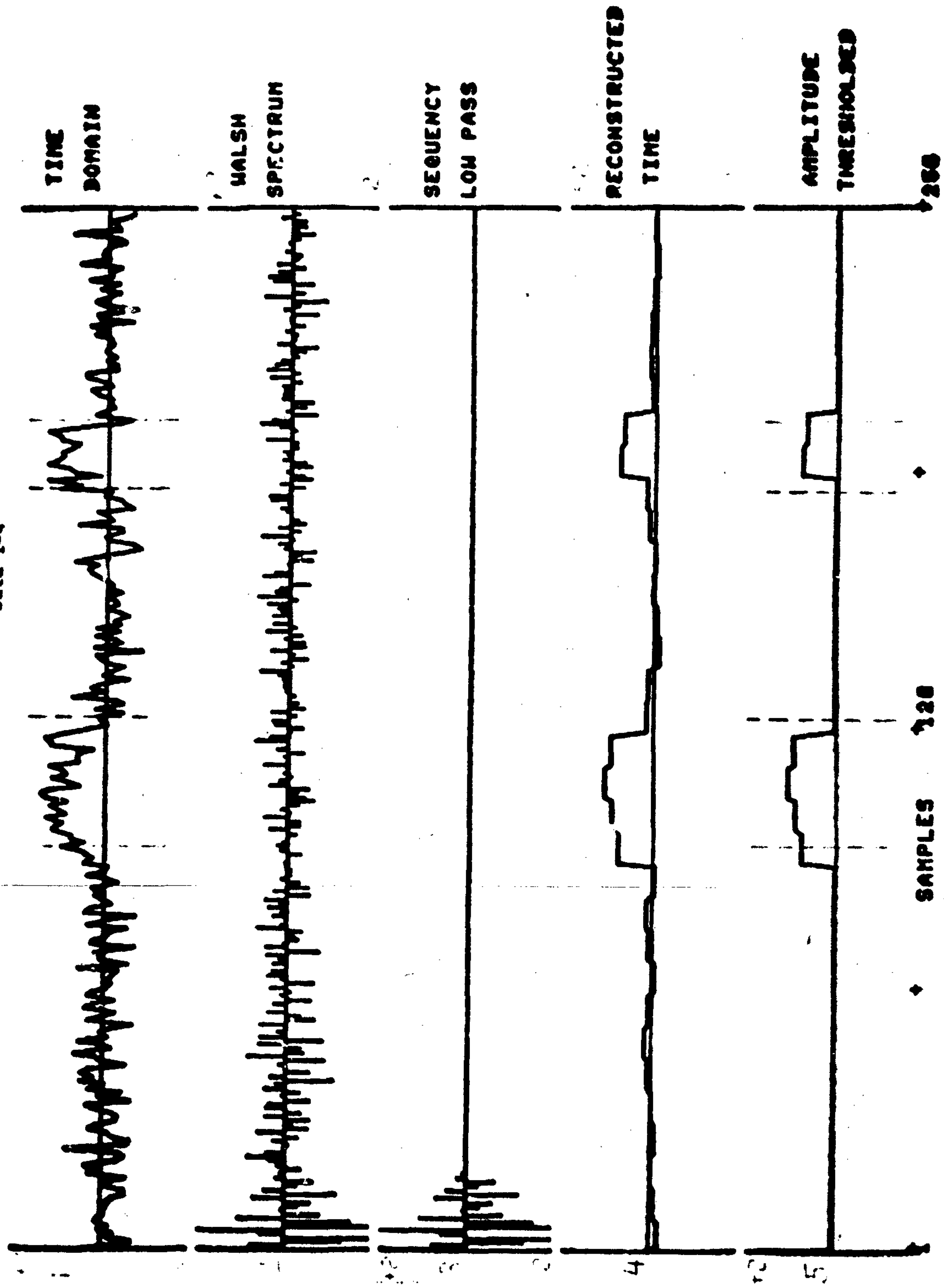
Data 1-2


+2
1
-2
2
3
4
5
TIME
DOMAIN
WALSH
SPECTRUM
SEQUENCY
LOW PASS
RECONSTRUCTED
TIME
AMPLITUDE
THRESHOLDED
SAMPLES
128
256

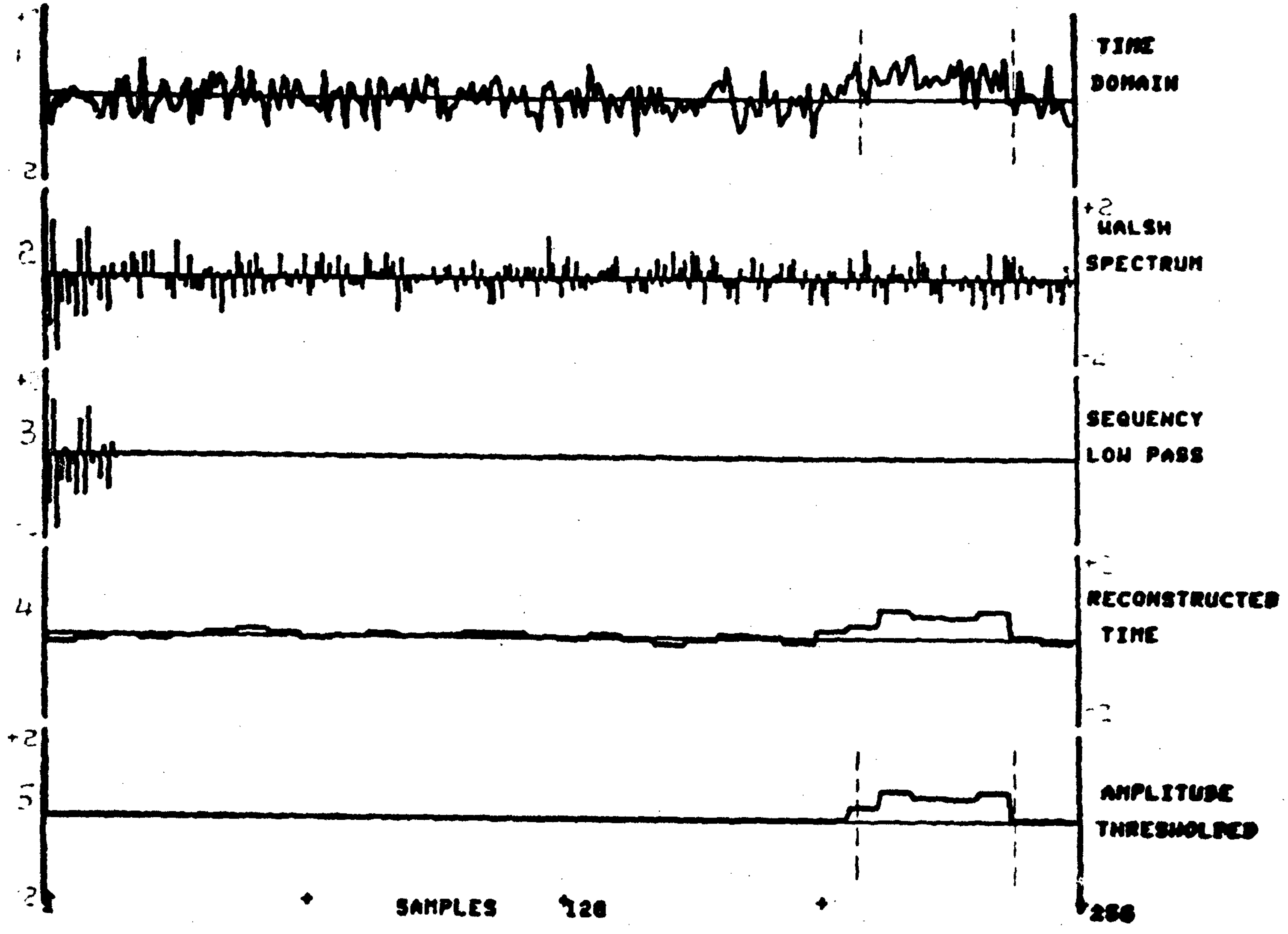
Data 1-3


TIME
DOMAIN
WALSH
SPECTRUM
SEQUENCY
LOW PASS
RECONSTRUCTED
TIME
AMPLITUDE
THRESHOLDED
SAMPLES
128
256

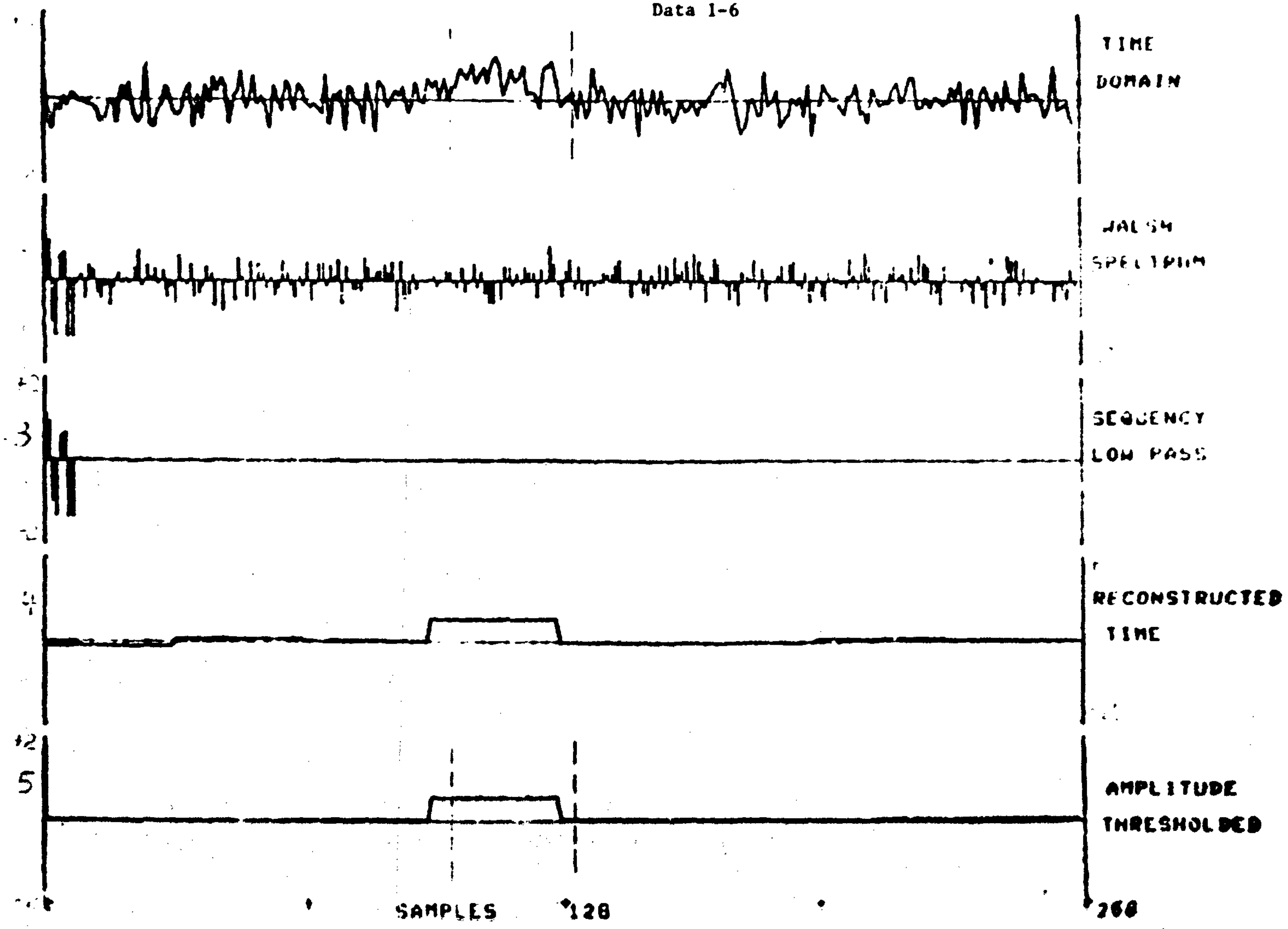
Data 1-4

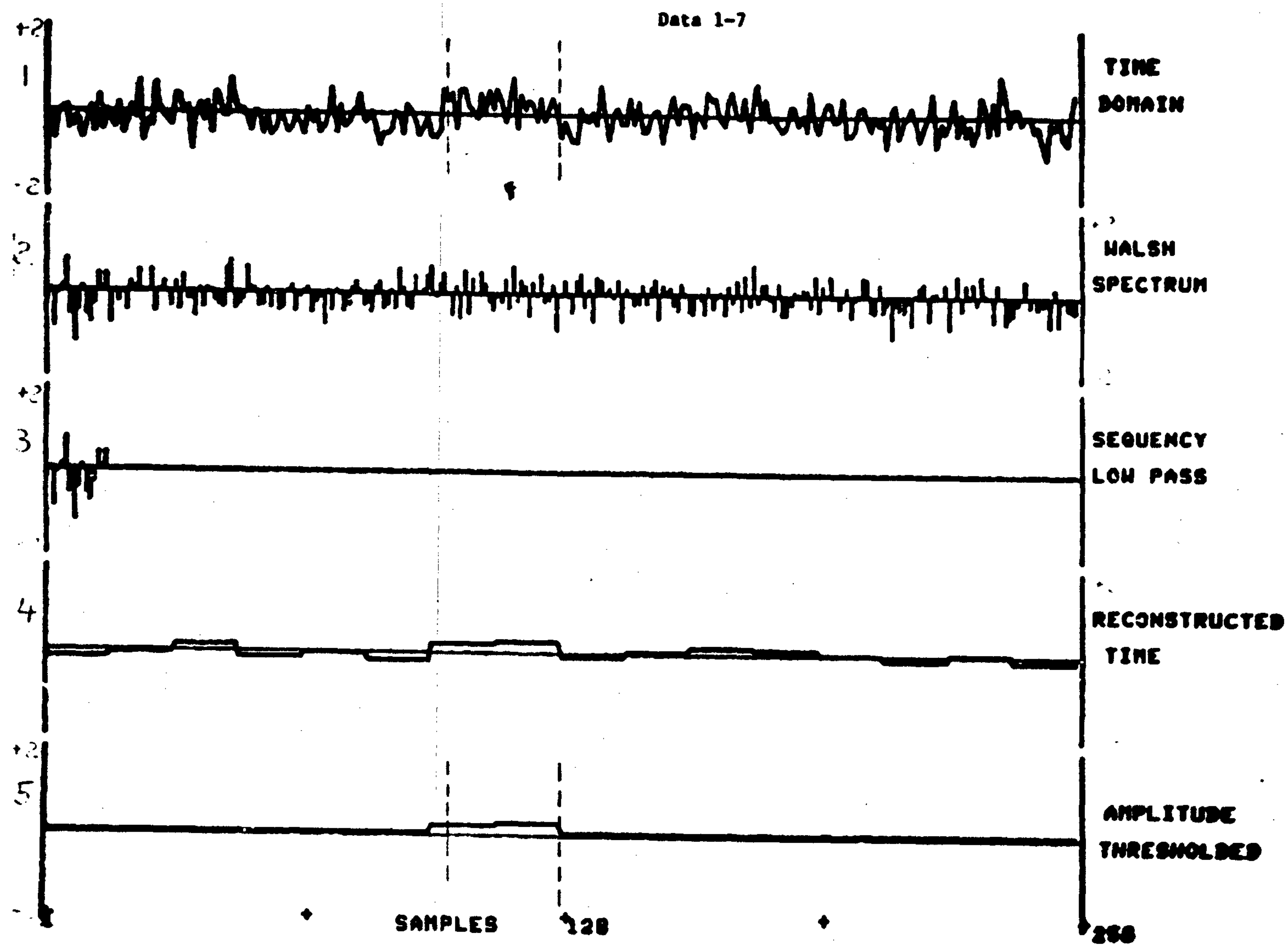

TIME DOMAIN
WALSH SPECTRUM
SEQUENCY LOW PASS
RECONSTRUCTED TIME
AMPLITUDE THRESHOLDED
256
128
SAMPLES

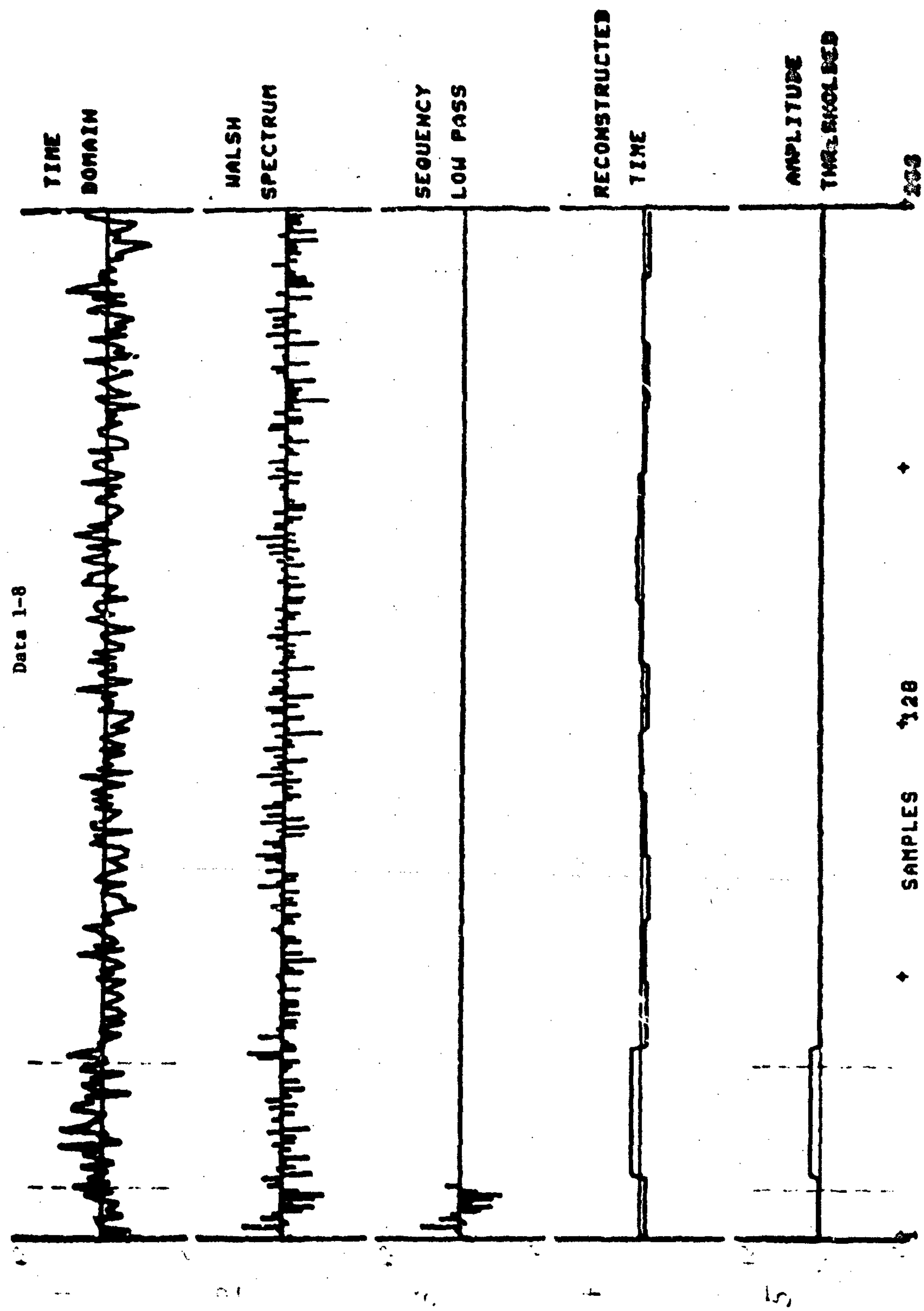
Data 1-5

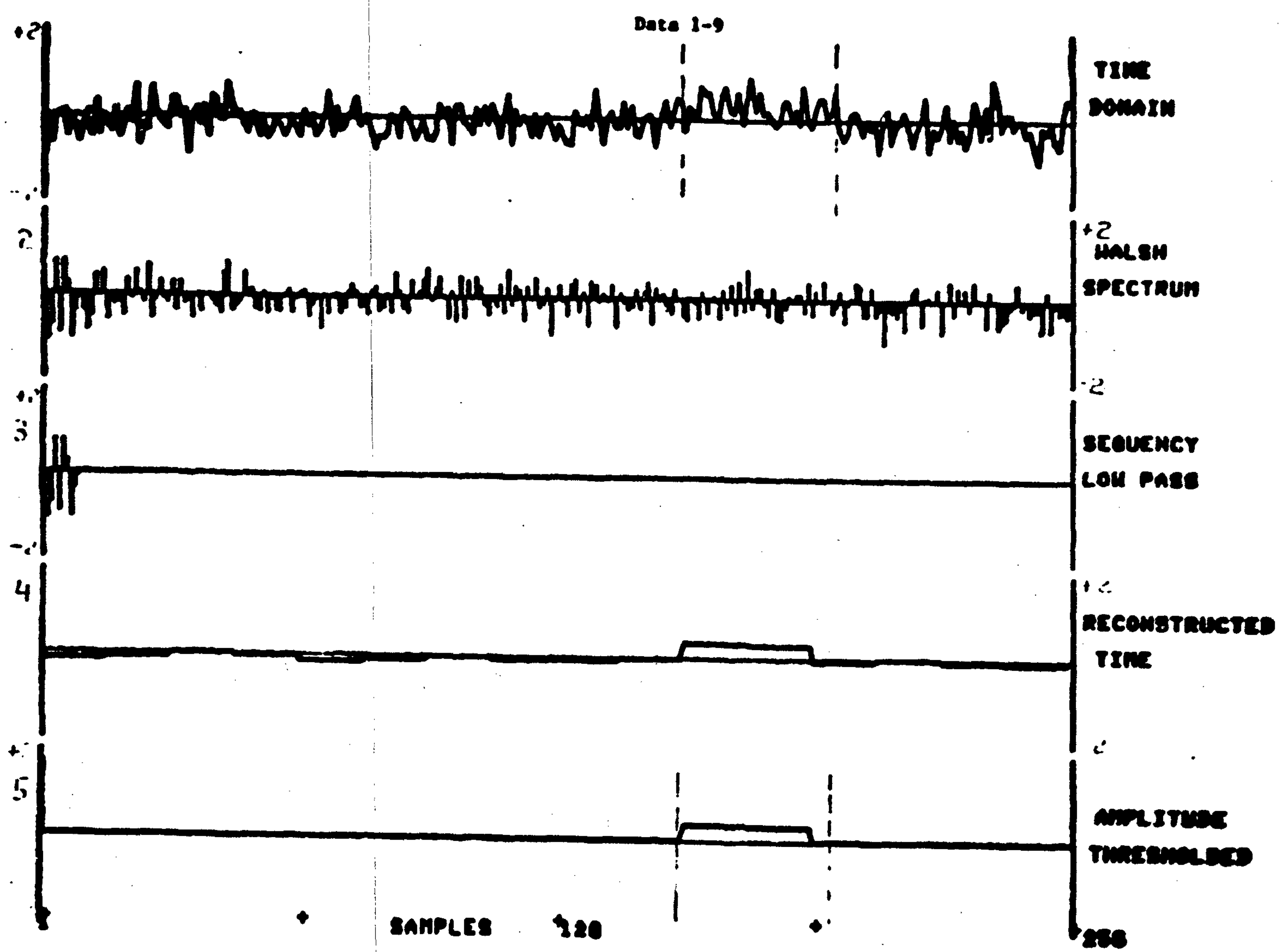

TIME
DOMAIN
WALSH
SPECTRUM
SEQUENCY
LOW PASS
RECONSTRUCTED
TIME
AMPLITUDE
THRESHOLDED
SAMPLES
128
256

Data 1-6

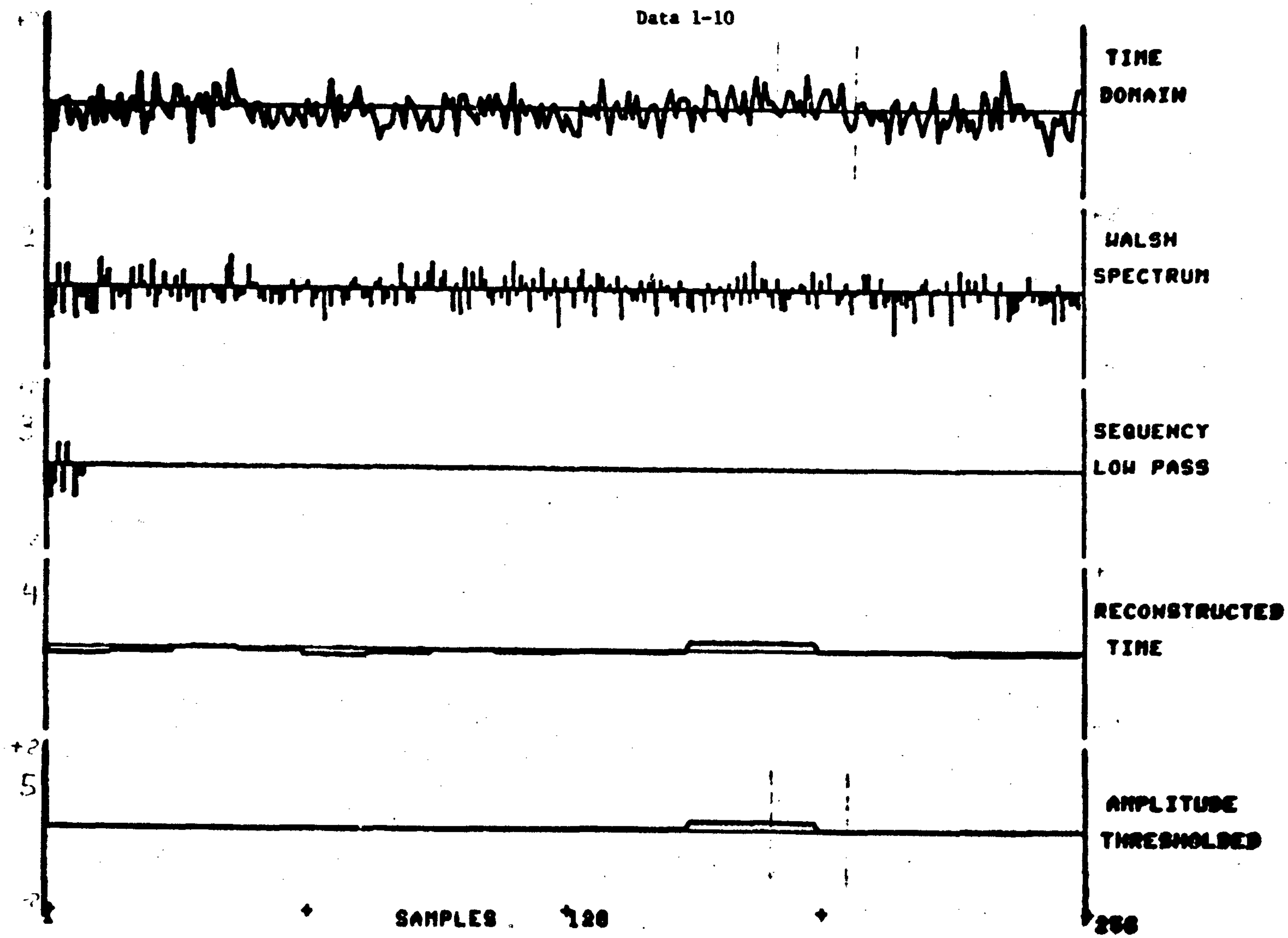

TIME
DOMAIN
WALSH
SPECTRUM
SEQUENCY
LOW PASS
RECONSTRUCTED
TIME
AMPLITUDE
THRESHOLDED
+2
3
4
5
SAMPLES
128
256


Data 1-7
+2
1
-2
2
+2
3
4
+2
5
TIME
DOMAIN
WALSH
SPECTRUM
SEQUENCY
LOW PASS
RECONSTRUCTED
TIME
AMPLITUDE
THRESHOLDED
SAMPLES
128
256

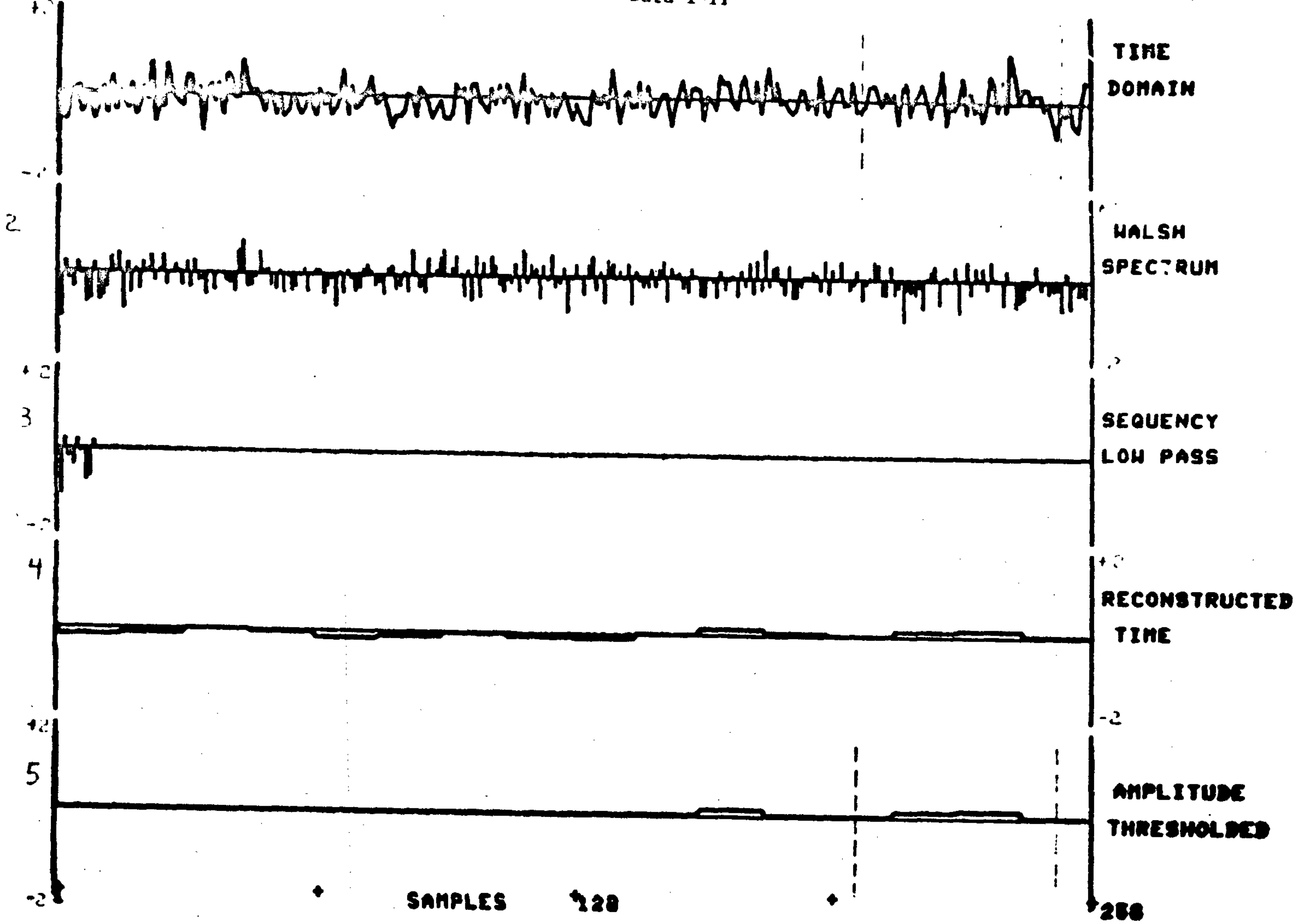

Data 1-8
TIME DOMAIN
WALSH SPECTRUM
SEQUENCY LOW PASS
RECONSTRUCTED TIME
AMPLITUDE THRESHOLDED
SAMPLES
128
256

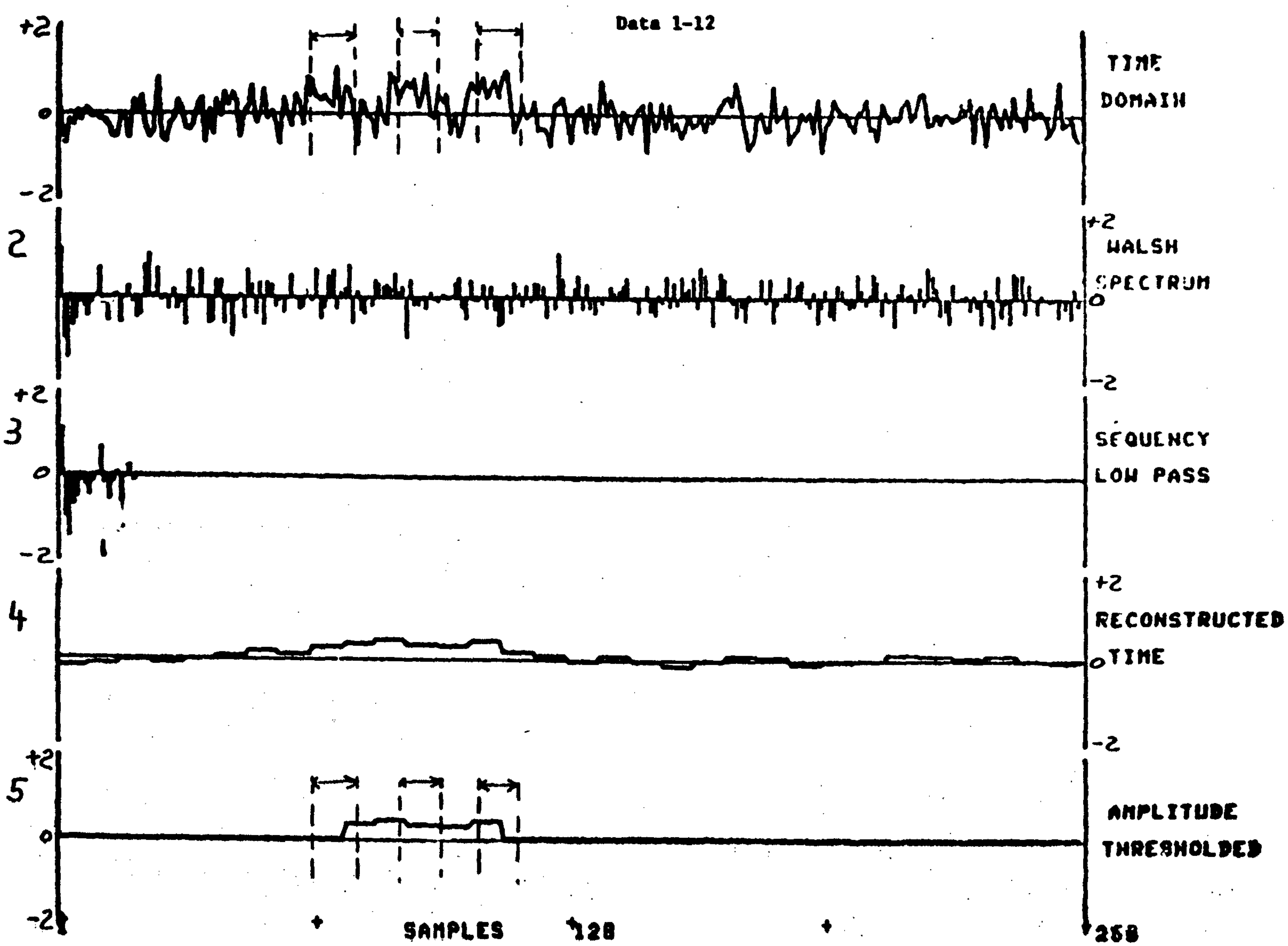

Data 1-9
+2
TIME
DOMAIN
2
+2
WALSH
SPECTRUM
-2
3
SEQUENCY
LOW PASS
4
RECONSTRUCTED
TIME
5
AMPLITUDE
THRESHOLDED
SAMPLES
128
256

Data 1-10


TIME DOMAIN
WALSH SPECTRUM
SEQUENCY LOW PASS
4
RECONSTRUCTED TIME
+2
5
AMPLITUDE THRESHOLDED
SAMPLES
128
256

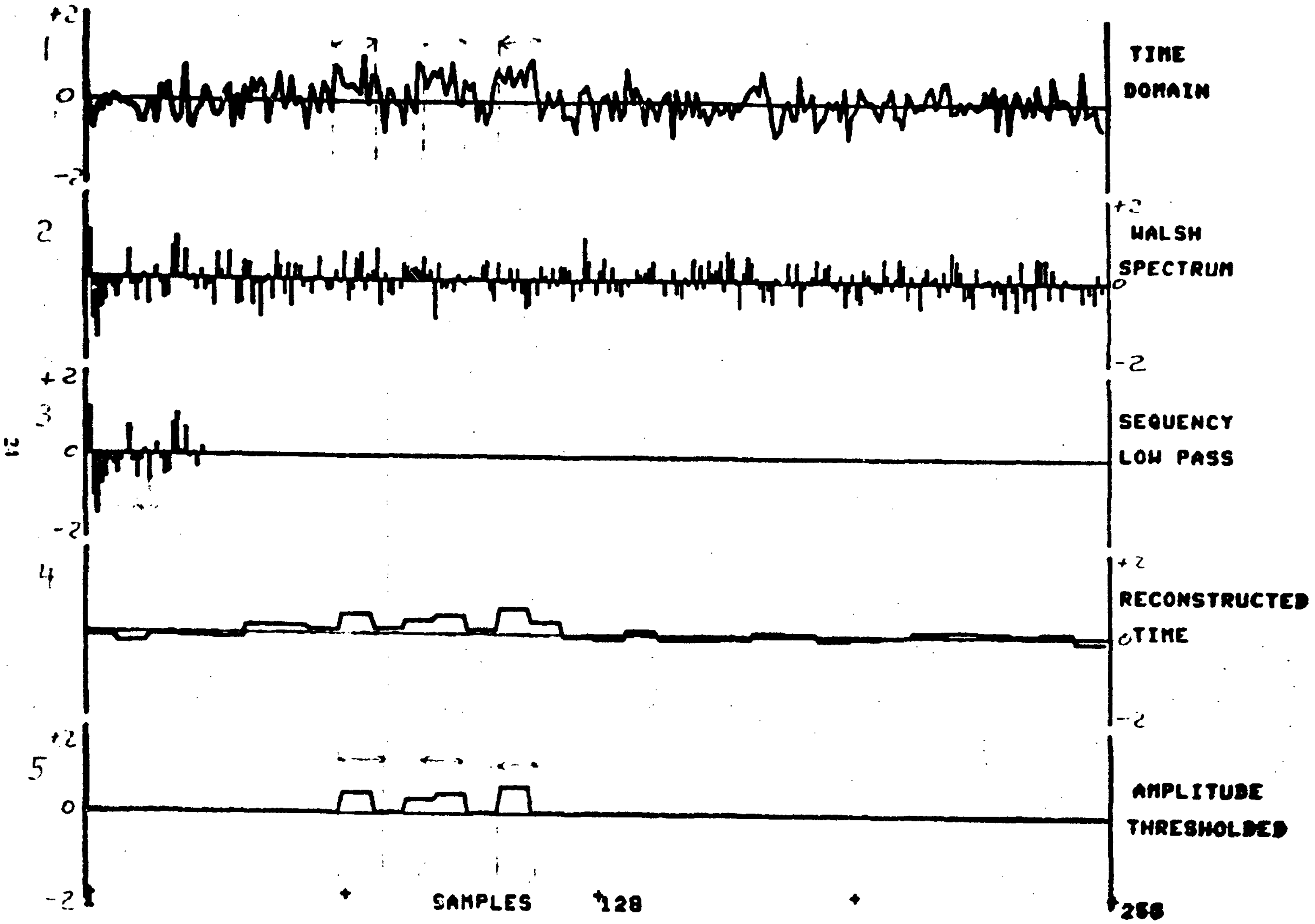
Data 1-11


TIME DOMAIN
WALSH SPECTRUM
SEQUENCY LOW PASS
RECONSTRUCTED TIME
AMPLITUDE THRESHOLDED
SAMPLES
128
256
-2
2
3
4
5



Data 1-12
+2
0
-2
TIME
DOMAIN
2
+2
WALSH
SPECTRUM
0
-2
+2
3
0
-2
SEQUENCY
LOW PASS
4
+2
RECONSTRUCTED
0
TIME
-2
+2
5
0
-2
AMPLITUDE
THRESHOLDED
SAMPLES
128
256

Data 1-13



+2
1
0
-2
2
+2
3
0
-2
4
+2
5
0
-2
TIME
DOMAIN
+2
WALSH
SPECTRUM
0
-2
SEQUENCY
LOW PASS
+2
RECONSTRUCTED
0 TIME
-2
AMPLITUDE
THRESHOLDED
SAMPLES
128
256

SECTION IV

Data 2 Description

Data Table 2 lists all the parameters which pertain to data plots 2-1 through 2-7. In this data set, no low pass filtering of the Walsh spectrum was performed. Instead, amplitude thresholding of the entire Walsh spectrum takes place. All sequency terms which have amplitude values below the listed threshold are set equal to zero. This threshold value was varied from .3 to 1.

Notes on Data 2

Data 2-1, 2-2, and 2-3 - These data plots have identical input data. In each one, however, the sequency amplitude threshold is set at a different level. A large S/N ratio improvement results in each case. This, however, could have been accomplished by simply low pass filtering the Walsh spectrum since the low order (0 - 4) terms dominate in the reconstruction of the pulse.

Data 2-5, 2-6, and 2-7 - These data plots have the same input data but different threshold values are used. No S/N ratio improvement was obtained in 2-5. In 2-6 the pulse falls deeper into the noise and in 2-7, the pulse is no longer present after processing has taken place. For this input data, the pulse lies deep enough in the noise so that the higher Walsh terms have amplitude values which are in the same amplitude range as the low order Walsh terms. The high order Walsh functions have many zero crossings within the width of this one pulse. They contribute to the corruption rather than the enhancement of the flat-topped pulse characteristics. By high order Walsh functions, I am referring to the functions of order greater than 32.

DATA TABLE 2

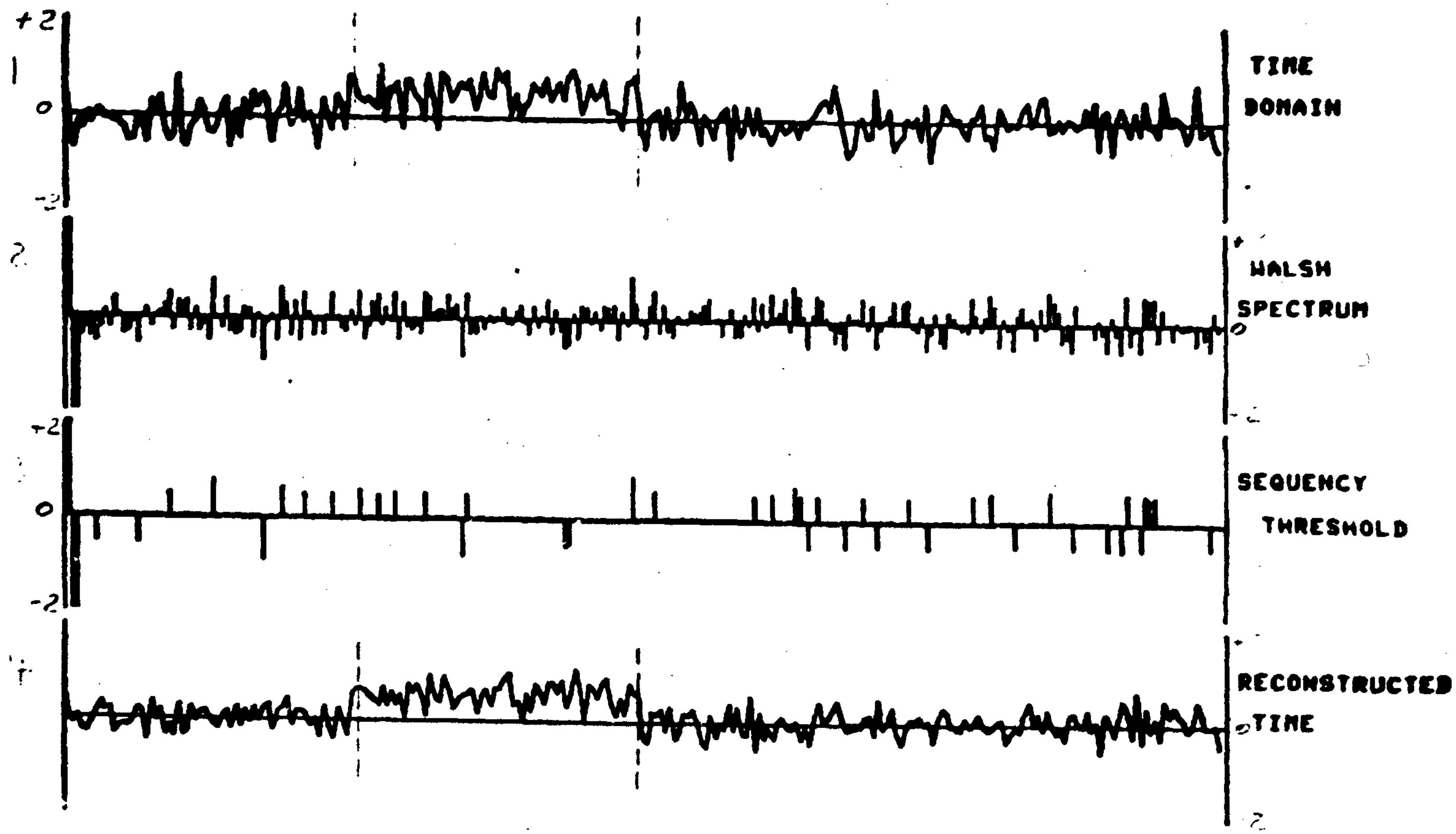
Data Number	Beginning of Pulse	End of Pulse	Sequency Threshold	Input Data S/N db	Output S/N(db) Line 5	S/N Improvement
2-1	64	128	.5	4	9	5
2-2	64	128	.7	4	14	10
2-3	64	128	1.	4	26	22
2-4	100	120	.7	4	6	2
2-5	200	220	.3	1	1	0
2-6	200	220	.5	1	0	*
2-7	200	220	.7	1	*	*

*See Data 2 Notes

Sequencies above this value rarely, if ever, contain any pulse information. Since the high order terms contribute in this example as much as do the low order terms, we cannot expect an S/N improvement by just amplitude thresholding. In data 2-7, the amplitude threshold is set too high. As a result, many of the low order terms are not processed and the pulse is lost.

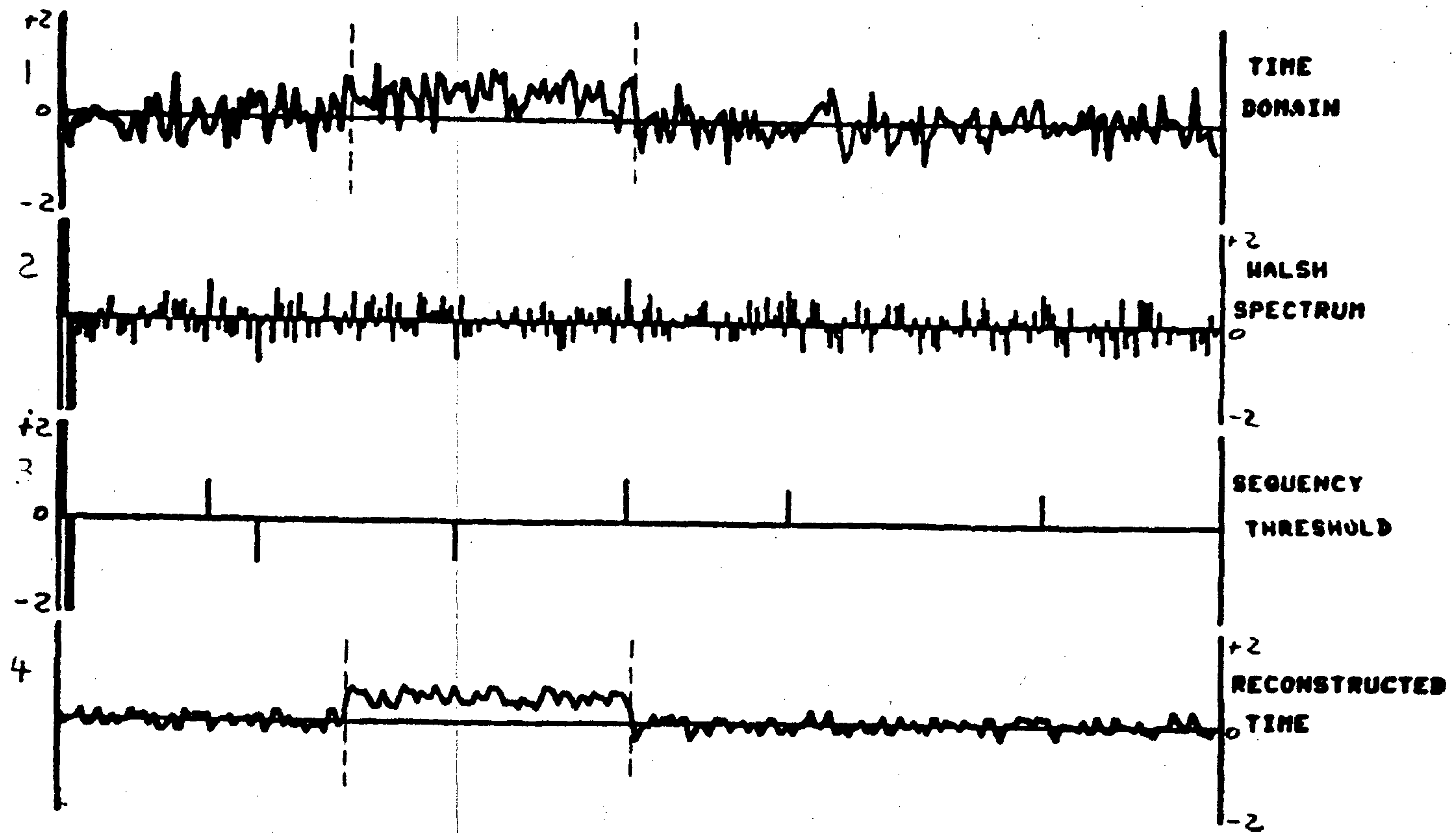
In general, amplitude thresholding of the Walsh spectrum will not give any improvement or increased likelihood of pulse detection at low S/N ratios.

Data 2-1



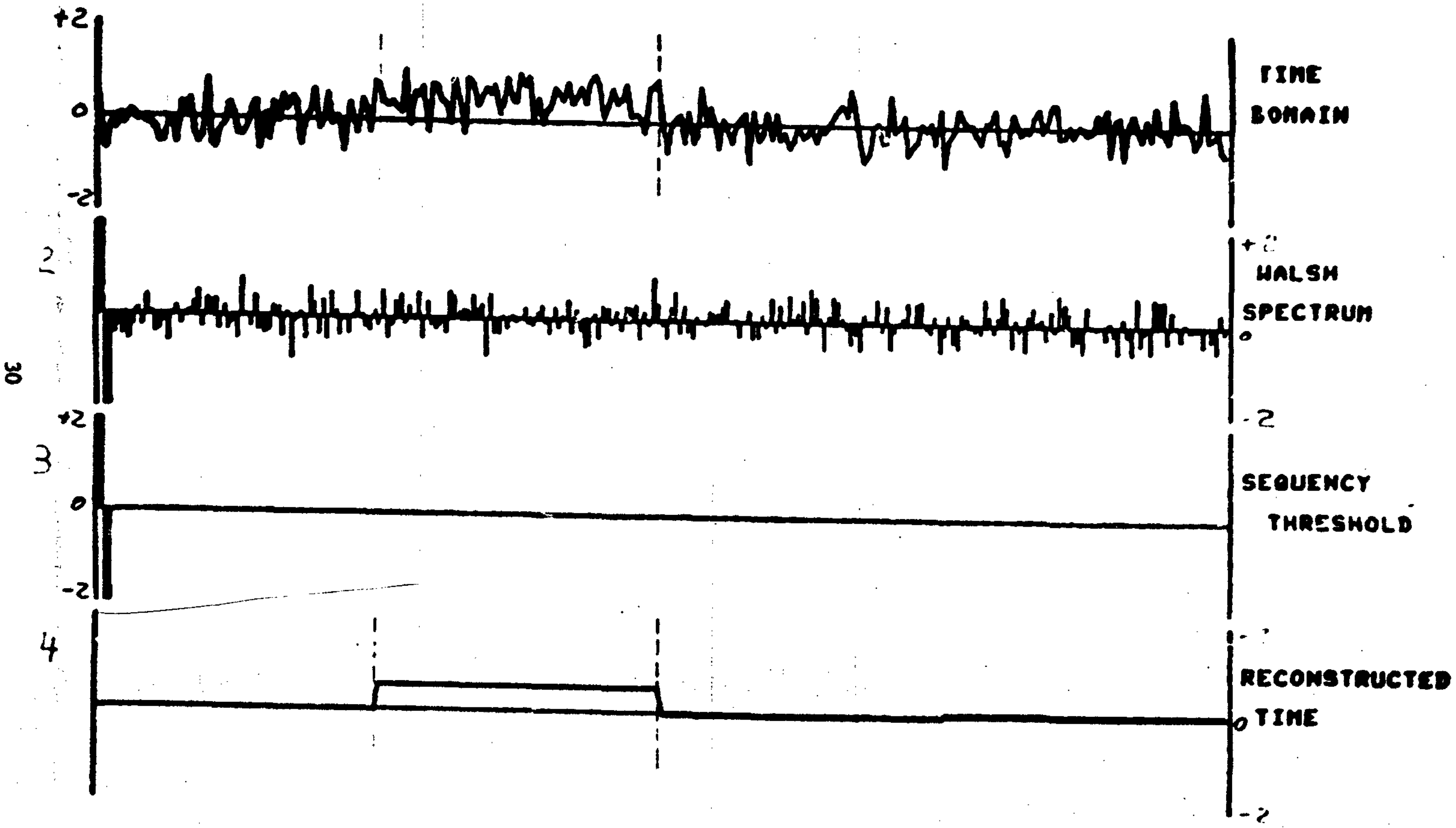
+2
0
-2
TIME
DOMAIN
WALSH
SPECTRUM
SEQUENCY
THRESHOLD
RECONSTRUCTED
TIME

Data 2-2



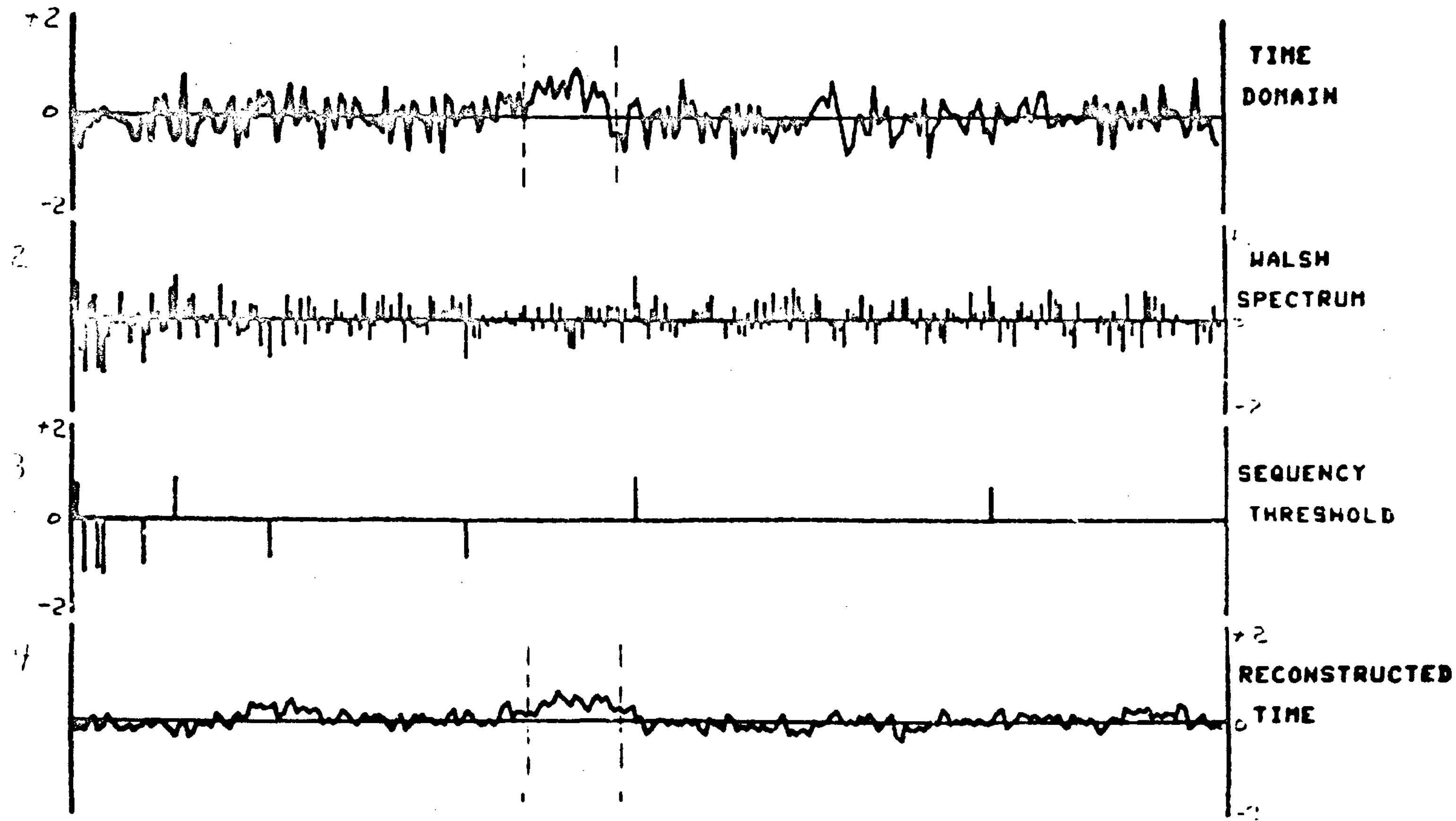
+2
1
0
-2
TIME
DOMAIN
2
+2
WALSH
SPECTRUM
0
-2
+2
3
0
-2
SEQUENCY
THRESHOLD
4
+2
RECONSTRUCTED
TIME
0
-2

Data 2-3



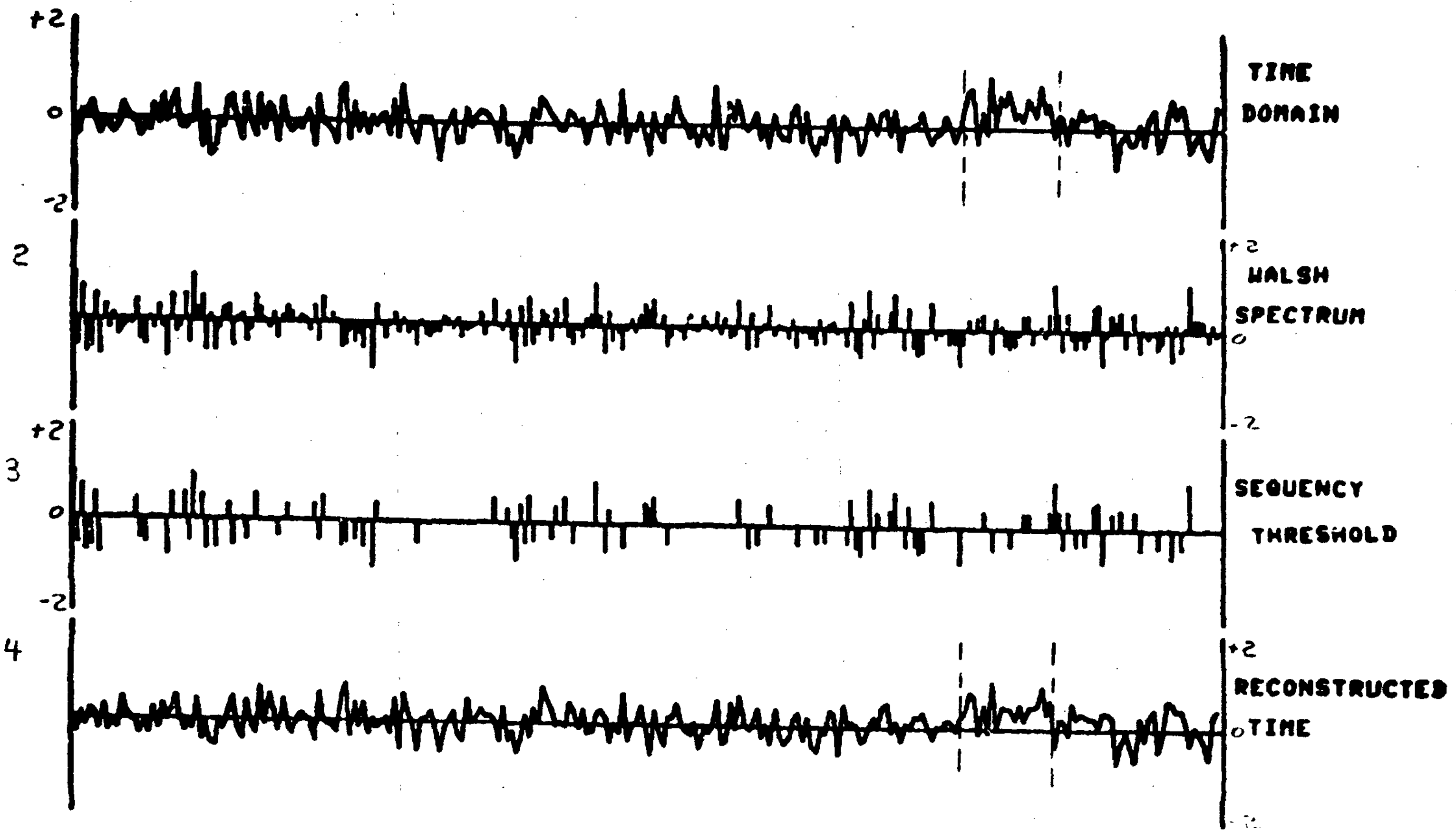
+2
0
-2
TIME
DOMAIN
+2
WALSH
SPECTRUM
0
-2
+2
0
-2
SEQUENCY
THRESHOLD
RECONSTRUCTED
0 TIME
-2
2
3
4

Data 2-4



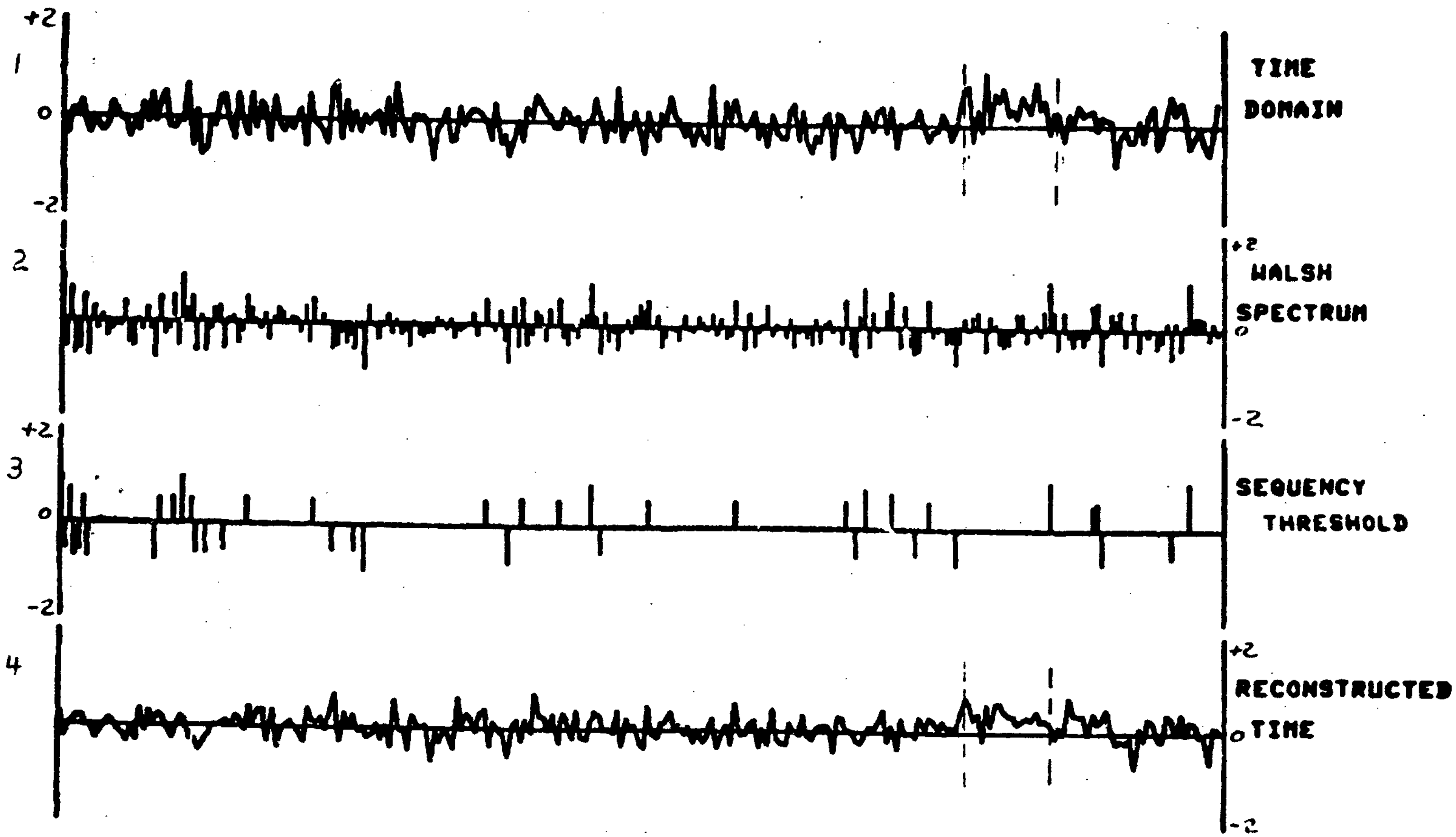
TIME
DOMAIN
WALSH
SPECTRUM
SEQUENCY
THRESHOLD
RECONSTRUCTED
TIME
+2
0
-2

Data 2-5



+2
0
-2
2
+2
3
0
-2
4
TIME
DOMAIN
+2
WALSH
SPECTRUM
0
-2
SEQUENCY
THRESHOLD
+2
RECONSTRUCTED
TIME
0

Data 2-6



+2
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-2
TIME
DOMAIN
2
+2
WALSH
SPECTRUM
0
-2
+2
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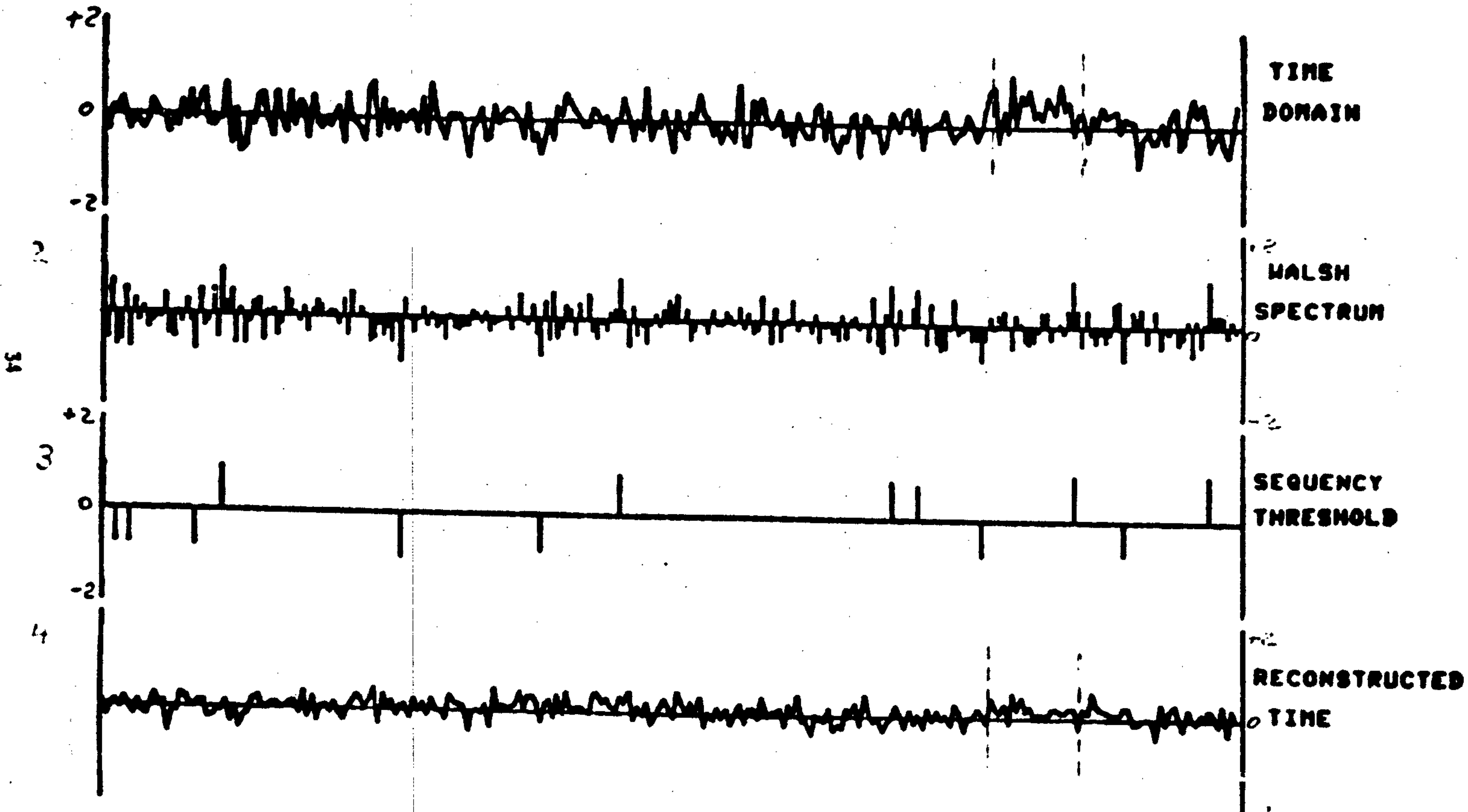
0
-2
SEQUENCY
THRESHOLD
4
+2
RECONSTRUCTED
0
TIME
-2

Data 2-7



+2
0
-2
TIME
DOMAIN
WALSH
SPECTRUM
+2
0
-2
SEQUENCY
THRESHOLD
RECONSTRUCTED
TIME

SECTION V

Data 3 Description

Both low pass filtering and amplitude thresholding of the Walsh spectrum are performed. The accuracy of pulse reconstruction is dependent upon the number of terms passed by the low pass filter and to the sequency threshold level setting. The sequency low pass filter setting is varied from 20 to 30 terms which are passed and contribute to the reconstructed data of line 4.

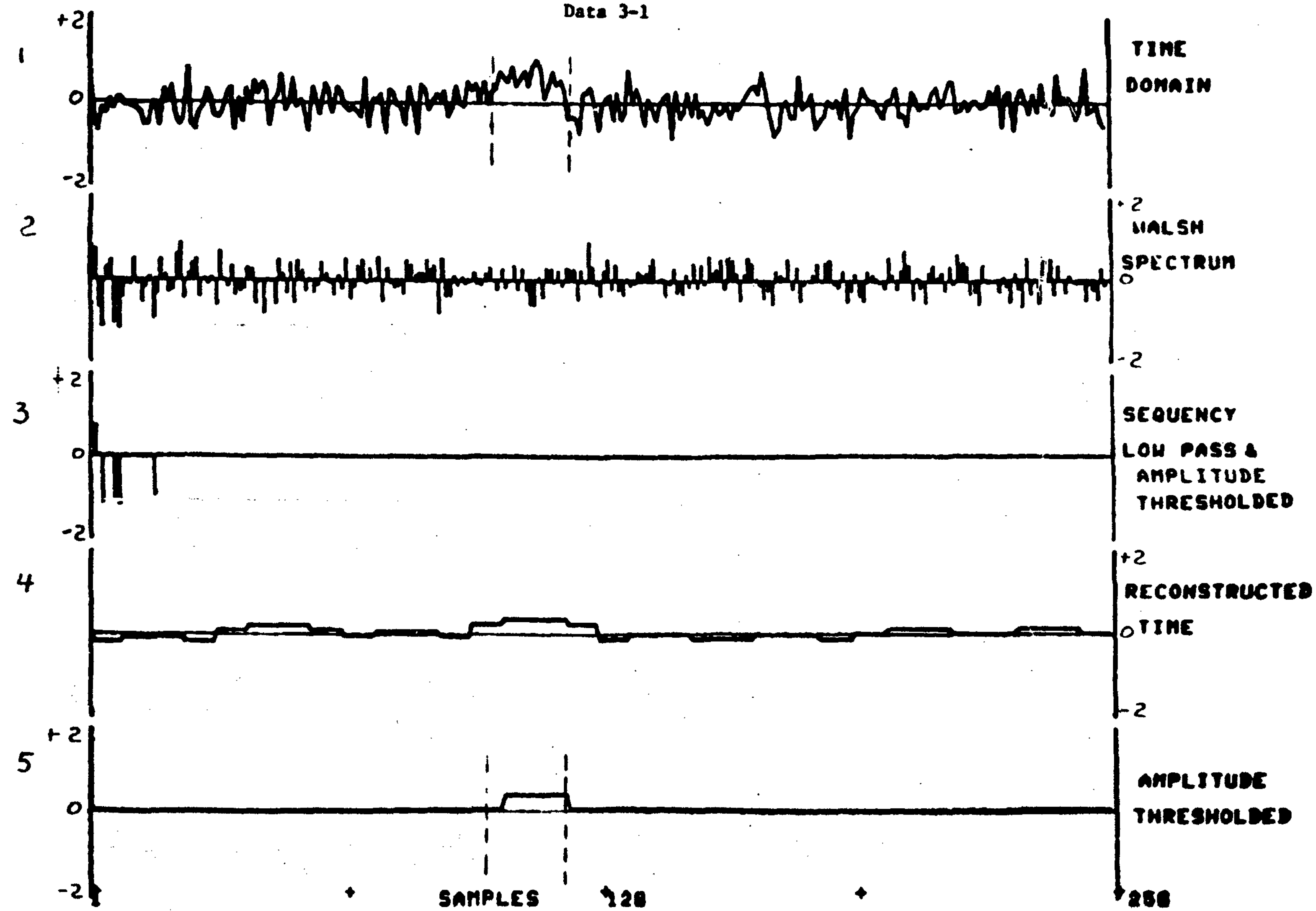
Notes on Data 3

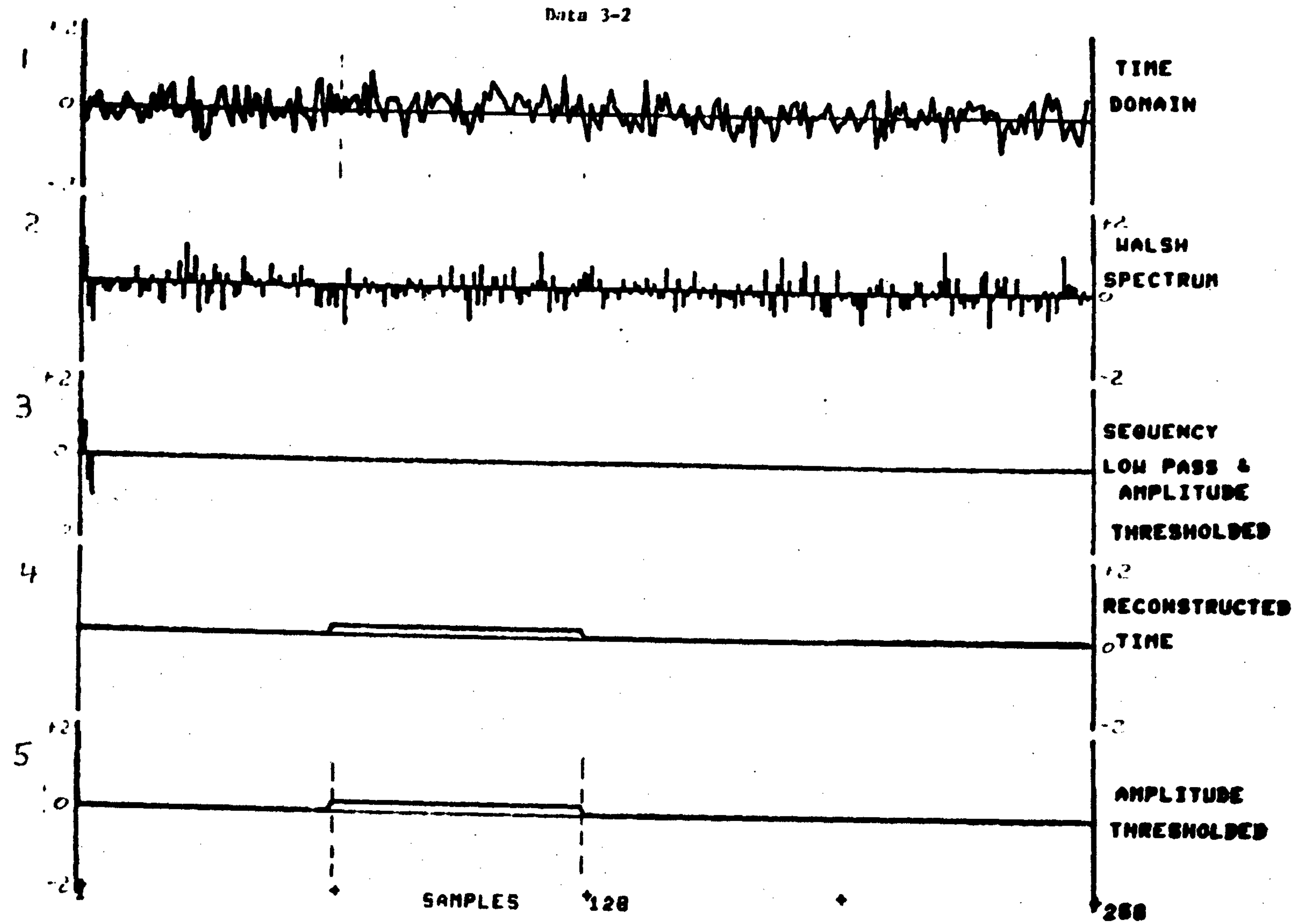
Data 3-6 and 3-7 have identical input data, but 3-6 does not amplitude threshold the sequency terms. 3-7 has a sequency threshold of .3; this results in smoother line 4 data and avoids the false pulse indication of 3-6, line 5. At S/N ratios of -5 db, a pulse can still be filtered from the noise.

DATA TABLE 3

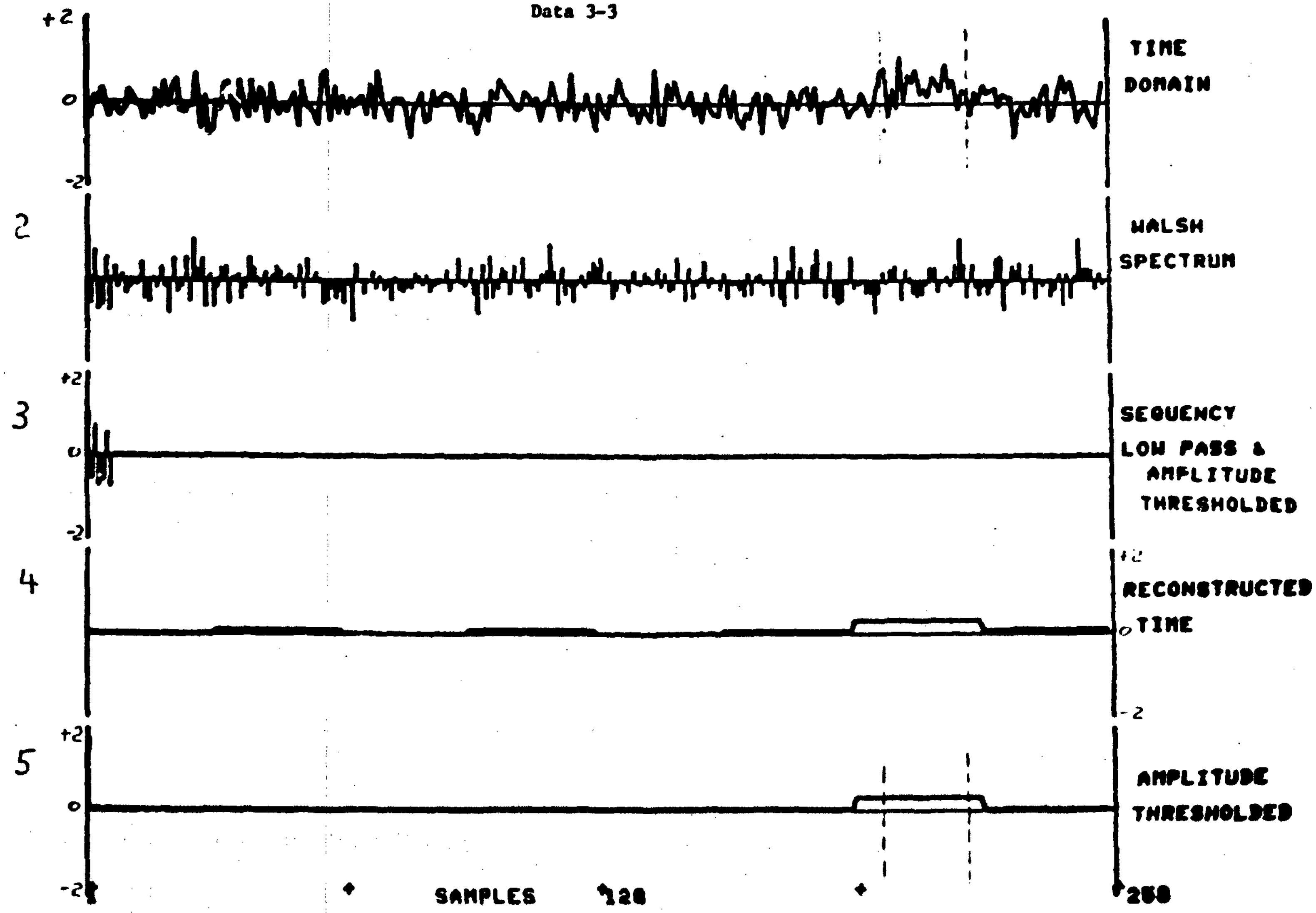
Data Number	Beginning of Pulse	End of Pulse	Sequency Terms Amplitude Threshold	Sequency Low Pass Filter Setting	Time Domain Amplitude Threshold	Input Data S/N db	PW Error Leading Edge	PW Error Trailing Edge
3-1	100	120	.7	20	.25	5	8	2
3-2	64	128	.4	20	.2	-2	0	0
3-3	200	220	.5	20	.2	2	5	3
3-4	100	140	.4	30	.2	-2	6	3
3-5	40 70	60 120	.4	20	.25	2	3 4	6 5
3-6	200	250	0	25	.15	-5	4	8
3-7	200	250	.3	25	.15	-5	7	3

Data 3-1


TIME
DOMAIN
WALSH
SPECTRUM
SEQUENCY
LOW PASS &
AMPLITUDE
THRESHOLDED
RECONSTRUCTED
TIME
AMPLITUDE
THRESHOLDED
SAMPLES
128
256
+2
0
-2
1
2
3
4
5

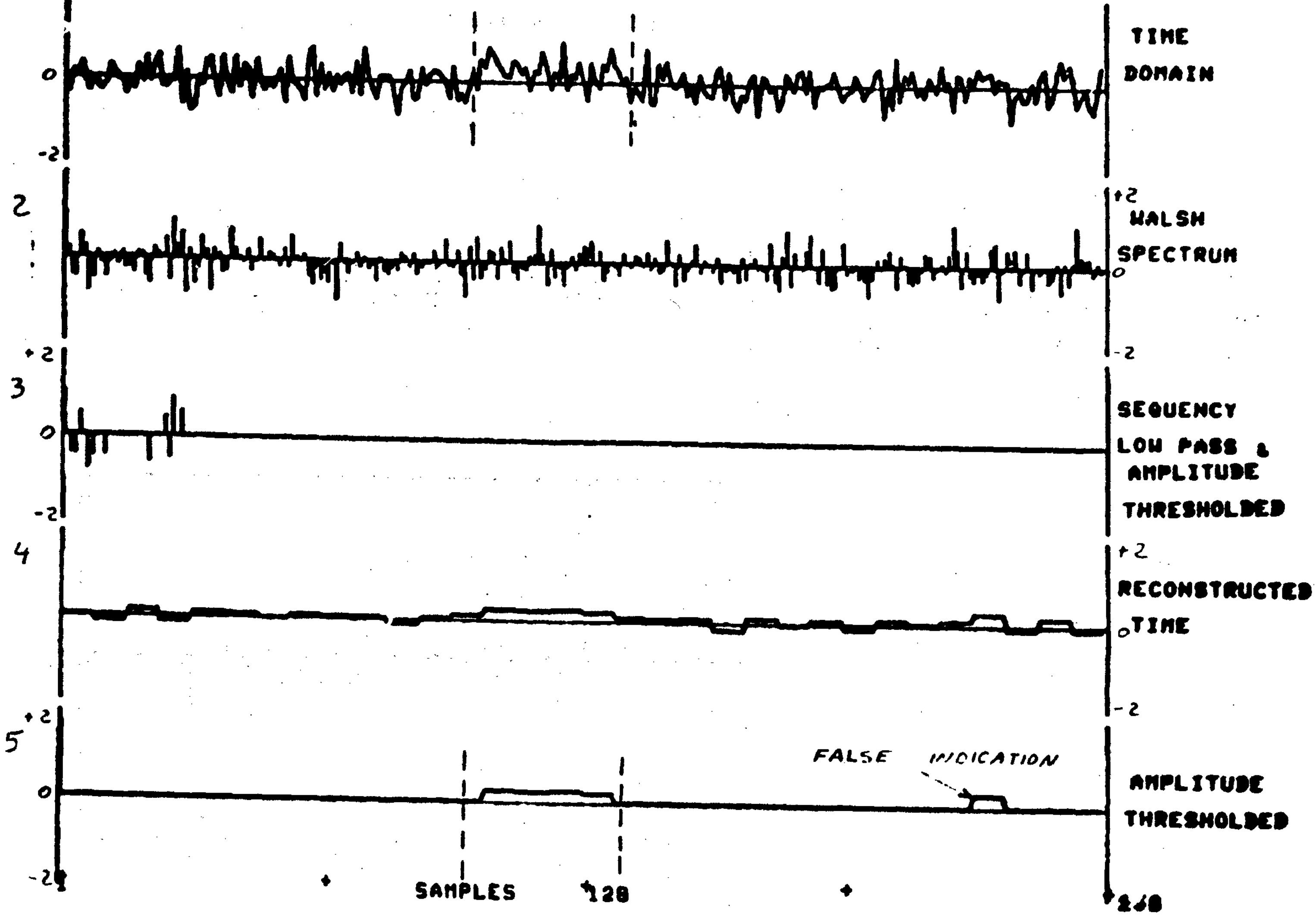

Data 3-2
1
2
3
4
5
TIME DOMAIN
WALSH SPECTRUM
SEQUENCY LOW PASS & AMPLITUDE THRESHOLDED
RECONSTRUCTED TIME
AMPLITUDE THRESHOLDED
SAMPLES
128
256

Data 3-3

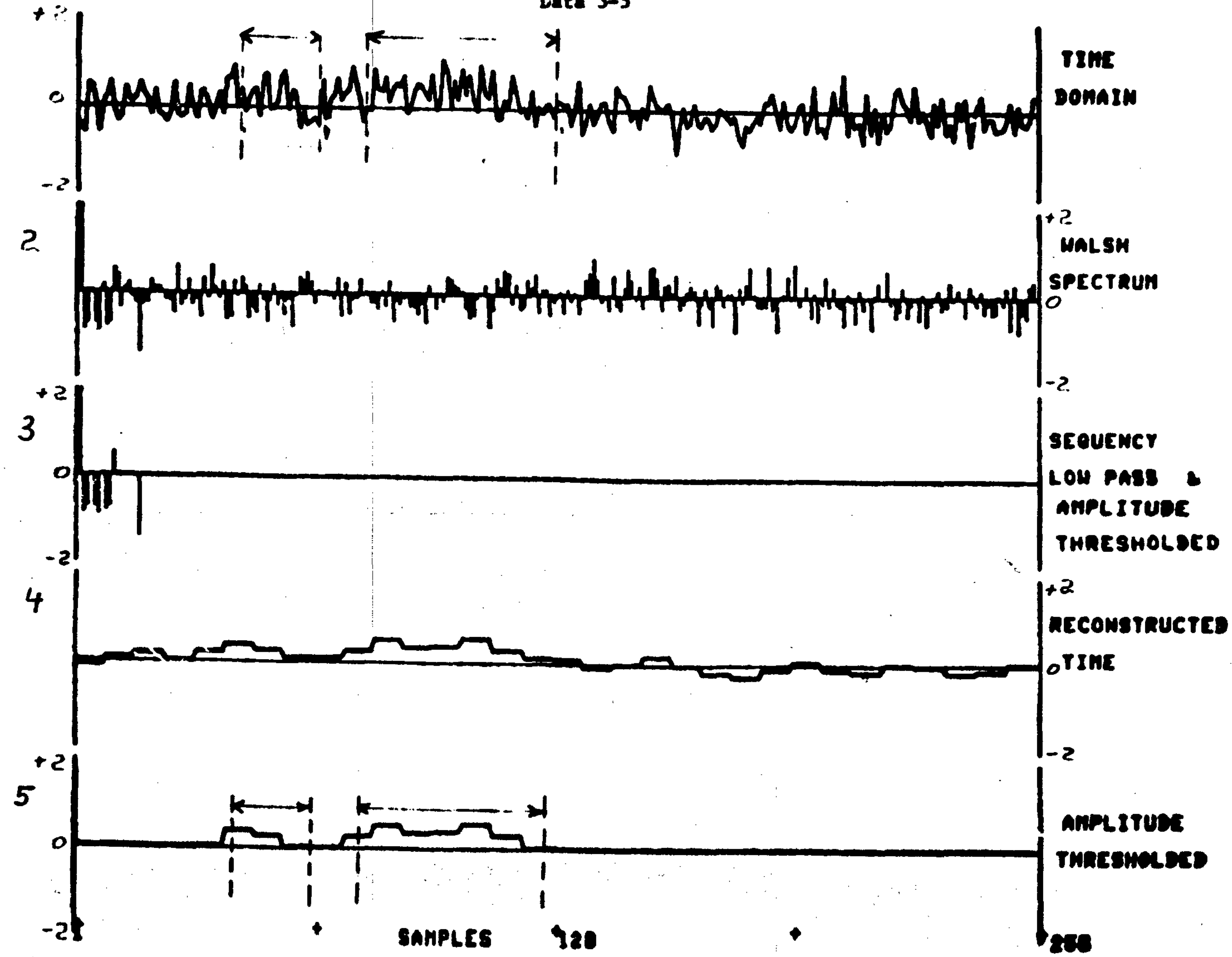


+2
0
-2
TIME DOMAIN
2
WALSH SPECTRUM
3
SEQUENCY LOW PASS & AMPLITUDE THRESHOLDED
4
RECONSTRUCTED TIME
5
AMPLITUDE THRESHOLDED
SAMPLES
128
256

Data 3-4

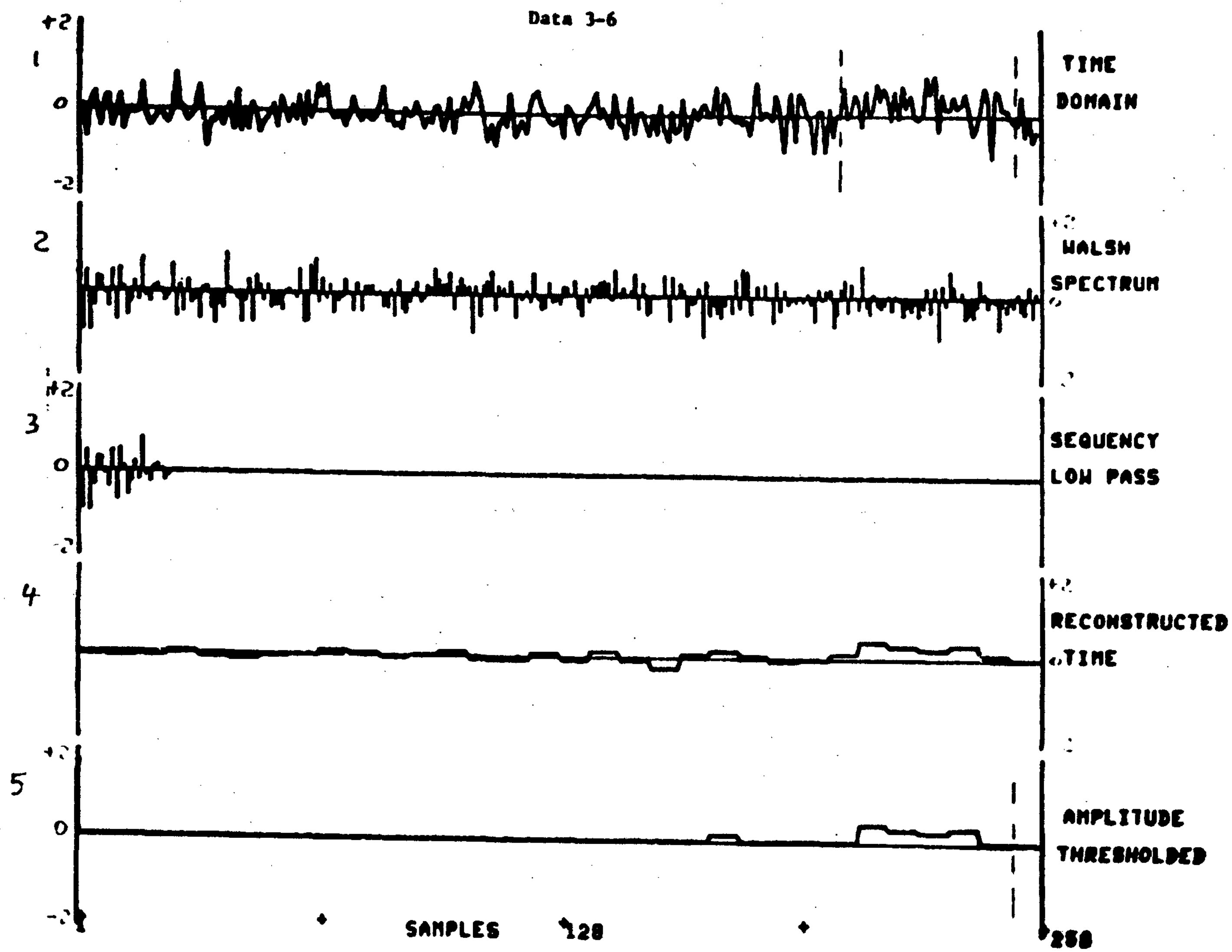

+2
0
-2
TIME
DOMAIN
2
+2
WALSH
SPECTRUM
0
-2
+2
3
0
-2
SEQUENCY
LOW PASS &
AMPLITUDE
THRESHOLDED
4
+2
RECONSTRUCTED
0 TIME
-2
5
+2
0
-2
FALSE INDICATION
AMPLITUDE
THRESHOLDED
SAMPLES
128
256

Data 3-5



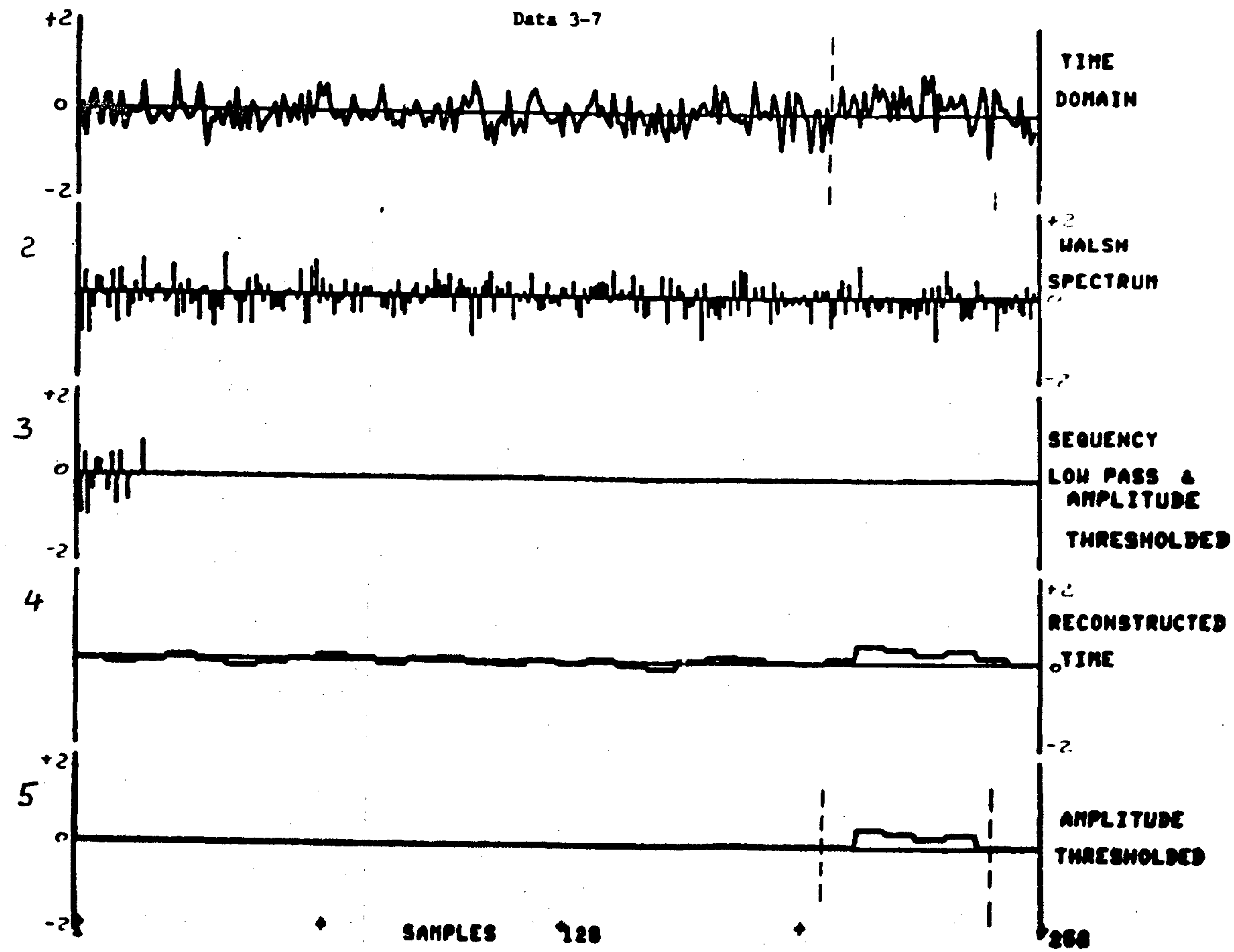
+2
0
-2
2
+2
3
0
-2
4
+2
5
0
-2
SAMPLES
128
256
TIME
DOMAIN
+2
WALSH
SPECTRUM
0
-2
SEQUENCY
LOW PASS &
AMPLITUDE
THRESHOLDED
+2
RECONSTRUCTED
0 TIME
-2
AMPLITUDE
THRESHOLDED

Data 3-6



TIME
DOMAIN
WALSH
SPECTRUM
SEQUENCY
LOW PASS
RECONSTRUCTED
TIME
AMPLITUDE
THRESHOLDED
SAMPLES
128
256
+2
0
-2
1
2
3
4
5

Data 3-7



+2
0
-2
TIME
DOMAIN
2
+2
WALSH
SPECTRUM
-2
3
+2
0
-2
SEQUENCY
LOW PASS &
AMPLITUDE
THRESHOLDED
4
+2
RECONSTRUCTED
0 TIME
-2
5
+2
0
-2
AMPLITUDE
THRESHOLDED
SAMPLES
128
256

SECTION VI

CONCLUSIONS

The square corner, flat-topped characteristics of Walsh functions make them ideally suited for the processing and detection of rectangularly shaped pulses buried in noise. Most types of noise are sinusoidal or spiked in shape. If a pulse is buried in this noise, the inverse Walsh transform requires fewer terms to reconstruct a pulse than the Fourier transform does.

The higher the order of a Walsh function, the more zero crossings it will have. Thus, the higher order Walsh sequencies contribute to the reconstruction of the noise rather than the pulse. By low pass filtering the sequency spectrum, much of the noise can be removed while still retaining pulse information. Low pass spectrum filtering followed by amplitude thresholding of the low order spectrum elements provides the optimum pulsed-signal detection capability at low S/N ratios.

APPENDIX I

Software Description

All of the information presented in the data sections of this report was obtained from computer programs written in Multics Fortran. These programs were implemented on the RADC Multics time sharing computer system. The FWT subroutine used to generate the Walsh transform can be found in reference 3. In this subroutine, x is the 256 point array containing the real data samples before the call to this subroutine is made. After the subroutine is called, x becomes the array of Walsh coefficients. Two successive calls to this subroutine operating on x will yield the original data since the inverse FWT is obtained in the same way as the forward FWT.

The noise used in the generation of the time domain input data was obtained from the Multics random number generator through the call:

```
call random_$normal (iseed,a)
```

where (a) is the random number generated and iseed is the starting seed for the generation of the random noise. This noise has an approximately Gaussian distribution.

The graphics were produced on a Tektronix 4002 CRT graphic computer terminal. Many of the Tektronix Plot 10 subroutines were used. These subroutines depend on the output of certain ASCII control characters which the Multics system would not output directly at the time this report was written. This made it necessary to write a PL/1 subroutine utilizing the Multics users subroutines of ios_rawoutput and changemode. This subroutine is called TOUTPT and it will output all control characters. The Plot 10 graphics could then be used to plot the data. TOUTPT does not pertain directly to this report. It is included here as reference material for users of the Multics system.

In this subroutine, k is the desired ASCII control character in decimal number form.

```
toutpt: proc(k) ;
dcl ios_$write_ptr entry (ptr,fixed bin,fixed bin);
dcl buff(3) bit(36) aligned;
dcl k bit(36);
dcl ios_$changemode entry(char(*),char(*),char(*),bit(72)aligned);
dcl ninebits bit(9);
ninebits=substr(k,28,9);
dcl  status bit(72) aligned;
dcl gmode char(32);
dcl sysprint file;
call ios_$changemode("user_i/o","rawo",gmode,status);
buff(1)=(ninebits) || "000000000"b ||"000000000"b ||"000000000"b;
call ios_$write_ptr (addr(buff(1)),0,1);
call ios_$changemode("user_i/o",gmode,gmode,status);
end toutpt;
```

APPENDIX II

References

1. Beauchamp, K. G., 1975, Walsh Functions and Their Applications by Academic Press

2. Gabel and Roberts, 1968, Signals and Linear Systems by Wiley

3. Kunt, Murat, November 1975, IEEE Transactions On Computers Fast Walsh-Hadamard Transform Algorithm

4. Corrington, Murlan S., April 1962, RADC-TDR-62-200, "Advanced Analytical and Signal Processing Techniques (AD 277 942)

METRIC SYSTEM

BASE UNITS:

Quantity	Unit	SI Symbol	Formula
length	metre	m	...
mass	kilogram	kg	...
time	second	s	...
electric current	ampere	A	...
thermodynamic temperature	kelvin	K	...
amount of substance	mole	mol	...
luminous intensity	candela	cd	...

SUPPLEMENTARY UNITS:

Quantity	Unit	SI Symbol	Formula
plane angle	radian	rad	...
solid angle	steradian	sr	...

DERIVED UNITS:

Quantity	Unit	SI Symbol	Formula
Acceleration	metre per second squared	...	m/s^2
activity (of a radioactive source)	disintegration per second	...	(disintegration)/s
angular acceleration	radian per second squared	...	rad/s^2
angular velocity	radian per second	...	rad/s
area	square metre	...	m^2
density	kilogram per cubic metre	...	kg/m^3
electric capacitance	farad	F	A·s/V
electrical conductance	siemens	S	A/V
electric field strength	volt per metre	...	V/m
electric inductance	henry	H	V·s/A
electric potential difference	volt	V	W/A
electric resistance	ohm		V/A
electromotive force	volt	V	W/A
energy	joule	J	N·m
entropy	joule per kelvin	...	J/K
force	newton	N	$kg·m/s^2$
frequency	hertz	Hz	(cycle)/s
illuminance	lux	lx	lm/m^2
luminance	candela per square metre	...	cd/m^2
luminous flux	lumen	lm	cd·sr
magnetic field strength	ampere per metre	...	A/m
magnetic flux	weber	Wb	V·s
magnetic flux density	tesla	T	Wb/m^2
magnetomotive force	ampere	A	...
power	watt	W	J/s
pressure	pascal	Pa	N/m^2
quantity of electricity	coulomb	C	A·s
quantity of heat	joule	J	N·m
radiant intensity	watt per steradian	...	W/sr
specific heat	joule per kilogram-kelvin	...	J/kg·K
stress	pascal	Pa	N/m^2
thermal conductivity	watt per metre-kelvin	...	W/m·K
velocity	metre per second	...	m/s
viscosity, dynamic	pascal-second	...	Pa·s
viscosity, kinematic	square metre per second	...	m^2/s
voltage	volt	V	W/A
volume	cubic metre	...	m^3
wavenumber	reciprocal metre	...	(wave)/m
work	joule	J	N·m

SI PREFIXES:

Multiplication Factors	Prefix	SI Symbol
$1\ 000\ 000\ 000\ 000 = 10^{12}$	tera	T
$1\ 000\ 000\ 000 = 10^{9}$	giga	G
$1\ 000\ 000 = 10^{6}$	mega	M
$1\ 000 = 10^{3}$	kilo	k
$100 = 10^{2}$	hecto*	h
$10 = 10^{1}$	deka*	da
$0.1 = 10^{-1}$	deci*	d
$0.01 = 10^{-2}$	centi*	c
$0.001 = 10^{-3}$	milli	m
$0.000\ 001 = 10^{-6}$	micro	μ
$0.000\ 000\ 001 = 10^{-9}$	nano	n
$0.000\ 000\ 000\ 001 = 10^{-12}$	pico	p
$0.000\ 000\ 000\ 000\ 001 = 10^{-15}$	femto	f
$0.000\ 000\ 000\ 000\ 000\ 001 = 10^{-18}$	atto	a

* To be avoided where possible.

*U.S. GOVERNMENT PRINTING OFFICE: 1977-714-[illegible]