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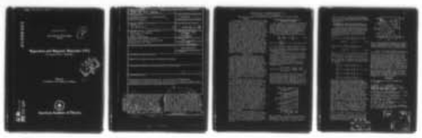

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MAGNON MODES OF A GYROTROPIC MAGNETIC BAR

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ABSTRACT

We present a theory of magnetostatic modes which are wavelike in the direction parallel to the axis of an infinitely long, ferromagnetic bar of square cross-section, and localized at its surface. We solve second order partial differential equations for the magnetic scalar potential by assuming a solution for φ of the form $\varphi(r,\theta,z)=e^{iqz}f(r,\theta)$. We expand $f(r,\theta)$ in terms of basis functions which transform according to the irreducible representations of C_4, the proper point group of the bar. The boundary conditions of continuity of φ and $\vec{B}_n$ across the surface of the bar therefore need to be applied at only one of the four faces of the bar and are automatically satisfied at the remaining faces. This is done by making φ and B_n continuous at a discrete set of points along this face. The solvability condition for the resulting set of homogeneous equations for the expansion coefficients in the expressions for φ yields the dispersion curves for the magnetostatic surface modes. We also present the dispersion curves for the magnetostatic surface modes for a gyrotropic right circular cylinder.

INTRODUCTION

We have recently studied surface[1] and edge[2] localized magnons or spin waves for a simple cubic Heisenberg ferromagnet with exchange interactions between nearest and next nearest neighbor spins. Such modes may play a role in phase transitions[3] and chemical reactions[4] at magnetic surfaces, and may have technological applications in surface or topographic wave guides for magnetic excitations. Inasmuch as the spins in a real ferromagnet interact through long range dipole interactions, as well as through short range exchange interactions, the former of which dominate in the long wavelength limit, it is of interest to see how the results of Refs. 1 and 2 are modified by the inclusion of dipolar interactions between spins.

In a recent study[5] the spin wave spectrum was obtained for a discrete, infinite bar of a simple cubic ferromagnet of square cross section, assuming nearest and next nearest neighbor short-range exchange interactions as well as long-range dipolar interactions. In the presence of the dipolar interactions structure was found in this spectrum which was absent when only the short range exchange interactions were taken into account. The nature of the structure is depicted schematically in Fig. 1, and appears to be of the type, observed in other, similar, contexts,[6] of the repulsion of levels which occurs when the dispersion curve for a localized excitation (a resonance mode) falls in the region of a continuous spectrum. Since this structure appears in the long wavelength limit, in the presence of the dipolar interactions, in the absence of retardation, it is natural to assume that the localized excitation is a magnetostatic mode of the ferromagnetic bar, localized at its surface or edges.

To ascertain the reasonableness of this assumption we obtained the dispersion relations for magnetostatic modes localized at the surface of a ferromagnet in the form of an infinite, right circular cylinder and for magnetostatic modes in a ferromagnet in the form of an infinite bar possessing a square cross section.

MAGNETOSTATIC SURFACE MODES ON A GYROTROPIC CYLINDER

We consider a right circular cylinder of radius a whose axis is parallel to the z-axis. In the magnetostatic approximation we introduce a scalar potential $\varphi(\vec{x},\omega)\exp(-i\omega t)$, in terms of which $\vec{H}=-\nabla\varphi$ and $\vec{B}$ is related to $\vec{H}$ by the magnetic permeability tensor appropriate to a ferromagnet in the presence of a dc magnetic field along the z-axis.

We find that $\varphi(\vec{x},\omega)$ obeys the equation

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\nu(\omega)\,\frac{\partial^2\varphi}{\partial z^2}=0\,, \qquad (2.1)$$

where $\nu(\omega)=\dfrac{\omega_o^2-\omega^2}{\omega_v^2-\omega^2}$ for $o\le r<a$, and $\nu(\omega)\equiv 1$ for $r>a$; $\omega_o=\gamma H_o$, and $\omega_v=\gamma[H_o(H_o+4\pi M_o)]^{\frac{1}{2}}$, where H_o is the dc magnetic field strength and M_o is the saturation magnetization, while γ is the gyromagnetic ratio.

We assume solutions of Eq. (2.1) which are wavelike along the axis of the cylinder. Inside the cylinder the solution is finite at the origin, and increases exponentially with increasing r; outside the solution decreases exponentially with increasing r.

The continuity of φ and of B_r at the boundary r=a yields the pair of dispersion equations whose solutions give the relation between ω and q:

$$(\nu(\omega))^{\frac{1}{2}}\mu_1(\omega)\,\frac{I_n'((\nu(\omega))^{\frac{1}{2}}qa)}{I_n((\nu(\omega))^{\frac{1}{2}}qa)}-\frac{K_n'(qa)}{K_n(qa)}\pm\frac{n\mu_3(\omega)}{qa}=0, \qquad (2.2)$$

where the upper sign obtains for $\omega>\omega_v$, and the lower for $0<\omega<\omega_o$. The $I_n(x)$ and $K_n(x)$ are the modified Bessel functions of the first and second kinds, respectively. It is found that there are no solutions in the frequency range $0<\omega<\omega_o$. Solutions of Eq.(2.2) for $\omega>\omega_v$ have been obtained numerically for several values of n, and are displayed in Fig.

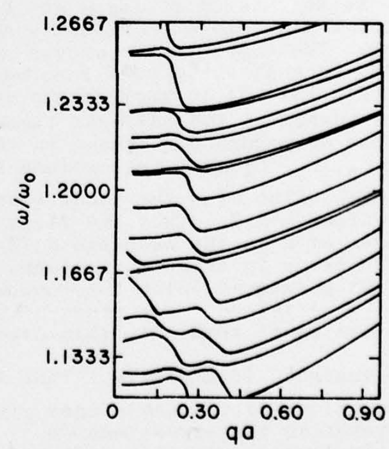

Fig. 1. Schematic depiction of the dispersion relations for spin waves in a discrete bar of a ferromagnet with square cross section, in the long wavelength limit. (ω_v/ω_o=1.00167)

2 for n = 1,2,3,4. (There are no solutions for n = 0.) It should be noted that they have the qualitative form suggested by the results shown in Fig. 1.

MAGNETOSTATIC SURFACE AND EDGE MODES ON A FERROMAGNETIC BAR OF SQUARE CROSS SECTION

We now turn to a solution of Eq. (2.1) for a bar of square cross section, whose axis is parallel to the z-axis. The edge of the square cross section of the bar is 2a, and its center is at x=y=0. The interior and exterior solutions for $\varphi(r,\theta,z;\omega)$ can be written as

$$\varphi_{in}^{(j)}(r,\theta,x;\omega) = e^{iqz}\sum_{n=o}^{\infty}\sum_{\lambda=1}^{2}a_{n\lambda}^{(j)}I_n((\nu(\omega))^{\frac{1}{2}}qr) \times$$

$$\times f_{n\lambda}^{(j)}(\theta) \qquad 0 \le r \le a \qquad (3.1a)$$

$$\varphi_{out}^{(j)}(r,\theta,z;\omega) = e^{iqz}\sum_{n=o}^{\infty}\sum_{\lambda=1}^{2}b_{n\lambda}^{(j)}K_n(qr) \times$$

$$\times f_{n\lambda}^{(j)}(\theta) \qquad r \ge a\ , \qquad (3.1b)$$

where $j = \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4$ labels the irreducible representations of the point group C_4 of the bar, and $f_{n1}^{(j)}(\theta)$ and $f_{n2}^{(j)}(\theta)$ are the parts of $\cos n\theta$ and $\sin n\theta$, respectively, which belong to each of these irreducible representations, for example:

$$f_{n1}^{(\Gamma_1)}(\theta) = \cos\frac{n\pi}{2}\cos^2\frac{n\pi}{4}\cos n\theta$$

$$f_{n1}^{(\Gamma_2)}(\theta) = \cos\frac{n\pi}{2}\sin^2\frac{n\pi}{4}\cos n\theta \qquad (3.2)$$

$$f_{n2}^{(\Gamma_1)}(\theta) = \cos\frac{n\pi}{2}\cos^2\frac{n\pi}{4}\sin n\theta$$

$$f_{n2}^{(\Gamma_2)}(\theta) = \cos\frac{n\pi}{2}\sin^2\frac{n\pi}{4}\sin n\theta$$

With the interior and exterior potentials expanded in terms of these symmetry adapted functions it suffices to satisfy the boundary conditions at only one face of the square bar, e.g., the face defined by $x = +a, -a \le y \le +a$: the boundary conditions are then automatically satisfied at the three remaining faces. In practice we do this by making φ and B_x continuous at a discrete set of points along this face. These points are chosen so as to divide the face $x=+a$, $-a \le y \le +a$ into segments of equal length but not to include the corners, which are shared by two adjacent faces. An even number of points was chosen in the case of modes of $\Gamma_2, \Gamma_3, \Gamma_4$ symmetry because here n>0, and for each value of n the index λ assumes the two values $\lambda=1,2$. Thus the first N nonzero values of n in the expansions (3.1) give rise to 2N terms in these expansions, which requires 2N points at which the boundary conditions are satisfied. For modes of Γ_1 symmetry, n can equal zero. In this case the $f_{n2}^{(\Gamma_1)}(\theta)$ vanishes identically. Thus the first N nonzero values of n in this case give rise to 2N-1 terms in the expansions (3.1), which requires 2N-1 points at which the boundary conditions are satisfied.

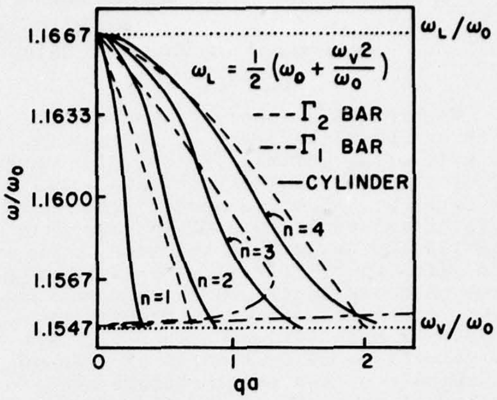

Fig. 2. The dispersion relations for magnetostatic surface modes on a gyrotropic right circular cylinder, and on a gyrotropic bar of square cross section. ($\mu_o=1$, $\omega_v/\omega_o=1.15467$)

In Fig. 2 we present dispersion curves for modes of Γ_1 and Γ_2 symmetry, obtained with N=4 in each case. The convergence of this method seems to be quite good. When these curves were recalculated with N=5 the lower frequency mode of Γ_2 symmetry shifted a maximum of 0.7%, while the higher frequency mode of this symmetry shifted a maximum of 1.1%. The higher frequency mode of Γ_1 symmetry shifted a maximum of 2.5%, while the lower frequency mode shifted by 10%. In each case the shift was in the direction of lower frequency. The higher frequency mode of Γ_1 symmetry and the two modes of Γ_2 symmetry have the form to explain the results shown in Fig. 1, which lends support to the latter advanced in the Introduction. Results for modes of Γ_3 and Γ_4 symmetry are now being obtained.

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