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ICE REGIME OF HYDROELECTRIC STATION PIPELINES

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Conditions favorable for ice formation in water supply lines are analyzed. Methods of calculating the probable ice thickness within pipes are suggested. The thermal regime of water flowing in a pipe, its dependence on outer and inner factors, and measures for the prevention of ice damages are discussed. Special attention was paid to the problem of ice formation in water turbines.

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THE ICE REGIME OF HYDROELECTRIC STATION PIPELINES

By: P. A. Bogoslovskiy

(Conditions favorable for ice formation in water supply lines are analyzed.

The book examines questions of the ice regime of pipelines. Methods of quantitatively calculating the (probable) thickness of the ice layer that forms on the inside surface of the pipe walls are suggested. Means of combatting the harmful consequences of internal icing are evaluated. ~~In passing,~~ the temperature regime of the water moving through the pipes ~~is illuminated~~ (considered) and certain concepts are given regarding the icing phenomena that occur in (water) turbines.

The book is intended for engineer-hydraulic technicians working in the field of designing, operating and investigating hydroelectric stations.

Foreword

Investigations on questions of internal icing of pipes conducted by the author are the basis of the work submitted for the reader's attention. During this process the results of investigations relative to the same questions and cited in literature sources or listed by the author in the appended list of utilized literature, were also considered.

The questions examined in this book are of significance when designing and operating pipelines under comparatively severe climatic conditions. The material on this question published until now, initially in the foreign literature, is totally inadequate with respect to the completeness of examination of the questions. The goal of this work is to fill this significant gap.

The author expresses deep gratitude to Doctor of Technical Sciences, Professor M. M. Grishin, Doctor of Technical Sciences M. F. Menkel', and Candidate of Technical Sciences, Assistant Professor L. G. Skritskiy, as well as to all who have given their suggestions to the author on the subject of his conducted investigations. These critical remarks were taken into account in the process of the author's additional work on the manuscript during its preparation for printing.

The author will be grateful to readers who send in their ideas on the subject of this book addressed to the publisher (Moscow, 114, Shlyuzovaya naberezhnaya, 10, Gosenergoizdat). Signed the author.

Basic Abbreviations, Accepted Values of Physical Characteristics and Units of Measurement

C_w = 1.0 kcal/kg degrees - thermal capacity of water.

C_i = 0.50 kcal/kg degrees - thermal capacity of ice.

E = 427 kgm/kcal - mechanical equivalent of heat.

H m - water pressure in height of water column.

h cm - funicular distance on radial scales.

J - hydraulic gradient.

J_d - design gradient.

L = 79.6 kcal/kg - latent heat of ice melting (heat of change of aggregate state).

m_a - scale in which size of a is depicted on the blueprint; the scale is determined by the number of units of value of a, consisting in 1 cm.

n - coefficient of roughness;

Q m^2/sec - flow rate of water.

$R_{in\ m}$ - inside radius of pipe.

$R_{o\ m}$ - outside radius of pipe.

$R_{cr\ m}$ - radius of free cross-section of pipe during critical icing.

$R_{lim\ m}$ - radius of free cross-section of pipe with limit icing.

r m - radius of free cross-section of frozen pipe.

$r_* = \frac{r}{R_{cr}}$ - relative radius of cross-section of frozen pipe.

t day - time.

w m/sec - velocity of wind blowing pipes.

x m - coordinate along the axis of the pipe; the origin of the coordinate coincides with the entry cross-section.

x_{bl} m - distance from the beginning of the pipe to the point of appearance of icing, i.e., the length of the stretch of pipes free of ice.

a_w kcal/m^2 hr degree - coefficient of heat transfer from water to the inside surface of a layer of ice or the sides of the pipe.

a_n kcal/m^2 hr degree - coefficient of heat transfer from the outside surface of the pipe to the air.

γ_w = 1.0 g/cm^3 - volumetric weight of water.

γ_i = 0.917 g/cm^3 - volumetric weight of ice.

ϑ degree - temperature of water.

ϑ_{in} degree - temperature of water at entry into water.

ϑ_o degree - temperature of outside air surrounding pipe.

ϑ_0 degree - temperature of ice melting.

δ m - thickness of ice layer.

kcal/m hr degree - coefficient of heat conductivity.

Certain Indexes

* - relative value.

w - applicable to water.

i - internal.

int - relative to intake to pipeline.

cri - critical.

i - pertinent to ice.

o - outside.

ini - relative to initial moment of time.

pl - pertinent to the point of appearance of ice on the inside surface of the pipe wall of a stretch free of ice.

w - pertinent to walls.

The following system of units was used for the calculations: meter, day, ton, and degree according to the Celsius scale. In this system of units heat should be measured in megacalories (the megacalorie is the amount of heat required to heat 1 ton of water 1°). However, certain values entering into the calculation are conventionally measured in other units and during their use one must introduce the transfer coefficients. Certain of these coefficients are given below:

1000 kcal = 1 mgkal (amount of heat W);

1 kcal/m hr degree = 0.024 mgcal/m day degree (coefficient of heat conductivity λ);

1 kcal/m^2 hr degree = 0.024 mgcal/m^2 day degree (coefficient of heat transfer α);

1 m^3/sec = 86400 m^3/day (flow rate of water Q);

1 g/cm^3 = 1 t/m^3 (volumetric weight γ);

1 kcal/kg degree = 1 mgcal/t degree (heat capacity C);

1 kcal/kg = 1 mgcal/t (latent heat of change in aggregate state L).

Introduction

The internal surfaces of the walls of hydroelectric station pipelines are covered in certain cases by a layer of ice under the effect of low Winter temperatures. Open pipes that are most often encountered at hydroelectric stations are particularly subject to such internal icing. Internal icing of pipes has been observed in the USSR both in regions with cold winters - at Nivages II (the Kol'sk Peninsula), at the Ul'binsk hydroelectric station (Altay), as well as in the southern regions with comparatively warm winters, for example, the valley of the Terek River on the sag pipes of the Alkhanchurtsk Canal (7). In some cases, icing of the inside surface of pipes does not create operating difficulties and even remains unnoticed; but sometimes it leads to complete halting of the station, as occurred at one of the inter-collective farm's hydraulic stations in the Kirgiz SSR, where measures had to be taken to heat the pipe (17).

Icing of the inside surface of the walls occurs with particular intensity if the pipe is filled with water but the water is not running through it. In this case the thickness of the layer of ice gradually increases and the ice can fill the entire cross-section of the pipe. If a certain flow of water is run through the pipe, then this protects it against complete freezing for the following reasons.

First, during its movement the water liberates a certain quantity of friction heat which prevents freezing. Second, the water running in the pipe can contain a certain amount of heat that causes the same effect. Third, during movement through the pipe the water undergoes pressure changes; the thermodynamic processes that arise in case of an increase in pressure, which usually occurs in the turbines of pipes of hydroelectric stations, and also prevent freezing. The reserves of heat in the water at the time of its entry into the pipe are sometimes quite great (for example, if the water is coming in from a reservoir), which entirely prevents icing on a certain stretch of the pipes adjacent to the entry. The reverse phenomenon is also possible, when the water entering the pipe has a temperature near the freezing point, as the result of which very strong icing of the internal surfaces of the walls near the intake to the pipeline proportional to recession from the intake, this icing diminishes.

The entry of brash ice together with water into pipelines that receive water from open streams (rivers and canals) is possible in Winter. One can assume that brash ice will facilitate an intensive increase in the thickness of the ice layer on the inside surfaces of the wall. With a large content of brash ice in the water, the brash ice can cause complete blockage of the pipe by ice.

If slight icing of the pipe causes no interference in operations and even remains unnoticed, then heavy icing can entail various complications. Internal icing causes a reduction in the handling capacity of the pipe, which is occasionally extremely significant. With internal icing the velocity of the water increases, which facilitates an increase in pressure with hydraulic shock. The increase in pressure with hydraulic shock is particularly noticeable in wooden pipes, since icing of the inside surfaces of the walls prevents the escape of water through the grooves between the staves that is possible under conditions of no icing. During thaws or with strong insulation, the layer of ice can separate from the walls and form an ice-gang in the pipe, which has a ruinous effect on the mechanisms located downstream. Icing disrupts the normal operation of pipeline valves, ventilation devices and compensaters.

Various methods of thermal insulation are employed for the purpose of combatting internal icing of pipes. Wooden and reinforced concrete superstructures are built over pipelines which do not hinder inspection and preventive maintenance. Pipes are buried in the ground, covered with concrete sections laid in covered trenches, covered on the outside by a layer of ice, piles of snow, etc. In order to reduce heating by radiant energy, pipes are painted white or are placed under screens. Stretches of pipe with mechanisms are covered with heated housings.

It becomes obvious from all of the above that the range of questions involved with planning the pipeline should include the analysis of its ice regime.

Ice phenomena in the broad meaning of the word already long ago interested Soviet scientists and engineers who worked in the field of hydraulic technology. Thus, in 1929 Professor V. Ye. Timonov suggested a plan for constructing an ice-testing station on the Neva River (21). In 1930, a group of ice technologists was organized under his supervision as a part of the hydraulic sector of the State Institute of Facilities (22).

Finally, the second All-Union Conference on the Operation of Hydroelectric Power Stations, held in the Summer of 1946 in Leningrad, directed attention to the significant dimensions of ice interference and suggested that scientific research institutes undertake studies in the field of ice phenomena and give practical instructions for conducting normal winter operation of hydroelectric stations (6).

Questions of the internal icing of pipes are of interest not only to hydraulic technicians, but also to the petroleum industry, where hardening of petroleum products similar to the freezing of water is possible.

Soviet engineers and scientists concerned with the indicated fields of technology have done a great deal for studying the question of internal icing of pipes and for combatting the harmful consequences of icing. We shall list several of these works.

Professor M. Ya. Chernyshev, in a book published in 1933 (25), posed the problem of calculating the heat of hydrodynamic resistances in pipes.

In 1931, Professor L. S. Leybenzon (14) published an extremely valuable solution to the problem of hardening of petroleum products in the pipeline for practical use, with respect to a line through which there was no liquid flow. The obtained conclusions are entirely applicable to hydrotechnical pipelines. From a theoretical standpoint, a similar solution was published in the same year by V. S. Yablonskiy, P. P. Shumilov and V. M. Pokrovskiy (27), on the subject of hardening of benzene in railroad tank cars.

In 1935, A. Ya. Popkov published (19) the results of experiments on the freezing of water in wooden and cast iron pipes, and the suggestions of L. S. Eygenson regarding the thermal calculation of small diameter pipes were published in that same year.

In 1939, Professor A. M. Yestifeyev published (8) a formula for calculating the icing of a pipeline that takes into account the heat of hydrodynamic resistances of moving water, and in 1941 (9) published the derivation of this formula.

In 1939, Projects of Technical Conditions and Standards of Hydrotechnical Planning (20) were published with a section on hydrothermal calculations and measures of combatting ice difficulties in hydraulic power plants, compiled by Engineer M. M. Dasin, Professor G. K. Lotter and Engineer B. V. Proskuryakov. Among a veritable number of questions, recommendations are given there concerning the quantitative calculation of internal icing of pipes. This work is the most extensive one on the given question.

In 1941, engineers N. N. Petrunichev and engineer G. S. Shadrin (18), followed by Candidate of Technical Sciences, G. S. Shadrin in 1947 (26), published works of a theoretical character concerning thermal processes involved with the operation of a pipeline laid in permafrost.

The results of many of these works will be cited below.

Besides works of a theoretical character, there are a number of articles in Soviet journals about different ice phenomena that were observed during the operation of pipelines, i.e., which illuminate the practical aspect of this question.

In addition to the investigations listed above, carried out in the USSR and directed toward clarifying the mechanism of internal icing of the pipeline and mastering its regulation, only a few recommendations of a "recipe" character appeared abroad by 1929 on the given questions. These linked the normal function of the pipeline first with the flow rate of water, then with its temperature and gave results several times too high (23). By 1937, the situation abroad regarding the investigation of the question of the ice regime of pipelines had not improved, which is indicated by certain materials of foreign consultation (10), where previous recommendations are cited and new ones are given that differ little in their character and essence from the former ones. Such a weak study of the given problem abroad cannot be explained by the absence of requirements of practice. Cases of accidents and interference involving the internal icing of pipelines and turbines are known (2 and 12). Especially interesting in this regard is the case of strong icing in the Winter of 1924/1925 on a 13 km stretch of wooden pipeline 71 cm in diameter forming a part of the water supply system of the city of Everett (Washington state, USA).

As the result of this icing, the city was without water for 5 days. The warming that ensued caused separation of the layer of ice from the walls and the formation of an ice-gang in the pipeline.

The achievements of Soviet engineers and scientists in investigating the internal icing of pipelines is of certain value from the standpoint of knowledge of this question and cannot be compared with the recommendations of the foreign literature referenced above.

However, despite these achievements, today a widely accepted method and practical ways of planning the ice regime of pipelines are lacking. This is particularly acutely perceived when planning pipelines of hydroelectric stations located in regions with cold winters.

This work has the purpose, on the one hand, of making a certain contribution to the investigation of internal icing of pipelines, and on the other hand, of suggesting methods and practical ways of quantitative calculations of icing that correspond to the modern level of knowledge. Its pressing nature is determined by the decision of the second All-Union Conference on the Operation of Hydroelectric Power Stations referenced above.

Chapter 1

The Heat Balance Equation

The thickness of the layer of ice that forms on the inside surfaces of the walls of a pipe depends on different factors that determine the thermal regime of the walls and the water running through the pipe. These factors are calculated in the heat balance equations which include the available reserves of heat as well as its intake and release. The heat balance equations are formulated below both for the entire pipeline as a whole (for the walls and the water), and for the frozen walls and running water separately. These equations serve as the basic relationships on whose basis all subsequent calculations are made.

In No. 1 of this chapter, the heat balance equations are formulated only in schematic notations. In the next paragraph the quantitative values of separate components of heat balance are determined. In No. 3 the substitution of these values in the heat balance equations is carried out and therefore differential equations are formulated,to whose solution the next chapters are devoted.

1. The Heat Balance Equations in the Schematic Notations

The heat balance equation shows that the intake, release and change of the heat balance of a given heat that occurs over a certain period of time is zero in the algebraic sum. Mathematically, this can be written as follows:

$$a_1 + a_2 + a_3 + \ldots + a_n = \sum_{i=1}^{i=n} a_i = 0, \tag{1}$$

where a_1, a_2, a_3, ..., a_n represent the different components of the heat balance.

The heat balance equation can be viewed as an expression of conservation of energy for processes that are accompanied by thermal phenomena (the first law of thermodynamics). In its form, the heat balance equation is similar to the continuity equation in hydraulics.

The basic task in formulating the heat balance equation in schematic notations is the calculation of all possible thermal changes.

Below, in many cases, after the schematic notations of the values their dimensionality is given in the units of measurement accepted for the subsequent calculations. This is done exclusively for the clearer representation of the character of the given value. It is entirely obvious that the general theoretical conclusions do not in this case lose their significance with choice of another system of units of measurement.

A. The General Heat Balance of a Pipeline with Ice on the Inside Surfaces of the Walls and with Water Running Through the Pipeline

Figure 1 schematically shows the cross- and longitudinal sections of a frozen pipe. In order to determine the positions of the cross-sections, we conventionally considered the coordinate axis OX as coinciding with the axis of the pipe and running in the direction of the flow of water. The origin of the coordinate axis is assumed to coincide with the beginning of the pipe, i.e., with the section through which water enters the pipe.

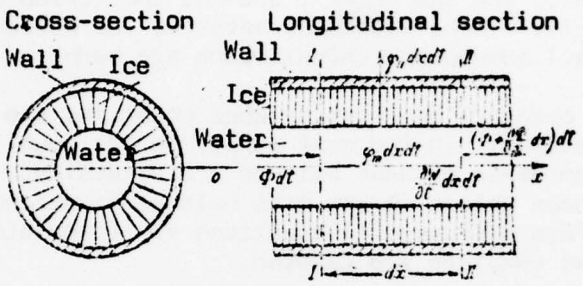

Figure 1. Diagram of components of the general heat balance of a pipe.

We isolate two planes I - I and II - II normal to axis OX, an elementary stretch of pipe having a length dx (Figure 1), and for this stretch of pipe formulate the heat balance equation that is the general one for the walls of the pipe, ice on the walls, and water running through the pipe. This balance pertains to an elementary span of time dt.

Water, as the heat carrier, caries heat in the amount Φ mgcal/day through the cross-section I/I per unit of time. Over the elementary period of time dt days, the water introduces an amount of heat Φdt mgcal into the examined stretch of pipe having a length dx m through cross-section I - I, and over the same span of time carries away heat through cross-section II - II in the amount $\left(\Phi + \frac{\partial \Phi}{\partial x} dx\right)$ mgcal. Hence, the examined elementary stretch of pipe receives an increment of heat in the amount

$$\Phi dt - \left(\Phi + \frac{\partial \Phi}{\partial x} dx\right) dt = -\frac{\partial \Phi}{\partial x} dx dt \text{ mgcal},$$

which should enter the heat balance equation as a component.

The movement of water through the pipe causes hydraulic friction. The energy expended on friction is converted into heat. We designate the amount of heat developed during friction per unit of length of the pipe per unit of time as q_m mgcal/m day. The examined elementary stretch of pipe over the elementary span of time receives friction heat in the amount $q_m dxdt$ mgcal , which should enter the heat balance equation.

The layer of ice on the inside surface of the pipe walls is a unique heat accumulator because of the latent heat of melting (the heat of change of the aggregate state). We shall designate the reserves of heat contained in the layer of ice in the form of latent heat as W mgcal/m. Calculation of the changes of these reserves of heat in the layer of ice in the elementary stretch of pipe and over the elementary span of time is carried out in the heat balance equation by the following component $-\frac{\partial W}{\partial t} dxdt$ mgcal . The minus sign is placed there because the increase in the reserve of heat of the change in the aggregate state that occurs during the melting of ice is accompanied by the absorption of heat.

The walls of the pipe are either directly or through some additional covering layer (thermal insulation, filler, etc.) in contact with the air surrounding the pipe , which has a low temperature and imparts a certain amount of heat to it that can be viewed as thermal losses of the pipe. We shall designate the value of thermal losses that fall per unit of length of the pipe per unit of time as q_n dxdt mgcal. Then it is necessary to take into account thermal losses in the amount of $q_n dxdt$ mgcal in the heat balance equation for the elementary stretch of pipe over the elementary period of time. The minus sign shows that heat is lost.

As the result of the possible changes of temperatures, the reserves of heat determined by the property of heat capacity of the bodies will change. Losses of heat on the change in temperature per unit of time per unit of length of pipeline are designated as follow: for the walls (and insulation, if there is any), as q mgcal/m day, as q_{est} mgcal/m day day for the layer of ice on the inside surface of the walls, and as q_{ev} for water in the pipe. Total losses of heat on the change in temperature will be the following:

$$q_e = q_{est} + q_{el} + q_{ev} \text{ mgcal/m day}$$

Hence, the amount of heat spent over the elementary period of time on the change in reserves of heat within the examined elementary

stretch of pipe will be expressed in the following way as the result of the property of heat capacity: $\varphi_c dx dt$ mgcal . The minus sign shows that the increase in heat content involves the absorption of heat.

Finally, the heat that appears during the compression of water as the result of the increase in pressure which it can experience in running through the pipe should be considered in the heat balance equation. We shall designate the amount of this heat that arises per unit of length of the pipe per unit of time as φ_d mgcal/ m day . The component $\varphi_d dx dt$ mgcal should enter the heat balance equation of the elementary section of the pipeline over the elementary period of time. The list of components of heat balance concludes with this. In Figure 1 the arrows show the heat fluxes between the element of the pipe and the air surrounding it, as well as the stretches of pipe to the left of cross-section I - I, and to the right of cross-section II - II. The heat balance components themselves that do not exceed the limits of the examined stretch of pipe are not shown by arrows. Components φ_e and φ_d are not noted in the diagram, since their role in the balance is extremely slight, which will be shown subsequently.

In accordance with equation (1), we shall formulate the algebraic sum of the listed elements and equate it to zero:

$$-\frac{\partial \varphi}{\partial x} dx dt + \varphi_m dx dt - \varphi_n dx dt - \\ -\frac{\partial W}{\partial t} dx dt - \varphi_e dx dt + \varphi_d dx dt = 0.$$

By removing dxdt, we obtain:

$$-\frac{\partial \varphi}{\partial x} + \varphi_m - \varphi_n - \frac{\partial W}{\partial t} - \varphi_e + \varphi_d = 0. \qquad (2)$$

This equation is the sought expression of the general heat balance of the pipeline with ice on the inner surface of the walls and with running water. The equation has been formulated in the symbols that determine the separate components of heat balance.

B. The Heat Balance of Water Running Through the Pipeline

The heat balance equation for water running through the pipeline is formulated in exactly the same way. This equation should include the following components, calculated for brevity per unit of length of the pipe and per unit of time:

$-\frac{\partial \Phi}{\partial x}$ mgcal/m day - heat lost by a section of pipeline and defined as the difference in heat introduced and carried away by the water;

φ_d mgcal/m day - heat of friction;

φ_m mgcal/m day - heat necessary as the result of the property of heat capacity for changing the temperature of the water within the examined section;

$-\varphi_{ev}$ mgcal/m day - heat that appears during pressure changes.

In addition to these components already known, one must still introduce the component - φ_{vl} mgcal/m day into the heat balance of water. This component defines the amount of heat exchanged from the water to the layer of ice that forms on the inside surfaces of the walls.

According to the principle expressed in equation (1), the equation of the heat balance for water moving through the pipeline is formulated:

$$-\frac{\partial \Phi}{\partial x} + \varphi_m - \varphi_{ev} + \varphi_d - \varphi_{vl} = 0. \qquad (3)$$

C. The Heat Balance of the Frozen Walls of the Pipeline

The heat balance equation of the walls together with the layer of ice that has formed on their inside surfaces is formulated from the following components:

- φ_n mgcal/m day - heat losses;
- $\frac{\partial W}{\partial t}$ mgcal/m day - change in reserves of heat in the layer of ice due to the latent heat of change of the aggregate state;
- φ_{st} mgcal/m day - heat necessary as the result of the property of heat capacity for a change in the temperature of the walls of the pipeline;
- φ_{el} mgcal/m day - heat similar to the latter, required for the layer of ice;
- φ_{vl} mgcal/m day - heat exchanged from water to the layer of ice.

The heat balance equation of the walls and the ice on them has the following appearance in the symbolic notations:

$$-\varphi_n - \frac{\partial W}{\partial t} - \varphi_{est} - \varphi_{el} + \varphi_{vI} = 0. \qquad (4)$$

The formulated heat balance equations are interrelated. Thus, by means of addending the heat balance equations for water (3) and the walls (4), one obtains the equation for the pipeline as a whole (2). Therefore, from the three-component equations (2), (3), and (4), only two are properly equations, and one will be an identity deriving from the other two.

Having thereby formulated the heat balance equations in the schematic notations, we proceed to determining the separate components.

2. The Separate Components of Heat Balance

The purpose of this section is to explain the relationships that determine the separate elements of the heat balances formulated earlier. Before proceeding to accomplish this task, we shall settle certain general concepts.

All relationships will be derived with the assumption that the pipeline has a circular section and is under identical conditions in the sense of heat exchange from all directions. Under these conditions one should anticipate that icing of the inside surfaces of the pipe at any moment in time and in any section will be a ring bounded by concentric circumferences. If one uses the cylindrical system of coordinates with a system axis that coincides with the axis of the pipe, then the degree of icing corresponding to each cross-section with a coordinate x can be determined only by the single radius of cross-section r without reference to the central angle of the cylindrical coordinate system.

It is extremely simple to determine the thickness of the ice layer that formed on the inside surface of the walls along the radius of the cross-section of a frozen pipe:

$$\delta = R_{in} - r \text{ м}, \qquad (5)$$

where δ m - thickness of ice layer;

R_{in} m - inside radius of pipe;

r m - cross-section radius.

We shall agree to consider that all bodies except the ice have no heat reserve at a temperature of 0°, that at a temperature higher than 0° they have a positive reserve as the result of properties of heat capacity, and that at a temperature below 0° they have a negative reserve (a reserve of "cold"). For ice the indicated condition is compounded by the fact that ice always has a negative reserve of the latent heat of change in the aggregate state.

Below values enter into the calculations which determine the physical properties of the ice. These numerical values are taken on the basis of data cited in the work of Professor B. P. Veynberg (5).

A. Heat Carried by Water Through the Pipeline

As a heat carrier, water carries a certain amount of heat that is in it due to the property of heat capacity during its movement. By subtracting, as was agreed upon above, the reserves of heat of water from the state at 0°, we obtain the expression for the amount of heat carried by the stream of water through a certain cross-section in the following form:

$$\Phi = 86400 Q \gamma_w C_w \vartheta \text{ mgcal/day}$$

where

Q m^3/sec - flow rate of water running through the pipeline;

γ_w = 1 t/m^3 - volumetric weight of water;

C_w = 1 kcal/kg degree = 1 mgcal/t degree - heat capacity of water;

ϑ degree - temperature of water.

The factor 86400 is introduced for the conversion to days from seconds that exists in the dimensionality of the flow rate.

A partial derivative with respect to length (along coordinate x) from the heat flux carried by the water enters the equations of heat balance. In order to calculate this derivative, it is vital to clarify which values in the latter formula are variable with respect to x. Actually, one should examine two values: Q - flow rate and ϑ - water temperature.

The flow rate of water Q running through the pipe, although it is a variable as the result of melting or freezing of ice on the inside surfaces of the walls of the pipeline, are such small changes that they

can be ignored and one can consider the value of the flow rate to be constant throughout the length of the pipeline.

Relative to water temperature ϑ of the water running through the pipe, one can certainly conclude that in the general case it will be a variable value both with respect to the length of the pipeline and in time.

On the basis of the concepts given above, one finds the partial derivative of the heat flux carried by water according to x:

$$\frac{\partial \Phi}{\partial x} = 86\,400 Q \gamma_w C_{w/v} \frac{\partial \vartheta}{\partial x} \text{ mgcal/ t day} \qquad (6)$$

This expression will be substituted in the heat balance equations in the future.

B. The Heat of Friction

a) Basic relationships. The movement of water along the pipeline causes certain losses of energy on hydrodynamic friction. In this case heat is released in the water, but not in the walls of the pipeline, and rapidly spreads throughout the free cross-section due to the intensive turbulent mixing. An exception is the thin layers of water adjacent to the walls of the pipeline, where mixing is limited. Consequently, one should expect that the entire area of the free cross-section of flow, with the exception of the indicated thin layers of water, should have an identical temperature ϑ degrees.

The energy lost on hydrodynamic losses is calculated according to the ordinary hydraulic formulas. The amount of heat that arises in the pipeline as a consequence of friction during the movement of water is determined by the following relationship:

$$q_m = 86\,400 \frac{1}{E} \gamma_w Q J \text{ mcal/m day} \qquad (7)$$

where

E = 427 kg m/kcal = 427 t m/mgcal - mechanical equivalent of heat;

γ_w = 1 t/m^3 - volumetric weight of water;

Q m^3/sec - flow rate of water in the pipeline;

J - hydraulic gradient in fractions of a unit.

Having used the known formula:

$$v = C\sqrt{RJ} = \frac{1}{\gamma_g} C \sqrt{rJ} \text{ m/sec},$$

one can formulate the following two relationships:

$$Q = \frac{1}{\sqrt{2}} \pi r^{2,5} C J^{0,5} \text{ m}^3/\text{sec}$$

and

$$J = \frac{2v^2}{C^2 r}.$$

In the last three formulas, the following symbols are used:

v m/sec - velocity of water in the pipeline;

$R = \frac{r}{2}$ m - hydraulic radius;

r m - radius in the inside of a pipe frozen on the inside;

C - coefficient.

The known formula is used to calculate this coefficient:

$$C = \frac{1}{n} R^{1/6} = \frac{1}{n} \left(\frac{r}{2}\right)^{1/6},$$

where n - coefficient of roughness.

By introducing this formula into the two previous ones, we obtain the following identical equations:

$$Q = \frac{\pi}{2^{2,5} n} r^{2,5} \sqrt{J} = 0{,}633 \frac{\pi}{n} r^{2,5} J^{0,5} \text{ m}^3/\text{sec} \tag{8}$$

$$J = \frac{2^{1,5} n^2 Q^2}{\pi^2 r^{5,5}} = 2{,}52 \frac{n^2 Q^2}{\pi^2 r^{5,5}}. \tag{9}$$

By substituting the expressions for Q for J from the latter formulas in equation (7), one can obtain the relationship of the heat of friction and the flow rate of water or hydraulic gradient:

$$\varphi_m = 218 \cdot 10^3 \frac{n^2 Q^3}{E \pi^2 r^{5,5}} \text{ mgcal/m day}, \tag{10}$$

or

$$\varphi_m = 54\,600 \frac{\pi r^{2,5} J^{1,5}}{En} \text{ mgcal/m day}. \tag{11}$$

Both of these expressions deserve attention during the engineering calculation, since the hydraulic regime of the pipeline can be given both by the flow rate that must run through the pipeline and by the hydraulic gradient, which cannot be exceeded (of course, there can be

hydraulic conditions that are combinations of these two basic conditions). Depending on any of the hydraulic conditions, it is necessary to choose formula (10) or (11) to determine the amount of heat of friction.

b) Choice of the roughness coefficient. n - the roughness coefficient of the ice surface - enters into both formulas (10) and (11) which determine the heat of friction. Its value is extremely indeterminate. Judging by the data cited by engineer P. N. Belokon' (3), the value of roughness of the subice surface in rivers is significantly greater in fresh ice at the beginning of the period of stable ice on open water in comparison with that at the end of this period. This is because at the beginning of Winter the lower surface of the ice can be extremely uneven, since the ice cap forms from the material of the Fall ice-gang and separate floes freeze together along inclined planes. Furthermore, brash ice accumulates in the beginning of Winter under the ice cover. This brash ice is very rough. The appearance of brash ice is most probable at increased speeds; this circumstance is perhaps an explanation of the fact that the high degrees of roughness correspond to the high speeds.

Turning to the ice regime of the pipeline, one can assume that the ice layer on the inside surfaces of the walls forms from the water directly frozen. In the presence of brash ice the layer of ice on the walls can be formed from water that freezes in place with the inclusion of ice crystals brought in from outside (particles of brash ice) that adhere to the layer. In any case, the participation of large chunks of floating ice (material of the broken surface of the ice cover of streams and reservoirs) in the formation of the ice layer is improbable. Therefore, one can assume that in the period of growth of the ice layer roughness of the ice surface in the pipeline will be less than in rivers and canals and will not depend on the flow velocity of the water.

A decrease in thickness of the ice layer is due to its melting as the result of the heat in the water. In this case a "polishing" of the surface of the ice in contact with the water occurs. Under conditions of open streams of water, polishing of the ice cover occurs at the end of winter or in early spring, when the influx of heat from water proves to be greater than the heat losses of ice to the atmosphere. In a pipeline, melting of the ice layer and the polishing of its internal surface can ensue much earlier than in open streams of water, and are repeated significantly more often, for example, due to an increase in flow rate in the pipeline, i.e., smoothing of the ice surface in the pipeline will occur much more energetically than in rivers and canals.

All of the above leads to the conclusion that the ice surface inside pipelines changes its roughness depending on ice and temperature conditions. However, for all cases here of change in icing (freezing), only the coefficient of roughness has significance, since the introduction of certain

values of this coefficient without making significant refinements in the derivations significantly complicates the process of calculations.

For pipelines, the value of the coefficient of roughness is taken at n = 0.01, which is the minimum value for rivers and canals. This choice has been made on the basis that processes of melting in the pipeline are repeated more often than in a river, and smoothing of the surface of ice washed by the water therefore occurs more energetically. In order to evaluate the chosen value of the coefficient of roughness, it is interesting to report that a value of n = 0.012 is suggested in the Plan of the TUiN of Hydrotechnical Design (L. 20).

It is vital to note that the value of the coefficient of roughness was calculated by Engineer P. I. Belokon' (3) according to the formula given below:

$$C = \frac{1}{n} R^{1/6},$$

which justifies the use of this formula in this case as well.

Below it becomes necessary to calculate the temperature of water in stretches of pipeline without internal icing of the walls. Formulas (10) and (11), which determine the heat of friction, are quite suitable for this purpose. The value of the coefficient of roughness itself should be chosen corresponding to the walls of the pipeline. In the first approximation, one can accept n = 0.010, which corresponds to rigid boards (n = 0.011 -- new cast iron and steel pipes well-laid and joined), i.e., a value that coincides with the value of the coefficient of roughness selected for ice.

Here one can limit the discussion of the question of the quantitative estimate of the heat of friction that arises as the result of the flow of water through the pipeline.

C. LOSSES OF HEAT BY THE PIPELINE

a) The method of calculation. If water is in the pipeline (running or standing) and the temperature of the air is negative, then a temperature drop and the related flux of heat arise in the walls of the pipeline (including freezing and insulation). The heat flux moves from the water to the atmosphere. The heat lost into the atmosphere from the pipeline should be considered lost from the latter. In the general case, calculation of the amount of heat loss involves solving a problem of a non-steady state (time-variable) temperature field in the walls of the pipeline because the thickness of the layer of ice on the inside surfaces of the walls will change. This problem, relating to the field of mathematical physics, can be stated in differential equations, but the solution has not yet been found. Therefore, here one must use approximate solutions. Significant simplifications are obtained if one considers that the material of the pipeline walls and the ice layer do not have thermal capacity. Having made such an assumption, one can consider that

in any moment of time a steady state distribution of temperatures between ϑ_0 -- temperature of melting of ice on the surface of the boundary between water and ice and ϑ_o -- temperature of the outside air surrounding the pipeline is established in the walls of the pipeline (in the layer of ice, properly, in the walls and insulation). The laws of the steady state distribution of temperatures in the walls of a cylinder are known, and therefore calculation of the value of heat losses does not pose difficulties.

Since there is always a certain temperature ϑ_0 on the surface of the boundary of water and ice, then the water temperature will have no effect on the value of heat losses.

Having accepted a steady state distribution of temperatures in the walls, one can calculate heat losses by the pipeline according to the formula known in heat exchange (11), which, using our symbols, has the following form:

$$\varphi_n = \frac{0{,}024 \cdot 2\pi(\vartheta_0 - \vartheta_o)}{\frac{1}{\lambda_i}\ln\frac{r}{R_{a\,out}} + \sum_{\text{for all } i}\frac{1}{\lambda_i}\ln\frac{R_{i\,a\,Q}}{R_{i\,out}} + \frac{1}{\alpha_{out}R_{out}}}; \quad (12)$$

where φ_n mgcal/m day -- heat losses by one running meter of pipeline per day;

ϑ_o -- temperature of the air surrounding the pipeline;

ϑ_0 degrees -- temperature of melting of ice; it is so small with respect to the absolute value (see Table 5) in comparison with ϑ_o, that in many of the subsequent calculations it can be ignored;

λ_i = 2.0 kcal/m hr degree -- coefficient of heat conductivity of the ice;

a_o kcal/m^2 hr degree -- coefficient of heat exchange between the outside surface of the pipeline walls and the air surrounding the pipeline;

R_{in} m -- inside radius of pipe;

R_o m -- outside radius of pipe (along the edge in contact with the air);

i (index) -- number of different layers comprising the wall of the pipeline (properly, the walls and the various types of insulation with which the pipeline can be covered); each such layer is characterized by its own:

λ_i kcal/m hr degree -- coefficient of heat conductivity;

$R_{i\ in}$ -- internal radius and $R_{i\ out}$ -- outside radius;

r m -- inside radius of icing (radius of the free cross-section of the frozen pipeline).

0.024 -- coefficient for converting the values of λ and a from the dimensionality kcal/m hr degree and kcal/m^2 hr degree into the dimensionality mgcal/m day degree and mgcal/m^2 day degree, i.e., into the system of units used in this presentation.

Values of the coefficients of heat conductivity of the material of the walls entering into formula (12) should be taken according to the tabular data. Certain data are given below regarding the value of the coefficient of heat exchange from the inside surface of the pipeline walls to the air.

b) The coefficient of heat exchange from the outside surface of the pipeline walls to the air. In order to calculate the coefficient of heat exchange between the outside surface of the wall and the air, the following formula is given in the plans of TUiN of Hydrotechnical Design:

$$a_o = 4\frac{w^{0,7}}{d^{0,3}} \text{ kcal/m}^2 \text{ hr degrees;} \qquad (13)$$

where w m/sec -- velocity of wind blowing pipeline;

d m -- outside diameter of pipe.

M. V. Kirpichev, M. A. Mikheyev and L. S. Eygenson (11) cite a formula that generalizes the data of various experiments in the following form:

$$Nu = cRe^n \cdot Pr^{0,31};$$

where c and n -- values that depend on the value of criterion

$$Re = \frac{wd}{\nu}.$$

The other criteria have the value

$$Pr = \frac{\nu}{a} \text{ and } Nu = \frac{\delta_{out}}{\lambda},$$

where d m -- outside diameter of pipe;

λ kcal/m hr degree -- coefficient of heat conductivity of the air;

ν m^2/sec -- kinematic coefficient of viscosity of the air;

w m/sec -- velocity of wind;

a_o kcal/m^2 hr degrees -- coefficient of heat exchange between the outside surface of the walls and the air;

a m^2/sec -- coefficient of heat conductivity of the air.

Under ordinary conditions, values of criterion Re for pipelines lie with-

in limits of 10^3 -- $50 \cdot 10^3$ or somewhat exceed the upper limit. For this interval of values, c = 0.218 and n = 0.60 (11) are recommended.

The value of criterion Pr for the air with atmospheric pressure and temperature within limits of 0 -- -20° changes little and has a value of Pr = 0.724 (11). By substituting these values in the formula which determines the relationship between the criteria, we obtain:

$$\alpha_{out} = 0{,}197 \frac{\lambda}{\nu^{0,6}} \cdot \frac{w^{0,6}}{d^{0,4}} \text{ kcal/m}^2 \text{ hr degrees.}$$

With the pressure and temperature of the air indicated above, its coefficient of heat conductivity will be λ = 0.02 kcal/m hr degrees and the coefficient of kinematic viscosity $\nu = 12 \cdot 10^{-6}$ m/sec (see Table 1 below); by substituting these constants with consideration of dimensionality, we obtain:

$$\alpha_{out} = 3{,}55 \frac{w^{0,6}}{d^{0,4}} \text{ kcal/m}^2 \text{ hr degrees.}$$

The latter formula is similar to the one described earlier (13). For subsequent use, formula (13) is selected. This formula promises to yield higher values of the coefficient of heat exchange at which more icing will be obtained, which enters the reserve of calculation. When making calculations according to formula (13), one should bear in mind that the wind velocity given ordinarily in the meteorological manuals pertains to a wind sock placed on a column at a height of 5 -- 20 meters above ground level. The velocity of the wind blowing a pipeline laid a low height from the surface of the ground will be less. The formulas that permit one to determine the velocity of the wind at a low height according to the data of meteorological observations made with a wind sock are given in the meteorology courses.

For a windless case, the Plans of TUiN of Hydrotechnical Design (20) recommend the formula

$$\alpha_{out} = \frac{\lambda}{d} 0{,}468 \sqrt[4]{B} \text{ kcal/m}^2 \text{ hr degrees} \qquad (14)$$

where λ kcal/m hr degrees -- coefficient of heat conductivity of the air

$$B = \frac{d^4 \gamma}{9{,}81 \cdot \mu^2} \ln \left(\frac{273 + \vartheta_{cm}}{273 + \vartheta_{out}} \right);$$

ϑ_{cm} degrees -- temperature of the surface of the pipeline;
γ kg/m^3 -- volumetric weight of air;
μ kg sec/ m^2 -- coefficient of viscosity of the air.

The values of the physical characteristics of the air are given in Table 1.

Table 1

Certain Physical Characteristics of the Air and Atmospheric Pressure

Characteristic \ t_{out}	−20°	−10°	0°
Volumetric weight γ, kg/m^3	1.39	1.34	1.29
Coefficient of viscosity, μ, kg sec/m^2	$1.59 \cdot 10^{-6}$	$1.65 \cdot 10^{-6}$	$1.71 \cdot 10^{-6}$
Kinematic coefficient of viscosity ν, m^2/sec	$11.3 \cdot 10^{-6}$	$12.1 \cdot 10^{-6}$	$13.0 \cdot 10^{-6}$
Coefficient of heat conductivity of the air λ, kcal/m hr degrees	0.0190	0.0197	0.0203

The practical use of formula (14) is extremely limited, since a comparison of the values of a_{out} calculated according to it and according to formula (13) leads to the conclusion that "absence" of wind should be understood to mean a movement of air at velocities of less than 0.1 mm/sec, and such a state of calm of the air surrounding the pipeline can only exist in rare cases.

The cited formulas for the coefficient of heat exchange do not take into account the positions of the pipeline relative to objects surrounding it and relative to the horizontal plane, while the indicated factors do play a significant role. Therefore, these formulas are only approximate.

c) The corrected radius. For convenience of the subsequent calculations, it is suggested to introduce the concept of the corrected radius R_{cr}. The actual pipe is replaced by a certain fictitious one whose walls consist of ice. The internal radii of the fictitious pipe and the actual one are identical, but the outside radius of the fictitious ice pipe R_{cr} (corrected radius) is such that with it both pipes are equal from the viewpoint of heat loss through the walls, i.e., the value q_n for both pipes is identical. This condition is described as follows:

$$q_n = \frac{0{,}024 \cdot 2\pi (t_{in} - t_{out})}{\frac{1}{\lambda_i} \ln \frac{r_{in}}{R_{in}} + \sum_i \frac{1}{\lambda_i} \ln \frac{R_{io}}{R_{in}} + \frac{1}{\alpha_o R_o}} =$$

$$= \frac{0{,}024 \cdot 2\pi (t_{in} - t_{out})}{\frac{1}{\lambda_i} \ln \frac{r_{in}}{R_{in}} + \frac{1}{\lambda_i} \ln \frac{R_{out}}{R_{cr}}},$$

where the first expression for q_n corresponds to the real pipe and the second to the fictitious one. The expression for q_n, corresponding to the fictitious pipe, is conveniently reduced to the form:

$$q_n = \frac{0{,}024 \cdot 2\pi (t_{in} - t_0)}{-\frac{1}{\lambda_i} \ln \frac{r}{R_{cr}}} \text{ mgcal/m day} \qquad (15)$$

where R_{cr} m -- corrected radius determined from the following equation:

$$-\frac{1}{\lambda_{cm}} \ln \frac{R_{in}}{R_{cr}} = -\sum_i \frac{1}{\lambda_i} \ln \frac{R_i}{R_{i0}} + \frac{1}{\alpha_0 R_0}.$$

or

$$R_{cr} = R_{in} e^{\lambda_{cm} \left(-\sum_i \frac{1}{\lambda_i} \ln \frac{R_i}{R_{i0}} + \frac{1}{\alpha_0 R_0} \right)}. \qquad (16)$$

The convenience of using the concept of the corrected radius is that it expresses all of the heat conducting properties of the walls of the pipeline in the heat conducting properties of ice. This makes it possible to reduce the further complicated calculations of icing of the surfaces of walls of the pipeline merely to a single material -- ice -- which significantly simplifies the calculation formulas.

With the goal of easing the determination of the value of the corrected radius in Fig. 2 graphs are given for steel and wooden open pipelines. In order to plot these graphs, a value of the coefficient of heat conductivity of ice of λ_i = 2.0 kcal/m hr degrees was accepted. For steel pipelines heat resistance and the thickness of the walls were not taken into account. For wooden pipelines the value of the coefficient of heat conductivity of λ_{cm} = 0,434 kcal/m hr degrees was accepted with the assumption that the walls, being in a state of total moistness, are frozen. The thickness of walls of wooden pipelines accepted in accordance with the data given by Professor, Doctor of Technical Sciences A. A. Morozov (16) are given in Table 2.

Table 2

Thickness of Walls of Wooden Pipelines

R_{in}	δ_{cm}, m	R_{in}, m	δ_{cm}, m
0.10	0.030	1.00	0.071
0.25	0.040	1.25	0.086
0.35	0.040	1.50	0.086
0.50	0.041	1.75	0.096
0.75	0.062	2.00	0.096

If the indicated numerical values and the coefficient of heat exchange according to (13) are substituted in formula (16), then as the result of simple transforms one obtains the following expressions: for open steel pipelines

$$R_{cr} = R_{in} \cdot e^{\frac{0{,}615}{w^{0,7} R_{in}^{0,7}}} \text{ м};$$

for open wooden pipelines

$$R_{cr} = R_{in} \cdot e^{-4{,}61 \ln \frac{R_{in}}{R_{in} + \delta_{cm}} + \frac{0{,}615}{w^{0,7} (R_{in} + \delta_{cm})^{0,7}}} \text{ м}.$$

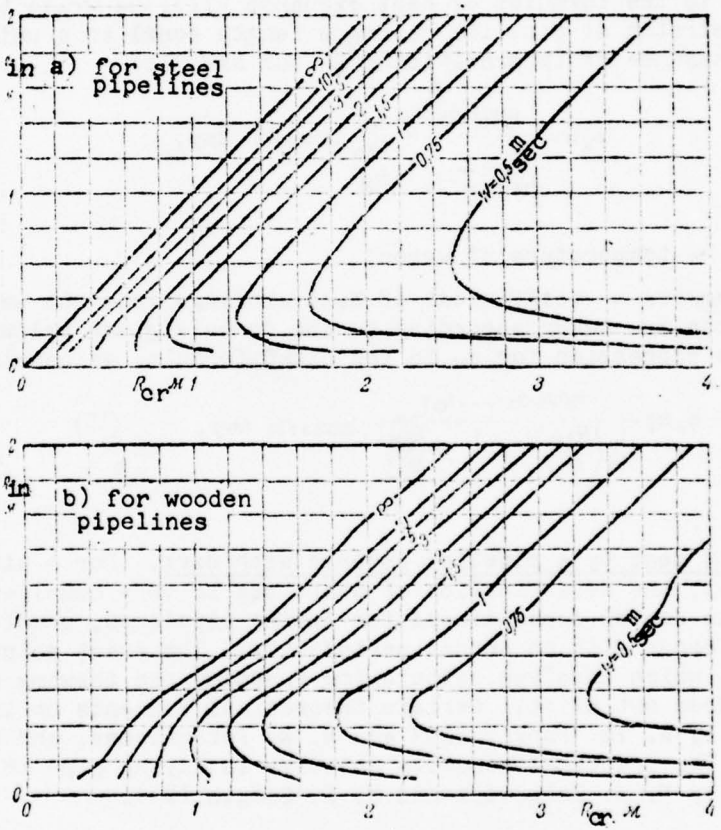

Fig. 2. Graphs of the corrected radii of open pipelines

The graphs in Fig. 2 have been plotted according to these two formulas.

The curvature of the lines for the wooden pipelines is due to the fact that the thicknesses of the walls introduced into the calculation have an

irregular relationship with the internal radius.

Turning to the estimate of the general thermal losses of the pipeline, we employ formula (15) as the final one for the subsequent calculations in a case when there is a layer of ice on the inside surface of the pipeline.

d) Heat losses in the absence of ice. Subsequently, during the determination of the heat loss by the pipeline under conditions of the absence of icing in it, for uniformity of the calculations the use of the pipe radius R_{cr} corrected for ice is also preserved. However, one should recall that in this case the temperature can be higher than the temperature of melting of ice ϑ_0 along the surface of contact of water and material of the walls. The value of heat loss in this case will depend both on the temperature of the outside air and on the temperature of the water inside the pipeline. Referring once again to the formulas of heat exchange (11), we write the amount of heat lost by a stretch of pipeline having a length equal to a unit per unit of time in the absence of internal icing in our symbols:

$$\varphi_n = \frac{0{,}024 \cdot 2\pi (\vartheta - \vartheta_0)}{\frac{1}{\alpha_0 R_{in}} + \frac{1}{\lambda_i} \ln \frac{R_{in}}{R_{cr}}} \text{ kcal/m day},$$

where ϑ degrees -- temperature of water;

a_w kcal/m^2 hr degrees -- coefficient of heat exchange from the water to the wall, whose values are taken according to the formula given below (33). By substituting the expression for a_w in the cited formula, we obtain:

$$\varphi_n = \frac{0{,}024 \cdot 2\pi (\vartheta - \vartheta_0)}{\frac{1}{358}\left(\frac{R_{in}}{Q}\right)^{3/4} + \frac{1}{\lambda_i} \ln \frac{R_{in}}{R_{cr}}} \text{ kcal/m day}, \qquad (17)$$

e) Losses of heat by a pipeline covered with dirt. For a pipeline covered with dirt, the determination of heat loss is very complicated. In the ground at the depths conventional for laying pipelines, constant changes in temperatures occur with an annual periodicity. The exact solution of this complex problem, which involves calculating freezing and thawing of moisture in the ground, does not exist. Certain theoretical concepts on this subject have been given by A. Ya. Popkov (19) and N. N. Petrunichev, and G. S. Shadrin (18) and (26). The practical concepts relative to laying pipe in permafrost have been given by N. A. Tsitovich and M. I. Sumgin (24).

The use of modeling can be useful in explaining the unsteady thermal and ice regimes of buried pipelines (4) and (15).

For approximate calculations one can use the solution to a problem in which all temperatures are assumed to be constant in time. Its solution (18) in our symbols has the following form:

$$q_n = \frac{0{,}021 \cdot 2\pi (t_n - t_0)}{\frac{1}{\lambda_d} \ln \left[\frac{h}{R_{in}} + \sqrt{\left(\frac{h}{R_{in}}\right)^2 - 1} \right]} \text{ kcal/m day,} \quad (18)$$

where λ_d kcal/m hr degree -- coefficient of heat conductivity of the dirt.

h m -- depth of burial of axis of pipeline below surface of ground.

Other symbols as before.

If the soil is covered by a layer of snow h_s, then it can be taken into account by the following method. One calculates the corrected value of burial of the pipeline in the dirt:

$$h = h_d + \frac{\lambda_d}{\lambda_s} h_s \text{ m,}$$

where the subscript d pertains to soil and subscript s to snow.

The heat conductivity of soil varies within wide limits depending on its composition, density, moistness and state (melted or frozen). Therefore the value of the heat conductivity coefficient of soil should be determined predominantly by special experimental investigations, without which one can only make extremely approximate calculations.

The heat conductivity of snow also varies within broad limits depending on its density. For settled and moderately wind-packed snow, one can (5) name a value $_s$ = 0.2 kcal/m hr degree.

The corrected depth of lie of the pipeline axis calculated according to the last formula is used for calculating heat losses according to expression (18). Thermal losses of a pipeline buried in soil are heterogeneous along its perimeter. The higher portions of the walls lose more heat than the lower ones. By averaging this process, we distribute losses uniformly over the entire perimeter. This makes it possible to simplify the calculation, having used the concept of corrected radius R_{cr} introduced earlier.

If one compares the formula (18) which determines the heat losses of a pipeline free of ice with formula (15), where, for identity of conditions (the absence of ice), r should be replaced by R_{in}, then one can obtain the equation:

$$\frac{1}{\lambda_d} \ln \left[\frac{h}{R_{in}} + \sqrt{\left(\frac{h}{R_{in}}\right)^3 - 1} \right] = -\frac{1}{\lambda_i} \ln \frac{R_{in}}{R_{cr}},$$

thence

$$R_{cr} = R_{in} \left[\frac{h}{R_{in}} + \sqrt{\left(\frac{h}{R_{in}}\right)^2 - 1} \right]^{\frac{\lambda_i}{\lambda_d}} \text{ m.} \quad (19)$$

Hence, calculations of heat losses of the buried and frozen pipeline can be made according to formula (15), by using expression (19) for determining the corrected radius. However, a question arises of the choice of the value of air temperature t_0 in formula (15). It is entirely obvious that small diameter pipes and little protected pipes will react more in the sense of freezing to short-term low atmospheric temperatures, while such short-term, albeit hard frosts will not have an effect on the ice regime of large diameter pipelines and well protected ones (for example, those enclosed in insulation). Hence, for different pipelines one should introduce atmospheric temperatures averaged over different periods of time into the calculation, which will also have an effect on the value of these temperatures.

f) <u>The critical value of the outside dimensions of thermal insulation of pipelines</u>. It is known from the theory of heat exchange (11) that certain determined outside dimensions of heat insulation of the pipeline exist called the critical dimensions, at which heat losses of the pipeline have the highest value. One can find the explanation for this in the following. In formula (12), which determines the heat losses of the pipeline, there are two members in the denominator which determine the properties and effect of the thermal insulation. We shall write these members of the denominator:

$$\frac{1}{\lambda_{ins}} \ln \frac{R_{in\,ins}}{R_0} + \frac{1}{a_0 R_0},$$

where λ_{ins} kcal/m hr degrees -- coefficient of heat conductivity of insulation;
$R_{in\,ins}$ m -- inside radius of thermal insulation;

R_0 m -- outside radius of insulation and of entire structure in general;

a_0 kcal/m^2 hr degrees -- coefficient of heat exchange from the outside surface of the structure to the air.

The outside radius R_0 increases proportional to the increase in the thickness of the insulating layer. R_0 also properly determines the outside dimensions of the insulation. This causes an increase in the first member and a decrease in the second member of the sum written above. The smaller the value of this sum, which consists of the members of the denominator of formula (12) that contain R_0, the higher the value of thermal losses Ψ_n mgcal/m day. Consequently, the critical value R_0 is then obtained when the sum of the indicated two members has the smallest value. By substituting the expression of the coefficient of heat exchange a_0 in this sum for the outside surface according to formula (13), we obtain:

$$\frac{1}{\lambda_{ins}} \ln \frac{R_{in\,ins}}{R_0} + \frac{1}{3{,}25w^{0,7}R_0^{0,7}}.$$

From the condition of the maximum of the sum described above, one can determine the value of the outside critical radius:

$$R_{o\,crf} = 0{,}112 \frac{\lambda_{ins}}{w} \approx 0{,}112 \frac{\lambda_{ins}}{w}. \qquad (20)$$

As is apparent, the value of the critical outside radius of a pipeline covered with thermal insulation does not depend on the thickness of this insulation and is only determined by the properties of heat conductivity of the insulating layer and by the conditions of heat exchange on the outside surface of pipeline, which is determined by the velocity of wind blowing the pipeline.

If the existing outside radius of the pipeline is less than the outside critical radius corresponding to the given thermal insulation and conditions of heat exchange, i.e., $R_o < R_{o\,cri}$, then placing a layer of insulation will not decrease but will increase the heat losses so long as this inequality exists. When $R_o = R_{o\,cri}$, heat losses will be highest. With a subsequent increase in the thickness of the layer of insulation, inequality $R_o > R_{o\,cri}$ will be observed, and the value of heat losses will decrease. Hence, in order to reduce heat losses one should apply a thickness of the insulating layer to the pipeline such that in any case $R_o \geq R_{o\,cri}$.

The value of the outside critical radius calculated according to the formula (20) for insulations with low heat conductivity such as snow, wood, soil, cinders, etc. is extremely small, less than the ordinary radii of hydroelectric station pipelines. Only in rare cases of low values a_o with a well protected pipeline with respect to the wind $R_{o\,cri}$ is a value obtained that limits the dimensions of the thermal insulation. Therefore, usually any thermal insulation with any of its dimensions reduces the thermal losses of the pipeline. The question only pertains to ensuring that this insulation is suitable for design and economic concepts. During its use as insulation, ice (λ_i = 2.0 kcal/m hr degrees), for example during external artificial freezing of the pipeline, has values of the critical radius according to formula (20) that are already practically significant dimensions.

g) Radiant heat exchange of the pipeline with the atmosphere. Radiant energy can participate in heat exchange between the pipeline and the atmosphere. The significance of radiant heat exchange depends upon a number of factors, of which the conventional are the following: material, color, and temperature of the pipeline surface, the temperature and state of the atmosphere, cloud cover, meridian, and the position of the pipeline relative to the direction of the sun's rays. In order to calculate all of these circumstances, one must refer to the appropriate literature; here the general characteristics of the effect of heat transferred by radiant energy on the ice regime of pipelines are given.

We shall arbitrarily assume a positive direction of the heat flux exchanged by radiant energy and consider a direction from the atmosphere to the outside surface of the pipeline. We shall assume that the average amount of heat exchanged by radiant energy per unit of time per unit of length of the

pipeline q_s = mgcal/m day, is known. For the purpose of simplifying the calculations, we assume that the flux of radiant energy over the surface of the pipeline is uniform. Then thermal losses with the participation of radiant heat exchange along the outside surface of the pipeline will be expressed by the relationship:

$$q_n = \frac{0{,}024 \cdot 2\pi \left(t_0 - t_\pi - \frac{\varphi_s}{0{,}024 \cdot 2\pi\alpha_0 R_0} \right)}{\frac{1}{\lambda_{cr}} \ln \frac{r}{R_{cr}}} \text{ kcal/m/day} \qquad (21)$$

This is similar to relationship (15), only here the value t_0 is conserved. In this case, t_0 can have practical value. A comparison of formulas (15) and (21) leads to the conclusion that the influx of heat to the pipeline by means of radiant energy can be view as a component of correction for the outside temperature of the air. The correction is made in the form of a component:

$$\frac{\varphi_s}{0{,}024 \cdot 2\pi\alpha_0 R_0} \text{ degrees}$$

If q_s is the influx of radiant energy to the pipeline (and not an efflux), then the correction is taken as is explicit from formula (21), with a negative value, and vice versa.

For winter conditions, the influx of radiant energy does not play a significant role and is not considered in the further calculations. But in spring this influx of heat can sharply complicate the operation of the pipeline and is a cause of separation of the layer of ice from the inside surfaces of the walls and the formation as a consequence of this of an ice-gang inside the pipeline. Separation of ice from the walls can only occur if the melting point of the ice t_0 appears in the point of contact between the wall and the ice. Since the same temperature also exist on the inside surface of the ice layer, then the previous condition is equal to that in which the separation of ice from the walls is only possible when in the walls and in the ice a temperature everywhere will exist that is uniform and equal to t_0. In this case the temperature gradients disappear, and consequently, there will be no loss of heat. Hence, if $q_n = 0$ is placed in (21), then one can find the intensity of radiant energy at which separation of ice from the walls is possible:

$$\varphi_s = 0{,}024 \cdot 2\pi\alpha_0 R_0 (t_0 - t_\pi) \text{ mgcal/m day.} \qquad (22)$$

As was stated above, q_s is the mean amount of heat transferred by radiant energy corrected to the entire surface of the pipeline. At the same time, the heat of radiant energy is distributed over the surface of the pipeline quite unequally. Some parts of the surface are irradiated (illuminated) more intensively and receive much more heat than others. If one designates the highest intensity of the influx of radiant energy per unit of time per unit

of pipeline surface area as S mgcal/m^2 day, then one can find the value of the S analogous to (22), at which the separation of ice from the wall will be possible only at a point of surface of the pipeline exposed to the most intensive radiation:

$$S = 0{,}024\alpha_0(\vartheta_0 - \vartheta_0) \text{ mgcal/m}^2 \text{ day}. \qquad (23)$$

From the cited concepts it becomes obvious that melting of ice in contact between the ice and the walls, and consequently, separation of the ice layer are even possible with a frost (with negative values of the temperature of the outside air ϑ_0), if radiation of the pipeline by the sun occurs with sufficient intensity. This is quite possible in mountainous conditions characterized by an extremely intensive insulation due to low humidity, purity and rarefaction of the air.

D. THE LATENT HEAT OF MELTING OF THE LAYER OF ICE THAT COVERS THE INSIDE SURFACE OF THE PIPELINE WALLS

An adequately low temperatures of the air and temperatures of water near the freezing point of ice, an ice layer forms on the inside surfaces of the pipeline walls. This ice layer forms as the result of freezing of water. In this case on the surface of the boundary between the water and the ice, with the assumption of the absence of supercooling of the water, the temperature of the melting point of ice should be conserved. During thermal equilibrium, when the influx of heat from the water to the surface of the boundary between the water and the ice is equal to the efflux of heat from that surface through the ice and thence through the walls to the atmosphere, no change in the thickness of the ice layer occurs. But as soon as the influx of heat from water to the indicated surface exceeds the efflux through the ice, then melting of the ice layer will occur. Melting should occur along the surface of the boundary between the water and the ice; the entire layer of ice will have a temperature lower than the temperature of melting of ice. This circumstance provides a basis to hypothesize that during melting of the ice layer particles of ice will not separate from it even with significant flow rates of water in the pipeline.

The heat in the layer of ice in the form of the latent heat of melting will be the following per unit of pipeline length:

$$W = -\pi\gamma_i L(R_{in}^2 - r^2) \text{ mgcal/m}, \qquad (24)$$

where γ_i = 0.917 t/m^3 -- volumetric weight of the ice;
L -- 79.6 mgcal/t -- latent heat of melting of the ice;
R_{in} m -- inside radius of pipe;
r m -- radius of free cross-section of the frozen pipe.

The minus sign has been introduced because earlier it was decided to subtract the reserves of heat from the liquid state of water at a tempera-

ture of 0°.

The change of the reserve of latent heat in the layer of ice in time will be:

$$\frac{dW}{d\tau} = 2\pi\gamma_i L r \frac{dr}{d\tau} \text{ mgcal/m day} \qquad (25)$$

where $\frac{dr}{d\tau}$ m/day -- rate of increase in radius of the free cross-section of the pipe as the result of melting of the ice layer.

E. HEAT LINKED WITH PROPERTIES OF HEAT CAPACITY OF BODIES

In the course of the thermal processes, the temperature of both the immobile part of the examined complex (the walls, the layer of ice, the insulation of the pipeline) and of the moving part (the water) changes. With a variable temperature, the heat content of these parts also changes due to the properties of the heat capacity of bodies.

Above, formula (15) was introduced with the assumption of the absence of heat capacity in the walls of the pipeline (including the layer of ice and the insulation), i.e., the following was accepted:

$$\varphi_{cm} = 0; \quad \varphi_i = 0. \qquad (26)$$

Taking into account the heat in the water by virtue of heat capacity, one should distinguish the volume of water running through the pipeline through a certain cross-section of it and the volume of water in the pipeline. The necessity of such a distinction becomes obvious from analysis of the components of thermal balance. Thermal balance was formulated for a stretch of pipeline between sections I -- I and II -- II (Fig. 1). The heat entering the section through cross-section I -- I and leaving from a section through cross-section II -- II was calculated. During the transfer of this heat, water participates as the heat carrier. In this case the properties of heat capacity of the water should be taken into account, as was done in formula (6). Now it remains to examine the heat linked with properties of heat capacity of the volume of water in the section lying between cross-sections I -- I and II -- II. The amount of this heat can be estimated by means of comparison with the amount of latent heat in the layer of ice of internal icing of the pipeline. We cite the following approximate calculations.

We determine the thickness of the layer of ice which would be equivalent to the amount of heat liberated or absorbed by water during a change in its temperature by a value $\Delta\vartheta$, according to the amount of the latent heat of melting within it. The calculations are made for a single running meter of pipeline. The water located in this stretch contains an amount of heat

$$\pi r_w^2 \gamma_w C_w \Delta\vartheta \text{ mgcal/m.}$$

The radial layer of ice having a thickness δ that formed on the inside surface of the walls requires an amount of heat $2\pi r\delta\gamma_i L$ mgcal/m for melting.

From the equality of these amounts of heat, one determines the sought layer of ice equivalent in heat content to a volume of water:

$$\delta = \frac{r\gamma_w c_w \Delta\vartheta}{2\gamma_i L} M;$$

after substitution of constants, we obtain:

$$\delta = \frac{r \cdot 1 \cdot 1 \cdot \Delta\vartheta}{2 \cdot 0{,}917 \cdot 79{,}6} \approx 0{,}007\, r\Delta\vartheta\ M.$$

If the temperature of the water will also change by $\Delta\vartheta = 1^\circ$, then one should consider it an extremely high value in a large pipeline as well, for example, when r = 3 m, and then the equivalent layer of ice proves to be a comparatively small value δ = 0.021 m, which can be ignored.

This calculation shows that changes resulting from the properties of heat capacity of reserves of the heat of water in the pipeline (not to be confused with the volume of water running through the pipeline) do not have a significant value in the heat balance and can be ignored in the engineering calculations, i.e., one can consider that

$$\varphi_{rw} = 0. \qquad (27)$$

Hence, all of the components of thermal balance which determine the amounts of heat due to the heat capacity of the separate elements are accepted to equal 0 in the given calculations:

$$\varphi_e = \varphi_{ecm} + \varphi_{ei} + \varphi_w = 0. \qquad (28)$$

F. HEAT TRANSFERRED FROM THE WATER TO THE ICE

a) <u>The general relationship</u>. On the surface of the boundary between the water and the ice there should be a certain temperature corresponding to the melting point of ice ϑ_0. Therefore, the amount of heat transferred from the water to the ice depends only on the temperature of the water and in no way depends on the temperature of the air surrounding the pipeline. The sought amount of heat falling per unit of length of the pipeline per unit of time will be:

$$\varphi_{wi} = 0{,}024 \cdot 2\pi r\alpha_w(\vartheta - \vartheta_0) \ \text{mgcal/m day}, \qquad (29),$$

where a_w kcal/m^2 hr degree -- coefficient of heat exchange between the water and the surface of the ice; ϑ degrees -- temperature of water; ϑ_0 degrees -- melting point of ice.

We shall separately examine the values entering into formula (29).

b) The coefficient of heat exchange from water to the surface of the ice. The coefficient of heat exchange a_w from water to the surface of the ice is a complex function of the dimensions of the cross-section and the thermal and velocity regimes of the pipeline.

In order to determine the value of this coefficient, the following formula is given in the Plan of TUiN of Hydrotechnical Design:

$$a_w = 13\,600 f_D f_\delta \cdot v^{0,75} \text{ kcal/m}^2 \text{ hr degrees}, \quad (30)$$

where v m/sec -- velocity of water; f_D -- coefficient that depends on the dimensions of the free cross-section of the pipeline;

f_δ -- coefficient that depends on the temperatures of water and the walls of the pipeline.

The same source gives the values of both of these coefficients (Tables 3 and 4).

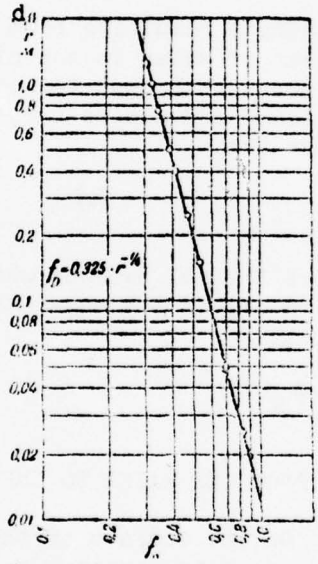

Fig. 3. Relationship of the value of coefficient f_D and the radius of the free cross-section r

It is convenient to convert the expression for a_w into a function of the radius of the free cross-section r. There is a good step relationship between f_D and r which is apparent from the logarithmic anamorphosis shown in Fig. 3:

$$f_D = 0,325\, r^{-1/4}. \quad (31)$$

The data for estimating the accuracy of the obtained formula are given in Table 3.

Table 3

Estimate of the Accuracy of the Formula $f_D = 0.325\ r^{-\frac{1}{4}}$

d, m	f_D according to the Table from (20)	r, m	$r^{-\frac{1}{4}}$	f_D according to the formula	Error, %
0,05	0,815	0,025	2,51	0,815	0,1
0,10	0,685	0,05	2,11	0,686	0,1
0,30	0,530	0,15	1,61	0,523	1,3
0,50	0,465	0,25	1,418	0,461	0,9
0,80	0,405	0,40	1,256	0,409	1,0
1,0	0,385	0,50	1,190	0,387	0,5
1,5	0,345	0,75	1,073	0,349	1,2
2,0	0,320	1,00	1,000	0,325	1,6
2,5	0,310	1,25	0,944	0,307	1,0

Note: Commas should be read as decimals

It is apparent from Table 3 that calculation according to formula (31) results in an error not in excess of 1.6% and therefore the formula can be considered entirely suitable.

The values of the coefficient f_ϑ are given in Table 4.

Table 4

Values of the Coefficient f_ϑ

Temperature of water \ Temperature of the wall	0°	5°
0°	0.184	0.180
10°	0.240	0.233

From the cited data it is apparent that coefficient f_ϑ depends little on the temperature of the wall, and depends significantly on the temperature of the water, changing for each degree of change in temperature of the latter by 3.1% of its value at 0°. However, one can assume that in hydrotechnical practice the temperature of water coming into the pipeline in winter will be comparatively low (within limits of 0 -- 3°), and therefore coefficient f_ϑ can be considered constant and equal to the following:

$$f_3 = 0{,}190 = \text{const}, \qquad (32)$$

By substituting the values of the coefficients (31) and (32) in formula (30), and also having expressed the velocity value through flow rate, we obtain the expression for the coefficient of heat exchange between water and ice in a function of the radius of the free cross-section:

$$\alpha_w = 358 \frac{Q^{1/4}}{r^{1/4}} \text{ kcal/m}^2 \text{ hr degrees.} \qquad (33)$$

It is useful to compare the obtained formula with the existing generalized formulas for heat exchange during the flow of different liquids (drop and gaseous) in pipes of different sizes and at different temperatures. One of these formulas, taken from the book by M. V. Kirpichev, M. A. Mikheyev, and L. S. Eygenson (11) has the following form:

$$Nu = 0{,}024\ Re^{0.8} Pr^{0.35},$$

where the criterion $Nu = \frac{\alpha_w d}{\lambda}$;

criterion $Re = \frac{vd}{\nu}$;

criterion $Pr = \frac{\nu}{a}$;

a_w kcal/m^2 hr degrees -- coefficient of heat exchange from the water to the walls of the pipeline;

d m -- inside diameter of pipeline;

λ_w kcal/m hr degrees -- coefficient of heat conductivity of water;

v m/sec -- velocity of water;

ν m^2/sec -- kinematic coefficient of viscosity of water;

a m^2/sec -- coefficient of temperature conductivity of water.

For water at 0°, criterion Pr = 13.3. By substituting this value, and the symbols for the other criteria, we obtain:

$$\frac{\alpha_w d}{\lambda_w} = 0{,}0595 \left(\frac{vd}{\nu}\right)^{0.8},$$

thence the sought coefficient of heat exchange between the water and the inside surface of the pipeline walls will be:

$$\alpha_w = 0{,}0595 \frac{\lambda_w v^{0.8}}{\nu^{0.8} d^{0.2}} \text{ kcal/m}^2 \text{ hr degrees.}$$

Here we substitute the values of the constants for water at 0°:

$$\lambda_w = 0{,}480 \text{ kcal/m hr degrees}$$

and $\nu = 1{,}790 \cdot 10^{-6}$ m^2/sec.

Then $\alpha_w = 1120 \frac{v^{0.8}}{d^{0.2}}$ kcal/m^2 hr degrees.

By transferring from velocity to flow rate and from the diameter to the radius of the free cross-section, we obtain the following formula for the value of the coefficient of heat exchange from the water to the inside surface of the pipeline walls:

$$\alpha_w = 390 \frac{Q^{0.8}}{r^{1.8}} \text{ kcal/m}^2 \text{ hr degrees,} \qquad (34)$$

where Q m^3/sec -- flow rate of water through the pipeline;

r m -- radius of free cross-section.

The latter formula is comparable to the formula (33) given earlier. Until conducting special investigations on this question, formula (33) is used for the calculations because it gives smaller values of the coefficient of heat exchange, i.e., provides for greater resistance to heat during its transfer from water to ice; in this case one should expect more icing of the pipeline than enters into the reserve of the calculation. Here the question of the change in values of the coefficient of heat exchange along the pipeline is not examined as it is not a significant value.

In a case when a build-up or melting of the ice layer does not occur, the hypothetical value of the coefficient of heat exchange is in no doubt. However, when a change in the aggregate state occurs, the value of the coefficient of heat exchange can deviate from that obtained according to formula (33). There are no experimental data about such a deviation for cases of freezing and thawing. One can assume that this deviation will be slight in both cases. Relative to the quality estimate of the effect of the change of the aggregate state on the value of the coefficient of heat exchange, one can cite the following concepts.

During melting the thermal resistance between water and ice can increase, since water is constantly appearing on the surface that separates them. This water is a product of melting of the ice with a temperature at the melting point. Although this water is carried away by the common flow, in a certain small layer of it adjacent to the ice the temperature gradient is either absent all together or is extremely small, which is also the cause for the hypothetical elevated heat resistances (reductions in the value of a_w). When the ice freezes, the ice seemingly advances into the water and reduces the thickness of the laminar layer adjacent to the ice which exists on the walls in the turbulent stream. This laminar layer creates the basic resistance to the heat flux between the liquid and the wall. With a reduction in thickness in the laminar layer, one should anticipate decreases in

the thermal resistance, i.e., increases in the coefficient of heat exchange between the water and the ice.

In these calculations a value of a_w independent of the character of change in icing is used.

c) The melting point of the ice. The temperature at which a change in the aggregate state occurs could be called both the melting point and the freezing point. However, water has the property of supercooling, which renders the concept of the freezing point indeterminate to a certain degree. Ice itself cannot exist in the "superheated" state, and therefore the melting point of ice under the given conditions can only have one entirely determined value. In view of such determinancy of the melting point of ice, for the sake of simplicity the value of the temperature of change in the aggregate state in general, of both melting and freezing is ascribed to the latter.

In order to determine the melting point of ice ϑ_0 dependent upon pressure, B. P. Veynberg (5) cites a relationship that is given below in the symbols used here:

$$p = -129{,}08\vartheta_0 - 1{,}52\vartheta_0^2 \text{ kg/cm}^2 \qquad (35)$$

where ϑ_0 degrees -- melting point of the ice;

p kg/cm^2 -- pressure in excess of atmospheric pressure.

This relationship encompasses an extremely broad range of high pressures (up to 2000 kg/cm^2). In the pipelines usually used in hydrotechnical operations, pressure will be much less. This makes it possible to simplify the relationship of the quadratic type given here and to reduce it to a linear one without significantly reducing the accuracy of the new formula in the narrower range of pressures. The following relationship gives the best results:

$$\vartheta_0 = -0{,}00784\, p \text{ degree.} \qquad (36)$$

For comparison of the original formula (35) and the simplified one (36), Table 5 gives the melting points of ice for certain pressures calculated according to them.

Table 5.

Comparison of the Results of Calculating the Melting Points of Ice According to the Original and Simplified Formulas

p, kg/cm²	ϑ_0 according to original formula (35)	ϑ_0 according to simplified formula (36)	Deviation of results of calculation, %
0	0	0	0
5	—0,01	—0,0392	2,0
10	—0,08	—0,0784	2,0
20	—0,16	—0,1568	2,0
50	—0,39	—0,392	0,5
100	—0,77	—0,784	2,0

Note: Commas should be read as decimals

It is apparent from Table 5 that the simplified formula (36) gives results that differ extremely little from the results of the original formula. This makes it possible to employ the simplified formula in our calculations. For its subsequent use, it is conveniently converted into an expression of pressure in heights of the water column in place of kg/cm². Then, formula (36) acquires the form:

$$\vartheta_0 = -0{,}000784\ H \text{ degree} \qquad (37)$$

where H m -- pressure expressed in height of the water column.

In this form the ralationship in used in future.

Salts dissolved in the water (the hardness of water) can also have an effect on the melting point of the ice, in addition to pressure. In hydrotechnical operations, water is usually used with an extremely small salt content, and therefore the factor of the change in the melting point of the ice as the result of water hardness can be ignored, considering $\vartheta_0 = 0^\circ$ at atmospheric pressure. However, if it becomes necessary to take into account the effect of salinity of water on the melting point of the ice, then in the first approximation one can accept:

$$\vartheta_0 = \vartheta_{0c} - 0{,}000784\ H \text{ degree},$$

where ϑ_{0c} degrees -- melting point of the ice at given salinity of water at atmospheric pressure.

It is assumed in all subsequent calculations that the water contains extremely little dissolved impurities, and therefore formula (37) is used.

Returning to the determination at the intensity of the heat flux from water to the ice, we substitute in the general formula (29) the value of the coefficient of heat exchange according to (33) and the value of the melting point of ice according to (37). As the result, we obtain:

$$q_{wi} = 17{,}2\pi\left(\frac{Q}{r}\right)^{3/4}(\vartheta + 0{,}000784\,H)\ \text{mgcal/m day}. \qquad (38)$$

This is also the final expression for the intensity of the thermal flux moving from the water to the ice, formed on the inside surfaces of the pipeline walls.

d) Water pressure in the pipeline. The value of water pressure in the pipeline enters the formulas given above. It is calculated by conventional methods of hydraulics. Here only the differential equation which determines pressure is given:

$$\frac{\partial H}{\partial x} = J_d - J, \qquad (39)$$

where H m -- pressure of water in the pipeline expressed in height of the water column; in a general case this pressure changes along the length of the pipeline and in time;

J_d -- design gradient of pipeline; its values are considered positive when the pipeline is sloped in the direction of the movement of water;

J -- hydraulic gradient;

$\frac{\partial H}{\partial x}$ -- gradient of pressures along the pipeline.

Here forces of inertia and local hydraulic resistances in the pipeline are not taken into account.

It the hydraulic gradient is expressed through flow rate according to formula (9), then the latter equation acquires the form:

$$\frac{\partial H}{\partial x} = J_d - 2{,}52\frac{n^2Q^2}{\pi^2 r^{5\frac{1}{3}}}. \qquad (40)$$

Integration of equation (40) is extremely simple and can be carried out, for example, in finite differentials. In all cases it is vital to have some boundary conditions for determining the integration constant.

e) The reserve of heat that appears in the water with an increase in pressure as the result of a change in the melting point of ice. Formula (29) takes into account the reserve of heat in the latent form that appears in the water as the result of a decrease in the melting point of the ice with the increase in pressure. Formula (37) shows that with a positive pressure

increment the melting point of ice ϑ_0 should have a negative value. Consequently, even water can have a negative temperature in the range from 0° to ϑ_0°. The heat liberated by water during cooling below 0 can be transferred to the ice and thereby facilitate a decrease in icing, i.e., heat the pipeline. With a negative increment in pressure (with a vacuum), the phenomenon will be the reverse. We shall take up these circumstances in greater detail.

Formula (29) for the amount of heat transferred from water to ice can be given in this form:

$$\varphi_{wi} = 0{,}024 \cdot 2\pi r a_w \vartheta - 0{,}024 \cdot 2\pi r a_w \vartheta_0 \text{ mgcal/m day},$$

from which it is explicit that the amount of this heat is a sum of two values: of the value determined by water temperature ϑ (the first member) and of the value determined by the melting point of the ice (the second member). In pressurized pipelines of hydroelectric power stations, pressure increases in the direction of motion of the water, and in connection with this, the melting point of the ice drops, always having a negative value as formula (37) shows. Hence, in moving along the pressurized pipeline, the water increases its capacity to transfer heat to the ice as the result of a decrease in the melting point of the ice (the second member in the last formula increases).

A unique heat reserve appears in the water, which can be transferred to the ice and thereby warm the pipeline and prevent the development of icing.

If the water undergoes a drop in pressure and the second member of the last equation decreases, then the reverse effect is obtained. The water's capacity to transfer heat to the ice decreases and the reserve of heat from pressure changes into a deficiency -- a deficit.

In order to clarify the practical significance of the reserve or deficit of heat that appears in the water during a change in pressure, the following example is offered. We shall assume that a pipeline has a valve that is in a position that somewhat constricts the flow of water, as the result of which a drop in pressure ΔH is created. We shall also assume that ahead of the valve and beyond it the radius of the free cross-section of the pipeline r, as well as the value of heat exchange a_w entering into formula (29) are identical. The water passing through the cross-section constricted by the valve undergoes a drop in pressure by a value ΔH m and heating as the result of friction heat. We shall explain how these changes are reflected in the capacity of the water to transfer heat to the ice. Heating of the water causes an increase in water temperature by a value

$$\Delta\vartheta = \frac{\Delta H}{c_w \gamma_w} = 0{,}00234\,\Delta H \text{ degrees.}$$

Judging by formula (29), this increase in temperature will facilitate an increase in the transfer of heat from water to the ice. On the other hand, the drop in pressure will cause an increase in the melting point of the ice by a value $\Delta\vartheta_0 = 0{,}00784\,\Delta H$ degrees, which will facilitate a decrease in the

transfer of heat from water to the ice.

The change in the value of the amount of heat transferred by water to the ice directly before the valve and beyond it is expressed via formula (29) by the value

$$\Delta\varphi_{wl} = 0{,}024 \cdot 2\pi r\alpha_w(0{,}00234\,\Delta H - 0{,}000784\,\Delta H) =$$
$$= 0{,}024 \cdot 2\pi r\alpha_w(0{,}00156\,\Delta H) \text{ mgcal/m day.}$$

An increase of this amount in the amount of heat transferred from water to the ice occurs. If it is taken as a unit, then the role of the heat of friction in the creation of this increase will be $\frac{0{,}00234}{0{,}00156} = 1{,}5$, and the role of the melting point of ice preventing the creation of this increase is correspondingly expressed by a fraction of $\frac{0{,}000784}{0{,}00156} = -0{,}5$.

In this example the change in the reserve of heat created in the water by pressure comprises one third of the heat of friction. This fraction can increase still more where a significant change in pressures is accompanied by small expenditures of energy on friction, for example, in pipelines with a high design gradient and in turbines. Therefore, the thermal reserve of pressure in the calculations cannot be ignored, i.e., the melting point of ice can never be assumed unchangeable and equal to 0°.

G. HEAT THAT ARISES DURING THE COMPRESSION OF WATER

With a change in pressure (load), the deformations of bodies occur which depend on elastic properties. In this case the body either releases or absorbs certain quantities of energy in the form of mechanical work and alters its reserves of heat, which is expressed in a change in temperature of the deformed body. With an increase in the pressure bodies heat and with a drop in pressure -- cool. Below a quantitative evaluation of this effect is given applicable to water running through a pressurized pipeline.

The compressibility of water can be expressed by the following formula:

$$\frac{\Delta V}{V} = \varepsilon = \frac{p}{E_w},$$

where V cm^3 -- volume of water at atmospheric pressure;
ΔV cm^3 -- volume lost by water with increment in pressure;
ε -- relative volumetric deformation;
p kg/cm^2 -- pressure increment that compresses water;
$E_w = 2 \cdot 10^4$ kg/cm^2 -- coefficient of compression of water at pressures conventional for hydroelectric station pipelines.

This formula expresses Hooke's law for water, and coefficient E_w is analogous to Young's modulus.

We shall calculate the amount of potential energy of deformation with which a 1 cm^3 volume of water is charged with an increase in pressure from 0 to p kg/cm^2;

$$A = \int_0^p p d\varepsilon = \int_0^p \frac{p}{E_w} dp = \frac{p^2}{2E_w} \frac{kg \cdot cm}{cm^3} = \frac{p^2}{2E_w} 10^{-2} \frac{kg \cdot m}{cm^3}.$$

If one assumes that all of this energy is converted into heat, then with an increase in pressure from 0 to p kg/cm^2 water should increase its temperature by the following value

$$\Delta\vartheta = -\frac{A}{E\gamma_w C_w} = \frac{10p^2}{2E_w E\gamma_w C_w} \text{ degrees},$$

where E = 427 kgm/kcal -- mechanical equivalent of heat;
γ_w = 1 g/cm^3 -- volumetric weight of water;
C_w = 1 cal/g. degree -- specific heat capacity of water.

By substituting the physical characteristics, we obtain:

$$\Delta\vartheta = 0{,}585 \cdot 10^{-6} p^2 = 0{,}0585 \cdot 10^{-9} H^2 \text{ degrees}, \quad (41)$$

where H m -- pressure expressed in height of the water column.

Hence, a quantitative characterization of the heating of water with an increase in pressure by a height of the water column H m has been obtained.

It is of interest to estimate the significance of such heating. With an increase in pressure accompanying this heating, a drop in the melting point of ice below 0° to a value expressed by formula (37) occurs. As a consequence of the decrease in the melting point of ice, a reserve of heat appears in the water which the water can yield, cooling below 0°, without altering its aggregate state. The value of this heat reserve is determined by the decrease in the temperature of water from 0° to ϑ_0°. We shall compare the amounts of heat acquired by water with an increase in pressure as the result of heating and as the result of a decrease in the melting point of ice. The comparison is conveniently made in temperatures of water expressed by the formulas (37) and (41). We shall compare the relationship of these temperatures:

$$\frac{\Delta\vartheta}{\vartheta_0} = \frac{0{,}0585 \cdot 10^{-9} H^2}{-0{,}000784 H} = -0{,}075 \cdot 10^{-6} H. \quad (42)$$

This relationship shows that the increase in the temperature of water even with a significant increase in pressure is extremely small in comparison with the drop in the melting point of ice. Since the reserves of heat acquired by water with an increase in pressure as the result of the drop in the melting point of ice and as the result of heating of water are being compared here, introduced by the components into the heat balance equation (2), then the value of heating can be ignored and can be considered

$$\varphi_0 = 0. \qquad (43)$$

3. THE HEAT BALANCE EQUATIONS IN THE DETAILED EXPRESSION

A. THE EQUATIONS

In the two previous sections, the equations of heat balance were formulated and the values of their separate components were explained. In this section, based on these data, the heat balance equations in the detailed expression are formulated.

In order to obtain the heat balance of a frozen pipeline as a whole with water running in it in the detailed expression, one must substitute the values of its components (6), (10), (15), (25), (28), and (43) in equation (2):

$$-86\,400\,Q\gamma_w C_w \frac{\partial \vartheta}{\partial x} + 218 \cdot 10^3 \frac{n^2 Q^3}{D\pi^2 r^{5,3}} - \frac{0,024 \cdot 2\pi(-\vartheta_0)}{-\frac{1}{\lambda_4}\ln\frac{r}{R_{cr}}} - 2\pi\gamma_1 Lr\frac{\partial r}{\partial t} = 0. \qquad (44)$$

Subsequently, it is convenient to introduce into the equations the radii of free cross-sections not in absolute values, but in relative ones. In this case, the value of the corrected radius is selected as the value relative to which the comparison is made. The ratio between the indicated radii is expressed as follows:

$$r_* = \frac{r}{R_{cr}}, \qquad (45)$$

where r m -- absolute radius of free cross-sections;
R_{cr} m -- corrected radius;
r_* -- relative radius of free cross-section of pipe.

The convenience of using the relative radius instead of the absolute one is felt in the calculations. The heat balance equation (44) includes complex functions of r. Their calculation requires a great deal of work which could be significantly eased by means of using corresponding tables. However, the compilation of these tables is difficulty and they are obtained in vast numbers because of the fact that the values of radii of free cross-sections can change depending on the size of the cross-sections of pipelines within broad limits. The value of the relative radius can change within limits of $0 < r_* < 1$. These defined limits make the auxiliary table for calculating functions of r_* convenient and compact. Such a table is in the appendix at the end of the book.

By introducing the relative radius into equation (44) instead of the absolute radius, we obtain:

$$-86\,400\,Q\gamma_w C_w \frac{\partial\vartheta}{\partial x} + 218\cdot 10^3 \frac{n^2Q^3}{D^{5.33}R_{cr}^{5.33}}\cdot\frac{1}{r_*^{5.33}} -$$

$$-\frac{0{,}024\cdot 2\pi(-\vartheta_0)}{-\frac{1}{\lambda_i}\ln r_*} - 2\pi\gamma_i LR_{cr}^2\cdot r_*\frac{\partial r_*}{\partial t} = 0. \qquad (46)$$

The values of the functions $\frac{1}{r_*^{5.33}}$ and $-\frac{1}{\ln r_*}$, encountered here, as well as those of the other functions of r_* in the subsequent equations are given in tabular form in the referenced appendix.

In order to obtain the heat balance of frozen walls of the pipeline (the walls and the layer of ice) in the detailed expression, one must substitute the values of its components according to (15), (25), (26), and (38) in equation (4):

$$17{,}2\pi\left(\frac{Q}{r}\right)^{3.33}(\vartheta + 0{,}000784H) - \frac{0{,}024\cdot 2\pi(-\vartheta_0)}{-\frac{1}{\lambda_i}\ln\frac{r}{R_{cr}}} -$$

$$-2\pi\gamma_i Lr\frac{\partial r}{\partial t} = 0 \qquad (47)$$

or, in the expression through the relative radius,

$$17{,}2\pi\left(\frac{Q}{R_{cr}}\right)^{3.33}(\vartheta + 0{,}000784H)\frac{1}{r_*^{3.33}} - \frac{0{,}024\cdot 2\pi(-\vartheta_0)}{-\frac{1}{\lambda_i}\ln r_*} -$$

$$-2\pi\gamma_i LR_{cr}^2 r_*\frac{dr_*}{dt} = 0. \qquad (48)$$

A result of these two equations is an equation of the heat balance of water running through the pipeline. The latter can be obtained in the detailed expression by means of substituting in (3) the values of the components according to (6), (10), (27), (38), and (43). After substitution we obtain:

$$-86\,400Q\gamma_w C_w\frac{\partial\vartheta}{\partial x} + 218\cdot 10^3\frac{n^2Q^3}{D^{5.33}r^{5.33}} -$$

$$-17{,}2\pi\left(\frac{Q}{r}\right)^{3.33}(\vartheta + 0{,}000784H) = 0 \qquad (49)$$

or, in the expression through the relative radius,

$$-86\,400Q\gamma_w C_w\frac{\partial\vartheta}{\partial x} + 218\cdot 10^3\frac{n^2Q^3}{D^{5.33}R_{cr}^{5.33}}\cdot\frac{1}{r_*^{5.33}} -$$

$$-17{,}2\left(\frac{Q}{R_{cr}}\right)^{3.33}(\vartheta + 0{,}000784H)\frac{1}{r_*^{3.33}} = 0. \qquad (50)$$

All of the equations formulated above pertain to a frozen pipeline whose hydraulic regime is set by the value of the flow rate of water running in it. If the hydraulic regime of the pipeline is determined by a drop in pressure, and specifically, by the hydraulic gradient, then in all of the equations for-

mulated above in this section it is vital to express flow rate Q via hydraulic gradient J, which can be done according to formula (8). The following equations are obtained as the result.

For a frozen pipeline with running water (on the whole):

$$-86\,400\cdot 0{,}633\,\frac{\pi}{n}\,r^{8/3}J^{1/2}\gamma_w C_w\frac{\partial\vartheta}{\partial x}+54\,600\,\frac{\pi r^{8/3}J^{1/2}}{En}-\frac{0{,}024\cdot 2\pi(-\vartheta_0)}{\frac{1}{\lambda_i}\ln\frac{r}{R_{cr}}}-2\pi\gamma_i Lr\frac{\partial r}{\partial t}=0 \qquad (51)$$

or, in the expression via relative radius,

$$-86\,400\cdot 0{,}633\,\frac{\pi}{n}\,R_{np}^{8/3}J^{1/2}\gamma_a C_a r_*^{8/3}\frac{\partial\vartheta}{\partial x}+54\,600\,\frac{\pi R_{cr}^{8/3}J^{1/2}}{En}\cdot r_*^{8/3}-\frac{0{,}024\cdot 2\pi(-\vartheta_0)}{\frac{1}{\lambda_i}\ln r_*}-2\pi\gamma_i LR_{cr}^2\cdot r_*\frac{\partial r_*}{\partial t}=0. \qquad (52)$$

For the iced walls:

$$911\,\pi r^{1/2}J^{1/2}(\vartheta+0{,}000784H)-\frac{0{,}024\cdot 2\pi(-\vartheta_0)}{\frac{1}{\lambda_i}\ln\frac{r}{R_{cr}}}-2\pi\gamma_i Lr\frac{\partial r}{\partial t}=0 \qquad (53)$$

or, in the expression via relative radius,

$$911\pi R_{cr}^{1/2}J^{1/2}(\vartheta+0{,}000784H)r_*^{1/2}-\frac{0{,}024\cdot 2\pi(-\vartheta_0)}{\frac{1}{\lambda_i}\ln r_*}-2\pi\gamma_i LR_{cr}^2 r_*\frac{\partial r_*}{\partial t}=0. \qquad (54)$$

For the water running through the pipeline:

$$-86\,400\cdot 0{,}633\,\frac{\pi}{n}\,r^{8/3}J^{1/2}\gamma_w C_w\frac{\partial\vartheta}{\partial x}+54\,600\,\frac{\pi r^{8/3}J^{1/2}}{En}-911\pi r^{1/2}J^{1/2}(\vartheta+0{,}000784H)=0, \qquad (55)$$

or, in the expression via relative radius,

$$-86\,400\cdot 0{,}633\,\frac{\pi}{n}\,R_{cr}^{8/3}J^{1/2}\gamma_w C_w\frac{\partial\vartheta}{\partial x}+54\,600\,\frac{\pi R_{cr}^{8/3}J^{1/2}}{En}\,r_*^{8/3}-911\pi R_{cr}^{1/2}J^{1/2}(\vartheta+0{,}000784H)r_*^{1/2}=0. \qquad (56)$$

The equations of heat balance in the detailed expression for stretches of a pipeline free of ice can be formulated in the same way. For brevity, these equations are not given here.

The obtained heat balance equations in the detailed expressions serve as the basis for all subsequent calculations.

B. THE GENERAL CHARACTERIZATION OF THE EQUATIONS AND THE EMPLOYED TERMINOLOGY

A pair of differential equations, for example, (44) and (47), and the differential hydraulic equation (40) comprise a system of three equations with the following five variables: r, ϑ, H, x, and t. By means of excluding two variables, for example, ϑ and H, one can obtain a single differential equation with three variables r, x, and t. As the result of solving this equation, one should try to obtain an expression of the function $r = f(x, t)$, which would determine the change in the radius of the free cross-section, i.e., the degree of icing of the pipeline with respect to both to its length and in time.

From the system of differential equations referenced above, one can also find the differential form of functions $\vartheta = f(x, t)$ and $H = f(x, t)$, which determine temperature and pressure of water by means of exclusion.

As is apparent, the degree of icing of the pipeline in the general case is a function of place and time $r = f(x, t)$. It is entirely obvious that partial cases can also exist.

Hence, there can be icing which does not change either according to length of the pipeline or in time. Such icing ensues following quite prolonged functioning of the pipeline under constant hydraulic and thermal conditions in an end stretch so remote from the beginning of the pipeline that the conditions of intake do not influence this stretch. We shall call such icing maximum and the radius of the free cross-section that corresponds to it will be designated R_{lim}. The radius of limit icing is a constant value $r = R_{lim} = const$, since it depends neither on x nor t.

There can be icing that changes with the course of time, but that does not change with respect to the length of the pipeline. Such a character of icing is obtained in an end stretch of a quite long pipeline where, due to remoteness from the beginning of the pipeline, intake conditions have no effect. We shall call such icing cylindrical because the inside surface of the layer of ice that has formed on the walls inside the pipeline comprises a cylindrical surface with a generatrix parallel to the axis of the pipeline. The radius of the free cross- section of the pipeline in that case will only be a function of time $r = f(t)$.

Finally, icing can be constant in time but variable with respect to the

length of the pipeline. Such a state of icing ensues during quite prolonged operation of the pipeline with constant conditions (constant flow rate and constant atmospheric temperature), when all processes are steady-state. We shall call such icing steady-state icing. The radii of the free cross-section of the pipeline in that case will be a function only of the distance from its beginning $r = f(x)$.

A similar classification of the processes also exists in hydraulics, where the following types of motion are distinguished: motion that is constant along the length of the flow of water and in time, called uniform motion; motion that varies in time, called unsteady motion, and motion constant in time but variable in length, called non-uniform motion. It does not seem possible to accept this terminology for determining the ice state of a pipeline because the definitions of hydraulics pertain to motion, but in this case one needs a definition of state. Page 60 has a table of the accepted terminology (Table 6).

Table 6

Accepted Terms Defining the Character of Icing of the Inside Surfaces of Pipeline Walls

Terms	Functional Relationship for r	Definition
Limit Icing	$r = R_{lim} = \text{const}$	Icing constant along the length of the pipeline and with the passage of time
Cylindrical Icing	$r = f(t)$	Icing constant along the length of the pipeline but variable with the passage of time
Steady-State Icing	$r = f(x)$	Icing variable along the length of the pipeline but constant with the passage of time
General Case of Icing	$r = f(x, t)$	Icing variable both with respect to pipeline length and the passage of time.

Analysis and solution of the differential equations of heat balance in the succeeding chapters will be conducted in the order of gradual complication of conditions. First,we shall examine limit icing, then cylindrical and steady-state icing, and finally, a general case of icing.

CHAPTER 2

LIMIT ICING OF THE PIPELINE

Above it was decided to call limit icing icing that does not change either along the length of the pipeline or with the passage of time. If icing is variable with respect to length and time, i.e., r = f (x, t), then with distance from the beginning of the pipeline, i.e., proportional to the increase in x and with an increase in operating time t, with all other conditions constant such icing would asymptotically approach limit icing. This can be described mathematically as follows:

$$\text{when } x \to \infty \text{ and } t \to \infty$$

$$\lim \frac{\partial r}{\partial x} = 0; \lim \frac{\partial r}{\partial t} = 0 \text{ and } \lim r = R_{\lim}. \qquad (57)$$

The expression for the radius of the free cross-section with limit icing can be obtained from the equations of thermal balance, if one assumes the following in them:

$$\frac{\partial r_*}{\partial x} = \frac{\partial r_*}{\partial t} = 0 \text{ and } r_* = R_{*\lim}, \qquad (58)$$

where R_{*lim} -- relative radius with limit icing.

Depending on what condition is set by the hydraulic operating regime of the pipeline, the expressions for the radius with limit icing are obtained variously.

In the subsequent sections, limit icing is examined both with a set flow rate of water and with a set hydraulic gradient.

4. LIMIT ICING WITH A SET FLOW RATE OF WATER

A. DETERMINING THE RADIUS OF THE FREE CROSS-SECTION WITH LIMIT ICING

In order to determine the radius of the free cross-section with limit icing under conditions of a set flow rate of water through the pipeline, it is vital to use the heat balance equations (46) and (48) and the hydraulic equation (40), having inserted conditions (58) in them. For this purpose, the time change for water temperature disappears and changes along the length of the pipeline are conserved; therefore the partial derivative of water temperature by length $\frac{\partial \vartheta}{\partial x}$ should be replaced by the total derivative $\frac{d\vartheta}{dx}$. Similar conditions will also exist for pressure, therefore $\frac{\partial H}{\partial x}$ is replace with $\frac{dH}{dx}$.

As the result of these substitutions and replacements, we obtain a system of the following three equations:

$$
\begin{aligned}
&-86\,400Q\gamma_w c_w \frac{d\vartheta}{dx} + 218\cdot 10^3 \frac{n^2Q^3}{E\pi^2 R_{cr}^{5\frac{1}{3}}} \cdot \frac{1}{R_{*\lim}^{5\frac{1}{3}}} - \\
&- \frac{0{,}024\cdot 2\pi(-\vartheta_b)}{-\frac{1}{\lambda_i}\ln R_{*\lim}} = 0; \\
&-17{,}2\pi\left(\frac{Q}{R_{cr}}\right)^{3\frac{1}{4}}(3 + 0{,}000784H)\frac{1}{R_{*\lim}^{3\frac{1}{4}}} - \\
&- \frac{0{,}024\cdot 2\pi(-\vartheta_0)}{-\frac{1}{\lambda_i}\ln R_{*\lim}} = 0; \\
&\frac{dH}{dx} = J_d - 2{,}52\frac{n^2Q^2}{\pi^2 R_{cr}^{5\frac{1}{3}}} \cdot \frac{1}{R_{*\lim}^{5\frac{1}{3}}}.
\end{aligned}
\qquad (59)
$$

This system of equations determines limit icing, however derivatives $\frac{d\vartheta}{dx} \neq 0$ and $\frac{dH}{dx} \neq 0$ enter into it. This shows that with limit icing the temperature and pressure of water can change along the length of the pipeline.

The system of equations (59) includes five variables: R_{*lim}, ϑ, H, x, and t. We exclude the variables ϑ and H in the following manner. From the second equation we find:

$$
\vartheta = \frac{0{,}024\lambda_i(-\vartheta_b)R_{cr}^{3\frac{1}{4}}}{8{,}59Q^{3\frac{1}{4}}} \cdot \frac{R_{*\lim}}{-\ln R_{*\lim}} - 0{,}000784H \text{ degree}. \qquad (60)
$$

By differentiating according to x, we find the derivative:

$$
\frac{d\vartheta}{dx} = -0{,}000784\frac{dH}{dx} \text{ degree/m} \qquad (61)
$$

Having substituted the value of the gradient of pressure along the axis of the pipeline $\frac{dH}{dx}$ according to the third equation of system (59) here, we find:

$$
\frac{d\vartheta}{dx} = -0{,}000784\left(J_d - 2{,}52\frac{n^2Q^2}{\pi^2 R_{cr}^{5\frac{1}{3}}} \cdot \frac{1}{R_{*\lim}^{5\frac{1}{3}}}\right) \text{degree/m} \qquad (62)
$$

Having substituted this expression in the first equation of system (59), we obtain:

$$
\begin{aligned}
&67{,}8Q\gamma_w c_w\left(J_d - 2{,}52\frac{n^2Q^2}{\pi^2 R_{cr}^{5\frac{1}{3}}} \cdot \frac{1}{R_{*\lim}^{5\frac{1}{3}}}\right) + \\
&+ 218\cdot 10^3 \frac{n^2Q^3}{E\pi^2 R_{cr}^{5\frac{1}{3}}} \cdot \frac{1}{R_{*\lim}^{5\frac{1}{3}}} - \frac{0{,}024\cdot 2\pi(-\vartheta_0)}{-\frac{1}{\lambda_i}\ln R_{*\lim}} = 0.
\end{aligned}
\qquad (63)
$$

From this equation, for the given pipeline and for the given conditions of its operation, i.e., with known values R_{cr}, J_d, Q, and ϑ_0, one can determine the value of the relative radius with limit icing R_{*lim}. The determina-

tion of the value R_{*lim} should be made by trial-and-error.

Having substituted the numerical values of the physical values in equation (63)

$$\gamma_w C_w = 1{,}0 \text{ mgcal m}^3 \text{ degree } n = 0{,}01; \quad E = 427 \text{ Tm/mgcal}$$
$$\lambda_i = 2{,}0 \text{ kcal/m hr degree},$$

we obtain

$$67{,}8QJ_d + 0{,}00343 \frac{Q^3}{R_{cr}^{5/3}} \cdot \frac{1}{R_{*lim}^{5/3}} - \frac{1}{}$$
$$- 0{,}302 (-\vartheta_o) \lg R_{*lim} = 0. \qquad (64)$$

This equation can serve for direct calculations of the value of the radius of the free cross-section with limit icing R_{lim}.

The temperature of the water in limit icing can be determined via R_{*lim} according to formula (60).

The problem of limit icing was solved by Professor A. M. Yestifeyev (8) and (9); in this case an equation was obtained for determining the diameter of the free cross-section during limit icing. In the symbols used by us, this equation has the following appearance:

$$695 dQ^3 = \frac{\pi D_{lim}^6 (-\vartheta_o)}{\frac{1}{\alpha_w} + \frac{D_{lim}}{\alpha_o D_o} + \frac{D_{lim}}{2\lambda_i} \ln \frac{D_{lim}}{D_o} + \frac{D_{in}}{2\lambda_{cm}} \ln \frac{D_{in}}{D_o}} \qquad (65)$$

where: D_{lim} m -- diameter of free cross-section with limit icing of pipeline;
a kcal/m^2 hr degree -- coefficient of heat exchange;
λ kcal/m hr degrees -- coefficient of heat conductivity;
ϑ_o degrees -- temperature of the outside air;
o -- subscript indicating that the given value pertains to the outside surface of the pipeline;
in and w -- subscripts that show that the given value pertains to the inside surface of the pipeline;
d -- coefficient in the Darsy formula for determining the hydraulic gradient in the pipeline;

$$J = \frac{d}{D_{lim}} \cdot \frac{v^2}{2g},$$

where v m/sec -- velocity of water along the pipeline.

Formula (65) has been derived without consideration of the reserve of heat that appears in the water with an increase in pressure as the result of the drop in the melting point of ice. Therefore, the practical use of this formula pertains only to a partial case in which the hydraulic gradient in the pipeline is equal to its design gradient, i.e., when $J = J_d$.

Furthermore, calculation of formula (65) of heat resistance between water and an ice surface determined by member $\frac{1}{a_w}$ and the denominator of its right hand side is disputable. During derivation of formula (65), apparently, it was assumed that the melting point of ice is 0°, and this temperature is ascribed to water flowing through the pipeline. Calculation of the thermal resistance between water and the surface of the ice in formula (65) is the same as recognizing that the temperature on the surface of the boundary between the water and the ice is below 0°, i.e., lower than the melting point of the ice. The presence of water supercooled by a value determined by the conditions of heat transfer should be considered doubtful.

B. THE VALUE OF THE DESIGN GRADIENT OF THE PIPELINE ENSURING A SET LIMIT ICING

The problem of the radius of limit icing R_{lim} can be posed somewhat differently than was done above. If one assigns a value R_{*lim}, then one can determine the value of J_d -- the design gradient of the pipeline at which such icing is obtained. Practically, this problem can be encountered during the design of a pressure derivation realized by the pipeline.

From equation (63), we find:

$$J_d = \left(2{,}52 - \frac{3\,220}{\gamma_w C_w P}\right)\frac{n^2 Q^2}{\pi^2 R_{cr}^{5{,}33}} \cdot \frac{1}{R_{*lim}^{5{,}33}} + \\ + \frac{0{,}024 \cdot 2\pi(-\vartheta_0)}{67{,}8 Q \gamma_w C_w \left(-\frac{1}{\lambda_i}\ln R_{*lim}\right)}, \qquad (66)$$

The very same expression in the numerical coefficients can be obtained via equation (64):

$$J_d = -50{,}7 \cdot 10^{-6}\frac{Q^2}{R_{cr}^{5{,}33}}\frac{1}{R_{*lim}^{5{,}33}} + \\ + 4{,}15 \cdot 10^{-3}\frac{(-\vartheta_a)}{Q} \cdot \frac{1}{-\ln R_{*lim}}. \qquad (67)$$

To illustrate this relationship, we cite the example of a calculation. We shall assume that there is a metal pipe with R_{in} = 0.60 m blown by the wind at a velocity w = 4.96 m/sec. We do not take the thickness of the wall into account, i.e., in the calculations we accept $R_o = R_{in}$. The coefficient of heat exchange from the outside surface of the wall to the air according to (13) will have a value a_o = 11.58 kcal/m^2 hr degrees. The value of the corrected radius according to (16) will be R_{cr} = 0.80m. Determination of the value of R_{cr} can be carried out according to the graphs in Fig. 2 as well. We shall further assume that the pipeline passes a flow rate Q = 2.0 m^3/sec and that atmospheric temperature $_o$ = -10°. By substituting the known values in (67), we obtain:

$$J_d = -0{,}667 \cdot 10^{-3} \frac{1}{R_{*lim}^{5/3}} + 0{,}0222 \frac{1}{\ln R_{*lim}}.$$

by assigning a value of the relative limit radius of icing R_{*lim}, one can determine the value of the design gradient J_d at which this limit icing is ensured. The results of calculations for certain values of the limit radius of icing and the design gradient are given in Table 7.

Table 7

Values of the Limit Radius of Icing and the Corresponding Values of the Design Gradient for a Pipeline with R_{in} = 0.60 m, R_{cr} = 0.80 m when Q = 2.0 m^3/sec and $\vartheta_o = -10^\circ$

R_{lim}, m	0.60	0.50	0.40	0.39	0.30	0.20
R_{*lim}	0.750	0.625	0.500	0.487	0.375	0.250
J_d	0.0740	0.0389	0.0051	0	-0.103	-1.069

It is apparent from Table 7 that the value of the limit radius of icing depends to a greater extent on the value of the design gradient. Thus, the design gradient with a value J_d = 0.0740 = 7.4% does not yield entirely limit icing, since at this gradient $R_{lim} = R_{in}$ = 0.60 m. With a horizontal pipeline J_d = 0, the limit radius of icing proves to be R_{lim} = 0.39, i.e., thickness of the ice layer $\delta_{lim} = R_{in} - R_{lim}$ = 0.21 m.

Stronger limit icings already appear at negative design gradients of the pipeline, i.e., under the conditions at which the route of the pipeline rises in the direction of flow of the water. Negative design gradients are characteristic of pipelines of hydrostorage electric power stations and of pumping stations in general.

The examined numerical example specifically shows that pressurized pipelines which run with a positive gradient are under the better conditions in the sense of icing than pipelines that run with a negative gradient.

5. Limit and Critical Icing with an Assigned Hydraulic Gradient in the Pipeline

During limit icing a state of thermal equilibrium ensues that is expressed in the fact that the influx of heat from the water to the ice is equal to heat losses. During the operation of a pipeline with a certain constant hydraulic gradient there can be two states of icing that differ from each other in which thermal equilibrium is observed.

One state with a greater radius of the free cross-section will be characterized by the same qualitative properties as limit icing obtained with a constant flow rate (examined in the preceding section). A pipeline tends toward such a state of icing with the passage of time. Therefore, the indicated state of icing can be called limit icing and the radius of the free cross-section during it can be symbolized by R_{lim}.

Another state of thermal equilibrium with a smaller radius will have an unsteady character and the following are ascribed to it: the name critical icing and a radius of free cross-section during it is symbolized through R_{cri}.

These two types of icing are described in greater detail below.

The ice regime of the pipeline with an assigned hydraulic gradient is determined by the heat balance equations (52) and (54) and by the hydraulic equation (39). For obtaining the relationships that correspond to limit and critical icing, one should introduce conditions (58) into the equations indicated above. In this case these conditions are complicated by the fact that instead of r the symbol $R_{lim,cri}$ is introduced which shows that the formulated equations determine both the limit and critical icing. Furthermore, in view of the absence of changes in time of temperature and water pressure, their partial derivatives along the length of the pipeline $\frac{\partial \vartheta}{\partial x}$ and $\frac{\partial H}{\partial x}$ change into total derivatives $\frac{d\vartheta}{dx}$ and $\frac{dH}{dx}$. After all of these substitutions, we obtain a system consisting of three equations:

$$
\begin{aligned}
&-86\,400\cdot 0{,}633\,\frac{\pi}{n}\,R_{cr}^{2\frac{2}{3}}J^{1/2}\gamma_w C_w R_{*\,lim,cri}^{2\frac{2}{3}}\frac{d\vartheta}{dx}+\\
&+51\,600\,\frac{\pi R_{cr}^{2\frac{2}{3}}J^{1/2}}{En}R_{*\,lim,cri}^{2\frac{2}{3}}-\frac{0{,}024\cdot 2\pi(-\vartheta_0)}{-\frac{1}{\lambda_i}\ln R_{*\,lim,cri}}=0; \qquad (68)\\
&911\pi R_{cr}^{1\frac{1}{4}}J^{3/4}(\vartheta+0{,}000784H)R_{*\,lim,cri}^{1\frac{1}{4}}\\
&\quad-\frac{0{,}024\cdot 2\pi(-\vartheta_\kappa)}{-\frac{1}{\lambda_i}\ln R_{*\,lim,cri}}=0;\\
&\frac{dH}{dx}=J_d-J.
\end{aligned}
$$

This system of equations is similar to the system examined earlier (59). Therefore, by using the exclusion of variables ϑ_c and H used earlier in the same way, from the second equation of the system we find the expression for water temperature:

$$\vartheta = \frac{0{,}024\lambda_i(-\vartheta_o)}{455R_{cr}^{11/6}J^{8/9}} \cdot \frac{1}{-R_{*lim,cri}^{11/6}\ln R_{*lim,cri}} - 0{,}000784H \text{ degrees} \quad (69)$$

from which we determine the derivative of water temperature according to x:

$$\frac{d\vartheta}{dx} = -0{,}000784\frac{dH}{dx} \text{ degree/m},$$

i.e., an expression identical to the one obtained earlier.

We substitute the expression for the pressure gradient from the third equation of system (68) here:

$$\frac{d\vartheta}{dx} = -0{,}000784(J_d - J) \text{ degree/m}, \quad (70)$$

Having substituted this value of the derivative in the first equation of system (68), we obtain:

$$42{,}9\frac{\pi}{n}R_{cr}^{8/3}J^{1/2}wC_w R_{*lim,cri}^{2/3}(J_d - J) + 54\,600\frac{\pi R_{cr}^{8/3}J^{11/6}}{Bn} - R_{*lim,cri}^{8/3} - \frac{0{,}024\cdot 2\pi(-\vartheta_w)}{-\frac{1}{\lambda_i}\ln R_{*lim,cri}} = 0. \quad (71)$$

In fact, this equation acquires the following form if the values of the physical characteristics are substituted in it

$$R_{cr}^{8/3}J^{1/2}(13\,500J_d + 26\,600J)R_{*lim,cri}^{2/3} - 0{,}302(-\vartheta_o)\frac{1}{-\ln R_{*lim,cri}} = 0. \quad (72)$$

From the equation for the given pipeline, characterized by R_{cr} - corrected radius and J_d - design gradient, and for the given conditions of its operation which are determined by J - hydraulic gradient and ϑ_o - air temperature, one can by selection find the value R_{*lim} - of relative radius of limit icing and R_{*cri} - relative radius of critical icing. Values R_{*lim} and R_{*cri} are two roots of equation (72).

Critical icing is similar to limit icing because during critical

icing there are no changes in relative radius of the cross-section R_{*cri} either with respect to length or its time. In both cases icing of the pipeline will be in thermal equilibrium, i.e., the amount of heat proceeding from water to the ice will be equal to the amount of heat lost to the atmosphere by the pipeline. However, when R_{*lim} has a steady thermal equilibrium, then when R_{*cri} thermal equilibrium is unstable, and that will be shown during the discussion of the non-steady state processes, icing with R_{*cri} can change to icing with R_{*lim} or complete freezing of the entire cross-section. Usually, $R_{*lim} > R_{*cri}$, but it can happen that $R_{*lim} = R_{*cri}$, and finally, formula (72) may not yield the actual solutions at all. In both of the latter cases, the pipeline is doomed to freezing of the entire free cross-section.

During planning one can also encounter another problem : to determine J - hydraulic gradient or R_{cr} and J_d - the parameters that determine the design elements of the pipeline according to a sign R_{*lim} or R_{*cri}. One can also use equation (72) to solve this problem.

Water temperature with limit and critical icing with a known value R_{*lim} or, respectively, R_{*cri} can be determined according to formula (69).

Chapter Three

CYLINDRICAL ICING OF THE PIPELINE

Cylindrical icing has already been decided to be the partial case of total freezing, when the free cross-section changes with the passage of time but remains constant along the length of the pipeline. Cylindrical icing ensues over quite a distant from the beginning of the pipeline where intake conditions no longer have an effect. The effect of the latter falls off gradually along the length of the pipeline and a cylindrical layer of ice is created at the limit. This circumstance can be described in the following way. In the general case of freezing determined by radius of the free cross-section, freezing is a function of place and time $r = f(x, t)$; but $\lim_{x \to \infty} \frac{\partial r}{\partial x} = 0$ and therefore, when $x \to \infty$

$$r = f(x, t). \tag{73}$$

In the limit expression, r m is the radius of the free cross-section during cylindrical icing.

In order to obtain the relationships that describe cylindrical icing from the equations of heat balance, it is vital to introduce the following condition in them:

$$\frac{\partial r}{\partial x} = 0. \tag{74}$$

Below, cases will be examined when the operation of the pipeline is set by the flow rate or the hydraulic gradient. Furthermore, a case will be examined that corresponds to a variable atmospheric temperature and changing flow rate of water.

6. Cylindrical Icing with a Set Flow Rate of Water

A. The Basic Equations

The heat balance equations (46) and (48) and the hydraulic equation (40) serve as the basic relationships originally. Entering condition (74) in them, which defines cylindrical icing, does not seem possible directly, since they do not contain the partial derivative $\frac{\partial r_*}{\partial x}$ in the explicit form. However, by virtue of condition (73), in these equations one should replace partial derivative $\frac{\partial r_*}{\partial t}$ with total derivative $\frac{dr_*}{dt}$. We shall write these equations with the indicated replacement and we shall view them as a system of equations:

$$\begin{gathered}
-86\,400Q\gamma_w C_w \frac{\partial \vartheta}{\partial x} + 218 \cdot 10 \frac{n^2Q^3}{\pi^2 R_{cr}^{5,3}} \cdot \frac{1}{r_*^{5,3}} - \\
- \frac{0{,}024 \cdot 2\pi(-\vartheta_0)}{-\frac{1}{\lambda_i}\ln r_*} - 2\pi\gamma_i L R_{cr}^2 r_* \frac{dr_*}{dt} = 0; \\
17{,}2\pi \left(\frac{Q}{R_{cr}}\right)^{3/4} (\vartheta + 0{,}000784H) \frac{1}{r_*^{3/4}} - \\
- \frac{0{,}024 \cdot 2\pi(-\vartheta_0)}{-\frac{1}{\lambda_i}\ln r_*} - 2\pi\gamma_i L R_{cr}^2 r_* \frac{dr_*}{dt} = 0; \\
\frac{\partial H}{\partial x} = J_d - 2{,}52 \frac{n^2Q^2}{\pi^2 R_{cr}^{5,3}} \cdot \frac{1}{r_*^{5,3}}.
\end{gathered} \tag{75}$$

This system of equations has five independent variables, r_*, ϑ, H, x, and t. We exclude the variables ϑ and H. For this purpose, we find from the second equation of system (75):

$$\vartheta = \frac{0{,}024\lambda_i R_{cr}^{3/4}(-\vartheta_0)}{8{,}60Q^{3/4}} \cdot \frac{r_*^{3/4}}{-\ln r_*} + \frac{\gamma_i L R_{cr}^{3/4}}{8{,}60Q^{3/4}} \cdot r_*^{13/4} \frac{dr_*}{dt} - \\ - 0{,}000784H = 0 \text{ degree}. \tag{76}$$

From this one can find the partial derivative of water temperature according to x.

By using condition (74) in this case, we obtain:

$$\frac{\partial \vartheta}{\partial x} = -0{,}000784 \frac{\partial H}{\partial x} \text{ degree/m.} \tag{77}$$

Here, we substitute the expression for the pressure gradient according to the third equation of system (75):

$$\frac{\partial \vartheta}{\partial x} = -0{,}000784 \left(J_d - 2{,}52 \frac{n^2 Q^2}{\pi^2 R_{cr}^{5\frac{1}{3}}} \cdot \frac{1}{r_*^{5\frac{1}{3}}} \right) \text{degree/m,} \tag{78}$$

We substitute this value of the partial derivative in the first equation of system (75). As the result of simple transforms, we obtain:

$$\frac{dr_*}{dt} = \frac{67{,}8 Q \gamma_w C_w J_d}{2\pi \gamma_i L R_{cr}^2} \cdot \frac{1}{r_*} + \left(\frac{218 \cdot 10^3}{B} - 170{,}6 \right) \frac{n^2 Q^3}{2\pi^3 \gamma_i L R_{cr}^{7\frac{1}{3}}} \times$$
$$\times \frac{1}{r_*^{6\frac{1}{3}}} - \frac{0{,}024 \lambda_i (-\vartheta_o)}{\gamma_i L R_{cr}^2} \cdot \frac{1}{-r_* \ln r_*} \cdot \frac{1}{\text{day}}. \tag{79}$$

This equation is the basic relationship which makes it possible to analyze the cylindrical ice regime of the pressurized pipeline. With respect to its mathematical form it is a quite complex differential equation of the first order which cannot be analytically solved.

The final goal of solving this equation is to explain the regime of the change in r - radius of the free cross-section of the pipeline in time -, i.e., to solve it one must determine the function r = f(t).

In this case it is assumed that values: Q - flow rate of water; ϑ_o - temperature of the air around the pipeline; R_{cr} - corrected radius, and J_d - design gradient of the pipeline - are known and constant.

In preparing equation (79) for the future calculations, we substitute the numerical values of the physical characteristics:

$\gamma_w C_w$ = 1 mgcal/m^3 degree - specific heat capacity of water with respect to volume;

n = 0.01 - coefficient of roughness of the ice;

λ_i = 2.0 kcal/m hr degrees - coefficient of heat conductivity of ice;

$\gamma_i L = 0.917 \cdot 79.6 = 73.0$ mgcal/m^3 - latent heat of change in the aggregate state of ice expressed with respect to volume;

E = 427 tm/mgcal - mechanical equivalent of heat.

After substitution, equation (79) acquires the following form:

$$\frac{dr_*}{dt} = 0{,}148\frac{QJ_0}{R^2_{cr}}\cdot\frac{1}{r_*} + 7{,}49\cdot10^{-6}\frac{Q^3}{R^{7/3}_{cr}}\cdot\frac{1}{r_*^{6/3}} - 0{,}660\cdot10^{-3}\frac{(-i_0)}{R^2_{cr}}\cdot\frac{1}{-r_*\ln r_*}\frac{1}{\text{day}}. \quad (80)$$

The solution of this differential equation is conducted in the subsequent divisions of this section.

During planning, it may prove necessary to know the temperature of the water during cylindrical icing. The formula for calculating this temperature is obtained from (76), if one substitutes the value $\frac{dr_*}{dt}$ according to (79) there. Conducting the necessary operations, we obtain :

$$\vartheta = \frac{67{,}8Q^{1/3}R^{2/3}_{cr}\gamma_i L}{17{,}2\pi}\cdot\frac{1}{r_*^{2/3}} + \left(\frac{218\cdot10^3}{P} - 170{,}6\right)\frac{n^2Q^{2/3}}{17{,}2\pi^2R^{4/3}_{cr}}\times \quad (81)$$
$$\times\frac{1}{r_*^{4/3}} - 0{,}000784H = 0 \text{ degree}.$$

Having substituted the numerical values of the physical characteristics, we obtain:

$$\vartheta = 1{,}254Q^{1/3}R^{2/3}_{cr}J_0 r_*^{2/3} + 0{,}0636\cdot10^{-3}\frac{Q^{2/3}}{R^{2/3}_{cr}}\cdot\frac{1}{r_*^{4/3}} - 0{,}000784H \text{ degree}. \quad (82)$$

The pressure value H is determined by solving the third equation of system (75).

At this, one can conclude preparation of the necessary relationships for the further analysis of cylindrical icing of the pipeline in the case of an assigned flow rate of water.

B. Icing of a Pipeline Filled with Standing Water

We shall examine a partial case of the problem of icing of a pipeline with an assigned flow rate when the flow rate equals zero:

Q = 0; in this case water is in the pipeline.

With standing water in the pipeline and an atmospheric temperature below the melting point of ice, a gradual increase in the thickness of the layer of ice on the inside surfaces of the pipeline walls will occur. Hence, the pipeline is doomed to unavoidable freezing of the water throughout the entire cross-section. The question only pertains to the duration of the period of time over which freezing occurs.

a) Derivation of the formula. With standing water, the thermal balance of the pipeline is formulated from only two components: from heat losses and from changes in reserves of the latent heat of change in aggregate state locked in the layer of ice. The heat of the change in the aggregate state is expended by means of thermal losses. Reserves of the latent heat of aggregate state are dissipated by the time of complete freezing of the pipeline.

In order to reduce the basic equation of this chapter (79) to a case of standing water, it is necessary to consider Q = 0 in it. Then this equation acquires the form:

$$\frac{dr_*}{dt} = -\frac{0{,}024\lambda_i(-\vartheta_0)}{\gamma_i L R_{cr}^2} \cdot \frac{1}{-r_* \ln r_*} \cdot \frac{1}{day}. \tag{83}$$

After factoring the variables, this equation will appear as follows:

$$A dt = r_* \ln r_* dr_*,$$

where

$$A = \frac{0{,}024\lambda_i(-\vartheta_0)}{\gamma_i L R_{cr}^2} = 0{,}000658 \frac{(-\vartheta_0)}{R_{cr}^2} \frac{1}{day}. \tag{84}$$

After integration, we obtain:

$$At = \frac{1}{2} r_*^2 \ln r_* - \frac{1}{4} r_*^2 + c.$$

If one accepts that t = 0 when r* = 1, then the arbitrary integration constant will be c = 1/4 and the equation acquires the form:

$$At = \frac{1}{2} r_*^2 \ln r_* - \frac{1}{4} r_*^2 + \frac{1}{4}. \tag{85}$$

This can be considered the basic expression for the gradual decrease in radius of the free cross-section during freezing of a pipeline filled with standing water. In Figure 4, the equation (85) is graphically depicted. It is apparent from the graph that near $r_* = 1$, icing develops extremely intensively.

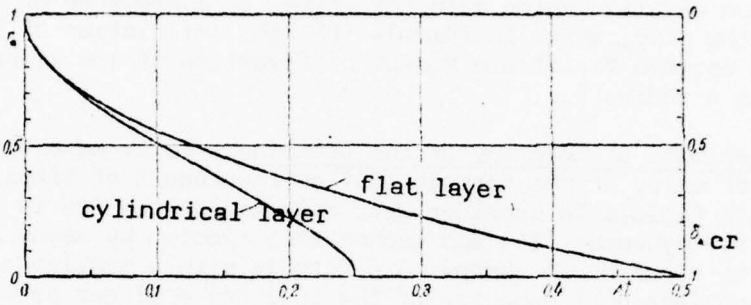

Figure 4. Increase in the thickness of the ice layer with standing water.

Then thickening of the ice layer occurs almost uniformly, but near the axis of the pipeline the intensity of freezing once again increases, reaching infinity when $r_* = 0$. Formula (85) was published by Professor L. S. Leybenzon (14) and V. S. Yablonskiy, P. P. Shumilov and V. M. Pokrovskiy (27) in 1931. The very same relationship is given a slightly different form in the plans of TUiN of Hydrotechnical Design (20) and in the article by Professor A. M. Yestifeyev (9). Applicable to the symbols used here, it has the following form:

$$t = -\frac{\gamma_i L}{0{,}024 \cdot 8\,(-t_0)}\left[\frac{d^2}{\lambda_i}\ln\frac{d}{D_{in}} + \right.$$

$$\left. + \left(\frac{1}{2\lambda_i} - \frac{1}{\lambda_{cm}}\ln\frac{D_{in}}{D_o} + \frac{2}{a_o D_o}\right)(D_{in}^2 - d^2)\right] \text{ days}, \qquad (86)$$

where d m - internal diameter of free cross-section of the ice ring;

D_{in} m - inside diameter of the pipeline;

D_o m - outside diameter of the pipeline;

λ_{cm} kcal/m hr degrees - coefficient of heat conductivity of the pipeline wall;

a_o kcal/m^2 hr degrees - coefficient of heat exchange between the outside surface of the pipeline and the air.

If one accepts value $\gamma_i L = 72.0$ mgcal/m^3, then the numerator of the

member before the quadratic brackets is equal to 72.0: 0.024 = 3000. This number is given in the literature sources (9) and (20) referenced above.

Formula (86) differs from formula (85) by the fact that it does not take into account possible thermal insulation and by the fact that the calculation of time begins with the moment of appearance of ice on the walls of the pipe, while in formula (85) the calculation of time begins with a certain fictitious moment of formation of ice along a surface having a radius R_{cr}.

b) The estimate of accuracy of the formulas. Below an estimate of the value of error of the formula (85) as the result of simplification and linked with failure to consider heat capacity of the ice is given. Professor L. S. Leybenzon (14) approaches this problem by means of determining the error of an approximate formula with a similar assumption (failure to consider heat capacity of the ice) for a linear problem (the ice freezes along a plane), for which there is an accurate solution from the viewpoint of mathematical physics.

Supporting this method, we shall make the necessary calculations.

For a case of constant temperatures of a thick layer of water and ice on its upper surface, the exact solution has the following appearance:

$$\delta = k\sqrt{t}, \tag{87}$$

where δ m - thickness of the ice;

t day - time of accretion;

km/day$^{1/2}$ - coefficient of proportionality determined from the transcendent equation:

$$\frac{0{,}024\lambda_i\vartheta_0 e^{-\frac{k^2}{4a_i^2}}}{a_i\Phi\left(\frac{k}{2a_i}\right)} + \frac{0{,}024\lambda_w\vartheta_w e^{-\frac{k^2}{4a_w^2}}}{a_w\left[1-\Phi\left(\frac{k}{2a_w}\right)\right]} = -\gamma_i L k\sqrt{\frac{\pi}{2}},$$

where ϑ_0 degrees - temperature of the upper surface of the ice;

ϑ_0 degrees - temperature of water at a significant depth;

$a = \sqrt{\frac{0{,}024\lambda}{\gamma C}}$ m/day$^{1/2}$ - coefficient of temperature conductivity;

i - subscript designating the relationship of any value for ice;

o - subscript designating the relationship of any value to water;

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} d\xi$$

- the Gaussian integral, whose numerical values are taken according to the appropriate tables.

If one ignores the heat capacity of the ice, i.e., considers that at any moment the ice has a linear temperature distribution as with a steady-state thermal flux, then the following approximate relationship is obtained:

$$\delta = \sqrt{\frac{0{,}024 \cdot 2\lambda_i}{\gamma_i L}(-\vartheta_o)t} = k\sqrt{t}\ м, \tag{88}$$

where δ m - thickness of the flat ice layer.

The expression for the proportionality coefficient has the following form:

$$k = \sqrt{\frac{0{,}024 \cdot 2\lambda_i}{\gamma_i L}(-\vartheta_o)} \quad \text{m/day}^{1/2}.$$

Hence, the general form of the relationship of the thickness of the ice and the time of its accretion in the precise (87) and approximate (88) formulas proves to be identical. However, the value of the coefficients of proportionality k varies.

If one assumes

$$\lambda_i = 2{,}0 \text{ kcal/m hr deg.} \quad C_i = 0{,}50 \text{ mg/cal/t deg.}$$
$$\gamma_i = 0{,}917 \text{ t/m}^3,$$
$$a_i = \sqrt{\frac{0{,}024\lambda_i}{\gamma_i C_i}} = 0{,}324 \text{ m/day}^{1/2},$$
$$L = 79{,}6 \text{ mgcal/t and } \vartheta_o = 0°,$$

then the formulas for calculating the proportionality coefficient k has the form:

the accurate formula

$$0{,}00228\ (-\vartheta_0)\, e^{-2{,}38k^2} = k\varphi(1{,}541k);$$

and the approximate formula

$$k = 0{,}0362\sqrt{(-\vartheta_0)} \quad \text{m/day}^{1/2}.$$

Below, in Table 8, the values of coefficient k are given according to both formulas for certain temperatures and the relative error in calculating the thickness of the ice layer due to the assumption of the absence of heat capacity of the ice is also given.

Table 8

A Comparison of the Values of Proportionality Coefficients Calculated According to the Precise and Approximate Formulas

ϑ_0, degrees		-1^0	-5^0	-10^0	-20^0	-40^0
k, $\text{m/day}^{1/2}$	Precise Formula	0.036	0.0805	0.113	0.158	0.221
	Approximate Formula	0.0362	0.0810	0.115	0.162	0.229
Error in determining thickness of the ice layer according to the approximate formula, %		0	1	2	3	4

As is apparent, the error in calculating the thickness of a flat ice layer according to the approximate formula is slight and is quite permissible during engineering calculations. One can assume that error will be just as small for a case of a cylindrical layer of ice that forms in the pipeline.

The existing solution to the problem of freezing of the pipeline containing standing water (85) has been simplified. Thus, in the article by V. S. Yablonskiy, P. P. Shumilov and V. M. Pokrovskiy referenced above (27), relationship (85) is replaced with a quadratic parabola:

$$\delta_{cr} = R_{cr} - r = R_{cr}\sqrt{2A\tau} \quad \text{m},$$

where δ_{cr} m - corrected thickness of the ice layer;

A $\frac{1}{\text{day}}$ - coefficient determined according to formula (84).

This formula is identical to formula (83), which determines the accretion of a flat layer of ice.

In order to compare the intensity of accretion of a cylindrical and a flat layer of ice, we shall proceed in the latter equation to the relative values:

$$\frac{\delta_{cr}}{R_{cr}} = \delta_{*cr} = 1 - r_* = \sqrt{2A\tau}. \tag{89}$$

Here, and in equation (85), values are figured identically for a cylindrical layer, and therefore it is convenient to compare these equations. Such a comparison is graphically carried out in Figure 4. On the left-hand side of this graph is a scale of relative radii r_* and on the right a scale of relative thicknesses of the ice layer δ_{*cr}, calculated with a cylindrical layer from the inside surface of a fictitious ice pipe with an outside radius R_{cr} and a flat layer from surface contact with cold air moving at very high velocity.

It is apparent from Figure 4 that with small thicknesses of the corrected ice layer the accretion curves of the layers lie near each other, but that with thickening of the ice they significantly diverge. Thus, with all other conditions being equal the period of time required for complete freezing of a pipe is twice as short as the period of time necessary for the formation of a flat layer of ice having a thickness equal to the radius of the pipe. Therefore, the replacement of formula (85) with formula (89) should be considered undesirable.

c) The time of complete freezing of the pipeline. Having relationship (85), one can calculate the time required for complete freezing of all water in the pipeline. If one substitutes $r_* = 0$, then we obtain t_{cr} day - the corrected freezing time, i.e., the time required for freezing of a fictitious (between R_{cr} and R_{in}) and the real (between R_{in} and the axis) parts of the pipeline cross-section. If however one substitutes $r_* = R_{*in}$, then we obtain t_f day - freezing time only of the fictitious part of the pipeline cross-section between the end having radii R_{cr} and R_{in}.

The period of time required for the freezing of water along the entire free cross-section of the pipeline is expressed by the following relationship

$$t = t_{cr} - t_f = \frac{1}{2A} R_{*cr}^2 \left[\frac{1}{2} - \ln R_{*in} \right] \quad \text{day}. \tag{90}$$

C. The Method of Graphic Solution

In the partial case of freezing of a pipeline with standing water discussed above, an analytical solution of the problem was obtained. Attempts have also been made to find an analytical solution for the general case when $Q \neq 0$. The differential equation (79), although permitting separation of variables, in this instance only enables one to obtain such a complicated function that one cannot directly integrate it. Attempts have been made to expand this function into a series. In this case, the solution of an extremely complicated type was obtained which is practically unsuitable for engineering use. This solution was still more complicated by the fact that in each partial case one had to estimate the convergence of the series and choose the necessary number of its members for the calculations.

Another attempt at the analytical solution consisted that in complex functions of the differential equation (79) were replaced by simpler ones. However, this method also failed to yield the simple solutions and the picture of the ice processes themselves were strongly distorted in this case. Finally, in the Plans of TUiN of Hydrotechnical Design (20), an analytical solution is offered whose idea consists in that analytical relationships that determine the growth or decrease in the thickness of a plane layer of ice are introduced into the calculation. For the transition to the cylindrical layer of ice that forms inside the pipeline a correction coefficient is introduced. Such a simplification, of course, leads to large deviations from the correct solution. The calculations made according to the recommendations of the Plans of TUiN of Hydrotechnical Design lead to incorrect results. Since no analytical solution to the problem of the changes in pipeline icing in time has been found as yet, then below the following method of graphic solution is offered.

The original equation is a differential equation with numerical coefficients (80), in which time t and relative radius r_* are variables and all other values are constant. In this case we shall consider t to be an independent variable and r_* a function of it. This differential equation has the following structure:

$$\frac{dr_*}{dt} = f(r_*) \frac{1}{\text{day}}. \tag{91}$$

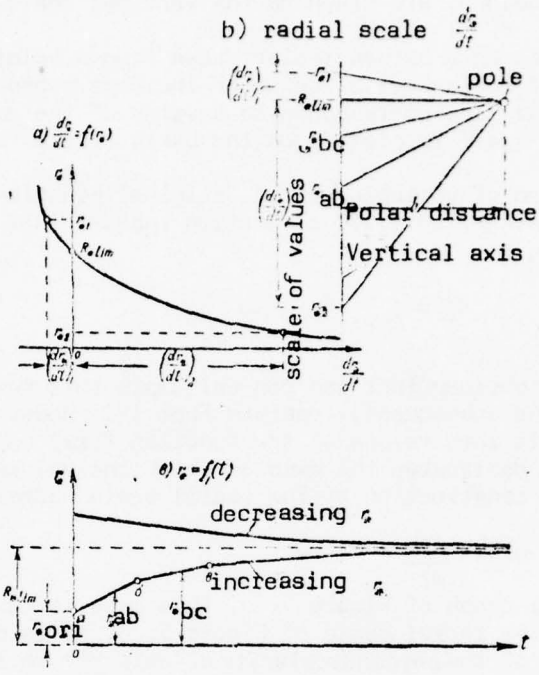

Figure 5. Diagram of graphic integration.

For any value r*, one can calculate the value f(r*). The value f(r*) = = const used in a certain short interval of changes in r* can be used for integration. All of these operations are conveniently conducted graphically.

Figure 5 shows a diagram of graphic integration. Figure 5, a is a curve $\frac{dr_*}{dt} = f(r_*)$ that can be plotted according to formula (80). It is entirely obvious that where the curve intersects the axis or*, the value of the derivative is $\frac{dr*}{dt} = 0$, i.e., in this case the relative radius of the free cross-section acquires a limit value $R*_{lim}$. The curve in Figure 5, a is the original one for the subsequent graphic integration. Figure 5, b depicts the radial scale of the same original function $\frac{dr_*}{dt} = f(r*)$. The value of derivative $\frac{dr*}{dt}$ is determined by the gradient of the corresponding ray. Values of r_* to which the rays, and consequently, the determined values

of derivative $\frac{dr*}{dt}$ belong, are drawn on the vertical scale. The pole of the radial scale lies on a perpendicular drawn from a point on the vertical axis with a value of the relative radius of the free cross-section $R*_{lim}$. Hence, the horizontal line corresponds to a value of the derivative $\frac{dr*}{dt} = 0$. The radial scale is plotted on the basis of the following concepts. During the separation of variables in the original equation (80), or its schematic representation (91), a function appears that is the reverse with respect to $f(r*)$:

$$dt = \frac{1}{f(r_*)}\, dr_*.$$

It is entirely obvious that one can calculate this reverse function for any value $r*$, and subsequently operate from it. However, the operation of dividing the unit into values of the function $f(r*)$ can be carried out graphically, having designated the axis on which the values of derivative $\frac{dr*}{dt}$ are plotted for construction of the radial scale normal for the axis $\frac{dr*}{dt}$ on the original graph $\frac{dr*}{dt} = f(r*)$.

In the original graph of Figure 5, a, this axis is taken to be horizontal, but in the radial scale of Figure 5, b, it is plotted in the vertical. The idea of the perpendicularity of axis $\frac{dr*}{dt}$ in the original graph and on the radial scale in both drawings consists in the following. The value of the derivative is determined by the tangent of the angle of inclination of the ray in the radial scale.

If the axes were parallel, then $\frac{dr_*}{dt} - \operatorname{tg}\alpha$, where α - angle of inclination of the ray. With perpendicular axes, the angle changes by 90°. In this case one obtains a value for the tangent of the new angle of inclination:

$$\operatorname{tg}(\alpha \pm 90^\circ) = \mp \operatorname{ctg}\alpha = \mp \frac{1}{\operatorname{tg}\alpha} = \mp \frac{1}{f(r_*)}.$$

Hence, the perpendicularity of the axes leads to functions reverse with respect to $f(r*)$, which is required for integration. The sign in the given case does not have significant value, since it can be changed by selecting a position of the pole from the right or left sides of the vertical axis of the radial scale.

An arbitrary point corresponding to $\frac{dr_*}{dt} = 0$ is chosen on the vertical axis of the radial scale and near it one inscribes the corresponding

numerical value of the relative radius of the free cross-section of limit icing R_{*lim}. From this point downward, one plots the positive values $\frac{dr*}{dt}$ for certain values of $r*$. This operation is carried out by simple transfer (with a measuring device) from the original drawing of Figure 5, a, of segments of a line determined in a certain scale of positive values $\frac{dr*}{dt}$. Opposite each point obtained by such plotting on the vertical axis of the radial scale one places a numerical value $r*$ to which the point corresponds. In exactly the same way, from a point R_{*lim} upward along the axis of the radial scale one plots the segments that determine the negative values $\frac{dr*}{dt}$, and inscribes the corresponding numerical values of $r*$. Figure 5, b, shows the plotting of only two points, one with a negative value $(\frac{dr_*}{dt})_1$ and the other with a positive value $(\frac{dr*}{dt})_2$.

Having thereby plotted the scale $r*$ on the vertical axis, one chooses a value of the polar distance h cm and plots the pole on a horizontal line that intersects the vertical axis at a point R_{*lim}. By connecting the points of the vertical axis with the pole, we shall obtain rays whose angles of inclination determine the values of derivatives $\frac{dr*}{dt}$. This completes plotting of the radial scale, which is an auxiliary graph. Now one can proceed properly to graphic integration and to plotting the resulting graph $r* = f_1(t)$, shown in Figure 5, c. We shall assume that at an initial moment of time $t = 0$, the relative radius of the free cross-section has a value $r* = r_{*ini}$. This radius is plotted from the origin of the coordinates of Figure 5, c, along axis or* and a point a is obtained. If the radii will increase, then one chooses a radius $r*_{ab}$, somewhat greater than r_{*ini}, and, on the contrary, if the radii will decrease, then one chooses a somewhat smaller radius than r_{*ini}. A ray corresponding to r*ab is sought in the radial scale, and from a point a a segment of line ab is drawn parallel to this ray. Segment ab should encompass the range of values $r*$ for which the value of the selected radius r*ab would be the mean. Then one chooses a new radius $r*_{bc}$, finds the new ray and the segment of line bc running from point b parallel to it, etc. From the segments of lines a broken line is formed giving at the limit a curve $r* = f_1(t)$. This curve determines the sought function of the change in relative radius in time. It is entirely obvious that the shorter the segments comprising the broken line, the more accurate will be the result of integration.

This curve should have an asymptotic horizontal line that corresponds to relative radius R_{*lim} with limit icing. The value of the initial radius

of the free cross-section r_{*ini} can be both greater and lesser than R_{*lim} (the latter case is also shown in Figure 5, c). Therefore, curve $r_* = f_1(t)$ can have two branches. One branch will correspond to pipeline icing increasing with time, and consequently, to a decreasing r_*, and the other, to icing decreasing with time and increasing r_*. Both branches have a common asymptote r_{*lim}.

During graphic integration, the question of the interrelationship of scales acquires pressing significance. We shall arbitrarily consider some value a number of units making up 1 cm to be a scale. We shall designate the scale of this value m_a, and

$$m_a = \text{content of a certain number of units of a value } a_* \text{ in 1 cm.} \tag{92}$$

For the graphs depicted in Figure 5, one should designate the following scales:

m_{r*} - scale of the relative radius;

$m_{\frac{dr*}{dt}}$ - scale of the derivative of the relative radius with respect to time;

m_t - time scale.

These scales can be selected arbitrarily with respect to value. During their selection one should try to have the graphs which depict the functions $\frac{dr*}{dt} = f(r_*)$ and $r_* = f(t)$ to be clear, suitable for use, and corresponding to the accuracy demanded of the calculation.

Choice of the value of the indicated three scales determines the value of polar distance h cm, which is calculated according to the following formula:

$$h = \frac{m_{r_*}}{m_{\frac{dr_*}{dt}} \cdot m_t} \text{ cm.} \tag{93}$$

The information above is entirely adequate for finding the relationship of the change in time of the radius of the free cross-section of the frozen pipeline determined by differential equation (80).

D. Example of a Calculation

Below, the following example of graphic integration according to the presented method is offered. It is necessary to investigate cylindrical icing in a metal pipe with an inside radius R_{in} = 0.60 m with a gradient

J_d = 0.0150. The pipe has a constant flow rate Q = 2.0 m^3/sec running through it. The temperature of the outside air ϑ_o = -10°. Wind velocity w = 4.95 m/sec. According to the graphs of Figure 2, the corrected radius is determined by the value R_{cr} = 0.80 m. Thence, the relative internal radius of the pipe will be $R_{*in} = R_{in} : R_{cr} = 0.75$. Consequently, relative radii of free cross-sections within the interval from 0.75 > r* > 0 are subject to investigation. The region of 1.0 > r* > > 0.75 is a fictitious one as the result of replacing the actual walls of the pipe with arbitrary ice walls of a fictitious pipe having an outside radius R_{cr}. For the indicated conditions, formula (80) acquires the form:

$$\frac{dr_*}{dt} = 0{,}00694 \frac{1}{r_*} + 0{,}309 \cdot 10^{-3} \frac{1}{r_*^{6/5}} - 0{,}0103 \frac{1}{-r_* \ln r_*} \frac{1}{\text{day}}. \quad (94)$$

Table 9

Calculation of the values of derivative $\frac{dr*}{dt}$ with different values of r* for a pipeline with R_{in} = 0.60 m, R_{cr} = 0.80 m, J_d = 0.0150 when Q = 2.0 m^3/sec and ϑ_o = -10°.

r_* (1)	$\frac{1}{r_*}$ (2)	$\frac{1}{r_*^{6/5}}$ (3)	$\frac{1}{-r_* \ln r_*}$ (4)	$0{,}00694 \frac{1}{r_*}$ (5)	$0{,}309 \cdot 10^{-3} \frac{1}{r_*^{6/5}}$ (6)	$-0{,}0103 \frac{1}{-r_* \ln r_*}$ (7)	$\frac{dr_*}{dt}$ (8)
0,75	1,333	6,19	4,63	0,00925	0,00191	—0,0476	—0,0365
0,70	1,430	9,59	4,01	0,00991	0,00296	—0,0413	—0,0284
0,65	1,539	15,31	3,57	0,01067	0,00473	—0,0368	—0,0214
0,60	1,669	25,5	3,26	0,01158	0,00787	—0,0336	—0,01415
0,55	1,820	44,1	3,05	0,01261	0,01362	—0,0314	—0,00517
0,50	2,00	80,8	2,89	0,01386	0,0250	—0,0298	+0,00905
0,45	2,22	157,8	2,78	0,01540	0,0488	—0,0286	0,0356
0,40	2,50	332	2,73	0,01732	0,01026	—0,0281	0,0918
0,35	2,86	772	2,72	0,01982	0,238	—0,0280	0,230
0,30	3,33	$2{,}05 \cdot 10^3$	2,77	0,0231	0,634	—0,285	0,629
0,25	4,00	$6{,}51 \cdot 10^3$	2,88	0,0277	2,01	—0,0296	2,01
0,20	5,00	$26{,}8 \cdot 10^3$	3,10	0,0347	8,28	—0,0319	8,28
0,15	6,67	$0{,}1652 \cdot 10^6$	3,52	0,0463	51,1	—0,0362	51,1
0,10	10,00	$2{,}16 \cdot 10^6$	4,34	0,0694	667	—0,0446	667
0,05	20,0	$0{,}1740 \cdot 10^9$	6,68	0,1387	$53{,}8 \cdot 10^3$	—0,0688	$53{,}8 \cdot 10^3$
0	∞	∞	∞	∞	∞	—∞	∞

According to this formula we calculate the value of derivative $\frac{dr_*}{dt}$ for different values of r_* within the interval indicated above. During the calculations it is convenient to use the table of values of different functions of r* given in the appendix. The calculations are given in Table 9.

By using the data of this table, in Figure 6, a a graph $\frac{dr_*}{dt} = f(r_*)$. The scales on the drawings of Figure 6 are chosen as follow

$$m_{r_*} = 0{,}1;\ m_{\frac{dr_*}{dt}} = 0{,}01 \text{ and } m_t = 1{,}0.$$

Since the drawing in Figure 6 is given in reduced size, as opposed to its actual dimensions, then for the purpose of convenience of orientation a scale of the drawing is given on it. According to formula (93), polar distance h - 10 cm is determined. According to the intersection of the curve in Figure 6, a with axis or_* one determines the relative limit radius $R_{*lim} = 0.53$ (one can of course obtain the same number according to the formula (64) as well). In Figure 6, b, a radial scale is plotted. Figure 6, c, depicts the sought value $r_* = f_1(t)$, plotted for the two extreme initial conditions:

$$r_{*ini} = 0 \text{ and } r_{*ini} = R_{*in}.$$

These initial conditions produce the origin of the two branches of curve $r_* = f_1(t)$. Curve 1 shows how the relative radius of the free cross-section will increase. Curve 2 shows how it will decrease. Both curves have a common asymptot with $R_{*lim} = 0.53$.

For more convenient use of the resulting drawing, in Figure 6, c, next to the scale of relative radii, a scale of r - absolute radii is plotted - and on the right-hand side of the drawing a scale of δ - the absolute thicknesses of the ice layer - is plotted. When plotting the scale of δ, formula (5) was used.

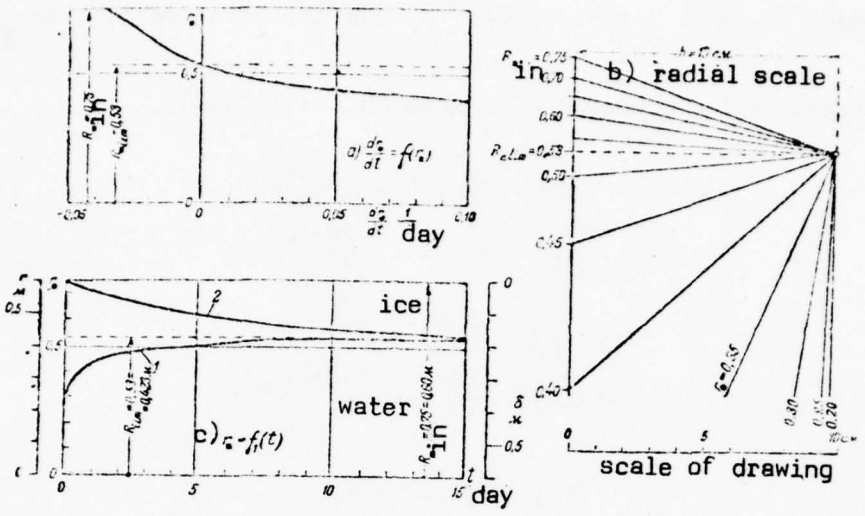

Figure 6. Graphic calculation of changes in time of cylindrical icing in a pipeline with an inside radius R_{in} = 0.60 m, corrected radius R_{cr} = 0.80 m, and a design gradient J_d = 0.0150 with a flow rate Q = 2.0 m^3/sec and a temperature of the outside air ϑ_o = -10°.

1 - branch of curve $r_* = f_1(t)$ with an increase in the relative radius from 0 to R_{*lim}; 2 - branch of curve $r_* = f_1(t)$ with a decrease in relative radius from R_{*in} to R_{*lim}.

The general aspects of the character of change in time of the radius of the free cross-section (curves 1 and 2) pertain to the fact that initially the strongly iced pipeline rapidly undergoes an increase in its free cross-section. This process occurs much more rapidly than the formation of an ice layer in the pipeline initially free of ice. Thus, in the given examples the pipeline even with the strongest initial icing comes to a state near the limit in 5 - 7 days, and from a state free of ice this occurs in 10 - 15 days.

7. Cylindrical Icing with a Set Hydraulic Gradient

A. The Basic Equations

For cylindrical icing with a set hydraulic gradient, the heat balance equations (52) and (54) and the hydraulic equation (39) are used as the initial relationships. As in the previous section, here it does not seem possible directly to use condition (74), which determines cylindrical icing. We shall write these equations with the

replacement of partial derivatives $\frac{dr_*}{dt}$ for the total derivative $\frac{dr_*}{dt}$

on the basis of the property of cylindrical icing (73):

$$
\begin{gathered}
-86\,400 \cdot 0{,}633 \frac{\pi}{n} R^{2/3} J^{1/3} \gamma_w C_w r_*^{2/3} \frac{\partial \vartheta}{\partial x} + \\
+ 54\,600 \frac{\pi R_{cr}^{2/3} J^{1/3}}{\lambda_i n} r_*^{2/3} - \frac{0{,}024 \cdot 2\pi(-\vartheta_0)}{-\frac{1}{\lambda_i} \ln r_*} - \\
- 2\pi \gamma_i L R_{cr}^2 r_* \frac{dr_*}{dt} = 0; \qquad (95) \\
911 \pi R^{1/3} J^{2/3} (\vartheta + 0{,}000784 H) r_*^{1/3} - \frac{0{,}024 \cdot 2\pi(-\vartheta_0)}{-\frac{1}{\lambda_i} \ln r_*} - \\
- 2\pi \gamma_i L R_{cr}^2 r_* \frac{dr_*}{dt} = 0; \\
\frac{\partial H}{\partial x} = J_d - J.
\end{gathered}
$$

This system of three equations with variables r_*, ϑ, H, x, and t is entirely analogous to system (75) in the preceding section. Here the same method of solution as before is selected. Variables ϑ and H are excluded. Variable ϑ is determined from the second equation of system (95):

$$
\begin{gathered}
\vartheta = \frac{0{,}024 \lambda_i (-\vartheta_0)}{455 R_{cr}^{1/3} J^{1/3}} \cdot \frac{1}{-r_*^{1/3} \ln r_*} + \\
+ \frac{\gamma_i L R_{cr}^{2/3}}{455 J^{1/3}} \cdot \frac{1}{r_*^{1/3}} \cdot \frac{dr_*}{dt} - 0{,}000784 H \text{ degree} \qquad (96)
\end{gathered}
$$

from which the partial derivative of water temperature along the length of the pipeline, remembering condition (74) and using the third equation of system (95), will be:

$$
\frac{\partial \vartheta}{\partial x} = -0{,}000784 \frac{\partial H}{\partial x} = -0{,}000784 (J_d - J) \text{ degree/m}. \qquad (97)
$$

After substitution of this expression in the first equation of system (95) and simple transforms, we obtain:

$$\frac{dr_*}{dt} = \frac{R_{cr}^{1/3} J^{1/2}}{2\gamma_i L_n} \left[42{,}9\gamma_w C_w J_d + \left(\frac{54600}{E} - 42{,}9\gamma_w C_w \right) J \right] r_*^{1/3} - \frac{0{,}024 k_i (-\vartheta_o)}{\gamma_i L R_{cr}^2} \cdot \frac{1}{r_* \ln r_*} \quad \text{day}^{-1}. \tag{98}$$

After substitution of values of the physical characteristics, this equation acquires the form:

$$\frac{dr_*}{dt} = R_{cr}^{1/3} J^{1/2} \left(29{,}5 J_d + 58{,}1 J \right) r_*^{1/3} - 0{,}659 \cdot 10^{-3} \frac{(-\vartheta_o)}{R_{cr}^2} \cdot \frac{1}{r_* \ln r_*} \quad \text{day}^{-1}. \tag{99}$$

The differential equation (98) or the differential equation (99) identical to it serves for determining changes in relative radius of the free cross-section during cylindrical icing with a set hydraulic gradient. The methods of solving this differential equation will be given somewhat later.

The expression for water temperature during cylindrical icing can be obtained from formula (96), if in it one substitutes the value of the derivative of the relative radius in time (99). After substitution and simple transforms, we obtain:

$$\vartheta = 0{,}001099 \frac{R_{cr}^{1/3} J^{1/2}}{n} \times \left[42{,}9\gamma_w C_w J_d + \left(\frac{54600}{E} - 42{,}9\gamma_w C_w \right) J \right] r_*^{1/3} - 0{,}000784 H \text{ degree}. \tag{100}$$

Upon introducing the numerical values of the physical characteristics, the same equation has the form:

$$\vartheta = 0{,}1099 R_{cr}^{1/3} J^{1/2} \left[42{,}9 J_d + 84{,}7 J \right] r_*^{1/3} - 0{,}000784 H \text{ degree}. \tag{101}$$

The value of water temperature can be calculated according to these formulas for each cross-section, having first determined pressure H according to the third equation of system (95).

B. Example of the Calculation

The differential equation (99), which is the key to determining relationship $r_* = f(t)$, cannot be solved analytically. Therefore it must

be solved graphically. The course of this solution is shown below in a specific example.

As before, a pipeline with R_{in} = 0.60 m and R_{cr} = 0.80 m laid with a design gradient J_d = 0.0150 and operating with a constant hydraulic gradient J = 0.0150 with $\vartheta_o = -10^o$ is examined. The assigned conditions are characteristic for a derivational pipeline.

By substituting these numerical values in (99), we obtain:

$$\frac{dr_*}{dt} = 0{,}1387\, r_*^{1/3} - 0{,}0103 \frac{1}{-r_* \ln r_*} \frac{1}{\text{day}} . \qquad (102)$$

Calculations of the value of derivative $\frac{dr_*}{dt}$ for different values of r_* are given in Table 10 according to this formula. The table given in the appendix was used during the calculations.

Table 10

Calculation of Values of Derivative $\frac{dr_*}{dt}$ for Different Values r_* for a Pipeline with R_{in} = 0.60 m, R_{cr} = 0.80 m, and J_d = 0.0150 when J = - 0.0150 and $\vartheta = -10^o$

r_* (1)	$r_*^{1/3}$ (2)	$\frac{1}{-r_* \ln r_*}$ (3)	$0{,}1387\, r_*^{1/3}$ (4)	$-0{,}0103 \frac{1}{-r_* \ln r_*}$ (5)	$\frac{dr_*}{dt} = (4) + (5)$ (6)
1,00	1,00	∞	0,1387	$-\infty$	$-\infty$
0,95	0,918	20,2	0,1273	—0,208	—0,081
0,90	0,839	10,59	0,1163	—0,109	+0,007
0,85	0,761	7,26	0,1055	—0,0748	0,0307
0,80	0,683	5,61	0,0956	—0,0578	0,0378
0,75	0,628	4,63	0,0871	—0,0476	0,0395
0,70	0,552	4,01	0,0766	—0,0413	0,0353
0,65	0,488	3,57	0,0677	—0,0368	0,0309
0,60	0,426	3,26	0,0591	—0,0336	0,0255
0,55	0,369	3,05	0,0512	—0,0314	0,0198
0,50	0,315	2,89	0,0436	—0,0298	0,0138
0,45	0,264	2,78	0,0366	—0,0286	0,0080
0,40	0,217	2,73	0,0301	—0,0281	+0,0020
0,35	0,174	2,72	0,0241	—0,0280	—0,0039
0,30	0,134	2,77	0,0186	—0,0286	—0,0100
0,25	0,0991	2,88	0,01372	—0,0297	—0,0160
0,20	0,0683	3,10	0,00948	—0,0319	—0,0224
0,15	0,0423	3,52	0,00587	—0,0363	—0,0304
0,10	0,0216	4,34	0,00300	—0,0447	—0,0417
0,05	0,00680	6,68	$0{,}943 \cdot 10^{-3}$	—0,0688	—0,0679
0	0	∞	0	$-\infty$	$-\infty$

Using Table 10, in Figure 7, a a graph of the relationship $\frac{dr_*}{dt} = f_1(r_*)$ is plotted. The curve of this graph intersects axis $0r_*$ at two points that determine the values $R_{*lim} = 0.91$ and $R_{*cri} = 0.38$. In this case it happens that $R_{*lim} > R_{*in} = 0.75$. Therefore, with a constant and quite prolonged passage of water the pipeline will be free of internal icing. The calculation could have been made within limits of $0 < r_* < R_{*in}$, but for demonstration of all of the theoretically possible changes in icing the calculation was made within limits of $0 < r_* < 1.0$. A radial scale is plotted in Figure 7, b. A double scale of r_* has been entered on its vertical axis, since two values of r_* correspond to each of its rays. The scale of r_* beginning at the top, drops downward a ng the right-hand side of the axis and at a value $r_* = 0.75$ crosses over to the left-hand side of the axis in order to rise.

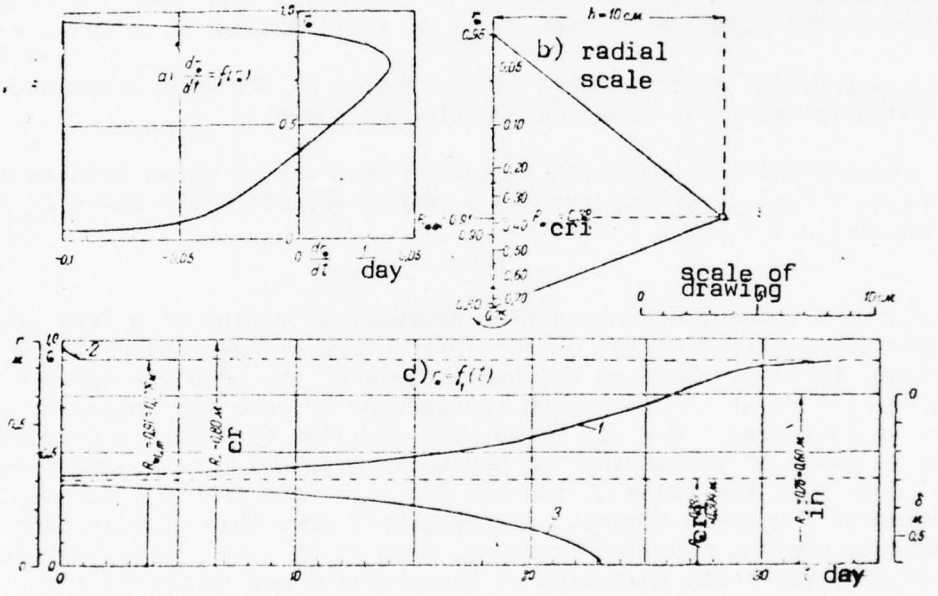

Figure 7. Calculation of changes in time of cylindrical icing of a pipeline with an inside radius $R_{in} = 0.60$ m, a corrected radius $R_{cr} = 0.80$ m, and a design gradient $J_d = 0.0150$ with water running through the pipeline with a hydraulic gradient $J = 0.0150$ and a temperature of the outside air $\vartheta_o = -10^o$.

1 - branch of curve $r_* = f_2(t)$, corresponding to an increasing radius of the free cross-section from $R_{* cri}$ to R_{*lim}; 2 - branch of curve $r_* = f_1(t)$, corresponding to a decreasing radius of the free cross-section from R_{*cr} to R_{*lim}; 3 - branch of curve $r_* = f_1(t)$, corresponding to the process of total freezing of the pipeline when $r_* < R_{*lim}$.

It is entirely obvious that R_{*lim} and R_{*cri} determine the common point on the scale. To the right along the horizontal line passing through this point, at a distance h = 10 cm from the vertical axis, a pole is marked. The polar distance of value h = 10 cm is determined according to formula (93) with the following values of scales used in the drawings of Figure 7: value of the scale of relative radius $m_{r*} = 0.1$ (i.e., 0.1 r_* is in 1 cm of drawing), value of the derivative scale $m_{\frac{dr*}{at}} = 0.01$ and value of the time scale $m_t = 1$. The drawings in Figure 7 are reduced in size; a scale of dimensions is given for orientation in these drawings.

By using the radial scale in Figure 7, c, a graph of the sought value $r_* = f_1(t)$ is plotted. Curve $r_* = f_1(t)$ has three branches. If at an initial moment in time t = 0 the value of the radius of the free cross-section lies within limits of $R_{*cri} < r_* < R_{*lim}$, then the free cross-section of the pipeline increases and at the limit has a value $r_* = \lim_{t \to \infty}$ $= R_{*lim} = 0.91$. These changes in the radius of the free cross-section for the given specific case are depicted by curve 1.

Curve 2 shows a reduction in radius of the free cross-section when $1 > r_{*ini} > R_{*lim}$. This curve has a common asymptote with curve 1, i.e., $\lim_{t \to \infty} r_* = R_{*lim} = 0.91$.

Curve 3 shows the process of a decrease in radius of a free cross-section leading to freezing of the entire free cross-section of the pipeline, totally. Such an emergency state of the pipeline appears when $R_{*cri} > r_{*ini} > 0$ and with the passage of time the intensity of its freezing increases. One can remove the pipeline from such a dangerous state by means of increasing the hydraulic gradient (increasing flow rate) such that the value of the new R_{*cri} is less than a value r_* existing at the given moment. If one cannot take this or some other measure to prevent pipeline freezing, then it is vital as rapidly as possible to terminate operation of the pipeline and to remove the water from it, having taken measures so that even a small quantity of water does not go inside and form ice.

From the physical point of view, the tendency of the radius of the free cross-section when $R_{*cri} < r_* < 1$ to take a value R_{*lim} is explained by the fact that the pipeline has a property to enter into thermal equilibrium independently, during which the amount of heat passing from the water to the layer of ice is equal to the amount of heat passing through the ice and the walls of the pipeline into the atmosphere. In this case the layer of ice is an automatic regulator. When there is a reduction for any reason in the amount of heat coming from water, the layer of ice increases; this reduces thermal losses and thermal equilibrium is restored. If however the influx of heat

from the water to the ice increases, then the thickness of the layer of ice decreases, losses increase, and thermal equilibrium again ensues. Hence, the state of thermal equilibrium is stable when R_{*lim}.

There is another state of thermal equilibrium when R_{*cri} exists - unstable. It is sufficient for the free cross-section of the pipeline to increase somewhat for the flow rate of water to increase and with it the influx of heat from water to the ice. The ice begins to melt, thence the influx of heat from the water to the ice dominates over losses. Melting of the ice occurs so long as r_* does not reach another state of thermal equilibrium with R_{*lim}. If the free cross-section somewhat constricts in comparison with the critical (r_* becomes less than R_{*cri}), then as the result of a reduction in the flow rate of water the influx of heat from water to the ice decreases. In this case heat losses begin to dominate over the heat influx, as the result of which the thickness of the layer of ice increases up to the point of complete freezing of the entire cross-section of the pipeline.

Returning once again to the resulting graph $r_* = f_1(t)$ depicted in Figure 7, c, we note that for transition from relative values of the radii of free cross-section r_* to absolute r, a scale r has been plotted next to the scale r_*. To the right of the graph, a scale of absolute thickness of the ice layer δ has been plotted.

Above, cases of icing of pipelines whose hydraulic regime was set either by flow rate or by hydraulic gradient were examined. On the basis of the imparted information one can calculate the icing of a pipeline whose hydraulic regime is set by the combined method; for example, a pipeline must handle a certain flow rate Q. But with strong icing, when hydraulic losses increase and the hydraulic gradient reaches a certain determined value J which cannot be overcome, the pipeline regime changes. It begins to function with a constant hydraulic gradient J that handles flow rates which vary depending on the degree of icing that are less than flow rate Q.

8. Cylindrical Icing with a Variable Flow Rate and Variable Atmospheric Temperature

Practically speaking, the pipelines of hydroelectric power stations operate non-uniformly (especially during daily regulation), handling different flow rates of water. Furthermore, they are under conditions of fluctuating atmospheric temperature. These circumstances of course have an influence on the ice regime of pipelines.

Below is a specific example which shows the methods of calculating the ice regime of a pipeline taking into account daily regulation and fluctuations in atmospheric temperature.

We shall assume that there is a pipeline with a corrected radius R_{cr} = 0.50 m and a design gradient J_d = 0.0150.

The temperature course of the air surrounding the pipeline in the Winter period is shown in Figure 9, b. It is necessary to estimate inside icing of the lower stretch of pipeline where the intake conditions already have no effect. Value R_{in} in the given case is not provided, since it does not have principal significance in the calculations cited below.

The hydraulic regime of the pipeline is assigned for comparison in the following three variations: with a constant flow rate having a value of 2.0 m^3/sec, with a constant flow rate having a value of 0.5 m^3/sec, and a mean daily flow rate having a value of 0.5 m^3/sec distributed over the length of the day such that for three-fourths of the day the flow of water is absent in the pipeline and for one-fourth of the day a flow rate having a value of 2.0 m^3/sec exists.

The basic calculations are preceded by certain preliminary calculations and constructions.

In Figure 8 the graphs of time changes of cylindrical icing r = f(t) are plotted with different hydraulic regimes of the pipeline.

In Figure 8, a, according to formula (85), a family of curves has been plotted which determine the course of freezing of a pipeline filled with water but handling no flow rate (Q = 0), at different temperatures of the outside air ϑ_0 degrees. The graph of Figure 4 can also be used for this construction. Figure 8, b, gives a family of curves of stabilization of cylindrical icing in the pipeline with a flow rate Q = 0.5 m^3/sec, also for different atmospheric temperature ϑ_0. This family of curves was plotted according to the methods presented in Section 6. Each curve of this family consists of two branches which determine the increase or decrease in the radius of free cross-section r. In Figure 8, c, the same family of curves is given for a flow rate Q = 2.0 m^3/sec. All of the auxiliary graphs in Figure 8 pertain to absolute values of the radii of free cross-section r.

During calculation of the pipeline under time-variable conditions of its operation, one proceeds from the assumption that for a certain short period of time these conditions are constant.

Thus, it is considered that over the course of the day an identical (mean daily) atmospheric temperature holds, and the flow rate of water during daily regulation sharply changes from 0 to 2.0 m^3/sec (the latter can quite closely correspond to reality).

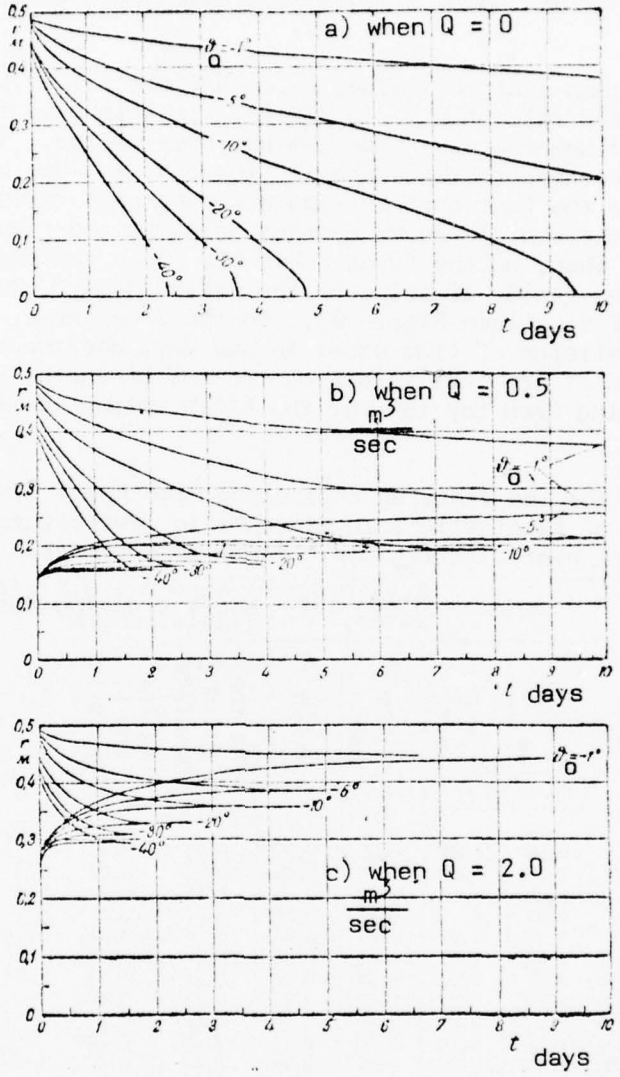

Figure 8. Graphs of changes in time of cylindrical icing r = f(t) at different atmospheric temperatures and with different hydraulic regimes of a pipeline with R_{cr} = 0.50 m.

The calculation is made in tabular form. Below, in Table 11, a calculation for one month (November) is given as an example. The first two columns of the table are occupied by dates and mean daily temperatures of the air in accordance with the graph in Figure 9, b.

The third and fourth columns give limit radii of icing R_{lim} for flow rates of Q = 2.0 m^3/sec and Q = 0.5 m^3/sec calculated according to formula (64).

The next column contains values of r that are set at the end of the day with a flow rate of Q = 2.0 m^3/sec. In this case it is assumed that on 3 November the pipeline was free of ice. The column was filled-in according to the graph in Figure 8, c. Use of the graph pertains to the fact that interpolation between curves enables one to find an intermediate curve corresponding to the temperature of the given days. Then, on the found curve one finds the point that corresponds to the radius of icing of the previous day. This radius plays the role of r_{ini} (see Figure 5). On the same curve, from this point through a stretch of time equal to one day, one reads the value of the new radius of ice that is set at the end of the calculation day. Thus, by proceeding from day to day, the fifth column is filled-in.

Table 11

Calculation of the ice state of a pipeline with R_{cr} = 0.50 m with consideration of fluctuations in atmospheric temperature during different hydraulic regimes. Month of November.

Date	t_0 degrees	R_{lim}, m		r with constant flow rates, m		r when Q = 0.5 m^3/sec and during daily regulation, m	
		Q = 2.0 m^3/sec	Q = 0.5 m^3/sec	Q = 2.0 m^3/sec	Q = 0.5 m^3/sec	At the end of Downtime	At the end of Operation
1	2	3	4	5	6	7	8
3	0	0,50	0,50	0,50	0,50	0,50	0,50
4	—3,5	0,40	0,24	0,44	0,43	0,44	0,43
5	—4,0	0,40	0,23	0,42	0,40	0,40	0,40
6	—7,0	0,38	0,21	0,38	0,36	0,37	0,37
7	—11,9	0,42	0,26	0,40	0,35	0,36	0,37
8	—1,5	0,43	0,27	0,41	0,35	0,36	0,37
9	—10,8	0,35	0,20	0,38	0,30	0,32	0,33
10	—11,0	0,36	0,20	0,37	0,27	0,28	0,30
11	—11,6	0,36	0,20	0,36	0,24	0,26	0,31
12	—3,5	0,40	0,24	0,38	0,24	0,30	0,32
13	—5,0	0,39	0,22	0,39	0,24	0,30	0,32
14	—7,8	0,38	0,21	0,38	0,23	0,29	0,32
15	—6,0	0,39	0,22	0,38	0,23	0,30	0,32
16	—4,0	0,40	0,23	0,39	0,23	0,30	0,32
17	—2,0	0,42	0,26	0,41	0,24	0,31	0,33
18	—14,5	0,35	0,18	0,37	0,21	0,28	0,32
19	—11,0	0,35	0,20	0,36	0,20	0,28	0,31
20	—15,0	0,34	0,18	0,34	0,19	0,26	0,31
21	—18,2	0,34	0,18	0,34	0,18	0,25	0,31
22	—15,3	0,34	0,18	0,34	0,18	0,26	0,31
23	—15,0	0,34	0,18	0,34	0,18	0,26	0,31
24	—13,0	0,35	0,19	0,35	0,18	0,27	0,31
25	—15,9	0,34	0,18	0,34	0,18	0,26	0,31
26	—13,0	0,35	0,19	0,35	0,18	0,27	0,31
27	—9,0	0,37	0,22	0,36	0,19	0,28	0,31
28	—18,0	0,34	0,18	0,35	0,18	0,25	0,31
29	—16,5	0,34	0,18	0,34	0,18	0,26	0,31
30	—12,0	0,36	0,20	0,35	0,19	0,27	0,31

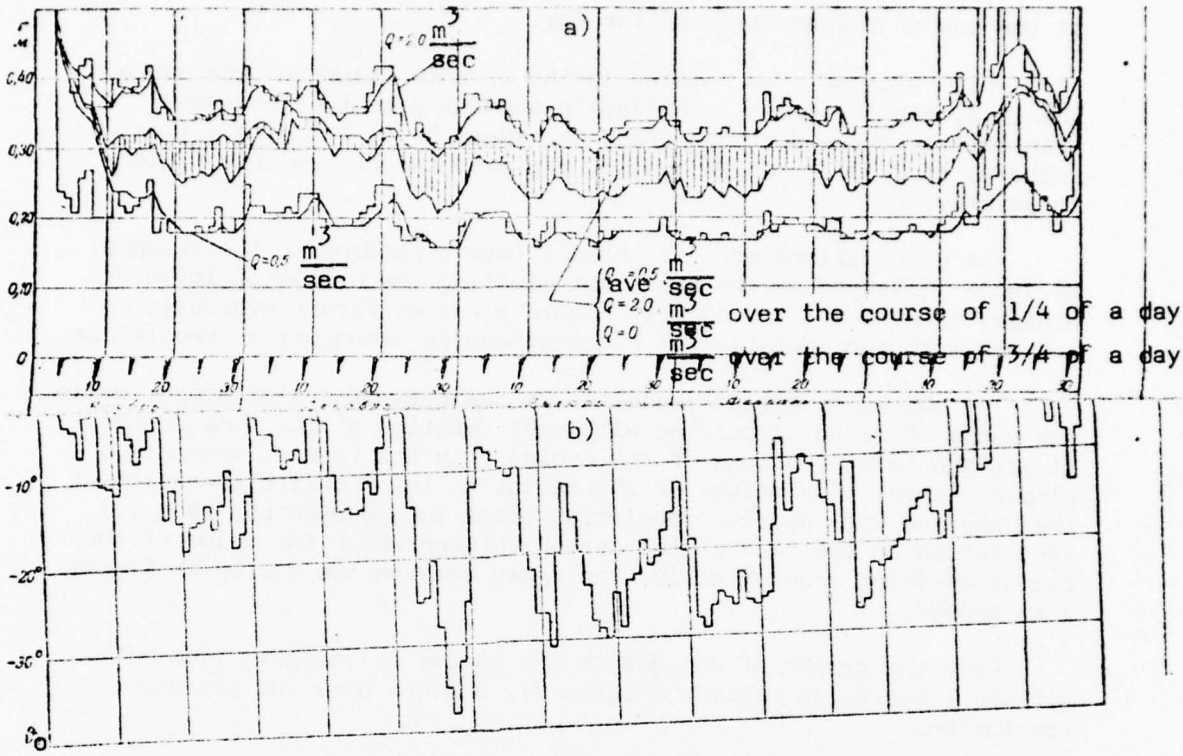

Figure 9. Change in pipeline icing with R_{cr} = 0.50 m over the course of a winter.
a - icing of pipeline with different hydraulic regimes; b - atmospheric temperature.

In the next (sixth) column, values of the time-variable radii of icing with a flow rate of water in the pipeline of Q = 0.5 m^3/sec are given. In this case, Figure 8, b is used as an auxiliary graph.

In the last two columns (the seventh and eighth), values of the radii of icing r are given for the mean daily flow rate Q_{ave} = 0.5 m^3/sec during daily regulation, with the provision that the first three quarters of the day has no flow of water through the pipeline, but that the pipeline remains full of water, and that in the last quarter of the day a flow rate having a value Q = 2.0 m^3/sec is handled by the pipeline. The penultimate (seventh) column of the table is filled-in by using the graph in Figure 8, a. In it, by interpolation, one notes a curve corresponding to atmospheric temperature on the given day; on this curve one notes the point corresponding to the value r_{ini} (this value r corresponds to the end of the previous period), and from that point through an interval of time of 0.75 days on the same curve, one finds the new radius that appears in the pipeline

at the end of the downtime of the day.

This value of r is entered in the seventh column of the table. The last, eighth column is filled-in according to the very same principle by the aid of the graph in Figure 8, c, but with a time interval of 0.25 days. Of course, the last two columns are filled-in jointly.

The calculations made in tabular form according to the example of Table 11 served as the basis for plotting the graphs of icing of a pipeline with R_{cr} = 0.50 m in Figure 9 for different hydraulic regimes with consideration of fluctuations in atmospheric temperature.

The values of R_{lim} are plotted in the form of a step graph, while the radii of icing calculated with consideration of the time change (third and fourth columns of the table) - in the form of segments of sloping lines. The values of the radius of icing during operation of the pipeline with diurnal regulation (last two columns of Table 11) are plotted in the form of two curves between which the value of the radius of icing changes daily (the space between the curves in Figure 9 is shaded).

From the graphs of changes in the radius of icing in time with different hydraulic regimes (Figure 9), one can draw the following conclusion.

The temperature regime of the air does not have such important significance for the degree of icing of the pipeline as the regime of flow rates of water within it.

Values of R_{lim} (step graphs) are extremely close to values of r (graph with slanted lines) calculated with consideration of the time change. (The greatest deviation between the values of R_{lim} and r is obtained at atmospheric temperatures near zero). The closeness of these values makes it possible with preliminary planning to characterize the icing of the pipeline constantly passing the same flow rate, and with a value of the radius of limit icing R_{lim}. In this case the atmospheric temperature can be introduced into the calculation averaged over a certain prolonged interval of time (over 10 days or even over a month).

The regime of daily regulation of flow rates has a great effect on the ice regime of the pipeline. Briefly running large flow rates through the pipeline can significantly reduce the layer of ice on the inner surfaces of the pipeline walls. The latter is extremely important for practical use in the Spring during preparation for the thaws, for the purpose of preventing an ice-gang which is possible in the pipeline at this time. The calculation of the limit ice state with

daily regulation according to the average daily flow rate has no real meaning.

Chapter Four

STEADY-STATE ICING OF THE PIPELINE

It was decided to call icing that is constant in time but variable along the length of the pipeline steady-state icing of the inside surface of the pipeline walls.

Steady-state icing ensues after a quite prolonged continuous operation of the pipeline with constant hydraulic and thermal conditions. It is the limit that icing tends toward with the passage of time.

Mathematically, it can circumstantially be described in the following way: $r = f(x, t)$, where r m - radius of the free cross-section, which determines the degree of pipeline icing. But $\lim_{t \to \infty} \frac{\partial r}{\partial t} = 0$. Therefore, when

$$r = f(x), \tag{103}$$

where r m is already the radius of the free cross-section during steady-state icing. This meaning for r is preserved in all subsequent formulas of this chapter.

Thus, the radius of the free cross-section during steady-state icing is only a function of x m - the distance along the axis of the pipeline from its entry.

Steady-state icing in the initial cross-section of the pipeline is determined by the temperature of the water entering the pipeline. Then along the pipeline icing changes and at a sufficient distance from the beginning acquires a limit value invariable with respect to length.

Hence, the investigation of steady-state icing is of interest only for a certain stretch of the pipeline lying adjacent to its beginning.

9. The Basic Relationships

The mathematical relationships for all cases of intake conditions (for different values of intake temperatures) are obtained from the heat balance equations (46) and (48) and the hydraulic equation (40), if the condition indicated above, that $\frac{\partial r}{\partial t} = 0$ is for steady-state icing, is inserted in them. In this case, according to (103), the partial derivatives according to x become total. After considering these circumstances,

we obtain a system of three equations:

$$\left.\begin{aligned} &-86\,400Q\gamma_w C_w \frac{d\vartheta}{dx} + 218\cdot 10^3 \frac{n^2Q^2}{\pi^2 R_{cr}^{5,33}} \cdot \frac{1}{r_*^{5,33}} - \\ &\quad - \frac{0{,}024\cdot 2\pi(\vartheta - \vartheta_0)}{\cdot -\frac{1}{\lambda_i}\ln r_*} = 0; \\ &17{,}2\pi\left(\frac{Q}{R_{cr}}\right)^{3/4}(\vartheta + 0{,}000784H)\frac{1}{r_*^{3/4}} - \\ &\quad - \frac{0{,}024\cdot 2\pi(\vartheta - \vartheta_0)}{-\frac{1}{\lambda_i}\ln r_*} = 0; \\ &\frac{dH}{dx} = J_d - 2{,}52\frac{n^2Q^2}{\pi^2 R_{in}^{5,33}} \cdot \frac{1}{r_*^{5,33}}. \end{aligned}\right\} \qquad (104)$$

This system of equations includes the basic relationships on which all further investigations of this chapter are based.

A. The Temperature of Water During Steady-State Icing

In any cross-section of pipeline during steady-state icing there is a certain quantitative relationship between ϑ degree - water temperature, H m - pressure above atmospheric pressure, r m - radius of the free cross-section. This relationship is determined by the second equation of system (104), which can be presented in the following form:

$$\vartheta = \frac{0{,}0242\lambda_i(-\vartheta_0)R_{cr}^{3/4}}{8{,}60Q^{3/4}} \cdot \frac{r_*^{3/4}}{-\ln r_*} - 0{,}000784H \text{ degrees.} \qquad (105)$$

Having substituted the numerical values of the physical characteristics here, we obtain:

$$\vartheta = 0{,}00559(-\vartheta_0)\left(\frac{R_{cr}}{Q}\right)^{3/4} \cdot \frac{r_*^{3/4}}{-\ln r_*} - 0{,}000784H \text{ degrees.} \qquad (106)$$

where R_{cr}, Q, and ϑ_0 are, according to the conditions of the problem, constant values, while ϑ, H, and r_* can change. Therefore, the relationship determined by formula (106) can be schematically represented in the form $\vartheta = f(r_*, H)$. This functional relationship will be widely used below.

Formula (106) can give different results for the value of the relative radius of the free cross-section within limits of $0 < r_* < 1$ during the calculations. If $r_* > R_{*in}$, then this means that in the

given cross-section there is no inside icing. Icing only exists where $r_* < R_{*in}$. If it proves that $r_* = R_{*in}$, then although icing is possible the thickness of the ice layer is extremely small.

B. Types of Steady-State Icing

Steady-state icing can develop with extreme variation under the effect of intake conditions.

The conditions of intake of water into the pipeline from the viewpoint of the thermal calculations are determined by ϑ_{int} degrees - temperature of water at the point of intake. If this temperature is high, then the thickness of the layer of ice along the length of the pipeline will increase to dimensions which correspond to limit icing. In this case there may be a stretch entirely free of ice at the beginning of the pipeline. If however the water temperature at the intake is low, then a thick layer of ice appears at the intake which will decrease along the length of the pipeline, also to the dimensions that characterize limit icing.

One can determine which of these types of steady-state icing will develop in the pipeline in the following way.

There is a completely determined water pressure H_{int} at the intake to the pipeline. Having substituted the value of this pressure in formula (106), we obtain the following relationship between water temperature and the relative radius of the free cross-section, which can simultaneously exist in the intake cross-section of the pipeline during steady-state icing:

$$\vartheta_{int} = 0{,}00559(-\vartheta_0)\left(\frac{R}{Q}\right)^{5/4} \frac{r_{*int}^{3/4}}{-\ln r_{*int}} - 0{,}000784 H_{int} \text{ degrees.} \qquad (107)$$

This relationship makes it possible to determine which of the types of steady-state icing develops in the pipeline.

For this purpose it is necessary to make a comparison among the following characteristic values of temperatures of the water or their corresponding values of relative radii:

ϑ_{int} or r_{*int} - temperature or relative radius of the free cross-section corresponding to it calculated according to formula (107);

$\vartheta_{int\ lim}$ or R_{*lim} - temperature corresponding to limit icing in the intake cross-section, or the limit relative radius of the free cross-section;

$\vartheta_{int\ pl}$ or R_{*in} - temperature corresponding to the appearance of an extremely thin layer of ice in the intake cross-section or the inside relative radius of the pipeline.

The following relationships can exist among these three values of characteristic temperatures for relative radii of free cross-sections. These values determine the character of steady-state icing.

If inequality $\vartheta_{int} > \vartheta_{int\ lim}$ exists, or the corresponding inequality $r_{*int} > R_{*lim}$, then at the beginning of the pipeline less icing is established than the limit and the thickness of the ice layer will increase proportional to pipeline length.

Moreover, if $\vartheta_{int} > \vartheta_{int\ pl}$ or, $r_{*int} > R_{*int}$, then a stretch exists at the beginning of the pipeline that is free of ice.

If however $\vartheta_{int} < \vartheta_{int\ lim}$ or $r_{*int} < R_{*lim}$, then there wil be more icing at the beginning of the pipeline than limit icing and the thickness of the layer of ice will decrease proportional to pipeline length.

It is entirely obvious that when $\vartheta_{int} = \vartheta_{int\ lim}$ and $r_{*int} = R_{*lim}$ a limit icing is established at the beginning of the pipeline that is unchangeable with respect to pipeline length, but when $\vartheta_{int} = \vartheta_{int\ pl}$ and $r_{*int} = R_{*in}$ in the intake cross-section of it a thin layer of ice appears which increases to the limit dimensions proportional to distance from intake.

Thus, the general aspects of steady-state icing, whose knowledge is vital in these subsequent calculations, are determined.

In all cases when $\vartheta_{int} < \vartheta_{int\ pl}$ or $r_{*int} < R_{*in}$, relative radius in the intake cross-section of the pipeline is determined by a value r_{*int}. This value is subsequently used as the boundary condition during the calculation of steady-state icing. In a case of $\vartheta_{int} > \vartheta_{int\ pl}$ or $r_{*int} > R_{*in}$ in the presence of a stretch free of ice, the boundary conditions become complicated and it becomes necessary to determine the length of the stretch that is free of ice. This question is examined in the next section.

C. The Length of a Stretch at the Beginning of the Pipeline Free of Ice As the Result of High Water Temperature

It was reported above that with adequately high temperatures of the water entering the pipeline, a stretch can exist at the beginning of the pipeline that is free of ice. In order to determine the length of that

stretch, we use the first equation of system (104), somewhat modifying it applicable to this case. The last member of this equation determines thermal losses of the frozen pipeline according to (15). This member should be taken from the equation and replaced by expression (17) which determines thermal losses of the pipeline without inside icing. Moreover, it is vital to make a substitution $r_* = R_{*in} = \frac{R_{in}}{R_{cr}}$ in the equation. After the indicated operation, the first equation of system (104) acquires the form:

$$-86\,400Q\gamma_w C_w \frac{d\vartheta}{dx} + 218\cdot 10^3 \frac{n^2Q^3}{B\pi^2 R_{in}^{5\frac{1}{3}}} - \frac{0{,}024\cdot 2\pi(\vartheta-\vartheta_n)}{0{,}00280\left(\frac{R_{in}}{Q}\right)^{3/4} - \frac{1}{\lambda_i}\ln\frac{R_{in}}{R_{cr}}} = 0. \tag{108}$$

After separation of variables, we obtain:

$$dx = \frac{A}{B - C(\vartheta - \vartheta_0)}\,d\vartheta,$$

where

$$A = 86\,400Q\gamma_w C_w;$$

$$B = 218\cdot 10^3 \frac{n^2Q^3}{B\pi^2 R_{in}^{5\frac{1}{3}}};$$

$$C = \frac{0{,}024\cdot 2\pi}{0{,}00280\left(\frac{R_{in}}{Q}\right)^{3/4} - \frac{1}{\lambda_i}\ln\frac{R_{in}}{R_{cr}}}.$$

Integration of this differential equation is conducted within limits from x to zero to x, i.e., from the beginning of the pipeline to a certain cross-section of the pipeline at a distance x from the beginning, and for ϑ within limits from ϑ_{int} to ϑ, where ϑ_{int} degrees - temperature of water entering the pipeline, and ϑ degrees - temperature of water in the pipeline at a distance x from its beginning.

After integration, we obtain:

$$x = \frac{A}{C}\ln\frac{B - C(\vartheta_{int} - \vartheta_0)}{B - C(\vartheta - \vartheta_0)}. \tag{109}$$

If one ignores the heat of friction and the thermal reserve that arises in the pipeline as the result of a change in the melting point of ice with a change in pressure, i.e., if one enters B = 0, then from (109) one obtains the formula for cooling of water running through a pipe (28) known in heat exchange.

Formula (109) can serve for calculating the temperature of the water

in any cross-section of the pipeline in a stretch free of ice.

In order to determine the length of a stretch of pipeline free of ice, it is vital first to clarify the significance of water temperature ϑ_{pl} degrees, at which icing of the inside surfaces of the walls appears. In order to determine this temperature, we substitute $r_* = R_{*in}$ in formula (106). As the result, we obtain:

$$\vartheta_{pl} = 0{,}00559(-\vartheta_0)\left(\frac{R_{cr}}{Q}\right)^{3/4} \frac{R_{*in}^{3/4}}{-\ln R_{*in}} - 0{,}000784\, H \text{ degree}. \tag{110}$$

where (with a constant design gradient of the pipeline and small local hydraulic resistances), pressure can be expressed by using the third equation of system (104) in the following form:

$$H = \left(J_d - 2{,}52\frac{n^2Q^2}{\pi^2 R_{in}^{5.33}}\right) x \text{ m}. \tag{111}$$

By substituting the value $\vartheta = \vartheta_{pl}$ in (109) according to (110), and by using (111), we obtain:

$$x_{pl} = \frac{A}{C}\ln \frac{B - C(\vartheta_{int} - \vartheta_0)}{B - C\left[0{,}00559\left(-\vartheta_0\right)\left(\frac{R_{cr}}{Q}\right)^{3/4} \cdot \frac{R_{*in}^{3/4}}{-\ln R_{*in}} - \right.} \tag{112}$$

$$\left. - 0{,}000784\left(J_d - 2{,}52\frac{n^2Q^2}{\pi^2 R_{in}^{5.33}}\right) x_{pl} - \vartheta_0\right]$$

In this transcendent equation, x_{pl} m is the sought value of the distance from the beginning of the pipeline to the cross-section in which icing of the inside surfaces of the walls appears.

If the atmospheric temperature is so low that in comparison with its absolute value (without consideration of sign) the value of water temperature can be ignored, then the differential equation from which formula (112) was obtained is simplified and acquires the form:

$$dx = \frac{A}{B - C(-\vartheta_0)}\, d\vartheta.$$

Following integration within the same limits as in the previous case, we obtain:

$$x = \frac{A(\vartheta_{int} - \vartheta_0)}{B - C(-\vartheta_0)} \text{ m}. \tag{113}$$

From this expression, just as in the previous case, through (110) and (111), one can obtain a formula for calculating x_{pl} m - the length of the stretch free of ice:

$$x_{pl} = \frac{A\left[0{,}00559(-\vartheta_0)\left(\frac{R_{in}}{Q}\right)^{3/4} \frac{R_{in}^{3/4}}{-\ln R_{*in}} - \vartheta_{int}\right]}{B - C(-\vartheta_0) + 0{,}000784A\left(J_d - 2{,}52\frac{n^2Q^2}{\pi^2 R_{in}^{5.33}}\right)} =$$

$$= \frac{86\,400 Q \gamma_w C_w \left[0{,}00559(-\vartheta_0)\left(\frac{R_{cr}}{Q}\right)^{3/4} \frac{R_{*in}^{3/4}}{-\ln R_{in}} - \vartheta_{in}\right]}{\left(\frac{218\cdot 10^3}{E} - 171\right)\frac{n^2Q^2}{\pi^2 R_{in}^{5.33}} - \frac{0{,}024\cdot 2\pi(-\vartheta_*)}{0{,}00280\left(\frac{R_{in}}{Q}\right)^{3/4} - \lambda_i \ln \frac{R_{in}}{R_{cr}}} + 67{,}8QJ_d} M. \tag{114}$$

This formula is a simplified expression for x_{pl}, in comparison with formula (112).

10. The Calculation of Steady-State Icing

A. The Basic Differential Equation of Steady-State Icing

From the system of three equations (104) that contain five variables r_*, ϑ, H, x, and t, we exclude the two variables ϑ and H. For this purpose, from equation (106), identical to the second equation of system (104), we find:

$$\frac{d\vartheta}{dx} = \frac{0{,}024\lambda_i(-\vartheta_0)R_{cr}^{3/4}\left[1 + \frac{3}{4}(-\ln r_*)\right]}{8{,}60Q^{3/4} r_*^{3/4}(-\ln r_*)^2} \cdot \frac{dr_*}{dx} -$$

$$-0{,}000784\frac{dH}{dx} \text{ degree/m}. \tag{115}$$

We substitute here the expression for the pressure gradient according to the third equation of system (104) :

$$\frac{d\vartheta}{dx} = \frac{0{,}024\lambda_i(-\vartheta_0)R_{cr}^{3/4}\left[1 + \frac{3}{4}(-\ln r_*)\right]}{8{,}60Q^{3/4} r_*^{3/4}(-\ln r_*)^2} \cdot \frac{dr_*}{dx} -$$

$$-0{,}000784\left(J_d - 2{,}52\frac{n^2Q^2}{\pi^2 R_{cr}^{5.33}} \cdot \frac{1}{r_*^{5.33}}\right). \tag{116}$$

After substitution of this expression in the first equation of system (104), we obtain:

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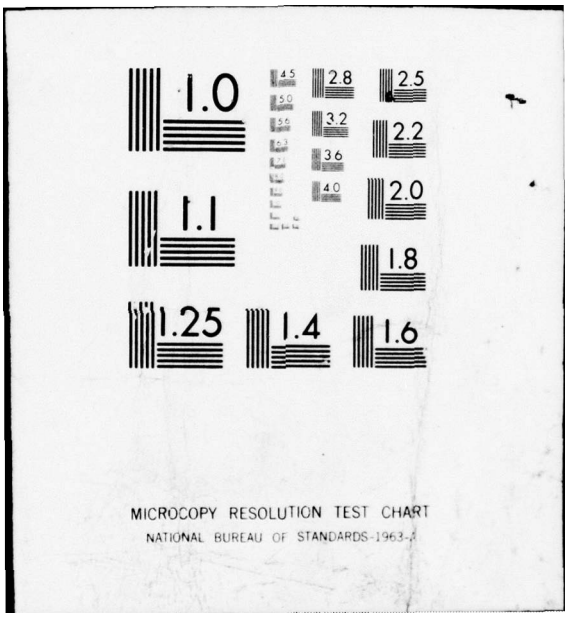
1.0
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$$-86\,400Q\gamma_w C_w\left[\frac{0{,}024\lambda_i(-\theta_0)R_{cr}^{3/4}}{8{,}60Q^{3/4}}\cdot\frac{1+\frac{3}{4}(-\ln r_*)}{r_*^{1/4}(-\ln r_*)^2}\cdot\frac{dr_*}{dx}-\right.$$

$$-0{,}000784\left(J_d-2{,}52\frac{n^2Q^2}{\pi^2R_{cr}^{5.33}}\cdot\frac{1}{r_*^{5/3}}\right)\Bigg]+$$

$$+218\cdot10^3\frac{n^2Q^2}{E\pi^2R_{cr}^{5.33}}\cdot\frac{1}{r_*^{5/3}}-\frac{0{,}024\cdot2\pi(-\theta_0)}{\frac{1}{\lambda_i}-\ln r_*}=0; \tag{117}$$

from which one can determine the derivative of the relative radius of the free cross-section according to x :

$$\frac{dr_*}{dx}=\left(\frac{1}{E}-0{,}000784\right)\frac{2{,}52\cdot8{,}60n^2Q^{2.33}}{\pi^2\cdot0{,}024\lambda_i(-\theta_0)R_{cr}^{5.33}}\times$$

$$\times\frac{(-\ln r_*)^2}{r_*^{5/3}\left[1+\frac{3}{4}(-\ln r_*)\right]}-\frac{8{,}60\cdot2\pi}{86\,400Q^{3/4}R_{cr}^{3/4}}\cdot\frac{r_*^{1/4}(-\ln r_*)}{1+\frac{3}{4}(-\ln r_*)}+ \tag{118}$$

$$+\frac{0{,}000784\cdot8{,}60Q^{3/4}J_d}{0{,}024\lambda_i(-\theta_0)R_{cr}^{3/4}}\cdot\frac{r_*^{1/4}(-\ln r_*)^2}{1+\frac{3}{4}(-\ln r_*)}.$$

After substitution of the numerical values of the physical characteristics, the same equation acquires the form:

$$\frac{dr_*}{dx}=0{,}00714\cdot10^{-3}\frac{Q^{2/3}}{(-\theta_0)R_{cr}^{5.33}}\cdot\frac{(-\ln r_*)^2}{r_*^{5/12}\left[1+\frac{3}{4}(-\ln r_*)\right]}-$$

$$-0{,}625\cdot10^{-3}\frac{1}{Q^{1/4}R_{cr}^{3/4}}\cdot\frac{r_*^{1/4}(-\ln r_*)}{1+\frac{3}{4}(-\ln r_*)}+ \tag{119}$$

$$+0{,}1407\frac{Q^{3/4}J_d}{(-\theta_0)R_{cr}^{3/4}}\cdot\frac{r_*^{1/4}(-\ln r_*)^2}{1+\frac{3}{4}(-\ln r_*)}\frac{1}{M}.$$

Calculation of steady-state icing of the pipeline is based on the integration of this differential equation.

B. An Example of the Calculation

a) The original data. The law of change in the size of the radius of the free cross-section along the axis of the pipeline during steady-state icing is determined by the differential equation (119). This equation cannot be solved analytically. Therefore, below methods of its graphic solution are employed similar to the methods already employed in the preceding chapter during the analysis of non-steady-state ice processes.

The structure of differential equation (119) can be depicted in the form:

$$\frac{dr_*}{dx} = f(r)\,\frac{1}{m}\,. \tag{120}$$

As the result of the calculations, it is necessary to find the relationship

$$r_* = f_1(x). \tag{121}$$

All of the arguments, methods and plots for solving the assigned problem are identical to those given in the previous chapter. The only difference is that in the previous chapter the independent variable was t - time (91), but here is x - the coordinate of the length along the axis of the pipeline. In view of the indicated analogy, all of the necessary information about methods of integration will be taken from the previous chapter, and here we shall proceed to explaining the solution to equation (119) based on the following specific example.

It is necessary to calculate steady-state icing of a pipeline having an inside radius R_{in} = 0.60 m, a corrected radius R_{cr} = 0.80 m, and a design gradient J_d = 0.0150 when running water at a flow rate Q = 2.0 m^3/sec through the pipeline and with temperature of the outside air $\vartheta_0 = -10^\circ$. Atmospheric pressure exists at the intake to the pipeline, i.e., H_{int} = 0. Calculations are made for two values of intake temperatures, $\vartheta_{int} = 0.10^\circ$ and $\vartheta_{int} = 0^\circ$.

For the sake of simplicity, R_{in}, R_{cr} and J_d in this example are accepted to be constant along the length of the pipeline. Under actual conditions, the values of these characteristics can be different in separate stretches of the pipeline. In this case, every stretch is subject to independent calculation, and terminal ice and temperature conditions of the previous section will serve as the intake conditions for the next one.

After substituting the numerical values of the magnitudes of the examined example in equation (119), we obtain:

$$\frac{dr_*}{dx} = 0{,}01867\cdot 10^{-3}\,\frac{(-\ln r_*)^2}{r_*^{5/16}\left[1+\frac{3}{4}(-\ln r_*)\right]} -$$

$$-0{,}619\cdot 10^{-3}\,\frac{r_*^{1/4}(-\ln r_*)}{1+\frac{3}{4}(-\ln r_*)} +$$

$$+0{,}417\cdot 10^{-3}\,\frac{r_*^{1/4}(-\ln r_*)^2}{1+\frac{3}{4}(-\ln r_*)}\,1/m. \tag{122}$$

The radii will be introduced within limits of the possible change in corrected radius of the free cross-section $0 < r_* < R_{*in} = 0.75$.

We calculate the value of derivative $\frac{dr*}{dx}$ for different values of relative radius r_* (Table 12).

According to this table, a graph $\frac{dr_*}{dx} = f(r_*)$ has been plotted (see below) in Figure 10, a, in which a curve intersects axis or$_*$ at a point that determines the value of the relative radius of limit icing $R_{*lim} = 0.53$.

Table 1

Calculation of the Values of $\frac{dr_*}{dx}$ with Different Values of r_* for a Pipeline with $R_{in} = 0.60$ m, $R_{cr} = 0.80$ m, and $J_d = 0.0150$ when $Q = 2.0$ m^3/sec and $\vartheta_o = -10°$.

r_*	$\frac{(-\ln r_*)^3}{r_*^{5/4}\left[1+\frac{3}{4}(-\ln r_*)\right]}$	$\frac{r_*^{1/4}(-\ln r_*)}{1+\frac{3}{4}(-\ln r_*)}$	$\frac{r_*^{1/4}(-\ln r_*)^2}{1+\frac{3}{4}(-\ln r_*)}$	$0{,}01867\cdot10^{-3}$ (2)	$-0{,}619\cdot10^{-3}$ (3)	$0{,}417\cdot10^{-3}$ (4)	$\frac{dr_*}{dx} = (5) - (6) - (7)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0,75	0,294	0,220	0,0534	$0{,}00549\cdot10^{-3}$	$-0{,}1361\cdot10^{-3}$	$0{,}0265\cdot10^{-3}$	$-0{,}1049\cdot10^{-3}$
0,70	0,618	0,258	0,0921	$0{,}01153\cdot10^{-3}$	$-0{,}1586\cdot10^{-3}$	$0{,}0384\cdot10^{-3}$	$-0{,}1087\cdot10^{-3}$
0,65	1,257	0,293	0,1252	$0{,}0234\cdot10^{-3}$	$-0{,}1813\cdot10^{-3}$	$0{,}0527\cdot10^{-3}$	$-0{,}1052\cdot10^{-3}$
0,60	2,53	0,324	0,1656	$0{,}0472\cdot10^{-3}$	$-0{,}200\cdot10^{-3}$	$0{,}0691\cdot10^{-3}$	$-0{,}0837\cdot10^{-3}$
0,55	5,16	0,355	0,212	$0{,}0964\cdot10^{-3}$	$-0{,}220\cdot10^{-3}$	$0{,}0885\cdot10^{-3}$	$-0{,}0351\cdot10^{-3}$
0,50	10,70	0,382	0,265	$0{,}1997\cdot10^{-3}$	$-0{,}236\cdot10^{-3}$	$0{,}1105\cdot10^{-3}$	$+0{,}1465\cdot10^{-3}$
0,45	23,2	0,409	0,326	$0{,}433\cdot10^{-3}$	$-0{,}253\cdot10^{-3}$	$0{,}136\cdot10^{-3}$	$0{,}316\cdot10^{-3}$
0,40	52,6	0,432	0,396	$0{,}982\cdot10^{-3}$	$-0{,}267\cdot10^{-3}$	$0{,}1652\cdot10^{-3}$	$0{,}880\cdot10^{-3}$
0,35	128,1	0,452	0,475	0,00239	$-0{,}280\cdot10^{-3}$	$0{,}198\cdot10^{-3}$	0,00231
0,30	347	0,468	0,564	0,00648	$-0{,}290\cdot10^{-3}$	$0{,}235\cdot10^{-3}$	0,00643
0,25	$1{,}086\cdot10^{3}$	0,481	0,667	0,0202	$-0{,}298\cdot10^{-3}$	$0{,}278\cdot10^{-3}$	0,0202
0,20	$4{,}47\cdot10^{3}$	0,488	0,836	0,0834	$-0{,}302\cdot10^{-3}$	$0{,}349\cdot10^{-3}$	0,0834
0,15	$22{,}9\cdot10^{3}$	0,487	0,922	0,427	$-0{,}301\cdot10^{-3}$	$0{,}384\cdot10^{-3}$	0,427
0,10	$0{,}236\cdot10^{6}$	0,475	1,093	4,40	$-0{,}294\cdot10^{-3}$	$0{,}456\cdot10^{-3}$	4,40
0,05	$11{,}39\cdot10^{6}$	0,436	1,308	212	$-0{,}270\cdot10^{-3}$	$0{,}546\cdot10^{-3}$	212
0	∞	0	0	∞	0	0	∞

(Note: commas should be read as decimal points.)

It should be noted that the values of the relative radius of limit icing R_{*lim} determined here and in the example of the preceding chapter during the investigation of changes in icing in time, are equal. This is because both examples pertain to the same pipeline and the limit radius of icing R_{*lim} is a common limit with a change in r_* both in time and with respect to length. R_{*lim} can also be calculated according to

formula (64).

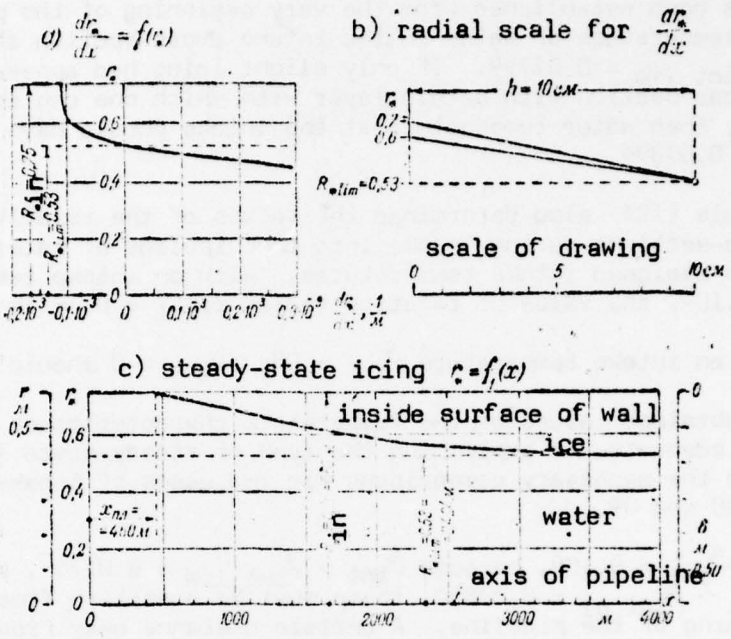

Figure 10. Graphic calculation of steady-state icing of a pipeline with an inside radius R_{in} = 0.60 m, a corrected radius R_{cr} = 0.80 m and a design gradient J_d = 0.0150 during passage of a flow rate of water Q = 2.0 m^3/sec through the pipeline with a water temperature at the point of intake into the pipeline of ϑ_{int} = 0.10° and a temperature of the outside air ϑ_o = -10°.

b) Determination of the forms of icing. Before proceeding to the graphic integration of the differential equation (122), it is vital to determine the forms of steady-state icing that should develop with the given values of intake temperatures. For this purpose, in accordance with Section B, No. 9 of this chapter, it is vital to find the values of the characteristic temperatures of water and radii of the free cross-sections characteristic for the intake cross-section. Each pair of values of the temperature and relative radius at the intake cross-section with steady-state icing are linked by the relationship (107). After substitution of the numerical values of magnitudes of the examined example in it, we obtain:

$$\vartheta_{int} = 0{,}0282 - \frac{r_{*int}}{\ln r_{*int}} \text{ degree.} \qquad (123)$$

This formula determines the following characteristic values of water temperatures in the intake cross-section. If limit icing with R_{*lim} = = 0.53 has been established from the very beginning of the pipeline, then the temperature of water in the intake cross-section should have a value $\vartheta_{int\ lim} = 0.0275^{\circ}$. If only slight icing had appeared in the intake cross-section with a thin layer with which one can consider that $r_* = R_{*in}$, then water temperature at the intake should have had a value $\vartheta_{int\ pl} = 0.0789^{\circ}$.

Formula (123) also determines the values of the relative radii of free cross-sections during intake into the pipeline of water corresponding to the assigned intake temperatures. With an intake temperature $\vartheta_{int} = 0.10^{\circ}$, the value of relative radius $r_{*int} = 0.79$ is obtained.

With an intake temperature $\vartheta_{int} = 0^{\circ}$, $r_{*int} = 0$ should exist.

The obtained values of the temperature characteristics and radii are quite adequate for explaining the type of steady-state icing. We shall make the necessary comparisons for two cases of intake temperatures, 0.10 and 0°.

When $\vartheta_{int} = 0.10^{\circ}$, because $\vartheta_{int} > \vartheta_{int\ lim} = 0.0275^{\circ}$, and furthermore, $\vartheta_{int} > \vartheta_{int\ pl} = 0.0789^{\circ}$, there must be a section free of ice at the beginning of the pipeline. A certain distance away from the intake one should anticipate the appearance of a thin layer of ice which, increasing along the length of the pipeline, tends to reach a value that corresponds to limit icing. It is quite obvious that one could come to the same conclusion by comparing the characteristic radii of free cross-sections between which the following inequalities have been established: $0.79 = r_{*int} > R_{*lim} = 0.53$, and, moreover, $r_{*int} > R_{*int} = 0.75$.

When $\vartheta_{int} = 0^{\circ}$, because $\vartheta_{int} < \vartheta_{int\ lim} = 0.0275^{\circ}$, or, which is the same thing, $0 = r_* < R_{*lim} = 0.53$, significant icing should appear at the intake into the pipeline. Such icing should decrease along the length of the pipeline, tending to reach the limit state.

c) The length of the stretch free of ice. Having explained the character of steady-state icing with two assigned values of intake temperatures, one can turn to the calculation of these states of icing.

We shall determine the length of a stretch of pipeline free of ice when $\vartheta_{int} = 0.10^{\circ}$ according to formula (112). The values of constants in this formula are the following for our example:

$$A = 86\,400 \cdot 2{,}0 \cdot 1{,}0 \cdot 1{,}0 = 173 \cdot 10^3;$$

$$B = 218 \cdot 10^3 \frac{0{,}01^2 \cdot 2{,}0^{\frac{1}{2}}}{427 \cdot \pi^2 \cdot 0{,}60^{5/2}} = 0{,}6297;$$

$$C = \frac{0{,}024 \cdot 2\pi}{0{,}00280\left(\frac{0{,}60}{2{,}0}\right)^{3/4} - \frac{1}{2{,}0} \ln 0{,}75} = 1{,}0402.$$

After substitutions of these values formula (112) acquires the form:

$$x_{pl} = \frac{173 \cdot 10^3}{1,0402} \ln \frac{0,6297 - 1,0402\,(0,10 + 10)}{0,6297 - 1,0402\,[0,0789 - 0,0105 \cdot 10^{-3} x_{pl} + 10]} =$$

$$= 166,2 \cdot 10^3 \ln \frac{-9,8763}{-9,8514 + 0,01091 \cdot 10^{-3} x_{pl}}\ м,$$

from which, by selection, one finds the value x_{pl} = 453 m.

For comparison, we make the same calculations according to the simplified formula (114):

$$x_{pl} = \frac{173 \cdot 10^3\,[0,0789 - 0,10]}{0,6297 - 1,0402 \cdot 10 + 173 \cdot 10^3 \cdot 0,0105 \cdot 10^{-3}} = 460\ м.$$

Both answers are so close that for our example one can use the simplified formula (114) instead of the complex formula (112).

d) Calculation of icing when $\vartheta_{int} = 0.10^{\circ}$. Having thereby explained the value x_{pl} which determines the location of the cross-section in which icing appears, we graphically integrate equation (122) in Figure 10 for a case of $\vartheta_{int} = 0.10^{\circ}$.

As was indicated above, in Figure 10, a, according to Table 12, a graph $\frac{dr_*}{dx} = f(r_*)$ was plotted; in this case, one pays attention to depicting the negative values of the derivative. In order to make the plots, the following scales were selected: $m_{1*} = 0.10$; $m\frac{dr_*}{dx} = 0.05 \cdot 10^{-3}$ and $m_x = 200$. The dimensions of the drawings in Figure 10 have been reduced in size, and therefore a graphic scale of dimensions is given there. Formula (93) for determining the value of polar distance in this case acquires the form:

$$h = \frac{m_{1*}}{m_{\frac{dr_*}{dx}} \cdot m_x}\ см. \qquad (124)$$

By substituting the values of the scales here, we obtain the value of polar distance h = 10 cm.

A radial scale has been plotted in Figure 10, b. Its axis has been chosen transverse to the vertical. The pole has been placed to the right of the axis on a horizontal line which intersects the axis in a point corresponding to R_{*lim}. Along the axis, moving upward, are plotted the segments that correspond to negative values of $\frac{dr_*}{dx}$, and the values of r_* that correspond to them are inscribed.

In Figure 10, c, using the radial scale, the sought graph $r_* = f_1(x)$ has been plotted.

Plotting it begins from a point $x = x_{pl} = 460$ and $r* = R*_{in} = 0.75$.

Calculation of the steady-state ice regime of a pipeline when $\vartheta_{int} = 0.10^{\circ}$ could conclude here. However, in certain cases it is necessary to determine hydraulic losses and the temperatures of water in the pipeline during steady-state icing. The calculation of these values is conducted in the following way.

e) Hydraulic calculation when $\vartheta_{int} = 0.10^{\circ}$. Hydraulic losses in the pipeline are calculated through the hydraulic gradient, which is determined to formula (9). In this case it is assumed that losses during intake into the pipeline and the velocity head are slight. Formula (9) can be presented in the form of a differential equation:

$$J = \frac{d(\Delta H)}{dx} = 2{,}52\ \frac{n^2Q^2}{\pi^2 R_{cr}^{5\frac{1}{3}}} \cdot \frac{1}{r_*^{5\frac{1}{3}}}. \tag{125}$$

After substitution of the numerical values of the constant values, this equation acquires the form:

$$\frac{d(\Delta H)}{dx} = 0{,}336 \cdot 10^{-3}\ \frac{1}{r_*^{5\frac{1}{3}}}. \tag{126}$$

The integration of this differential equation is also carried out by the method described in the previous chapter. According to equation (126), which has the following structure

$$\frac{d(\Delta H)}{dx} = f(r_*),$$

one plots a curve which serves for plotting the radial scale. The radial scale is plotted from the vertical scale, on which segments are laid which correspond to $\frac{d(\Delta H)}{dx}$ and the values of $r*$ are inscribed. The inclination of the rays determines the hydraulic gradient $J = \frac{d(\Delta H)}{dx}$ in a certain scale, with the given free cross-section of the frozen pipeline characterized by a relative radius $r*$.

By using relationship $r_* = f_1(x)$, depicted in the form of a graph in Figure 10, c, by the aid of this radial scale, one plots curve $\Delta H = f_3(x)$m. Hence, the value of r_* is excluded and the value of the total hydraulic losses ΔH becomes a function of x.

Calculation of polar distance, depending on the selected scales, is carried out according to the following formula:

$$h = \frac{m_{\Delta H}}{m_{\frac{d(\Delta H)}{dx}} \cdot m_x} \text{ см.} \qquad (127)$$

Integration begins from a point corresponding to the static level.

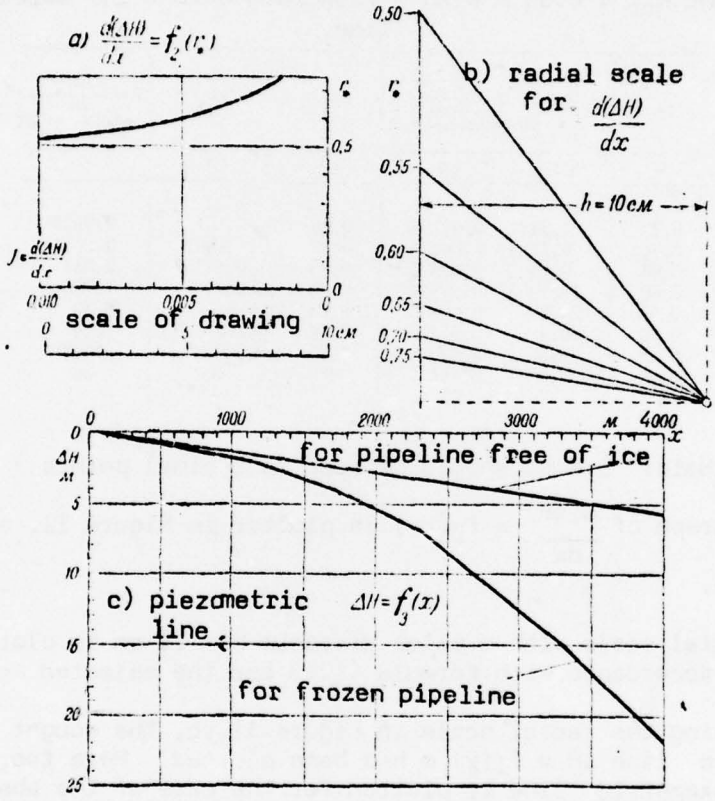

Figure 11. Graphic construction of the piezometric line during steady-state icing of a pipeline with an inside radius R_{in} = 0.60 m, a corrected radius R_{cr} = 0.80 m and a design gradient J_d = 0.0150 with a flow rate in the pipeline of Q = 2.0 m^3/sec and water temperature at the point of intake into the pipeline ϑ_{int} = 0.10° and a temperature of the outside air ϑ_o = -10°.

Calculation of hydraulic losses, i.e., plotting of the piezometric line for the examined numerical example when $_{int}$ = 0/10° is carried out in Figure 11. The scales for the graphic plots are accepted to be the following:

$$m_{\frac{d(\Delta H)}{dx}} = 0{,}001; \quad m_x = 200 \text{ and } m_{\Delta H} = 2{,}0.$$

The values of $J = \frac{d(\Delta H)}{dx} = f_2(r_*)$ are determined according to formula (126) in Table 13.

Table 13

Calculation of the values of the hydraulic gradient with different values of relative radius of the free cross-section for a pipeline with R_{in} = = 0.60 m and R_{cr} = 0.80 m with a flow rate of Q = 2.0 m^3sec in the pipeline.

r_*	$\frac{1}{r_*^{5/3}}$	$J = \frac{d(\Delta H)}{dx} = 0{,}336 \cdot 10^{-3}$ (2)
(1)	(2)	(3)
0,75	4,64	0,00156
0,70	6,71	0,00225
0,65	9,96	0,00335
0,60	15,28	0,00514
0,55	24,3	0,00816
0,50	40,4	0,0136
0,45	71,0	0,0238
0,40	132,8	0,0446
0,35	270	0,0907
0,30	615	0,206
0,25	$1{,}628 \cdot 10^3$	0,547
0,20	$5{,}35 \cdot 10^3$	1,796
0,15	$24{,}8 \cdot 10^3$	8,33
0,10	$0{,}216 \cdot 10^6$	72,6
0,05	$8{,}70 \cdot 10^6$	2 920
0	∞	∞

(Note: commas should be read as decimal points.)

The graph of $\frac{d(\Delta H)}{dx} = f_2(r_*)$ is plotted in Figure 11, a, according to this table.

A radial scale with a polar distance h = 10 cm is plotted in Figure 11, b, in accordance with formula (127) and the selected scale values.

By using the radial scale in Figure 11, c, the sought graph of the piezometric line $\Delta H = f_3(x)$ m has been plotted. Here too, for comparison, a piezometric line is plotted for the case of the absence of ice. Plotting of the latter line is extremely simple. On the radial scale one chooses a ray that corresponds to $r_* = R_{*in} = 0.75$, and parallel to it, from the origin of the coordinates of the graph, one draws a line the entire length of the pipeline. The piezometric line for a frozen pipeline in a stretch from x = 0 to $x = x_{pl}$ = 460 m is plotted in the same way. It is further assumed that in a stretch 460 m < x <1150 m there is a constant relative radius having a value $r_* = 0.70$ (see Figure 10, c). A new ray on the radial scale is selected for this value of the relative radius and, parallel to it, line $\Delta H = f_3(x)$ on stretch 460 m < < x < 1150 m is continued.

Table 14

Calculation of ϑ - steady-state temperature of water and ϑ_o - temperature of ice in a pipeline with R_{in} = 0.60 m, R_{cr} = 0.80 m, and J_d = 0.0150 when Q = 2.0 m^3/sec, ϑ_o = -10° and ϑ_{int} = 0.10°.

x, m	r_*	ΔH, m	$\frac{r_*^{3/4}}{-\ln r_*}$	$0{,}0182\frac{r_*^{3/4}}{-\ln r_*}$	$0{,}01176\cdot10^{-4}\cdot x$	$0{,}000784\cdot\Delta H$	$\vartheta = (5)+(6)+(7)$, deg.	$\vartheta_o = (6)+(7)$, deg.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	0,75	0	—	—	—	—	0,10°	0
460	0,75	0,7	2,80	0,0789	—0,00541	0,000549	0,0740°	—0,00486°
1 000	0,69	1,8	2,04	0,0575	—0,01176	0,001411	0,0471°	—0,01035°
1 500	0,64	3,2	1,611	0,0454	—0,01763	0,00251	0,0303°	—0,01512°
2 000	0,60	5,4	1,332	0,0376	—0,0235	0,00424	0,0183°	—0,01926°
2 500	0,57	8,5	1,169	0,0329	—0,0294	0,00666	0,0102°	—0,0227°
3 000	0,55	12,5	1,072	0,0302	—0,0353	0,00980	0,0047°	—0,0255°
3 500	0,54	17,0	1,022	0,0288	—0,0411	0,01332	0,0010°	—0,0278°
4 000	0,53	22,0	0,976	0,0275	—0,0470	0,01724	—0,0023°	—0,0298°

(Note: Commas should be read as decimal points.)

Thus, a broken line which at the limit yields the piezometric line of pipeline icing is gradually plotted.

f) Calculation of water temperatures when $\vartheta_{int} = 0.10^{\circ}$. The next partial problem is determining the steady-state temperature of water in any cross-section of the pipeline, i.e., determining function $\vartheta = f_4(x)$.

Formula (106) determines the steady-state temperature of water depending on r_* of the relative radius and H m - pressure at the height of the water column. For our example, this formula acquires the form:

$$\vartheta = 0{,}0282 \frac{r_*^{3/4}}{-\ln r_*} - 0{,}000784 H \text{ degrees}, \qquad (128)$$

where the pressure value can be determined by the relationship:

$$H = J_d x - \Delta H \text{ м}.$$

By substituting the last expression for H in formula (128), we obtain:

$$\vartheta = 0{,}0282 \frac{r_*^{3/4}}{-\ln r_*} - 0{,}01176 \cdot 10^{-3} \cdot x + 0{,}000784 \cdot \Delta H \text{ degrees}. \qquad (129)$$

By using the graphs of $r_* = f_1(x)$ and $\Delta H = f_3(x)$ in Figure 10, c, and 11, c, one can find the sought relationship $\vartheta = f_4(x)$ according to (129). The calculations for determining this relationship have been made in Table 14. In the same table calculations have been made of the temperature of the melting points of ice according to the formula:

$$\vartheta_0 = - 0.000784H = - 0.01176 \cdot 10^{-3} x + 0.000784\Delta H \text{ degrees}. \qquad (130)$$

Using this table, a graph of water temperatures has been plotted in Figure 12 in different cross-sections of a pipeline $\vartheta = f_4(x)$ degrees as has a graph of the melting points of ice $\vartheta_0 = f_5(x)$ degrees.

It is evident from this drawing that water can exist in the pipeline at a negative temperature, but at a higher temperature than the melting point of ice.

Data concerning the temperature of water in the pipeline make it possible to judge the ice processes that must occur in the turbine during the supply of water to it from the pipeline. We shall assume that water entering the turbine can have a lower temperature than the melting point of ice under the pressure that exists in the turbine. In this case, water is in the supercooled state when coming into the turbine, i.e., conditions appear that are favorable for crystallization.

Part of the ice formed in this place can be carried along by the flow of water through the turbine, while part of it can settle primarily on the metal parts of the controlling apparatus.

Returning to the example under examination, we shall calculate the amount of ice that should form in a turbine if the length of the pipeline l = 4000 m and if the vacuum in the turbine comprises 4 m of the water column.

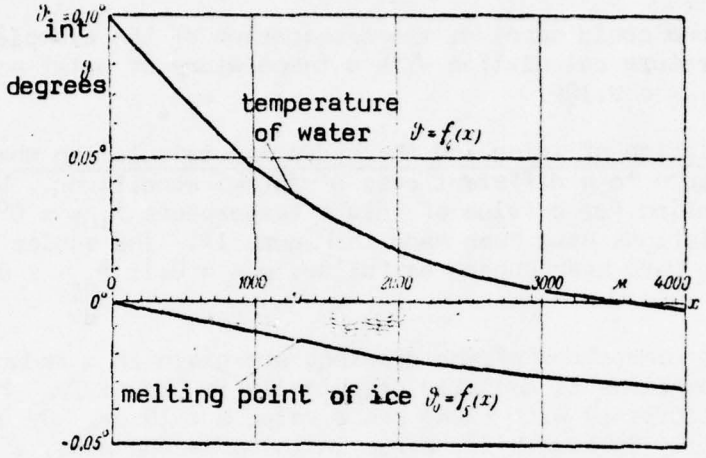

Figure 12. The temperature of water and the melting point of ice inside a pipeline having an inside radius R_{in} = 0.60 m, a corrected radius R_{cr} = 0.80 m, and a design gradient J_d = 0.0150 with a flow rate Q = 2.0 m^3/sec and temperature of water coming into the pipeline ϑ_{int} = 0.10° and a temperature of the outside air ϑ_o = -10°.

With a such vacuum, i.e., when pressure H = -4 m, the melting point of ice according to formula (37) should be ϑ_0 = 0.0031°. The pipeline supplies water to the turbine, judging according to the graph in Figure 12 or according to Table 14, at a temperature ϑ = -0.0023°. Hence, upon entering the turbine the water is supercooled by a value $\vartheta_0 - \vartheta$ = 0.0031 + 0.0023 = 0.0054°. In order that the water can take on the melting temperature of ice ϑ_0 = 0.0031°, the following amount of ice should form in it for each cubic meter of water:

$$v_{1i} = \frac{\gamma_w C_w (\vartheta_0 - \vartheta)}{\gamma_i L} = \frac{0{,}0054}{73{,}0} = 0{,}074 \cdot 10^{-3}\ м^3.$$

Since a flow rate Q = 2.0 m^3/sec through the turbine, then the amount of ice that appears in the turbine each second will be

$$Q_i = \frac{\gamma_w C_w (\vartheta_0 - \vartheta)}{\gamma_i L} Q\ m^3/sec. \qquad (131)$$

By substituting the numerical values, we obtain: $Q_i = 0.148 \cdot 10^{-3}$ - $m^3/sec = 0.148$ i/sec. Although this is only a small quantity of ice, by gradually accumulating on the vanes controlling the apparatus, it can cause a decrease in turbine power followed by a complete stop. Breaks of both the controlling apparatus by forces of the servomotor, which attempts to change the position of the frozen vanes, and of vanes of the operating wheels, which can become siezed on the ice accumulations and protruberances formed on the guiding apparatus can occur.

At this one could conclude the examination of the example of the ice and temperature calculation with a temperature of water entering the pipeline of $\vartheta_{int} = 0.10^o$.

g) Calculation of icing and the hydraulic calculation when $\vartheta_{int} = 0^o$. We shall now turn to a different case of intake conditions. We shall make a calculation for a value of intake temperature $\vartheta_{int} = 0^o$. The graphic calculations have been made in Figure 13. The scales for calculating icing have been chosen as follow: $m_{r*} = 0.1$; $m_{\frac{dr_*}{dx}} = 0.5 \cdot 10^{-3}$; $m_x = 20$.

Since the dimensions of the drawings are given in a reduced size, a scale of dimensions is depicted graphically in Figure 13. Polar distance in accordance with (124) has a value h = 10 cm. By using the data of Table 12, in Figure 13, a, as in the previous case, a graph $\frac{dr*}{dx} = f(r*)$ has been plotted, but here special attention has been paid to demonstrating the positive values of the derivative. In Figure 13, b, a radial scale has been plotted and in Figure 13, c, integration has been properly carried out, and the initial point of plotting the curve was taken to be the origin of the coordinates, since when $\vartheta_{int} = 0^o$ and $H_{int} = 0$, the relative radius in the intake cross-section, as was explained earlier, is $r*_{int} = 0$. The sought graph of the change in the value of the relative radii of icing along the length of a pipeline $r* = f_1(x)$ was obtained in Figure 13, c. This graph shows that greatest icing exists at the intake to the pipeline. Theoretically, when x = 0, $r*_{int} = 0$, i.e., the cross-section totally freezes. However, under practical conditions, complete freezing at the intake can hardly occur.

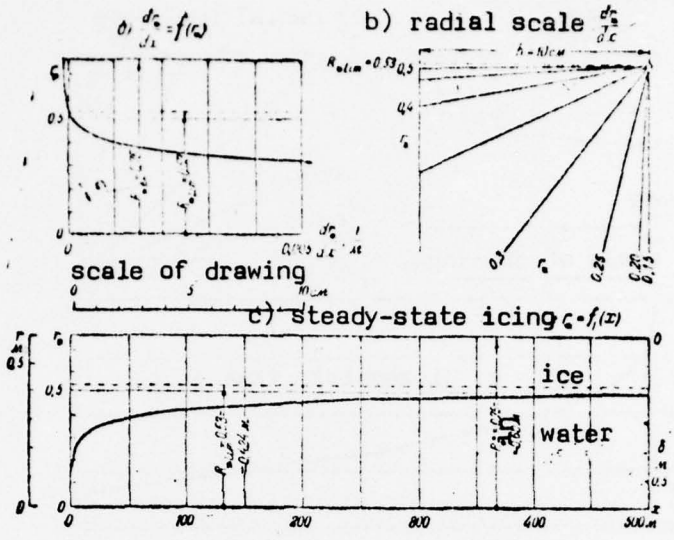

Figure 13. Graphic calculation of steady-state icing of a pipeline with an inside radius R_{in} = 0.60 m, a corrected radius R_{cr} = 0.80 m, and a design inclination J_d = 0.0150 when running a flow rate Q = 2.0 m^3/sec through the pipeline, with water temperature at the point of intake into the pipeline ϑ_{int} = 0, and with temperature of the outside air ϑ_o = -10°.

The graphs of Figure 14, a, b, c serve for calculating hydraulic losses. A graph $\frac{d(\Delta H)}{dx} = f_2(r*)$ has been plotted according to Table 13 and Figure 14, a. The chosen scales m_{r*} = 0.10; $m\frac{d(\Delta H)}{dx}$ = 0.01; m_x = = 20 and $m_{\Delta H}$ = 2.0; polar distance is determined according to (127) and has a value h = 10 cm. In Figure 14, b, the radial scale has been plotted for a hydraulic gradient $J = \frac{d(\Delta H)}{dx}$, depending on the value of the relative radius of the free cross-section. A piezometric line has been drawn in Figure 14, c by the aid of the radial scale. The piezometric line shows that with an intake temperature ϑ_{int} = 0, extremely significant hydraulic losses appear as the result of strong icing at the beginning of the pipeline. These losses can be the cause of vacuum formation in certain cases.

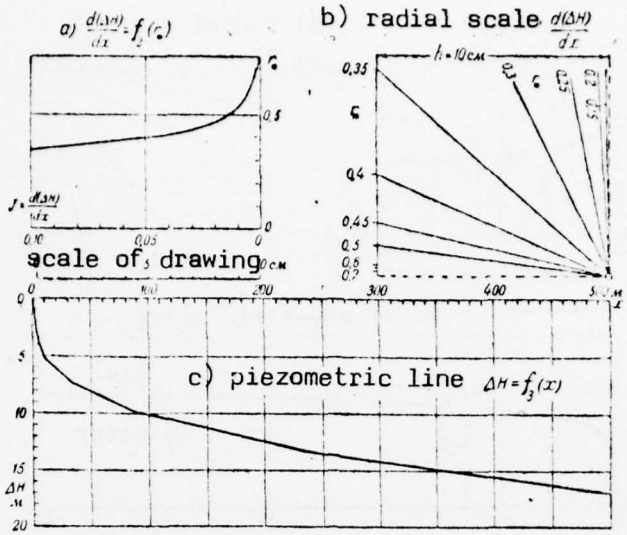

Figure 14. Graphic plotting of the piezometric line during steady-state icing of a pipeline with an inside radius R_{in} = 0.60 m, a corrected radius R_{cr} = 0.80 m, and a design gradient J_d = 0.0150 with a flow rate Q = 2.0 m^3/sec, water temperature entering the pipeline ϑ_{int} = 0°, and a temperature of the outside air ϑ_o = -10.0°.

Under practical conditions, the value of the intake temperature extremely often proves to be equal to the melting point of ice, which entails strong constriction of the free cross-section at the beginning of the pipeline by the layer of ice that formed on the inside surfaces of the walls. An increase in thermal insulation, i.e., an increase in R_{cr} reduces the length of a stretch with strong icing, but does not reduce icing in the intake cross-section, and judging by formula (107), the intake cross-section should be completely closed by ice since $r_* = 0$.

Even if continuous icing does not occur, then there will still be strong icing which causes higher hydraulic resistances. Therefore, one cannot agree with the existing opinion that burying a pipeline under a layer 1 m thick in Winter prevents freezing (12).

Good thermal insulation retards the growth of ice and the formation of the steady-state forms of icing, but the presence in the water of slush can significantly accelerate these processes.

In order to prevent strong icing at the beginning of the pipeline, one should artificially make the walls of the pipeline a higher temperature than the melting point of ice. This temperature, being even

slightly higher than the melting point of ice, prevents the formation of an ice layer in the inside surface of the pipeline on such a warmed stretch. One can practically heat the pipeline by building a heated shelter over it. On the heated stretch of the pipeline it is desirable to give the line as high a design gradient as possible in order to obtain a pressure in the pipeline which, having lowered the melting point of the ice, would create a certain differential between the intake temperature and the melting point of ice. The required value of this differential is determined by the suitable value of the relative radius in the cross-section directly following the heater and having the greatest amount of icing. We shall explain this by the use of an example.

We shall assume that in the case examined above with an intake temperature of $\vartheta_{int} = 0$, the relative radius in the cross-section with greatest icing is $r_* = 0.39$. In order to obtain this value of the radius of icing, one should have a pressure different than zero. The value of this pressure can be determined according to formula (106).

Having substituted the values of the example under discussion in it, we obtain:

$$0 = 0{,}00559 \cdot 10 \left(\frac{0{,}8}{2{,}0}\right)^{3/4} \cdot \frac{0{,}39^{3/4}}{-\ln 0{,}39} - 0{,}000784\, H,$$

from which the value of required pressure will be H = 18.8 m. It is extremely desirable to obtain this value of pressure on a short stretch with a large design gradient and to cover this stretch with a heated shelter. This measure protects the pipeline against strong icing of its intake segment. There will be no icing at all under the heated shelter, and in the cross-section immediately after the heated shelter icing will have a relative radius $r_* = 0.39$. Further icing will decrease, as shown in Figure 13, c, beginning with x = 50 m, where $r_* = 0.39$. The operating expenditures involved with heating the shelter will be comparatively small, since temperature in the shelter should only be slightly above the melting point of ice. It is unnecessary to heat water in this case.

Heating water with electricity for the purpose of combatting icing cannot be recommended under ordinary conditions, since more expenditures of energy are required than losses of energy as the result of increased icing of the pipeline are possible.

In certain cases, heating water with electricity can be permitted, for example, on derivation pipelines when they are not in a condition to allow the required flow rate of water to pass as the result of strong icing. With the presence of uses of heat on any industrial enterprise next to the hydroelectric power station, the use of these heat wastes for increasing the effectiveness of Winter operation of the hydroelectric power station is extremely desirable. The use of warm ground waters for

the same purpose is also extremely desirable.

11. Certain Concepts Relative to the Thermal Condition of Water Intake to the Pipeline

A. Evaluation of the Role of Local Hydraulic Resistance and the Drop in Pressure During Intake into the Pipeline

Above, during the calculations of icing, the heat of friction that arises during the movement of water along the pipeline was taken into account. The heat of friction of local hydraulic resistances was not introduced into the calculation however. Since when $t_{int} = 0^{\circ}$ one obtains very strong icing of the intake cross-section, then it seems of practical interest to estimate the role of the heat of friction of local hydraulic resistances, as well as other thermal phenomena that accompany the intake of water into the pipeline. Below an approximate qualitative evaluation of these thermal phenomena is given.

As is known, hydraulic resistances during the intake of water into the pipeline are expressed by the drop in pressure of a water column having a height $\Delta H_m = \zeta \frac{v^2}{2g}$ m, where the coefficient of resistance of intake into the pipeline $\zeta = 0.05 - 0.5$ and $\frac{v^3}{2g}$ m - the velocity head. However, the actual drop in pressure during intake has a value $\Delta H = (1 + \zeta) \frac{v^2}{2g}$ m as the result of the formation of a reserve of kinetic energy.

The hydraulic resistances cause the appearance of heat which warms the water and thereby facilitates a reduction in icing, while the actual drop in pressure causes an increase in the melting point of the ice, which facilitates, as was shown in No. 2, and increase in icing. In the same No. 2 it was explained that if the value of the heat of friction, determined by a certain drop in pressure, ia taken as 1, then the decrease in the reserve of heat as the result of an increase in the melting point of ice with the same drop in pressure is one-third. This makes it possible to qualitatively evaluate the thermal processes during intake. The heat of friction during intake into the pipeline is proportional to ΔH_m, while the decrease in the reserve of heat as the result of a drop in pressure is proportional to ΔH. If the heat of friction is equal to a decrease in the thermal reserve, then $\Delta H_m = \frac{1}{3} \Delta H$ or $\zeta = \frac{1}{3} (1 + \zeta)$. Hence, the value of the hydraulic coefficient of local resistance during intake should be $\zeta = \frac{1}{2}$. At this value thermal equilibrium ensues; the heat of friction is totally covered by the losses of the thermal reserve that appears as the result of an increase in the melting point of the ice with the drop in pressure. If $\zeta < \frac{1}{2}$, then this thermal equilibrium is disrupted.

Losses of the thermal reserve begin to dominate over the heat of friction. This creates conditions that strengthen icing. Since the coefficient of intake resistance usually has a value $\zeta \leqslant 0.5$ in pipelines, then during intake into the pipeline conditions are created that are unfavorable in the sense of icing.

B. The Required Accuracy of the Measurements of Water Temperatures During Hydroenergetic Research

The value of water temperature of the water entering the pipeline has great practical significance. Therefore, the investigation of the temperature regime of the source of water supply of the hydroelectric power station (river, lake, reservoir) and the water-carrying facilities (canal, adit, trough, pipeline) should be given serious attention both during field research and investigations and during planning. Specifically, it should be noted that during research and during the hydrological investigations one should study the temperature and ice regime of the source of water in greater detail than is usually done. It should be recognized that usually the thermometers that are employed during investigations (spring and tippable) do not satisfy the planning demands if only because their scales, having a division value of 0.2° or 0.1°, do not give the required precision. The numerical examples given above showed that the characters of icing of a pipeline differ totally with temperatures of water entering the pipeline of $\vartheta_{int} = 0.10^{\circ}$ and $\vartheta_{int} = 0^{\circ}$ (see Figures 10 c, and 13, c). During planning of the ice regime of pipelines one should know temperature with an accuracy of at least up to 0.001°. One should orient one's self toward this precision, as to a minimum, during the development of the design of a thermometer which should be adopted in wide practice of hydroenergetic field investigations.

The necessary calculations of water temperature in reservoirs, rivers, and canals can be made according to the methods of S. N. Kritskiy, M. F. Menkel', and K. I. Rossinskiy (13). Some information about the calculation of water temperature in troughs and tunnels can be found in the Plans of TUiN of Hydrotechnical Planning (20).

C. Estimating the Amount of Heat in the Stream of Water

The content of heat in a stream of water, and specifically, in the water entering the pipeline, is determined by its temperature. The heat content of a stream of water can be viewed as thermal energy. It is of practical interest to compare the value of this thermal energy with the energy of the hydroelectric station, since in certain cases the question arises of heating the water coming into the pipeline with electricity generated by the hydroelectric power station.

If one considers the reserves of thermal energy of water above 0°, then the power of the stream of water in the form of thermal energy carried by it through a certain free cross-section per unit of time will be:

$$P = 4190 \vartheta Q \text{ kw}, \tag{132}$$

where ϑ degress - temperature of water and
Q m^3/sec - flow rate of water.

If one calculates this thermal power of the stream entering the pipeline under the conditions of the example examined above when $Q = 2.0$ m^3/sec and $\vartheta_{int} = 0/10^{\circ}$, then it proves that $P = 838$ kw. This comprises about half the power of a hydroelectric power station operating on the indicated flow rate with a pressure head of about 100 m. Hence, heating water even by 0.1° under the conditions of the cited example involve the relatively high energy expenditure. It is entirely obvious that the greater the flow rate of water and the smaller the pressure head at which the hydroelectric power station operates, the greater the relative expenditures of energy required to heat the water.

12. Steady-State Icing with an Assigned Pressure Differential at the Intake and at the End of the Pipeline

All of the formulas and constructions given above pertained to the hydraulic regime of a pipeline assigned a constant flow rate. But in practice, another case can also be encountered when the operation of a pipeline is assigned a differential of pressures at its beginning and end, or, in other words, when a mean hydraulic gradient is set for the pipeline. Calculation of steady-state icing under these conditions is complicated and can be carried out by trial-and-error. In this case the following order of the calculation is suitable.

One begins with a certain flow rate of water and for it calculates icing and then hydraulic losses. These losses are compared with the assigned differential of pressures with respect to the ends of the pipeline. If the losses proved smaller than the assigned pressure differential, then the flow rate is increased, but if losses are greater, then flow rate is decreased. For a new flow rate one once again repeats the cycle of calculation, as the result of which the value of the new hydraulic losses becomes clear, which is once again compared with the set pressure differential.

Such trial calculations are repeated until an adequate correspondence of the value of hydraulic losses and the set value of the pressure differential on the ends of the pipeline is obtained. The value of flow rate and steady-state icing of the pipeline obtained in this case are the solution of the assigned problem.

Chapter Five

A GENERAL CASE OF PIPELINE ICING

A case in which icing changes along the pipeline and proportional to the passage of time has been defined as the general case of pipeline icing. Hence, such icing is unsteady and non-cylindrical. If the hydraulic operation of the pipeline is assigned a value of flow rate, then this general case of icing is expressed by the heat balance equations (46) and (50) and by the hydraulic equation (40). The indicated equations create the following system of equations :

$$\left.\begin{aligned}
&17{,}2\pi\left(\frac{Q}{R_{cr}}\right)^{3/4}(\vartheta+0{,}000784\,H)\frac{1}{r_*^{3/4}}-\\
&-\frac{0{,}024\cdot 2\pi(-\vartheta)}{-\frac{1}{\lambda_i}\ln r_*}\,Q-2\pi\gamma_i L R_{cr}^2 r_*\frac{\partial r_*}{\partial t}=0;\\
&-86\,400\,Q\gamma_w C_w\frac{\partial\vartheta}{\partial x}+218\cdot 10^3\frac{n^2Q^3}{\pi^2R_{cr}^{5,33}}\cdot\frac{1}{r_*^{5,33}}-\\
&-17{,}2\pi\left(\frac{Q}{R_{cr}}\right)^{3/4}(\vartheta+0{,}000784\,H)\frac{1}{r_*^{3/4}}=0;\\
&\frac{\partial H}{\partial x}=J_d-2{,}52\frac{n^2Q^2}{\pi^2R_{cr}^{5,33}}\cdot\frac{1}{r_*^{5,33}}=0.
\end{aligned}\right\}\qquad(133)$$

This system of three equations includes five variables: r_*, ϑ, H, x, and t. In order to reach the equation which determines the relationship between r_*, x, and t, one can take the following path for excluding variables. From the third equation after integration, one finds the pressure value H. It is substituted in the first and second equations and thereby completed excluded from the system. Then, from the first equation of the system one finds the value of water temperature and its partial derivative with respect to length $\frac{\partial\vartheta}{\partial x}$. Both of these values are substituted in the second equation. As the result of these substitutions, the second equation acquires the form:

$$A\frac{\partial r_*}{\partial x}+B\frac{\partial r_*}{\partial t}+C\frac{\partial r_*}{\partial x}\cdot\frac{\partial r_*}{\partial t}+D\frac{\partial^2 r_*}{\partial x\cdot\partial t}+E=0,$$

where A, B, C, D and E are functions of r_*. This equation with the partial derivatives is the sought function $r_* = f(x, t)$ in the differential form. It does not seem possible to integrate such a differential equation.

13. The Simplified Calculation

One can use a method of calculation whose general idea consists in the

following as the first approximation during planning. For the intake cross-section of the pipeline and for a cross-section in which icing acquires practically cylindrical forms, one can obtain an accurate solution to the problem of the change in icing in time. Between these cross-sections icing is known for an initial moment, as the assigned condition of the problem, and by means of calculation the limit icing can be determined. Hence, the following data should be prepared for calculating the general case of icing:

$r = f(x_{int}, t)$ - change of icing in the intake cross-section;

$r = f(x_c, t)$ - change of the cylindrical icing in time;

$r = f(x, t_0)$ - distribution of icing at an initial moment of time;

$r = f(x, t_\infty)$ - steady-state icing.

As the result of the calculation, one should determine icing in any cross-section and at any moment of time $r = f(x, t)$. However, the interval for distances x from $x_{int} = 0$ to x_c is of practical interest; when $x > x_c$ there will be cylindrical icing. For a time t there is also a period that is of practical interest from t_0 to t_∞, where t_∞ should be viewed as a certain determined moment of time at which icing extremely closely approaches the steady state that is obtained when $t \to \infty$. Consequently, explanation of the sought function $r = f(x, t)$ in the following intervals of independent variables $0 = x_{int} < x < x_c$ and $t_0 < t < t_\infty$ is of practical interest. For the extreme limit values of the independent variables x_{int}, x_c, t_0 and t_∞, icing is known and is determined by the four functions described above.

The sought icing will be determined by means of interpolation among these extreme known values. This is also properly the idea of the simplified calculation. Its details are given below.

The change of icing in time in the intake cross-section of the pipeline can be explained extremely simply. We employ the first equation of system (133) applicable to the intake cross-section. We make the following substitutions in it: $\vartheta = \vartheta_{int}$; $H = H_{int}$ and $\frac{\partial r_*}{\partial x} = \frac{dr_*}{dx}$. The latter substitution is possible because in this case the radius of the free cross-section pertains to a certain value $x = x_{int} = 0$.

After these substitutions, the indicated equation can be given the form:

$$\frac{dr_*}{dt} = \frac{8{,}60\,Q^{9/4}(\beta_{int} + 0{,}000784H_{int})}{\gamma_i LR_{cr}^{5/4}} \cdot \frac{1}{r_*^{19/4}} - \frac{0{,}024\lambda_i(-t_0)}{\gamma_i LR_{cr}^2} \cdot \frac{1}{-r_* \ln r_*} \cdot \frac{1}{\text{day}}. \quad (134)$$

The solution to this differential equation is difficult analytically. Graphically, however, it is solved the same as equation (80) of cylindrical icing. Therefore, its solution is not given here. In any case, for the intake cross-section one can find the change of the radius of the free cross-section in time $r_* = f(x_{int}, t)$.

A similar relationship can be obtained for a cross-section in which steady-state icing is quite near the limit. The position of this cross-section theoretically recedes to infinity, since steady-state icing approaches the limit asymptotically. However, one can practically always note a cross-section with a length coordinate x_c with some degree of accuracy in which steady-state icing approaches quite close to the limit. The selection of this cross-section should be made according to the graph of steady-state icing $r_* = f(x, t_\infty)$. As an example of graphs of steady-state icing, one can point out Figures 10, c, in Figure 13, c. The change of icing in time in a cross-section with the coordinate x_c, i.e., function $r_* = f(x_c, t)$, should be considered the same as during cylindrical icing. An example of the calculation and graphic representation of this function can be Figure 6, c. Thus does one prepare the three extreme values of the function of icing: $r = f(x_{int}, t)$, $r = f(x_c, t)$, and $r = f(x, t_\infty)$. The fourth extreme function $r = f(x, t_0)$, as was indicated above, should be assigned as a condition of the problem.

Interpolation among these extreme values can be carried out in the following way. For any cross-section with a length coordinate within limits of $0 = x_{int} < x < x_c$, one can accept the law of change of icing in time analogous to the corresponding laws in the extreme cross-sections with coordinates x_{int} and x_c. The transfer from one law to another is made according to a linear relationship.

We have decided to determine the width of the ice layer in a certain determined cross-section at any moment in time with the following linear equation:

$$\delta_m = \delta_0 + \Delta\delta = \delta_0 + k(\delta_{st} - \delta_0), \quad (135)$$

where δ_m m - thickness of the ice layer at any moment of time for the determined cross-section;

δ_0 m - thickness of the layer in this cross-section at an initial moment of time;

δ_{st} m - thickness of the ice layer in that cross-section during steady-state icing;

$\Delta\delta$ m - increment in ice layer that occurred in this cross-section by a moment of time t, reckoning from the initial moment of time;

k - coefficient of porportionality variable in length and in time, which is determined by the following formula:

$$k = \frac{\Delta\sigma}{\delta_{st} - \delta_o} . \tag{136}$$

The change in the coefficient of porportionality along the pipeline for a certain determined moment of time is also assumed to occur according to the linear relationship:

$$k = (k_c - k_{int}) \frac{x - x_{int}}{x_c - x_{int}} + k_{int}, \tag{137}$$

where k - coefficient of proportionality for a cross-section with a length coordinate x m in a certain determined moment of time;

k_{int} - coefficient of proportionality for the intake cross-section with $x_{int} = 0$ for the same determined moment of time;

k_c - coefficient of proportionality at the same moment of time and for a cross-section in which changes of steady-state icing have practically ceased along the length of the pipeline and where icing can be considered cylindrical; the length coordinate of this cross-section is x_c.

As an example of the described approximate method, calculation is made for a general case of pipeline icing with an inside radius R_{in} = 0.60 m, a corrected radius R_{cr} = 0.80 m, a design gradient J_d = 0.0150, with a flow rate Q = 2.0 m^3/sec, an air temperature ϑ_o = -10°, and temperature of water in the pipeline ϑ_{int} = 0°, and pressure at the point of intake H_{int} = 0. It is assumed that the pipeline is free of ice at an initial moment of time.

Under these conditions, the solution of equation (134), which determines the change of icing in time in the intake cross-section, leads to the results in Table 15. The same table gives data about the state of icing at different moments of time during cylindrical icing. The latter data are cited according to the graph in Figure 6, c.

Table 15

Build-up of Ice Layer in the Intake Cross-Section and During Cylindrical Icing for a Pipeline R_{in} = 0.60 m, R_{cr} = 0.80 m, J_d = 0.0150 when Q = 2.0 m^3/sec, ϑ_o = -10°, ϑ_{int} = 0°, and H_{int} = 0.

t, days	Intake cross-section x_{int} = 0			Cross-section with cylindrical icing x_c= 1000 m		
	r_o	$r_{i,M}$	$\delta = R_{in} - r_{i,M}$	r_o	$r_{i,M}$	$\delta = R_{in} - r_{i,M}$
0	0,75	0,600	0	0,75	0,600	0
1	0,708	0,566	0,034	0,713	0,571	0,029
3	0,625	0,500	0,100	0,660	0,528	0,072
5	0,560	0,448	0,152	0,618	0,494	0,106
10	0,400	0,320	0,280	0,564	0,451	0,149
20	0,245	0,196	0,404	0,530	0,424	0,176
∞	0	0	0,600	0,530	0,424	0,176

(Note: commas should be read as decimal points.)

The steady-state icing condition is given in Table 16 according to calculations made earlier as an example. The results of these calculations are presented graphically in Figure 13, c.

Table 16

Steady-State Icing of the Pipeline with R_{in} = 0.60 m, R_{cr} = 0.80 m, J_d = 0.0150 when Q = 2.0 m^3/sec, ϑ_o = -10°, ϑ_{int} = 0° and H_{int} = 0.

$x_{i,M}$	r_o	$r_{i,M}$	$\delta_{st} = R_{in} - r_{i,M}$
0	0	0	0,600
1	0,170	0,136	0,464
5	0,280	0,224	0,376
10	0,310	0,248	0,352
30	0,365	0,292	0,308
50	0,388	0,310	0,290
100	0,427	0,342	0,258
200	0,460	0,368	0,232
300	0,480	0,384	0,216
400	0,488	0,390	0,210

(Note: commas should be read as decimal points.)

The last two tables contain all of the necessary data for calculating the coefficient of proportionality k, which determines the thickness of the ice layer at any moment of time in any cross-section. For the given example in all cross-sections at an initial moment of time, there is no layer of ice: $\delta_0 = 0$ and $x_{int} = 0$. Therefore, the formulas (135), (136), and (137) acquire the form:

$$\delta = \Delta\delta = k\delta_{st}^{M};$$

$$k = \frac{\delta}{\delta_{st}};$$

$$k = (k_c - k_{int})\frac{x}{x_c} + k_{int}.$$

Calculation of the values of coefficient the proportionality of k and thicknesses of the ice layer for certain cross-sections and certain moments of time are given in Table 17 and were made according to these formulas.

In this table, the second and fourth columns are completed according to Table 15. The formulation of the formula for calculating the coefficient of proportionality is so simple that it does not require and explanation. This formula corresponds to every moment in time. The formulas are written in the sixth column. The following columns are paired. Each pair corresponds to a certain value of x and δ_{st} - the steady-state thickness of the ice layer. These thicknesses have been taken from Table 16. The first column of each pair belongs to k, calculated according to the formula from the sixth column that corresponds to a given moment of time. The second column belongs to the sought value of thickness of the ice layer δ m.

By using the data of this table, a graph is plotted in Figure 15, c, of a general case of the change in icing in the pipeline. The scale of thicknesses of the ice layer is placed on the right-hand side of the graph. On the left-hand side is the scale of radii of the free cross-section. On the axis of the pipeline ox is plotted the scale of distances from the intake cross-section. Each curve of the graph represents a division line between water and ice at a certain moment in time.

Having thereby obtained the distribution of icing with respect to length and in time, i.e., having obtained function r = f(x, t), one can find the changes in pressure and temperature of water with respect to length and time, i.e., find the functions H = f(x, t) and ϑ = t(x, t). These functions are determined in the following way. For a given moment of time, the relative radius of the free cross-section is a certain function of length.

Table 17

Approximate Calculation of a General Case of Pipeline Icing with R_{in} = 0.60 m, R_{cr} = 0.80 m, J = 0.0150 when Q = 2.0 m^3/sec, ϑ_0 = -10°, and ϑ_{int} = 0° at an Initial Moment when t = 0, the Pipeline is Free of Ice.

(1)		Value of x and δ_{st}	Designation of lines	t, days 0	1	3	5	10	20	∞
(2)	Compilation of formula for k	x_{int} = 0	δ, m	0	0,034	0,100	0,152	0,280	0,404	0,600
(3)		δ_{st} = 0,60 m	$k_{int} = \frac{\delta}{\delta_{lim}}$	0	0,057	0,167	0,252	0,466	0,674	1,00
(4)		x_c = 1 000 m	δ, m	0	0,029	0,072	0,106	0,149	0,176	0,176
(5)		δ_{lim} = 0,176 m	$k_c = \frac{\delta}{\delta_{lim}}$	0	0,165	0,409	0,602	0,846	1,00	1,00
(6)		Formula $k = (k_c - k_{int})\frac{x}{x_c} + k_{int}$	k=0	$k=0{,}108\cdot10^{-3}x+0{,}057$	$k=0{,}242\cdot10^{-3}x+0{,}167$	$k=0{,}350\cdot10^{-3}x+0{,}252$	$k=0{,}380\cdot10^{-3}x+0{,}466$	$k=0{,}326\cdot10^{-3}x+0{,}674$	k=1	
(7)	Calculation of values of k and δ	x = 0	k	0	0,057	0,167	0,252	0,446	0,674	1
(8)		δ_{st} = 0,60 m	δ, m	0	0,034	0,100	0,151	0,280	0,404	0,600
(9)		x = 1 m	k	0	0,057	0,167	0,252	0,466	0,674	1
(10)		δ_{st} = 0,464 m	δ, m	0	0,026	0,077	0,117	0,218	0,313	0,464
(11)		x = 5 m	k	0	0,058	0,168	0,254	0,468	0,676	1
(12)		δ_{st} = 0,376 m	δ, m	0	0,022	0,064	0,095	0,176	0,253	0,376
(13)		x = 10 m	k	0	0,058	0,169	0,255	0,470	0,677	1
(14)		δ_{st} = 0,352 m	δ, m	0	0,020	0,060	0,090	0,165	0,238	0,352
(15)		x = 30 m	k	0	0,060	0,174	0,262	0,477	0,684	1
(16)		δ_{st} = 0,308 m	δ, m	0	0,018	0,054	0,081	0,147	0,210	0,308
(17)		x = 50 m	k	0	0,061	0,179	0,270	0,485	0,690	1
(18)		δ_{st} = 0,290 m	δ, m	0	0,018	0,052	0,078	0,141	0,200	0,290
(19)		x = 100 m	k	0	0,068	0,191	0,287	0,504	0,706	1
(20)		δ_{st} = 0,258 m	δ, m	0	0,018	0,049	0,074	0,130	0,182	0,258
(21)		x = 200 m	k	0	0,079	0,195	0,332	0,542	0,739	1
(22)		δ_{st} = 0,232 m	δ, m	0	0,018	0,045	0,077	0,125	0,171	0,232
(23)		x = 300 m	k	0	0,088	0,240	0,357	0,580	0,772	1
(24)		δ_{st} = 0,216 m	δ, m	0	0,019	0,052	0,077	0,125	0,167	0,216
(25)		x = 500 m	k	0	0,100	0,264	0,392	0,618	0,804	1
(26)		δ_{st} = 0,210 m	δ, m	0	0,021	0,055	0,082	0,130	0,169	0,210

(Note: Commas should be read as decimal points.)

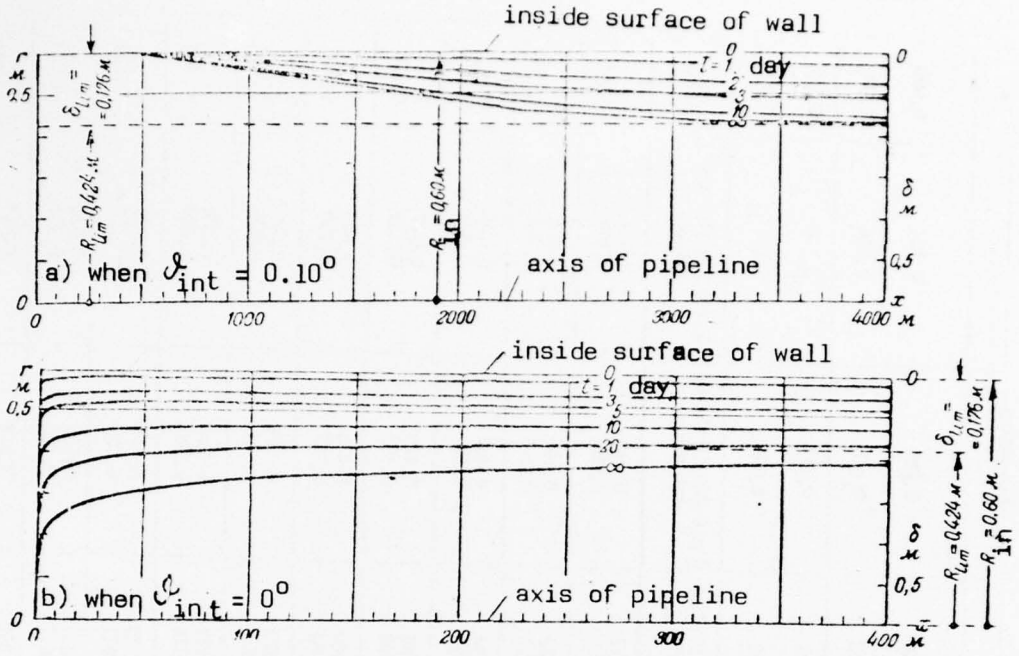

Figure 15. The general case of change in icing with respect to length and in time in a pipeline with R_{in} = 0.60 m, R_{cr} = 0.80 m, and J_d = 0.0150 when Q - 2.0 m^3/sec and ϑ_0 = -10°. At the initial moment of time the pipeline is free of ice.

By using this function, one can integrate the third equation of system (133) by the method of finite differentials, i.e., in the final analysis find the function H = f(x, t). By exactly the same method of finite differentials one can integrate the second equation of system (133) and find the relationship of water temperature with time and place, i.e., function ϑ = f(x, t).

Figure 15, a, also shows the graph of a general case of pipeline icing at water temperature entering the pipeline of ϑ_{int} = 0.10°.

This case differs from the previous one because icing here appears only in the cross-section at a distance x_{pl} = 460 m. In this cross-section in any moment of time, the thickness of the ice layer will be δ = 0. As the first approximation for this case, it is suggested instead of formula (137) to assume that in a given moment of time for all cross-sections the coefficient of proportionality k is the same. However the value of coefficient k is taken as for cylindrical icing. Calculation of the thickness of the ice layer in this case is made in Table 18.

The columns of this table, exactly as the preceding Table 17, correspond to certain moments of time. The second and third lines serve

for calculating k according to cylindrical icing. The second line is filled-in according to Table 15. Lines 4 - 12 correspond to different cross-sections determined by coordinate x. The value of thickness of the layer with steady-state icing δ_{st}, which corresponds to the data of x taken according to the graph in Figure 10, c.

The data of Table 18 served as a basis for plotting the graph in Figure 15, a, which showed the general case of icing for the given initial conditions.

14. The Method of Solution in Finite Differentials

The system of equations (133) determining the general case of icing can be solved in finite differentials more accurately than was offered above. The method of this solution is the following. For an initial moment of time t_1, icing should be assigned a certain function $r_* = f(x, t_1)$. The pipeline should be divided into stretches having lengths $\Delta_1 x$, $\Delta_2 x$, $\Delta_3 x$, ..., $\Delta_n x$. On each of these stretches a value of the relative radius of the free cross-section r_{*1}, r_{*2}, r_{*3}, ..., r_{*n} invariable with respect to length is accepted. For each such stretch one can find the drop in pressure $\Delta_1 H$, $\Delta_2 H$, $\Delta_3 H$, ..., $\Delta_n H$ according to the third equation of system (133), and knowing the hydraulic conditions of intake, find relationship $H = f(x, t_1)$. This relationship will correspond to a moment of time t_1.

Table 18

Approximate Calculation of a General Case of Icing of a Pipeline with R_{in} = 0.60 m, R_{cr} = 0.80 m, J_d = 0.0150 when Q = 2.0 m^3/sec, $\vartheta_0 = -10.0^\circ$, $\vartheta_{int} = 0.10^\circ$; at the Initial Moment of Time when t = 0, the Pipeline is Free of Ice.

(1)	t, days		0	1	3	5	10	∞
(2)	Calculation of values of k for cylindrical icing δ_{lim} = 0.176 m	δ, m	0	0,029	0,072	0,106	0,149	0,176
(3)		$k = \frac{\delta}{\delta_{lim}}$	0	0,165	0,409	0,602	0,846	1,00
(4)	Calculation of values δ_m	$x = 0$; $\delta_{st} = 0$	0	0	0	0	0	0
(5)		$x = 460$ m; $\delta_{st} = 0$	0	0	0	0	0	0
(6)		$x = 1\,000$ m; $\delta_{st} = 0,048$ m	0	0,008	0,020	0,029	0,041	0,048
(7)		$x = 1\,500$ m; $\delta_{st} = 0,087$ m	0	0,014	0,036	0,052	0,074	0,087
(8)		$x = 2\,000$ m; $\delta_{st} = 0,122$ m	0	0,020	0,050	0,073	0,103	0,122
(9)		$x = 2\,500$ m; $\delta_{st} = 0,145$ m	0	0,024	0,059	0,087	0,122	0,145
(10)		$x = 3\,000$ m; $\delta_{st} = 0,160$ m	0	0,026	0,065	0,096	0,135	0,160
(11)		$x = 3\,500$ m; $\delta_{st} = 0,172$ m	0	0,028	0,070	0,104	0,145	0,172
(12)		$x = 4\,000$ m; $\delta_{st} = 0,176$ m	0	0,029	0,072	0,106	0,149	0,176

For this same moment of time one can find the function which determines the distribution of water temperatures along the pipeline $\vartheta = f(x, t_1)$ from the second equation of the system (133). There are no obstacles to such determination, since the second equation of the system is extremely simply interpolated according to variable ϑ. By knowing the temperature of the water in any cross-section of the pipeline, by using the first equation of system (133) one can find $\frac{\partial r*}{\partial t} = f(x, t_1)$, i.e., the distribution of rates of change in radius of the free cross-section along the pipeline. We shall set a certain period of time which will be viewed as the increment of time Δt, such that $t_2 = t_1 + \Delta t$. Over this period of time the relative radii of the free cross-section obtain an increment $\Delta r* = \frac{\partial r*}{\partial t} \Delta t$ and at a moment of time t_2 the new relative radii of the free cross-section will have a value $r* = r*_1 + \Delta r*$, where $r*_1$ designates the value of relative radii at the preceding moment of time t_1. Hence, for a moment of time t_2 one can obtain a new distribution of icing with respect to length $r_* = f(x, t_2)$. However, this distribution should be viewed as the first approximation. Actually, according to the first equation of system (133) a function $\frac{\partial r*}{\partial t} = f(x, t_1)$ was determined with values of ϑ and H that correspond to a moment of time t_1, i.e., the beginning of the elementary period of time Δt. However the results of calculations are distributed over the entire period Δt. In order to obtain the values of H and ϑ that are average for the period Δt, one should determine H according to the third equation of the system (133), and ϑ according to the second equation of the system according to the function obtained in the first approximation for t_2, $r_* = f(x, t_2)$, according to the method described above, and take the average values H and ϑ for moments t_1 and t_2, i.e., take their average values for the period ΔT. With these average values one should repeat the calculations of $\frac{\partial r*}{\partial t} = f(x, t_2)$ according to the first equation of the system (133), and then obtain the function $r* = f(x, t_2)$ for t_2, already in the second approximation. Such repeated calculations should be made until one obtains the suitable approximation of values of the functions $r_* = f(x, t_2)$ in the adjacent calculations.

Having thereby calculated the distribution of relative radii of free cross-sections for a moment of time t_2, one can proceed to calculations of the distribution of ice for the next moment of time t_3, etc.

These calculations are extremely laborious. They can more readily be used in research work than during engineering planning.

Chapter Six

FRAZIL ICE IN THE PRESSURE TRACTS OF THE HYDROELECTRIC POWER STATIONS

15. The Effect of Slush on the Ice Regime of Pipelines

All of the concepts and quantitative estimates cited above pertain to a case of running water free of ice crystals through a pipeline. However, during the operation of hydroelectric power stations the entry of slush into the pipeline together with water is extremely possible, and therefore an analysis of running slush through the pipeline is of practical interest. Unfortunately, up to now information about slush has been so sparse that there is no possibility to make even an approximate quantitative analysis. Therefore, below only the qualitative characteristics of the processes and phenomena that accompany running water with a slush content through the pipeline are given.

If water enters the pipeline together with slush, then one can consider that this mixture has a temperature near the melting point of ice. This assumption introduces determinancy into the estimate of the intake conditions.

The movement of a mixture of water and slush is accompanied by losses of energy on friction different from losses during the motion of water alone. With a high content of slush these losses should significantly increase. In any case, during the movement of the mixture inside the pipeline a heat of friction arises which prevents the complete freezing of the entire cross-section. Another factor that prevents freezing is, as was shown earlier, the increase in pressure experienced by water in moving through the pipeline. This circumstance has great significance for pressure pipelines of hydroelectric power stations in which the pressure gradients are great with respect to length. In the derivation pipelines this factor is more weakly involved and in those stretches where the gradient has a negative value, i.e., where the moving water experiences a decrease in pressure, it causes the opposite effect, facilitating freezing. The heat of friction and the thermal reserve that arise with the increase in pressure are expended in two directions: on heating the immobile ice layer that has formed on the inside surface of the walls, and on melting the slush that is moving along with the water.

The fraction of heat proceeding from the water to the ice layer prevents the development of pipeline icing. Another fraction of heat transferred from water to the slush is expended on melting it. If the first fraction of heat is of entirely obvious use, then the second is expended without bringing practically any use. Reserves of heat of the water are so small that they cannot melt slush in noticeable quantities. Therefore, one cannot count on having slight melting of the slush that occurs in the initial stretch of the pipeline improve the ice regime in its end stretch.

It does not seem possible to identify from the total amount of thermal energy expended by the water that fraction which is transferred to the immobile ice layer on the walls of the pipeline and to the moving slush with the modern state of knowledge.

It is entirely obvious that the more slush there is in the water, the more heat the slush will take upon itself and the less heat reaches the immobile layer of ice, and this will cause greater constriction of the free cross-section of the pipeline, i.e., its greater icing. If the slush consisting of separate ice crystals merges into chunks, then this reduces its capacity to carry-off heat from the water.

It is clear from the above that the presence of slush in water increases inside icing of the pipeline.

The presence of slush causing large hydraulic losses can lead to such a state in a derivational pipeline that with the existing pressure head it will be in no state to handle the required flow rate. In that case the derivational pipeline transfers from the operating regime with the given flow rate to an operating regime with a given drop in pressure, or, which is the same thing, with a given hydraulic gradient. Such a hydraulic regime is dangerous in the sense that during it critical icing and complete freezing of the entire cross-section are possible.

If the layer of ice on the inside surface of the walls is in the process of increasing its thickness, then adhesion and freezing together of separate crystals of slush ice in contact with this layer are probable. One can assume that as the result of this adhesion of the slush, the increase in thickness of the ice layer will occur rapidly, because this is observed in litter screens. However, if the area of the free cross-section of the trash screens is completely blocked by ice, then in pipelines that handle a certain constant flow rate of water, this cannot occur, since with a decrease in the free cross-section the heat of friction increases, which slows and then terminates the increase in thickness of the ice layer. In a case when the layer of ice is in the process of reducing its thickness, the adhesion of crystals of slush ice to it is hardly possible, since in that case there are no conditions for the supercooling of water at the inside surface of the ice layer.

Above, mention was made of ice crystals in the form of slush entering the pipeline with the water.

The possibility of formation of ice crystals inside the pipeline in the layer of water moving along it is not excluded either. Such free ice crystals can only arise if the water at the melting point of ice has a tendency to receive a still lower temperature, i.e., to become supercooled. In the pipeline, supercooling of moving water can occur

with a drop in pressure. If the drop in pressure along the direction of the water flow is so great that it will raise the melting point of ice to a greater value than that of heat that can be produced during friction, conditions are created that are favorable for the supercooling of water and the appearance of ice crystals.

If the temperature of the water was near the melting point of ice ahead of a stretch with such a sharp drop in pressure, then crystallization of the water occurs.

16. The Ice Regime of Turbines

In certain cases, favorable conditions for the precipitation of ice are created in turbines. Water is fed to the turbine under pressure along a pressurized pipeline, which corresponds to a certain melting point of ice that is lower than the melting point of ice in the vacuum which exists in the turbine. Thus, over the comparatively short path covered by water and determined by the dimensions of the blades of the controlling apparatus and the operating wheel, a sharp change in the conditions of the thermodynamic state of water occurs as the result of which crystallization of the water is made possible in certain conditions. We shall assume that in moving along the pressurized pipeline water has dropped to a temperature near the melting point of ice corresponding to pressure at the end of the pressurized pipeline. In this case, at the intake to the turbine and in moving between its blades, the water can be in a supercooled state as a consequence of the high drop in pressure, which entials crystallization. In this case some of the ice crystals can form inside the water, and be carried through the turbine and outtake pipe into the tail bay. Another fraction of the crystals will form on the surfaces of components of the turbine, blocking the flow of water: on walls, on vanes of the controlling apparatus, and on the working wheel of the turbine. Turbine icing is obtained. In this case, the stream has no possibility of preventing such icing, since the heat of hydrodynamic friction that arises in this process is inadequate. Therefore, the icing that has begun in the turbines should usually lead to their complete occupation by ice. Cases of ice stoppage of controlling apparatus and breaks of the vanes of the working wheels are known in this case. In all probability, these emergencies are caused by a supercooled state of water during its movement through the turbine. The ice that forms on the control vanes first interferes with their rotation, i.e., disrupts regulation, and then, by constricting the free cross-section between them, totally blocks the access of the water to the turbine. Buildups of ice on the controlling apparatus and the unmoving walls, which form the cavity in which the working wheel rotates, as well as icing of the vanes of the working wheel itself can cause impacts during rotation of the turbine and damage it. The basic role of slush in these emergencies consists in the fact that it facilitates cooling of water during movement along the pipeline and thereby brings the temperature of the water closer to the temperature of the melting point of ice before intake into the turbine.

Furthermore, separate crystals of slush carried in by water from outside, can adhere to the surfaces bounding the streams, and this accelerates the ice blockage of the turbine. However, blockage of the turbine by ice is also possible without the entry of slugh into the pipeline with the water. It is also quite probable that intake of slush stone can be a cause of breakage of vanes of the working wheel. It is known that in handling the very same water pipelines are subject to the significant effect of frost but remain capable of handling the operating flow rate of water, but turbines, being under the solid thermal protection of the station building, ice on the inside such that one is forced to employ special measures to remove ice from the internal cavities. These ice formations in turbines make the complications comprehensible from the thermodynamic situation cited above and which occur in the turbines comprehensible.

On the basis of the explanation of turbine icing given above, one can first foresee during planning the possibility of ice difficulties (see, for example, a calculation of steady-state icing in Chapter Four, No. 10), and second, to plan measures of combatting turbine icing. The latter pertains to the uptake of heat to surfaces in contact with the water of parts of the turbine approximately as is done for trash-retaining screens.

The amount of transferred heat can be small in comparison with the total heat deficit caused by turbine icing. Experiments of such heating have been carried out and produced positive results.

Hence, with respect to the pressure regime in regard to internal icing, the pressurized pipeline and the intake pipe at the hydroelectric power station are under better conditions than the turbine. At the pumping station the situation changes; the intake and pressurized pipeline are under worse conditions in the sense of possible icing, since along them pressure drops and the pump in which pressure is sharply changed is under better conditions. Therefore, it is no chance that in operation practice of pumping stations ice difficulties are known in the intake pipelines but are unknown in the pumps (confirmation of this can be the reports of V. Ya. Al'tberg about the operation of the Leningrad pumping station (1)).

For the possibility of a quantitative estimate of the role of slush in the ice regime of pipelines as well as in hydraulic machinery, one should conduct special research, and primarily clarify the conditions of heat exchange between water and slush.

Conclusion

In order to conduct an effective struggle with internal icing of pipelines, a correct understanding of the processes of icing and the development of a method of its quantitative evaluation are vital. As

the result of conducting theoretical investigations, the following basic circumstances became clear.

1. Icing of the inside surfaces of the walls changes both with respect to the length of the pipeline and with the passage of time. With constant operating conditions of the pipeline, internal icing tends gradually to take on steady-state forms that are also invariable in time. Steady-state icing is characterized by the fact that proportional to its distance from the beginning of the pipeline it more and more approaches a certain state constant with respect to length called limit icing. The ice layer during limit icing is bounded on the inside by a cylindrical surface.

2. In by-pass pipelines where the value of the handled flow rate is limited by the difference in pressures at its end, besides limit icing critical icing can also exist. Critical icing is characterized by greater thickness of the ice layer than limit icing. If for some reasons the icing of the pipeline is more than critical, then subsequently the layer of ice increases up to complete freezing of water throughout the entire cross-section.

3. An increase in thickness of the layer of ice in the walls of the pipeline with standing water occurs more rapidly than the increase of a flat layer of ice (for example, the ice cover of water surfaces) placed under the same conditions.

4. A great role in the ice regime of the pipeline is played by the character of the longitudinal profile of the pipeline route. The greater the gradient of the pipeline toward the movement of water, the less icing should be anticipated in the pipeline. Stretches of by-pass pipelines running with a reverse gradient are under disadvantageous conditions in the sense of icing.

During sharp pressure drops without significant energy losses on friction (for example, in turbines), water can be in the supercooled state, which causes crystallization of a certain volume of the water and ice obstruction of the water lines.

By appropriate choice of the pipeline route one can limit the value of its inside icing.

5. The basic factor that determinates the character of steady-state icing is the temperature of the water entering the pipeline. Depending on the value of this temperature, icing at the beginning of the pipeline can be either extremely significant or totally absent. At a sufficient distance from the beginning of the pipeline, icing acquires a limit state which does not depend on water temperature at intake.

6. Icing increases proportional to the drop in outside atmospheric temperature. With all other conditions equal, icing increases more if there were not especially low atmospheric temperatures before cooling; a drop of an already quite low temperature of the atmosphere does not cause a significant increase in icing.

7. With the same mean daily flow rate, but different regimes of running it through the pipeline in time, thicknesses of the ice layer on the inside surfaces of walls vary. This circumstance makes it possible to regulate the degree of icing, which is extremely important when preparing the pipeline for the anticipated Spring thaws.

8. Direct solar radiation of the frozen pipeline is a basic cause of a possible ice-gang inside the pipeline as the result of separation of the layer of ice from the walls. Therefore, at the beginning of Spring and especially in mountainous regions, the appearance of such an ice-gang is most probable.

9. With the presence of slush in the water entering the pipeline, internal icing becomes greater than in the absence of slush. Individual ice crystals of slush can participate in the formation of the ice layer and thereby hasten the process of its growth. The slush melts during movement along the pipeline, but this melting is so slight that the flow rate of slush does not practically change. Slush facilitates in maintaining water temperature in the pipeline near the melting point of ice. The indicated circumstance prepares conditions for supercooling of water in turbines.

10. In the investigated ice processes of pipelines one perceives a shortage of experimental data, especially of full-scale observations. Setting up such investigations will make it possible to check and refine the theoretical conclusions and to develop reliable planning methods.

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APPENDIX

CERTAIN FUNCTIONS OF r* ENCOUNTERED DURING CALCULATIONS OF THE ICE REGIME OF THE PIPELINE

r_*	$-\ln r_*$	$\frac{1}{-\ln r_*}$	$r_*^{5/3}$	$\frac{1}{r_*^{5/3}}$	$r_*^{2/3}$	$\frac{1}{r_*}$	$\frac{1}{r_*^{6/3}}$	$\frac{1}{-r_* \ln r_*}$
1,00	0	∞	1,00	1,00	1,00	1,00	1,00	∞
0,99	0,010	100	0,946	1,058	0,972	1,010	1,069	101,0
0,98	0,020	50	0,896	1,118	0,948	1,020	1,140	51,0
0,97	0,030	33,3	0,850	1,178	0,920	1,031	1,213	34,3
0,96	0,041	24,4	0,804	1,244	0,895	1,041	1,294	25,4
0,95	0,052	19,22	0,759	1,320	0,871	1,052	1,390	20,2
0,94	0,062	16,14	0,719	1,392	0,848	1,063	1,481	17,19
0,93	0,073	13,70	0,678	1,476	0,821	1,074	1,587	14,72
0,92	0,083	12,04	0,639	1,566	0,800	1,087	1,702	13,09
0,91	0,094	10,63	0,605	1,654	0,779	1,099	1,820	11,70
0,90	0,105	9,53	0,570	1,755	0,753	1,111	1,950	10,59
0,89	0,116	8,62	0,536	1,868	0,732	1,123	2,10	9,70
0,88	0,128	7,81	0,505	1,982	0,711	1,138	2,25	8,89
0,87	0,139	7,20	0,476	2,10	0,688	1,150	2,42	8,28
0,86	0,151	6,62	0,447	2,24	0,668	1,162	2,60	7,70
0,85	0,162	6,17	0,420	2,33	0,649	1,177	2,74	7,26
0,84	0,174	5,75	0,394	2,54	0,628	1,190	3,02	6,84
0,83	0,186	5,38	0,370	2,70	0,607	1,205	3,26	6,49
0,82	0,198	5,05	0,346	2,89	0,588	1,220	3,53	6,16
0,81	0,211	4,74	0,325	3,08	0,569	1,234	3,80	5,85
0,80	0,223	4,49	0,304	3,29	0,551	1,250	4,11	5,61
0,79	0,236	4,24	0,284	3,52	0,531	1,266	4,46	5,37
0,78	0,248	4,04	0,265	3,77	0,513	1,281	4,84	5,19
0,77	0,261	3,83	0,248	4,03	0,497	1,300	5,24	4,98
0,76	0,274	3,65	0,232	4,31	0,480	1,318	5,68	4,81
0,75	0,288	3,47	0,216	4,64	0,464	1,333	6,19	4,63
0,74	0,301	3,32	0,200	5,00	0,448	1,351	6,76	4,49
0,73	0,315	3,18	0,1865	5,36	0,431	1,370	7,35	4,36
0,72	0,328	3,05	0,1731	5,78	0,416	1,390	8,01	4,24
0,71	0,342	2,92	0,1608	6,22	0,401	1,410	8,76	4,11
0,70	0,357	2,80	0,1491	6,71	0,385	1,430	9,59	4,01
0,69	0,371	2,70	0,1380	7,25	0,371	1,450	10,51	3,92
0,68	0,385	2,60	0,1278	7,83	0,357	1,471	11,52	3,83
0,67	0,400	2,50	0,1180	8,49	0,343	1,493	12,67	3,74
0,66	0,415	2,41	0,1089	9,19	0,330	1,516	13,92	3,66
0,65	0,431	2,32	0,1005	9,96	0,316	1,539	15,31	3,57
0,64	0,446	2,24	0,0925	10,80	0,304	1,562	16,88	3,50
0,63	0,462	2,16	0,0850	11,78	0,291	1,588	18,70	3,43
0,62	0,478	2,10	0,0777	12,89	0,279	1,614	20,8	3,39
0,61	0,494	2,03	0,0716	13,96	0,266	1,640	22,9	3,33
0,60	0,511	1,958	0,0657	15,28	0,256	1,669	25,5	3,26
0,59	0,527	1,900	0,0599	16,70	0,245	1,696	28,3	3,22
0,58	0,545	1,836	0,0546	18,34	0,234	1,727	31,7	3,17
0,57	0,562	1,780	0,0500	20,0	0,223	1,755	35,1	3,12
0,56	0,580	1,727	0,0454	22,0	0,212	1,787	39,3	3,09
0,55	0,598	1,675	0,0412	24,3	0,202	1,820	44,1	3,05
0,54	0,616	1,624	0,0374	26,8	0,193	1,851	49,6	3,01
0,53	0,635	1,575	0,0338	29,6	0,184	1,887	55,9	2,97
0,52	0,654	1,530	0,0305	32,8	0,175	1,924	63,1	2,94
0,51	0,673	1,486	0,0275	36,4	0,166	1,961	71,3	2,92

continuation:

$r_*^{1/6}$	$\frac{-\ln r_*}{1+\frac{3}{4}(-\ln r_*)}$	$\frac{r_*^{1/4}(-\ln r_*)}{1+\frac{3}{4}(-\ln r_*)}$	$\frac{r_*^{1/4}(-\ln r_*)^2}{1+\frac{3}{4}(-\ln r_*)}$	$\frac{(-\ln r_*)^2}{r_*^{5/16}[1+\frac{3}{4}(-\ln r_*)]}$	$\frac{r_*^{3/4}}{-\ln r_*}$	$\frac{1}{r_*^{3/4}}$	$r_*^{1/4}$
1,00	0	0	0	0	∞	1,00	1,00
0,982	0,00994	0,00992	$0,0992\cdot10^{-3}$	$0,1049\cdot10^{-3}$	99,2	1,009	0,998
0,966	0,0197	0,0196	$0,392\cdot10^{-3}$	$0,438\cdot10^{-3}$	49,2	1,016	0,996
0,950	0,0293	0,0291	$0,874\cdot10^{-3}$	0,001029	32,5	1,025	0,993
0,932	0,0397	0,0392	0,001606	0,00200	23,7	1,030	0,990
0,918	0,0501	0,0494	0,00260	0,00344	18,50	1,040	0,988
0,900	0,0592	0,0584	0,00364	0,00506	15,40	1,049	0,986
0,884	0,0691	0,0679	0,00496	0,00731	12,96	1,058	0,983
0,869	0,0780	0,0764	0,00635	0,00995	11,31	1,063	0,980
0,852	0,0878	0,0856	0,00805	0,01332	9,90	1,074	0,978
0,839	0,0974	0,0947	0,01022	0,01778	8,79	1,081	0,974
0,823	0,1069	0,1039	0,01206	0,0225	7,90	1,090	0,972
0,807	0,1168	0,1128	0,01442	0,0286	7,10	1,100	0,969
0,792	0,1218	0,1176	0,01636	0,0344	6,49	1,110	0,966
0,777	0,1333	0,1284	0,01941	0,0435	5,91	1,120	0,963
0,761	0,1441	0,1382	0,0224	0,0522	5,46	1,130	0,961
0,747	0,1538	0,1472	0,0256	0,0650	5,05	1,140	0,958
0,732	0,1621	0,1548	0,0288	0,0778	4,68	1,150	0,955
0,718	0,1722	0,1639	0,0324	0,0938	4,35	1,160	0,952
0,702	0,1822	0,1730	0,0365	0,1123	4,05	1,170	0,949
0,689	0,1911	0,1808	0,0403	0,1326	3,81	1,180	0,946
0,675	0,200	0,1888	0,0446	0,1570	3,55	1,191	0,943
0,660	0,209	0,1962	0,0486	0,1830	3,36	1,203	0,940
0,648	0,218	0,204	0,0534	0,215	3,14	1,215	0,936
0,632	0,225	0,210	0,0576	0,248	2,97	1,229	0,933
0,628	0,237	0,220	0,0634	0,294	2,80	1,241	0,930
0,603	0,246	0,228	0,0681	0,340	2,66	1,251	0,927
0,591	0,255	0,236	0,0744	0,398	2,51	1,269	0,925
0,578	0,264	0,243	0,0798	0,461	2,38	1,280	0,921
0,564	0,272	0,250	0,0859	0,534	2,26	1,293	0,918
0,552	0,282	0,258	0,0921	0,618	2,14	1,307	0,914
0,538	0,290	0,264	0,0980	0,711	2,04	1,321	0,911
0,526	0,299	0,271	0,1042	0,816	1,950	1,336	0,908
0,511	0,308	0,278	0,1111	0,946	1,850	1,351	0,905
0,499	0,316	0,285	0,1182	1,088	1,765	1,367	0,902
0,488	0,326	0,293	0,1262	1,257	1,678	1,382	0,898
0,474	0,334	0,299	0,1333	1,441	1,611	1,400	0,895
0,462	0,343	0,306	0,1413	1,662	1,532	1,416	0,892
0,450	0,352	0,313	0,1494	1,925	1,460	1,433	0,889
0,438	0,360	0,318	0,1570	2,19	1,398	1,450	0,885
0,426	0,369	0,324	0,1656	2,53	1,332	1,466	0,880
0,414	0,378	0,331	0,1742	2,91	1,272	1,483	0,876
0,403	0,387	0,338	0,1840	3,38	1,221	1,506	0,873
0,392	0,395	0,343	0,1924	3,85	1,169	1,522	0,869
0,380	0,404	0,349	0,202	4,45	1,117	1,542	0,865
0,369	0,412	0,355	0,212	5,16	1,072	1,561	0,861
0,358	0,421	0,360	0,222	5,96	1,022	1,584	0,856
0,347	0,430	0,367	0,234	6,93	0,976	1,610	0,853
0,336	0,439	0,373	0,244	8,00	0,936	1,635	0,850
0,325	0,447	0,378	0,254	9,24	0,900	1,659	0,845

continuation

r_*	$-\ln r_*$	$\frac{1}{-\ln r_*}$	$r_*^{5/3}$	$\frac{1}{r_*^{5/3}}$	$r_*^{3/3}$	$\frac{1}{r_*}$	$\frac{1}{r_*^{6/3}}$
0,50	0,693	1,442	0,0248	40,4	0,157	2,00	80,8
0,49	0,713	1,402	0,0222	45,1	0,149	2,04	92,1
0,48	0,734	1,363	0,01985	50,4	0,141	2,08	104,9
0,47	0,755	1,326	0,01780	56,2	0,133	2,13	119,6
0,46	0,776	1,289	0,01585	63,1	0,126	2,18	137,6
0,45	0,798	1,252	0,01410	71,0	0,119	2,22	157,8
0,44	0,821	1,219	0,01253	79,8	0,112	2,28	181,8
0,43	0,844	1,186	0,01109	90,4	0,105	2,32	210
0,42	0,867	1,153	0,00979	102,1	0,0988	2,38	244
0,41	0,891	1,122	0,00860	116,2	0,0928	2,44	284
0,40	0,916	1,091	0,00754	132,8	0,0868	2,50	332
0,39	0,941	1,062	0,00659	151,8	0,0810	2,56	389
0,38	0,967	1,036	0,00574	174,2	0,0758	2,63	459
0,37	0,994	1,008	0,00498	201	0,0704	2,70	544
0,36	1,021	0,980	0,00430	233	0,0655	2,78	649
0,35	1,050	0,952	0,00370	270	0,0609	2,86	772
0,34	1,079	0,928	0,00317	315	0,0562	2,94	926
0,33	1,109	0,904	0,00270	371	0,0520	3,03	$1,125 \cdot 10^3$
0,32	1,139	0,879	0,00230	435	0,0479	3,13	$1,361 \cdot 10^3$
0,31	1,171	0,855	0,001938	516	0,0440	3,23	$1,666 \cdot 10^3$
0,30	1,204	0,830	0,001628	615	0,0404	3,33	$2,05 \cdot 10^3$
0,29	1,238	0,809	0,001359	737	0,0368	3,45	$2,54 \cdot 10^3$
0,28	1,273	0,785	0,001130	885	0,0335	3,57	$3,16 \cdot 10^3$
0,27	1,309	0,765	$0,927 \cdot 10^{-3}$	$1,079 \cdot 10^3$	0,0304	3,71	$4,00 \cdot 10^3$
0,26	1,347	0,743	$0,759 \cdot 10^{-3}$	$1,320 \cdot 10^3$	0,0275	3,85	$5,09 \cdot 10^3$
0,25	1,386	0,721	$0,615 \cdot 10^{-3}$	$1,628 \cdot 10^3$	0,0247	4,00	$6,51 \cdot 10^3$
0,24	1,427	0,701	$0,495 \cdot 10^{-3}$	$2,02 \cdot 10^3$	0,0222	4,16	$8,40 \cdot 10^3$
0,23	1,470	0,680	$0,394 \cdot 10^{-3}$	$2,54 \cdot 10^3$	0,0198	4,35	$11,04 \cdot 10^3$
0,22	1,514	0,660	$0,311 \cdot 10^{-3}$	$3,21 \cdot 10^3$	0,0176	4,55	$14,60 \cdot 10^3$
0,21	1,560	0,641	$0,243 \cdot 10^{-3}$	$4,12 \cdot 10^3$	0,0156	4,76	$19,60 \cdot 10^3$
0,20	1,609	0,621	$0,1870 \cdot 10^{-3}$	$5,35 \cdot 10^3$	0,0137	5,00	$26,8 \cdot 10^3$
0,19	1,661	0,602	$0,1421 \cdot 10^{-3}$	$7,04 \cdot 10^3$	0,0119	5,26	$37,0 \cdot 10^3$
0,18	1,715	0,584	$0,1065 \cdot 10^{-3}$	$9,40 \cdot 10^3$	0,0103	5,56	$52,3 \cdot 10^3$
0,17	1,772	0,565	$0,0785 \cdot 10^{-3}$	$12,75 \cdot 10^3$	0,00888	5,89	$75,1 \cdot 10^3$
0,16	1,832	0,545	$0,0569 \cdot 10^{-3}$	$17,60 \cdot 10^3$	0,00754	6,25	$0,1100 \cdot 10^6$
0,15	1,897	0,528	$0,0404 \cdot 10^{-3}$	$24,8 \cdot 10^3$	0,00636	6,67	$0,1652 \cdot 10^6$
0,14	1,966	0,509	$0,0278 \cdot 10^{-3}$	$36,0 \cdot 10^3$	0,00529	7,14	$0,257 \cdot 10^6$
0,13	2,040	0,490	$0,0188 \cdot 10^{-3}$	$53,2 \cdot 10^3$	0,00434	7,70	$0,410 \cdot 10^6$
0,12	2,125	0,471	$0,01223 \cdot 10^{-3}$	$81,8 \cdot 10^3$	0,00350	8,34	$0,684 \cdot 10^6$
0,11	2,207	0,453	$0,00771 \cdot 10^{-3}$	$0,130 \cdot 10^6$	0,00277	9,10	$1,182 \cdot 10^6$
0,10	2,303	0,434	$0,00464 \cdot 10^{-3}$	$0,216 \cdot 10^6$	0,00464	10,00	$2,16 \cdot 10^6$
0,09	2,408	0,415	$0,00264 \cdot 10^{-3}$	$0,379 \cdot 10^6$	0,00163	11,11	$4,21 \cdot 10^6$
0,08	2,526	0,396	$0,001412 \cdot 10^{-3}$	$0,708 \cdot 10^6$	0,00119	12,50	$8,86 \cdot 10^6$
0,07	2,656	0,377	$0,693 \cdot 10^{-6}$	$1,442 \cdot 10^6$	$0,832 \cdot 10^{-3}$	14,29	$20,6 \cdot 10^6$
0,06	2,803	0,357	$0,305 \cdot 10^{-6}$	$3,28 \cdot 10^6$	$0,551 \cdot 10^{-3}$	16,68	$54,7 \cdot 10^6$
0,05	2,996	0,334	$0,115 \cdot 10^{-6}$	$8,70 \cdot 10^6$	$0,339 \cdot 10^{-3}$	20,0	$0,1740 \cdot 10^9$
0,04	3,219	0,311	$0,0250 \cdot 10^{-6}$	$40,0 \cdot 10^6$	$0,187 \cdot 10^{-3}$	25,0	$1,000 \cdot 10^9$
0,03	3,506	0,285	$0,00755 \cdot 10^{-6}$	$0,1325 \cdot 10^9$	$0,0869 \cdot 10^{-3}$	33,3	$4,41 \cdot 10^9$
0,02	3,912	0,256	$0,869 \cdot 10^{-9}$	$1,150 \cdot 10^9$	$0,0295 \cdot 10^{-3}$	50,0	$57,5 \cdot 10^9$
0,01	4,605	0,218	$0,0215 \cdot 10^{-9}$	$46,5 \cdot 10^9$	$0,00463 \cdot 10^{-3}$	100,0	$4,65 \cdot 10^{12}$
0,00	∞	0	0	∞	0	∞	∞

continuation:

$\frac{1}{-r_\bullet \ln r_\bullet}$	$r_\bullet^{1/6}$	$\frac{\ln r_\bullet}{1+{}^5/_6(-\ln r_\bullet)}$	$\frac{r_\bullet^{1/6}(-\ln r_\bullet)}{1+{}^5/_6(-\ln r_\bullet)}$	$\frac{r_\bullet^{1/6}(-\ln r_\bullet)^2}{1+{}^5/_6(-\ln r_\bullet)}$	$\frac{(-\ln r_\bullet)^2}{r_\bullet^{5/6}[1+{}^5/_6(-\ln r_\bullet)]}$	$\frac{r_\bullet^{1/6}}{-\ln r_\bullet}$	$\frac{1}{r_\bullet^{1/6}}$	$r_\bullet^{1/6}$
2,89	0,315	0,455	0,382	0,265	10,70	0,855	1,680	0,840
2,86	0,304	0,465	0,389	0,278	12,52	0,822	1,710	0,836
2,84	0,294	0,474	0,394	0,289	14,56	0,788	1,730	0,833
2,82	0,284	0,481	0,398	0,300	16,84	0,754	1,761	0,829
2,80	0,274	0,490	0,404	0,313	19,72	0,719	1,790	0,824
2,78	0,264	0,499	0,409	0,326	23,2	0,688	1,819	0,820
2,77	0,254	0,509	0,414	0,340	27,1	0,659	1,852	0,816
2,75	0,244	0,516	0,418	0,353	31,9	0,628	1,889	0,810
2,74	0,235	0,526	0,424	0,367	37,5	0,600	1,923	0,806
2,74	0,226	0,535	0,428	0,381	44,3	0,576	1,951	0,800
2,73	0,217	0,543	0,432	0,396	52,6	0,554	1,974	0,796
2,72	0,209	0,551	0,435	0,410	62,2	0,526	2,02	0,790
2,72	0,199	0,560	0,439	0,424	73,9	0,503	2,06	0,785
2,72	0,191	0,569	0,444	0,441	88,6	0,480	2,10	0,780
2,72	0,182	0,579	0,449	0,459	106,9	0,456	2,15	0,775
2,72	0,174	0,588	0,452	0,475	128,1	0,433	2,20	0,770
2,73	0,166	0,596	0,456	0,491	154,4	0,412	2,25	0,765
2,74	0,158	0,605	0,459	0,509	188,5	0,393	2,30	0,759
2,75	0,150	0,614	0,462	0,526	228	0,374	2,35	0,752
2,76	0,142	0,624	0,465	0,545	281	0,354	2,41	0,746
2,77	0,134	0,633	0,468	0,564	347	0,336	2,47	0,740
2,79	0,127	0,641	0,470	0,582	429	0,320	2,53	0,734
2,81	0,120	0,651	0,473	0,603	534	0,302	2,60	0,727
2,84	0,113	0,661	0,476	0,623	672	0,286	2,67	0,721
2,86	0,106	0,670	0,478	0,644	851	0,270	2,75	0,714
2,88	0,0991	0,680	0,481	0,667	$1,086 \cdot 10^3$	0,255	2,82	0,708
2,91	0,0926	0,690	0,483	0,689	$1,392 \cdot 10^3$	0,240	2,92	0,700
2,96	0,0863	0,700	0,485	0,713	$1,810 \cdot 10^3$	0,225	3,02	0,693
3,00	0,0802	0,709	0,486	0,736	$2,36 \cdot 10^3$	0,212	3,11	0,685
3,05	0,0741	0,719	0,487	0,760	$3,13 \cdot 10^3$	0,1991	3,22	0,677
3,10	0,0683	0,729	0,488	0,836	$4,47 \cdot 10^3$	0,1862	3,33	0,670
3,16	0,0629	0,739	0,489	0,812	$5,71 \cdot 10^3$	0,1736	3,47	0,660
3,24	0,0572	0,750	0,489	0,838	$7,87 \cdot 10^3$	0,1613	3,62	0,651
3,32	0,0521	0,760	0,488	0,866	$11,04 \cdot 10^3$	0,1494	3,78	0,642
3,41	0,0472	0,771	0,488	0,894	$15,72 \cdot 10^3$	0,1376	3,96	0,633
3,52	0,0423	0,783	0,487	0,922	$22,9 \cdot 10^3$	0,1278	4,15	0,623
3,63	0,0378	0,794	0,486	0,956	$34,4 \cdot 10^3$	0,1168	4,36	0,611
3,77	0,0334	0,806	0,485	0,990	$52,6 \cdot 10^3$	0,1060	4,62	0,601
3,92	0,0291	0,820	0,484	1,029	$84,2 \cdot 10^3$	0,0960	4,91	0,590
4,12	0,0252	0,832	0,480	1,060	$0,1379 \cdot 10^6$	0,0865	5,24	0,576
4,34	0,0216	0,845	0,475	1,093	$0,236 \cdot 10^6$	0,0772	5,61	0,562
4,61	0,0181	0,859	0,471	1,135	$0,430 \cdot 10^6$	0,0680	6,10	0,548
4,96	0,0149	0,872	0,464	1,172	$0,831 \cdot 10^6$	0,0638	6,21	0,541
5,39	0,0119	0,886	0,456	1,212	$1,750 \cdot 10^6$	0,0513	7,35	0,514
5,95	0,00920	0,903	0,447	1,252	$4,11 \cdot 10^6$	0,0431	8,24	0,494
6,68	0,00680	0,923	0,436	1,308	$11,39 \cdot 10^6$	0,0354	9,43	0,473
7,77	0,00468	0,941	0,421	1,354	$54,4 \cdot 10^6$	0,0278	11,20	0,447
9,50	0,00290	0,968	0,402	1,412	$0,871 \cdot 10^9$	0,0205	13,89	0,416
12,80	0,00147	0,994	0,373	1,460	$1,678 \cdot 10^9$	0,0136	18,80	0,383
21,8	$0,464 \cdot 10^{-3}$	1,031	0,326	1,502	$69,8 \cdot 10^9$	0,00690	31,6	0,416
∞	0	1,333	0	0	∞	0	∞	0