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Two-Dimensional, Two-Phase Flow Through a Rocket Exhaust Nozzle: A Progress Report

Jane M. Cousins

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Aylesbury
Bucks

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6 TWO-DIMENSIONAL, TWO-PHASE FLOW THROUGH A ROCKET EXHAUST NOZZLE: A PROGRESS REPORT,

by

10 Jane M./Cousins

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SUMMARY

This memorandum describes progress in the development of a set of computer programs which solve the equations of gas/particle flow through an axisymmetric nozzle. Two programs have been written to solve the equations in the transonic throat region of the nozzle. The first treats the two-phase fluid as a heavy perfect gas with modified isentropic exponent and molecular weight, and solves the equations of isentropic transonic flow. An initial flow field configuration is obtained for input to the second program, which incorporates non-equilibrium effects of the gas/particle mixture.

The programs provide accurate initial line data for starting a supersonic calculation, and will be used as input to a third program, soon to be written, for solving the two-phase flow equations in the supersonic region of the nozzle.

Further copies of this report can be obtained from the Defence Research Information Centre, Station Square House, St. Mary Cray, Orpington, Kent BR5 3RE.

CONTENTS

1 INTRODUCTION

Accurate prediction of the properties of a rocket exhaust plume necessitates detailed analyses of the nozzle and plume flow fields. The Rocket Exhaust Plume program (REP)[1] and the Base Flow program (BAFL)[2], which calculate the structure of the external (plume) flow field, incorporate the latest developments in gas dynamics and chemical kinetics, and are the most advanced models available. However, the accuracy of the predictions depends upon the accuracy of the input nozzle exit parameters, and the programs currently used at the RPE for calculating the flow from the combustion chamber to the nozzle exit plane[3], while able to take account of non-equilibrium chemical reactions, can treat only one-dimensional, gas phase flow.

Many investigators (e.g. Ref. 5) have shown that the flow in a typical rocket nozzle is far from one-dimensional, and it has also been shown (e.g. Ref. 6) that condensed metal oxides in the rocket exhaust nozzle can significantly affect the performance of the motor. These factors must be considered when the nozzle calculations are performed, and the ultimate aim of current work is to develop a set of computer programs to solve the equations governing the flow of a gas/particle mixture through an axisymmetric rocket exhaust nozzle.

In the solving of the equations for flow through a convergent/divergent nozzle, many mathematical difficulties are encountered in the region of transition from subsonic to supersonic flow. Although the transition is smooth the properties in the two regions are quite different. In the subsonic region the governing equations are elliptic, whereas in the supersonic region they are hyperbolic. The supersonic flow field may be solved by the method of characteristics, but accurate initial line data are needed to start the calculation. Since the flow near the nozzle throat is far from one-dimensional, the sonic velocity at the wall being reached well upstream of the throat, while that at the axis is reached downstream of the throat, a method of solution which describes the properties of both subsonic and supersonic flow segments must be devised. This memorandum describes the method adopted for solving the flow in this transonic throat region.

2 TRANSONIC TWO-PHASE FLOW IN AXISYMMETRIC NOZZLES

"To solve the equations governing the transonic flow of a gas-particle mixture is an extremely formidable task" - Kliegel[6].

Of the many approaches which have been made to the solution of the two-phase flow problem[4], the earliest was to treat the nozzle expansion processes as uncoupled - that is, the particle velocity and thermal lags were considered to be independent of each other, and their effects on the gas properties were ignored. More recently, Kliegel and Nickerson[7] have developed the method of constant fractional lag, in which the ratio of the gas to particle velocities and the ratio of the temperatures are represented by constants. Their program is the most widely used, but the method of solution is restricted to nozzles of a particular shape, and is inapplicable to nozzles of sharp wall curvature.

It was decided to use the technique employed by Regan, Thompson and Hoglund[9], as this leads to a fully coupled solution, and is valid for a much wider range of nozzle shapes. The method proceeds in two stages, and a separate computer program has been written for each.

In the first stage, the gas/particle mixture is assumed to be in equilibrium - that is, there are no particle heat or velocity lags, and the two-phase fluid is treated as a heavy perfect gas with modified molecular weight

$$\bar{m} = m(1 + \phi) \tag{2.1}$$

based on the particle-to-gas mass ratio ϕ, and effective isentropic exponent

$$\bar{\gamma} = \gamma(1 + \phi_c)/(1 + \gamma\phi_c) \tag{2.2}$$

where

$$\phi_c = \phi \, Cp_p/Cp_g .$$

The equations for isentropic transonic flow are solved, and initial flow field data obtained for input to the second stage. The equilibrium program may be used separately to provide starting values for a supersonic solution in the absence of particles, or when the non-equilibrium particle effects can be ignored.

In the second program, the non-equilibrium effects of the two-phase fluid are incorporated. The partial differential equations governing the gas/particle flow are rewritten as algebraic replacement equations and, using the results from the initial stage as a first approximation, are solved iteratively.

3 STAGE 1: TRANSONIC EQUILIBRIUM PARTICLE/GAS MIXTURE PROGRAM (TEPGAS)

There are several recognised methods for solving the equations of transonic, axisymmetric flow, but most are applicable only to nozzles of very restricted geometry. One method is to reduce the problem to that of solving a set of ordinary differential equations by expanding the flow variables in inverse power series in the radius of curvature of the throat (Ref. 8 and Appendix A). However, this method is limited to nozzles with a throat wall radius of curvature, R, greater than twice the throat radius, r_t (e.g. Fig. 1). This can be overcome by using an inverse approach[5], in which the boundary geometry is not specified but is obtained from the solution of a prescribed velocity distribution along a suitable reference line. The velocity is then modified until a streamline of the flow approximates to the shape of the nozzle. This method depends on the ability to specify, a priori, the velocity distribution which yields the desired nozzle boundary. It is useful for nozzles whose geometry can be defined by the radius of curvature of the throat, but is unlikely to give accurate results for nozzles of complicated geometry.

The method adopted was that described by Stow[10], which can be applied to a nozzle of general geometry. It is based on the "streamline curvature" method used for subsonic flows.

3.1 Governing equations

The equations governing the steady axisymmetric motion of an inviscid compressible fluid are, in the coordinates of Fig. 1:

continuity

$$\frac{\partial}{\partial x}(r \rho u) + \frac{\partial}{\partial r}(r \rho v) = 0 \quad , \tag{3.1}$$

conservation of momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial x} \tag{3.2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} \quad . \tag{3.3}$$

Equation (3.1) implies the existence of a stream function ψ satisfying

$$\frac{\partial\psi}{\partial r} = r\rho u \quad , \quad \frac{\partial\psi}{\partial x} = -r\rho v$$

or

$$\psi(x,r) = \int^r (r\rho u)_{x=\text{const}}\, dr + \text{const} \quad . \tag{3.4}$$

By introducing the speed of sound, defined by $c = (\partial P/\partial\rho)^{1/2}$, using the relationship $v = u \tan\theta$, and the identity

$$\left(\frac{\partial}{\partial x}\right)_\psi = \left(\frac{\partial}{\partial x}\right)_r + \tan\theta \left(\frac{\partial}{\partial r}\right)_x{}^*$$

it is possible to rewrite equations (3.1) to (3.3) as

$$\frac{\sec^2\theta}{\rho}\left(\frac{\partial P}{\partial r}\right)_x = -u^2\left(\frac{\partial\tan\theta}{\partial x}\right)_\psi + \frac{\tan\theta}{\rho}\left(\frac{\partial P}{\partial x}\right)_\psi \tag{3.5}$$

and

$$\frac{\tan\theta}{\rho}\left(\frac{\partial P}{\partial r}\right)_x = -u^2\left(\frac{\partial\tan\theta}{\partial r}\right)_x - \frac{u^2\tan\theta}{r} + \frac{1}{\rho}\left(1-\frac{u^2}{c^2}\right)\left(\frac{\partial P}{\partial x}\right)_\psi \quad . \tag{3.6}$$

Eliminating $(\partial P/\partial x)_\psi$ between equations (3.5) and (3.6) gives the radial equilibrium equation:

$$\left(1-\frac{u^2}{c^2}\sec^2\theta\right)\frac{1}{\rho}\left(\frac{\partial P}{\partial r}\right)_x = -u^2\left(1-\frac{u^2}{c^2}\right)\left(\frac{\partial\tan\theta}{\partial x}\right)_\psi$$

$$+\, u^2\tan\theta\left(\frac{\partial\tan\theta}{\partial r}\right)_x + \frac{u^2\tan^2\theta}{r} \tag{3.7}$$

relating the static pressure field to the streamline geometry. The term $\left(\frac{\partial\tan\theta}{\partial x}\right)_\psi$ is referred to as the "curvature" of a streamline.

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*Subscripts ψ , r , x refer to differentiation along a line of constant ψ , r or x .

3.2 Method of solution

Equation (3.7) is solved numerically in an iterative manner. The velocity field and the shape of the streamlines are approximated from either a series or a one-dimensional solution (see Appendix A). Since all other quantities are known, equation (3.7) then becomes an ordinary differential equation in the static pressure, and if an approximation is made to the mid-radial pressure, it may be integrated to the wall and axis. The mass flux is determined from equation (3.4) and the mid-radial pressure is changed until the calculated mass flux agrees with the true mass flux (see Appendix B). All the axial stations are solved in this way, and the other flow variables are evaluated from isentropic relations. A new approximation to the streamline geometry is made and the calculation is repeated until the solution converges.

Equation (3.7) has an indeterminacy at a sonic point. If the values of the calculated streamline slope and curvature were exact the equation would have a zero on both sides and the indeterminacy could be removed. However, since the procedure is iterative, only approximate values of the slope and curvature are known, and the equation has a singularity at such a point. This problem is overcome by using equation (3.5) in the first few iterations to find the approximate locations of the sonic points, the term $(\partial P/\partial x)_{\psi}$ being determined from the solution in the previous iteration. Equation (3.5) is then used close to the sonic points while equation (3.7) is used elsewhere.

In practice relaxed values of the calculated slope and curvature must be used in certain cases to ensure convergence. For example,

$$(\tan\theta)^{n}_{\text{used}} = (\tan\theta)^{n-1}_{\text{used}} + \text{relaxation factor} \times \left\{ (\tan\theta)^{n}_{\text{found}} - (\tan\theta)^{n-1}_{\text{used}} \right\} .$$

The relaxation factor depends upon the nozzle geometry and the closeness of the initial estimate to the final solution, as does the number of steps required, convergence being assumed when the change in the flow parameters between successive iterations is smaller than some desired degree of accuracy.

4 STAGE 2: TRANSONIC GAS/PARTICLE FLOW PROGRAM (TGPF)

4.1 Governing equations

The equations governing the flow of an ideal gas/particle mixture through an axisymmetric nozzle are[6]:

gas phase and particle phase continuity equations*

$$(\rho_g u_g)_x + \frac{1}{r}(r \rho_g v_g)_r = 0 \tag{4.1}$$

$$(\rho_p u_p)_x + \frac{1}{r}(r \rho_p v_p)_r = 0 \tag{4.2}$$

mixture axial and radial momentum equations

$$\rho_g \left[u_g u_{g_x} + v_g u_{g_r} \right] + P_{g_x} + F\rho_p (u_g - u_p) = 0 \tag{4.3}$$

$$\rho_g \left[u_g v_{g_x} + v_g v_{g_r} \right] + P_{g_r} + F\rho_p (v_g - v_p) = 0 \quad , \tag{4.4}$$

mixture energy equation

$$u_g P_{g_x} + v_g P_{g_r} - \gamma R T_g \left[u_g \rho_{g_x} + v_g \rho_{g_r} \right]$$

$$- F\rho_p (\gamma - 1) \left[(u_g - u_p)^2 + (v_g - v_p)^2 + (G/F)(T_p - T_g) \right] = 0 \quad , \tag{4.5}$$

gas phase equation of state

$$P_g = \rho_g R T_g \quad , \tag{4.6}$$

particle drag equations

$$u_p u_{p_x} + v_p u_{p_r} = F (u_g - u_p) \tag{4.7}$$

$$u_p v_{p_x} + v_p v_{p_r} = F (v_g - v_p) \quad , \tag{4.8}$$

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*Subscripts x and r denote differentiation w.r.t x or r .

particle heat transfer equation

$$u_p T_{p_x} + v_p T_{p_r} = - G (T_p - T_g)/Cp_p \tag{4.9}$$

where

$$F \equiv \frac{9 \mu_g f_p}{2 m_p r_p^2} \tag{4.10}$$

and

$$G \equiv \frac{3 \mu_g g_p}{m_p r_p^2} \frac{Cp_g}{Pr} . \tag{4.11}$$

The drag and heat transfer factors, f_p and g_p, vary with local flow field properties in accordance with empirical relationships. The empirical formulae used are discussed in Appendix C.

4.2 Basic assumptions

The following basic assumptions are made in deriving equations (4.1) to (4.9) (Ref. 9):

1) there are no mass or energy losses from the system;

2) the gas is inviscid except for its interactions with the condensed particles;

3) the volume occupied by the condensed particles is negligible;

4) the thermal (Brownian) motion of the particles does not contribute to the pressure of the system;

5) the condensed particles do not interact directly;

6) the drag and heat transfer characteristics of an actual shape and size distribution of particles can be represented by spherical particles of a single size;

7) the internal temperature of the particles is uniform;

8) energy exchange between the gas and the particles is controlled only by convection;

9) the only forces on the particles are viscous drag forces;

10) there is no mass transfer between the gas and the condensed phase during the nozzle expansion;

11) the particles do not undergo a phase change in the region of the calculation;

12) the gas phase is thermally and calorically perfect and the flow is chemically frozen.

The possibility of eliminating some of these limitations is discussed in Section 6.

4.3 Method of solution

An initial flow field is computed from the TEPGAS program using a modified molecular weight (see equation (2.1)) and a modified isentropic exponent (see equation (2.2)). The gas and particle velocities and temperatures are then equated to the TEPGAS values, and the gas and particle densities are determined from knowledge of the mixture density and particle loading. It is assumed that the static pressure field and the locations of the gas streamlines remain constant at the TEPGAS values, and that the gas velocity and density gradients do not vary from their initial values.

The partial differential equations (4.3) to (4.9), which are quasi-linear and of first order, are rewritten in finite difference form as linear algebraic equations, and used to derive a set of algebraic replacement equations in which a dependent variable is isolated on the left hand side of the equation and its value determined numerically from the right.

The equations are then solved iteratively by performing the following sequence of operations.

1) By rewriting equations (4.7) and (4.8), the particle velocities are calculated from

$$u_p = \frac{F\left[u_g\,(F + v_{p_r}) - v_g\,u_{p_r}\right]}{\left[(F + u_{p_x})(F + v_{p_r}) - u_{p_r}\,v_{p_x}\right]}$$

$$v_p = \frac{F\left[v_g\,(F + u_{p_x}) - u_g\,v_{p_x}\right]}{\left[(F + u_{p_x})(F + v_{p_r}) - u_{p_r}\,v_{p_x}\right]} .$$

The quantities on the right of the equations are evaluated from the initial flow field values.

2) The particle trajectories are integrated through the particle velocity field via the equation

$$\tan \theta_p = v_p/u_p$$

and the new particle streamlines determined. The outermost particle streamline constitutes a boundary between the two-phase and particle-free regions. Interpolation is carried out to find the particle properties at the new particle grid points.

3) The particle density is calculated from equation (4.2) to ensure continuity.

4) The gas properties at the particle grid points are found by interpolation and the particle temperature is calculated from equation (4.9).

5) Steps 1 to 4 are repeated.

6) The gas velocity components and the gas temperature and density at the gas phase coordinate system grid points are calculated.

7) The gas phase total pressure p_{g_o} and total temperature T_{g_o} are calculated for each gas streamline at its intersection with the limiting particle streamline. These quantities remain constant along the gas streamlines in the particle-free region, in which the gas is assumed to be strictly adiabatic, and this permits computation of the gas velocities, temperature and density from isentropic relationships.

8) This terminates one iteration of the relaxation process, and the solution is tested for convergence. Generally, absolute convergence is not possible, and a decrease in the change in all variables between successive iterations is considered equivalent to convergence. If this has not been reached, the gas properties are found at the particle coordinate system grid points and the sequence of operations is repeated.

5 RESULTS OF COMPUTATIONS

The equilibrium particle/gas mixture program (TEPGAS) and the non-equilibrium gas/particle program (TGPF) were run with the input data shown in Tables 1 and 2. The nozzle chosen (Fig. 1) has a relatively large normalised throat wall radius of curvature (>5), but even so the results from TEPGAS (Fig. 2 and Table 3) show significant variation in the flow variables across the nozzle, which becomes more pronounced as the calculation proceeds downstream. This two-dimensional effect produces the bent sonic line shown in Fig. 3, Curve I, which corresponds to a fairly large transonic region. When account is taken of the two-phase effects, the particle properties change very little from the equilibrium values (Table 4). There is a slight increase in the particle temperatures over the TEPGAS values, and the particle axial velocities are marginally lower. There is also very little change in the gas properties away from the nozzle wall, the gas temperatures falling slightly from the equilibrium values, and the axial velocities increasing slightly. However, there is a very marked change in the gas properties close to the nozzle wall. Fig. 3 compares the gas sonic line predicted by the TEPGAS program (Curve II), and that given after three iterations of TGPF (Curve III). The equilibrium assumption gives the sonic line location on the axis quite well, but provides a very poor estimate near the wall. This is because the particles depart from the gas streamlines, leaving a particle-free region adjacent to the wall. This departure is only slight close to the axis, but becomes more significant where the curvature of the gas streamlines is greater. Figs 4 and 5 show the effect of the isentropic nature of the gas expansion in the particle-free region, the gas temperature and axial velocity differing by up to 15 per cent from the values calculated with the equilibrium assumption.

The programs were also run for a range of nozzle shapes and particle sizes, and for different particle loadings. A decrease in the normalised throat wall radius of curvature of the nozzle leads to increased variation in the properties across the nozzle, and results in a larger particle-free region close to the nozzle wall. The particle-free region is also extended if bigger particles are introduced (Fig. 6). With 1 μm diameter particles, the flow is very close to gas/particle equilibrium, but as the particle size is increased, the limiting particle streamline departs from the nozzle wall further upstream, changing the local stagnation conditions in the isentropic region, and resulting in the gas sonic velocity close to the wall being reached much sooner. This leads to greater differences between the gas properties predicted with the equilibrium assumption and those calculated taking into account the particle velocity and thermal lags (Fig. 7). Changing the particle-to-gas mass ratio alters the position of the gas sonic line. If the number of particles is reduced the gas sonic line moves closer to the mixture sonic line, but if it is increased the gas sonic line, in the two-phase region, is displaced further downstream, thus extending the transonic region.

6 CONCLUSIONS

Two computer programs to solve the two-dimensional, two-phase flow equations in the transonic throat region of a rocket exhaust nozzle have been developed. The first treats the gas/particle mixture as a heavy perfect gas with modified isentropic exponent and molecular weight, and solves the equations of isentropic transonic flow through an axisymmetric nozzle. An initial flow field estimate is obtained for input to the second program, which incorporates the non-equilibrium effects of the two-phase fluid. The equations governing the transonic, two-dimensional flow of a gas/particle mixture are expressed as finite difference replacement equations and are solved by a numerical relaxation technique. The method is limited by the use of finite difference approximations, and by the assumptions listed in Section 4.2. Future work will be concerned with lifting some of these restrictions. The assumptions that the particles do not undergo a phase change and are of a single size will be studied first as these are of some importance and may be dealt with relatively easily. The next, and most difficult, step will be to remove the restriction of a frozen gas composition. Equilibrium chemistry (infinite reaction rates) will be assumed in the transonic throat region of the nozzle. When account has been taken of the chemical reactions, the possibility of allowing for particle interactions, and for the transfer of mass between the gas and the condensed phase, will be considered.

The programs have been used in exemplary calculations of the flow through an axisymmetric nozzle for a variety of wall shapes, particle sizes and particle-to-gas mass ratios. The results show that if accurate initial line data for the start of a supersonic calculation are to be obtained, the radial variations, and the non-equilibrium effects of all but the smallest particles (<1 μm), must be included. When the modifications discussed above have been implemented, the final stage of the work will be undertaken. This is to write a computer program to solve the two-phase flow equations in the supersonic region of the nozzle, taking into account the finite, non-zero rates of chemical reactions.

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APPENDIX A

Approximate solution of isentropic transonic flow by series solution method

This method, developed by Kliegel and Quan[10], depends upon the nozzle wall shape being expressed in terms of R, the nozzle throat wall radius of curvature, normalised with respect of the nozzle throat radius r_t. Thus, an R is chosen such that the nozzle wall may be approximated by the hyperbola

$$y^2 = 1 + z^2$$

where $z = \sqrt{\frac{1}{R}}\,\frac{x}{r_t}$ and $y = \frac{r}{r_t}$.

The axial and radial velocity components, u and v, may be expressed in inverse powers of R by the expansions

$$u = u_o(y,z) + \frac{u_1(y,z)}{R} + \frac{u_2(y,z)}{R^2} + \ldots\ldots\ldots$$

and

$$v = \sqrt{\frac{1}{R}}\,v_o(y,z) + \frac{v_1(y,z)}{R} + \frac{v_2(y,z)}{R^2} + \ldots\ldots\ldots$$

It may be shown that

$$u_o(y,z) = a_o(z)$$

$$v_o(y,z) = a_1(z).y$$

where

$$a_1 = \frac{a_o}{y_w}\,\frac{dy_w}{dz}$$

and

$$(1 - a_o^2)\frac{da_o}{dz} + 2\left(1 - \frac{\gamma - 1}{\gamma + 1}a_o^2\right)\frac{a_o}{y_w}\,\frac{dy_w}{dz} = 0$$

with boundary conditions

$$a_o\ (0) = 1$$

$$a_1\ (0) = 0 \quad ,$$

since at the throat

$$\frac{dy_w}{dz} = 0 \quad .$$

Similarly, the first order velocity components are defined by

$$u_1\ (y,z) = b_o\ (z) + b_2\ (z)\ y^2$$

$$v_1\ (y,z) = b_1\ (z)\ y + b_3\ (z)\ y^3$$

where $b_o - b_3$ are found by solving four ordinary differential equations similar to those relating a_o and a_1 . These equations are algebraic at the throat and can be solved directly to find the throat boundary conditions. The second order velocity components are defined by

$$u_2\ (y,z) = C_o\ (z) + C_2\ (z)\ y^2 + C_4\ (z)\ y^4$$

$$v_2\ (y,z) = C_1\ (z)\ y + C_3\ (z)\ y^3 + C_5\ (z)\ y^5$$

where $C_o - C_5$ are related by six ordinary differential equations which can also be solved algebraically to determine the throat boundary conditions.

For a nozzle whose shape may be approximated by a hyperbola with a normalised radius of curvature greater than three, the second order series solution yields a very good first approximation. However, for nozzles of sharp wall curvature the estimates are not so reasonable, particularly if R is less than two. For a normalised throat wall radius of curvature of less than one, the second order solution predicts that the throat axis velocity is supersonic, which is physically impossible. The third and higher order equations may be obtained, but the mathematics involved becomes extremely cumbersome. Thus, for nozzles where the second order solution is not feasible, a one-dimensional approximation is used.

APPENDIX B

Determination of the mass flux

The mass flux $\dot{m}$ is calculated at the throat in each iteration by choosing the mid-radial static pressure which maximises the mass flux through the throat. This is equivalent, in one-dimensional flow, to finding the point where the Mach number becomes unity.

At the throat

$$\dot{m} = 2\pi \int_o^1 (r\,\rho\,u)_{x\ \mathrm{const}}\, dr \quad . \qquad \text{(B.1)}$$

By writing

$$\rho = \rho_o\,(P/P_o)^{1/\gamma} = \rho_o\, z^{1/\gamma} \quad \text{where} \quad z = P/P_o$$

and

$$u = Mc\cos\theta$$

$$= \sqrt{\frac{2\gamma\,R\,T_o}{\gamma - 1}}\cos\theta \left[1 - z^{\frac{\gamma-1}{\gamma}}\right]^{1/2}$$

equation (B.1) may be converted into

$$\dot{m} = 2\pi K \int_o^1 \left\{ r\cos\theta\, z^{1/\gamma} \left(1 - z^{\frac{\gamma-1}{\gamma}}\right)^{1/2} \right\} dr \ ,$$

where

$$K = \sqrt{\frac{2\gamma\,\rho_o\,P_o}{\gamma - 1}} \ .$$

For $\dot{m}$ to be a maximum, z is such that

$$\frac{d\dot{m}}{dz} = 2\pi K \int_o^1 \left\{ \frac{r\cos\theta}{2} \left[z^{2/\gamma} - z^{\frac{\gamma+1}{\gamma}}\right]^{-1/2} \left[2/\gamma\, z^{2/\gamma - 1} - \frac{\gamma+1}{\gamma}\, z^{1/\gamma}\right] \right\} dr = 0 \quad .$$

When the required mass flux has been found, the other axial stations are solved by determining the mid-radial pressure which satisfies

$$f(z) = 2\pi K \int_0^{r_w} \left\{ r \cos\theta \left[z^{2/\gamma} - z^{\frac{\gamma+1}{\gamma}} \right]^{1/2} \right\} dr - \dot{m} = 0 \quad . \quad (B.2)$$

Since equation (B.2) has a turning point at the throat, at axial stations upstream and downstream of the throat two possible solutions exist. These correspond to the subsonic and supersonic solutions of one-dimensional flow, although two-dimensional flow can be mixed in the radial direction. The problem of which solution to choose arises only at stations downstream of the throat, since upstream the equivalent of the one-dimensional subsonic solution occurs. Physically there is continuous change in the mid-stream Mach number for isentropic flows which serves as a basis for choosing the solution downstream of the throat.

APPENDIX C

Particle drag and heat transfer factors

Definitions

The empirical relationships used to calculate the particle drag and heat transfer factors are the same as those used by the TRW program[7] and by Regan[9]. They are

$$f_p = K_D (1 + 2.52\,\overline{C})/(1 + 3.78\,\overline{C})$$

and

$$g_p = K_D/(1 + 3.42\,K_D\,\overline{C}/\gamma^{1/2}\,Pr)$$

where

$$\overline{C} = \mu_g/\rho_g\,r_p\,(R\,T_g)^{1/2}$$

and

$$K_D = C_D/(C_D)_{Stokes}$$

where C_D is the particle drag coefficient.

The gas viscosity is a function of temperature

$$\mu_g = \mu_{g_o}\,(T_g/T_{g_o})^{0.67}$$

and the Prandtl number is computed from Euken's relationship

$$Pr = 4\gamma/(9\gamma - 5) \quad .$$

Drag coefficient

Because the condensed particles are assumed to be spherical the drag coefficient for a sphere is used to estimate the particle drag coefficient. Experimental work to establish the magnitude of the drag forces on spheres has been going on since Newton began his experiments in 1710, but until recently there were no experimental data to give the drag coefficient of a sphere in the flow regime of interest. In 1850 Stokes solved the Navier-Stokes equation by neglecting the inertial terms ($Re \rightarrow 0$) and obtained the simple drag law

$$(C_D)_{Stokes} = 24 / Re \tag{C.1}$$

where Re is the particle Reynolds number based on the speed of the particle relative to the gas and is defined by

$$Re = 2 \, r_p \, (W_g - W_p) \, \rho_g / \mu_g \quad . \tag{C.2}$$

However, it has been shown[4] that the Stokes drag law has only limited applicability to gas/particle nozzle flows since it is restricted to continuum, incompressible flow and particle Reynolds numbers less than 1. The flow regimes encountered by the micron-sized particles extend from continuum to free-molecule flow, and particle Reynolds numbers of up to 100 and particle Mach numbers exceeding 1. It was therefore necessary to derive expressions which agreed with the available data, conformed with theoretically predicted trends, and provided reasonably smooth variations with Mach and Reynolds numbers.

More recently, drag coefficient data have become available, and Crowe[13] compares the equations devised by Kliegel[14], Carlson[11] and Crowe[12] with an empirical expression which he developed to fit the new data. He concludes that the differences between nozzle performance predictions made with each devised drag law, and those made using the law validated by experiment, are negligible. Thus, since the choice of drag law has little effect on the predicted results, it was decided to use the relationship derived by Carlson[11] as this is the simplest expression. It is based on the Stokes drag law, equation (C.1), and includes terms to correct for rarefaction, inertial effects and compressibility effects:

$$C_D = \frac{24}{Re} \left[\frac{(1 + 0.15 \, Re^{0.687}) \left[1 + \exp(-0.427/M^{4.63} - 3/Re^{0.88}) \right]}{1 + (M/Re) \left[3.82 + 1.28 \exp(-1.25 \, Re/M) \right]} \right]$$

where Re is the particle Reynolds number and M is the particle Mach number defined by

$$M = (W_g - W_p) \Big/ \sqrt{\gamma R T_g} \; .$$

Table 1

Input data for TEPGAS*	
Throat radius, r_t	0.4328 cm
Radius of curvature of circular arc section, R	2.54 cm
Distance from nozzle throat to start of conic section	0.6574 cm
Angle of conic section	15^o
Modified isentropic exponent, $\overline{\gamma}$	1.157
Chamber pressure, P_o	3.507 E 06 Nm^{-2}
Chamber temperature, T_o	2472 K
Modified gas constant, $R/\bar{m}$	239.0 $Nm\ K^{-1}\ kg^{-1}$
No. of streamlines	16
No. of axial stations	40
Axial step length	0.04 cm

Table 2

Input data for TGPF*	
Gas isentropic exponent, γ_g	1.232
Gas reference viscosity, μ_{g_o}	8.0 E-05 $kg\ m^{-1}\ s^{-1}$
Gas reference temperature, T_{g_o}	2500 K
Gas specific heat, Cp_g	1500 $J\ kg^{-1}\ K^{-1}$
Particle radius, r_p	1.0 E-06 m
Particle density, m_p	3.97 E 03 $kg\ m^{-3}$
Particle specific heat, Cp_p	1355 $J\ kg^{-1}\ K^{-1}$
Particle-to-gas mass flow ratio, ϕ	3/7

*1.0 E 06 = 1.0×10^6

Table 3

Sample of results from TEPGAS (after 9 iterations)						
Streamline number	1	16	1	16	1	16
x/r_t	0.0	0.0	0.8318	0.8318	1.6636	1.6636
r/r_t	0.0	1.0	0.0	1.05924	0.0	1.23873
u	7.666 E 02	8.280 E 02	1.009 E 03	1.102 E 03	1.235 E 03	1.327 E 03
v	0.0	0.0	0.0	1.577 E 02	0.0	3.555 E 02
M	9.600 E-01	1.043 E 00	1.298 E 00	1.453 E 00	1.644 E 00	1.877 E 02
P	2.096 E 06	1.917 E 06	1.404 E 06	1.133 E 06	8.490 E 05	5.801 E 05
T	2.306 E 03	2.278 E 03	2.184 E 03	2.121 E 03	2.040 E 03	1.937 E 03
ρ	3.803 E 00	3.521 E 00	2.690 E 00	2.234 E 00	1.742 E 00	1.253 E 00

Table 4

Sample of results from TGPF (after 3 iterations)

Streamline number	1	16	1	16	1	16
x/r_t	0.0	0.0	0.8318	0.8318	1.6636	1.6636
r_g/r_t	0.0	1.0	0.0	1.05924	0.0	1.23873
u_g	7.666 E 02	9.213 E 02	1.009 E 03	1.257 E 03	1.235 E 03	1.522 E 03
v_g	0.0	0.0	0.0	1.800 E 02	0.0	4.079 E 02
M_g	7.786 E-01	9.499 E-01	1.053 E 00	1.376 E 00	1.334 E 00	1.818 E 00
P_g	2.096 E 06	1.917 E 06	1.404 E 06	1.133 E 06	8.490 E 05	5.801 E 05
T_g	2.305 E 03	2.236 E 03	2.182 E 03	2.025 E 03	2.037 E 03	1.785 E 03
ρ_g	2.663 E 00	2.510 E 00	1.885 E 00	1.638 E 00	1.220 E 00	9.515 E 01
r_p/r_t	0.0	0.998867	0.0	1.05618	0.0	1.23227
u_p	7.640 E 02	8.244 E 02	1.005 E 03	1.096 E 03	1.230 E 03	1.320 E 03
v_p	0.0	-1.440 E 00	0.0	1.534 E 02	0.0	3.532 E 02
T_p	2.307 E 03	2.279 E 03	2.185 E 03	2.124 E 03	2.042 E 03	1.941 E 03
ρ_p	1.145 E 00	1.064 E 00	8.101 E-01	6.789 E-01	5.243 E-01	3.814 E-01

Nomenclature

c	speed of sound
C_D	particle drag coefficient
C_p	specific heat at constant pressure
F, f_p	particle drag factors
G, g_p	particle heat transfer factors
K_D	$C_D/(C_D)_{Stokes}$
m	molecular weight
$\overline{m}$	molecular weight of gas particle mixture
m_p	density per unit volume of particle
$\dot{m}$	mass flow rate
M	gas Mach number
p	pressure
Pr	Prandtl number
r	radial coordinate measured from nozzle axis
r_p	particle radius
R	gas constant; nozzle throat wall radius of curvature, normalised with respect to throat radius r_t
Re	Reynolds number
T	temperature
u	axial velocity component
v	radial velocity component
W	speed
x	axial coordinate measured from nozzle throat
γ	isentropic exponent
$\overline{\gamma}$	isentropic exponent of gas/particle mixture
θ	streamline angle with respect to nozzle axis
μ	viscosity coefficient
ρ_g	gas density
ρ_p	particle density in the gas (based on the gas volume)
ψ	stream function
ϕ	particle-to-gas mass flow rate

Subscripts

g	gas property
o	total condition
p	particle property
w	nozzle wall condition
t	throat condition

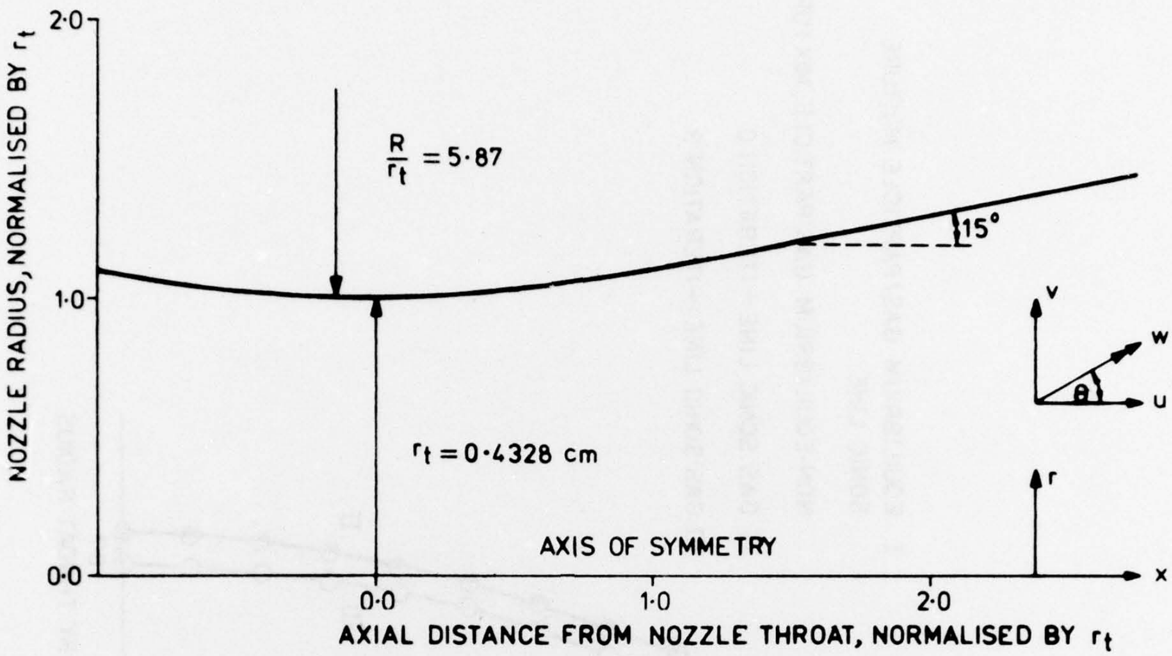

FIG. 1 NOZZLE GEOMETRY.

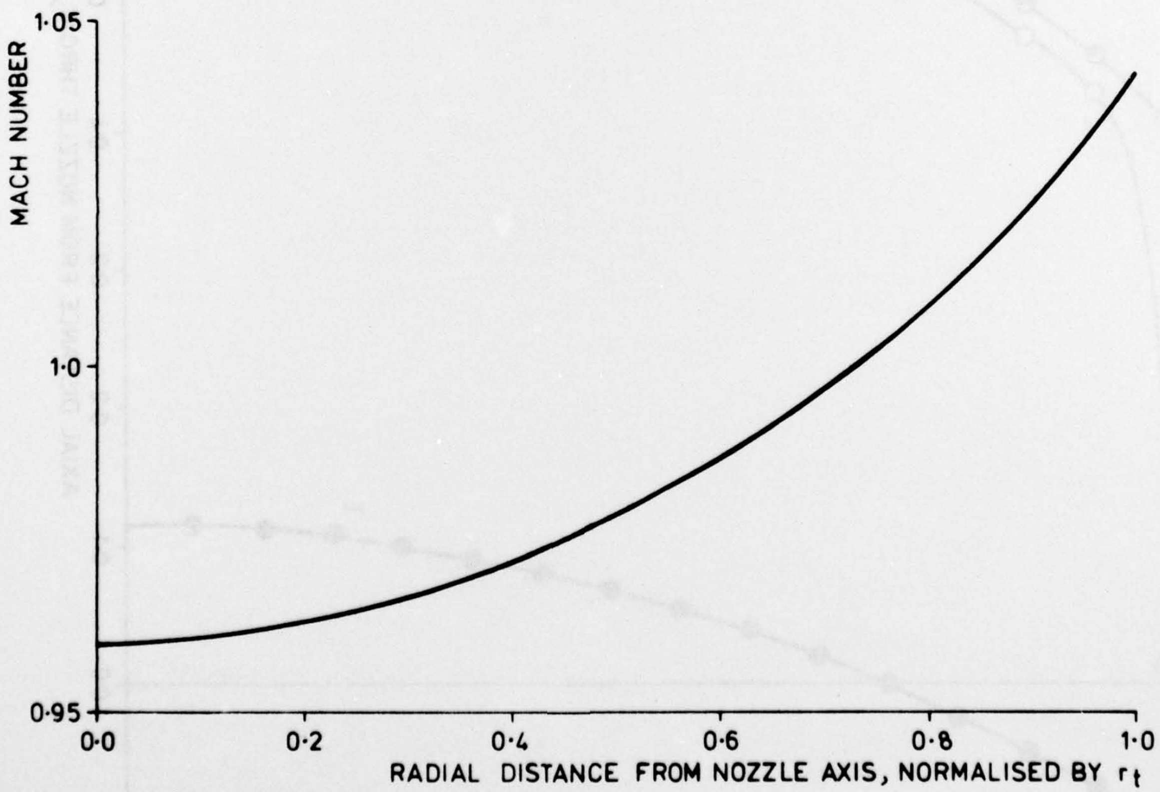

FIG. 2 EQUILIBRIUM GAS/PARTICLE MIXTURE. PLOT OF MACH NUMBER AT THE THROAT.

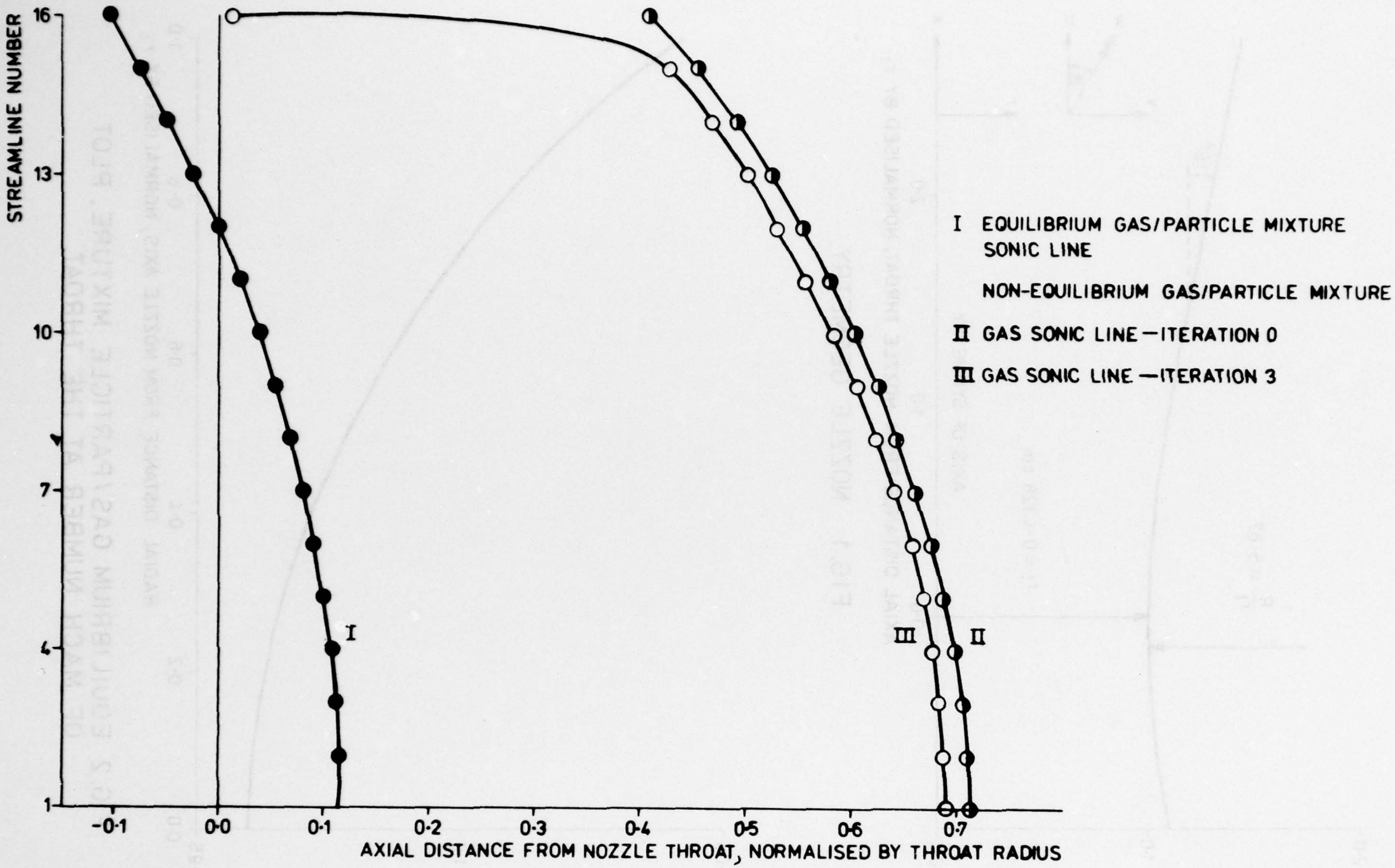

FIG. 3 COMPARISON OF THE SONIC LINE LOCATION FOR 30% PARTICLE LOADING, OF 2μm DIAMETER.

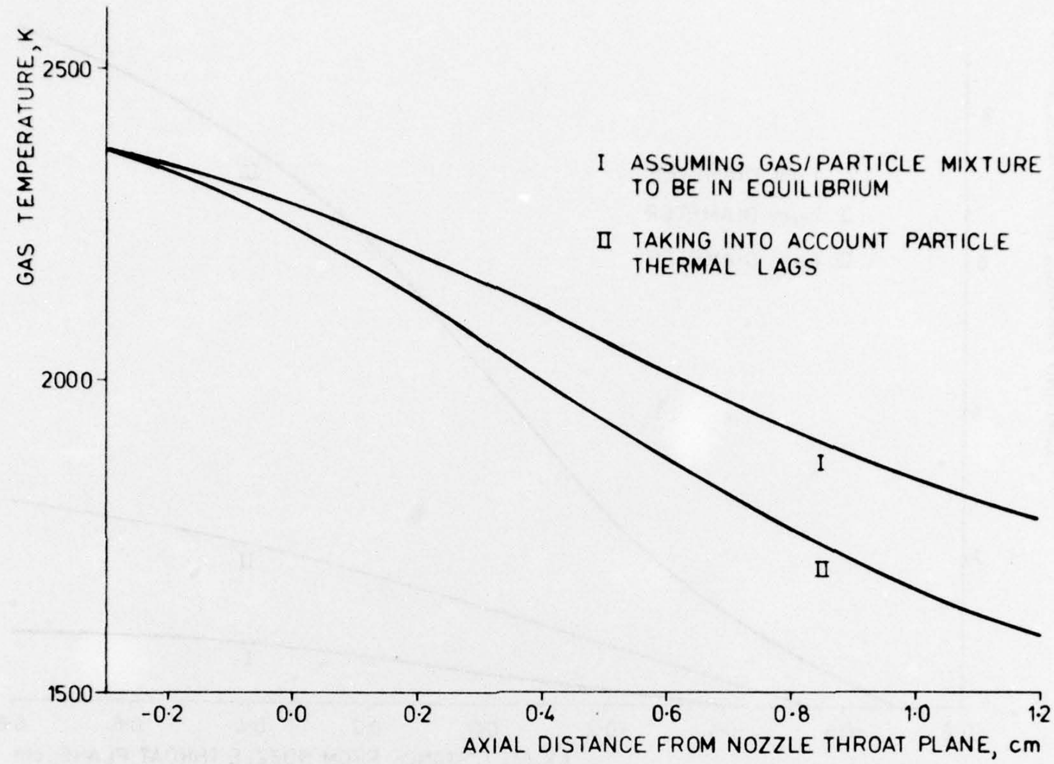

FIG. 4 COMPARISON OF GAS TEMPERATURES ALONG NOZZLE WALL FOR 2μm PARTICLES.

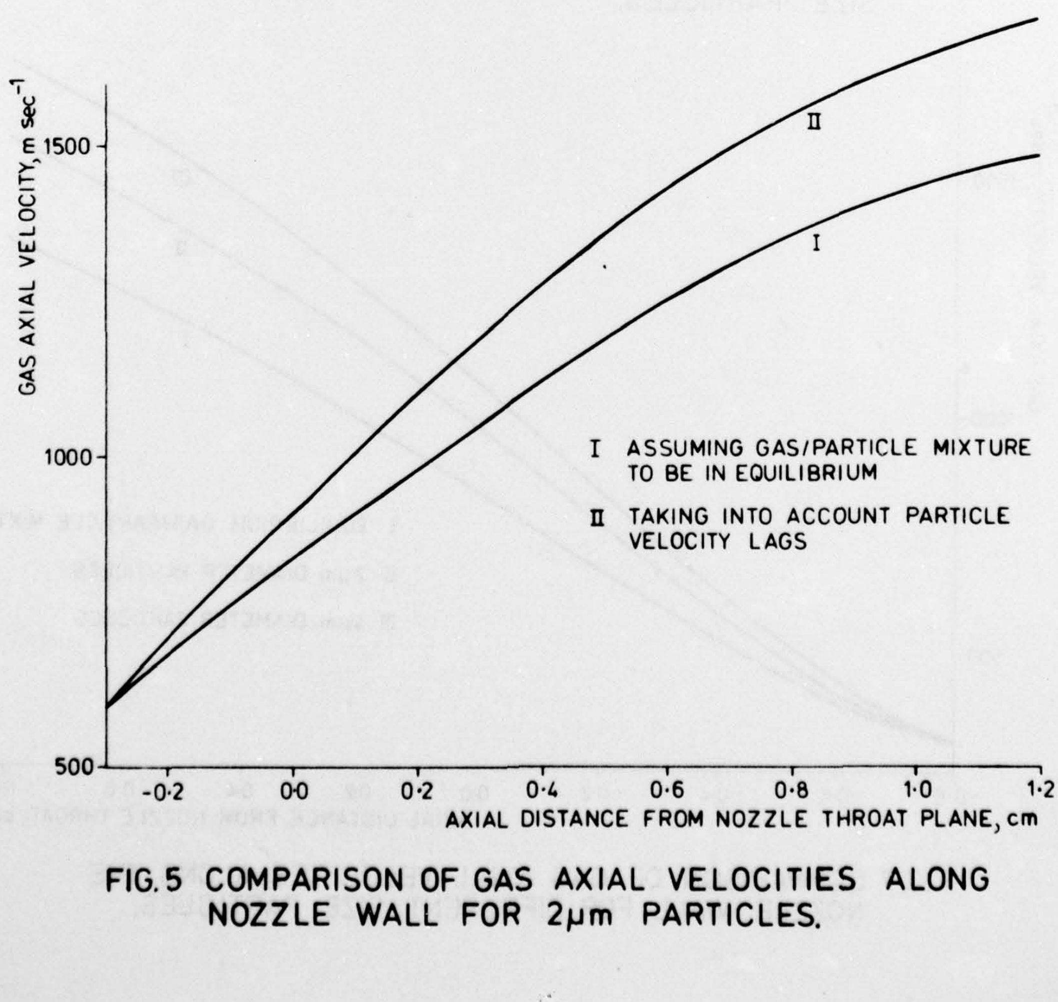

FIG. 5 COMPARISON OF GAS AXIAL VELOCITIES ALONG NOZZLE WALL FOR 2μm PARTICLES.

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FIG. 6 & 7

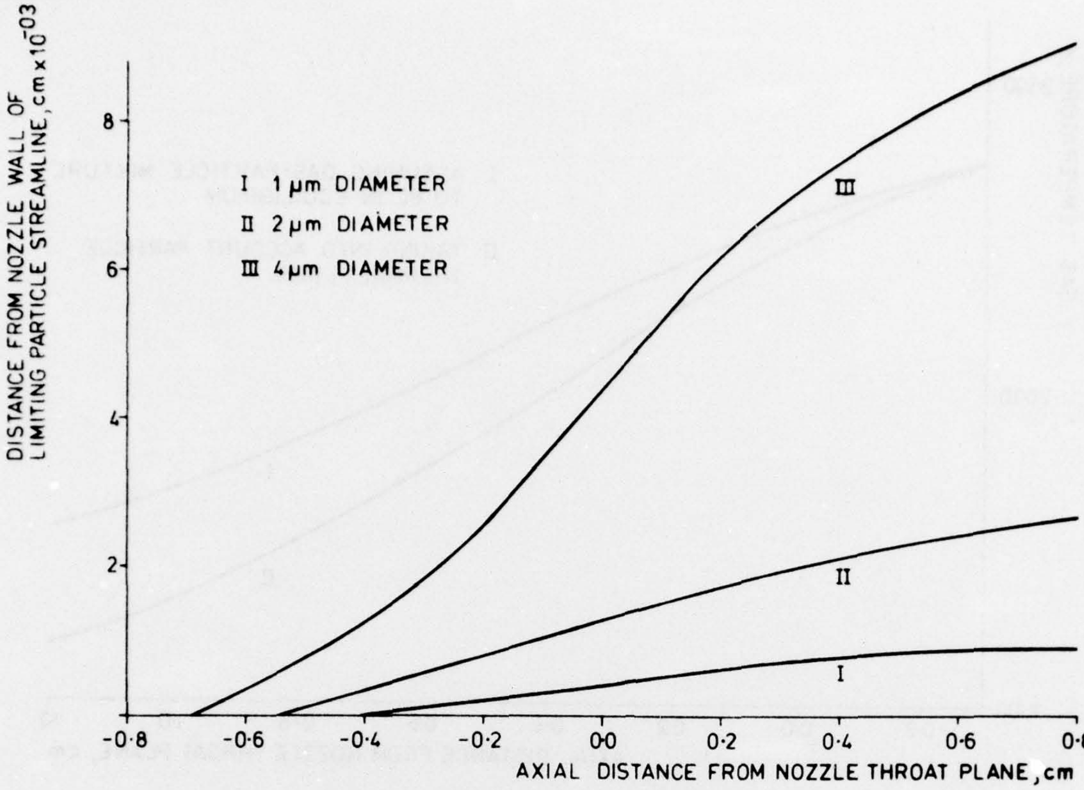

FIG. 6 DISPLACEMENT FROM THE NOZZLE WALL OF THE LIMITING PARTICLE STREAMLINE, FOR DIFFERENT SIZE PARTICLES.

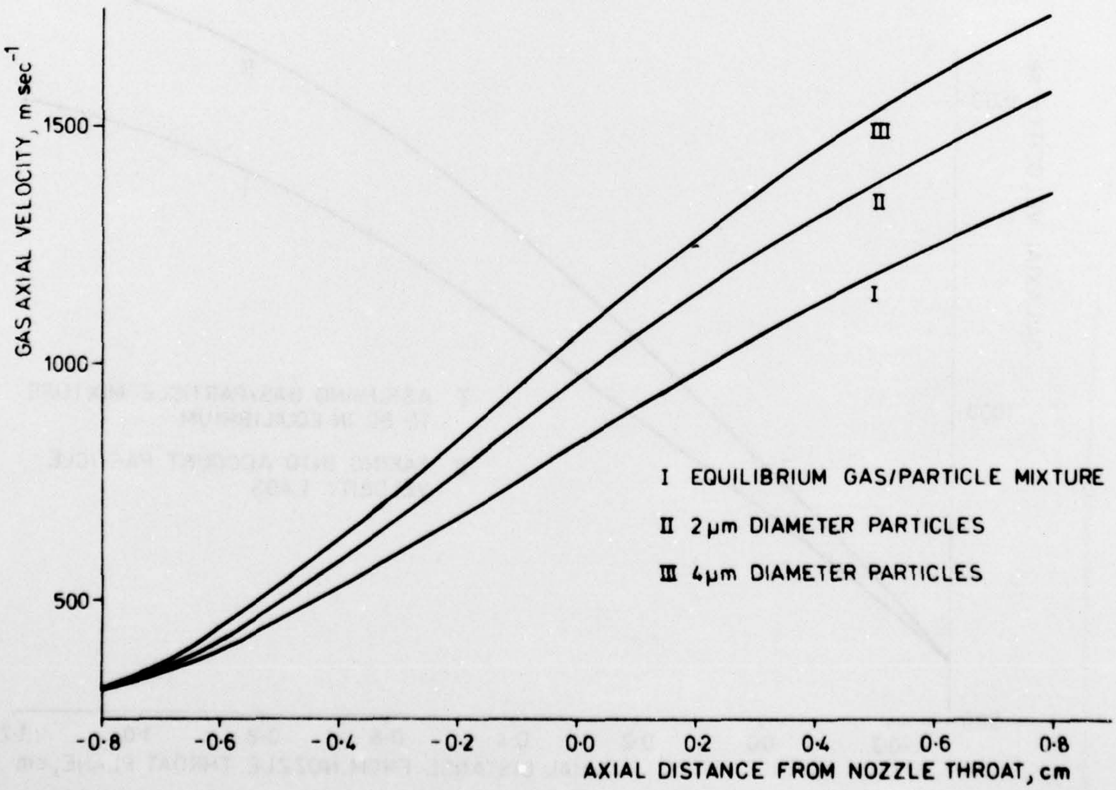

FIG. 7 COMPARISON OF GAS AXIAL VELOCITIES ALONG THE NOZZLE WALL FOR DIFFERENT SIZE PARTICLES.

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7. Title

TWO-DIMENSIONAL, TWO-PHASE FLOW THROUGH A ROCKET EXHAUST NOZZLE: A PROGRESS REPORT

7a. Title in Foreign Language (in the case of translations)

7b. Presented at (for conference papers). Title, place and date of conference

8. Author 1. Surname, initials	9a Author 2	9b Authors 3, 4...	10. Date	pp	ref
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Abstract This memorandum describes progress in the development of a set of computer programs which solve the equations of gas/particle flow through an axisymmetric nozzle. Two programs have been written to solve the equations in the transonic throat region of the nozzle. The first treats the two-phase fluid as a heavy perfect gas with modified isentropic exponent and molecular weight, and solves the equations of isentropic transonic flow. An initial flow field configuration is obtained for input to the second program, which incorporates non-equilibrium effects of the gas/particle mixture.

The programs provide accurate initial line data for starting a supersonic calculation, and will be used as input to a third program, soon to be written, for solving the two-phase flow equations in the supersonic region of the nozzle.