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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This work is concerned with a new method for testing material properties under uniform plane stress conditions by means of specially designed plane specimen. Photoelastic analysis showed that in the significant section of the specimen it is possible to produce uniform plane stress with high accuracy subject to the limitation that the principal stresses are of different signs. An important special case of loading produces shear on the significant section. The method is of particular importance for fiber composite testing. Preliminary experimental results presented are encouraging.

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by

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This work is concerned with a new method for testing material properties under uniform plane stress conditions by means of specially designed plane specimen. Photoelastic analysis showed that in the significant section of the specimen it is possible to produce uniform plane stress with high accuracy subject to the limitation that the principal stresses are of different signs. An important special case of loading produces pure shear on the significant section. The method is of particular importance for fiber composite testing.

Preliminary experimental results presented are encouraging.

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* Associate Professor

** Professor and Chairman

*** Graduate Assistant

1. Introduction

One of the important problems in testing materials is to devise specimens and loading devices to produce states of uniform plane stress. The problem has assumed increased significance with the advent of composite materials.

The main reason for this is that fiber composites are frequently in the form of laminates in which the laminae are unidirectionally reinforced. In most engineering applications the laminae are in states of plane stress. It is therefore necessary to test single laminae in plane stress in order to obtain mechanical properties and failure criteria.

It appears that the best method for such tests is to use thin-walled filament wound circular cylindrical specimens which are subjected simultaneously to axial and torsional loading [1]. Desired orientations of fibers are produced by helical winding; but the severe disadvantage of this method is the high price of such specimens.

A simple method to produce a class of plane stress loadings is off-axis specimens, that is: plane reinforced specimens in which the fibers are oriented at some desired angle α with respect to the load [2]. In such a specimen there is produced a plane state of stress

$$\begin{aligned} \sigma_A &= \sigma\cos^2\alpha \\ \sigma_T &= \sigma\sin^2\alpha \qquad\qquad (1.1) \\ \tau &= \sigma\sin\alpha\cos\alpha, \end{aligned}$$

where A and T denote fiber and transverse to fiber directions, respectively and τ is shear stress. Proper care must be taken to eliminate edge effects. Limitations of the stress system (1.1) are that the normal stresses always have the same sign and pure shear cannot be produced.

A conventional method for shear testing is the "rail shear method" [3] whose disadvantages are as follows:

- The grips strongly influence the state of stress.
- The exterior forces produce a moment on the sample in addition to the shear force. Therefore the state of stress produced is shear <u>and</u> normal stress and not pure shear.
- There is no significant section.
- The isostatics are not inclined at 45° to the shear section at the free ends of the "rail-shear" sample. (The state of stress is non-homogeneous).
- The uniform part of the shear diagram is spread only over 75% of the section length.

Some recent works in which the "rail-shear test" was used for composite materials [4,5] illustrate the disadvantages listed above.

Another method for shear testing is the picture frame system [6], where the sample has no significant section, the state of stress is not homogeneous, and only a rather small central part of the sample is subjected to pure shear.

A recent method for shear testing uses the fact that in a ±45° fiber composite laminate uniaxially loaded in 0° direction the lamina longitudinal shear stress in fiber direction is statically determinate and equal to half of the externally applied uniaxial stress [7]. It should, however, be noted that the lamina state of stress is not pure shear, normal stresses in fiber direction and transverse to fiber being also present. These normal stresses are statically indeterminate, that is they are dependent on the material properties. Consequently, this method's primary importance is to determine the elastic shear modulus. It's use for shear strength and inelastic shear behaviour is limited because of interaction effects of shear with normal stresses.

The present work is concerned with a new method of plane stress testing in which it is possible to produce pure shear and normal principal stresses which are of different signs and arbitrary ratio. Part of the work has been presented in [8].

2. The Specimen

A picture of the specimen used is shown in Fig. 1. The specimen is plane circular with antisymmetric cutouts. The significant section of the specimen is AB and the specimen must be designed in such a manner that the state of stress on AB shall be as uniform as possible. The specimen is loaded by equal and opposite compression or tension (Fig. 1a). If the stresses on AB are uniform, it follows from equilibrium that on AB

$$\sigma_{xx} = \frac{P_\alpha}{S} \sin\alpha$$
$$\sigma_{xy} = \frac{P_\alpha}{S} \cos\alpha \qquad (2.1)$$

where S - the area of the section AB.

The rectilinear portions of the cutouts are oriented at ±45° and therefore the principal stresses in the vicinity are also in these direction. It will be shown below, on the basis of photoelastic analysis, that the principal stresses throughout AB are oriented at ±45°. It follows that σ_{xy} on AB as given by (2.1) is a principal shear stress. Therefore on AB

$$\sigma_{xx} = \sigma_{yy} = \frac{P_\alpha}{S} \sin\alpha \qquad (2.2)$$

and the principal stresses are

$$\sigma_1 = \sigma_{xx} + \sigma_{xy} = \frac{P_\alpha}{S} (\sin\alpha + \cos\alpha)$$
$$\sigma_2 = \sigma_{xx} - \sigma_{xy} = \frac{P_\alpha}{S} (\sin\alpha - \cos\alpha) \qquad (2.3)$$

It follows from (2.3) that the ratio between the principal stresses is given by

$$\frac{\sigma_1}{\sigma_2} = K_1 = \frac{\tan\alpha + 1}{\tan\alpha - 1} \qquad (2.4)$$

For angles α in the range 0° - ±90° this ratio assumes all values between $-\infty$ and $+\infty$, but the specimen is not suitable for the angles whose absolute value is larger than 45° for in that case the desired uniform plane stress on AB is not achieved and stress concentrations at A and B become significant. Therefore the angular range of loading will be limited to

$$|\alpha| \leq 45°$$

and consequently the ratio (2.4) is negative.

When $\alpha = 0$ a state of pure shear is produced:

$$\sigma_{xy} = \frac{P}{S}$$

$$\sigma_{xx} = \sigma_{yy} = 0 \qquad (2.5)$$

The specimen geometry was designed so as to approach uniform stresses along AB as closely as possible. Uniformity of the stresses was investigated by photoelastic methods by means of specimens made of Polycarbonate (for isochromatics) and acrylic resin (for isoclinics).

For a better understanding of this circular sample (its geometry and its mechanical behaviour) one may consider it to consist of two parts:

- The central part (between the dashed lines) called afterwards the "significant sample" with the significant section AB.
- The two exterior parts transmitting the load to the central part, functioning as grips and directing the isostatics into ±45° directions.

Significant geometrical parameters are d_1, d_2, d_3 and radii r_1,r_2(Fig. 2). Optimal values of these parameters were found to be governed by the ratios

$$\frac{d_1}{d_2} = \frac{d_1}{\sqrt{2}\, r_1} = 5$$

$$\frac{d_1}{d_3} = \frac{d_1}{2r_2} = \frac{1}{2}$$

The dimensions used were

$d_1 = 10$ mm
$d_2 = 2$ mm
$d_3 = 20$ mm
$r_1 = 1.42$ mm
$r_2 = 10$ mm

The diameter of the entire specimen was chosen as

$$D = 10d_1 = 100 \text{ mm}$$

3. Analysis of the Specimen

Experimental analysis of the specimen has been performed by photoelastic and strain gauge methods for the cases of pure shear and general plane stress.

A. Beginning with the case of pure shear ($\alpha = 0°$ loading) one may analyse by aid of photoelastic models the state of stress and check if the conditions to obtain uniform pure shear on the whole significant section are fulfilled.

Loading of a PSM - 1* model with symmetric forces P_φ , produces the isochromatic pattern (Fig. 3a). Similar loading of a Perpex model produces the isoclinics. The first condition to ensure that there is pure shear stress along AB is fulfilled if σ_1 and σ_2 are ±45° oriented with respect to the section AB and this is clearly proved by the 45° isoclinic (Fig. 3b) along the whole section. A second condition is fulfilled if AB is on an isochromatic, which implies that the principal shear stress is uniform. The slight existing non-uniformity is insignificant as is illustrated by the diagram of σ_{AB}^{max} (Fig. 4). Strain gauge measurements subsequently described have shown that σ_1 and σ_2 in the significant section AB are uniform with high accuracy. Since σ_{xy} in the section has also experimentally shown to be uniform (see above) it follows that σ_{xx} and σ_{yy} on the section are uniform. Therefore the results (2.5) apply. It follows that σ_{xy} is a pure shear.

A diagram of the shear stresses in the significant section has also been

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* Photolastic Inc., 67 Lincoln HWY, Malvern, PA. 19355.

obtained by strain gauge measurements at 5 points (Fig. 4) on a 3 mm thick Aluminium 2024 - T3 sample. At these 5 points on both sides of the significant section, at ±45°, 10 KYOWA** strain gauges KFS - C1 - 23 of length 1 mm were applied, thus obtaining the principal strains ε_1 and ε_2. The resulting diagram of shear strain γ_{AB} exhibits two symmetrically located maximum values, thus explaining the slight non-uniformity of the isochromatics (Fig. 3). The average value of γ_{AB}

$$\gamma_{avr} = \frac{\int \gamma_{AB}\, d\ell}{\ell_{AB}} \tag{3.1}$$

may be obtained directly by just measuring the shear strain in the middle of AB (the error will be in the range of errors of measurements).

One may conclude, that the proposed circular pure shear sample indeed produces a homogeneous state of pure shear on the significant section AB, with sufficient accuracy.

B. The second possibility - to obtain an arbitrary ratio of σ_1/σ_2 (by P_α loading) - may be also examined by a photoelastic model. The state of stress in the significant section will be characterized for every angle α by one isochromatic and by 45° isoclinic along AB (Fig. 5). This implies that the principal stresses σ_1 and σ_2 are directed as in the previous case at ±45° with respect to the significant section and that

$$\sigma_1 - \sigma_2 = 2\tau_{max} \tag{3.2}$$

is constant along AB. But unlike the pure shear case (A) the loads P_α

** KYOWA Electronic Instruments Co., Ltd., 5 - 1, Chofugaoka, 3 - chome, Chofu - shi, Tokyo, Japan.

are not symmetrically oriented and so $\sigma_1 \neq \sigma_2$. The uniformity of state of stress along AB also implies uniformity of the ratio

$$\sigma_1/\sigma_2 = K_1 < 0$$

or of the ratio (3.3)

$$\sigma_{xx}/\sigma_{xy} = K_2$$

These ratios depend on the direction α of the forces P_α.

The measurements were aimed at finding out the necessary P_α for different values of α (-40° - +40°) in order to obtain an isochromatic of some specified order along AB. The sample was again made from PSM - 1, 3.2 mm thickness. Table 1 presents the measured values P_α as well as the computed coefficients K_1, K_2 and the $\sigma_1 - \sigma_2$ values; the average

$$(\sigma_1 - \sigma_2)_{avr} = 22.515 \text{ kg/cm}^2$$

is, as expected, the model stress fringe value σ_0.

The series of data presented in Table 1 demonstrate the possibility of obtaining on the significant section principal stresses with a wide range of negative ratio K_1. This is important for experiments characterizing the mechanical response of materials to biaxial states of stress. The usual method of tubular samples subjected to axial loading and internal pressure is complicated by the need of special testing machines and the expense and labor required to produce the tubular specimens. These difficulties are further augmented for the case of fiber composites.

TABLE I

MEASURED VALUES P_α AND COMPUTED PRINCIPAL STRESSES IN THE SIGNIFICANT SECTION AB

α	-40°	-30°	-20°	-10°	0	$+10^\circ$	$+20^\circ$	$+30^\circ$	$+40^\circ$
P_α Kg	4.90	4.30	4.00	3.80	3.70	3.80	4.00	4.30	4.90
σ_{xx} Kg/cm^2	-9.54	-6.52	-4.15	-2.00	0	2.00	4.15	6.52	9.54
σ_{xy} Kg/cm^2	11.37	11.29	11.39	11.34	11.21	11.34	11.39	11.29	11.37
$K_2 = \dfrac{\sigma_{xx}}{\sigma_{xy}}$	-0.84	-0.58	-0.36	-0.18	0	+0.18	+0.36	+0.58	+0.84
σ_1 Kg/cm^2	1.83	4.77	7.24	9.34	11.21	13.34	15.54	17.80	20.92
σ_2 Kg/cm^2	-20.92	-17.80	-15.54	-13.34	-11.21	-9.34	-7.24	-4.77	-1.83
$K_1 = \dfrac{\sigma_1}{\sigma_2}$	-0.087	-0.27	-0.47	-0.70	-1.00	-1.43	-2.14	-3.73	-11.40
$\sigma_1 - \sigma_2 = \sigma_c$ Kg/cm^2	22.81	22.57	22.78	22.68	22.42	22.68	22.78	22.57	22.81

4. Applications and Conclusions

Some preliminary experiments have been performed to test material properties with the aid of the specimen described.

The shear modulus G of Aluminium 2024 - T3 was determined from the specimen by measuring shear strain with rosettes and specifying the shear stress by (2.5). The same modulus G was also determined indirectly by measuring E and ν with aid of an aluminium coupon and calculating G by the usual formula. The results were

specimen	$G = 3.06 \times 10^5$ kg/cm^2
coupon	$G = 2.95 \times 10^5$ kg/cm^2

These results may be considered to be in good agreement.

Speciments of unidirectional Glass/Epoxy fiber composite with 52% fiber volume concentration were fabricated, the fibers being in the AB direction. The specimens were cut out of unidirectionally reinforced plates. Loading with $\alpha = 0$ determines the longitudinal shear modulus G_A which was again determined with strain gauges. The value found was

$$G_A = 3.88 \times 10^4 \text{ kg/cm}^2$$

A formula for this shear modulus derived by Hashin and Rosen [9] gave the value

$$G = 3.71 \times 10^4 \text{ kg/cm}^2$$

The specimen may be used to determine the transverse shear modulus G_T, a modulus which is notoriously difficult to measure. This will be reported elsewhere.

The possibility to produce combined uniform plane stress states is particulary important to test elastic, inelastic and failure parameters, especially for fiber reinforced materials. Shear failure is shown for a fiber reinforced specimen in Fig. 6. It is seen that failure occured on the significant section AB.

Limitations of the specimen are that σ_1 and σ_2 have opposite sign and that special grips are needed to prevent out of plane bending of the specimen which would modify the plane state of stress.

In conclusion, it is believed that the specimen described provides a relatively simple procedure to test mechanical properties in plane stress conditions. It is of particular importance for fiber composites.

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LIST OF CAPTIONS

1. The circular sample (a)

   The state of stress induced by forces P_o in the significant section AB $\sigma_1/\sigma_2 = -1$ or pure shear σ_{xy} (b)

   The state of stress induced by forces P_α, $\sigma_1/\sigma_2 = K_1$ (c).

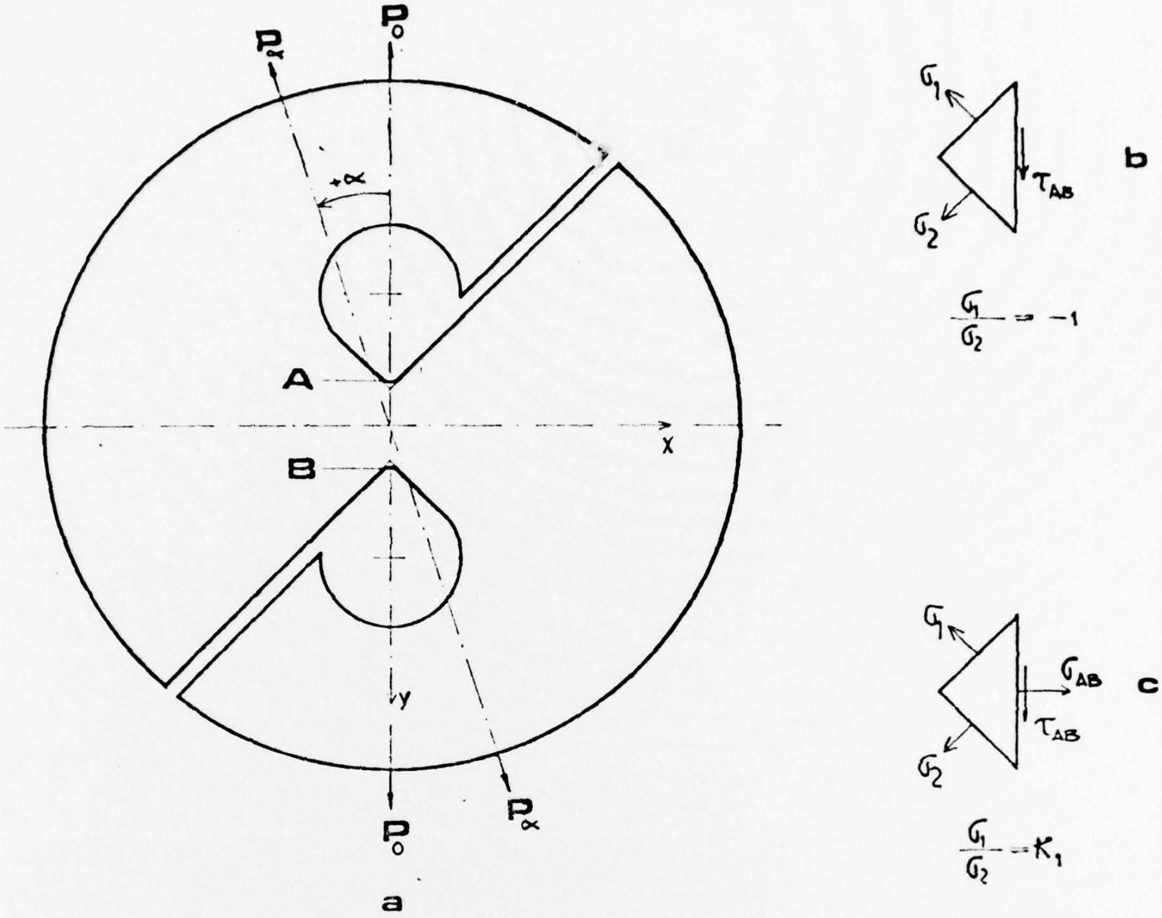
2. The two constitutive parts of the sample: the significant sample (between the dashed lines) and the exterior one with the zone governing isostatics (a); dimensions (b).

3. The circular sample loaded with forces P_o to obtain pure shear Isochromatic pattern (a); 45° Isoclinic (b).

4. Strain gauge measurements of shear stresses in the section AB. The 5 strain gauges at 45° on one face of the sample (a). The shear stress diagram (b).

5. The circular sample loaded with forces P_α to obtain σ_1/σ_2 arbitrary. Isochromatic pattern for $\alpha = 22°30'$ (a); 45° Isoclinic for $\alpha = 22°30'$ (b).

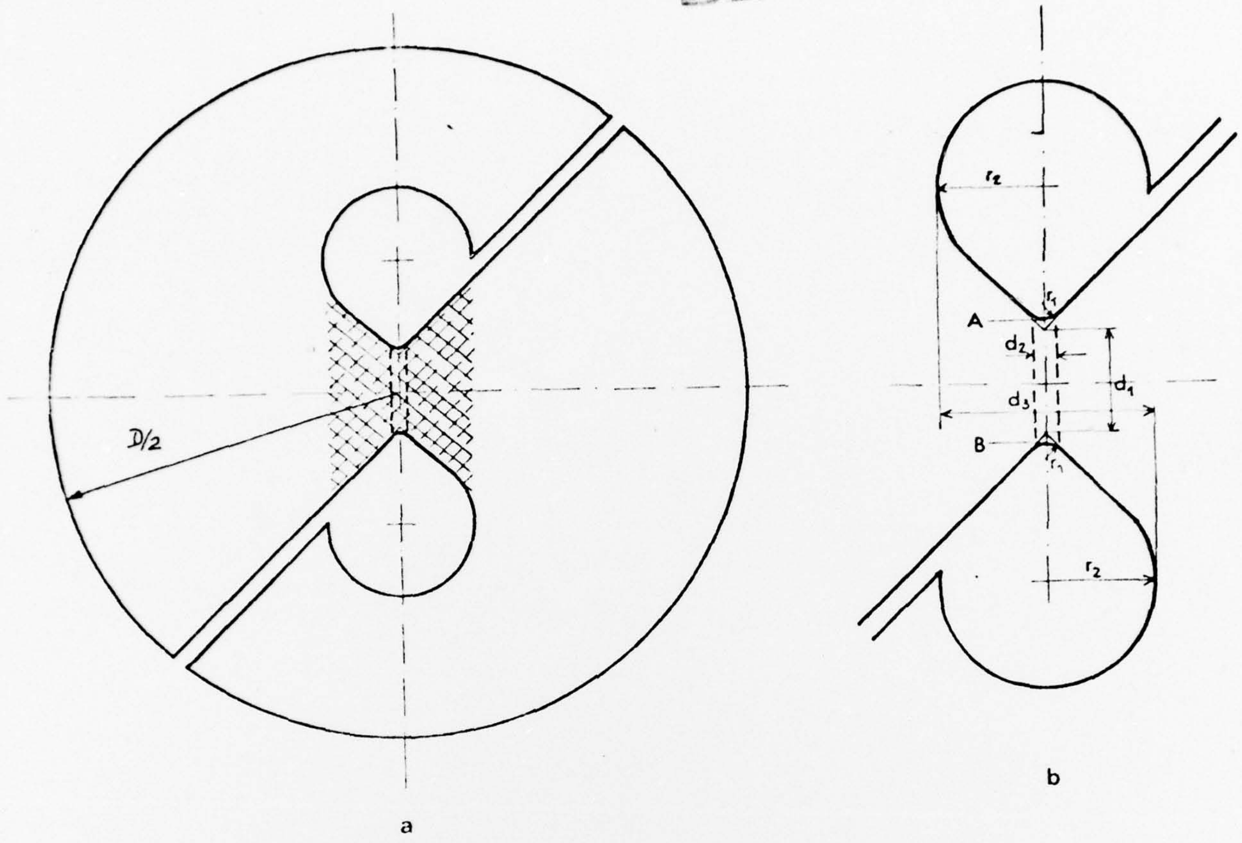
6. Failure on unidirectional FRM circular sample.



1. The circular sample (a)

The state of stress induced by forces P_0 in the significant section AB $\sigma_1/\sigma_2 = -1$ or pure shear σ_{xy} (b)

The state of stress induced by forces P_α, $\sigma_1/\sigma_2 = K_1$ (c).



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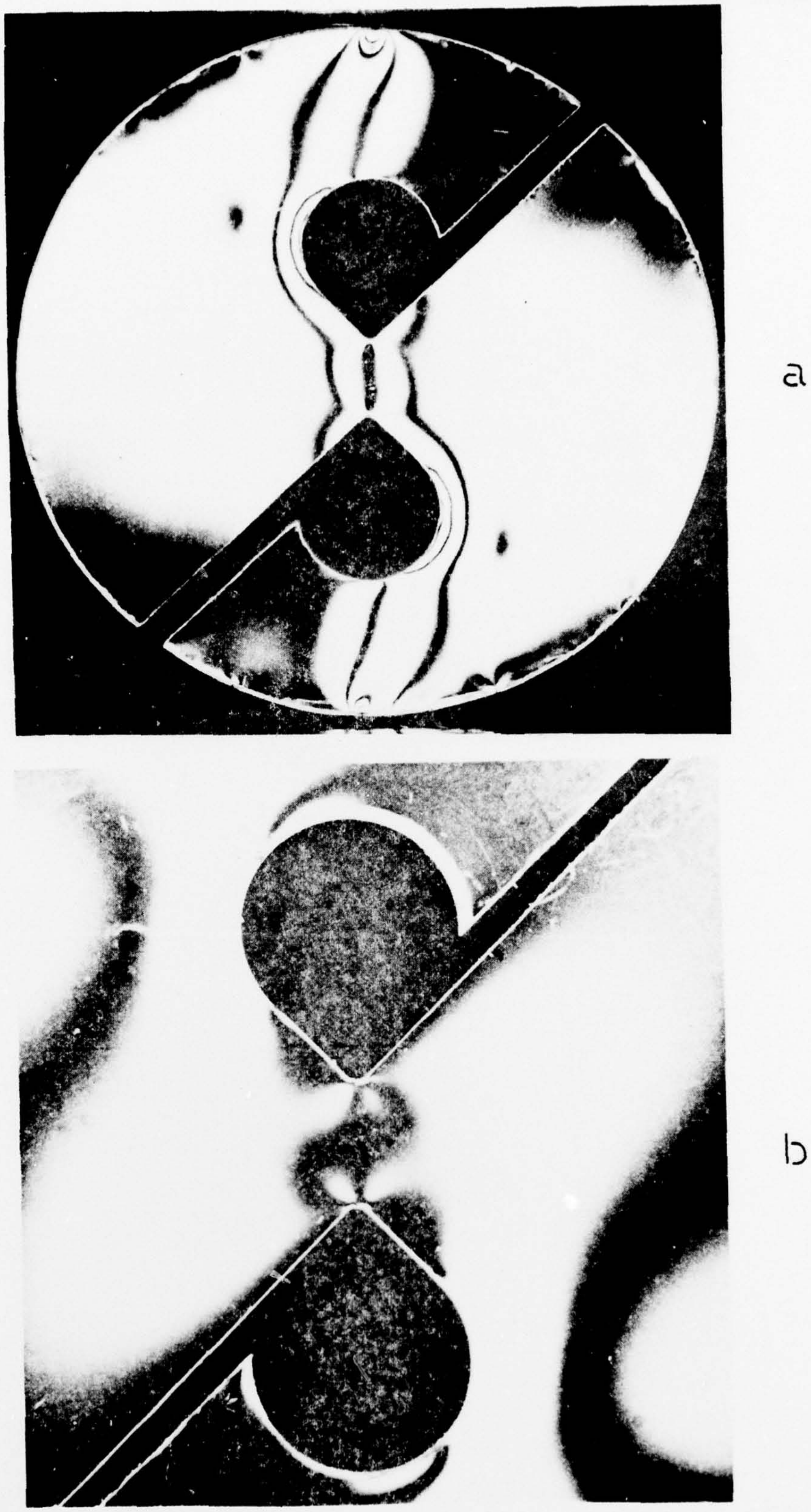

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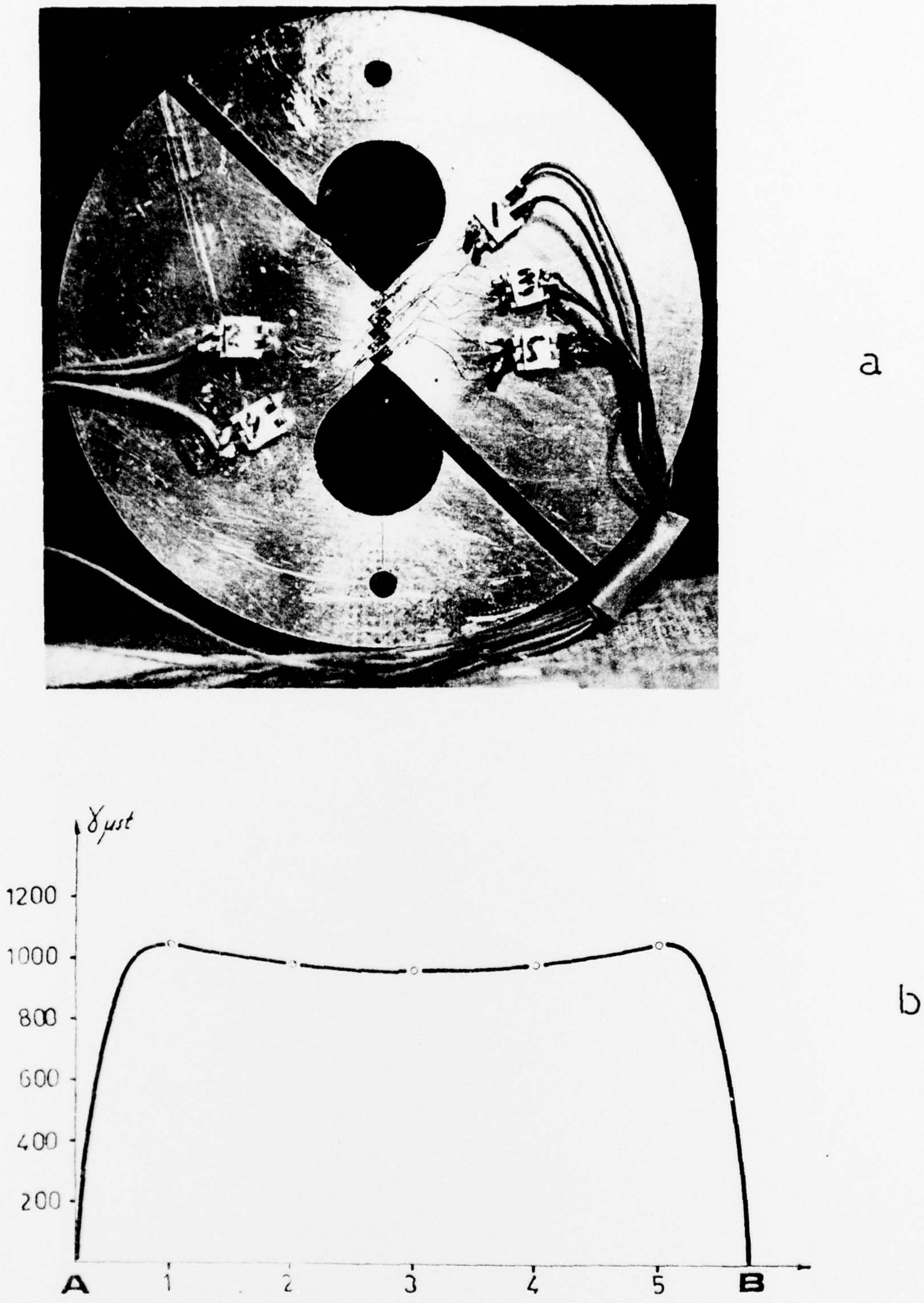
Isochromatic pattern (a)

45° Isoclinic (b).



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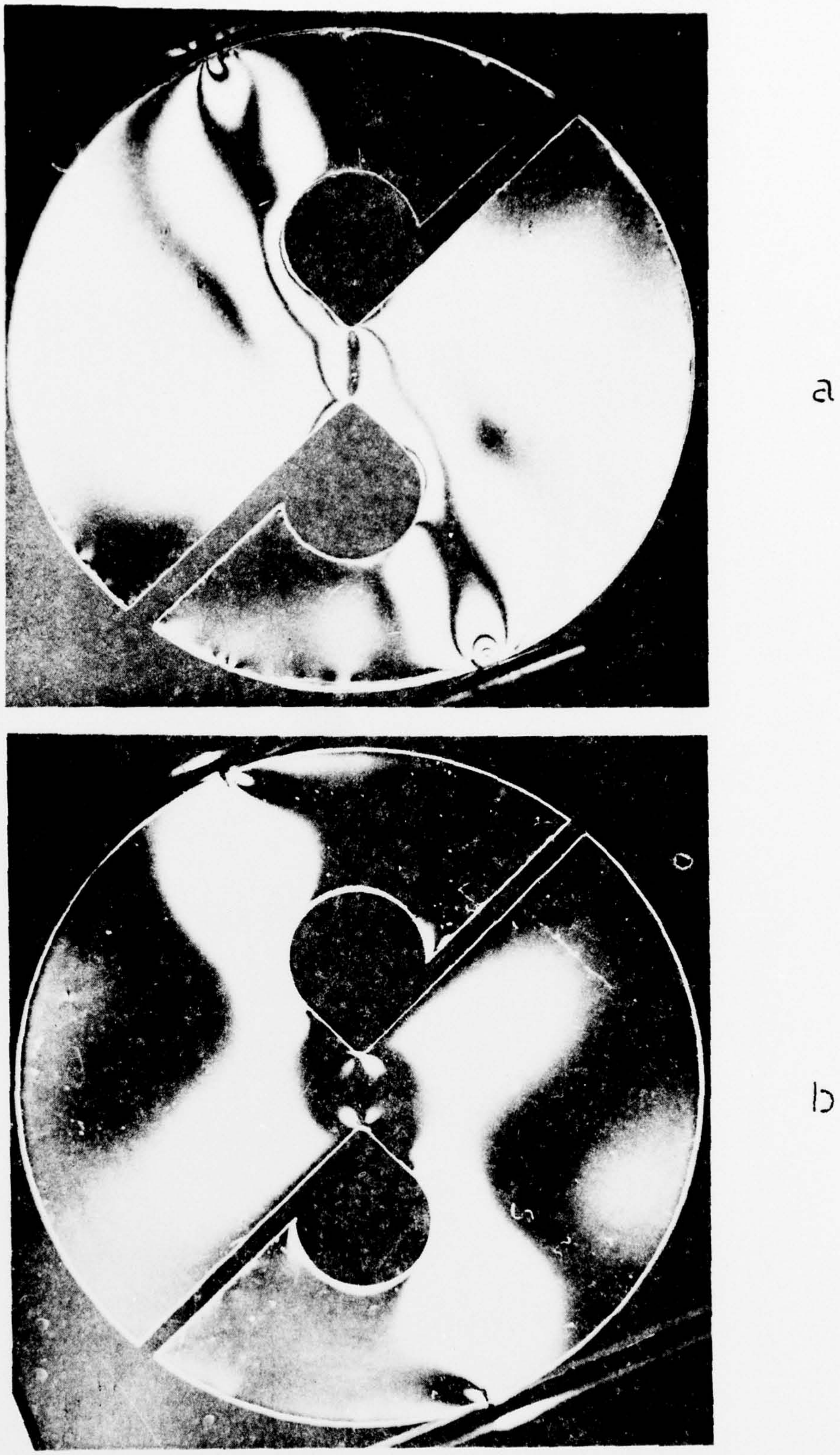
The 5 strain gages at 45° on one face of the sample (a)

The shear stress diagram (b).



5. The circular sample loaded with forces P_α to obtain σ_1/σ_2 arbitrary
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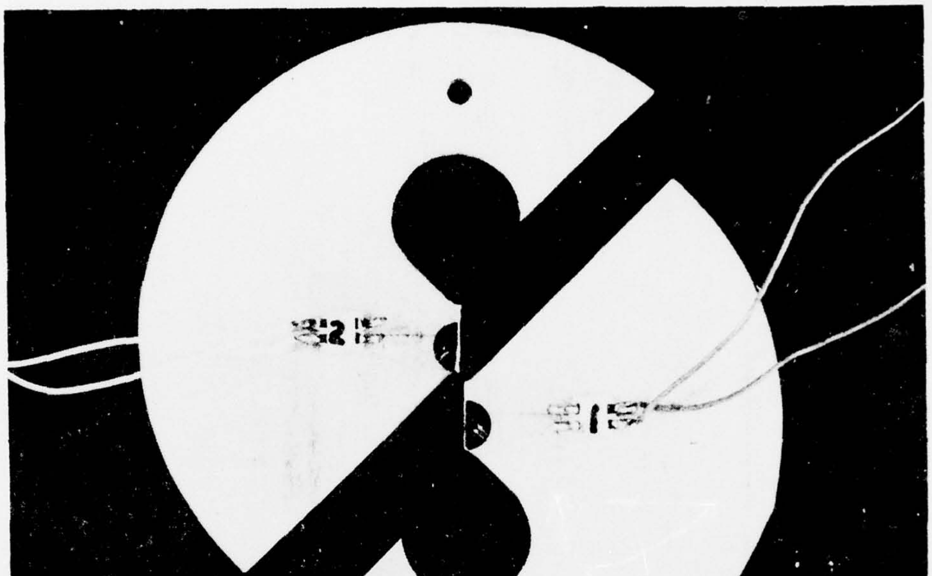

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