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for

CONTROL OF NONLINEAR SYSTEMS

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New methods were developed for stabilizing a "time-varying" linear differential system. Three technical notes involving feedback stabilization of a discrete-time constant linear system, derivation new lower bounds on the solution matrix, to the algebraic matrix Riccati equation, and new sufficient conditions for the linear constant differential-difference system were published. Results have been obtained concerning a minimum energy regulator problem for linear time-invariant systems in which the control variable is subject to an "average-power" constraint on the response time interval.

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I. *INTRODUCTION*

This progress report concerns research completed to date under grant AFOSR-75-2793. The personnel listed in Section *II* received at least partial support from the grant during this period. Completed research is discussed in Section *III*. Publications, including papers submitted for publication, are listed in Section *IV*.

II. *SUPPORTED PERSONNEL*

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III. RESEARCH COMPLETED

(a) Control of Linear Systems

Although the synthesis of feedback control laws for linear differential systems has been actively researched for many years, there exist very few methods for stabilizing a "time-varying" linear differential system. New results on this class of problems were obtained by Kwon and Pearson, [1-3], relative to the linear time varying system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad x(t_o) = x_o$$
$$y(t) = C(t)x(t)$$

where the matrices $(A(t), B(t), C(t))$ are assumed to be piecewise continuous functions for all $t \geq t_o$. The feedback control law synthesized for this system is of the form

$$u(t,x) = -R^{-1}(t)B'(t)P^{-1}(t,t+T)x$$

where the nonsingular symmetric matrix $P(t,t+T)$ is obtained by integrating the matrix Riccati equation

$$\frac{-\partial P(\tau,\sigma)}{\partial \tau} = -A(\tau)P(\tau,\sigma) - P(\tau,\sigma)A'(\tau) - P(\tau,\sigma)C'(\tau)Q(\tau)C(\tau)P(\tau,\sigma) + B(\tau)R^{-1}(\tau)B'(\tau) \quad , \quad \tau \leq \sigma$$

backward in time from $\tau=\sigma = t+T$ to $\tau = t$, subject to the boundary condition $P(\sigma,\sigma) = 0$. It is shown in [3] that the above feedback control law is optimal for the moving cost function

$$J(u) = \int_t^{t+T} [y'(\tau)Q(\tau)y(\tau) + u'(\tau)R(\tau)u(\tau)]d\tau$$

subject to the moving terminal constraint $x(t+T) = 0$, where $(Q(\cdot), R(\cdot))$ are nonnegative definite symmetric weighting matrices with $R(t) > 0$ for all t, and T is a chosen positive scalar. More importantly, the above control law has been shown to be uniformly asymptotically stable under some mild technical conditions involving controllability and observability of the matrix pairs $(A(t), B(t))$ and $(A(t), C(t))$ and the choice in the parameter $T > 0$. A major advantage of the above control law in comparison with the standard regulator problem is that the integration interval is finite for the Riccati equation of this formulation, while it is infinite for the solution to the standard regulator problem. Also, it is shown in [3] that the minimal values of the cost functions for the above receding horizon problem and the standard regulator problem are identical for $T = \infty$, thus providing a link between the two types of linear state variable feedback control law solutions.

In the case of time invariant systems with constant weighting matrices, i.e., (A, B, C, Q, R) all constant, the above control becomes a fixed gain feedback control law and, as shown in [3], generalizes a well-known method for stabilizing a linear fixed system given by Kleinman.* In particular, Kleinman's result is obtained as a special case by choosing $Q = 0$ for the weighting matrix on the state.

The dual problem in filtering theory corresponding to the above control problem is shown in [3] to yield an asymptotically stable Kalman-Bucy filter for the case of completely unknown statistics involving the initial state $x(t_o) = x_o$, i.e., the case in which the mean $E\{x_o\} = \bar{x}_o$ = unknown and the variance $E\{(x_o - \bar{x}_o)(x_o - \bar{x}_o)'\} = \infty$.

Three technical notes pertaining to the control of linear systems have also been completed during this period. The first, [4], extends a result due

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*Kleinman, D. L., "An Easy Way to Stabilize A Linear Constant System," IEEE Trans. Auto. Contr., AC-15, 692, December, 1970.

to Kleinman* in the feedback stabilization of a discrete-time constant linear system, $x(i+1) = Ax(i) + Bu(i)$, by a feedback control of the form

$$u(i) = -R^{-1}B'A'^{N}[\varepsilon I + \sum_{k=m}^{N} A^{k}BR^{-1}B'A'^{k}]^{-1} A^{N+1} x(i)$$

where the choice in the integers (m,N) and the nonnegative scalar ε depend on the multiplicities of the zero eigenvalue as a root of the characteristic and minimal polynomials of the matrix A. The main result of this note is the removal of the nonsingularity condition on the A matrix and a weakening of the controllability assumptions pertaining to the pair (A,B).

The second technical note, [5], derives new lower bounds on the solution matrix K to the algebraic matrix Riccati equation, $A'K + KA - KBB'K + Q = 0$, and shows how these bounds are sharper than those appearing previously in the literature, as well as providing exact estimates in certain special cases. Extensions to the discrete algebraic matrix Riccati equation are also included in [5].

A third technical note, [6], provides new sufficient conditions for the linear constant differential-difference system

$$\dot{x}(t) = Ax(t) + A_h x(t-h) + Bu(t)$$

to be memoryless stabilizable by a feedback control law of the form

$$u(t) = F\, x(t).$$

More importantly, the results in [6] are constructive in that the gain matrix F can be easily computed if it is determined that the various derived sets of sufficient conditions for stabilizability are upheld. One such set of sufficient conditions is the existence of a positive definite matrix Q and a

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*Kleinman, D. L., "Stabilizing a Discrete, Constant, Linear System With Application to Iterative Methods for Solving the Riccati Equation," IEEE Trans. on Auto. Contr., AC-19, pp. 252-254, June 1974.

positive scalar T such that the matrix inequality

$$4A_h Q^{-1} A_h' < 2BB' + P(T)QP(T)$$

is upheld, where P(T) is the solution (at a fixed time T) of the matrix Riccati differential equation

$$\dot{P}(t) = -AP(t) - P(t)A' - P(t)QP(t) + BB', \quad P(o) = 0.$$

In this case, the stabilizing gain matrix F is given by $F = -B'P^{-1}(T)$. Another (simpler) set of sufficient conditions is given by the selection of the matrix Q according to the inequality

$$Q \geq 2\lambda_{max}(HH')I$$

where H is a matrix such that $A_h = BH$. This case applies to that special situation in which the columns of A_h are linear combinations of the columns of B. The gain matrix F is defined the same as in the first case after defining Q so as to satisfy the above inequality. This special case is not devoid of representation since it is shown by way of example in [6] that such a Q can be constructed for the linear constant differential difference system

$$\sum_{i=o}^{n} \alpha_i y^{(i)}(t) + \sum_{i=o}^{n} \beta_i y^{(i)}(t-h) = u(t)$$

where $y^{(i)} \triangleq i^{th}$ derivative of y.

Results have been obtained during this period concerning a minimum energy regulator problem for linear time-invariant systems in which the control variable is subject to an "average-power" constraint on the response time interval [7]. The optimization problem considered is that of minimizing the control energy cost function,

$$J(u) = \int_{t_o}^{T} u'(t)u(t)dt$$

for the linear system $\dot{x} = Ax + Bu$, while regulating the state of the system from the given initial state $x(t_o) = x_o$ to the origin $x(T) = 0$ in a fixed time $T-t_o$. After solving this problem (the solution of which is well-known), the response time $(T-t_o)$ is chosen so that the optimal control signal $\hat{u}$ satisfies the average-power constraint

$$\frac{1}{T-t_o}\int_{t_o}^{T} \hat{u}(t)\hat{u}(t)dt = 1.$$

This is an optimal controller with transient response characteristics comparable to a time optimal "bang-bang" controller in which each control signal is subject to the hard constraint $|u_i(t)| \leq 1$ on the response time interval $t_o \leq t \leq T^*$, and T^* is the minimum time solution to reach the origin. The above average-power constraint is also satisfied (coincidentally) by the bang-bang controller, which accounts for the similarity in their transient response characteristics. However the minimum energy average-power contrained regulator is easier to obtain in feedback form due to the softer constraint on the control variable. Although also implicitly defined, the main advantage of this control law over the time optimal control is that a suboptimal, explicitly defined, feedback control can be constructed, as shown in [7], with whatever degree of accuracy is desired for a general n^{th} order system, while this is practically impossible for the time optimal bang-bang controller when $n \geq 3$. It is noted that the control law for the minimum energy average power constrained regulator is nonlinear, and that the asymptotic stabilization of this nonlinear control law has been established in [7]. Simulation results for second and fourth order examples are also summarized in [7] which illustrate the restraining effect of the average power constraint on the control signal while regulating the state of the system over a wide range of initial conditions in the state space. This is the major advantage of this nonlinear controller over a linear feedback controller designed to achieve the same settling time.

(b) Control of Nonlinear Systems

Sufficient conditions for the controllability of the class of nonlinear systems described by

$$\dot{x}(t) = A(t,x(t),u(t))x(t) + B(t)u(t) + f(t,x(t),u(t)), t_o \leq t \leq t_1$$

have been obtained by Wei [8] during this period. These conditions involve the nonsingularness of the controllability Gramian associated with the parametrized matrix pair $\{A(t, \zeta(t),v(t)),B(t)\}$, where $\zeta(t)$ and $v(t)$ are regarded as elements (parameters) in a product space, $C_{nm}[t_o,t_1]$, of vector valued continuous function pairs, $(\zeta(t),v(t))$, on the time interval $t_o \leq t \leq t_1$. Using the Schauder's fixed point theorem in $C_{nm}[t_o,t_1]$, sufficient conditions for both local and global controllability are derived involving the boundedness and continuity of the quantities $(A(t,x,u), B(t), f(t,x,u))$ and their partial derivatives, in addition to the nonsingularness of the aforementioned controllability Gramian. These results remove some assumptions previously needed in earlier publications on this problem and, generally, extend these earlier results to a broader class of nonlinear systems.

A number of results have been obtained during this period concerning the bilinear regulator problem for the class of nonlinear systems described by

$$\dot{x}(t) = (A + \sum_{i=1}^{m} B_i u_i(t))x(t)$$

where $u = (u_1, \cdots u_m)'$ is the control variable and $(A,B_1, \cdots B_m)$ are given nxn matrices, [9-10]. First, existence of an optimal control has been established for the minimization of the quadratic cost

$$J(u) = x'(T)Qx(T) + \int_{t_o}^{T} [x'(t)W(t)x(t) + u'(t)R(t)u(t)]dt$$

where $(Q,W(t),R(t))$ are symmetric nonnegative definite weighting function matrices with $R(t) > 0$ for all $t \in [t_o, T]$. Next, the regulator problem for the special class of commutative bilinear systems has been considered in some detail. This is the class for which every pair of matrices in the set $\{A, B_1, \cdots B_m\}$ commute with each other. Within the context of this class, it has been shown that the optimal control which minimizes the above quadratic cost, without any terminal constraint on the state, is in the form of a constant vector which satisfies a certain nonlinear algebraic equation. Furthermore, for a single input commutative bilinear system (m=1), it is shown in [10] that this optimal control is unique if (as a sufficient condition) the matrix $B_1'Q + B_1'QB_1$ is non-negative definite. Also, sufficient conditions have been obtained in the multi-input case which involve the nonnegative definiteness for all $v \in R^n$ of the mxm matrix Z(v) defined by

$$Z_{ij} = v'(B_j'B_i'Q + B_i'QB_j)v, \quad i,j = 1, \cdots m.$$

The implication of these results for the regulator problem associated with a commutative bilinear system is that the optimal control can be computed by well-known iterative methods in finite dimensional (R^m) spaces, and that this control vector is unique if certain additional conditions involving the system matrices are upheld.

Concerning the same class of regulator problems for commutative bilinear systems, but with a fixed terminal state constraint, i.e., $x(T) = x_1$ = a given terminal vector, it has also been shown in [10] that if x_1 belongs to the reachable set, then there exists a constant optimal control which does the job, and that this optimal control vector satisfies a certain nonlinear algebraic equation which depends on the given boundary conditions: $x(t_o) = x_o$ and $x(T) = x_1$. In the terminal constraint problem such optimal controls are not generally unique and a simple example is given in [10] to illustrate this fact.

Application of the above theory for the regulator problem of a commutative bilinear system has been obtained in the companion paper [11]. Here it is shown how the two dimensional missile intercept problem for a maneuverable target and a pursuing missile can be formulated in the present context through the introduction of some auxiliary states. The kinematic equations are

$$\dot{x}_1(t) = -v_T \sin x_3(t) + x_2(t)u_p(t)$$
$$\dot{x}_2(t) = v_T \cos x_3(t) - x_1(t)u_p(t) - v_p(t)$$
$$\dot{x}_3(t) = u_T - u_p(t)$$

where (v_T, v_p) are the line speeds of the target and pursuer, (x_1, x_2) are the position coordinates of the target relative to the pursuer, x_3 is the relative angle between the headings of the two missiles, and (u_T, u_p) are the angular rates of the target and pursuer. Introducing auxiliary states $x_4 = \sin x_3, x_5 = \cos x_3$ and $x_6 = 1$, and making the crucial assumption that the line speed v_p of the pursuer can be modeled proportional to u_p, i.e., $v_p(t) = \gamma u_p(t)$, it is first shown in [11] that the resulting equations of motion are in the bilinear form, $\dot{x} = (A + Bu_p)x$, and that the 6x6 matrices (A,B) commute. This implies that the optimal control for the quadratic cost problem is a constant vector satisfying a certain nonlinear algebraic equation. Furthermore, it is possible to solve these nonlinear equations explicitly, i.e., in closed-form, for the terminal constraint case: $x_1(T) = x_2(T) = 0$, i.e., zero missed distance, given that the intercept angle $\beta = x_3(T)$ is allowed to be selected with some degree of latitude. Specifically, it has been shown that there exists a triple (γ, β, T) for every set of initial data $(x_1(t_o), x_2(t_o), x_3(t_o))$ such that the desired zero missed distance terminal constraint can be upheld, and that the optimal control u_p^* which minimizes the quadratic cost

$$J(u) = \int_{t_o}^{T} u^2(t)dt$$

subject to $x_1(T) = x_2(T) = 0$, is a constant given by

$$u_p^* = u_T + \frac{x_3(t_o)-\beta}{T-t_o}$$

Inasmuch as this solution has been obtained in closed-form, it is potentially feasible that the result might be used on-line for obtaining a closed-loop control law for the missile intercept problem assuming that the target speed, acceleration and initial heading, $(v_T, u_T, x_3(t_o))$, can be estimated from the given measurements. A least squares estimate of the pair of quantities $(v_T, x_3(t_o))$ has been derived, and the entire step-by-step estimation and control sequence, which defines the closed-loop control law, has been simulated under a variety of initial conditions. A summary of these simulation runs is given in [11].

A singular perturbation problem has also been considered in [11] relating to the practical situation in which the missile turn rate is furnished by a motor with actuator dynamics. First order dynamics were assumed for the analysis and simulation studies, but the results actually apply to higher order actuator dynamics are well. An interesting feature of these results is that a closed-form solution can be obtained for the higher order singularly perturbed system of this paper in contrast with the approximate solutions for general nonlinear systems.

(c) Parameter Identification

A deterministic least squares identification of the coefficient matrices in the differential operator model

$$P(D)y(t) = Q(D)u(t), \quad D = \frac{d}{dt}$$

where

$$P(D) = D^n + \sum_{i=o}^{n-1} P_{n-i} D^i, \qquad Q(D) = \sum_{i=o}^{n-1} Q_{n-i} D^i,$$

has been developed in [12,13] which differs from more traditional uses of least squares theory in the following respects: (i) input-output data $[u(t),y(t)]$ is assumed to be given on a finite time interval, $0 \leq t \leq t_1$, of arbitrarily short (but non-zero) duration, (ii) unknown disturbance inputs and measurement noises on $0 \leq t \leq t_1$, are modeled implicitly in the above model by arbitrary solutions to a homogeneous linear differential equation of assumed order, but with no assumptions about the characteristic modes of this equation, (iii) no attempt is made to estimate either the initial state of the system or the initial conditions giving rise to the disturbance inputs on $0 \leq t \leq t_1$.

One advantage of this approach, which might be termed parameter identification without initial state estimation, is that the potential exists for obtaining very accurate estimates of the system parameters, based on input-output data observed over a relatively short time interval, even for very small signal-to-noise ratios, eg. -20db. or less. The main reason for this lies in the technique developed in [12,13] for circumventing the need to estimate the unknown initial conditions, which reduces this aspect of the computational burden associated with other approaches. Another reason is that the disturbances are modeled deterministically as uncontrollable modes, and the frequencies associated with these modes on $0 \leq t \leq t_1$ are identified along with the system parameters.

Theoretical conditions for the uniqueness of solutions to the above least squares estimation problem have also been obtained in [13]. These conditions involve the linear independence of the given input-output data, together with a certain number of their derivatives on $[0,t_1]$. Simulation results are reported in [13] which illustrate that highly accurate estimates for the parameters of a fourth order system can be obtained on a time interval comparable to the time constants in the system even in the presence of very large disturbance signals.

Subsequent to the results reported in [12,13], important extensions have been obtained which enlarge the class of systems and provide for computational advantages in a variety of situations [14]. These extensions arise principally by viewing the identification problem in terms of finding a parameter vector θ which satisfies a differential operator equation of the form

$$P(D)v(t) + Q(D)V(t)f(\theta) = 0, \quad 0 \leq t \leq t_1$$

where $(P(D),Q(D))$ are given polynomial matrices in the differential operator $D = \frac{d}{dt}$, $(v(t),V(t))$ are vector and matrix valued functions of the given input-output data, $f(\theta)$ is a given vector valued function (possibly nonlinear) of the parameter vector θ, and the observation time interval, $0 \leq t \leq t_1$, is again of arbitrarily short duration. Some attributes of this formulation in relation to the results reported in [12,13] are the following: (i) the parameters θ may enter nonlinearly into the basic model, i.e., the function $f(\cdot)$ may be a nonlinear function of θ, (ii) the disturbances on $0 \leq t \leq t_1$ are modeled exactly the same as in [12,13], i.e., by arbitrary solutions to a homogeneous differential equation of assumed order, but with no assumptions about the characteristic modes of this equation; however, the parameters associated with the disturbances are modeled <u>explicitly</u> in the formulation in [14], in contrast with the implicit modeling of disturbances in [12,13], (iii) the coefficent matrix of the highest derivative on the input-output data is allowed to be singular for the formulation in [14], while this condition was previously ruled out due to the particular state variable representation used in [12,13].

The computational aspects of the formulation in [14] involve minimizing an explicitly defined function $J(\theta)$ of the form

$$J(\theta) = f'(\theta)\Phi f(\theta) + 2\zeta' f(\theta) + \alpha$$

where the nonnegative definite matrix Φ, the vector ζ and the scalar α are determined by integrating a certain set of differential equations driven by the input-output data on $0 \leq t \leq t_1$. Moreover, this minimum is known to correspond to the sought value of $\theta=\theta^*$ if $J(\theta^*) = 0$ and some other nondegeneracy conditions are upheld involving the input-output data (see the Assertion on p. 846 of [14]).

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