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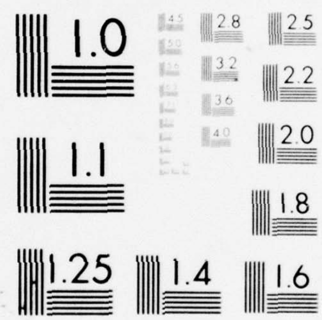
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Magnetic-Field-Induced Enhancement of Relativistic-Electron-Beam Energy Deposition

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Plasma Technology Branch
Plasma Physics Division

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The collisional transport of relativistic electron beams in dense, magnetized, high-Z plasmas is characterized by a perturbation solution of the relativistic Boltzmann equation with a Fokker-Plank collision term in the limit of small beam-electron mean-free-path or gyroradius. The general formulation allows for arbitrary magnetic-field, electric-field, and plasma geometries. Analytic solutions in 1-D yield energy-deposition profiles for a magnetized plasma slab.

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MAGNETIC-FIELD-INDUCED ENHANCEMENT OF RELATIVISTIC-ELECTRON-BEAM ENERGY DEPOSITION

An important area of research associated with the goal of relativistic-electron-beam-initiated fusion in pellet targets has been the investigation of techniques to reduce the energy-deposition length of the beam in dense matter.[1] Attention has been devoted to this question because the long deposition lengths of MeV-energy electrons require beam powers for fusion[2] which are technologically difficult to achieve. A reduction in deposition length in the target plasma might be affected by the presence of a strong self- or external-magnetic field transverse to the beam-current direction.[3]

Here, a perturbation procedure[4] is used to solve the relativistic Boltzmann equation with a Fokker-Planck collision term[5] in order to characterize the collisional transport of relativistic electron beams in magnetized, high-atomic-number plasmas. The analysis is valid for arbitrary magnetic-field and plasma-parameter variations provided that macroscopic scale lengths are large compared to either a beam-electron scattering length or gyroradius. The ordering of smallness parameters results in a diffusion-like equation for the energy distribution function of beam electrons involving the electric field and dynamic friction. Explicit one-dimensional solutions are then used to determine

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the manner in which the deposition of electron energy changes with transverse magnetic field strength. Finally, the applicability of this mechanism to current experiments and fusion systems is discussed.

The equation describing the momentum distribution function of relativistic electrons interacting with a high-atomic-number plasma may be written[5]

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m\gamma}\cdot\vec{\nabla} f - e\left(\vec{E} + \frac{\vec{p}\times\vec{B}}{m\gamma}\right)\cdot\vec{\nabla}_p f = \vec{\nabla}_p\cdot[\nu_S(p)(p^2\overleftrightarrow{I} - \vec{p}\vec{p})\cdot\vec{\nabla}_p f] + \vec{\nabla}_p\cdot[\nu_E(p)\vec{p}f] \tag{1}$$

where $\gamma^2 = 1 + p^2/(mc)^2$. The quantities ν_S and ν_E are scattering and energy-loss frequencies:

$$\nu_S = \Omega_S\gamma/(\gamma^2 - 1)^{3/2}, \qquad \nu_E = \epsilon\gamma\nu_S, \tag{2}$$

where $\Omega_S = 2\pi n_i r_o^2 c(Z^2 + Z)\ln\Lambda$ and $\epsilon = 2/(Z+1) \ll 1$. Here, n_i is the plasma ion density, r_o is the classical electron radius, c is the velocity of light, Z is the plasma atomic number, and $\ln\Lambda$ is of order 10. Since electron diffusive-relaxation times through the dense plasma are much shorter than the time over which macroscopic beam parameters vary, the time-derivative term may be neglected. In keeping with the diffusion ordering, the $\vec{\nabla} f$ and electric-field terms are considered to be of order $\epsilon^{\frac{1}{2}}\nu_S$. The magnetic-field term is taken as comparable to ν_S for generality.

With the ordering just discussed, the zero-order equation takes the form

$$-\frac{e}{m\gamma}\vec{p}\times\vec{B}\cdot\vec{\nabla}_p f_o = \nu_S\vec{\nabla}_p\cdot[(p^2\overleftrightarrow{I} - \vec{p}\vec{p})\cdot\vec{\nabla}_p f_o] \tag{3}$$

where $f = f_0 + f_1 + f_2 + \ldots$, f_1 is of order $\epsilon^{\frac{1}{2}} f_0$, f_2 of order ϵf_0, etc. Equation (3) is satisfied only by a function which is isotropic in momentum space, i.e. $f_0(\vec{p},\vec{x}) = f_0(p,\vec{x})$. To see this, $\vec{p}$ is written in terms of the spherical coordinates (p,θ,φ) with the polar axis parallel to $\vec{B}$. Expanding f_0 in terms of the surface harmonics $Y_\ell^m(\theta,\varphi)$ and substituting to Eq. (3) results in a solution only for $m = \ell = 0$. Thus, f_0 is isotropic. Isotropic f_0 is confirmed by sophisticated particle codes which have demonstrated that tightly-pinched electron beams have this character.[6]

The first-order equation then is

$$\vec{p} \cdot \left(\frac{1}{m\gamma} \vec{\nabla} f_0 - \frac{e\vec{E}}{p} \frac{\partial f_0}{\partial p} \right) = \nu_S \vec{\nabla}_p \cdot \left[(p^2 \overleftrightarrow{I} - \vec{p}\vec{p}) \cdot \vec{\nabla}_p f_1 \right] + \frac{\Omega_0}{\gamma} \vec{p} \times \hat{b} \cdot \vec{\nabla}_p f_1 \tag{4}$$

where $\hat{b} = \vec{B}/B$ and $\Omega_0 = eB/m$. The non-relativistic form of Eq. (4) has been solved.[4] The solution follows from inspection of reference 4.

$$f_1 = - \frac{1}{2\nu_S} \vec{p} \cdot \overleftrightarrow{M} \cdot \left(\frac{1}{m\gamma} \vec{\nabla} f_0 - \frac{e\vec{E}}{p} \frac{\partial f_0}{\partial p} \right) \tag{5}$$

where

$$\overleftrightarrow{M} = \hat{b}\hat{b} + \frac{1}{1+\alpha^2} (\overleftrightarrow{I} - \hat{b}\hat{b}) + \frac{\alpha}{1+\alpha^2} \hat{b} \times \overleftrightarrow{I} \tag{6}$$

and $\alpha = \Omega_0 / 2\gamma\nu_S$. The equation for $f_0(p,\vec{x})$ is determined by integrating Eq. (1) over the momentum-space solid angle and keeping terms to lowest significant order.[4] This procedure results in

$$\left(\frac{1}{m\gamma}\vec{\nabla} - \frac{e\vec{E}}{p}\frac{\partial}{\partial p}\right)\cdot\left[\frac{p^3}{6\nu_S}\overset{\rightrightarrows}{M}\cdot\left(\frac{1}{m\gamma}\vec{\nabla}f_o - \frac{e\vec{E}}{p}\frac{\partial f_o}{\partial p}\right)\right] + \frac{1}{p}\frac{\partial}{\partial p}\left(\nu_E p^3 f_o\right) = 0 \qquad (7)$$

In the limit of $\Omega_o = 0$, Eqs. (5) and (7) reduce to published results.[7]

The interesting macroscopic quantities derivable from solution of Eqs. (5) and (7) are the particle flux $\vec{\Phi}$ and heat flux $\vec{q}$. These are defined by

$$\begin{Bmatrix}\vec{\Phi}\\ \vec{q}\end{Bmatrix} = \int \begin{Bmatrix}1\\ mc^2(\gamma-1)\end{Bmatrix}\frac{\vec{p}}{m\gamma}f_1 d^3p \qquad . \qquad (8)$$

Taking the divergence of Eq. (8), substituting from Eq. (7), and integrating by parts over p results in the relations

$$\vec{\nabla}\cdot\vec{\Phi} = -4\pi\epsilon\Omega_S(mc)^3 f_o(o,\vec{x}) \qquad (9)$$

$$Q = -\vec{\nabla}\cdot\vec{q} = e\vec{E}\cdot\vec{\Phi} + 4\pi\epsilon\Omega_S(mc)^3\cdot mc^2\int\gamma^2 f_o d\gamma \qquad . \qquad (10)$$

The particle "sink" in Eq. (9) represents the merging of beam electrons with the plasma background once they have been slowed by dynamic friction to low energies. The quantity Q is the volumetric heating rate of plasma due to both ohmic and collisional beam-energy losses.

One-dimensional solutions in a slab geometry from which $\vec{\Phi}$ and Q can be determined explicitly are now considered. A monoenergetic ($\gamma = \gamma_o$) electron beam of particle flux Φ_o is assumed incident on a uniform plasma occupying the half-space $x \geq 0$. A uniform magnetic field of arbitrary strength aligned parallel to the plane $x = 0$ is imbedded in the plasma.

Equation (9) in one dimension states that the beam flux decays with x so that in order to maintain charge conservation, a plasma-electron current-density $j(x) = e(\Phi-\Phi_o)$ must flow. An electric field $E(x) = \eta j$, η being the plasma resistivity, must then exist. The importance of E to evaluation of f_o is determined by comparing the magnitude of the $\vec{\nabla} f_o$ and electric-field terms in Eq. (4). It is now shown that E is negligible for problems of interest, i.e. that $eE \ll m_o c^2 \gamma_o \chi^{-1}$, where χ is the scale over which f_o varies. Since $E \leq e\eta\Phi_o$, and $\chi \leq c/\epsilon^{\frac{1}{2}}\Omega_S$ for electron energies of interest[8], the above condition is satisfied if $e\eta\Phi_o \ll \epsilon^{\frac{1}{2}} m_o c^2 \gamma_o \Omega_S/e$. Substituting numerical values and assuming classical resistivity it is found that the electric-field is negligible when

$$e\Phi_o \ll 3 \times 10^{10} \left(\frac{n_i}{n_S}\right) \theta^{3/2}/Z_i \quad \text{Amps/cm}^2 \quad . \qquad (11)$$

In this expression, n_i/n_S is the ratio of plasma-ion to solid density, θ is the plasma temperature in eV and Z_i is it's ionization level. The numerical coefficient is appropriate for a gold plasma. This condition is satisfied for electron beams of interest so that the electric field can be ignored for determination of f_o.

With the simplifications discussed, Eq. (7) reduces to

$$a^2 \frac{\partial^2}{\partial x^2}(\gamma^2 f_o) + \left[\frac{\gamma^4}{(\gamma^2-1)^3} + \frac{\Omega_o^2}{4\Omega_S^{\,2}}\right] \frac{\partial}{\partial \gamma}(\gamma^2 f_o) = 0 \qquad (12)$$

where $a^2 = c^2/6\epsilon\Omega_S^2$. This relation takes the form of a simple diffusion equation with the change of variable

$$\tau(\gamma) = \int_{\gamma}^{\gamma_o} \left[\frac{\gamma^4}{(\gamma^2-1)^3} + \frac{\Omega_o^2}{4\Omega_S^2} \right]^{-1} d\gamma \quad . \tag{13}$$

for which the second term in Eq. (12) reduces to $-\partial(\gamma^2 f_o)/\partial\tau$. The solution corresponding to monoenergetic f_o at $x = 0$ is[8]

$$\gamma^2 f_o = \Psi_o \frac{x}{(2\pi)^{\frac{1}{2}} a \tau^{3/2}} \exp\left(- \frac{x^2}{4a^2\tau} \right)$$

with Ψ_o determined from the solution of Eq. (9) and the condition that $\Phi(o) = \Phi_o$.

In the limit of large $\Omega_o^2/4\Omega_S^2$, τ can be approximated by $\tau \simeq 4\Omega_S^2(\gamma_o-\gamma)/\Omega_o^2$ for which Eq. (10) can be integrated to give the deposition profile in a strong magnetic field

$$\frac{Q(x)}{\int_0^\infty Q dx} = \pi^{\frac{1}{2}} k \cdot \text{erfc}(kx) \; ; \qquad k = \frac{\Omega_o}{4\Omega_S(\gamma_o-1)^{\frac{1}{2}} a} \quad . \tag{14}$$

This form is accurate for $\Omega_o/\Omega_S \geqslant 8\gamma_o^2/(\gamma_o^2-1)^{3/2}$. For lower magnetic fields, $Q(x)$ is determined by numerical integrations of Eqs. (10) and (13). Results are displayed in Fig. 1 for $\gamma_o = 3$. The case $\Omega_o/\Omega_S = 3$ results in a profile indistinguishable from Eq. (14).

The magnetic-field-induced enhancement in peak heating rate (or reduction in deposition length) above that due to dynamic friction alone can be estimated from Eq. (12) by identifying the characteristic deposition length Δ

$$\Delta(\Omega_o) = a \left[\frac{\gamma_o^4}{(\gamma_o^2-1)^3} + \frac{\Omega_o^2}{4\Omega_S^2} \right]^{-\frac{1}{2}} \quad . \tag{15}$$

Then,

$$\frac{Q_M(\Omega_o)}{Q_M(0)} \approx \frac{\Delta(0)}{\Delta(\Omega_o)} = (1+A)^{\frac{1}{4}} \; ; \quad A = \frac{\Omega_o^2(\gamma_o^2-1)^3}{4\Omega_S^2\gamma_o^4} \; . \qquad (16)$$

The numerical results show that this relation is valid to within about 10% for all values of Ω_o for 1 MeV incident electrons.

It is of interest to determine whether magnetic enhancement of energy deposition is operational in current experiments and scalable to fusion conditions. This is best accomplished by numerically solving Eq. (7) with $\overset{\Rightarrow}{M}$ determined from the azimuthal self-field of the pinched beam or by employing Monte-Carlo techniques.[3] However, when the beam radius R exceeds Δ, Eq. (16) with Ω_o determined by B_θ can provide an estimate. Magnetic enhancement is important when $A \geqslant 1$. This condition is satisfied in the target-plasma blow-off when $B_\theta \geqslant 2 \times 10^6 n_i/n_S$ gauss for a 1 MeV beam incident on gold. In this case, R exceeds Δ when the beam current exceeds 50 kA. The condition $A > 1$ is then satisfied in current experiments[9,10] provided that the plasma is sufficiently resistive to allow B_θ to diffuse a distance Δ into it. Thus, a necessary condition for deposition enhancement due to self-fields is $\tau > \tau_D = 4\pi\Delta^2/\eta c^2$, where τ is the beam duration. For $A > 1$ in gold, this condition may be written $\tau > 128\theta^{3/2}/Z_i B_\theta^2$. Evaluating τ_D for $\theta = 10$ eV, $Z_i = 5$, and $B_\theta = 2 \times 10^5$ g results in a value of 20 ns. Note that although the diffusion-time condition is satisfied in current experiments, it is not sufficient to insure B-field penetration since plasma expansion can convect field lines away from the dense region where enhanced deposition is desired. Simultaneous solution of the plasma MHD, deposition, and magnetic-diffusion equations in a realistic geometry is required for a definitive answer to the role of magnetic enhancement in current experiments. However, at the elevated temperatures required for fusion-pellet targets, these considerations indicate that it is unlikely that sufficient self-fields would penetrate the plasma unless anomolous resistivity were present or unless a significant fraction of the field could be made to penetrate before plasma heating occurred. This last situation might occur if the beam were focussed onto the pellet only after its current has risen close to the maximum value, as is done in certain experimental configurations.[9] In that case, the unpenetrated portion of the field must confine the heated plasma in a way which does not disrupt symmetric implosion of the pellet.

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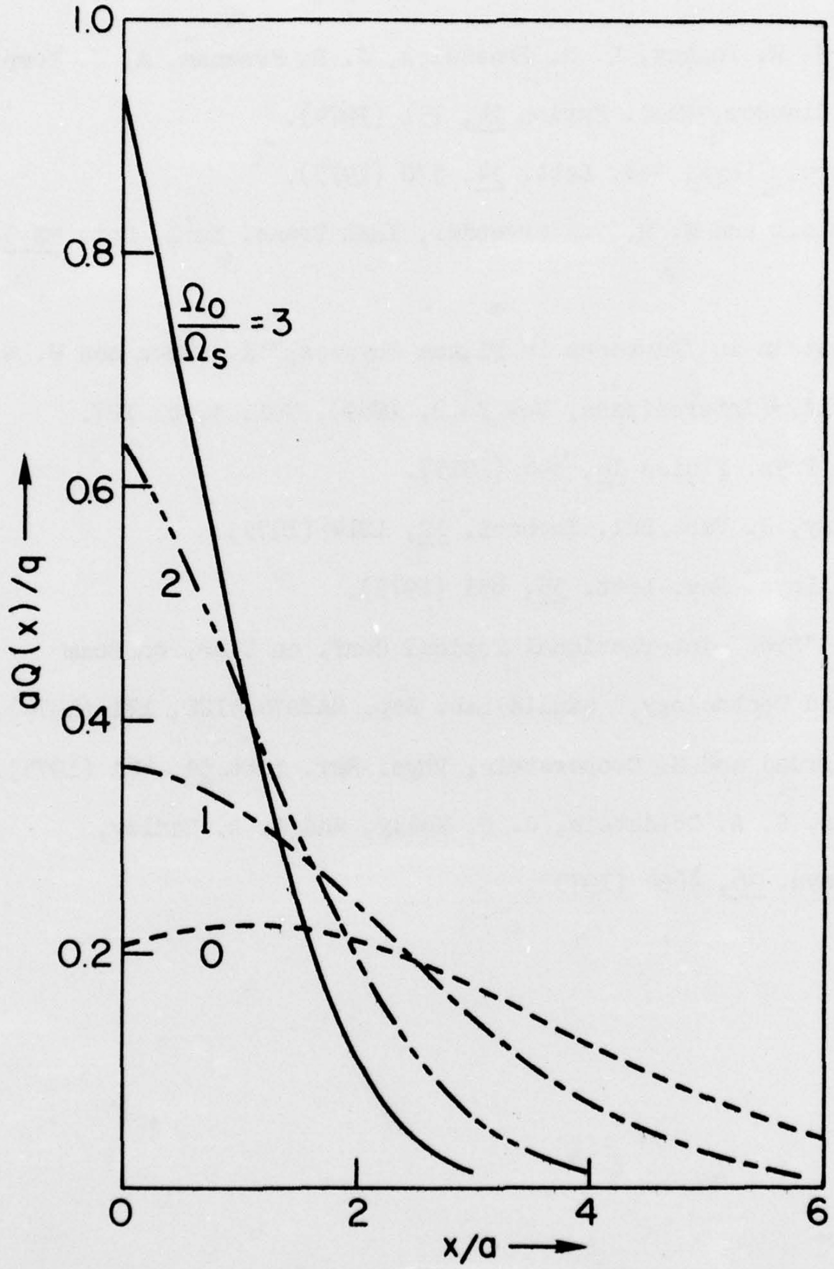

Fig. 1 — Energy deposition profiles of a 1 MeV electron beam incident on a gold plasma slab for various values of transverse magnetic-field strength. The quantity q is the integral of Q over x.

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