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# STATIC AND DYNAMIC BEHAVIOR OF NONCIRCULAR CYLINDRICAL SHELLS

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### Subject: STATIC AND DYNAMIC BEHAVIOR OF NONCIRCULAR CYLINDRICAL SHELLS

#### Introduction

During the past contract period efforts have been placed upon the buckling, postbuckling, and vibrations of cylindrical shells with variable curvature. In the analytical work, progress has been made in four areas, namely, the nonlinear (large deflection) postbuckling analysis of oval cylinders under combined loads, the free vibration of finite oval cylindrical shells with free ends, the free vibration of finite open cylindrical shells, and the linear buckling analysis of reinforced oval cylindrical shells of finite length. In all four areas, the effects of variable curvature have been taken into account. In addition, it is worth reporting that the work on the bifurcation buckling analysis of oval cylinders, as it appeared in Poly-AE/AM Report No. 75-7, AFOSR-TR-75-1425, entitled "Buckling of Oval Cylindrical Shells Under Compression and Asymmetric Bending", by Y.N. Chen and Joseph Kempner has been published in the AIAA Journal. In this paper bifurcation buckling of oval cylindrical shells under the interaction of uniform compressive forces and bending moments of arbitrary orientation is investigated. Buckling loads as well as the asymmetric buckling modes are determined by a matrix iterative solution aided by a sequence of successive approximate solutions.

### Status of Work

(1) The first area of research mentioned above is concerned with the postbuckling behavior of noncircular cylindrical shells subjected to combined loading of axial compressive forces and bending moments. This problem is highly nonlinear in nature and, hence, a numerical approach was judged to be most promising. Nevertheless, certain analytical ground work must be carefully prepared. Specifically, in contrast to the cases of pure bending, which now have been completed, the loading technique must be precisely defined. This consideration involves the question of whether the generalized loading forces (axial force and moment) or the generalized boundary deformations (end shortening and end rotations) should be prescribed. Since the present problem has two pairs of such quantities, there exist four possible permutations, each of which must be analyzed separately, since the superposition principle is not valid for a nonlinear equilibrium problem. It was found that the four permutations do not represent similar degrees of mathematical complexity. For example, when bending moments are prescribed, the ends of the cylinder undergo both a rotation and an axial translation, while prescribing the axial forces results only in an end shortening, but no rotation. As a result, it was decided that efforts be concentrated on the case which prescribes the end rotations and axial forces.

2.

Similar to the completed cases of pure compression and pure bending, the process of minimization of the nonlinear energy functional is being carried out via a numerical iterative method. Each iterative cycle requires the numerical solution of the compatability equation in finite difference form, while the nonlinear equilibrium configurations are "improved" by direct minimization of the total energy, again, numerically, in the sense of optimal, but not in the sense of steepest descent. The required programming work has now been completed, and two degenerate cases of pure bending and of uniform compression have been successfully reproduced.

2) The work on the free vibrations of complete, oval cylindrical shells is a part of an ongoing research program aiming at the systematic development of analytical methods which stem from the general theory of eigenfunction expansions and which are applicable to complex problems of structural dynamics. One such application has been completed and reported on in the POLY-AE/AM Report No. 75-14, AFOSR-TR-76-1067, "Modal Method for Free Vibrations of Oval Cylindrical Shells with Simply Supported or Clamped Ends." Emphasis of work of this type is placed on the complicating effect in the analysis caused by the exact enforcement of edge conditions within the context of modal expansion. When the method of modal expansion developed in the aforementioned report for cylindrical shells with supported ends was carried over to deal directly with finite shells with free ends, this procedure proved to be inadequate. Accordingly, efforts were applied to the development of the analytical technique required to handle this class of problems (free ends), and its usage has proved successful. A technical report, POLY-M/AE Report No. 76-1, AFOSR-TR-76-1203, entitled "Modal Method for Free Vibration of Oval Cylindrical Shells with Free Ends", by Y. N. Chen and Joseph Kempner has been approved and will be distributed shortly. This work differs from the work on free vibrations of supported oval cylinders mainly in two aspects. First of all, the completeness of the eigenfunctions employed requires the incorporation of the first symmetric modes and the first antisymmetric modes, which, in common practice, were vaguely identified with the Rayleigh-Love approximate

modes. These modes can be treated separately, as had been done heretofore when dealing with the dynamics of circular cylindrical shells. However, it was found during the course of the present work that the omission of these modes would result in completely erroneous behavior of the higher modes due to the strong coupling stemming from the presence of variable curvature through both the energy functional and the boundary conditions. Secondly, although each of the eigenfunctions included in the set satisfies the edge conditions of free ends exactly, the resulting tractions at the ends of the "free" oval cylinder do not vanish identically in contrast to the cases of supported ends. In the present work the unbalanced tractions (forces and moments) are forced to vanish through the utilization of Lagrange multipliers.

Similar to the cases of supported oval shells, two modes of deformation, corresponding to a "higher" and a "lower" frequency were also observed to exist for every pair of nominal axial and circumferential wave numbers, depending upon the degree of circumferential symmetry in the deformed pattern. For the reason of economy in computer time, numerical results were presented only for the "lower" modes. It is believed, however, that the existence of the "higher" and "lower" modes for a given oval geometry and given axial and circumferential wave numbers is a general characteristic of oval shells in contrast to circular shells. Such a behavior has been observed in both the vibration and linear buckling of such configurations.

3) As a natural extension of the work on the supported and unsupported oval cylindrical shells, another important area of application of the modal technique is the free vibration of open cylindrical shells (or panels) having variable radii of curvature. In fact, even the problem of the circular cylindrical panel is not trivial, unless the circular arc length coincides with a circumferential half-wave length. As is well known, the encountering of additional boundaries in any boundary value problem results in an increase of mathematical complexity.

Presently, the method being developed is restricted to open cylindrical shells supported at the ends of the cylinder. Such a restriction enables one to concentrate on the straight edges parallel to the generators, since the appropriate boundary conditions are satisfied at the supported ends upon the incorporation of the eigenfunctions in the variational functional. The unbalanced tractions and/or the excessive displacements on the straight edges are now functions of the axial coordinate. This behavior departs from that encountered when treating the free edges at the ends of a closed oval

cylinder since, in the latter case, terms of "residual" stresses were expanded circumferentially, and are not functions of the coordinates. The present approach chooses to enforce the boundary conditions pointwise along the straight edges. The resulting additional conditions are then posed as constraints. The remaining steps in handling the Lagrange multipliers are straight forward. This approach includes the ultimate elimination of the Lagrange multipliers as well as the "suppressed modes." In a physical sense, these modes that are eliminated mathematically are still present and contribute to the overall dynamic coupling of the problem. Currently, efforts are being made to develop a suitable computer program. The required modal information has been determined from the work on vibrations previously reported. (Only the eigenvalues are retained while the eigenfunctions are re-computed when needed). It is worth pointing out that in the adopted approach care must be taken to eliminate those constraint conditions that are linearly dependent. For example, when the noncircular arc of the cross section is symmetric, the boundary conditions on one straight edge are identical to those on the other straight edge and, hence, half of these conditions must be eliminated. Such a feature has been built into the computer program being developed and debugged. Since the program is in the testing stage, no final results are available at the present time.

4) The fourth area of analytical research involves the bifurcation type of buckling of reinforced noncircular cylindrical shells compressed by axial forces in the presence of internal or external pressure. The reinforcement includes rings and/or stringers placed either inside or outside of the shell. The shell in question is finite in length and, thus, the influence of various types of end supports must also be investigated. To date, the parametric study initiated during the preceding year has mostly been accomplished. This includes the cases of reinforced shells, with or without lateral pressure, for the types of simple supports known as S2 and S4, the clamped supports of classes C1 and C4, and the cases of reinforced shells with similar edge conditions but without lateral pressure.

5) With regard to the experimental studies, because of the complex behavior of the variable curvature cylinder undergoing vibrations, difficulties have developed in the procedures for detecting high frequency displacement patterns. The attempt to use a speckle pattern interferometric method to observe displacements in real time and to obtain photographs of

the wave shapes in this manner has turned out to be more difficult than was anticipated. A moiré method of the detection and observation is now being adapted, and it is hoped that this technique will serve to accomplish the desired determination of vibration characteristics of the oval cylindrical shells.

Reports and Publications (January 1976 - December 1976)

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