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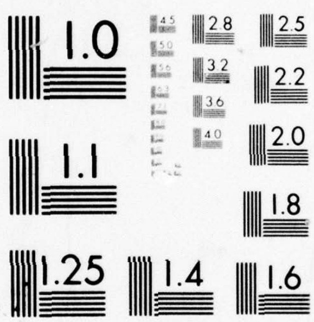

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APPLICATION OF THE EQUIVALENCY PRINCIPLE TO THE SOLUTION OF HEAT CONDUCTIVITY PROBLEMS

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Several possibilities have been demonstrated for applying the equivalency principle to solving heat conductivity problems. In this process at the boundaries of a body the sources and resistances of one type are replaced by others in such a way that the dynamics of the body's temperature field remains unchanged. In particular it has been demonstrated that the solution of the problem of the temperature conditions in a semi-limited body covered with a layer of agitated fluid, in the presence of temperature resistance between the →

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fluid and the body surface and in the presence of given heat flows from the outside and within the liquids, can be expressed as the sum of solutions for two problems with third-order boundary conditions at the surface of a body without fluid.

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NOTICE

§1. If we apply several principles such as superposition, symmetry, equivalency and reciprocity, then by utilizing the solution to known problems it is possible to solve a large number of complex heat conductivity problems in a relatively simple way and at the same time with complete rigor.

The proposed problem solution method, as presented by us, is distinguished by simplicity and by accessibility to persons in various specialties and makes it possible to reanalyze phenomena and discover their physical essence, and the practical use of this method for calculation purposes requires relatively little time.

In work [1] primary attention was devoted to the application of the superposition principle, and article [6] was specially devoted to the application of the reciprocity principle. In this article we will demonstrate possibilities for applying the equivalency principle to solving heat conductivity problems.

§2. The essence of the equivalency principle is that the replacement of one of the similarity conditions which determines the event under consideration with another similarity condition causes no change in the course of the event at any point involved in the given phenomena; this substitution leads to equivalency of the problems, but does not make it possible to model the event. It is necessary, however, to point out that identity may not include all the qualitative characteristics, but simply one under study, for instance, that of heat; the other characteristics, for instance magnetic or mechanical ones, may change in this process.

The equivalency principle is applied in various areas of science, in particular in electronics.

Thus, L. R. Neyman and K. S. Demirchyan ([2], p. 137) write: "... for the sake of convenience in designing electrical circuits it is extremely useful to replace the electromotive force source with an equivalent source of current or to carry out their reverse substitution, to replace the current source with an equivalent EMF source, ...", and also "sources of EMF and current are equivalent if they possess the same external characteristic... in other words, the regime in the receiver should not change when the EMF source is replaced by an equivalent source of current, and vice versa."

As applied to thermal problems, the equivalency principle states that replacing some identity condition has no influence on the thermal conditions of the body under consideration: the course of temperature remains the same at all points.

The equivalency principle indicates the possibility of making an equivalent exchange between heat sources and heat resistances, as well as thermophysical characteristics, geometric form and body dimensions.

§3. There exists two types of heat sources: I_t -- sources of a given temperature and I_s -- sources of a given heat flow intensity. Sources I_s can be both external and internal. Sources I_t are only external. Therefore we can only examine the problem of making equivalent exchanges between external heat sources of one type and external heat sources of another type or external heat sources for internal sources, and vice versa, replacing internal sources with external sources. It is impossible to make an equivalent change of external sources for internal sources since internal sources can only be of one type (I_s).

In the general case the action of external heat sources on the body body is inhibited by thermal resistance at the body surface. Let us recall that two types of thermal resistance are distinguished: temperature resistance R_t and resistance to a heat flow (heat capacity resistance) R_s. Resistance R_t "extinguishes" the action of only temperature sources I_t and does not inhibit the action of sources I_s, where

$$R_t = \frac{1}{\alpha} = \frac{h_t}{\lambda}. \quad (1)$$

On the other hand, resistance R_s "suppresses" only the action of sources I_s; it is equal to

$$R_s = c'\rho' h'. \quad (2)$$

The relationship between heat sources and resistances can be clarified by examining the heat balance equation and the boundary of the body (boundary condition). If at the body surface there is a layer of well agitated liquid, then by setting the origin of the coordinates on the body surface and by directing the x axis into the body perpendicular to the surface, we will have

$$-\lambda \frac{\partial t}{\partial x}\bigg|_{x=+0} = \alpha(\vartheta - t') + S + S' - c'\rho' h' \frac{\partial t'}{\partial \tau}, \quad (3)$$

$$-\lambda \frac{\partial t}{\partial x}\bigg|_{x=+0} = \alpha' (t' - t_{x=+0}). \quad (4)$$

The physical essence of equation (3) is the following. The amount of heat which passes into the body is equal to the algebraic sum of the terms which express the convection between the surface of the liquid and the surrounding medium, the external sources of heat flow including sources located within the liquid layer and finally the change (reduction) in the liquid's heat content.

Equation (4) supplements equation (3) and expresses the law of heat exchange between the liquid and the body when α' is not equal to ∞.

In equation (3) the temperature source is characterized by the temperature of the surrounding medium ϑ, and the sources of heat flow are characterized by values S and S'.

Equation (3) can be represented schematically in the following form:

$$-\lambda \frac{\partial t}{\partial x}\Big|_{x=+0} = \left(\frac{I_t - t'}{R_t}\right) - \left(I_s - R_s \frac{\partial t'}{\partial \tau}\Big|_{x=-0}\right). \qquad (5)$$

The values of the heat sources and resistances are usually given. Equivalency is expressed by replacing one combination of values I_t, I_s, R_t and R_s by another set in such a way that the left-hand portion of equation (5) remains unchanged. Subsequently we will examine certain cases for which we know how to carry out such an equivalent exchange.

§4. The simplest example of an equivalent exchange is the switch from the case where $R_t > 0$ and $R_s = 0$ and external sources of both types act to the case in which the heat resistances remain the same but $I_s = 0$; equivalent compensation for removing source I_s will be a change in I_t, for which instead of ϑ it is necessary to adopt (Figure 1)

$$\vartheta_e = \vartheta + \frac{S}{\alpha}. \qquad (6)$$

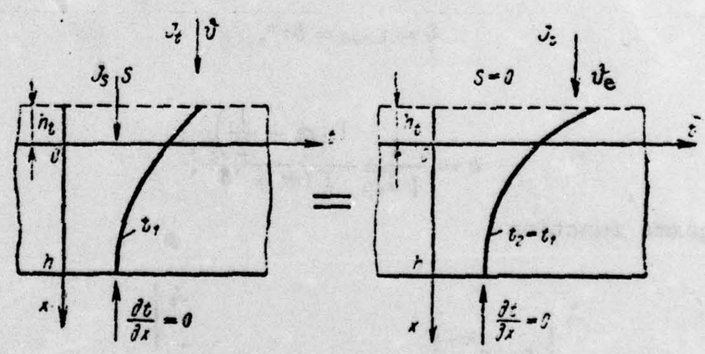

Figure 1. Equivalent Replacement of a Temperature Source and a Heat Flow Source Which Act Simultaneously With One Temperature Source.

Thus, equation (6) expresses an equivalent replacement of sources of two types which act simultaneously on one surface by one temperature source. In the more general case replacing ϑ and S with other equivalent sources should satisfy the relationship

$$\alpha\vartheta_e + S_e = \alpha\vartheta + S,$$

from which

$$\vartheta_e = \vartheta + \frac{S - S_e}{\alpha}. \tag{7}$$

Equation (7) shows the possibility of making an equivalent replacement of one pair of sources by another pair. It is obvious that in the special case where $S_e = 0$, instead of (7) we have relationship (6). During these transitions the thermal resistances remain unchanged: $R_s = 0$ and $R_t = 1/\alpha$.

§5. We will now show the transitions during which not only equivalent sources, but also equivalent resistances are introduced. Our examples will be problems with semi-limited bodies.

Assume that at the surface of a semi-limited body there acts only heat source I_s, where $R_s = 0$:

$$-\lambda \frac{\partial t}{\partial x}\bigg|_{x=0} = S. \tag{8}$$

A source of type I_s can be replaced by a source of type I_t, and in this case instead of $R_s = 0$ it is necessary to assume that $R_t = 0$ (Figure 2).

Thus, if

$$S = k\tau^{m-\frac{1}{2}}. \tag{9}$$

then the equivalent transition will occur if instead of S a temperature source acts which, when $t_0 = 0°C$, is equal to [3]:

$$\vartheta = t_{x=0} = b\tau^m, \tag{10}$$

where

$$b = \frac{k}{\sqrt{\lambda c \rho}} \frac{\Gamma\left(m + \frac{1}{2}\right)}{\Gamma(m+1)}; \tag{11}$$

$\Gamma(z)$ is a gamma function.

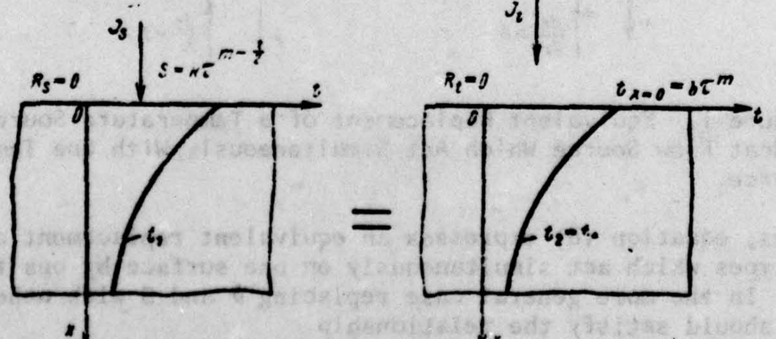

Figure 2. Equivalent Replacement of a Heat Flow Source by a Temperature Source.

§6. Let us now examine the case where at the surface of a semi--limited body there is a layer of liquid ($R_s > 0$) of thickness h' and external sources of type I_s act (Figure 3). The boundary condition will have the form:

$$-\lambda \left.\frac{\partial t}{\partial x}\right|_{x=+0} = S + S' - c'\rho' h' \left.\frac{\partial t}{\partial \tau}\right|_{x=-0}. \quad (12)$$

We will call (12) a fifth-order boundary condition (BC-V). We first replace the liquid with a volumetric heat capacity $c'\rho'$ with a liquid with a volumetric body heat capacity $c\rho$; then, in order for the heat resistance R_s to remain unchanged, the thickness of the liquid layer should change:

$$h_s = h' \frac{c'\rho'}{c\rho}. \quad (13)$$

In addition, it is convenient to represent external sources of type I_s in the form of volumetric sources located in the liquid; their intensity should be equal to:

$$q_v = \frac{S - S'}{h'} \frac{c\rho}{c'\rho'}. \quad (14)$$

By making allowance for (13) and (14), it is possible to write boundary condition (12) in the following form:

$$-\lambda \left.\frac{\partial t}{\partial x}\right|_{x=+0} = q_v h_s - c\rho h_s \left.\frac{\partial t}{\partial \tau}\right|_{x=-0}. \quad (15)$$

The solution to such problems can be obtained as follows. First it is necessary to divide the problem into two component problems, and then to transform these problems into problems with I and III-order BC with the aid of the equivalency principle. The solutions to the latter are usually known and their sum is the solution to the initial problem.

The means of dividing the initial problem into two components and their equivalent transformation are shown in Figure 3. In both component problems the thickness of the liquid layer at the body surface is very (infinite) great. In the first component problem the sources are located throughout the liquid ($-\infty < x < 0$).

Since in the initial problem (the one to be solved) the sources act only in the area $-h_s \leq x < 0$, then in the second component problem the sources of the reverse sign ($-q_v$) are located in the area $-\infty < x < h_s$. We will now complete the equivalent transitions in each component problem after replacing in them the heat flow sources with temperature sources and the heat capacity resistances with temperature resistances. In both problems the equivalent temperature sources are the same with regard to absolute value and are equal to:

$$\vartheta = t_{ad} = \frac{1}{c\rho}\int_0^{\tau}(\pm q_v)\,d\tau = \frac{\mp 1}{h'c'\rho'}\int_0^{\tau}(S+S')\,d\tau. \qquad (16)$$

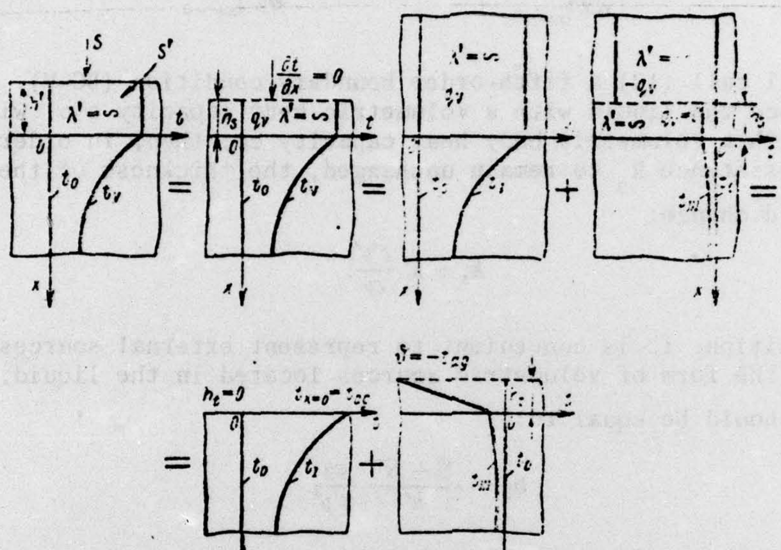

Figure 3. Equivalent Replacement of Heat Sources and Thermal Resistances.

The equivalent temperature resistances can be determined on the basis of the necessity for observing the equality

$$h_t = h_s, \qquad (17)$$

where

$$h_t = \frac{\lambda}{\alpha}. \qquad (18)$$

Considering (1), (2), (13), (17) and (18), we find the condition for shifting to the equivalent problem

$$R_t = \frac{R_s}{\lambda c\rho} \qquad (19)$$

or according to (13), (17) and (18)

$$\alpha = \frac{\lambda}{h_s} = \frac{\lambda c\rho}{h'c'\rho'}. \qquad (20)$$

In the first component problem $R_s = 0$ and $h_s = 0$, and therefore according to (19) and (20) after the equivalent transition $R_t = 0$, $\alpha = \infty$, and consequently $h_t = 0$, i.e., I-order BC occur.

In the second component problem $R_s > 0$ and $h_s > 0$, and therefore $R_t > 0$, $\alpha < \infty$ and $h_t > 0$, which corresponds to III-order BC.

Thus, a solution to the problem for a semi-limited body with a layer of agitated liquid on its surface and with external sources of heat flow (IV-order BC) is found as the algebraic sum of the solutions to the two component problems of which one is the course of the surface temperature (I-order BC) and the other gives the temperature course of the medium (III-order BC)

$$t_{V} = t_{I} + t_{III}. \tag{21}$$

The temperatures of the surface and the medium are equal in absolute value, but they have different signs (16).

Let us examine two special cases.

First case: S + S' = const. According to (16), it is necessary to assume the surface temperature of the body and the temperature of the medium change linearly:

$$t_{x=0} = b\tau, \tag{22}$$

$$\vartheta = -b\tau, \tag{23}$$

where

$$b = \frac{S+S'}{h'c'\gamma'}. \tag{24}$$

The value of the factor of heat emission from the body surface is determined from formula (20). The solutions for the two component problems are known, and for each of them there are computation graphs ([1], pp. 113 and 122); according to (21) their sum is a solution to the problem at hand.

Second case: S + S' = 0, the initial temperature in the body is equal to 0 but in the liquid layer h' differs from zero, $t_x = 0 = t'_0 \neq 0$.

The initial heat content of the liquid may be represented as the result of the action at moment $\tau = 0$ of an instantaneous source I_s which emits (per unit of body surface area) a quantity of heat equal to $W = c\rho h_s t'_0$. It is obvious that the integral in (16) is equal to W and consequently in the given special case

$$t_{x=0} = t_0', \qquad \vartheta = -t_0'. \tag{25}$$

Therefore t_I and t_{III} are solutions to component problems at constant body surface $t_x = 0 = t'_0$ and medium $\vartheta = -t_0$ temperatures.

As in the first case, the solutions to both component problems are known (see, for instance [1], pp. 103 and 110, or [4], pp. 76 and 183), and therefore according to (21) the solution to this second case can also be regarded as known.

We have selected these two special cases since solutions have already been found for them, and this makes it possible to determine whether our formula (21) is correct. Solutions for both cases are given in [5] (p. 301, formulas 4.11 and 4.12). A careful examination makes it possible to determine that each solution actually is the sum of the solutions of the two problems and completely coincides with solution (21).

However, if the method of solving the thermal conductivity problems of semi-limited bodies covered with a layer of water is correct when I_s = const and when there is instantaneous emission of heat, then it is obvious that according to the superposition principle it is also correct with any law of I_s change. Thus, the correctness of solution (21) is confirmed for more general cases as well.

Example. The following are given: on the surface of the ground (λ = 1.49 kcal/m·hr·deg; c = 0.321 kcal/kg·deg; ρ = 1960 kg/m^3; a = 2.37·10^{-3} m^2/hr) there is a layer of well agitated water ($\lambda' = \infty$, c' = 1 kcal/kg·deg; ρ' = 1000 kg/m^2) of thickness h' = 0.5 m. The initial temperature of the ground and the water is $t_0 = t_0' = 6°C$. A heat flow, the intensity of which is equal to S = 20 kcal/m^2/hr, enters the water.

Find the temperature of the water and the temperature of the ground at depth x = 1.0 m after τ = 1 month.

Solution. According to the equivalency principle this task with II-order BC and with a layer of liquid can be represented as the sum of two problems with I and III-order BC, but without liquid (see formula 21). The intensity of the temperature sources of heat in the equivalent problems is found from formulas (22) and (23):

$$t_{x=0} = \frac{S\tau}{c'\rho'h'} = \frac{20\tau}{1\cdot 1000\cdot 0{,}5} = 0{,}04\tau,$$

$$\vartheta = -\frac{S\tau}{c'\rho'h'} = -0{,}04\tau;$$

according to (20):

$$\alpha = \frac{\lambda c\rho}{h'c'\rho'} = \frac{1{,}49\cdot 0{,}321\cdot 1960}{0{,}5\cdot 1\cdot 1000} = 1{,}87 \text{ kcal/m}^2\cdot\text{hr}\cdot\text{deg}.$$

With regard to conditions the component (equivalent) problems coincide with problems no. 4 and no. 7 according to [1], and therefore for practical calculations it is possible to utilize computation graphs on other materials given in [1] on pages 113-115 and 122-124.

The computation formula for the temperature of the ground surface has the form:

$$t = t_0 + b\tau(1 - \theta),$$

and the initial argument for determining temperature parameter θ is

$$M = \frac{\alpha^2 a\tau}{\lambda^2} = \frac{(1{,}87)^2 \cdot 2{,}37 \cdot 10^{-3} \cdot 720}{(1{,}49)^2} = 2{,}68.$$

We find $\theta = 0.57$ and we calculate the desired temperature:

$$t = 6 + 0{,}04 \cdot 720\,(1 - 0{,}57) = 18{,}4°\mathrm{C}.$$

The ground temperature at depth $x = 1.0$ m is found according to the formula

$$t = t_0 + b\tau(\theta_1 - \theta_2),$$

and the initial arguments for determining temperature parameters θ_1 and θ_2 are equal to:

for value θ_1

$$\mathrm{Fo} = \frac{a\tau}{x^2} = \frac{2{,}37 \cdot 10^{-3} \cdot 720}{1} = 1{,}71;$$

for value θ_2

$$\mathrm{Fo} = \frac{a\tau}{x^2} = 1{,}71;\ \mathrm{Bi} = \frac{\alpha x}{\lambda} = \frac{1{,}87 \cdot 1}{1{,}49} = 1{,}3.$$

Finding that $\theta_1 = 0.4$ and $\theta_2 = 0.2$ from the computation graphs, we calculate the required temperature:

$$t = 6 + 0{,}04 \cdot 720\,(0{,}4 - 0{,}2) = 11{,}8°\mathrm{C}.$$

If beginning at $\tau = \tau_1 = 0.5$ months, $S = 50$ kcal/m^2·hr had occurred, then the solution would have to have been sought as follows. It is necessary to divide the problem into two components. One completely coincides with the solution of the above-given problem. In the second it is necessary to begin calculating time from τ_1, i.e., to accept instead of τ the value $(\tau - \tau_1)$ and to assume that the initial temperature is $t_0 = t'_0 = 0°$C and $S = 50 - 20 = 30$ kcal/m^2·hr.

§7. Solution (21) was obtained above based on general considerations of the possibility of dividing a complex problem into the sum of two simple problems and then the possibility that equivalent conversions could be carried out. However, from the methodological point of view it would be equally correct to obtain solution (21) in another way, specifically by making a structural analysis of one of the available solutions to the special cases and by subsequently generalizing the solution obtained. The first method can probably be regarded as the deduction method, and the second as the induction method. To solve the following more complex problem, we will utilize this second investigation method.

§8. We will examine a problem which is distinguished from the one examined above in §6 by the fact that between the liquid and the semi-limited body there is temperature resistance R_t, i.e., the factor of heat emission from the liquid to the body surface α is not infinitely great. The boundary condition in this case is expressed by the above-described system of equations (3) and (4).

An analytical solution to this problem is known for the simplest case where S + S' = 0 and the initial temperature of the body is equal to zero, while the temperature of the liquid is equal to t_0' (see [5], pp. 301 and 302, formulas 4.17 and 4.18). In our writing system this solution after very simple transformations has the following form:

$$t = t_0' \frac{1/h_t}{\beta - \gamma}\left[\exp(\gamma x + \gamma^2 x^2 \mathrm{Fo})\,\mathrm{erfc}\left(\frac{1}{2\sqrt{\mathrm{Fo}}} + \gamma x\sqrt{\mathrm{Fo}}\right) - \exp(\beta x + \beta^2 x^2 \mathrm{Fo})\,\mathrm{erfc}\left(\frac{1}{2\sqrt{\mathrm{Fo}}} + \beta x\sqrt{\mathrm{Fo}}\right)\right], \tag{26}$$

here the values β and γ are the roots of the quadratic equation

$$q^2 + \frac{1}{h_t} q + \frac{1}{h_t h_s} = (q + \gamma)(q + \beta)^*. \tag{27}$$

We will compare solution (26) with the solution to the problem of heat propagation in a semi-limited body with a zero initial condition and a constant ambient temperature which, as is known, has the following form (see [5], p. 77):

$$\frac{t}{\vartheta} = \mathrm{erfc}\frac{1}{2\sqrt{\mathrm{Fo}}} - \exp\left(\frac{x}{h_t} + \frac{x^2}{h_t^2}\mathrm{Fo}\right)\mathrm{erfc}\left(\frac{1}{2\sqrt{\mathrm{Fo}}} + \frac{x}{h_t}\sqrt{\mathrm{Fo}}\right). \tag{28}$$

We note that the first right-hand term describes the temperature change in the case of I-order BC, and the second term is a correction equal to the retardation of the heating (cooling) process due to the fact that III-order BC occur, i.e., at the surface there exists temperature resistance. It is easy to see that solution (26) is the

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*In [5] there is a misprint: instead of (q + γ) there stands (q - γ); in the English-language original there is no misprint.

arithmetic difference between solutions (28) for the two problems; in these two problems

$$\vartheta = t_0' \frac{1/h_t}{\beta - \gamma}, \quad (29)$$

and consequently in the first problem

$$h_{t_1} = \frac{1}{\beta}, \quad (30)$$

and in the second problem

$$h_{t_2} = \frac{1}{\gamma}. \quad (31)$$

Thus it is proven that solution (26) can be expressed as:

$$t_V = t_{III.1} - t_{III.2}, \quad (32)$$

where $t_{III,\,1}$ and $t_{III,\,2}$ are correspondingly solutions (28) to problems with zero initial conditions, constant and uniform ambient temperatures and various heat emission factors α_1 and α_2. According to (18), (30) and (31) values α_1 and α_2 are equal to

$$\alpha_1 = \lambda\beta \quad (33)$$

$$\alpha_2 = \lambda\gamma. \quad (34)$$

We still have to find what β and γ are equal to; for this purpose we will utilize equation (27).

It is obvious that

$$\frac{1}{h_t} = \gamma + \beta, \quad (35)$$

$$\frac{1}{h_t h_s} = \gamma\beta. \quad (36)$$

Consequently

$$h_t = \frac{1}{\gamma + \beta} = \frac{1}{\frac{1}{h_{t_1}} + \frac{1}{h_{t_2}}} = \frac{\lambda}{\alpha_1 + \alpha_2}, \quad (37)$$

$$h_s = \frac{1}{\beta} + \frac{1}{\gamma} = h_{t_1} + h_{t_2} = \lambda\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right), \quad (38)$$

from which we find an equation system which determines the values α_1 and α_2:

$$\alpha' = \alpha_1 + \alpha_2, \quad (39)$$

$$\frac{h_s}{\lambda} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \quad (40)$$

substituting (39) into (40):

$$\alpha_1^2 - \alpha'\alpha_1 - \frac{\lambda}{h_s}\alpha' = 0, \quad (41)$$

from which we finally find

$$\alpha_1 = \frac{\alpha'}{2}\left(1 + \sqrt{1 - \frac{4\lambda}{\alpha' h_s}}\right) \quad (42)$$

and

$$\alpha_2 = \frac{\alpha'}{2}\left(1 - \sqrt{1 - \frac{4\lambda}{\alpha' h_s}}\right). \quad (43)$$

The value of the ambient temperature is found by substituting (18), (23) and (34) into (29):

$$\vartheta = t_0' \frac{\alpha'}{\alpha_1 - \alpha_2} \quad (44)$$

or, by determining the values of the heat emission factors we obtain

$$\vartheta = t_0' \frac{1}{\sqrt{1 - \frac{4\lambda}{\alpha' h_s}}}. \quad (45)$$

From the superposition principle it follows that when S + S' ≠ 0 and $t_0 \neq 0$, it is necessary to assume that the ambient temperature for the two component problems is equal to

$$\vartheta = \left(t_0' - t_0 + \frac{1}{h'c'\gamma'}\int_0^{\tau}(S + S')\,d\tau\right)\frac{1}{\sqrt{1 - \frac{4\lambda}{\alpha' h_s}}}. \quad (46)$$

Thus, the solution to the problem concerning the temperature conditions in a semi-limited body covered with a layer of agitated liquid, in the presence of temperature resistance between a liquid and the body surface (α' ≠ ∞) and at given thermal flows from the outside and within the liquid can be expressed as the difference between the solutions to two problems with III-order BC and the temperature of the medium is determined by equation (46), while the heat emission factors are determined by functions (42) and (43), (Figure 4).

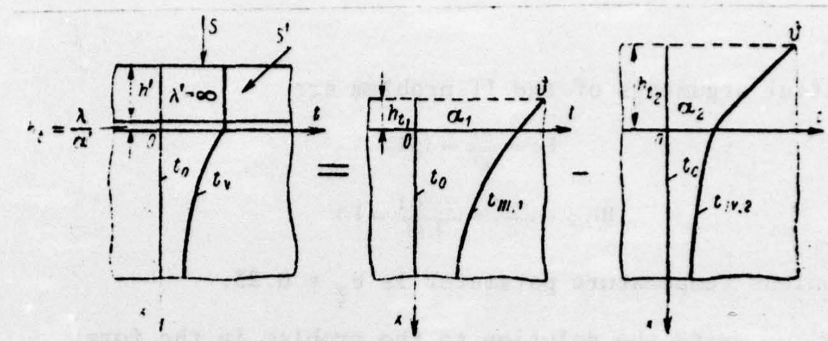

Figure 4. Diagram of the Solution to the Problem of the Temperature Regime of a Semi-Limited Body, Covered with a Layer of Agitated Liquid.

Example. The following are known: the conditions of the problem are the same as in §6, but the heat emission factor from the water to the ground is not infinitely great, but equals $\alpha' = 14$ kcal/m^2·hr·deg.

Find the temperature of the ground at depth $x = 1.0$ m after $\tau = 1$ month.

Solution. In accordance with §8 above the solution to the problem under consideration with IV-order BC can be presented as the difference between the solutions to two problems with III-order BC. The boundary conditions in the component problems can be found from formulas (46), (42) and (43). We will determine the equivalent thickness of the heat capacity resistance from formula (13):

$$\vartheta = \frac{S\tau}{c'\rho'h'} \frac{1}{\sqrt{1 - \frac{4\lambda}{\alpha' h_s}}} = b\tau = 0{,}0588\tau;$$

$$h_s = h' \frac{c'\rho'}{c\rho} = \frac{0{,}5 \cdot 1{,}0 \cdot 1000}{0{,}321 \cdot 1960} = 0{,}794 \text{ м};$$

$$\alpha_{\text{III},1} = 7\,(1 + 0{,}681) = 11{,}77 \text{ kcal/m}^2\cdot\text{hr}\cdot\text{deg},$$

$$\alpha_{\text{III},2} = 7\,(1 - 0{,}681) = 2{,}23 \text{ kcal/m}^2\cdot\text{hr}\cdot\text{deg}.$$

With regard to conditions the component problems are identical to problem No. 7 [1], and therefore in solving the problem it is possible to utilize the computation graphs and other materials given in [1] on pages 122-124.

The initial arguments of the I problem are:

$$\text{Fo}_x = \frac{a\tau}{x^2} = \frac{2{,}37 \cdot 10^{-3} \cdot 720}{1} = 1{,}71.$$

$$\text{Bi}_{1,x} = \frac{\alpha_1 x}{\lambda} = \frac{11{,}77 \cdot 1}{1{,}49} = 7{,}9.$$

The dimensionless temperature parameter is $\theta_1 = 0.35$.

The initial arguments of the II problem are

$$Fo = \frac{a\tau}{x^2} = 1{,}71,$$

$$Bi_{2.x} = \frac{\alpha_2 x}{\lambda} = \frac{2{,}23 \cdot 1}{1{,}49} = 1{,}5.$$

The dimensionless temperature parameter is $\theta_2 = 0.23$.

Finally, we write the solution to the problem in the form:

$$t_V = t_0 - (t_{III,1} - t_{III,2}),$$

$$t_{III,1} = \theta_1 b\tau,$$

$$t_{III,2} = \theta_2 b\tau,$$

$$t_V = t_0 + b\tau(\theta_1 - \theta_2) = 6 + 0{,}0588 \cdot 720 \cdot 0{,}12 = 6 + 5{,}08 = 11{,}08°C.$$

§9. It is possible to make an equivalent replacement of one body with some thermophysical characteristics by means of the same body having other thermophysical characteristics.

An examination of known analytical solutions presented in criterional form makes it possible to readily perceive the equivalent exchanges which are possible. For instance, problem solutions (see, for example [1], problem no. 16) which have the form $\theta = \frac{\lambda(t - t_0)}{Sh} = f(Fo)$ make it possible to state that we can make an equivalent transition from a body with λ_1 to a body with λ_2, but on the condition that there is a corresponding change in the values of S at which θ = idem occurs, while of course keeping the temperature conductivity value a constant by changing the factor of volumetric heat capacity $c\rho$.

§10. An equivalent exchange of a body of one form with a body of another form or other dimensions can be carried out by arbitrarily changing (reducing, increasing, deforming) that portion of the body which in practical terms is not reached by the action of heat sources. For instance, if the action of heat sources located at the surface of the plate $x = 0$ penetrate only to $x = x_1$ and if there are no other heat sources, then thickness of the plate can be changed arbitrarily within the limits $x_1 < x \leq \infty$, and the adiabatic surface of the plate can be deformed as desired, but no point should come closer than distance x_1 to surface $x = 0$.

None of these changes should influence the thermal conditions of the plate.

Conclusions

The concept of equivalency is utilized successfully in various areas of science. The equivalency problem has become widely utilized, for instance in electronics. In heat conductivity theory and especially in its practical application (in design) unjustifiably little use is made of equivalent transitions. It is necessary to direct attention to the broader utilization of already known equivalent transitions, as well as to the discovery of new laws. Equivalent replacements of some identity conditions by means of others may prove to be extremely useful not only for new problems and for analyzing the physics of phenomena, but also in establishing experimental investigations.

Designations

x -- coordinate;
τ -- time;
λ -- heat conductivity factor of body;
c -- specific heat capacity of body;
ρ -- body density;
λ' -- heat conductivity factor of agitated liquid ($\lambda' = \infty$);
c' -- specific heat capacity of liquid;
ρ' -- density of liquid;
α -- heat exchange factor from air to body surface or to surface of liquid layer;
α' -- heat exchange factor of body from layer of agitated liquid to surface of solid;
h -- plate thickness;
h' -- thickness of agitated liquid layer;
h_t -- thickness of temperature resistance layer;
h_s -- thickness of heat capacity resistance layer;
R_t -- temperature resistance;
R_s -- resistance to heat flow;
t -- body temperature;
t_0 -- initial body temperature;
t' -- temperature of agitated liquid layer;
t'_0 -- initial temperature in layer of agitated liquid;
ϑ -- temperature of medium;
b -- constant which is proportional to the temperature at the body surface or the temperature of the medium;
q_v -- the temperature of a volumetric uniformly distributed source of heat (per unit of body surface area);
S -- density of heat flow from the outside;
S' -- total intensity of heat sources in agitated liquid layer (per unit of body surface area);
k -- constant proportional to the value of heat flow;
t_{ad} -- adiabatic temperature change under the action of internal heat sources;
W -- amount of heat, heat content;
m -- real number;

I_t -- heat source of given temperature;

I_s -- heat source of given intensity;

BC -- boundary condition.

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