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COST BENEFIT ANALYSIS AND THE NATIONAL AVIATION SYSTEM - A GUID--ETC(U)

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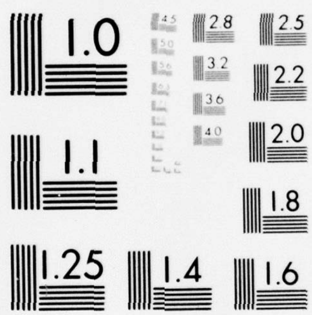

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COST-BENEFIT ANALYSIS AND THE NATIONAL AVIATION SYSTEM

- A GUIDE -

February 1977

FINAL REPORT

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16. Abstract

This manual contains a discussion of cost-benefit methodology as it applies to the national aviation system, an explanation of selected values for potential use in Federal Aviation Administration studies, and the principles, concepts, and techniques appropriate to estimating benefits and life-cycle costs. In addition, parameters useful for valuing changes in capacity, delay, and aviation safety are presented.

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PREFACE

The Federal Aviation Administration decided that a guide to the conduct of cost-benefit analysis would be most useful. The guide would contain concepts and techniques, and recommended values for reducing passenger and aircraft delays, accidents and fatalities, and other relevant impact variables. J. Watson Noah Associates, Inc., under contract #DOT-FA76WA-3769 dated December 30, 1975, reviewed relevant literature and compiled this guide for the FAA Office of Aviation Policy (AVP-210).

This manual contains a discussion of cost-benefit methodology as it applies to the national aviation system, an explanation of selected values recommended for use in FAA studies, and the principles, concepts and techniques appropriate to estimating benefits and life-cycle costs.

A GUIDE TO COST-BENEFIT ANALYSIS IN THE FEDERAL AVIATION ADMINISTRATION

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INTRODUCTION

Background

Two of the Federal Aviation Administration's most important functions are: (a) provision of airport and airway navigation facilities and traffic control, and (b) administration of the airport-airway program. The primary regulatory role of the FAA is to set and maintain minimum standards for air safety. The Administrator of the FAA -- often through formal rulemaking proceedings -- issues and enforces rules, regulations, and minimum standards pertaining to the manufacture, operation, and maintenance of the civil air fleet. The FAA also certifies new aircraft, inspects flight navigation facilities, and certifies private and commercial pilots. Registration of civil aircraft, and research and development activities pertaining primarily to aircraft, airport, and airway safety also fall within FAA responsibility.

The problem of resource allocation is directly related to FAA's fulfillment of its responsibilities. The principles of cost-benefit analysis form a basis for efficiently allocating resources. The cost-benefit method is particularly useful to the analysis of alternatives proposed for research and development with respect to FAA navigation facilities; to its installations and operations; and to acquiring and operating a system of air traffic control and navigation. Cost-benefit analysis is a formal procedure for comparing the costs and benefits of alternative policies and investment projects. The formal procedure and basis of comparison rely on specialized techniques and principles, most of which derive from economic theory.

This report draws on that economic theory to develop guidance useful to FAA analysts charged with the responsibility for evaluating the preferredness of investment alternatives. The goal of this report is to provide guidance in a practical manner. The controversy surrounding the theory and measurement of certain parameter values necessitates a judicious selection from among theoretical bases while at the same time attempting to enhance the measurement of parameter values. It is our opinion that immeasurable but correct theory is no more useful than precise measurements based on incorrect theory. It is our hope that the parameter values prescribed in this report are about right rather than precisely wrong.

Purpose of the Guide

As mentioned above, the primary purpose of this guide is to provide FAA analysts with a useful reference for the conduct of cost-benefit studies. The guide stresses analysis of alternative investment projects. However, as a logical extension, the method of cost-benefit analysis is equally relevant to assessing consequences of alternative public policies. For example, a particular service might be encouraged either by a system of taxation or by direct regulation. Neither of these options may involve significant public investment, but a cost-benefit comparison may be useful in making the final choice between the alternatives.

Organization of the Remainder of the Guide

A discussion of special topics relating to cost-benefit methodology is followed by specific techniques for comparing alternative series of costs and benefits, giving appropriate recognition to time preference. Three techniques, each representing an investment criterion, are presented: equivalent uniform annual value, present value, and internal rate of return.

Within the framework of estimating benefits, some useful values relating to capacity, delay, and safety are presented in Section 4. Section 5 discusses problems related to time preference, including a statement of a position taken by the Office of Management and Budget. Techniques for normalizing data to account for varying quantities and rates of output, and varying price levels are followed by a discussion of estimating relationships. Section 8 presents the framework for an FAA life cycle cost model in general terms, and that is followed by a summary of mathematical/statistical techniques potentially useful to the cost-benefit analyst. Two of those techniques -- statistical regression analysis and linear programming -- are examined more thoroughly in Appendices A and B, respectively.

Summary

The basic concept underlying this guide is that cost-benefit analysis is a formal procedure for comparing costs and benefits of alternative investment actions to isolate the preferred action. Our discussion of concepts and techniques for applying investment criteria notes that the present value technique is generally advocated by economists writing on the subject of cost-benefit analysis.

Benefits accruing to users of the national aviation system and to society as a whole are dependent upon the provision of adequate capacity, minimal delay, and reasonable safety. Techniques for measuring those benefits quantitatively are discussed, and the values are summarized in Table 4.9.

We recommend that 10 percent be used as the discount rate in evaluating FAA alternatives on a cost-benefit basis, in keeping with OMB Circular A-94.

The remainder of the guide discusses specific techniques for normalizing data, for deriving estimating relationships, and for developing a life-cycle cost model. Finally, a brief examination of mathematical and statistical techniques related to cost-benefit analysis suggests the two most useful techniques are statistical regression analysis and linear programming.

2

THE COST-BENEFIT METHOD

Basic Concepts

The concept of cost-benefit analysis used in this guide is one of analyzing government decisions to use resources. The purpose of cost-benefit analysis is to aid the government decisionmaker in the fundamental economic task of allocating scarce resources to alternative uses. An improvement in economic efficiency of the national aviation system is attainable if it is possible to increase the value of the output of that system for any given amount of resource input.

Cost-benefit analysis is, therefore, a formal procedure for comparing the costs and benefits of alternative investment actions. Benefits and costs of a single investment action may be assessed and compared to the "do nothing" alternative. The spectrum of resource allocation problems is very broad. At one extreme is the anarchistic approach of allocating resources by whim, completely foregoing equity and consideration of the whole of society. On the other extreme, the ideal prescription for resource allocation is to maximize a weighted sum of all society's objectives by an efficient allocation of resources. However, such an ideal is, and will remain, unattainable. We cannot know how to weigh one objective against all others, nor could so huge a policy analysis be undertaken even if we knew the appropriate weights to place upon objectives. (Fisher, 1970).

Conceptual problems in cost-benefit studies stem from these inevitable analytic deficiencies. We are forced through imperfection to address narrower, more tractable questions, ones which fall somewhere between chaos and perfect order. The result is that the spillovers, incidental or unintentional effects of resource allocation, cannot be taken into account, although they may be quite important. For example, it is extremely difficult, if not impossible, to place monetary value on the aesthetic quality of a new airline terminal. Therefore, we must carefully structure our studies to make appropriate, although necessarily imperfect, allowances for beneficial or detrimental spillovers which raise difficult conceptual and practical issues.

A related activity, cost-effectiveness analysis, estimates the costs of alternative methods to achieve a given policy objective. In assessing alternatives, the cost-effectiveness procedure may take either one of two fundamental forms: a desired level of effectiveness may be specified, and the analysis seeks the most economical way to achieve it; or a level of expenditure may be specified, and the analysis explores the effectiveness offered by system variations. (Breckner and Noah, 1967). The cost-effectiveness framework may be applied in the case of a cost-benefit analysis. However, it is not essential that one or the other -- costs or benefits -- be fixed at a given level while examining variations in the other. The reason is that we are able to measure the value of benefits and costs in terms of dollars that are commensurable.

There is no all-purpose criterion, or test, for preferred policies. The appropriate test depends upon what alternatives are open to the decisionmaker, upon what aspects of the situation must be taken as given, and even upon what kinds of measurements are feasible (McKean, 1958). Briefly, however, to maximize the difference between benefits and costs is certainly an acceptable criterion -- the equivalent of making the most out of whatever actions can be taken. In reality, there are constraints which must be taken into account. In many cost-benefit analyses, as in most cost-effectiveness analyses, a constraint is that a particular scale of benefit or cost is fixed. This reality forces us to <u>fix</u> either the costs or the benefits, seeking the way to get the most for a given cost, or to achieve a specified objective at least cost.

These two criteria are equally <u>acceptable</u>. The benefit/cost ratio, on the other hand, is useful for ranking a list of possible actions when the scale of activity is fixed, and the actions are not interdependent.

However, the ratio of benefits to costs is inherently an incorrect criterion. (McKean, 1958; Grant, 1964). More will be said later about the fallacy of using benefit/cost ratios.

Finally, it is inherent in government enterprises that market prices cannot be used in appraising their social contribution. And yet, some economic basis is needed for judging which potential government undertakings are worthwhile and which are not. Cost-benefit analysis provides this basis; it is closely analogous to methods of investment analysis used in the market place. The essential difference is that estimates of social value are used in place of estimates of revenues. Cost concepts useful to the evaluation of government undertakings are virtually identical to those used in business.

The Analytical Process

The key elements of a cost-benefit analysis are shown in Exhibit 2.1. After defining the problem and the objective and scope of the analysis -- a critically important step -- the assumptions on which the analysis will rest must be specified. That second step, specifying the assumptions, usually cannot be done exhaustively as a second step. It must wait until we have gained knowledge that can only be obtained by attempts at many of the subsequent steps. However, some major assumptions can and should be specified at the outset.

Implicit in the definition of the problem, objective, and scope of the analysis is often some indication of the alternatives to be examined. A complete list of feasible alternatives requires, however, considerably more thought. Furthermore, some alternatives may surface only after conducting a first iteration of the cost-benefit process examining those alternatives that come to mind immediately.

Having described the first set of feasible alternatives, the process next entails estimating relevant costs and benefits for each alternative. Those costs and benefits should be estimated, whenever reasonably possible, in dollar amounts.

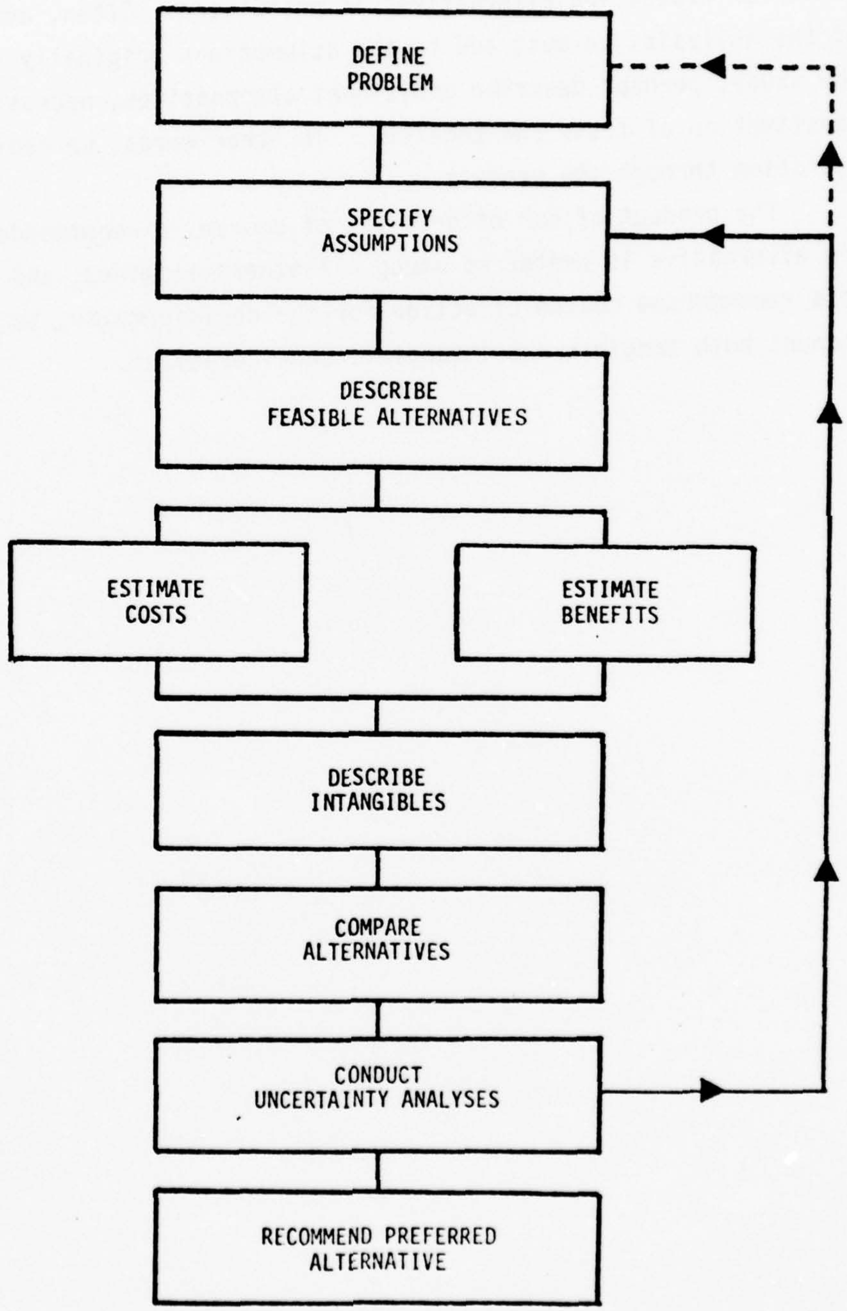
A natural fallout of attempts to estimate costs and benefits quantitatively is the identification of intangibles -- those things we cannot reasonably reduce to dollar amounts. Those intangible considerations should be listed and described for the decisionmaker. They should not be neglected, for likely, they will be extremely important to the final analysis.

Our next step is to compare on the basis of an acceptable criterion (as discussed in the previous section) the costs and benefits of alternatives examined. At this point, or perhaps before this step, the uncertainty involved in both cost and benefit estimates should be examined. The sensitivity of results may be tested for high and low estimates in the uncertain parameters.

After the initial comparison of alternatives, it is useful to con-

EXHIBIT 2.1

THE COST-BENEFIT PROCESS



duct a contingency analysis; that is, examine how the ranking of the alternatives under consideration holds up when a relevant change in criteria for evaluating alternatives is postulated. Often, at this stage of the analysis, we must add to the assumptions originally specified for the study, perhaps describe additional alternatives, necessitating a reestimation of costs and benefits. In other words, we begin our first iteration through the process.

The product of our efforts is, of course, a recommendation that one alternative is preferred among all others examined, and this leads to a recommended course of action for the decisionmaker, taking into account both tangible and intangible considerations.

Issues in Valuation

Those effects of a project which are characterized by increases in consumer satisfaction or decreases in the amount of resources required to produce goods and services may be referred to as benefits. Increases in well-being resulting from a project are usually taken as primary benefits, while demand-inducing effects and other effects generated by the direct output are considered secondary benefits. For example, reduced delays in the terminal control area comprises a primary benefit, while the increased profit accruing to airport concessionaires is a demand-induced secondary benefit.

The valuation of benefits resulting from a project may be thought of in terms of the affected parties' willingness to pay. Implicit in the willingness-to-pay concept is that such benefits should be measured in dollar terms, and that they are therefore tangible benefits. Benefits which cannot be valued in monetary terms or any other real measure are called intangible benefits.

The lack of available data and empirical methods have nothing to do with whether or not a benefit is tangible or intangible. Furthermore, failure to value in monetary terms certain project effects does not mean that they should be excluded from the analysis. The analyst should describe such unmeasured effects as quantitatively as possible, though perhaps not in monetary terms. As mentioned above, such quantitative information is a significant product of a cost-benefit analysis, since there will always exist intangibles which a decisionmaker should consider.

The value of the aesthetic qualities of an airport is, for example, an intangible benefit. Though we are unable to conceptualize a monetary value to assign as a benefit stemming from added aesthetics, we most certainly can -- and should -- ascribe to them the costs they add to the project.

Assessing the true opportunity costs -- the costs of foregone opportunities -- of additions to the national aviation system is a complex matter; the need for understanding some of the underlying analytical problems is essential.

Costs that will be incurred no matter what choice we make, or that must be borne regardless of the decision at hand, are not costs of that particular choice or decision. Those costs are irrelevant. Perhaps the most common distinction drawn between relevant and irrelevant costs is between past (sunk) and future costs. Costs that have already been incurred -- past costs -- are costs resulting from past decisions. For example, the cost of constructing the Air Route Traffic Control Center in Leesburg, Virginia, is a sunk cost. It has already been incurred, and the amount is irrelevant to the analysis of whether or not that facility should continue to operate in the future. All costs associated with its continued operation in the future are relevant, and should be considered in any analysis of alternatives to its continued operation.

Distinctions drawn between various classifications of costs are often more confusing than useful. Dean (1951) distinguishes between outlay and opportunity costs, past and future costs, short-run and long-run costs, variable and constant costs, traceable and common costs, out-of-pocket and book costs, incremental and sunk costs, escapeable and unavoidable costs, controllable and non-controllable costs, and finally replacement and historical costs. Fisher (1971) says that some of those terms -- fixed, variable, sunk, incremental, recurring, nonrecurring, internal, external, and so on -- are useful in distinguishing relevant from irrelevant costs. In the analysis of proposed government projects, we believe the distinction between relevant and irrelevant costs can best be made by keeping in mind that it is always <u>prospective differences</u> between alternatives that are significant in making a choice (Grant, 1964). This means that all past costs and many future ones will be unaffected by a particular choice. If this concept is <u>not</u> kept firmly in mind, an analyst may inadvertently employ average costs per unit when he should be using incremental costs per unit, or the cost for a specific lot of units. This subject is discussed more thoroughly in Section 6.

Benefit-Cost Ratio Pitfall

There is a tendency within various Federal agencies to use the ratio of benefits to costs as the major criterion in evaluating a new project. This tendency stems from the Flood Control Act of June 22, 1936, which recognized that the Federal government should improve watersheds if "the benefits to whomsoever they may accrue are in excess of the estimated costs..." Resultant practice was that engineers divided total benefits by total costs in their search for benefit/cost ratios exceeding one. The use of ratios usually poses no problem as long as the analysis is conducted in the framework mentioned earlier; i.e., fixing the level of benefits or the level of costs for all alternatives under consideration. However, it is common to encounter studies where that framework has not been adhered to, and meaningless comparisons were made on the basis of benefit/cost ratios.

Consider the following example. Several nondirectional radio beacons (NDBs) provide instrument approaches to several airports. The users of those NDBs enjoy benefits equal to $200,000 per year. The costs of maintaining the NDBs equals $100,000 per year. It is expected that the NDBs can continue in service another 30 years with negligible increases in operating and maintenance costs.

Four alternatives are being considered. Alternative A is to replace the existing NDBs with instrument landing systems (ILS) costing $30,000,000 to acquire and $1,000,000 to maintain annually. Alternative B is also an ILS costing only $25,000,000 to acquire and $1.3 M to maintain, the difference owing to its geographic location. Alternatives C and D are VHF Omnirange (VOR) installations. Costs are shown in the Table below.

TABLE 2.1. ALTERNATIVE NAVIGATION AIDS (millions of constant dollars)

Alternative	Initial Costs	Annual O&M	Annual Benefits
1. Existing NDB	-0-	0.1	0.2
2. A -- ILS_1	30.0	1.0	5.0
3. B -- ILS_2	25.0	1.3	4.5
4. C -- VOR_1	20.0	1.6	4.0
5. D -- VOR_2	15.0	2.0	3.0

The varying amounts of benefits for each alternative, including the existing site, result from the fact that more users will be better served by the more sophisticated ILS system than the VOR system and more users better served by both the ILS and the VOR installations than by the existing NDB.

Assuming the appropriate rate of discount is 10 percent, and the lifetime of all alternatives is 30 years, we find the present value of costs and benefits as tabulated below, and reduce the results to a benefit/cost ratio for each alternative. See Section 3 for a discussion of the present value criterion and method.

TABLE 2.2. PRESENT VALUE OF COSTS AND BENEFITS (millions of constant dollars)

Alternative	Initial Costs	Annual Costs	Total Costs	Total Benefits	B/C Ratio
1. Existing NDB	-0-	0.94	0.94	1.89	2.01
2. A -- ILS_1	30.00	9.43	39.43	47.14	1.20
3. B -- ILS_2	25.00	12.26	37.26	42.42	1.14
4. C -- VOR_1	20.00	15.08	35.08	37.71	1.08
5. D -- VOR_2	15.00	18.85	33.85	28.28	0.84

Note that the existing NDB system provides a benefit/cost ratio of 2.01 significantly better than Alternatives A through D. If we were preoccupied with the use of benefit/cost ratios, we would choose the existing NDB system in favor of the alternatives. If, on the other hand, we accept the criterion that it is the excess of benefits over costs that determines the preferred alternative, we would select Alternative A, the most expensive ILS installation. This is shown by the tabulation of benefits minus costs, below.

TABLE 2.3 SUMMARY OF BENEFITS MINUS COSTS (millions of constant dollars)

Alternative	Benefits Minus Costs
1. Existing NDB	0.95
2. A -- ILS_1	7.71
3. B -- ILS_2	5.16
4. C -- VOR_1	2.63
5. D -- VOR_2	-5.57

Note for Alternatives A, B, C, and D, the scale of costs and benefits is of the same order of magnitude. The first alternative, continuing to operate the existing NDB, differs significantly from the others. This is the typical situation that mitigates against the use of benefit/cost ratios as the proper criterion for making a choice from among alternative proposals.

Oftentimes an analyst conceives an idea for improving an existing situation, but has no time to search for alternative solutions. He may estimate the benefits and costs of his single investment proposal, and assess that proposal in terms of its benefit/cost ratio. From his point of view, any proposal resulting in benefits in excess of its cost would seem desirable. This method of comparing benefits to cost is certainly acceptable as a means for surfacing proposals for top FAA management consideration. The problem of overall programming of capital supply and total capital outlay, and of conscientious screening to choose among rival proposals should not, however, rely solely upon a comparison of benefit/cost ratios.

Benefits To Whom

Governmental activity deals with the satisfaction of fundamental group wants that can be satisfied best by the association of all these individuals in a particular community. Activities of this sort from which there are no specific measurable benefits to any individual are necessarily financed by taxes of some sort, presumably levied more or less on the principle of ability to pay, and with no particular relation to benefits received by the individual taxpayer.

Governmental bodies also undertake the satisfaction of individual desires where the social interest is somehow involved. Such activities are frequently financed by a tax or user-charge that is essentially a price to be based as nearly as possible on the benefits received, or perhaps on the cost of providing those benefits. This often creates a problem of allocating joint costs and benefits to particular individuals or groups. A good example of this type of activity is the recent controversy over the assessment of airway user charges; another example is the construction of highways financed largely or entirely by user taxes.

When conducting a cost-benefit analysis, it is essential that the analyst decide from the outset whose point of view should be taken. It is feasible to consider the economy of a proposed improvement to the national aviation system from several viewpoints:

- That of a particular user group (air carriers, general aviation, or the Department of Defense).
- That of the users of a particular set or group of navigational aids, as in the example in the previous section.
- That of society; i.e. all of the people in the United States.

We emphasize that it is absolutely essential to have clearly in mind whose viewpoint is being taken before proceeding with the cost-benefit study.

It is natural, and in many cases much simpler, for an FAA analyst to take the viewpoint of a particular user group, considering only the prospective benefits to be received by that group and the prospective costs to be incurred by the FAA and that group in providing the benefits. This is analogous to a study for the private corporation in which the relevant matters are the prospective receipts and disbursements of that corporation. It should be clear that this viewpoint is a sound one in cost-benefit studies only when the alternatives to be compared provide identical services to the people whom the government is organized to serve. For instance, this viewpoint might be correct in the choice between a concrete and an asphalt runway for a municipal airport; the differences between the alternatives would then merely be differences in costs to the city, assuming no Federal aid.

Where there are differences in the service, or benefits, provided by the alternatives, we must recognize the broader viewpoint that what the government does is simply something done collectively by all the people. Following our objective to "promote the general welfare," we must consider the probable effects of alternative governmental policies and programs on <u>all of the people</u>, not merely on the Federal Aviation Administration and/or its users.

Admittedly, all of the effects on the people of a nation resulting from improvements to the national aviation system may be hard to trace, and doubly hard to evaluate quantitatively. Nevertheless, the viewpoint of all of the people in the United States seems to be the correct one in all Federally-financed programs (Grant, 1964). Although the prospective "local" effects are relatively clear, and noticeably simpler to evaluate, we should not lose sight of the goal that it is all of the people in the country whose viewpoint should be considered when Federal programs are being assessed.

Federally-financed improvements to the national aviation system provide benefits to some people, are a matter of indifference to others, and possibly a detriment to still others. This raises the questions of <u>what</u> are the benefits, and <u>who</u> gets them.

In recent years there have been efforts made to allocate taxes, or user charges, according to a price principle that recognizes who gets the benefits and who is responsible for the costs. These efforts have necessarily involved somewhat arbitrary allocations of joint benefits and joint costs. Whenever possible, we should strive to identify separable costs and benefits, and clearly understand that joint costs are inseparable; by definition they must be allocated in an arbitrary fashion, if allocated at all. (DOT Cost Allocation Study, 1973).

The contrast between economic studies for private enterprise and economic studies for governmental activities is strikingly illustrated in the difference among three transportation systems: railways, highways, and airways. A railway company owns both the roadway and the vehicles that operate over it; if it spends money to make improvements in the roadway -- for instance by reducing grades or shortening the length of its line -- it is compensated by saving money in vehicle operating costs. In the cases of highway and airway systems, the money for improvements is spent by many governmental units -- federal, state, county, and city; operating costs, however, are paid by the many individuals who own aircraft and vehicles rather than by these governmental units as such. In the case of railways, their design and utilization are under the control of one group of officials; in the case of highways and airways, their design is under the control of many groups of officials none of whom has direct control over their utilization, although it is much more direct in the case of airways than it is for highways.

Although government officials cannot control highway utilization and, to a lesser extent airway utilization, they must consider prospective utilization in arriving at economical highway and airway plans. The general principle that all differences between alternatives are relevant to their comparison makes it necessary to consider the probable consequences to the highway and airway users, and to the general public, from any proposal for the expenditure of funds.

Treatment of Residual Values

Oftentimes the analyst attempts to estimate the value of resources remaining at the end of a program where the time horizon has been set somewhat arbitrarily, say 10 or 20 years in the future. Those amounts are sometimes called "residual values." Because it is usually impractical to attempt to trace out the costs or benefits of a program year by year until the end of its existence, or even for any extended period into the future, the analyst attempts to summarize the status of the program alternatives as of the end of the study -- i.e., the time horizon. In comparing alternative programs, we make note of the possibility that the resources of one may have greater residual value at the end of our arbitrary planning and analysis period than the other. As a practical matter, adoption of some procedure for making estimates of residual values is inescapeable. Explicit estimates of residual values are, however, seldom made in practice. The analysis always implies a value whether made explicitly or not. Because of the value of time and the discount rate, the costs and benefits of a program 15 or 20 years hence are, dollar-for-dollar, much less important than those of the next 5 or 10 years. Estimation of residual value is not really a way of escaping the task of looking into the long-run future, because there is no way of estimating this value at the end of, say 10 years, except by looking further into the future. The estimated residual value of an item at the end of a given planning period is often treated as a credit against its cost, and that value should represent future avoidable expenditures (Noah, 1965). That is, we should estimate the value of an item in its most likely use at the end of the planning period.

The concept pertaining to the value remaining at the end of the planning period also pertains to the value of assets that may be inherited from some other use at the beginning of the planning period. That is, assets on hand and "available" for use in one or more of the alternatives being compared have some value or we would not suggest their continued use. As a matter of practice, such assets are generally treated as free, and termed inherited assets. Ignoring the value of inherited assets and the residual values of assets at the end of the planning period are simplifying assumptions that, as a matter of practice, seldom cause the analyst a great deal of trouble. Nonetheless, there are situations where both considerations could be quite important.

See Section 3, Approximate Capital Recovery Methods, for a discussion of techniques sometimes used to estimate residual values.

Treatment of Uncertainty

Explicit treatment of uncertainty should be provided in cost-benefit analyses. The kinds of uncertainty that should be treated may be distinguished as follows:

- o Uncertainty about the state of the world in the future; e.g., factors influencing the state of the technological art, supply and demand relationships, and significant world events.
- o Statistical uncertainty; i.e., uncertainty stemming from chance elements in the real world having a more or less objective or calculable probability of occurrence.

Uncertainties of the second type are usually the least troublesome to handle in cost-benefit studies. When necessary, Monte Carlo, sensitivity analysis, or other techniques may be used to deal with statistical fluctuations. But these perturbations are usually dwarfed by uncertainties of the first type, which are dominant in most long-range planning problems (Fisher, 1970).

Uncertainties about the state of the world in the future are typically present, and they are most difficult to take into account in a cost-benefit study. Fisher classifies techniques most often used as sensitivity analysis, contingency analysis, and *a fortiori* analysis.

Suppose in a given study there are a few key parameters about which the analyst is very uncertain. Instead of using mean or expected values for these parameters, the analyst may successively use several values ranging from high to low in an attempt to see how sensitive the results are to variations in the uncertain parameters. A certain amount of judgment must be used to define the full range of uncertainty; i.e. the range from low to high for an uncertain parameter. The analyst, having successively used several values, may observe how the ranking of alternatives changes.

If a certain alternative is superior in all of these sensitivity investigations, it is referred to as a dominant solution. Dominance is a characteristic that the analyst is always seeking, but its existence is rare in the types of problems of concern in this guide.

Contingency analysis investigates how the ranking of the alternatives under consideration holds up when a relevant change in criteria for evaluating the alternatives is postulated, or a major change in the general environment is assumed. Suppose, for example, that a basic analysis is conducted assuming that traffic to and from a developing country will be non-existent. We might want to investigate what would happen if that developing country adds significantly to our estimated traffic.

Suppose a cost-benefit analysis results in the selection of alternative A over B. The basic analysis, however, contains a number of uncertainties. If we resolve major uncertainties in favor of B, and find that A is still preferred, we have developed a very strong case to support the selection of alternative A. This is called *a fortiori* analysis.

These three techniques are useful in a direct analytical sense, and they also contribute indirectly. For example, through sensitivity and contingency analyses, the analyst may gain a good understanding of the really critical uncertainties and issues in a given problem. On the basis of this knowledge, he might then be able to conceive a new alternative that will provide a reasonably good hedge against a range of the more significant uncertainties.

See Section 7 for a more complete discussion of uncertainty, and especially how the problem may be handled in actual studies.

Treatment of Problems Associated with Time

In dealing with problems associated with time, much depends upon the design of the cost-benefit study. In many instances the comparison among alternatives made in cost-benefit studies is conducted in a static framework; i.e., without regard to the timing of costs to be incurred and benefits to be received. Timing considerations are, of course, taken into account to some extent in the work leading up to the static comparisons. For example, estimates of operational capability dates have to be examined to help insure that the proposed future capabilities being compared are really relevant alternatives in terms of the time period of interest. Likewise, the time period must be given some attention to distinguish incremental from sunk costs.

Time-phasing of costs and benefits of alternatives offers several advantages. It gives decisionmakers explicit knowledge of the points in time when the heaviest resource commitments of various alternatives might occur, and tells them when benefits might be received. Also, estimates of costs and benefits are likely to be given more careful attention when they have to be time-phased. Finally, developing cost and benefit streams through time for the various alternatives provides the basis for a definitive treatment of time preference.

The problem of time preference is discussed in more detail in Sections 3 and 5. Briefly, to introduce the subject, we note that Congressional committees have conducted special hearings on discounting and related matters, seminars have been held on what discount rate should be used, papers have been written, chapters of books have been devoted to this subject, and controversy still reigns.

We do not propose to become engaged in the controversy in this Guide. In Section 5 we cite the discount rates to use in cost-benefit studies, and suggest strongly that the analyst's time can better be spent improving his estimates of costs and benefits rather than in discussions of the appropriate rate of discount.

In short, a positive rate of discount is recommended because one generally prefers to defer the incurrence of costs and hasten the receipt of benefits. The mathematical application of a positive discount rate is merely a technique for introducing that preference into our formulations of investment criteria.

Intangible Considerations

An underlying theoretical rationale of benefit measurement involves the concept of willingness to pay. Even when other techniques are used in practice, they are validated by reference to willingness to pay. Extending the scope of benefits to intangibles, or indirect or externality impacts, does not change the underlying concept of benefits.

The problem that intangibility creates for the analyst is not a result of the vagueness or indefinability of the benefit described. The problem, rather, is a consequence of the lack of markets in which the benefit is sold. Markets automatically provide the analyst with an unambiguous measure of the tradeoff people express between money and the intangible benefit.

Unfortunately, a reference by many analysts to intangibility means that those benefits are difficult to measure for one reason or another. That should not be the definition placed on intangible benefits or costs. Conversely, the mere act of deriving a monetary measurement for a benefit or cost does not necessarily eliminate the philosophical question of whether you should come up with a monetary measure. There are theoretical reasons why there might be no dollar value for certain benefits and costs. In those cases, the benefit or cost is truly an intangible, and no dollar value should be placed on its measurement.

With respect to environmental protection, administrative agencies are required by national law to expand their concept and application of cost-benefit analysis. Thus, the National Environmental Policy Act now mandates that many impacts of government projects that were previously neglected as intangible be explicitly included in the planning process.

Perhaps this is fortunate. A major benefit of cost-benefit analysis is the learning process induced by having to justify value placed on particular hard-to-measure outputs. Making assumptions explicit facilitates the debate. The challenge for economist and practitioner alike is to expunge subjectivity to the greatest extent possible and at the same time increase the breadth and application of cost-benefit analysis.

In summary, cost-benefit analysis must first measure those factors which can be measured in dollars. The remaining factors can then be measured either by a sensitivity analysis from which the value per unit can be inferred or by calculating break-even values. For those factors which are not quantifiable in any monetary or physical form, such as aesthetic values, this information should be presented as a side display and contrasted to net measurable benefits.

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3

INVESTMENT CRITERIA

The cost-benefit analysis is intended to help us choose among alternative means to our ends. Among those alternatives is the "do-nothing" alternative, which in most cases means to continue doing what we are doing. In choosing among alternatives, we must adopt tests of preference. The process of choosing a preferred alternative involves, as one step, predicting the consequences of alternative actions. Another vital step is distinguishing preferred combinations of consequences from less desirable ones. This step entails the use of criteria, either explicitly or implicitly.

There is no all-purpose test of preference, for the appropriate criterion depends upon what alternatives are open to the decisionmaker, upon what aspects of the situation must be taken as given, and even upon what kinds of measurements are feasible. Nonetheless, a few general observations about suitable criteria can be made.

When benefits and costs can be measured in the same unit, say dollars, to maximize benefits minus costs is certainly an acceptable criterion; it is the equivalent of making the most out of whatever actions can be taken. We emphasize that this test is a possibility only when benefits and costs are commensurable. To make time streams of costs and benefits commensurable when we prefer to delay costs and hasten benefits, the simple sum of all benefits minus the sum of all costs is *not* the answer to this test. One way to allow for our time preference is to discount future amounts and convert each stream to its present value. For example, if we discount future amounts at 10 percent, the present value (the value today) of a dollar one year hence is 90 cents.

Another test of preference depends on a calculation of the internal rate of return provided by feasible alternatives. "Internal rate of return" is a technical term meaning the rate of discount which makes the present value of the project's receipt stream equal to the present value of its cost stream. Oftentimes this procedure is used to rank alternatives according to their rates of return; then we proceed down the list until the budget is exhausted. The internal rate of the first project not covered by the budget would then be the "marginal internal rate of return;" if the net benefit streams from all projects in the list were then discounted at this rate, all those with higher internal rates would have positive present worths (present value of benefits minus present value of costs), and would be preferred to those with lower internal rates. The latter would, of course, have negative present worths.

The internal rate of return criterion has a shortcoming. It leaves the following important question unanswered: If several ventures are interdependent, which combination should be chosen? Interdependent ventures are projects whose costs or benefits depend upon whether or not certain others are undertaken. This type of question is beyond the scope of the Guide; we have limited our discussion to a selection of one preferred action from among a number of competing alternatives.

Generally speaking, proposals for appropriation of funds to acquire physical plant are subject to review and approval by top management in the FAA as part of its system of budgetary control. As mentioned earlier, this Guide is designed primarily for use by the FAA analyst developing proposals for the appropriation of funds, not for top management use in allocating its capital budget.

The Capital Recovery Criterion

To compare alternative nonuniform series of outlays and/or receipts, in those cases where we have a preference regarding their timing, it is necessary to make those series commensurable. One way is to reduce each to an equivalent uniform annual value. How may this be done?

The goal of the initial step is a year-by-year tabulation of estimated outlays and receipts associated with the alternatives being examined. Admittedly, the estimating process necessary to arrive at year-by-year cash flows is frequently very time-consuming, and results are often uncertain. Setting these difficulties aside for the sake of exposition, we illustrate with a tabulation of cash flows for two alternatives, A and B, in the table below.

TABULATION OF CASH FLOW
(Thousands of Constant Dollars)

YEAR	ALTERNATIVE A	ALTERNATIVE B	B-A
0		-1500	-1500
1	- 800	- 500	+ 300
2	- 800	- 500	+ 300
3	- 800	- 500	+ 300
4	- 800	- 500	+ 300
5	- 800	- 500	+ 300
6	- 800	- 500	+ 300
7	- 800	- 500	+ 300
8	- 800	- 500	+ 300
9	- 800	- 500	+ 300
10	- 800	- 500	+ 300
TOTAL	-$ 8000	-$ 6500	+1500
R at 10%	- 800	- 744	+ 56

The tabulation is hypothetical; experience suggests it is almost inevitable that certain outlays will vary from year to year. Maintenance costs fluctuate and tend to increase with equipment age; wage rates change; property tax rates and assessed valuations usually increase. Nevertheless, often there is no rational basis for making different estimates for each year. When this is the case, as illustrated in the table, only capital costs require conversion by appropriate compound interest factors.

Cost stream totals may be thought of as giving the present value of the cash flows using a discount rate of zero percent. Using a positive rate, such as 10 percent, we may find the equivalent uniform annual cost (R) of Alternatives A and B, and compare them. The comparison provides a basis for choosing between the alternatives.

If we let R equal the annual cost of capital recovery, P the initial outlay, n the life or study period in years, L the expected net salvage value at the end of n years, and i the discount rate, then:

$$R = (P-L)\ (crf - i - n) + Li \qquad (1)$$

Capital recovery factors, indicated by "crf" in the equation above, may be found for various discount rates and periods in Table 3.1.

Examining the tabulation of cash flows once again, note that Alternative A has no initial outlay. The equivalent uniform annual cost associated with Alternative A is $800 thousand. Alternative B, on the other hand, has an initial outlay of $1.5 million, and that must be capitalized. The tabulation shows no indication of salvage values, we have suggested the use of a 10 percent discount rate, and the period is 10 years. The capital recovery factor for 10 years and 10 percent is 0.16275; when multiplied by the initial outlay, we find the amount necessary to recover Alternative B's capital outlay to be $244,125. When added to the annual cost of Alternative B, we find that the equivalent uniform annual cost for B is $744,125. Therefore, B is to be preferred; it costs less than A when compared on the basis of equivalent uniform annual values.

Now, suppose Alternative A's cost stream allows a number of VHF receivers to operate so, at the end of five years, they will have a salvage value of $200 thousand. The cash flow for Alternative B includes the initial outlay necessary to replace the existing receivers, a reduced annual operations and maintenance cost, an expected useful life of 14 years, and a salvage value of $300 thousand.

TABLE 3.1. CAPITAL RECOVERY FACTORS: UNIFORM END-OF-YEAR AMOUNT, R, SECURED FROM PRESENT AMOUNT, P.

PERIOD	INTEREST RATE 5.0%	7.5%	10.0%	12.5%	15.0%	20.0%	25.0%
0- 1	1.05000	1.07500	1.10000	1.12500	1.15000	1.20000	1.25000
1- 2	0.53780	0.55693	0.57619	0.59559	0.61512	0.65455	0.69444
2- 3	0.36721	0.38454	0.40211	0.41993	0.43798	0.47473	0.51230
3- 4	0.28201	0.29857	0.31547	0.33271	0.35027	0.38629	0.42344
4- 5	0.23097	0.24716	0.26380	0.28085	0.29832	0.33438	0.37185
5- 6	0.19702	0.21304	0.22961	0.24668	0.26424	0.30071	0.33882
6- 7	0.17282	0.18880	0.20541	0.22260	0.24036	0.27742	0.31634
7- 8	0.15472	0.17073	0.18744	0.20483	0.22285	0.26061	0.30040
8- 9	0.14069	0.15677	0.17364	0.19126	0.20957	0.24808	0.28876
9-10	0.12950	0.14569	0.16275	0.18062	0.19925	0.23852	0.28007
10-11	0.12039	0.13670	0.15396	0.17211	0.19107	0.23110	0.27349
11-12	0.11283	0.12928	0.14676	0.16519	0.18448	0.22526	0.26845
12-13	0.10646	0.12306	0.14078	0.15950	0.17911	0.22062	0.26454
13-14	0.10102	0.11780	0.13575	0.15475	0.17469	0.21689	0.26150
14-15	0.09634	0.11329	0.13147	0.15076	0.17102	0.21388	0.25912
15-16	0.09227	0.10939	0.12782	0.14739	0.16795	0.21144	0.25724
16-17	0.08870	0.10600	0.12466	0.14451	0.16537	0.20944	0.25576
17-18	0.08555	0.10303	0.12193	0.14205	0.16319	0.20781	0.25459
18-19	0.08275	0.10041	0.11955	0.13993	0.16134	0.20646	0.25366
19-20	0.08024	0.09809	0.11746	0.13810	0.15976	0.20536	0.25292
20-21	0.07800	0.09603	0.11562	0.13651	0.15842	0.20444	0.25233
21-22	0.07597	0.09419	0.11401	0.13512	0.15727	0.20369	0.25186
22-23	0.07414	0.09254	0.11257	0.13392	0.15628	0.20307	0.25148
23-24	0.07247	0.09105	0.11130	0.13287	0.15543	0.20255	0.25119
24-25	0.07095	0.08971	0.11017	0.13194	0.15470	0.20212	0.25095
25-26	0.06956	0.08850	0.10916	0.13113	0.15407	0.20176	0.25076
26-27	0.06829	0.08740	0.10826	0.13042	0.15353	0.20147	0.25061
27-28	0.06712	0.08641	0.10745	0.12980	0.15306	0.20122	0.25048
28-29	0.06605	0.08550	0.10673	0.12925	0.15265	0.20102	0.25039
29-30	0.06505	0.08467	0.10608	0.12876	0.15230	0.20085	0.25031
30-31	0.06413	0.08392	0.10550	0.12833	0.15200	0.20070	0.25025
31-32	0.06328	0.08323	0.10497	0.12795	0.15173	0.20059	0.25020
32-33	0.06249	0.08259	0.10450	0.12762	0.15150	0.20049	0.25016
33-34	0.06176	0.08201	0.10407	0.12732	0.15131	0.20041	0.25013
34-35	0.06107	0.08148	0.10369	0.12706	0.15113	0.20034	0.25010
35-36	0.06043	0.08099	0.10334	0.12683	0.15099	0.20028	0.25008
36-37	0.05984	0.08055	0.10303	0.12662	0.15086	0.20024	0.25006
37-38	0.05928	0.08013	0.10275	0.12644	0.15074	0.20020	0.25005
38-39	0.05876	0.07975	0.10249	0.12628	0.15065	0.20016	0.25004
39-40	0.05828	0.07940	0.10226	0.12613	0.15056	0.20014	0.25003
40-41	0.05782	0.07908	0.19205	0.12601	0.15049	0.20011	0.25003
41-42	0.05739	0.07878	0.10186	0.12589	0.15042	0.20009	0.25002
42-43	0.05699	0.07850	0.10169	0.12579	0.15037	0.20008	0.25002
43-44	0.05662	0.07825	0.10153	0.12571	0.15032	0.20007	0.25001
44-45	0.05626	0.07801	0.10139	0.12563	0.15028	0.20005	0.25001
45-46	0.05593	0.07779	0.10126	0.12556	0.15024	0.20005	0.25001
46-47	0.05561	0.07759	0.10115	0.12549	0.15021	0.20004	0.25001
47-48	0.05532	0.07741	0.10104	0.12544	0.15018	0.20003	0.25001
48-49	0.05504	0.07723	0.10095	0.12539	0.15016	0.20003	0.25000
49-50	0.05478	0.07707	0.10086	0.12535	0.15014	0.20002	0.25000

The example may now be summarized as follows:

	Alternative	
	A	B
Initial Outlay, $ x 10^3	Sunk	1500
Life, Years	5	14
Salvage Value, $ x 10^3	200	300
Annual Outlay, $ x 10^3	800	500

Since the initial outlay for A is sunk -- i.e., irrelevant because it is past and has nothing to do with the comparison of prospective costs -- formula (1) above as applied to A becomes:

$$R = Li = \$200\ (0.10) = +\ \$20$$

When $20 thousand (a benefit) is subtracted from the annual cost of $800 thousand, the equivalent uniform annual value, R, is $780 thousand.

For Alternative B we find:

$$R = (1500\text{-}300)\ (\text{crf} - 10\% - 14\ \text{yrs}) + 300\ (0.10)$$
$$= (1200)\ (0.13575) + 30 = \$193$$

To B's capital recovery amount we add its annual outlay of $500 thousand to find R, or $693 thousand. Alternative B is still preferred.

Historically, capitalized costs were widely used for many years, particularly by civil engineers. The widespread use of capitalized costs probably had its origin in Wellington's classic work, *The Economic Theory of Railway Location* (1887). Wellington published during a time when most engineers worked for railways during at least part of their career, and he influenced the thinking of the entire engineering profession. Grant, whose first book on engineering economy was published in 1930, still prefers to use capitalized costs when making comparisons. He prefers that method over the present value method which has in more recent years been advocated by economists writing on the subject of cost-benefit analysis. Grant recognizes that the present value method is somewhat easier to compute in problems where irregular series of outlays are involved.

The Present Value Criterion

We emphasize that, given the same interest rate and the same estimated series of outlays, comparisons by the capital recovery criterion lead to the same conclusion as comparisons by the present value criterion; i.e., reducing or discounting future outlays/receipts to an equivalent present amount using a stipulated rate of preference. Two techniques for treating the present value criterion are discussed below: discrete and continuous compounding.

A common convention in mathematics of finance assumes that cash flow occurring throughout a year is concentrated at year end. An alternative convention is to assume that cash flow taking place during a year occurs uniformly throughout the year. Tables giving present value factors for both assumptions are included in this section.

To explain the tables it is necessary to develop the formula for present value. The following symbols and meanings are used:

i = an interest rate per interest period

n = a number of interest periods

P = a present sum of money, the principal

S = a sum of money at the end of n periods from the present date that is equivalent to P with interest i.

If P is invested at interest rate i, the interest for the first year is Pi and the total amount at the end of the first year is $P + Pi$, and that is equal to $P(1 + i)$. This is the formula for the compound amount, S, that may be obtained in n years from a principal, P.

$$S = P(1 + i)^n \tag{2}$$

Now, if we express P in terms of S, i, and n,

$$P = S\left[\frac{1}{(1 + i)^n}\right] \tag{3}$$

or,

$$P = S(1 + i)^{-n}$$

P may then be thought of as the principal that will give a required amount S in n years; in other words, P is the present value of a payment of S to be made n years in the future. The discrete compounding table, 3.2, is based on formula (3) above.

Before explaining the continuous compounding table, it is necessary to discuss the relationship between nominal and effective interest rates. Many loan transactions stipulate that interest is computed and charged more often than once a year. For example, interest on deposits in savings banks may be computed and added to the deposit balance four times a year, or compounded quarterly. A loan transaction in which interest is charged at one percent per month is sometimes described as having an interest rate of 12 percent per annum. This is misleading, however, and in actuality, this rate should be described as the nominal rate per annum compounded monthly. The monthly compounding at one percent has the same effect on the year-end compound amount as charging a rate of 12.7 percent compounded annually. The effective interest rate is therefore 12.7 percent.

The more frequent the number of compoundings during the year, the greater the differences between the values of nominal and effective rates per annum. This difference is greatest in continuous compounding, where an infinite number of compoundings is assumed. In the language of the mathematics of finance, the nominal rate r used in continuous compounding is referred to as the *force of interest*. The mathematical relationship between nominal and effective interest is described using the following symbols:

m = number of compoundings per annum

r = the nominal interest rate per annum

Therefore, the effective interest rate per annum is,

$$\left(1 + \frac{r}{m}\right)^m - 1 \qquad (4)$$

The tabulation below shows the values of force of interest to yield various integral values of effective interest rates.

EFFECTIVE RATE PER ANNUM, %	FORCE OF INTEREST, %
3.500	3.44014267
4.875	4.75989788
7.500	7.23206615
10.000	9.53101798
12.500	11.77830357

The force of interest rates shown above are calculated by taking the natural logarithm of the effective rate plus one; e.g., ln (1 + .035) = .0344014267 or 3.44 percent.

To be specific about the figures in the continuous compounding table, 3.3, note the first period's figure under the 10 percent column. It is 0.95382. It is calculated using the following formula from Grant (1964):

$$P = \frac{e^r - 1}{re^r} \qquad (5)$$

Substituting the force of interest, r, as given in the above tabulation, yields 0.95382. All subsequent figures in the 10 percent column are the product of 0.95382 and the appropriate present value factor given in the discrete compounding table. For instance, the figure of 0.71662 for the period "three to four" in continuous compounding, at 10 percent, is equal to 0.95382 times the discrete compounding value at 10 percent, at the end of the third year, 0.75131.

Where the series involves a uniform set of outlays/receipts per period, R, the present value may be found more readily with this equation,

$$P = R \frac{(1 + i)^n - 1}{i\,(1 + i)^n} \qquad (6)$$

Table 3.4 contains present value factors for uniform series. For example, the present value of $1000 laid out at the end of each year for 10 years is $6,144.60 if the appropriate rate of discount is 10 percent.

TABLE 3.2. DISCRETE COMPOUNDING: PRESENT VALUE OF ONE DOLLAR FLOWING AT END OF STATED PERIODS.

PERIOD	INTEREST RATE 5.0%	7.5%	10.0%	12.5%	15.0%	20.0%	25.0%
0- 1	0.95238	0.93023	0.90909	0.88889	0.86957	0.83333	0.80000
1- 2	0.90703	0.86533	0.82645	0.79012	0.75614	0.69444	0.64000
2- 3	0.86384	0.80496	0.75131	0.70233	0.65752	0.57870	0.51200
3- 4	0.82270	0.74880	0.68301	0.62430	0.57175	0.48225	0.40960
4- 5	0.78353	0.69656	0.62092	0.55493	0.49718	0.40188	0.32768
5- 6	0.74622	0.64796	0.56447	0.49327	0.43233	0.33490	0.26214
6- 7	0.71068	0.60275	0.51316	0.43846	0.37594	0.27908	0.20972
7- 8	0.67684	0.56070	0.46651	0.38974	0.32690	0.23257	0.16777
8- 9	0.64461	0.52158	0.42410	0.34644	0.28426	0.19381	0.13422
9-10	0.61391	0.48519	0.38554	0.30795	0.24718	0.16151	0.10737
10-11	0.58468	0.45134	0.35049	0.27373	0.21494	0.13459	0.08590
11-12	0.55684	0.41985	0.31863	0.24332	0.18691	0.11216	0.06872
12-13	0.53032	0.39056	0.28966	0.21628	0.16253	0.09346	0.05498
13-14	0.50507	0.36331	0.26333	0.19225	0.14133	0.07789	0.04398
14-15	0.48102	0.33797	0.23939	0.17089	0.12289	0.06491	0.03518
15-16	0.45811	0.31439	0.21763	0.15190	0.10686	0.05409	0.02815
16-17	0.43630	0.29245	0.19784	0.13502	0.09293	0.04507	0.02252
17-18	0.41552	0.27205	0.17986	0.12002	0.08081	0.03756	0.01801
18-19	0.39573	0.25307	0.16351	0.10668	0.07027	0.03130	0.01441
19-20	0.37689	0.23541	0.14864	0.09483	0.06110	0.02608	0.01153
20-21	0.35894	0.21899	0.13513	0.08429	0.05313	0.02174	0.00922
21-22	0.34185	0.20371	0.12285	0.07493	0.04620	0.01811	0.00738
22-23	0.32557	0.18950	0.11168	0.06660	0.04017	0.01509	0.00590
23-24	0.31007	0.17628	0.10153	0.05920	0.03493	0.01258	0.00472
24-25	0.29530	0.16398	0.09230	0.05262	0.03038	0.01048	0.00378
25-26	0.28124	0.15254	0.08391	0.04678	0.02642	0.00874	0.00302
26-27	0.26785	0.14190	0.07628	0.04158	0.02297	0.00728	0.00242
27-28	0.25509	0.13200	0.06934	0.03696	0.01997	0.00607	0.00193
28-29	0.24295	0.12279	0.06304	0.03285	0.01737	0.00506	0.00155
29-30	0.23138	0.11422	0.05731	0.02920	0.01510	0.00421	0.00124
30-31	0.22036	0.10625	0.05210	0.02596	0.01313	0.00351	0.00099
31-32	0.20987	0.09884	0.04736	0.02307	0.01142	0.00293	0.00079
32-33	0.19987	0.09194	0.04306	0.02051	0.00993	0.00244	0.00063
33-34	0.19035	0.08553	0.03914	0.01823	0.00864	0.00203	0.00051
34-35	0.18129	0.07956	0.03558	0.01621	0.00751	0.00169	0.00041
35-36	0.17266	0.07401	0.03235	0.01440	0.00653	0.00141	0.00032
36-37	0.16444	0.06885	0.02941	0.01280	0.00568	0.00118	0.00026
37-38	0.15661	0.06404	0.02673	0.01138	0.00494	0.00098	0.00021
38-39	0.14915	0.05958	0.02430	0.01012	0.00429	0.00082	0.00017
39-40	0.14205	0.05542	0.02209	0.00899	0.00373	0.00068	0.00013
40-41	0.13528	0.05155	0.02009	0.00799	0.00325	0.00057	0.00011
41-42	0.12884	0.04796	0.01826	0.00711	0.00282	0.00047	0.00009
42-43	0.12270	0.04461	0.01660	0.00632	0.00245	0.00039	0.00007
43-44	0.11686	0.04150	0.01509	0.00561	0.00213	0.00033	0.00005
44-45	0.11130	0.03860	0.01372	0.00499	0.00186	0.00027	0.00004
45-46	0.10600	0.03591	0.01247	0.00444	0.00161	0.00023	0.00003
46-47	0.10095	0.03340	0.01134	0.00394	0.00140	0.00019	0.00003
47-48	0.09614	0.03107	0.01031	0.00350	0.00122	0.00016	0.00002
48-49	0.09156	0.02891	0.00937	0.00312	0.00106	0.00013	0.00002
49-50	0.08720	0.02689	0.00852	0.00277	0.00092	0.00011	0.00001

Assuming discrete compounding (end of year convention) of interest at various stated rates per annum.

TABLE 3.3. CONTINUOUS COMPOUNDING: PRESENT VALUE OF ONE DOLLAR FLOWING UNIFORMLY THROUGHOUT STATED PERIODS.

	INTEREST RATE						
PERIOD	5.0%	7.5%	10.0%	12.5%	15.0%	20.0%	25.0%
0- 1	0.97600	0.96470	0.95382	0.94335	0.93326	0.91414	0.89628
1- 2	0.92952	0.89739	0.86711	0.83854	0.81153	0.76178	0.71703
2- 3	0.88526	0.83478	0.78828	0.74537	0.70568	0.63482	0.57362
3- 4	0.84310	0.77654	0.71662	0.66255	0.61364	0.52901	0.45890
4- 5	0.80296	0.72236	0.65147	0.58893	0.53360	0.44084	0.36712
5- 6	0.76472	0.67197	0.59225	0.52349	0.46400	0.36737	0.29369
6- 7	0.72830	0.62509	0.53841	0.46533	0.40348	0.30614	0.23496
7- 8	0.69362	0.58148	0.48946	0.41363	0.35085	0.25512	0.18796
8- 9	0.66059	0.54091	0.44497	0.36767	0.30509	0.21260	0.15037
9-10	0.62914	0.50317	0.40451	0.32682	0.26529	0.17717	0.12030
10-11	0.59918	0.46806	0.36774	0.29050	0.23069	0.14764	0.09624
11-12	0.57065	0.43541	0.33431	0.25822	0.20060	0.12303	0.07699
12-13	0.54347	0.40503	0.30392	0.22953	0.17443	0.10253	0.06159
13-14	0.51759	0.37677	0.27629	0.20403	0.15168	0.08544	0.04927
14-15	0.49294	0.35049	0.25117	0.18136	0.13190	0.07120	0.03942
15-16	0.46947	0.32603	0.22834	0.16121	0.11469	0.05933	0.03154
16-17	0.44712	0.30329	0.20758	0.14330	0.09973	0.04944	0.02523
17-18	0.42582	0.28213	0.18871	0.12737	0.08672	0.04120	0.02018
18-19	0.40555	0.26244	0.17155	0.11322	0.07541	0.03434	0.01615
19-20	0.38624	0.24413	0.15596	0.10064	0.06558	0.02861	0.01292
20-21	0.36784	0.22710	0.14178	0.08946	0.05702	0.02384	0.01033
21-22	0.35033	0.21126	0.12889	0.07952	0.04958	0.01987	0.00827
22-23	0.33364	0.19652	0.11717	0.07068	0.04312	0.01656	0.00661
23-24	0.31776	0.18281	0.10652	0.06283	0.03749	0.01380	0.00529
24-25	0.30263	0.17005	0.09684	0.05585	0.03260	0.01150	0.00423
25-26	0.28821	0.15819	0.08803	0.04964	0.02835	0.00958	0.00339
26-27	0.27449	0.14715	0.08003	0.04413	0.02465	0.00799	0.00271
27-28	0.26142	0.13689	0.07276	0.03922	0.02144	0.00665	0.00217
28-29	0.24897	0.12734	0.06614	0.03487	0.01864	0.00555	0.00173
29-30	0.23711	0.11845	0.06013	0.03099	0.01621	0.00462	0.00139
30-31	0.22582	0.11019	0.05466	0.02755	0.01410	0.00385	0.00111
31-32	0.21507	0.10250	0.04969	0.02449	0.01226	0.00321	0.00089
32-33	0.20483	0.09535	0.04518	0.02177	0.01066	0.00267	0.00071
33-34	0.19507	0.08870	0.04107	0.01935	0.00927	0.00223	0.00057
34-35	0.18579	0.08251	0.03734	0.01720	0.00806	0.00186	0.00045
35-36	0.17694	0.07675	0.03394	0.01529	0.00701	0.00155	0.00036
36-37	0.16851	0.07140	0.03086	0.01359	0.00609	0.00129	0.00029
37-38	0.16049	0.06642	0.02805	0.01208	0.00530	0.00107	0.00023
38-39	0.15285	0.06178	0.02550	0.01074	0.00461	0.00090	0.00019
39-40	0.14557	0.05747	0.02318	0.00954	0.00401	0.00075	0.00015
40-41	0.13864	0.05346	0.02107	0.00848	0.00348	0.00062	0.00012
41-42	0.13203	0.04973	0.01916	0.00754	0.00303	0.00052	0.00010
42-43	0.12575	0.04626	0.01742	0.00670	0.00263	0.00043	0.00008
43-44	0.11976	0.04304	0.01583	0.00596	0.00229	0.00036	0.00006
44-45	0.11406	0.04003	0.01439	0.00530	0.00199	0.00030	0.00005
45-46	0.10863	0.03724	0.01309	0.00471	0.00173	0.00025	0.00004
46-47	0.10345	0.03464	0.01190	0.00418	0.00151	0.00021	0.00003
47-48	0.09853	0.03222	0.01081	0.00372	0.00131	0.00017	0.00002
48-49	0.09383	0.02998	0.00983	0.00331	0.00114	0.00014	0.00002
49-50	0.08937	0.02789	0.00894	0.00294	0.00099	0.00012	0.00002

Assuming continuous compounding of interest at various stated effective rates per annum.

TABLE 3.4. PRESENT VALUE FACTORS -- UNIFORM SERIES PRESENT AMOUNT, P, SECURED FROM UNIFORM END-OF-PERIOD AMOUNT, R.

	INTEREST RATE						
PERIOD	5.0%	7.5%	10.0%	12.5%	15.0%	20.0%	25.0%
0- 1	0.9524	0.9302	0.9091	0.8889	0.8696	0.8333	0.8000
1- 2	1.8594	1.7956	1.7355	1.6790	1.6257	1.5278	1.4400
2- 3	2.7232	2.6005	2.4869	2.3813	2.2832	2.1065	1.9520
3- 4	3.5460	3.3493	3.1699	3.0056	2.8550	2.5887	2.3616
4- 5	4.3295	4.0459	3.7908	3.5606	3.3522	2.9906	2.6893
5- 6	5.0757	4.6938	4.3553	4.0538	3.7845	3.3255	2.9514
6- 7	5.7864	5.2966	4.8684	4.4923	4.1604	3.6046	3.1611
7- 8	6.4632	5.8573	5.3349	4.8820	4.4873	3.8372	3.3289
8- 9	7.1078	6.3789	5.7590	5.2285	4.7716	4.0310	3.4631
9-10	7.7217	6.8641	6.1446	5.5364	5.0188	4.1925	3.5705
10-11	8.3064	7.3154	6.4951	5.8102	5.2337	4.3271	3.6564
11-12	8.8633	7.7353	6.8137	6.0535	5.4206	4.4392	3.7251
12-13	9.3936	8.1258	7.1034	6.2698	5.5831	4.5327	3.7801
13-14	9.8986	8.4892	7.3667	6.4620	5.7245	4.6106	3.8241
14-15	10.3797	8.8271	7.6061	6.6329	5.8474	4.6755	3.8593
15-16	10.8378	9.1415	7.8237	6.7848	5.9542	4.7296	3.8874
16-17	11.2741	9.4340	8.0216	6.9198	6.0472	4.7746	3.9099
17-18	11.6896	9.7060	8.2014	7.0398	6.1280	4.8122	3.9279
18-19	12.0853	9.9591	8.3649	7.1465	6.1982	4.8435	3.9424
19-20	12.4622	10.1945	8.5136	7.2414	6.2593	4.8696	3.9539
20-21	12.8212	10.4135	8.6487	7.3256	6.3125	4.8913	3.9631
21-22	13.1630	10.6172	8.7715	7.4006	6.3587	4.9094	3.9705
22-23	13.4886	10.8067	8.8832	7.4672	6.3988	4.9245	3.9764
23-24	13.7986	10.9830	8.9847	7.5264	6.4338	4.9371	3.9811
24-25	14.0939	11.1469	9.0770	7.5790	6.4641	4.9476	3.9849
25-26	14.3752	11.2995	9.1609	7.6258	6.4906	4.9563	3.9879
26-27	14.6430	11.4414	9.2372	7.6674	6.5135	4.9636	3.9903
27-28	14.8981	11.5734	9.3066	7.7043	6.5335	4.9697	3.9923
28-29	15.1411	11.6962	9.3696	7.7372	6.5509	4.9747	3.9938
29-30	15.3725	11.8104	9.4269	7.7664	6.5660	4.9789	3.9950
30-31	15.5928	11.9166	9.4790	7.7923	6.5791	4.9824	3.9960
31-32	15.8027	12.0155	9.5264	7.8154	6.5905	4.9854	3.9968
32-33	16.0025	12.1074	9.5694	7.8359	6.6005	4.9878	3.9975
33-34	16.1929	12.1929	9.6086	7.8542	6.6091	4.9898	3.9980
34-35	16.3742	12.2725	9.6442	7.8704	6.6166	4.9915	3.9984
35-36	16.5469	12.3465	9.6765	7.8848	6.6231	4.9929	3.9987
36-37	16.7113	12.4154	9.7059	7.8976	6.6288	4.9941	3.9990
37-38	16.8679	12.4794	9.7327	7.9089	6.6338	4.9951	3.9992
38-39	17.0170	12.5390	9.7570	7.9191	6.6380	4.9959	3.9993
39-40	17.1591	12.5944	9.7791	7.9281	6.6418	4.9966	3.9995
40-41	17.2944	12.6460	9.7991	7.9361	6.6450	4.9972	3.9996
41-42	17.4232	12.6939	9.8174	7.9432	6.6478	4.9976	3.9997
42-43	17.5459	12.7385	9.8340	7.9495	6.6503	4.9980	3.9997
43-44	17.6628	12.7800	9.8491	7.9551	6.6524	4.9984	3.9998
44-45	17.7741	12.8186	9.8628	7.9601	6.6543	4.9986	3.9998
45-46	17.8801	12.8545	9.8753	7.9645	6.6559	4.9989	3.9999
46-47	17.9810	12.8879	9.8866	7.9685	6.6573	4.9991	3.9999
47-48	18.0772	12.9190	9.8969	7.9720	6.6585	4.9992	3.9999
48-49	18.1687	12.9479	9.9063	7.9751	6.6596	4.9993	3.9999
49-50	18.2559	12.9748	9.9148	7.9778	6.6605	4.9995	3.9999

Internal Rate of Return Criteron

The internal rate of return is defined as the interest rate that equates the present vale of the stream of expected future net benefits to zero.

In his dissertation, Sutton (1968) notes that the present value and the internal rate of return criteria are seen to be consistent decision algorithms in the sense that both lead to maximization of the present value of returns. He says that the internal rate of return criterion is often rejected on grounds that there may be no unique value of the internal rate of return associated with an investment option. He concludes that rejection of the criterion on grounds of non-uniqueness of the internal rate results from efforts to extend a two-period definition of the criterion to a multiperiod analysis in general.

Grant (1964) also recognizes the non-uniqueness problem. He notes that the calculation of rate of return may be expressed by an algebraic equation in which the interest rate is unknown, and that certain of these equations have two or more roots. Although analysts should be aware of the circumstances under which two or more different rates of return may be computed from the same cash flow, Grant's experience is that those circumstances are rarely a source of difficulty in actual studies.

The formula for finding the internal rate of return is simply the present value formula (given in the preceding section) solved for that particular value of interest, i, that causes the present value to equal zero. In other words, the same basic equation is used for both methods, but in the present value method the discount rate is specified and the present value found, while in the internal rate of return method the present value is specified to equal zero and the value of i that forces the present value to equal zero is found. Perhaps the fact that the internal rate of return is found by trial and error accounts for its secondary use when compared to the present value method.

Under ordinary circumstances, the present value and the internal rate of return criteria give identical rankings to mutually exclusive projects.

Therefore, using either of the criteria will result in the same selection when choosing among competing projects. The internal rate of return criterion is especially useful when comparing three or more alternatives, particularly where the alternatives relate to different proposed levels of investment. However, it will give spurious results when one (or more) of the alternatives has a positive net benefit stream and no initial costs, the case when initial costs of an existing alternative are sunk.

In the example in Section 2, Benefit-Cost Ratio Pitfall, the present values of cost and benefit streams were found for five alternatives. The alternatives were ranked on the basis of the excess of benefits over costs, and ILS_1 was found to be preferred. Using the same example, we may find the internal rate of return for each alternative, as in the table below.

Table 3.5. Summary Comparison of Alternatives

Alternative	Benefits Minus Costs	Internal Rate of Return, %
1. Existing NDB	0.95	∞
2. A -- ILS_1	7.71	12.992
3. B -- ILS_2	5.16	12.418
4. C -- VOR_1	2.63	11.548
5. D -- VOR_2	- 5.57	5.217

Note that the internal rate of return method, when applied to this particular example, gives a spurious answer for the first alternative. This is because the rate of return necessary to equate the present value of a stream of net benefits offset by zero initial cost is infinite. Thereafter, however, the ranking of alternatives by all three methods -- benefit/cost ratio, present value, and internal rate of return -- is identical.

The peculiar nature of this example insofar as the rate of return method is concerned has to do with the description of the first alternative.

In effect we are asking what the return is when we get something for nothing. Of course, it is infinite. Because the internal rate of return *is designed to treat return on investment, it is inappropriate to a problem* in which one of the alternatives requires no investment, as in Alternative A.

Approximate Capital Recovery Methods

In the section above on capital recovery, we showed how to convert an initial cost, P, into an equivalent uniform annual figure (the capital recovery amount) over a stipulated period of time. To make this conversion, we must assume an opportunity rate of interest, i, an estimated life or study period, n, and an estimated salvage value, L, at the end of the life or study period. This equation, which involves the use of the capital recovery factor, is independent of depreciation accounts. However, in some economics studies that use the method of annual costs, various combinations of depreciation and interest figures are used. The total of depreciation plus interest is intended to serve the same purpose as our annual cost of capital recovery with a return. For example, this concept was employed in a recent Department of Transportation Study (1974). The method in that study treated the cost of capital as a flow rather than as a stock measure, and included two components: opportunity costs and depreciation costs.* The interest rate is assumed to be the measure of the opportunity costs incurred when funds are used to purchase capital. This rate reflects what the funds could earn in another investment, *ceteris paribus*. An appropriate estimate of the rate paid for funds by government agencies was the U.S. Government taxable bond rate. The depreciation component reflects the loss of economic value of the capital assets due to a loss of efficiency and loss of remaining life. The depreciation rate was calculated using a 1.5 declining balance formula, expressed as follows:

$$\text{Annual Depreciation Rate} = \frac{1.5}{\text{Estimated Life}}$$

Applying the method of the DOT study to find ARTCC and tower capital

*This study actually included three components of direct capital cost: interest, depreciation, and revaluation. The revaluation rate was defined as the change in the market value of assets caused by factors other than aging. For purposes of illustrating our example, their treatment of revaluation is not included.

costs, a task conducted as Phase 1 of this contractual effort, we found the annual rate per dollar of capital stock as in Table 3.6.

Using the declining balance method of depreciation, with a rate of 1.5, the DOT method overstates the capital recovery amount; for example at an opportunity rate of 7 percent, the overstatement is by more than 50 percent. Fortuitously, the method overstates the capital recovery amount by much less when the capital recovery is calculated at the OMB-suggested opportunity rate of 10 percent. If the straight-line depreciation method had been used, the results would have closely approximated those found by the capital recovery method using an opportunity rate of 10 percent.

The method of depreciation plus opportunity costs gives satisfactory results in some cases and misleading results in others. An example of misleading results was given above. The analyst using depreciation methods to approximate capital recovery amounts, or to estimate the portion of original costs to be charged during the time period of his study (for purposes of estimating residual values), is interested in calculating the cumulative depreciation charge through the year representing his time horizon. The equations given below allow one to calculate that charge directly.

The symbols used in the equations are defined as follows:

n = number of years from present to time horizon
F = first or original cost
S = estimated salvage value at end of useful life
L = useful life in years
r = depreciation rate expressed as a decimal
i = interest rate

In the straight-line method the full useful service life and prospective net salvage value are estimated. Given the first cost of the asset and the number of years to the time horizon, the relationship for the cumulative straight-line depreciation charge through year n (SL_n) is:

$$SL_n = \frac{F - S}{L} n \qquad (1)$$

TABLE 3.6. DIRECT COST OF CAPITAL IN AIRWAYS

	Structures	Equipment
A. DOT Method		
(1) Interest Rate, *i* (Bond Rate)	0.0699	0.0699
(2) Depreciation Rate, *r* (1.5 Declining Balance)	0.0600	0.0905
(3) Life, years	25	16.57
B. Straight-Line		
(1) Interest Rate, *i* (Bond Rate)	0.0699	0.0699
(2) Depreciation Rate, *r*	0.0400	0.0600
C. Capital Recovery		
(1) Interest Rate, *i* (Bond Rate)	0.0699	0.0699
(2) Capital Recovery Factor, crf*	0.0857	0.1037
D. Capital Recovery		
(1) Interest Rate, *i* (OMB A-94)	0.1000	0.1000
(2) Capital Recovery Factor, crf	1.1102	0.1259
E. Sums		
(A) DOT Method	0.1299	0.1604
(B) Straight Line	0.1099	0.1299
(C) crf @ 7%	0.0857	0.1037
(D) crf @ 10%	0.1102	0.1259

$$^{*}\text{crf} = \frac{i\,(1+i)^n}{(1+i)^n - 1}$$

One rationale given for the use of the declining balance method is that assets reaching their final years of usefulness are generally employed in a standby or other secondary status. With this method a given rate is applied each year to the unamortized cost (i.e., that portion of original cost not already written off). The rate is sometimes expressed as a multiple of the straight-line rate. For example, an asset with an estimated life of 20 years and zero salvage value has a straight-line rate of five percent per year (100%/20 years); the double-rate declining balance method would apply a rate r of 10 percent for that asset. Unlike the other methods of depreciation, the declining balance method does not write off all the original cost of the asset, even for those assets estimated to have zero salvage value. That is, the prospective salvage value is disregarded; the rate r is calculated as:

$$r = \frac{\text{multiple of straight-line rate}}{\text{estimated life in years}} \qquad (2)$$

and the equation for DB_n, the cumulative depreciation charge through year n, is:

$$DB_n = F - F(1 - r)^n \qquad (3)$$

The sum-of-the-years digits (SOYD) method adds the corresponding to the number of years of the estimated useful life. In the first year the write off is equal to the fraction of original cost found by multiplying by the estimated useful life divided by the sum of the digits, and in the second year by the estimated useful life less one divided by the sum of the digits, etc. The equation for $SOYD_n$, the cumulative depreciation charge through year n, is:

$$SOYD_n = (F - S) \left[\frac{n(2L + 1 - n)}{L(L + 1)}\right] \qquad (4)$$

The above method writes off about three-fourths of the cost in the first half of the estimated useful life.

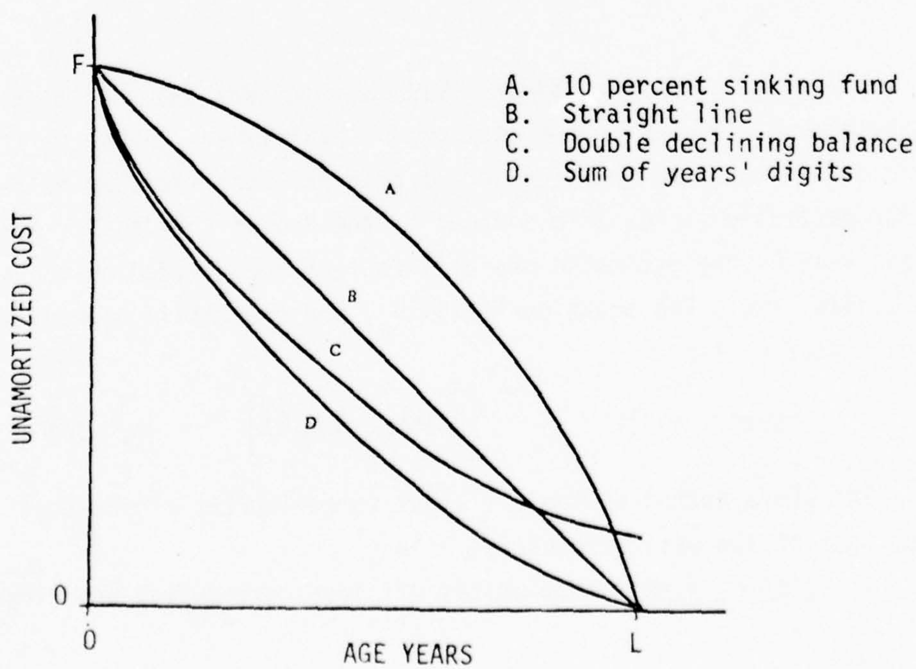
The sinking fund method writes off less cost during the first half of life than in the last half. Imagine a sinking fund into which uniform end-of-year payments are made during the lifetime of the asset. Assuming the deposits draw interest at rate i, the depreciation charge in any one year is equal to the sinking-fund payment plus interest on the imaginary accumulated fund. The cumulative sinking fund depreciation charge through year n (SF_n) is found by solving:

$$SF_n = (F - S)\left[\frac{i}{(1+i)^L - 1}\right]\left[\frac{(1+i)^n - 1}{i}\right] \qquad (5)$$

Exhibit 3.1 shows the general shape of the curve describing each method of depreciation discussed above. In summary, we do not recommend the indiscriminate use of methods of depreciation for estimating capital recovery amounts or for estimating residual values. In the former case, the use of the proper capital recovery technique is no more difficult than the approximation methods. In the latter case -- estimating residual values -- the use of an appropriately selected depreciation method is often the best one can do, and in some cases far better than ignoring the problem.

EXHIBIT 3.1

DEPRECIATION FUNCTIONS



Analysis for Retirement and Replacement - An Example

Of particular interest to the FAA is the retirement and replacement of air navigation facilities. Various causes of real property retirement include: (a) improved alternatives, (b) changes in service requirements, (c) changes in the old assets themselves, (d) changes in FAA requirements, and (e) casualties. These causes are not necessarily mutually exclusive; for instance, an asset may be retired partly because of obsolescence, partly because of inadequacy, and partly because of increasing annual costs for repairs and maintenance.

Generally, assets are retired even though still physically capable of continued service. The disposal of an asset by its owner is referred to as a retirement. Not all retirements involve scrapping the retired asset. Many assets retired by the FAA may be used by other owners before reaching the scrap heap. If an asset is retired and another asset is acquired to perform the same service, we call this a replacement.

Frequently new assets are acquired to perform the services of existing assets, with the existing assets not retired but merely transferred to some other use -- frequently an inferior use such as standby service. In those cases, the acquisition of the new asset is also described as a replacement.

Terborgh (1949) coined two terms that simplify explanation of the principles of replacement decisions. An existing old asset, considered as a possible candidate for replacement, is called the defender. The proposed new replacement asset is called the challenger. The terms are appropriate and we have adopted them for use in this discussion. In many replacement studies a common assumption regarding the defender is, if retained in service at all, it will be kept for a relatively short time. In contrast, the appropriate assumption regarding the challenger may be, if acquired, it will be kept for its full economic life. These alternatives have different service lives, a usual feature of studies in replacement economy.

An example taken from the early 1960s will serve to illustrate the special characteristics of replacement studies, a type of study not uncommon within FAA.

EXAMPLE 3.1*

Given: The FAA operates 7,015 tube-type receivers at an annual cost of $1,558,000. The initial cost, including installation, of replacing the existing receivers (the defender) with solid-state receivers (the challenger) is $2,456,400. The new receivers may be operated and maintained for $519,000 per annum. It is estimated that the tube-type receivers have an expected additional lifetime of five years, and the solid-state receivers 20 years. The present net salvage value for the existing receivers is $50,000, for the new receivers $100,000. If the existing receivers are retained another five years, their net salvage value is estimated to be negligible. The appropriate discount rate is 10 percent.

The Alternatives: (A) Retain the existing receivers for five more years, or (B) replace them now with solid-state receivers.

Find: Equivalent uniform annual costs of the two alternatives.

Solution: From the viewpoint of a replacement economics study, the capital recovery costs of the defender must be based on its present salvage value, an opportunity foregone by Alternative A. Therefore,

Retain Existing Receivers

(A) R	= ($50,000) (crf - 10% - 5 yrs)		
	= ($50,000) (0.2638)	=	$ 13,190
Equivalent uniform annual costs		=	1,558,000
			$1,571,190

* Taken from a case study prepared during the 1960s by Professor R.F. Vancil, Harvard University, for the U.S. Civil Service Commission.

(B) R = ($2,456,400 - $100,000)(crf - 10% - 20 yrs)
+ $100,000 (0.10)
= ($2,346,400) (0.11746) + $10,000
= $286,783

Annual O&M = 519,000

Equivalent Uniform Annual Cost = $805,783

In this comparison of total or life cycle costs, service over the same number of years is an implied assumption. The assumption is concealed by the use of equivalent uniform annual costs (Noah, 1965). A similar comparison using the present value criterion may be made, but it would be necessary to describe the alternatives so that service is provided for the lowest common multiple of lifetimes. The alternatives may be described as follows: (A) retain the existing receivers for five more years, then replace them with solid-state receivers having a useful life of 20 years (a 25-year life cycle), or (B) replace existing receivers now with solid state receivers (a 20-year life cycle).

The lowest common multiple of lifetimes associated with the two alternatives is quite long -- 100 years -- and their comparison on the basis of present value, while providing a ranking identical to the capital recovery criterion, would be cumbersome.

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4

SOME USEFUL VALUES

R.A. Groemping

A Framework for Estimating Benefits

The benefits of a government project can be considered as the sum of the benefits accruing to all individuals affected by that project. Benefits accruing to the users of the national aviation system and to society as a whole are dependent upon planning and action to ensure adequate capacity, minimal delay, and reasonable safety.

Adequate capacity of an ATC system is its ability to accommodate the demands of its users while maintaining a reasonable safety level without imposing undue delay. Inadequate capacity can lead to a restriction of the number of flights which can use the airport's facilities. The cost of passengers being denied service by a restricted number of flights can be measured using the concept of consumers' surplus.

Aircraft delay in the air terminal control area causes unexpected operating costs for airlines and lost time for passengers. These are measurable costs of delay. Delay also creates undesirable externalities such as noise, air pollution, and excessive energy consumption. Avoidance of the costs of delay predicted for future levels of airline activity is measured as a benefit to future users.

The value of improvements in safety is approximated by the value of aircraft not damaged or destroyed and passengers not injured or killed.

Capacity

The concept of consumers' surplus as a measure of the value of adequate capacity in regulated industries goes back to the work of Hotelling in the late 1930's. It has recently been applied to air transport pricing by de Neufville and Mira (1974) and was considered by the Aviation Cost Allocation Study team (1972).

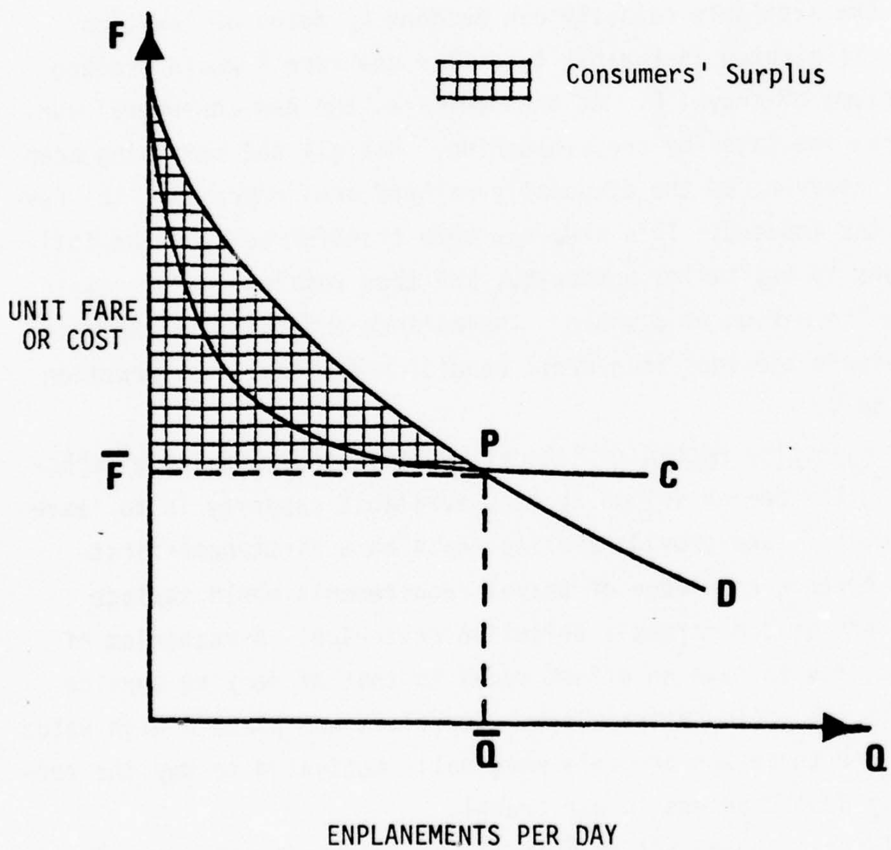
Consumers' Surplus Applied to an Individual Air Terminal. In the case of an air terminal which is unable to provide the quantity of service demanded, the concept of consumers' surplus can be used to assess the magnitude of the cost of consumer satisfaction foregone, and accordingly the benefit of increasing capacity to provide the quantity of service demanded. Exhibit 4.1 shows how the concept of consumers' surplus can be applied to an individual air terminal. Enplanements are designated Q and the fare F. The curve D is the demand curve. It shows that as the price of a good or service increases, less of that good or service is demanded by consumers. The curve C is the average cost of providing the good or service as a function of quantity offered. This cost is assumed to include all indirect costs and profits to the supplier. The cost curve for air transportation is assumed to be horizontal in the region of interest. In the absence of constraints such as capacity, equilibrium would occur at the point P, where an amount $\bar{Q}$ is bought at a fare $\bar{F}$. Scheduled air fares are, however, not determined by market forces. They are regulated by the CAB. It is assumed here that fares are being set at what would be the free-market price.

The demand curve slopes upward to the left of point P indicating that there are customers who would be willing to pay more than $\bar{F}$ for air transportation. These customers are receiving a net benefit which is the difference between what they would be willing to pay and what they are actually charged. This difference is the consumers' surplus. The cross-hatched area in Exhibit 4.1 is the total of all the individual surpluses. It has the units of dollars per day, and it represents the net benefit, as perceived by the customers, of the existence of air transportation service in the city pair market.

If an airport involved in serving a city pair has a capacity less than unconstrained demand, the full quantity of unconstrained demand, $\bar{Q}$,

EXHIBIT 4.1

UNCONSTRAINED DEMAND AND COST CURVES



cannot be served. The change in total consumers' surplus for this city pair which occurs with the reduction of available capacity is dependent on the mechanism used to reduce the travel volume. A pricing mechanism will be shown to be clearly superior to a random rejection mechanism, a "do nothing" approach.

A Pricing Mechanism. Raising the price of the service until the demand equals the available capacity can be done by means of taxation. The effect is illustrated in Exhibit 4.2. The new fare $\tilde{F}$ would produce the desired volume of travel $\tilde{Q}$. At the new fare, the new consumers' surplus is the area indicated by cross-hatching. Not all the remaining area has been lost, however, as the diagonally-hatched area represents the revenue from the tax imposed. This area has been transferred from the individual travelers to the taxing authority, and thus remains in the domain of benefits to the nation as a whole. The balance indicated by the dotted area represents the loss in overall benefit resulting from reduction in capacity $\bar{Q}$ to $\tilde{Q}$.

A Random Rejection Mechanism. An alternative to the pricing mechanism for cutting the demand volume to meet available capacity is to leave the price fixed at $\bar{F}$, and provide airline seats on a first-come-first-served basis. Advance knowledge of travel requirements would replace willingness to pay as the market's selection criterion. A mechanism of this type is assumed to have an effect equal to that of denying service to randomly rejected would-be travelers. Travelers who place a high value on the service and those who are only marginally motivated to buy the service are equally denied access to air travel.

This would have the effect of multiplying the horizontal coordinate of the demand curve by a fraction, producing the new equilibrium point $\tilde{P}$ of Exhibit 4.3. The demand has been reduced to match the new capacity value $\tilde{Q}$; there is no tax revenue, as in the pricing mechanism case; and the consumer disbenefit (with respect to the uncongested case) is again indicated by the dotted area in Exhibit 4.3. The remaining consumers' surplus accrues to the individual travelers who do obtain service.

The disbenefit area for the random-rejection mechanism is always larger than that for the previously described pricing mechanism, because the price mechanism area is always contained within the larger random-rejection area. Accordingly, this alternative is of use primarily to show the costs of a "do nothing" approach to air terminal congestion.

Quantification of the Cost of Inadequate Capacity. To determine the value of consumer disbenefit, it is necessary to make an assumption about the shape of the demand curve. There is agreement as to the slope of this curve in the vicinity of the equilibrium point, but the shape of the curve at much higher fare levels is a matter of speculation. Two alternative shapes for the demand curve are derived from assumptions of constant elasticity and constant slope. Representive curves are shown in Exhibit 4.4 with their generative equations. An elasticity[1] of -1.05, an average of elasticities of -1.02 developed by Brown and Watkins (1971) and -1.07 estimated by DeVaney (1974), is used to specify Exhibit 4.4 and for the following quantification. The constant slope line is tangential to the constant elasticity curve at equilibrium in Exhibit 4.4 at 600 enplanements, at a fare of $60.

Neither the constant elasticity nor the constant slope formulation of the demand curve is entirely satisfactory. The constant slope assumption implies that of 600 passengers paying a $60 fare, not one would be willing to pay $120 for that flight. The constant elasticity formulation of demand implies that almost half of the passengers would be willing to pay twice the initial $60 fare, and 10 percent would be willing to pay more than $500 for the flight. The actual demand at greatly reduced service levels is likely to be understated by the constant slope assumption

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[1] Elasticity $= \dfrac{dQ/Q}{dP/P}$ where Q = Quantity, P = Price

For a discussion see Samuelson (1970).

EXHIBIT 4.2

DISBENEFIT OF PRICING MECHANISM

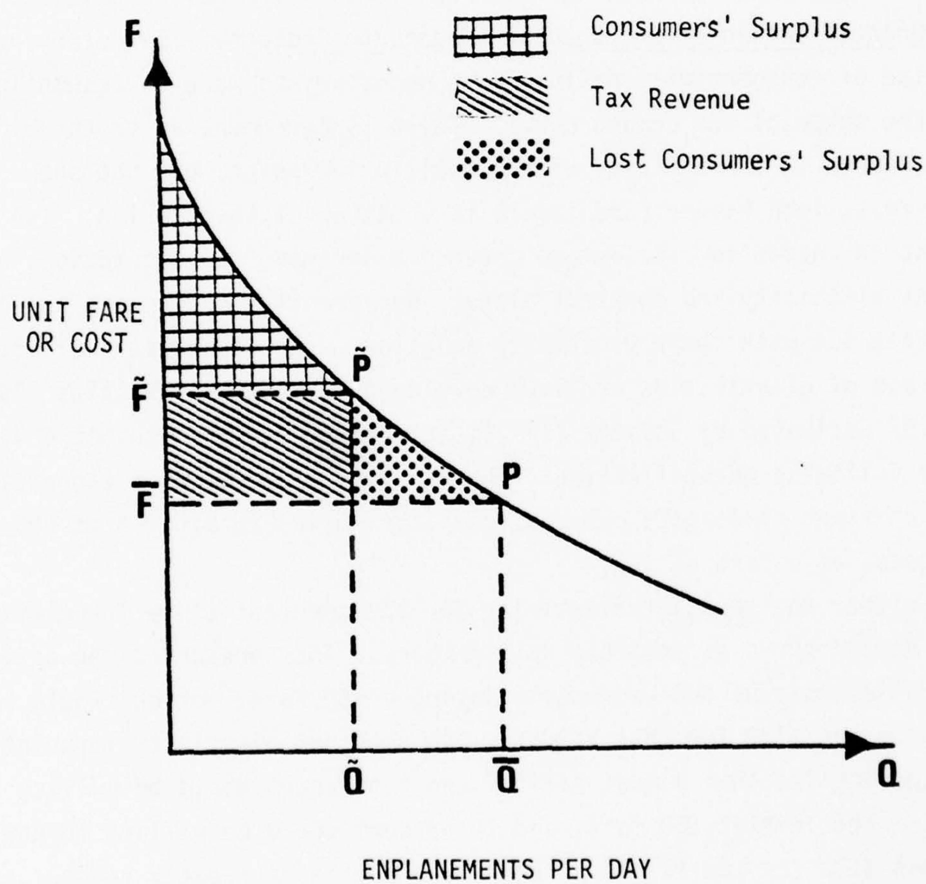


EXHIBIT 4.3

DISBENEFIT OF RANDOM-REJECTION MECHANISM

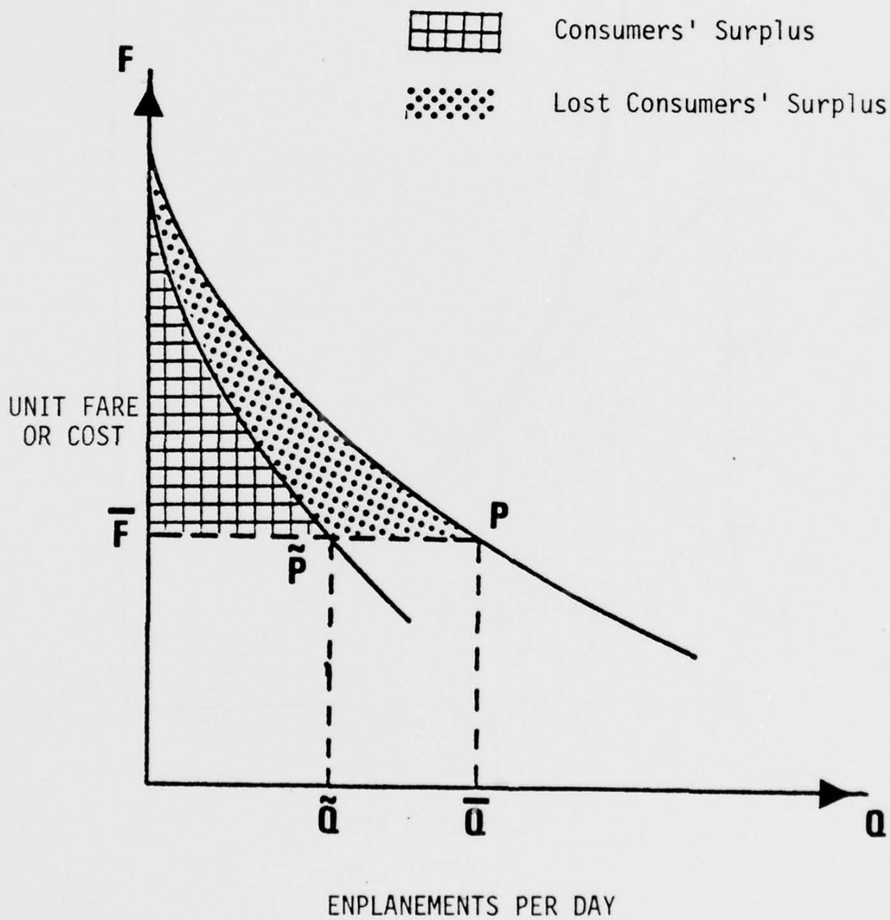
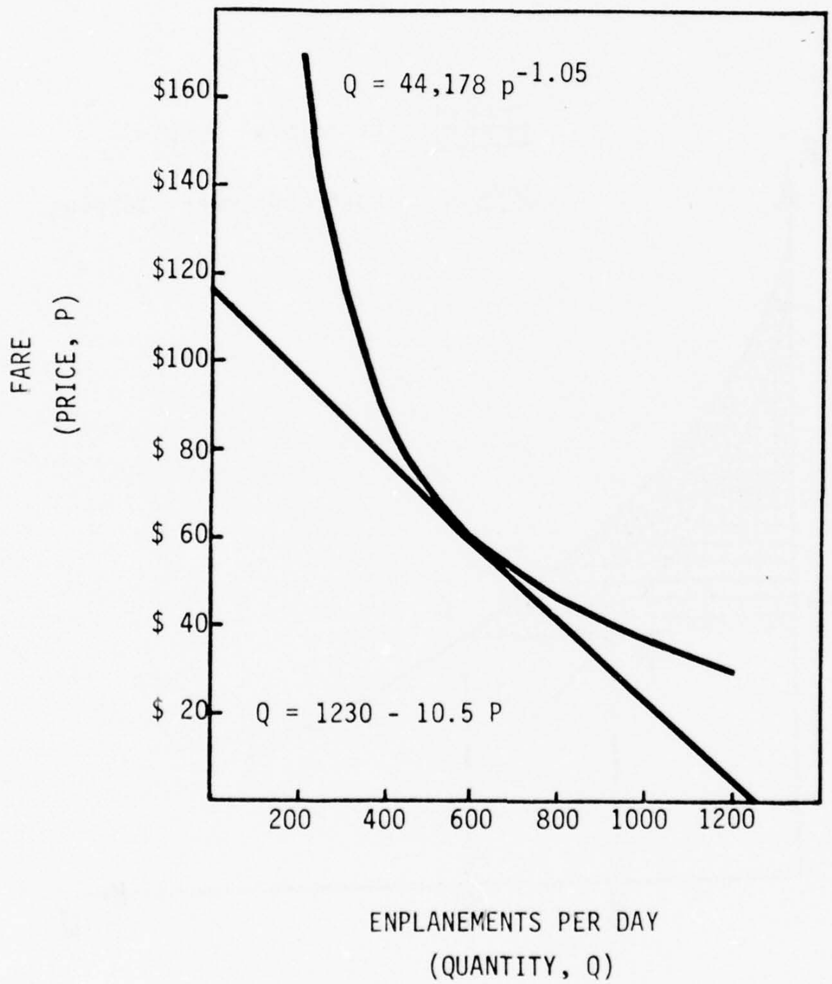


EXHIBIT 4.4

CONSTANT ELASTICITY AND
CONSTANT SLOPE DEMAND CURVES



and overstated by the assumption of constant elasticity. The two assumed shapes of the demand curve thus allow estimates of the lower and upper bounds of consumers' surplus. Table 4.1 shows consumer disbenefit and tax revenue measured as percentages of initial total revenue for capacity limitations of up to 50 percent. The constant slope figures can be used as lower bounds on the estimates, and the constant elasticity as upper bounds.

Exhibit 4.5 shows the consumers' surplus lost as a function of capacity lost and Exhibit 4.6 the tax revenue generated by the pricing mechanism. For up to a 30 percent capacity decrease, the tax revenue is more than five times as large as the consumer disbenefit, which is relatively small, no more than 5 percent of the total revenue at full capacity.

Exhibit 4.7 shows random rejection as being both more costly by a factor of 3.5 at a 30 percent capacity reduction with the constant slope assumption, and more sensitive to that assumption. The constant elasticity assumption at a 30 percent capacity restriction creates a consumer disbenefit more than one hundred times that of the price mechanism. Clearly, random rejection as a means of adjusting to restricted air terminal capacity is to be avoided.

Delay

The cost of aircraft delay may be considered as the sum of its two principal components. The first of these is direct operating costs incurred by the airlines. These costs are quite easy to ascertain in comparison to those of the second component, passenger delay. There are a number of approaches to the quantification of the cost of passenger delay, which yield somewhat different estimates. Accordingly, a range of values for passenger delay time is presented here.

The similarity of direct operating costs per passenger minute among various types of aircraft makes possible the presentation of total costs of delay in terms of dollars per passenger-minute. This approach is simpler than those involving fleet mix projections (Rogers, et al., 1975) and is likely to be no less accurate. An assumption of load factor is implicit

TABLE 4.1. THE COST OF INADEQUATE CAPACITY AS A PERCENTAGE OF TOTAL REVENUE AT UNCONSTRAINED EQUILIBRIUM.

	PRICING MECHANISM				RANDOM REJECTION MECHANISM	
	Consumer Disbenefit		Tax Revenue		Consumer Disbenefit	
Capacity Lost %	Constant Slope	Constant Elasticity	Constant Slope	Constant Elasticity	Constant Slope	Constant Elasticity
0	0 %	0 %	0 %	0 %	0 %	0 %
5	.11	.17	4.53	4.75	2.38	104.2
10	.44	.53	8.56	9.49	4.76	208.3
15	1.00	1.25	12.14	14.22	7.14	312.5
20	1.81	2.17	15.23	18.93	9.52	416.7
25	2.81	3.47	17.85	23.63	11.90	520.8
30	4.06	5.36	20.00	28.30	14.28	625.0
35	5.50	7.64	21.66	32.96	16.63	729.2
40	7.19	10.47	22.85	37.58	19.04	833.3
45	9.11	13.89	23.56	42.17	21.42	937.5
50	11.25	18.25	23.80	46.73	23.80	1041.7

EXHIBIT 4.5

CONSUMERS' SURPLUS LOST
USING THE PRICING MECHANISM

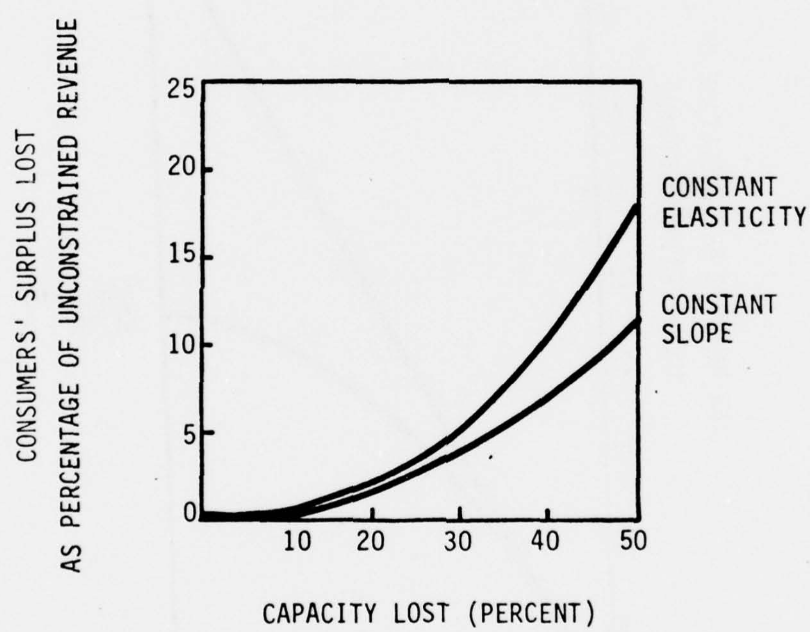


EXHIBIT 4.6

TAX REVENUE GENERATED BY THE PRICING MECHANISM

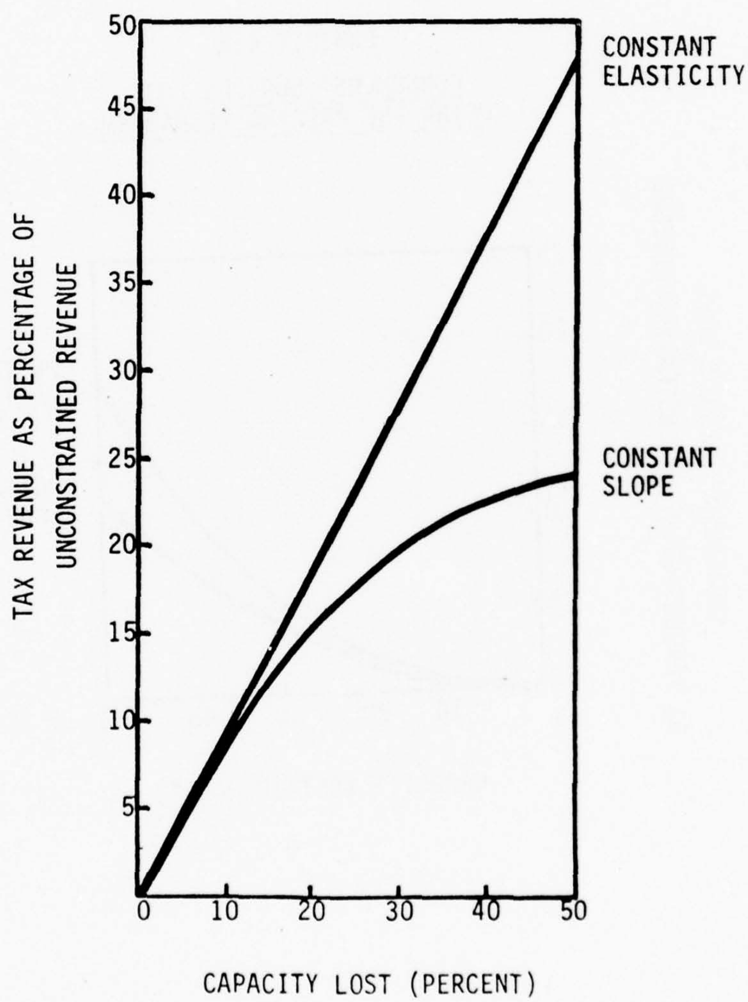
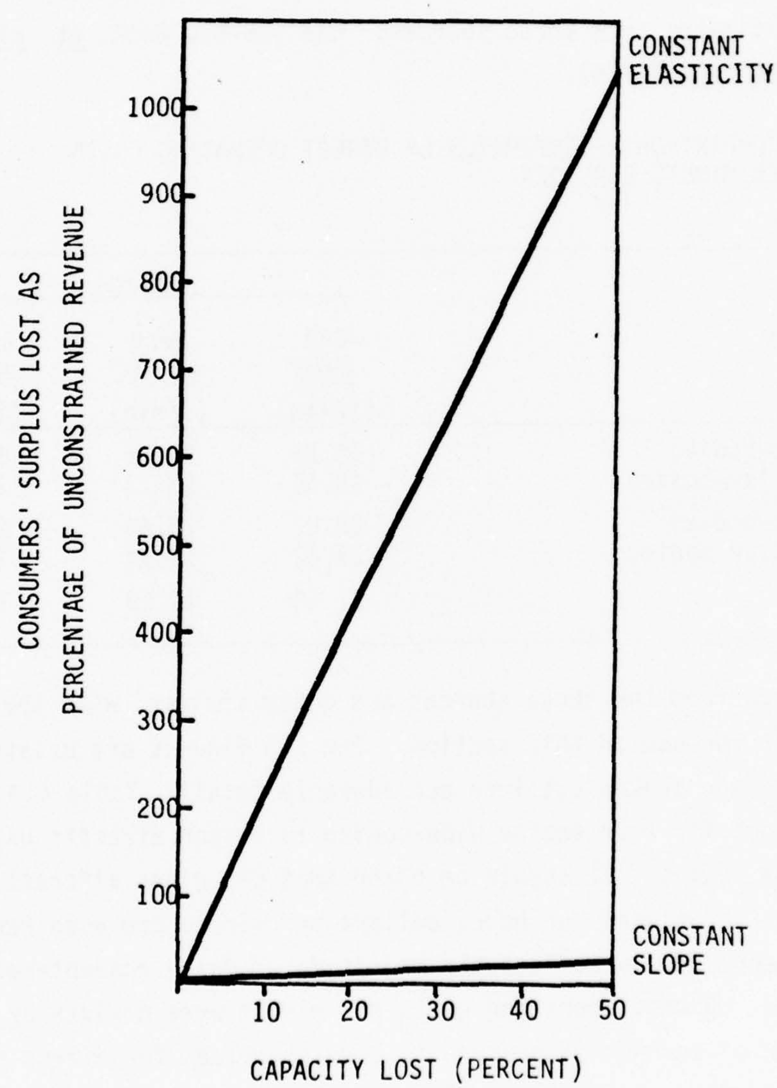


EXHIBIT 4.7

CONSUMERS' SURPLUS LOST USING THE RANDOM REJECTION MECHANISM



in dollar per passenger-minute estimation, and passenger-minute values are given as a function of load factor.

Direct Operating Costs. Direct operating costs to the airlines of additional flying time due to delay include the cost of fuel, crew time, maintenance, and depreciation. Direct operating costs differ by aircraft type, with larger aircraft costing more to fly. Table 4.2 compares dollar-per-minute estimates from three sources: CAB (1975), Reck, et al. (1975), and Rogers, et al. (1975).

TABLE 4.2. COMPARISON OF ESTIMATES OF DIRECT OPERATING COSTS IN DOLLARS PER MINUTE FOR 1974

Aircraft Type	Source		
	CAB 1975 ($/Min)	Reck et al. ($/Min)	Rogers et al. ($/Min)
4 Engine Wide-bodied	40.26	38.26	33.00
4 Engine Regular-bodied	18.54	18.22	18.00
3 Engine Wide-bodied	28.82	28.64	27.00
3 Engine Regular-bodied	14.52	14.34	13.00
2 Engine	11.58	11.19	11.00

Estimates from the three sources are quite similar, with the CAB figures chosen for use in this section. The CAB figures are updated regularly, and are broken out into considerable detail. Table 4.3 shows the CAB breakout for four engine wide-bodied turbo-fan aircraft used on domestic trunk routes. It should be noted that CAB gives aircraft operating expenses in dollars per hour; dollars per minute are used here because they more nearly reflect the magnitude of delay encountered by most airplanes. Direct operating costs per minute were divided by the average number of revenue passengers to develop values for direct operating costs per passenger-minute. These are shown in Table 4.4.

TABLE 4.3. AIRCRAFT OPERATING COST AND PERFORMANCE REPORT

LINE No.	EXPENSES, PERFORMANCE and CHARACTERISTICS	TRUNKS - DOMESTIC OP. PASSENGER CABIN CONFIG. T.FAN, 4-ENG., WIDE-BODIED	
		12 MONTHS ENDED DEC. 31, 1974	1973
	AIRCRAFT OPERATING EXPENSES		
	PER BLOCK HOUR (ALL SERVICES) (IN DOLLARS)		
	FLYING OPERATIONS (LESS RENTALS)		
1	CREW	353.38	303.66
2	FUEL AND OIL	793.99	446.55
3	INSURANCE	25.03	25.31
4	OTHER	0.77	0.82
5	TOTAL FLYING OPERATIONS (LESS RENTALS)	1183.16	776.34
	MAINTENANCE-FLIGHT EQUIPMENT		
6	DIRECT MAINTENANCE-AIRFRAME AND OTHER	149.27	120.93
7	DIRECT MAINTENANCE-ENGINE	206.03	242.39
8	MAINTENANCE BURDEN	215.16	148.59
9	TOTAL MAINTENANCE-FLIGHT EQUIPMENT	570.46	511.91
	DEPRECIATION AND RENTALS-FLIGHT EQUIPMENT		
10	DEPRECIATION-AIRFRAME AND OTHER	300.50	260.65
11	DEPRECIATION-ENGINE	67.09	54.91
12	OBSOLESCENCE AND DETERIORATION-EXPENDABLE PARTS	12.32	7.41
13	RENTALS	281.79	218.33
14	TOTAL DEPRECIATION AND RENTALS-FLIGHT EQUIPMENT	661.70	541.30
15	TOTAL AIRCRAFT OPERATING EXPENSES	2415.33	1829.55
16	PER AIRBORNE HOUR (ALL REVENUE SERVICES) (IN DOLLARS)	2708.19	2058.55
17	PER AIRCRAFT MILE (ALL REVENUE SERVICES) (IN DOLLARS)	5.43	4.08
18	PER REVENUE TON-MILE (SCHEDULED REVENUE SERVICE) (IN CENTS)	24.611	21.548
19	PER AVAILABLE TON-MILE (SCHEDULED REVENUE SERVICE) (IN CENTS)	10.442	7.922
20	PER REVENUE PASSENGER-MILE (SCHEDULED REVENUE SERVICE) (IN CENTS)	3.090	2.718
21	PER AVAILABLE SEAT-MILE (SCHEDULED REVENUE SERVICE) (IN CENTS)	1.586	1.244
	PERFORMANCE and CHARACTERISTICS		
	UTILIZATION		
22	AVERAGE AIRCRAFT ASSIGNED TO SERVICE (CARRIER'S ROUTES/EQUIPMENT)	47.4	51.8
23	TOTAL AIRBORNE HOURS (ALL REVENUE SERVICES)	142548.	177841.
24	REVENUE HOURS PER AIRCRAFT PER DAY (CARRIER'S ROUTES/EQUIPMENT) (IN HOURS)	8.24	9.41
25	PERCENT TOTAL AIRBORNE TO REVENUE AIRBORNE HOURS	101.3	101.9
26	PERCENT BLOCK-TO-BLOCK TO AIRBORNE HOURS	110.7	110.5
27	PERCENT SCHEDULED TO TOTAL REVENUE AIRCRAFT MILES	99.6	99.4
28	AVERAGE STAGE LENGTH (ALL REVENUE SERVICES) (IN MILES)	1728.	1858.
	AIRCRAFT CAPACITY		
29	AVERAGE AVAILABLE TONS PER AIRCRAFT MILE (ALL REVENUE SERVICES)	52.0	51.6
30	AVERAGE AVAILABLE SEATS PER AIRCRAFT MILE (ALL REVENUE SERVICES)	342.4	328.4
	SPEED		
31	AVERAGE AIRBORNE SPEED (ALL REVENUE SERVICES) (IN MPH)	499.	504.
32	AVERAGE BLOCK-TO-BLOCK SPEED (ALL REVENUE SERVICES) (IN MPH)	450.	456.
	PRODUCTIVITY		
33	AVERAGE AVAILABLE TON-MILES PER AIRBORNE HOUR (ALL REVENUE SERVICES)	25932.	25987.
34	AVERAGE AVAILABLE SEAT-MILES PER AIRBORNE HOUR (ALL REVENUE SERVICES)	170731.	165552.
	FUEL		
35	GALLONS OF FUEL CONSUMED PER BLOCK HOUR (ALL SERVICES)	3335.	3388.
36	COST OF FUEL PER GALLON (ALL SERVICES) (IN CENTS)	23.268	12.780
	TRAFFIC		
37	AVERAGE REVENUE TONS PER AIRCRAFT MILE (SCHEDULED REVENUE SERVICE)	22.1	19.0
38	AVERAGE REVENUE PASSENGERS PER AIRCRAFT MILE (SCHEDULED REVENUE SERVICE)	175.8	150.2
39	TON LOAD FACTOR (SCHEDULED REVENUE SERVICE) (PERCENT)	42.4	36.8
40	SEAT LOAD FACTOR (SCHEDULED REVENUE SERVICE) (PERCENT)	51.3	45.7
41	FIRST CLASS PASSENGER LOAD FACTOR (SCHEDULED REVENUE SERVICE) (PERCENT)	40.8	35.3
42	COACH PASSENGER LOAD FACTOR (SCHEDULED REVENUE SERVICE) (PERCENT)	52.8	47.3
43	PERCENT COACH TO TOTAL REVENUE PASSENGER-MILES (SCHEDULED REVENUE SERVICE)	90.1	90.1

Source: Aircraft Operating Cost and Performance Report, Volume IX, Civil Aeronautics Board, Washington, D.C., July 1975.

TABLE 4.4. DIRECT OPERATING COSTS PER PASSENGER-MINUTE BY AIRCRAFT TYPE

Turbo-Fan Aircraft	Direct Operating Cost Per Minute ($/Min)	Average Number of Passengers (No. of Pax)	Direct Operating Cost Per Pax-Min ($/Pax-Min)
4 Engine Wide-bodied	40.26	175.8	.229
4 Engine Regular-bodied	18.54	78.2	.237
3 Engine Wide-bodied	28.82	112.6	.256
3 Engine Regular-bodied	14.52	64.5	.225
2 Engine	11.58	56.9	.204

Source: Aircraft Operating Cost and Performance Report, Volume IX, Civil Aeronautics Board, Washington, D.C., July 1975.

Passenger Delay Costs. In addition to the direct operating costs to the airlines quantified above, there are costs associated with passengers being delayed. After a review of the literature (236 references) on the subject of the value of time to travelers, Haney (1975) concluded that "the views about the value of time are about as numerous as there are writers willing to express them." There are at least three methods which can be used to estimate the value of avoiding delay in air travel.

One approach is to determine the travelers' willingness to pay to avoid delay. Such a method would require extensive surveying of the flying population, and be open to questions of validity as to the passengers' estimation of the value or cost of their being delayed. This method does not play a prominent role in the current literature.

Another approach is to value an air traveler's time as a function of his wage. Some authors consider delay time to be only 50 percent lost, while others hold that overhead rates should be applied to the basic wage rate. Accordingly, given agreement on a $12.50 median wage for air travelers, a range of $6.25 per hour (50 percent lost time) to $25.00 per hour (100 percent overhead rate) could be developed. An assumption underlying this analysis is that an average hourly wage rate can be adjusted to be applicable to those people whose time often does not have the uniform value implied by an hourly wage; e.g., salaried employees and vacationers.

A more sophisticated development on the scene of air traveler time valuation is the use of derived elasticities of the demand function for air travel to compute the value air travelers place on their time. This procedure yields results quite similar to the average wage of air travelers. This similarity prompted De Vaney (1974) to suggest that "air travelers value their time at their wage." This may very well be true, but it does not necessarily follow that passengers place the same value on unexpected delay time as on expected travel time.

Two estimates of the value of passenger time are summarized in Table 4.5. These estimates are averages of other estimates. Zaidman (1975) found the average of the 1974 dollar equivalents of five estimates from the 1962-1967 period, each based on a different assumption as to the disutility of delay. Reck (1975) averaged two estimates based on calculated elasticity of demand with an estimate of the value of the time of general aviation travelers.

The estimates of the value of air passengers time developed by Brown and Watkins (1971) and De Vaney (1974) as cited by Reck, are in close agreement. Their method of calculating the value of time from the derived elasticity of the demand function avoids the subjectivity which characterizes the factors used to weight an average hourly wage figure. Assuming that 90 percent of air passengers travel coach and 10 percent first class (CAB 1975), Brown and Watkin's coach and first class values of time average to \$11.07 per hour. This compares to \$11.71 estimated by De Vaney. The average of these two numbers is \$11.39 per hour, or \$0.190 per minute.

Application. A consideration in the application of a value to delay time is that passengers are not likely to perceive the cost of delay as a linear function. That is, delay of only a few minutes is likely to be ignored, and after a considerable amount of delay, some passengers may consider a range of \$11.39 per hour to be a distinct undervaluation of their time.

TABLE 4.5. SUMMARY OF ESTIMATES OF THE VALUE OF PASSENGER TIME

Source	Method	1974 $ Per Hour
Zaidman, Steve[1]	average	$13.89
Gansle (1967)	wage factor	$24.35
Baxter (1967)	wage factor	$13.98
Fromm (1962)	wage factor	$11.99
Skaggs (1965)	wage factor	$ 6.94
Reck, Robert[2]	average	$12.50
Brown and Watkins (1971)	elasticity	$15.64/$10.57[3]
De Vaney (1974)	elasticity	$11.71
Kirkwood, et. al. (1973)	delphi	$12.65

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[1]Zaidman, Steve, Internal Memorandum, Office of Aviation System Plans, Federal Aviation Administration, 1975.

[2]Reck, Robert, et al., "Cost Analyses Supporting the Cost-Benefit Study of the Upgraded Third Generation ATC System." Department of Transportation, Cambridge, MA, December 1975.

[3]first class/coach

The dollar values developed in this section are intended for use with estimates of passenger delay time. Estimates of delay time often involve projected air traffic loads between hundreds of city pairs. (Reck, et al. 1975; Rogers, et al. 1975; Kirkwood, et al. 1973). There is considerable uncertainty in these estimates because in addition to the uncertainty inherent in any forecasting, there is no firm definition of what constitutes an aircraft delay imposed by an airport or air traffic control system.

One possibility is to include only those delays caused by the probabilistic nature of aircraft demands, the finite rate at which an airport or ATC system can service aircraft, and the queues of aircraft that result when more aircraft demand service than the airport or ATC system can simultaneously accommodate. It is this class of congestion delays on which an ATC system has the greatest impact.

Given the probabilistic nature of aircraft demands and weather conditions, airports frequently operate at some level of delay, regardless of their capacity. For example, if two aircraft request clearance to land at the same time on the same runway, one aircraft will be forced to absorb a delay. Even if these were the only two aircraft to arrive during the day, a rate far below any airport's capacity, the random nature of their arrival pattern imposes delays. Likewise, a thunderstorm passing an airport may create delays on all arrivals, perhaps forcing some to seek alternate airports.

The technique for measuring benefits due to reduced delays is to estimate the costs imposed on the users of the aviation system, including direct aircraft operating costs and costs to passengers resulting from those delays. The benefits associated with an alternative scheme for reducing those delays are equal to the costs of the delays avoided.

Safety

Introduction. There are two cost components to safety estimation. The first is the cost of aircraft damaged or destroyed in accidents. The second is the cost of passengers injured or killed. Estimates of the value of aircraft damaged or destroyed are based on average aircraft replacement values obtained from airline industry statistics. This estimate is a straightforward calculation.

There are a number of conceptually different approaches to measuring the cost of passenger fatalities. One is to develop an estimate of the present value of the average passenger's lifetime earnings stream. This approach attempts to measure the value of the passenger to society. A second method uses his wealth and utility functions to develop an estimate of the dollar value at which an individual can maximize his utility. This approach estimates a passenger's value to himself, and results in a figure generally higher than his discounted earnings stream. A third method of valuing the life of an air passenger is to determine the average value of settlements to relatives or to the estate in the event of a passenger's death due to negligence. This is an estimate of the value of a passenger's life to his family and relatives.

The costs of injury have often been estimated as the sum of income lost and medical expenses. The estimates for serious injuries are significantly less than the average settlement for these injuries.

Property Damage. Because of FAA certification requirements, all aircraft within a given fleet -- air carrier, general aviation, or air taxi -- are assumed to be equally safe. The probability of accident is further assumed to be uniform throughout the fleet. The average value of the various elements of that fleet can therefore be considered the basis for replacement or repair of any element of that fleet involved in an accident. (Reck, 1975)

The Airline Statistical Annual (1974) sets the total air carrier (trunk and local jet) flight equipment replacement value at slightly over $12.6 billion on January 1, 1974. The fleet consisted of 2091 aircraft, with an average value per aircraft of about $6.04 million. This value is similar to the $6.0 million air carrier replacement value estimate put forth by Ashby (1975) in the ILS Establishment Criteria Study. Thus, the recommended air carrier replacement value is taken as $6.0 million in constant 1974 dollars.

The replacement cost of General Aviation (GA) aircraft is averaged over an extensive and diverse fleet. The value of $50 thousand developed by Ashby (1975) is consistent with that of $47 thousand estimated by RAND (1973) and $47.6 thousand used in a GA cost impact study (1975). The value of $50 thousand is recommended for use as the replacement value of GA aircraft.

Air taxi replacement costs were estimated at $200 thousand by Ashby in the ILS study. Because of the overall consistency of that study and its compatibility with other sources, the estimated replacement cost of $200 thousand is recommended here.

The CAB recognizes several categories of damage to aircraft -- destroyed, substantial damage, minor damage, and none. The cost of a destroyed aircraft is taken to be the replacement cost given above. Insurance experience indicates that average repair cost of a substantially damaged aircraft is one-third of the replacement cost. Repair costs are negligible for aircraft with minor damage. (Reck, et al., 1975)

Reck found no firm values for third party property damage arising from aircraft accidents. However, estimates made by RAND (1973) based on an earlier study by Fromm were cited. In 1974 dollars, these estimates are $40 thousand for air carriers and $400 for general aviation. No estimate is available for air taxi third party property damage costs.

These values are negligible in comparison to the costs of aircraft damage and destruction, and are not included in Table 4.8 summarizing property damage values.

Injury. The basic estimates of the cost of injury to airline passengers were developed by Fromm (1968), and have been used extensively since then. (Kirkwood, 1973; Mitre, 1975; Reck, et al., 1975). Reck updated Fromm's estimates to 1974 dollars.

Fromm assumed the average seriously-injured passenger requires about six months to fully recuperate from the accident with a per injury cost of $45 thousand for air carrier and air taxi accidents and $38 thousand for GA accidents (1974 dollars). The lower cost attributable to the GA victim, despite a generally higher income, reflects much lower per incident accident investigation costs. For minor injuries, Fromm assumed the victim is incapacitated for one month. The per injury cost (in 1974 dollars) is estimated at $6,000.

TABLE 4.6. SUMMARY OF RECOMMENDED VALUES OF PROPERTY DAMAGE, THOUSANDS OF 1974 DOLLARS

	Destroyed	Substantially Damaged	Minor or No Damage
Air Carrier	$6,000	$2,000	0
Air Taxi	200	67	0
General Aviation	50	17	0

Table 4.7 shows average settlements for serious injury resulting from aircraft accidents. For the 1964 to 1974 period, the average of 341 settlements was $78,180 (1974 dollars), significantly higher than Fromm's estimates. However, the $78,180 figure represents an actual cost to air carrier insurers, and ultimately, to the flying public. This estimate, rounded to $80,000, is the value recommended for use in cost-benefit studies.

TABLE 4.7. NON-WARSAW AVERAGE SETTLEMENTS FOR SERIOUS INJURY RESULTING FROM AIRCRAFT ACCIDENTS

Year	Number of Settlements	Thousands of Current Year Dollars	Thousands of 1974 Dollars*
1964	6	6.1	9.5
1965	11	7.0	10.7
1966	25	40.8	60.9
1967	23	48.5	70.2
1968	25	39.1	54.4
1969	35	42.2	56.0
1970	38	63.5	79.9
1971	41	23.7	28.5
1972	25	28.7	33.4
1973	37	52.4	57.8
1974	75	171.3	171.3
Totals	341		78.2

* developed using the GNP deflator.

Source: Civil Aeronautics Board

The Value of Life. There are a number of approaches to establishing a dollar value for the lives of airline passengers. One approach is to calculate the present value of a typical passenger's expected earnings stream.

The typical airline passenger is about 40 years old* and has an income of about $24 thousand per year.** Salary levels tend to stabilize by age 40. Assuming that a passenger's average annual salary is $24 thousand constant 1974 dollars, and that his salary increases at 2½ percent per annum for the next 25 years, the present value of his earnings stream at a 10 percent discount rate is about $300 thousand.

Recent work by Conley (1976), indicates the value of an individual's life is greater than the present value of his lifetime labor income. Conley extends the traditional model of individual maximization to include the effects of choices involving a changed probability of living. The value of human life is estimated with reference to an individual's wealth and utility function characteristics. An individual is expected to maximize the present value of his lifetime utility stream. The utility stream includes the individual's estimation of his probability of living during future time periods. His utility maximization is constrained by his expected discounted labor income, which he can schedule throughout his lifetime by borrowing money against future income. The potential difference between the commercially based discount rate of money, and an individual's discount rate of his future utility allows the present value of utility to be maximized at a level higher than that of his income.

Conley summarizes his findings by presenting estimates for the value of life as a function, α, of the present value of an earnings stream.

---

*Courtesy United Airlines Corporate Affairs Office, Washington, D.C.

**Based on $11.39 per hour developed in preceding "Capacity" section, corroborated by Zaidman 1975.

Present Value of Earnings Stream	$30,000	$70,000	$100,000	$200,000	$300,000
α	3	1	2/3	1/2	3/7 (est.)
"Value of Life"	$10,000	$70,000	$150,000	$400,000	$700,000

By Conley's method, the typical passenger with an expected earnings stream having a present value of $300,000 values his life at $700,000.

Fromm (1966) suggests increasing the value of an individual's life to himself by the following values to others:

Family	$225,000
Community	$ 52,000
Employer + Government + Airlines	$ 23,000

The passenger by these standards is worth about $1 million.

Fourth, the judicial process, as approximated by the average of settlement agreements and court decisions, has over the 11-year period of 1964 to 1974 established a trend leading to an estimated average settlement of about $195 thousand per fatality (1974 dollars). This is a considerably lower number than that previously estimated for the average value of settlements. Data obtained from the Civil Aeronautics Board based on non-Warsaw payments during the 1966-1970 period were used by the FAA Office of Aviation Policy and Plans to develop a value of $300 thousand per aircraft accident fatality.

This number was developed by extrapolating the trend line indicated by the air accident fatality settlement data from 1966 to 1970. For those years the upward trend in the constant year dollar values was expected to lead to an average settlement value of $300 thousand constant 1974 dollars.

The increases forecast for the 1971 to 1974 period did not occur to the extent predicted. Table 4.8 shows average settlements as reported by the Civil Aeronautics Board inflated to constant 1974 dollars using the GNP deflator. Exhibit 4.8 presents this data, the FAA projected trend line, and a trend line developed for the years 1964 to 1974. The years

TABLE 4.8. NON-WARSAW AVERAGE SETTLEMENTS FOR AIRCRAFT ACCIDENT FATALITIES

Year	Number of Settlements	Average Settlement Current Year Thousands of Dollars	Average Settlement Constant 1974 Thousands of Dollars*
1964	1	49.0	76.6
1965	23	60.2	92.4
1966	46	102.4	152.9
1967	29	61.8	89.4
1968	117	114.2	158.8
1969	128	140.6	186.6
1970	112	165.3	207.9
1971	170	123.7	148.9
1972	165	122.4	142.5
1973	99	148.2	163.4
1974	141	233.2	233.2
Totals	1,031		171.0

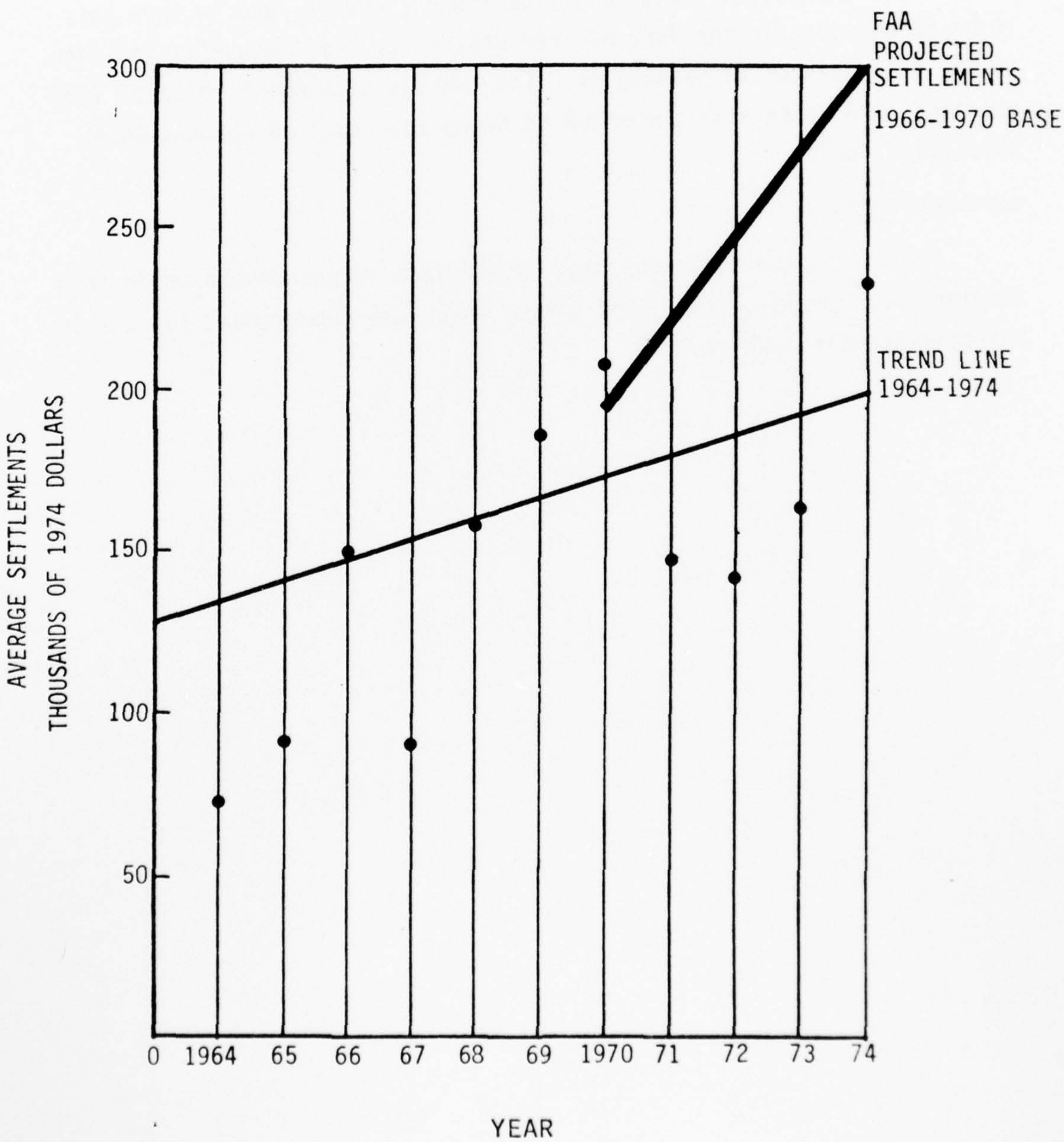
*The GNP deflator was used.

Source: Civil Aeronautics Board

EXHIBIT 4.8

NON-WARSAW AVERAGE SETTLEMENTS FOR AIRCRAFT ACCIDENT FATALITIES

(1974 DOLLARS)



1964 to 1967 were combined into a single data point of 1965½, with a value of $126.4 thousand, because of the fewer settlements in these early years. This trend line estimates a $195,000 average settlement in 1974.

The expected earnings, expected utility, and expected utility plus value to others criteria are theoretical constructs. The average settlement value is an actual cost to air carriers insurers, who in turn pass it on to airlines in the form of premiums, which are eventually included in the fare charged to passengers. The average settlement value of $195 thousand 1974 dollars is preferred as being an actual as opposed to a theoretical cost.

Summary of Values

Table 4.9, the following page, summarizes values developed in this chapter for capacity, delay, and safety which are recommended for use in FAA cost-benefit analysis.

TABLE 4.9. SUMMARY OF VALUES RECOMMENDED FOR USE IN FAA COST-BENEFIT ANALYSIS

CAPACITY	Percent of Total Revenue	
Capacity Lost	Consumers' Surplus Lost*	Tax Revenue Produced*
0%	0 %	0 %
5	.17	4.75
10	.53	9.49
15	1.25	14.22
20	2.17	18.93
25	3.47	23.63
30	5.36	28.30
35	7.64	32.96
40	10.47	37.58
45	13.89	42.17
50	18.25	46.73

DELAY	Cost Per Passenger-Minute of Delay
	$0.420 Per Passenger-Minute of Delay (1974 Dollars)

SAFETY	Amount of Damage (1974 Dollars)		
Property Damage	Destroyed	Substantial Damage	Minor or No Damage
Air Carrier	$6,000,000	$2,000,000	0
Air Taxi	200,000	67,000	0
General Aviation	50,000	17,000	0
Personal Injury	Fatality	Seriously Injured	Minor Injury
All Air Travelers	$ 195,000	$ 80,000	$6,000

*As percentage of dollar value of Total Revenue at unconstrained equilibrium, assuming a constant elasticity demand curve.

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5

PROBLEMS RELATED TO RATE OF DISCOUNT

The necessary instrument in present value and capital recovery criteria is the appropriate rate of interest or rate of discount by which the net benefit or cost at any point in time is weighted. It is commonly assumed that the correct rate of interest is that which reflects society's rate of time preference. Generally, there are two broad approaches to determining a social discount rate:[1]

- o The Social Rate Approach
- o The Private Opportunity Rate Approach

The Social Rate of Time Preference

The "social time preference approach," associated prominently with Marglin (1963), notes the dependence of some individual's utility on future consumption by others, and deduces from this dependence a difference between social time preference and an aggregation of individual preferences as reflected in private capital markets. Consumption by future generations is thought of as a public good. In providing that public good, present individuals collectively agree to some amount of current capital formation beyond that which would be undertaken by each acting in isolation. The bond rate, in this view, could not then serve even as an appropriate measure of pure (social) time preferences. Rather, some explicit political decisionmaking procedure, perhaps democratic voting as suggested by Marglin,

[1] While it is possible to distinguish between two broad approaches -- the market opportunity and the social time preference approaches -- and there is also some reason to believe that public and private discount rates might differ, even in theory, the controversy surrounding this subject among eminent economists is so great as to be beyond the scope of this Guide.

would be required to determine the overall rates of growth, investment, and discount in a society. However, the morass of difficulties in designing a theoretical mechanism by which a social consensus would emerge from the welter of individual values has been well noted. (See Arrow, 1951, and others).

The Private Opportunity Rate of Time Preference

The "private opportunity rate approach" is identified with theoretical arguments by Hirshleifer (1965, 1966), Mishan (1967) and Baumol (1968), among others. According to this view, the source of funds for any public project is ultimately the private sector, so that net returns to the public project ought to be discounted by the private opportunity rate, i.e., by the rate of return on investment in the private sector.

Eugene Grant, considered the father of engineering economy and whose book has been used as a standard text and reference since the 1930s, considers interest as a cost "in the sense of an opportunity foregone, an economic sacrifice of a possible income that might have been obtained by investment elsewhere." The Office of Management and Budget presently prescribes a private opportunity rate of 10 percent with certain exceptions.[2] For example, the Water Resources Council was permitted to use 4 7/8 percent until 1972. The Department of Defense has a directive permitting the use of range of rates from 5 to 12 percent.[3]

Choosing a Discount Rate

We recommend that 10 percent be used as the discount rate in evaluating FAA alternatives on a cost-benefit basis. This rate is in keeping with the OMB Circular, and represents a rate that is a proxy for the private

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[2] Circular A-94, revised, Office of Management and Budget, "Discount Rates to be Used in Evaluating Deferred Costs and Benefits," March 27, 1972.

[3] Department of Defense, Instruction 7041.3, "Economic Analysis of Proposed Department of Defense Investments," February 16, 1969.

opportunity rate. Tables 3.2 and 3.3 are provided to permit discounting streams of costs and benefits at rates varying from 5.0 percent to 25.0 percent using discrete (end-of-year convention) or continuous compounding techniques.

The OMB Position

The 1969 version of OMB Circular A-94 refers to a study of the interest rate representing opportunities foregone in the private sector. The Assistant Director of Program Evaluation, when contacted, suggested the use of a 10 percent discount rate, and referred to a study by Stockfisch (1969). A summary of that study is quoted:

> *This paper accepts the position that the rate-of-return, or capital cost measure, employed in the evaluation or costing of government programs (or cost-benefit and cost-effectiveness studies) should equal the before-tax rate of return generated by private, physical investment. Such a cost-of-capital measure is similar to the economists conception of "the marginal productivity of capital," or "the marginal efficiency of investment." Such theoretical concepts, however, imply that the economy is in a state of "general equilibrium," in which rates of return on all margins of investment are equal, and that uncertainty or risk associated with physical investment exerts no effect on the rate of return. Because of the limitations of such assumptions, there is no readily available single measure revealed by private sector capital markets that provides a measure of capital costs that can be used for evaluating capital costs of government projects. Security yields -- especially government debt yields -- are most deficient as a measure of capital cost, particularly when viewed in the opportunity cost sense. Security yields are deficient measures because they express returns from private, physical investment activity after corporate and property taxes; and because they are claims against combinations of physical assets and cash holdings of business organizations. The presence of cash in asset portfolios causes security yields to be lower than the earnings imputable to physical asset investment.*
>
> *An attempt is made to estimate rates of return from different margins of physical investment in the private sector by use of financial accounting information*

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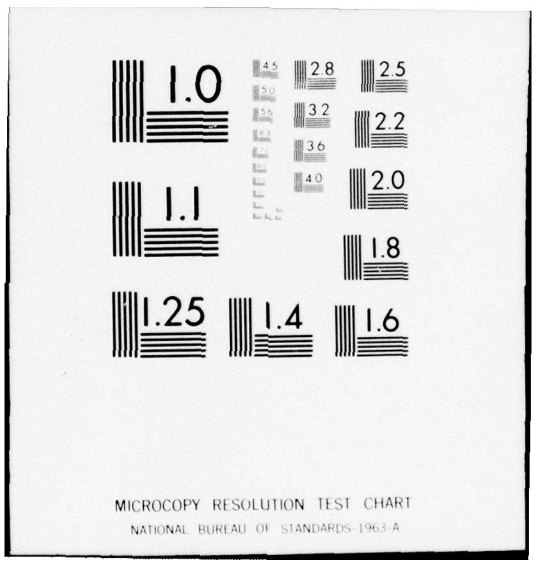
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sources, to adjust those returns by factors based on tax data, and to weight the different returns by appropriate weights reflecting the relative importance of major activities.

It is shown that the before-tax rate of return in corporate manufacturing is 16.5 percent; in public utilities, 11.5 percent; and in the non-corporate sector, 10 percent. If these rates are weighted in proportion to their relative importance, the before tax rate of return -- unadjusted for inflation -- is 12 percent. An adjustment for inflation is estimated to be 1.6 percent for the period to which the analysis applies. The "real" opportunity rate of return is thus estimated to be 10.4 percent.

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6

PROBLEMS IN NORMALIZING COSTS

To be of use to the analyst, raw data collected for the purpose of estimating benefits and costs must be made consistent and comparable. Yet often they are neither. Hence, before estimating relationships can be derived, raw data have to be adjusted; examples being production quantity differences, production rate differences, price level changes, physical and performance characteristics, and the sometimes arbitrary distinction between recurring and nonrecurring costs.

This section focuses on quantitative adjustments that should be made for production quantity, production rate, and price level differences.

Cost-Quantity Relationships

Near the turn of the century, industrial engineers developed a technique called the "learning" curve which related production manhours per unit to quantity of units. Engineers observed that as the quantity of units produced increased, the manhours per unit necessary to produce decreased. Just prior to World War II, this technique was extended and used for estimating the cost of producing airframes. This estimating technique has been called by various names -- improvement curve, progress curve, and learning curve. The technique has since been applied to production of many kinds of items. (Asher, 1956)

The theory of the progress curve, although developed for labor manhours, has been applied to costs including labor, material, and overhead. In its most popular form, the theory states: as the total quantity of units produced doubles, the cost per unit declines by some constant percentage. There are two cost per unit variants to this popular form of progress curve; one treats cost per unit as the average cost of a given number of units, and the other form treats it as the cost of a specific unit. Either formulation results in a linear function when plotted on logarithmic grids. When the cost per unit is assumed to be the average cost, the form of the progress curve is known as log-linear cumulative average curve; when the cost per unit is treated as the cost of a specific unit, it is known as the log-linear individual unit curve.

It has not been established that one form of progress curve will more closely approximate the cost of future production of a particular item than will some other form. However, a convenient technique is based on the log-linear cumulative average function. Therefore, this section is limited to a discussion of that formulation, a tabulation of factors for its application, and a brief example.

When it is assumed that a percentage increase in production results in a constant percentage decline in the <u>average</u> cost, the cumulative average curve is described by

$$\bar{Y}_n = ax^b \qquad (1)$$

which is a function having the characteristic of linearity on logarithmic grids (see Exhibit 6.1). This characteristic probably accounts in large measure for its continued popularity as a form of progress curve.

Although developed on the basis of direct man-hour data and most appropriately used to explain the principle of direct man-hour reduction versus quantity produced, common usage today extends the theory to apply to the sum of all elements of hardware cost. As such,

$\bar{Y}_n$ = cumulative average cost of "n" items
x = cumulative output
a = cost of the first article
b = slope of the logarithmic curve

When the cumulative average curve is assumed to be log-linear as in equation (1), the resulting unit curve is expressed by the function

$$Y_i = a \left[x_i^{1+b} - x_{i-1}^{1+b} \right] \qquad (2)$$

where

Y_i = the cost per unit for the i^{th} unit
x_i = the cumulative unit number

Table 6.1 gives values of equations (1) and (2) for selected slopes and quantities when "a" is equal to one.* For convenience, the ratios for these selected points are also given. The ratios are useful for quickly determining either the unit or cumulative average cost when one or the other is known at a given quantity and slope. The unit curve values allow one to find the unit cost for any combination of quantity and slope tabled, given the cost at some other quantity appearing in the table. Likewise, the cumulative average values allow one to find the cumulative average cost

*Slope is given in percentage terms; e.g., an 80 percent slope means that with a doubling of cumulative output, costs reduce by a factor of 0.80.

EXHIBIT 6.1

PROGRESS CURVES RESULTING FROM THE ASSUMPTION OF A LINEAR CUMULATIVE AVERAGE CURVE ON LOGARITHMIC GRIDS

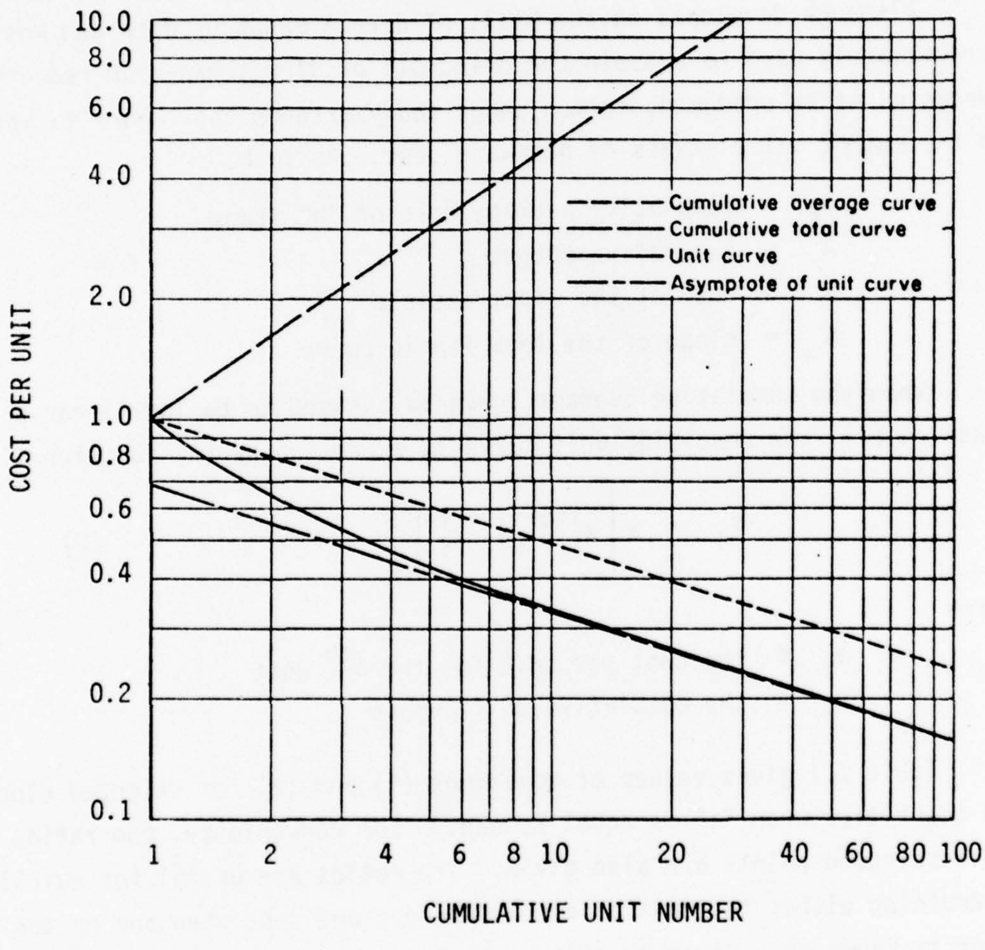

TABLE 6.1. SLOPE-QUANTITY FACTORS FOR THE LOG-LINEAR CUMULATIVE AVERAGE CURVE.

		QUANTITY								QUANTITY			
	SLOPE	2	5	10	25	50	100	250	500	1000	2000	5000	10000
	95	.90000	.82859	.78392	.73082	.69376	.65882	.61549	.584679	.555424	.527643	.493046	.468392
	90	.80000	.67493	.60229	.52143	.46860	.42142	.36646	.329768	.296768	.267081	.232351	.209115
UNIT CURVE	85	.70000	.53835	.45162	.36162	.30664	.26034	.20986	.178341	.151572	.128628	.103919	.088330
	80	.60000	.41813	.32855	.24214	.19308	.15421	.11470	.091736	.073377	.058697	.043700	.034959
Y'I	75	.50000	.31372	.22985	.15509	.11582	.08668	.05919	.044374	.033274	.024952	.017058	.012793
	70	.40000	.22423	.15246	.09360	.06518	.04550	.02835	.019839	.013884	.009717	.006063	.004244
	65	.30000	.14894	.09346	.05185	.03348	.02169	.01225	.007961	.005173	.003361	.001902	.001236
LOG-LINEAR	95	.95000	.88771	.84333	.78804	.74864	.71121	.66458	.631356	.599782	.569799	.532444	.505821
CUMULATIVE	90	.90000	.79293	.70468	.61306	.55176	.49658	.43202	.388319	.349937	.314943	.273996	.246596
AVERAGE	85	.85000	.68567	.58232	.47014	.39962	.33967	.27400	.232908	.197972	.168276	.135743	.115382
CURVE	80	.80000	.59563	.47650	.35478	.28382	.22706	.16905	.135246	.108197	.086557	.064446	.051556
	75	.75000	.51274	.38455	.26290	.19718	.14788	.10110	.075827	.056870	.042652	.029160	.021870
YBAR'N	70	.70000	.43684	.30579	.19083	.13358	.09350	.05835	.040849	.028594	.020016	.012491	.008743
	65	.65000	.36778	.23906	.13526	.08792	.05715	.03233	.021019	.013662	.008880	.005024	.003266
	95	1.055	1,071	1.075	1.078	1.079	1.079	1.079	1.079	1.079	1.079	1.079	1.079
	90	1.125	1.160	1.169	1.175	1.177	1.178	1.178	1.179	1.179	1.179	1.179	1.179
RATIO	85	1.214	1.273	1.290	1.300	1.303	1.304	1.305	1.305	1.306	1.306	1.306	1.306
	80	1.333	1.424	1.450	1.465	1.469	1.472	1.473	1.474	1.474	1.474	1.474	1.474
YBAR'N/Y'I	75	1.500	1.634	1.673	1.695	1.702	1.705	1.708	1.708	1.709	1.709	1.709	1.709
	70	1.750	1.943	2.005	2.033	2.049	2.054	2.057	2.058	2.059	2.059	2.059	2.059
	65	2.166	2.469	2.557	2.608	2.625	2.633	2.638	2.640	2.641	2.641	2.641	2.641

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for any combination of quantity and slope tabled, given the cost at some other tabled quantity. To determine approximate solutions for values not tabulated, one may interpolate between given values of quantity and slope.

For convenience in cost analysis, slope (S) has been defined as the ratio of the "y" values (be it cumulative average cost or unit cost) at two "x" values (cumulative outputs) which are different by a factor of two.* For example, if on a log-linear cost curve we find the cost to be $100 at quantity 10 and $80 at quantity 20, we say the slope of the progress curve is 80 percent. Symbolically, we may express this as

$$S = \frac{\bar{Y}_{2x}}{\bar{Y}_x} = \frac{a\,(2x)^b}{ax^b} \qquad (3)$$

or

$$S = 2^b \quad \text{or} \quad \log S = b \log 2 \qquad (4)$$

and $b = \log S/\log 2$

As is indicated by equation (4), there is associated with each "b" value a corresponding value for "S." Solving equation (4) for "b" when "S" takes on values from 65 percent to 100 percent, we obtain the "b" values tabulated in Table 6.2.

To illustrate the application of the above technique, this example is taken from the case study cited in the previous section.

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*The definition given "progress curve slope" is different from the mathematical definition. In mathematics, the slope of a function is, of course, defined as the first derivative of that function.

TABLE 6.2. PERCENTAGE SLOPES WITH CORRESPONDING b VALUES

Percent S	Tangent -b	Percent S	Tangent -b
100.0	0	80.0	0.322
99.0	0.0147	79.0	0.340
98.0	0.0293	78.0	0.358
97.0	0.0439	77.0	0.377
96.0	0.0589	76.0	0.396
95.0	0.0740	75.0	0.415
94.0	0.0896	74.0	0.434
93.0	0.105	73.0	0.454
92.0	0.120	72.0	0.474
91.0	0.136	71.0	0.494
90.0	0.152	70.0	0.515
89.0	0.168	69.0	0.535
88.0	0.184	68.0	0.556
87.0	0.201	67.0	0.578
86.0	0.218	66.0	0.600
85.0	0.234	65.0	0.622
84.0	0.252	64.0	0.644
83.0	0.269	63.0	0.667
82.0	0.286	62.0	0.690
81.0	0.304	61.0	0.713

EXAMPLE 6.1

Given: A contract to purchase 7,015 solid-state receivers was expected to result in an estimated unit cost of $340 per receiver; the cost would be increased to $385 per receiver if only 5,340 were ordered.

Find: The slope of the progress curve, the cumulative average cost for 100 receivers (assuming the cumulative average cost curve is log-linear), and the cost of the 100th unit.

Solution: Solving the equation, $Y = ax^b$, simultaneously for the two cost-quantity points given, we find that the "b" value is -0.4558, or about 73 percent (see Table 6.2, or solve $S = 2^b$). The simultaneous equations are

$$\log 340 = \log a + b \log 7015$$
$$\log 385 = \log a + b \log 5340$$

and

$$S = 2^{-0.4558} = 72.91\%$$

We may find the cost of the first unit, a, implied by the above slope to be

$$a = \frac{\bar{Y}_{7015}}{7015^{-0.4558}} = \$19,253$$

Given the cost of the first unit, the cumulative average cost of 100 units is

$$\bar{Y}_{100} = 19,253\ (100)^{-0.4558}$$
$$= \$2,360$$

and the cost of the 100th unit is

$$y_{100} = 19{,}253 \ (100)^{1-.4558} - (99)^{1-.4558}$$

$$= \$1{,}279$$

For approximate answers to the example problem, and for practical use of the technique, tables and slide charts are often used. (Noah, 1962; Boren and Campbell, 1970) Table 6.1 is a condensed version of a learning curve table; with interpolation, it may be used to solve the above example adequately.

Cost and Rate of Output

Few statistical studies of cost and output rate have been made, and they are not conclusive as to the shape of the cost function for a variety of reasons. Predominant among those studies are those conducted by Dean (1951) that covered only a few firms, and did not include years of over-full employment when production capacity was crowded to the limits imposed by cost behavior or physical size.

In view of his studies, Dean observes that, in the short run, a functional relation exists between cost and a set of independent variables which may include, for example, volume of production, size of production lot, prices of input services, and variety of output. The independent variables will be different for each type of manufacturing operation, although in general the most important variable is rate of output, according to Dean.

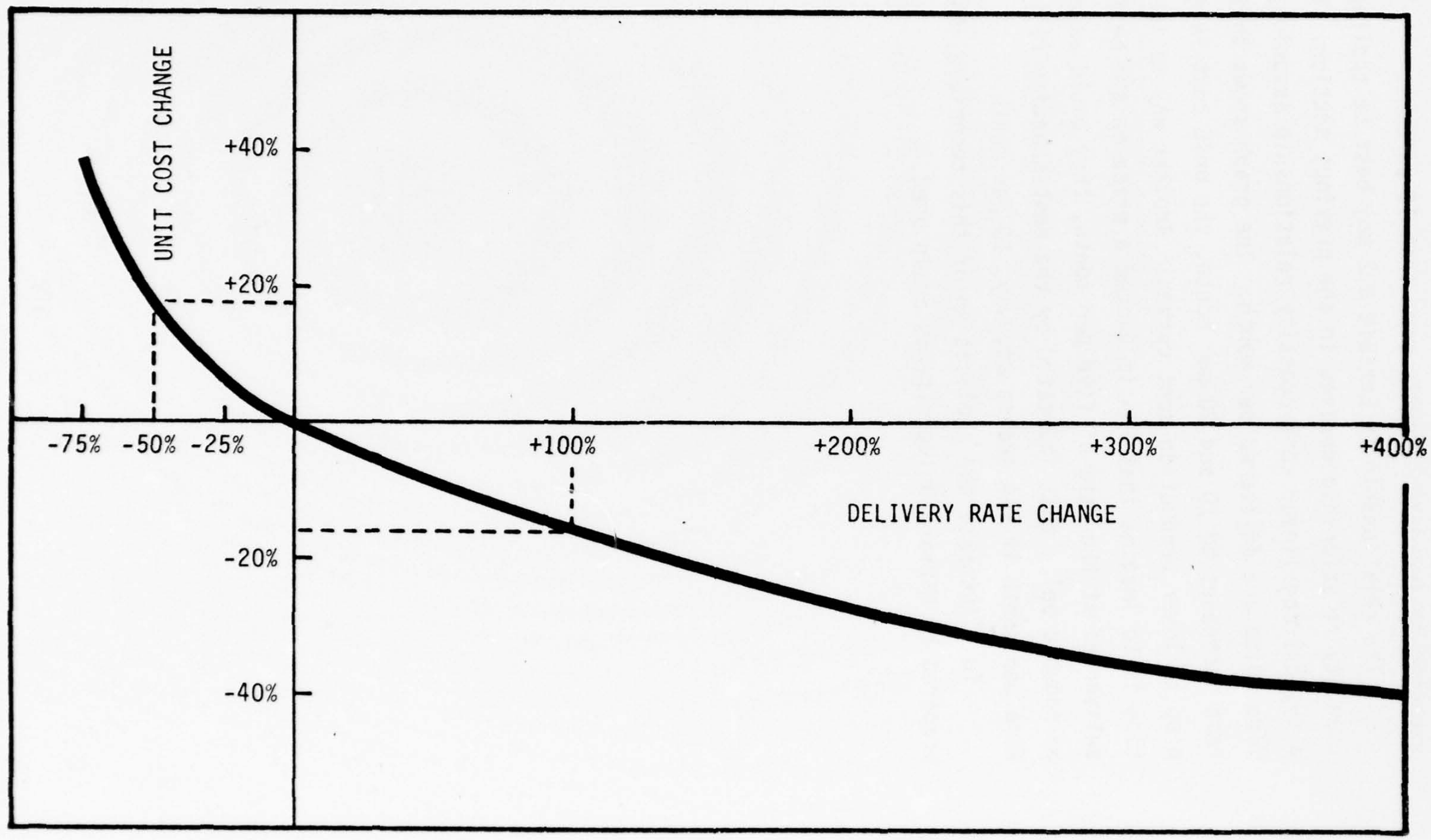
In partial support of this generally accepted doctrine, manufacturers engaged in the aerospace industry generally agree that marginal cost declines with increases in output rate over the output range of their experience. Noah (1974) examined that question among others, and found that marginal cost does indeed decline with increases in output rate over the output range of the experience gained during the production of two major aerospace items, and also found that output rate is a very important independent variable. At the risk of generalizing on the basis of a small sample, we are including values by which unit cost changes may be measured as a function of delivery rate changes.

Exhibit 6.2 shows that functional relationship. Delivery rate is used as a proxy for production rate. For example, unit cost may be expected to decline by about 15 percent when delivery rate is doubled. Conversely, when delivery rate is reduced by 50 percent, the unit cost may be

EXHIBIT 6.2

UNIT COST CHANGE
VERSUS
DELIVERY RATE CHANGE

(CUMULATIVE VOLUME IS CONSTANT)



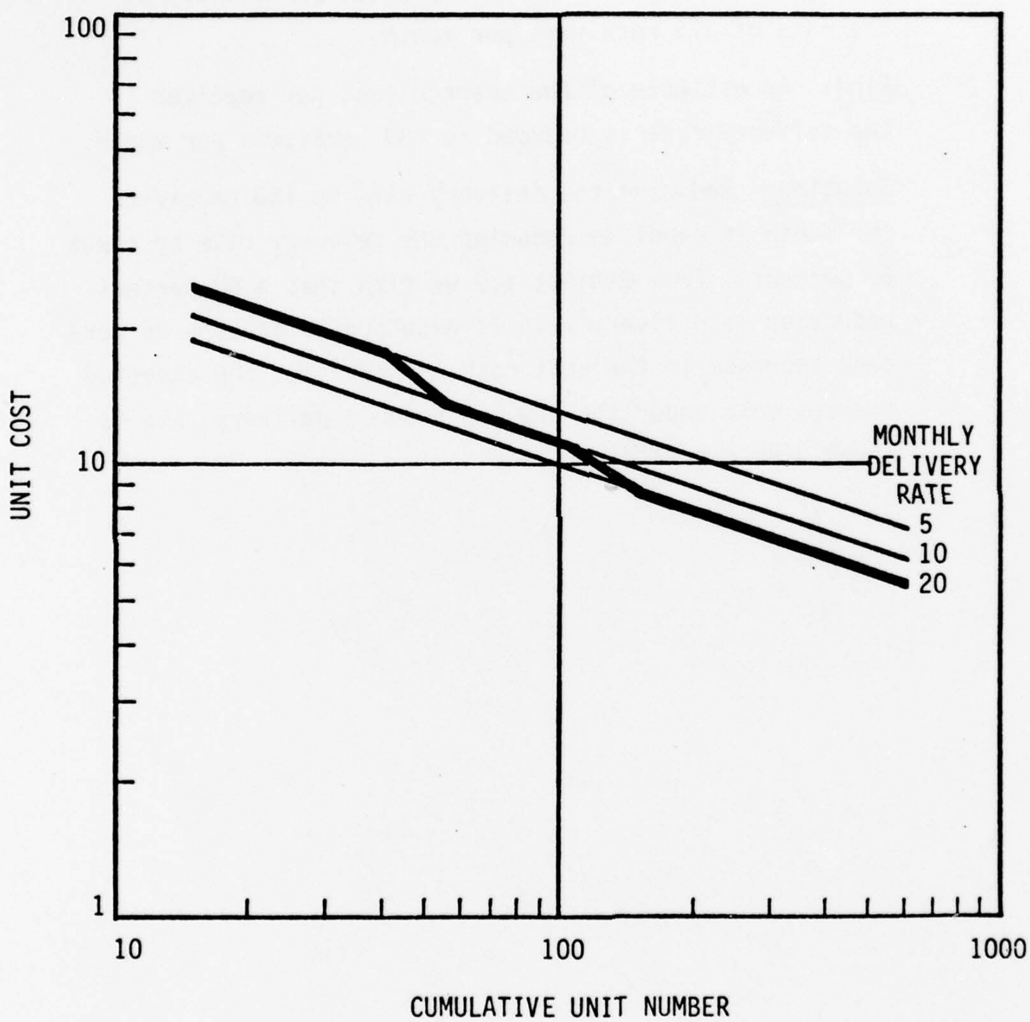
expected to increase slightly more than 15 percent.

The relationship in Exhibit 6.2 may best be applied to the cost-quantity relationship derived in the previous section. Exhibit 6.3 shows a typical log-linear cost-quantity relationship assuming, first, that five units are delivered per month. The graph shows that as the delivery rate increases to 10 and 20 per month, the unit cost is expected to drop to lower production cost curves. Another way to look at this question is to imagine that FAA is quoted a price on the basis of equipment deliveries at the rate of five per month, they could expect the price to be reduced below that indicated by the cost-quantity relationship for five per month if FAA orders at, say, 10 per month.

To illustrate the application of this technique, the case study prepared by Harvard University is again used.

EXHIBIT 6.3

EFFECT OF DELIVERY RATE ON TYPICAL
UNIT COST/CUMULATIVE QUANTITY RELATIONSHIP



EXAMPLE 6.2

Given: The contract to purchase 5,340 solid-state receivers at $385 per receiver was based on delivery at the rate of 325 receivers per month.

Find: An estimate of the average cost per receiver if the delivery rate is reduced to 130 receivers per month.

Solution: Reducing the delivery rate to 130 receivers per month is equal to reducing the delivery rate by about 60 percent. From Exhibit 6.2 we find that a 60 percent reduction in delivery rate is expected to cause a 25 percent increase in the unit cost. Therefore, the expected average cost under the new and reduced delivery rate is about $480 per receiver.

Price-Level Changes

To normalize the data base for price level changes, the analyst must find or construct appropriate price indexes, adjust them to the desired base year, and then deflate observed values in the data base. The result is that the data points are expressed in terms of constant dollars for the selected base year.

The FAA has constructed deflator indices for "All Structures" and "All Equipment" for the years 1945 to 1974. These and the GNP deflator are presented in Table 6.5.

The following example shows how the GNP deflator was used in Section 4 to transform current year dollar settlements into constant 1974 dollars. The relevant portion of GNP deflator is shown below in 1958 base year and again transformed into 1974 base year.

TABLE 6.3. GNP DEFLATOR SHOWN FOR 1958 AND 1974 BASE YEARS

	1958 Base Year	1974 Base Year
1964	108.85	63.99
1965	110.86	65.17
1966	113.94	66.98
1967	117.59	69.13
1968	122.30	71.89
1969	128.20	75.63
1970	135.24	79.50
1971	141.35	83.09
1972	146.12	85.90
1973	154.31	90.72
1974	170.11	100.00

To obtain constant 1974 dollar values for settlements from the years 1964 to 1974, current year dollars were divided by the 1974 based GNP deflator, as shown in Table 6.4.

Deflators are often given with the base year specified as 100 as shown in Table 6.5. This is a convenience in that it avoids long strings of decimal places, but it also can lead to slipped decimal points if it is not remembered that index numbers of a 100-based deflator stream are actually percentages, and must be treated accordingly during arithmetic manipulations.

TABLE 6.4. CONVERSION OF CURRENT YEAR DOLLARS TO 1974 CONSTANT DOLLARS.

Year	Current Year Dollars	1974 Based Deflator	1974 Constant Dollars
1964	49.0	63.99	76.6
1965	60.2	65.17	92.4
1966	102.4	66.98	152.9
1967	61.8	69.13	89.4
1968	114.2	71.89	158.8
1969	140.6	75.63	186.6
1970	165.3	79.50	207.9
1971	123.7	83.09	148.9
1972	122.4	85.90	142.5
1973	148.2	90.72	163.4
1974	233.2	100.00	233.2

Sometimes this task can be quite difficult, even when various data sources for the purpose are available, such as "Employment and Earnings" and "Wholesale Prices and Price Indexes," both published by the U.S. Bureau of Labor Statistics.[1/]

Price indexes are inherently inexact and their use, while necessary, can introduce errors into the data. For example, the average hourly earnings for all electronics production workers may increase by five cents in a given year, but at any particular company the increase may be more or less than that amount. Use of the average amount to adjust the data for a given company will inevitably introduce error. Also, for many specialized items of equipment, a good published price index does not exist.

A more fundamental problem, perhaps, is that the upward trend in

[1/] Index number theory and the process of statistical deflation are discussed in most standard texts on statistical analysis. (Spurr and Bonini, 1967).

TABLE 6.5. DEFLATORS APPLICABLE TO FAA COST-BENEFIT ANALYSIS

	All Structures	All Equipment	GNP
1945	40	72.4	59.66
1946	46	80.4	66.70
1947	53	95.3	74.64
1948	60	94.4	79.57
1949	62	94.5	79.12
1950	65	96.5	80.16
1951	71	100.5	85.64
1952	76	97.0	87.45
1953	79	93.3	88.33
1954	82	93.8	89.63
1955	84	93.6	90.86
1956	92	95.4	93.99
1957	98	99.0	97.49
1958	100	100.0	100.00
1959	102	100.9	101.66
1960	102	99.4	103.29
1961	100	99.1	104.62
1962	101	100.0	105.78
1963	102	100.7	107.17
1964	104	101.1	108.85
1965	107	100.9	110.86
1966	111	102.6	113.94
1967	115	106.4	117.59
1968	119	112.2	122.30
1969	126	117.8	128.20
1970	137	123.1	135.24
1971	147	131.1	141.35
1972	156	138.1	146.12
1973	166	139.9	154.31
1974	200	149.4	170.11

wage rates may have been accompanied by a parallel trend in the output per employee -- productivity rate. Practically speaking, the real dollar output per man is difficult to measure for industries in which continual change rather than standardization is the rule. Certainly the growth in productivity has not been uniform for electronics, and to develop a productivity index for that industry would be a difficult and contentious task. Present practice, therefore, is to apply the price-level adjustment factor to obtain constant dollars and, at the same time, to remain alert to inequities that may be introduced.

The Green Book (1950) officially recommended that changes in the general price level be taken into account in cost-benefit analyses. Today, however, cost-benefit analyses are usually made in terms of a constant price level, as indicated above. Our position is that cost-benefit estimates should help us see which choices are preferred. So far as efficiency in this sense is concerned, movements of the general price level are irrelevant; if the price level rises from 200 to 600 by the time the benefits are expected to accrue, it is simply incorrect -- and grossly misleading -- to say that benefits are trebled. It may, however, be advisable to predict movement of the general price level in order to foresee what groups will gain and how much they can be taxed or charged -- but not in order to gauge the worth of projects to the economy as a whole. (McKean, 1958).

The OMB position, stated in Circular A-94 dated March 1972, is that:

> *All estimates of the costs and benefits for each year of the planning period should be made in constant dollars; i.e., in terms of the general purchasing power of the dollar at the time of decision. Estimates may reflect changes in the relative prices of cost and/or benefit components, where there is a reasonable basis for estimating such changes, but should not include any forecasted change in the general price level during the planning period.*

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7

ESTIMATING RELATIONSHIPS

In discussing the subject of estimating relationships, it is imperative that certain fundamental points about their derivation and use be understood. The main purpose of this section is to discuss these points.

Some Fundamental Points

Estimating relationships may be thought of as analytic devices which relate various categories of costs or benefits, expressed in dollars or in physical units, to explanatory variables. Estimating relationships may take numerous forms, ranging from informal rules of thumb or simple analogies to formal mathematical functions derived from statistical analyses of empirical data. A critical step in the derivation of estimating relationships is to assemble and refine the data that constitute the empirical basis of the relationship to be developed. Typically, the raw data are at least partially in the wrong format for analytical purposes, and have various other irregularities and inconsistencies. Adjustments, therefore, almost always have to be made to insure a reasonable, consistent, and comparable data base. No degree of sophistication in the use of advanced mathematical statistics can compensate for a seriously deficient data base.

Given the data base, any of a wide variety of techniques may be used to derive appropriate estimating relationships. The range extends all the way from unaided judgment and simple graphical procedures through complex statistical techniques. Here, considerable judgment must be exercised. The particular method used is strongly related to the nature of the problem, and particularly to the nature of the data base. For example, it usually does not make sense to try to fit a complicated multivariate function to a data base having a very small sample size, reducing the degrees of freedom to a ridiculously small number. Even with a relatively large data base, one must avoid mechanically running large numbers of correlation analyses on the computer to determine that combination of explanatory variables which maximizes the correlation coefficient. As discussed in Appendix A on regression analysis, high correlation coefficients, in and of themselves, do not necessarily ensure statistically significant relationships.

Care must also be exercised in the <u>use</u> of estimating relationships. The user must have a good understanding of the data base and the procedures used in deriving the estimated relationship. This is particularly important when the user himself has not derived a relationship. Above all, one must exercise care in extrapolating beyond the range of experience (the sample) underlying the relationship. Scaling factors, for example, may have to be taken into account, especially when we are estimating the costs of future equipments or activities which are different from those of the past, present, and near future.

Types of Estimating Relationships

Estimating relationships exist in many forms and numerous possible types may be useful in practice. In this section we briefly discuss three types: simple linear functions, step functions, and multivariate functions.

Useful estimating relationships are not necessarily expressed in terms of complicated mathematical functions. In fact, a considerable number of the relationships used in cost-benefit analysis are of the form,

$$Y = bX \qquad (1)$$

Equation (1) is a special case of the linear form,

$$Y = a + bX \qquad (2)$$

The use of (1) is particularly prevelant. The numerical value of b may be determined by a simple averaging process, by using formal statistical regression analysis, or even by policy considerations. Examples of cases where linear homogenous estimating relationships are useful include personnel pay as a function of number of personnel, construction cost as a function of number of square feet, and the value of time saved as a function of the number of passenger-minutes. Most rule-of-thumb estimating relationships are in effect linear homogenous functions.

The two-variate simple linear form, equation (2), in which the location coefficient is not equal to zero, is illustrated in the context of a scatter diagram. See Exhibit 7.1. In two-dimensional cases, the use of scatter diagrams can be very useful in deriving estimating relationships. For example, from the following tabulation of ARTCC plus Tower Personnel

Year	ARTCC + Tower Personnel	Total FAA Personnel
1966	12,574	43,557
1967	12,278	44,328
1968	14,021	46,835
1969	16,059	49,106
1970	18,559	51,577
1971	19,974	54,550
1972	19,612	53,330
1973	19,873	53,679
1974	20,475	55,971

and total FAA personnel, for the years 1966 to 1974, we may wish to know how the numbers of ARTCC plus Tower personnel change with increases in total FAA personnel. Visual conception of the scatter diagram suggests that ARTCC and Tower personnel might well be estimated as a linear function of total FAA personnel. If we regress the data, the resulting equation is,

$$Y = -19546 + 0.72713 X$$

where,

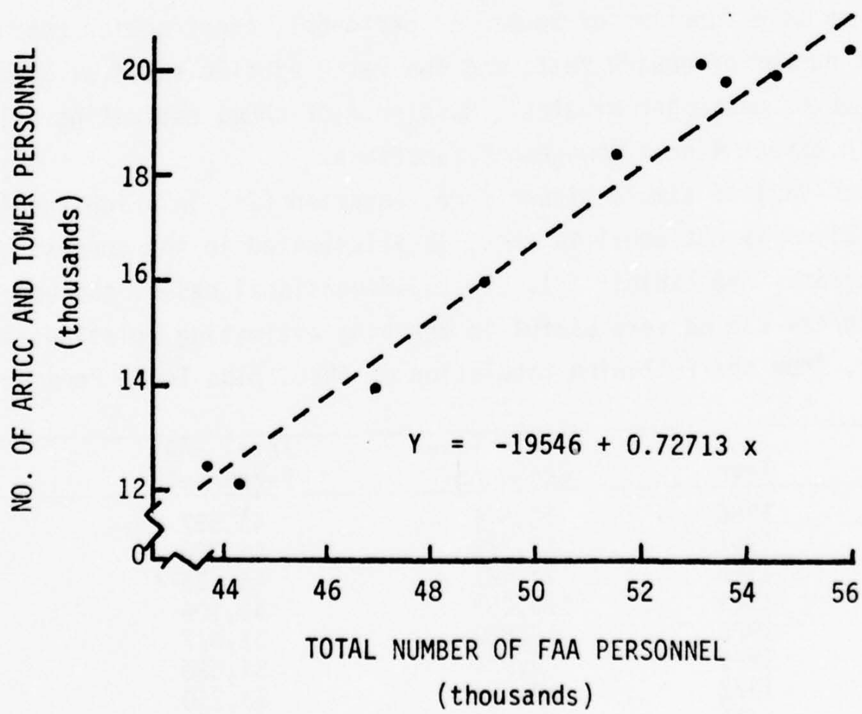
Y = Number of ARTCC and Tower Personnel

X = Total FAA personnel,

represented by the line on the graph.

EXHIBIT 7.1

ARTCC AND TOWER PERSONNEL AS A FUNCTION OF TOTAL FAA PERSONNEL



If a formal curve-fitting technique is used, the regression equation and certain statistical measures of uncertainty -- standard error, coefficient or correlation, coefficient of variation -- result. The statistical measures help the user in forming judgments about the reliability of the estimating relationship.

The above relationships assumed continuity between the costs or benefit measure -- the dependent variable -- and the explanatory or independent variable. This, however, need not be the case. Cost can be at a constant level over a certain range of the explanatory variable, then suddenly jump to a higher level at some point and remain constant for a time, then jump to another level, and so on. This kind of relationship is known as a step function, and is illustrated in Exhibit 7.2.

Step functions can be especially useful in portraying the behavior of, for example, support activities such as the recently-established procedures designed to thwart highjacking, and the more recent procedures being considered to guard against terrorist bombings.

Oftentimes, a measure of cost or benefit cannot be adequately explained in terms of one independent variable. In those cases estimating relationships will take the form of multivariate functions; i.e., estimating equations having more than one explanatory variable. An example where a multivariate estimating relationship has been derived is the production function relating the total number of aircraft handled to two independent variables -- direct labor and capital stock. (Eskew, 1975)

$$\ln Q = 5.974 + 0.684 \ln L + 0.337 \ln K \qquad (3)$$

where,

Q = Total Aircraft handled

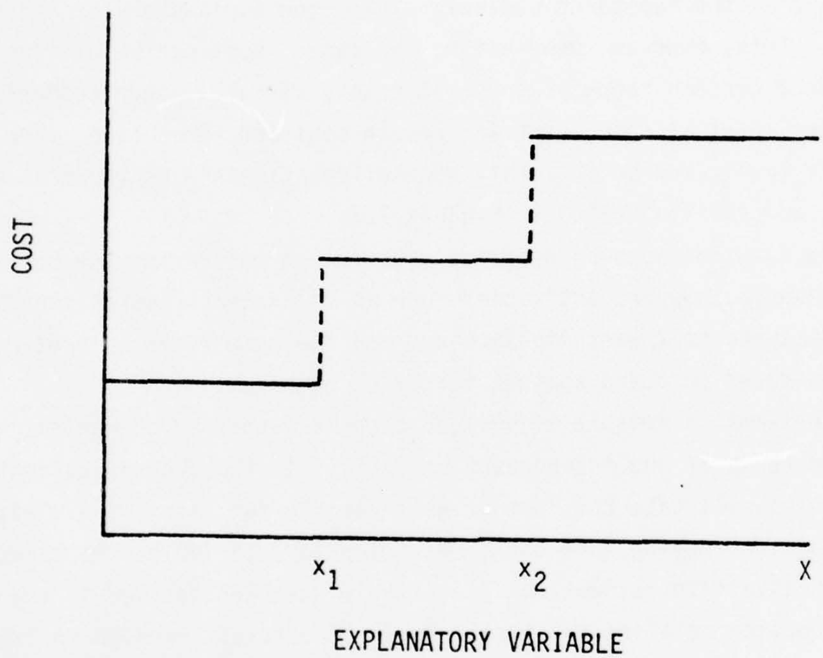
L = Direct Labor

K = Total Capital Stock

Statistical measures indicative of the "goodness of fit" of the estimating relationship generally result from a multivariate regression analysis. For a more complete discussion of formal statistical regression analysis, see Appendix A.

EXHIBIT 7.2

ILLUSTRATION OF A STEP FUNCTION



The Data Problem

As indicated above, one of the most vitally important steps in the derivation of estimating relationships is to assemble an appropriate data base. Since the data problem is fundamental, analysts typically devote a considerable amount of their time to collecting data, to making adjustments in the raw data to help insure consistency and comparability, and to providing for proper storage of information so that it may be retrieved rapidly when it is needed. With appropriate information in hand, the analytical task of deriving estimating equations is often relatively easy, given the analytical tools and powerful computational devices now available.

We have a data problem because information is generally given in an unsuitable format, the data needed for a given analysis is usually divided among several sources that do not necessarily match, definitions given to items contained in various data sources are not consistent, and so on.

The data problem can be overcome, not by adding reporting requirements to existing reporting requirements, but by persistence and ingenuity on the part of the analyst. The analyst, given no constraints on his desire for data, could not establish a universal, all-purpose information system that would satisfy his needs forever.

There is the problem of small samples, which arises from the fact that the FAA has to deal with a rapidly changing technology. This means that in many instances only a relatively small number of observations will be available for a certain era or class of technology. Here, even a near-perfect information system cannot increase the sample size.

There are two possibilities for alleviating the problem associated with small samples. The first is extremely simple, but it can help a great deal, particularly in deriving estimating relationships for use in long-range planning studies. The analyst should not necessarily restrict himself to historical data. In many cases he should seriously consider increasing the number of observations by including appropriate data points based on estimates made by experts for the very near-term future, or by taking advantage of certain kinds of qualitative information.

In summary, the techniques for assisting and handling the small sample problem are:

- Under certain conditions the size of the sample can be increased by judiciously using estimates for the near future to supplement the historical data base.

- It may be possible to use qualitative information to assist in deciding what kind of estimating relationship is most appropriate.

Occasionally the analyst finds there is simply a void in the existing data base. This is likely to be the case when planners are considering new proposals for distant future capabilities requiring major equipments, or operational concepts, markedly different from those of the past and the present.

One possible way to alleviate this problem is to see if any relevant experiments are being conducted pertaining to the subject at hand, and if not, to try to initiate such an experiment. Oftentimes the use of experiments to broaden the data base is quite expensive, in terms of both time and money. For example, a manufacturer set up a special shop to explore a variety of manufacturing operations on aluminum, stainless steel, and titanium structures. Taking aluminum as the base case, the objective of his experiment was to determine the probable incremental labor costs involved in working the other two materials for a representative sample of various types of manufacturing operations. Given data from the experiment, the analysts were then in a position to devise techniques for adjusting the historical data base, which included aluminum, so that it would be more appropriate for dealing with the stainless steel and titanium problems.

Adjustments to the raw data base, as in the example above, are necessary to make data consistent and comparable. Before useful estimating relationships can be derived, the raw data have to be adjusted for such things as price levels changes, definitional differences, production quantity differences, and the like. Adjustments of this sort were discussed in the previous section.

Use of Estimating Relationships

The analyst must be judicious in his use of generalized estimating relationships which synthesize our knowledge about past, present, and near-future capabilities by relating resource requirements to key structural characteristics of these capabilities. The main purpose of such synthesized descriptions is to help in assessing resource impacts of proposed new systems for the distant future. While generalized estimating relationships are quite useful to the analyst, they are by no means self-sufficient, and many problems arise in using them in the cost-benefit process.

Some of these problems are more or less mechanical in nature. For example, in cases where the analyst takes an existing estimating relationship from previous work, he must check to make sure that the definition of the categories of resource items built into the relationship is the same as that required in the problem at hand. Similarly, if the estimating relationship furnishes estimates of dollar costs, the analyst must check to see what kind of dollars are provided -- current or constant, and if constant, what year? If the base year built into the relationship is different from that required by the analyst's present study, an adjustment must be made.

No one has yet devised a standard procedure that will guarantee caution in the application of generalized estimating relationships. Basically what is required is a firm judgment on the part of the analyst, and this cannot be reduced to a mechanical process. However, a number of steps may be taken to facilitate proper use of estimating relationships in long-range planning studies.

In cases where generalized estimating relationships have been derived by formal statistical methods, the cost analyst may turn to the relative standard error of estimate (coefficient of variation), the confidence interval for an individual forecast, or some other measure, and use these statistics to help decide what should be done about statistical uncertainty. If, for example, the relative standard error of estimate is about 5 percent, the analyst may feel comfortable in taking an expected value estimate as provided by the regression equation and using it without further question. On the other hand, if the relative standard error of estimate is 30 or 40 percent, he should probably do some sensitivity testing. Here the objective would be to explore the impact on final results of possible estimating errors for the particular input category under consideration.

With respect to uncertainties about the values of explanatory variables, several precautionary steps may be taken. As an illustration, let us consider the major equipment area. Here, costs of future equipment proposals are often estimated from relationships having equipment performance or physical characteristics as explanatory variables. Where the analyst is uncertain as to what values of these variables should be inserted into the estimating equation, the first thing he should do is be skeptical about the numbers presented by advocates of the proposed new equipment. We know from past experience that participants in the advocacy process often tend to overstate performance characteristics, to understate certain physical characteristics, and to understate costs. Under these conditions, the analyst should consult with design engineers who are neutral, and seek their advice regarding appropriate values of equipment characteristics to be used as explanatory variables.

The greatest possible care should be exercised by the analyst when he has to project to the distant future and has good reason to doubt that the characteristics of that future are the same as those reflected in the data base underlying his estimating relationships. Here, rote application of generalized relationships can be especially hazardous. What usually happens in such instances is that the relationships are used primarily as a reproducible point of departure in the estimating process. Something else -- usually involving a considerable effort -- has to be done before the final estimates are obtained. As an example of something else that might be done, recall the experiment conducted by the manufacturer interested in the relative costs of producing aluminum, stainless steel, and titanium.

In summary, solution to parts of this data problem may be had through major overhaul of present information systems and through the establishment of new, complete systems. Neither, however, appears feasible as a general solution to the problem. Short of such major efforts are numerous alternatives. Some examples include use of sampling techniques on an *ad hoc* basis supplementing the existing historical data base by including estimated data points for the near future, statistical adjustment and manipulation of the existing data base, and obtaining additional information by conducting experiments.

Uncertainty

The lack of certainty that can be expected in many cost and benefit estimates is a key characteristic of cost-benefit analysis. Uncertainty, perhaps more than anything else, tends to compound the severity of analytical problems.

Uncertainties should be treated explicitly using concepts and techniques appropriate to the type of uncertainty with which one is dealing. A distinction between risk and uncertainty is useful to the explanation of how uncertainty should be treated. A risky situation is one in which the outcome is subject to an uncontrollable random event stemming from a known probability distribution. Unlike the risk in the toss of a true coin for example -- with the probability of a head turning up being 0.5 -- an uncertain situation on the other hand, is characterized by the fact that the probability distribution of the uncontrollable random event is unknown.

Probability distributions are sometimes assigned to uncertain situations, but these are necessarily subjective in nature. That is, they must to a degree be based on the personal judgment and experience of the analyst, rather than on incontrovertible empirical evidence. Rarely in cost-benefit analysis are objective probabilities available. Essentially all cost-benefit analysis involve situations of uncertainty rather than of risk.

Historical evidence suggests that early estimates of costs and benefits prepared for major public programs have missed the mark significantly. The analyst has underestimated costs and overestimated benefits for two major reasons. By far the larger contributor to poor estimation lies with changes in system configuration resulting from basic changes in "requirements" not contemplated at the time of the early estimate. A secondary cause of poor estimation may be traced to deficiencies inherent in the analyst's techniques, including his empirical data base. Hence, we may say that the two fundamental reasons for poor estimation are requirements uncertainty and estimating uncertainty. (Fisher, 1970)

In long-range planning studies involving broad, relative comparison of numerous alternatives exemplified by the Upgraded Third Generation Air Traffic Control System, it is useful to ascertain initially whether estimating uncertainties can be properly suppressed, or treated as a second order problem. If "expected values" will suffice during the early stages of a cost-benefit analysis, the overall study can be structured in a more straightforward manner than would otherwise be the case. It is, however, very important to treat both types of uncertainty.

To insure that estimating uncertainty can be suppressed safely, one may use statistical measures -- standard errors, prediction intervals, coefficients of determination -- for those estimating relationships derived from empirical evidence. Those measures may be used to help form judgments regarding estimating uncertainties for each cost and benefit element examined. Given such judgments, several approaches may be considered.

Perhaps the simplest approach is to single out those categories of cost and/or benefit measures deemed to be the subject of greatest estimating uncertainty. Then, for expected value estimates having significant error, conduct sensitivity tests to determine the impact on the cost-benefit comparison. If the resulting impact on the comparison is small, we may conclude that estimating uncertainty in those elements may safely be suppressed.

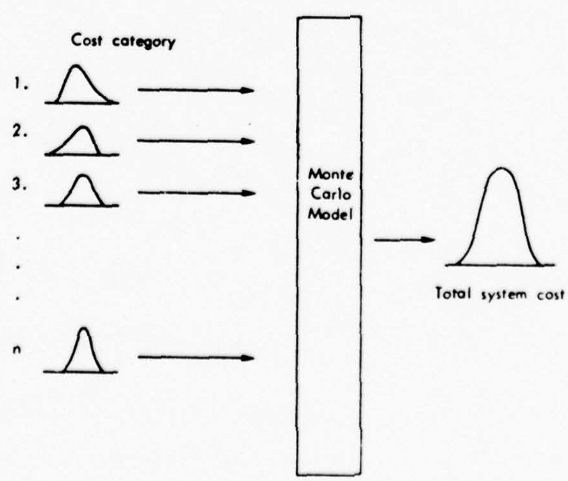
For those elements derived without the benefit of statistical measures, judgments about estimating uncertainty in terms of high, medium, and low estimates for each element can be made. Again, if these ranges are narrow relative to other uncertainties in the total cost-benefit problem, one should use the medium estimates in making the comparison among alternatives.

The two approaches discussed briefly above are most useful when a large number of alternatives are being compared. Once the number of alternatives has been narrowed, one may formalize subjective judgments about estimating uncertainty for each element in the cost-benefit structure. A subjective probability distribution - a beta function is often used - for each element may be specified, and the component distributions may be combined using a Monte Carlo technique to derive a distribution of total system costs on the one hand, and total system benefits on the other. (Klein and Jordon, 1975)

Perhaps the greatest advantage to using the formalized Monte Carlo technique to express subjective probability distribution functions is that its application places a certain amount of discipline on the analyst. It causes him to ask appropriate questions; it causes him to record answers in a consistent and understandable manner; it permits him to ascertain the impact on the final cost-benefit comparison.

EXHIBIT 7.3

USE OF MONTE CARLO METHOD TO
OBTAIN DISTRIBUTION OF TOTAL SYSTEM COST



Typically, estimating uncertainties are swamped by requirements uncertainties in studies of concepts such as the Upgraded Third Generation Air Traffic Control System. We believe that in most cases a limited amount of simple sensitivity testing can help a great deal in determining whether or not one should recommend that estimating uncertainties be suppressed. Simplifying procedures should be used whenever it is reasonable to do so, enabling the analyst to concentrate effort on requirements uncertainties.

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Gene H. Fisher, *Cost Considerations in Systems Analysis,* The RAND Corporation, R-490-ASD, December 1970.

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8

THE LIFE-CYCLE COST MODEL

The fundamental problem facing the cost analyst is developing and applying concepts and techniques for assessing economic costs of proposed alternative future actions under conditions of uncertainty. Alternative actions usually take the form of some combination of the following:

- o Proposed new capabilities for the future
- o Proposed modifications of existing or presently programmed capabilities
- o Proposed deletions from the presently programmed capabilities

Suppose that planners are considering alternative concepts of air traffic control that differ in the basing of the signal receiver (in space or on ground), the date of the initial operational capability, the level of automation, and whether the system is centralized in three centers or distributed among 21 centers. RAND (1973) conducted such a study of alternative Advanced Air Traffic Management Systems (AATMS) for operational use in the latter portion of this century. In addition to various measures of possible benefits of proposed alternatives, the planners must also know what the economic cost is likely to be -- the incremental cost to *develop, procure,* and *operate* the new capability over a period of years. The "cost of the system" includes the cost of everything directly related to the decision to achieve this proposed new capability; it excludes the cost of items not so related, such as the costs of administrative and support activities that would go on regardless of the decisions under consideration. (Fisher, 1971)

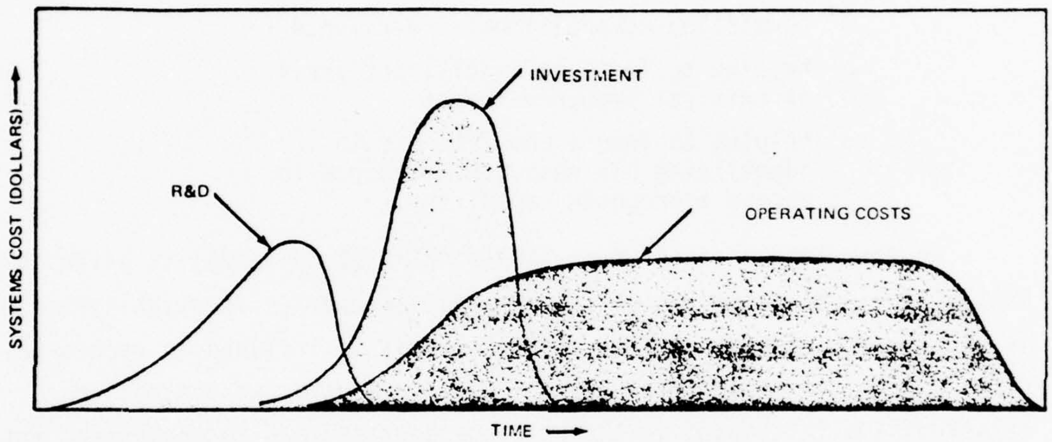
The cost implications of alternative concepts, such as those described above, can best be examined by segregating the costs of the proposed alternatives into three categories:

- o Research and Development Costs -- the outlays for resources required to develop the new capability to the point where it can be introduced into the operational inventory at some desired level of reliability.
- o Investment Costs -- the one-time or non-recurring outlays required to introduce the capability into the operational inventory.
- o Operating Costs -- the recurring outlays required year by year to operate and maintain the capability in service over a period of years.

An illustration of the relationship of these costs in the life of a system is presented in Exhibit 8.1. The life cycle identification is important for several reasons. One, it helps to insure identification of the total resource impact of a proposal. Oftentimes, decisionmakers may become preoccupied with investment costs, to the relative neglect of the annual operating costs which will be an inevitable consequence of their decision. Life-cycle costing helps to avoid such a pitfall. Two, the life-cycle identification facilitates the analytical process. Systems analysts and long-range planners usually must examine variations of the extent to which the system is to be introduced, the number of years various capabilities are assumed to be in the operational inventory, and the like. The life-cycle identification is essential for this kind of parametric examination. Research and development costs, for example, are largely independent of the number of units procured and the number of years a capability is assumed to be in the operating inventory. Investment costs are, of course, related to the number of units procured, but are essentially independent of the number of years of operation. Operating costs are a function of the number of units procured and the number of years of operation. (Fisher, 1971)

EXHIBIT 8.1

LIFE CYCLE COST HISTORY



Cost Categories in the Model

To generate costs of alternatives, the analyst must establish resource categories (for equipment, facilities, manpower, and so on) and functional categories (for maintenance, training, and so on). These categories must be meaningful and useful from several points of view:

- o easing the problem of data collection
- o permitting computational convenience
- o helping to indicate significant areas of critical resource impact
- o helping to insure completeness in identifying all resources required to obtain a proposed capability.

An example patterned after RAND's AATMS Study (1973) is given in Exhibit 8.2. No matter which set of input categories is established it is vitally important to define carefully what is included in each category. This is a fundamental prerequisite to developing estimating relationships (discussed in the previous section) and to consistent estimating of the cost implications of alternative proposals for future capabilities.

In long-range studies, attempts to structure problems in great detail are usually not rewarding; however, it is important to specify input structures in enough detail to allow those aspects of a proposal which are really new to be distinguished from those which are not. Even the most advanced system proposals contain many elements which are not significantly new. These should be separated from those which are new, so that the analytical effort can be concentrated on the latter. In the hardware area this usually means going down at least to the subsystem level, and perhaps even lower.

Given the desired set of resource and functional categories, and estimating relationships for each, we are now in a position to synthesize this information into a life-cycle cost model.

EXHIBIT 8.2. CATEGORIES FOR A TYPICAL FAA LIFE-CYCLE COST MODEL

RESEARCH AND DEVELOPMENT (R&D)

- Prototype Hardware
- Test Facilities
- Technical Experiments
- Modeling and Simulation
- Software
- Operational Tests
- System Design and Engineering

INVESTMENT (Including F&E)

- Land
- Facilities (F)
- Equipment (E)
- Initial Spares and Spare Parts
- Initial Training
- Initial Travel
- Initial Transportation
- Other Miscellaneous

OPERATIONS AND MAINTENANCE (O&M) For N Years

- Personnel
- Equipment Replacement
- Equipment Maintenance
- Replacement Training
- Recurring Travel and Transportation
- Other Miscellaneous

TOTAL SYSTEM COST

- R&D + Investment + O&M

Classes of Cost Models

The term "cost model" has a wide variety of meanings. In general, it connotes an integrating device designed to facilitate the analytic process. Cost models may be classified in several ways. One basis for classification is in terms of the extent to which the model manipulates the inputs. The simplest cost model of this sort summarizes the facts provided by the analyst; it may only consist of rules for subtotaling and totaling the information supplied as inputs. A slightly more complex model may require a minor amount of multiplication in order to turn out a few intermediate outputs to be summarized and displayed. Somewhat more complex models of this type may provide for making choices of estimating techniques depending upon specific inputs. The most complex of these input manipulators may involve the use of fairly sophisticated techniques such as nonlinear programming or probabilistic iterations.

Cost models can be categorized according to the function they serve. Some models are designed to assist long-range planners. Others are for use in programming, where this term implies a more detailed level of planning and application in the near future. Still others are designed for use in preparing next fiscal year's budget. Function influences the design of the model in several ways. The level of detail to be represented is one of the most obvious. For example, a model designed for use in near-future budgeting would not usually be useful for long-range planning, because it would require unavailable detail. It might also utilize categories and identifications in forms which are not of interest to long-range planners.

Cost models can also be classified in terms of their subject matter. For example, within FAA, cost models may be narrowly conceived to deal only with cost as a function of increasing reliability of a specific type of equipment, e.g., a non-directional beacon. At the other extreme, a model may be conceived to represent the entire FAA structure and its fiscal responsibility for the next 20 or 30 years.

Cost models useful to cost-benefit studies are designed to serve the long-range planning function, and usually deal with replacement proposals. The replacement of an existing ATC system or subsystem, for example, is a proposed action generating potential costs traceable to a so called "system." When modeling the costs of that system, only those costs that are believed to be a function of introducing the new system and removing the old are treated. All other costs are, by implication, assumed to be fixed and therefore not subject to variation as a function of the decision at hand. Models of this type are sometimes called individual system life-cycle cost models; their usefulness to the analysis of future FAA requirements is the subject of the next subsection.

The Individual System Life Cycle Cost Model

Life cycle cost models may be structured quite generally, with a view to automation, and with provision for storing a sizeable data bank of estimating relationships covering a wide range of hardware and operational concept configurations. Inputs to the model would comprise sets of subsystems descriptions, and the computer program would automatically select the appropriate combination of estimating relationships for use in any given case. For example, the AATMS may be divided among four subsystems including:

- o air traffic control equipment
- o data acquisition equipment
- o communications equipment
- o navigation equipment

One or more cost estimating relationships (CERs) may be derived for prototype hardware for each type of subsystem. The cost model would select the appropriate CER.

Individual system life-cycle cost models may also be structured in terms of the form of output that is desired. For example, if time-phased cost estimates are deemed useful, then explicit provision must be made for inputting major equipment delivery schedules, activation schedules, or some other form of projected time table. The model must then contain a procedure for relating the cost estimate to the specified time table and for generating alternative patterns of timing of the cost impacts.

A frequently used approach is to make the basic calculations on the basis of deliveries of equipment, and then to use a series of lead and lag factors to convert the basic calculations into time-phased estimates of total obligational authority, expenditures, or some other alternative form of time-phased output. Additionally, a procedure may be included in the model for treating alternative assumptions about time preference; that is, assumptions about alternative discount rates.

A principal reason life-cycle cost models are computerized is the high likelihood they will be used repetitively. Models of this sort may find repeated use during a single study effort because many options must be examined while seeking the preferred solution.

Large-scale, time-phased models dealing with considerable detail are quite expensive. They are expensive to develop and to run repetitively. However, a simplified cost model can often be designed for use with desk-top, time-sharing, terminals. While inadequate for portraying time-phased costs, they are generally adequate for comparing alternatives during the early conceptual phase of a study. Many options may be examined efficiently, discarding those that are clearly inferior. An example of a desirable form of output for the AATMS individual system life cycle cost summary is shown in Exhibit 8.3. Significant features of the option are indicated in the heading, non-recurring costs of R&D and investment are divided among subsystems, and recurring costs of operations and maintenance are shown for the system as a whole. In addition to the total life-cycle cost summation, summations are shown on the basis of crude* time-phasing for costs discounted at 10 percent and at 20 percent.

In summary, cost models are important integrating devices. They are designed to facilitate the analytical process by bringing together a wide range of factors on the input side and relating them to specific types of output capabilities. Successful cost-benefit analysis, in many ways, depends on the utility of the cost model employed.

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*For example, the R&D costs may be assumed to spread evenly, year by year, over the length of the R&D program.

EXHIBIT 8.3

OPTION:GSSA21 -- ATC

LIFE CYCLE COST SUMMARY
(MILLIONS OF CONSTANT FY77 DOLLARS)

R&D PROGRAM LENGTH, YRS	xx	NO. OF CENTERS	xx
NO. OF PROTOTYPES	x	NO. OF SUPER CONTROLLERS	x
NO. OF OPN'L TESTS	xx	NO. OF CONTROLLERS [illegible]	xx
		IOC DATE	xxxx

COST ELEMENT	ATC EQUIP	DATA ACQ EQUIP	COMM EQUIP	NAV EQUIP	SYSTEM REC COST	TOTAL
R&D						
PROTO HARDWARE	xxx	xxx	xxx	xxx		xxxx
TEST FACILITIES	xxx	xxx	xxx	xxx		xxxx
TECH EXPERMNTS	xxx	xxx	xxx	xxx		xxxx
MOD & SIM	xxx	xxx	xxx	xxx		xxxx
SOFTWARE	xxx	xxx	xxx	xxx		xxxx
OPN'L TESTS	xxx	xxx	xxx	xxx		xxxx
SYSTEM D&E	xxx	xxx	xxx	xxx		xxxx
INVESTMENT	xxx	xxx	xxx	xxx		xxxx
LAND	xxx	xxx	xxx	xxx		xxxx
FACILITIES	xxx	xxx	xxx	xxx		xxxx
EQUIPMENT	xxx	xxx	xxx	xxx		xxxx
INITIAL SUPP	xxx	xxx	xxx	xxx		xxxx
INITIAL TRNG	xxx	xxx	xxx	xxx		xxxx
INITIAL TRAV	xxx	xxx	xxx	xxx		xxxx
INITIAL TRAN	xxx	xxx	xxx	xxx		xxxx
OTHER MISC	xxx	xxx	xxx	xxx		xxxx
OPNS & MTCE					xxx	xxxx
PERSONNEL					xxx	xxxx
EQ REPLMNT					xxx	xxxx
EQ MTCE					xxx	xxxx
REPLMNT TRNG					xxx	xxxx
TRAV & TRAN					xxx	xxxx
OTHER MISC					xxx	xxxx
	--------	--------	--------	--------	--------	--------
TOTAL R&D+INV+O&M	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
LIFE CYCLE COST						xxxx
DISCOUNTED AT 10%						xxxx
DISCOUNTED AT 20%						xxxx

PREPARED:4/20/76

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9

SUMMARY OF RELATED MATH-STAT TECHNIQUES

Mathematical and statistical methods have been reasonably successful in dealing with many complex problems in the physical world and, hence, it is quite natural to hope that, by extension, they might perform equally well in dealing with the broader and even more complex questions which cost-benefit analysis tries to answer. Cost-benefit analysis is concerned with relationships between a large number of quantities, so it is not surprising that mathematics and statistics provide useful techniques.

A casual survey of operations research literature -- which is identical with cost-benefit analysis in so far as its analytical tools are concerned -- leaves the impression that success in this field depends on a thorough knowledge of certain rather specialized mathematical techniques. Quade (1964) notes that in its rather short life as a named discipline, operations research has so firmly adopted certain tools -- linear programming, Monte Carlo, and game theory to list a few -- that these techniques almost seem to be the complete activity. Johnston (1960) says one of the most important developments in economics has been the increasing amount of statistical and econometric work. In recent years the emphasis has been on the application of statistical techniques to data in order to estimate economic relationships and to test various hypotheses about such relationships. We have seen in the recent past an ever-accumulating body of empirically tested propositions covering many fields of economic activity.

The difficulties in problem solving range from the philosophical or conceptual to the analytic or mathematical. However, there is no clear-cut separation. Operations research techniques which we have mentioned are designed to overcome difficulties at the mathematical or analytic end of the range. Although clearly dwarfed by most at the other end, one should not conclude that these techniques are without significance or importance. For this reason, readers of this guide may find it profitable to learn something about the mathematical techniques that have proved extremely useful in dealing with a large and important class of problems. Moreover, even though an understanding of fundamental concepts may be more important than analytic techniques -- in part because more elementary methods will ordinarily serve, though less efficiently -- new analytic techniques frequently lead to new understanding.

The analytic aids associated with operations research and cost-benefit analysis range from tools like computers or tables of random numbers, to broad techniques, like regression analysis, linear programming, Monte Carlo, game theory, queuing theory, and many others. Some of the more widely used techniques are briefly discussed pointing out their strengths and limitations, and describing the part they can reasonably be expected to play in cost-benefit analysis.

Statistical Regression Analysis

Regression analysis is a classical procedure with wide use in the field of cost-benefit analysis. The hypothesis underlying this approach is that a dependent variable, like the cost of an item, bears a statistically stable relation to design and performance specifications or other characteristics of the item -- independent variables descriptive of the item. The stable portion of this relation is the regression function of cost, the dependent variable, on the independent variables. The regression function is the expected cost in the presence of error. Error is the unstable portion of the statistical relation.

The regression function, $f(x, a)$ combines known values x, like design and performance specifications, and coefficients a which are stable but which are not directly observable. Least squares is the most widely used method of estimating these coefficients. Its name derives from the criterion used to produce the estimators. In a sample of data, the mean square deviation of observed outcomes y from a candidate function $f(x, a)$ can be minimized over all possible assignments of coefficients a, since values x are known. The estimator $\hat{a}$ for the true coefficients a (both generally are vectors) is obtained from the assigned $\hat{a}$ that minimizes mean square error (around the regression function) in the population and from errors of estimate for $\hat{\alpha}$.

The method is used to test certain economic hypotheses about the character of the regression function or the coefficients estimated. These hypotheses might be concerned, for example, with the variation of production cost within a firm, or among firms within an industry, as the volume and rate of output change. It is important, from a practical as well as theoretical point of view, that such hypotheses be tested against available relevant evidence.

The statistical testing of economic hypotheses can be complex and hazardous. In cost-benefit analysis, the first major problem is often a paucity of published data relevant to the subject of investigation. Empirical work generally requires the collection of suitable data as a first step.

The second major problem, although present in most econometric investigations, is especially troublesome in cost analysis. It is the proper adjustment of the data into forms relevant to the subject of investigation.

The third major problem is the validity of the statistical techniques and tests applied, since most often data samples cannot be large. Statistics has developed rapidly in this century, largely stimulated by problems in the natural sciences. Thus the assumptions underlying statistical methods often rest on relevance to non-economic problems and, as well, on mathematical tractability. The econometrician who wishes to apply these methods to data on the behavior of a complex economic system must examine the potential for conflict between the economic model which is assumed to generate observations and the statistical models which underlie statistical methods. Fortunately, some assumptions are less important than others in that statistical techniques can be insensitive to departures from them.

Because of the importance of this subject to cost-benefit analysis, a detailed discussion with references is included as Appendix A.

Linear Programming

The word "program" as used here refers to a set of instructions which can be given to a man, or to a machine, that tells what to do next to move toward the objective when a certain stage is reached. If the activity can be represented by a mathematical model, then a computational method may be evolved for choosing the best schedule of actions; this is a mathematical program.

Many economic, industrial, and transportation activities can be expressed (or at least approximated) by systems of linear equations and inequalities. When this can be done we have linear programming, the best known and most widely used technique of operations research. Electronic computers, using linear programming, have solved problems involving 3200 equations and 600,000 variables. But linear programming gives a systematic and efficient way of finding the best case, or one of the better cases, without the need for examining each possibility separately. Furthermore, much of the analytic power of the linear program lies in assessing the sensitivity of the least case to changes in the variables, based on their shadow price, or marginal value.

The term "linear" in linear programs refers to the relations that must hold among the various activities for the plan to be consistent with available resources. The essence of the technique, when applied to transportation problems, lies in instructing the computer that if substituting one route for another lowers the over-all cost, this same substitution should be repeated as often as is consistent with the constraints as to the number of items which can use that route. The explicit cost calculation of most of the possible routings is avoided and just enough cases are examined to ensure that no profitable alternative has been overlooked.

Linear programming, because of the simple form of its associated mathematical model, appears to be more restricted as to the generality of the problems it can solve than is actually the case. Although the method requires that the problem be formulated to fit the linear programming format, systems of linear inequalities can approximate a wide variety of cases.

And while this formulation is frequently difficult if the model is to remain an adequate representation of reality, adequate approximations have usually been found. A great deal of effort by mathematicians is directed toward extending linear programming into such areas as non-linear programming, integer programming, and programming under uncertainty. The pressure to use the method is great, however, because the computational algorithm is so powerful that systems containing hundreds of equations can be solved. Appendix B contains a discussion of the method.

Game Theory

The theory of games is a mathematical treatment of planning under conditions of conflict. The types of behavior that appear in such situations, of course, have long been observed and recorded. However, aside from some attempts to set up models in which optimal courses of action can be dealt with by the calculus, or, in more sophisticated form, by the calculus of variations, the only mathematical theory so far put forth -- and that only relatively recently -- is the theory of games. This theory is concerned with the selection of an optimal course of action taking into account not only the possible actions of the planner himself but those of his adversaries as well. The principal modes of resolution are collusion and conciliation.

Game theory does not cover all the diverse factors which enter into behavior in the face of a conflict of interest. There are certain important limitations. First, the theory assumes that all the possible outcomes can be specified and that each participant is able to assign to each a measure of preference, or utility, so that the one with a larger numerical utility is preferred to one with a smaller utility. Second, all the variables which determine the payoff and the values of the payoff can be specified; that is, a detailed description of all possible actions is required.

Only an occasional problem associated with systems analysis has been simple enough to solve by actually computing the game theory solution -- and some of these were only marginally related to the real world. Recently, however, advances in our theoretical knowledge have given promise that the situation may be changing. Game theory is now being successfully applied to various tactical problems -- such as radar search and prediction, the allocating defense to targets of unequal value, studying missile penetration aids, scheduling missile fire under enemy pindown, and other problems as diverse as antisubmarine warfare and inspection for arms control.

In contrast to linear programming, which contributes mainly as a tool for solving specific problems, direct uses of game theory are relatively rare. But, its contributions to policy analysis are possibly far greater for it tells us how to think about situations of conflict with an intelligent and reacting opponent who may have common as well as opposing interests (Williams 1954).

Monte Carlo

Monte Carlo may be described as a method for estimating the answer to a problem by means of an experiment with random numbers. For example, suppose one wishes to determine the probability of winning a game of solitare. One might attempt to calculate this directly, but would quickly discover that the amount of computation required is staggering. Another approach would be simply to play the game a large number of times, N, count the number of successes, n, and then estimate the probability of winning as the quotient n/N.[1] This estimate would be in error, but the error could be decreased by increasing the number of trials. To speed up this process, the game could be programmed for a high-speed digital computing machine and the trials performed by machine rather than by humans. But even with a fast computer the number of trials required to get a good answer might still be overwhelming since the error may decrease very slowly. In any event, however, a judicious mixture of analysis with random trying is likely to be more effective, and this is called the Monte Carlo technique.

The origins of Monte Carlo lie in the random sampling investigations of statisticians. The Monte Carlo approach seeks answers to mathematical

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[1] For the solution of many problems in probability it is necessary to know how many different sets of r objects can be chosen from n objects. In general, the number of *permutations* of n things taken r at a time is given by the formula,

$$P^n_r \quad (n)(n-1)(n-2) \ldots (n-r+1) = \frac{n!}{(n-r)!}$$

From this expression may be written for the number of permutations of n things taken all at a time.

$$P^n_n = n!$$

Sets without regard to the order of drawing are called *combinations*. In general, the number of combinations of n things taken r at a time is given by the formula,

$$C^n_r = \frac{n!}{r!\,(n-r)!}$$

problems and is dealing with an abstract, rather than with a real, population. This circumstance, because it allows the population to be altered, makes many refinements in technique possible. (Quade, 1964)

The use of Monte Carlo is now widespread in operations research, basically because it is the easiest computational method to apply to the large and complicated problems typical of such investigations. These problems often have prominent random elements. They are frequently new and difficult to formulate mathematically. Even if they can be formulated, they almost never have known analytic solutions and the application of the traditional methods of numerical analysis is difficult, if not impossible. In order to apply Monte Carlo methods it is only necessary to be able to model the physical process. Since high-speed machines can take over the laborious part of the calculations, Monte Carlo often allows one to substitute brute force for mathematical ingenuity and thought. Furthermore, for a good many problems studied by operations researchers and systems analysts there is no feasible alternative to Monte Carlo -- especially if information on the probability distributions of the outcomes is required as well as information about the expected values. Traditional methods of analysis are ordinarily useless in such cases if the problem is at all complicated.

Queuing Theory

As the name implies, queuing theory is concerned with waiting times for customers who arrive for service as these are affected by the rate at which customers arrive and the rate at which they are served. Familiar examples are aircraft waiting to land, machines waiting to be repaired, telephone callers waiting for lines to clear, and postal patrons waiting to mail packages.

The theory is easiest developed for a steady state as regards the arrival of customers. The time between arrivals obeys an arbitrary distribution, fixed during the period of interest. The same is true of the time needed to serve a customer once he has arrived at the head of the queue for service. More than one server, a mix of interarrival distributions and service distributions, "balking" by customers eligible for service (as in lists) who refuse it, departure of customers before being served, and priorities among customers are all aspects of queuing which have been treated. Quantities of interest in the proper sizing of service capacity are the mean size of the queue, mean waiting time for a customer, the mean length of a busy period for a server, the mean number of servers idle and the mean idle time accumulated by servers. Other measures of location, variances, and distributions of indicated quantities can be found by the methods of renewal theory.

In many cases, the expected number of arrivals during a service of one customer yields a useful summary of the process. This number is the process intensity. It must be less than one for the whole process -- arrivals and service -- to even have a steady state. If process intensity is greater than one, then customers arrive faster than they can be served, so that queues tend to grow indefinitely.

When interarrival times are exponentially-distributed, the mean and variance of service time enable mean waiting time and queue size to be calculated. When service times are also exponentially-distributed, the probability that queue size is q is given by $(1-p)\,p^q$ where p is process intensity. Waiting times and other measures of customer delay tend to be least (for a fixed average service time) when service variation is at a minimum -- it is "deterministic." Delays tend to be greatest when service time is exponentially-distributed. These two cases are useful poles for comparison with more elaborately described distributions.

In most cases it is costly to keep service available. But delay in service is also costly, often within the same cost system as the service. This is the case with machine repair. When service is part of what is sold, then delay loses sales or customers, a real economic penalty although sometimes difficult to quantify as such. For regulated servers, like telephone companies and postal service, capacities which must be attained by capital investment are adjusted to requirements on the grade of service. Thus queuing theory is a generator of information for cost and cost-grade of service tradeoffs in a great variety of settings.

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APPENDIX A

STATISTICAL REGRESSION ANALYSIS

STATISTICAL REGRESSION ANALYSIS

John E. Berterman

The Linear Model

Statistical Basis. The hypothesis that underlies least squares analysis is that an observed outcome, Y, like the annual cost of tower operations at a given airport, has an expected value which depends linearly on variables like the annual number of VFR operations, the annual number of IFR operations, the peak distribution of those operations as a function of time, and so on. A disturbance (error) is added to this expected value to produce the observed outcome, the value of the dependent variable.

Symbolically,

$$Y = b_o + b_1 x_1 + \ldots + b_k x_k + u$$

where

Y is the observed cost, the value of the dependent variable.

$x_1, \ldots, x_k$ are observed values of the independent variables (regressors, predictors).

$b_o, b_1, \ldots b_k$ are regression coefficients, not known directly.

u is the random disturbance (error) contribution to Y.

The average value of u, the random disturbance is 0. (Any other value for the average is subsumed by b_o). Thus,

$$b_o + b_1 x_1 + \ldots + b_k x_k$$

represents the average value of Y given independent variables $x_1, \ldots, x_k$. It is the regression function of Y on these variables, by definition. The whole regression function $b_o + b_1 x_1 + \ldots + b_k x_k$ is exactly analogous to a mean. When the function is b_o alone, it is just a mean, unvarying with independent variables $x_1, \ldots, x_k$. Estimators $\hat{b}_o, \hat{b}_1, \ldots \hat{b}_k$ for the regression coefficients are derived from a sample by averaging methods which are essentially those by which the arithmetic average of a sample is used to estimate the population mean. The essentials are visible in the character of the sample mean as an estimator.

The sample mean emerges as the value of the single coefficient b_o that minimizes mean square error in the data sample around the regression function b_o. This mean square error is,

$$\frac{1}{n} \sum_i (y_i - b_o)^2 = \frac{1}{n} \Sigma (y_i - \bar{y})^2 + n (b_o - \bar{y})^2$$

where $\bar{y}$ is the sample mean. Setting $\hat{b}_o = \bar{y} = \hat{b}_o$ evidently minimizes the mean square error in the sample. It also yields the equation for $\hat{b}_o$ as $\hat{b}_o = \bar{y}$.

For more general regression functions, the procedure of minimizing mean square error in the sample produces just the same separation of mean square error into two parts, the first depending only on sample values, like $\frac{1}{n} \Sigma (y_i - \bar{y})^2$. The second part increases with departure of combinations of coefficients $b_o, b_1, \ldots b_k$ from what are really averages calculated from the data samples in a way similar to the term $(b_o - \bar{y})^2$. Equating the combinations of coefficients to these data means yields a set of equations for estimators $\hat{b}_o, \hat{b}_1, \ldots \hat{b}_k$.

$$\hat{b}_o + \hat{b}_1 x_1 + \ldots + b_k x_k = \bar{y}$$

$$\hat{b}_o \bar{x}_1 + \hat{b}_1 \overline{x_1^2} + \ldots + b_k \overline{x_1 x_k} = \overline{x_1 x_k}$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$\hat{b}_o \bar{x}_k + \hat{b}_1 \overline{x_k x_1} + \ldots + b_k \overline{x_k^2} = \overline{x_k^2}$$

The bar indicates averaging in the sample. The quantities averaged are values $x_1, \ldots, x_k$, y and, as in $\overline{x_1^2}$, $\overline{x_1 y}$, averages of quantities x_1^2 and $x_1 y$.

Estimation of coefficients $\hat{b}_o$, $\hat{b}_1$, ... $\hat{b}_k$ enables calculation of a 'predicted' value for each value y_i of the dependent variable in the sample,

$$\hat{y}_i = \hat{b}_o + \hat{b}_1 x_{1i} + \ldots + \hat{b}_k x_{ki}$$

The variance of the disturbance term can then be estimated by,

$$\hat{\sigma}^2 = \frac{\Sigma (y_i - \hat{y}_i)^2}{n - (k + 1)} .$$

Symbolizing the equations for $\hat{b}_o$, ..., $\hat{b}_k$ by,

$$S\ b_k = \overline{x^1 y}$$

Variances and covariances of estimators $\hat{b}_o$, ..., $\hat{b}_k$ are entries in the matrix $\hat{\sigma}^2 S^{-1}$.

Like the sample mean, estimators $\hat{b}_o$, ... $\hat{b}_k$ have "optimum" properties as the minimum variance unbiased linear estimators of the coefficients when disturbances in the data are independent of one another and all have the same variance. Cases in which error correlations occur and variances are unequal can be handled (when necessary scaling factors are available) by what is essentially a transformation of the data into the form presupposed by the linear model.

Application. Analysis of costs by use of the linear model involves the following steps:

(1) Collection of data on the outcomes, Y, and independent variables, x.

(2) Adjustment of the data for price level differences and other possible inconsistencies with the structure of postulated regression functions.

(3) Estimating coefficients $\hat{b}_o$, $\hat{b}_1$, ... $\hat{b}_k$ by application of the formulas relating them to the data.

(4) Assessing the precision of estimated coefficients by use of the error variance estimated from the sample and relations between it and error among values $\hat{b}_o$, b_1, ... $\hat{b}_k$.

(5) Based on the outcome of (4) perhaps reformulating the selection of x's and repeating the process.

In some ways this approach has great appeal. No matter how advanced a subsystem or component may be, it is usually possible to obtain data on costs and characteristic parameters of the system to use as independent variables on roughly similar terms. Computer programs for regression analysis are readily available; and a high coefficient of determination, R^2 -- a measure of the explanatory power of the estimated relation -- is not uncommon. There are, however, potentials for difficulty. The data may contain occasional large errors and may reflect correlated errors, the set of independent variables selected may be incomplete, or the index R^2 may be spuriously high owing to small sample size or to smoothing procedures utilized in producing the data.

Another possible difficulty is that prediction by use of the fitted coefficients may require going well beyond the ranges of some values of the independent variables which enter into the regression function. This effect is calculable (on the assumptions made to produce the estimated coefficients) but calculation can show that variation around most likely cost estimates is large -- that confidence intervals are wide. Departure from assumptions of the analysis, of course, can further reduce precision in prediction.

The methods applicable to deriving coefficients do enable deflating R^2 in a way that reflects sample size and the consequences of small samples. Similarly, t-statistics (to test hypothetical values for single coefficients) and F-statistics (to test hypothetical values for two or more coefficients) enable a corrected assessment of error in the estimation of coefficients and an assignment of suitable confidence intervals for predictors or for coefficients. Thus attention to these correctives for small-sample effects can avoid attaching spurious precision to results.

The calculation of indices of serial correlation among residual values $y_i - \hat{y}_i$ and of correlations between residuals and powers of $\hat{y}_i$ serve to indicate whether assumptions themselves must be questioned. While the calculations can be routinized, the subsequent revision of the model, to introduce new variables or, perhaps, assign a different error distribution (as by "weighting" errors) must be guided by subject-matter expertise.

The Log-Linear Model

Frequently it is reasonable to assume that a cost relationship is not linear but of the form:

$$Y = b_o X_1^{b_1} X_2^{b_2} \ldots X_k^{b_k}$$

where the error term is now multiplicative and assumed to have mean = 1. Note that if logarithms of both sides are taken, a new expression is obtained.

$$Y^* = b^*_o + b_1 X_1^* + \ldots + b_k X^*_k + u^*$$

where

$$Y^* = \log Y$$
$$X^*_i = \log X$$
$$u^* = \log u$$

which is linear in the logs of the variables. This means the regression coefficients can be estimated by the same (least-squares) procedure as before, which is a real convenience if not a necessity. Often we are in the situation of not knowing whether the linear or log-linear formulation is correct, and hence looking for a criterion by which to decide. It seems natural enough to select the function with the greater explanatory power (higher R^2), but there the difficulty arises. While the R^2 of the linear function measures the proportion of the variation of Y explained, the R^2 of the log-linear function measures the proportion of the log Y explained, which -- as Goldberger has colorfully put it -- "is not the same animal."[1] A comparable measure for the second function can be obtained by taking anti-logs of the predicted costs and performing one additional calculation. While this problem has been recognized for some time it is often overlooked in practice.

A related problem arising in this same context, and one that has only recently been pointed out -- again by Goldberger[2]-- is that least-squares

[1]A.S. Goldberger, "Topics in Regression Analysis." (Macmillan Co. London, 1968) p. 130.

[2]*Ibid.*, pp. 119-121.

regression when applied to a log-linear relationship, results in a systematic prediction bias that needs to be (and can be) corrected. The difficulty stems from taking the logarithm of a multiplicative error term[3] with mean = 1. While the log of the mean of u is indeed zero, the mean of log u is in fact not equal to zero, and it is the latter that must be zero in order to justify straightforward application of least squares. As suggested above, Goldberger has shown how the correction can be made, but it is disquieting to imagine how many estimating relationships are probably now in use where this difficulty has been left unattended.

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[3]Note that if the error term were additive as in the first formulation, the relationship would be mathematically intractable; i.e., it would not be possible to simply take the logs of both sides of the equation.

Ridge Analysis

A more serious and even more common problem arises in regression analysis when two or more of the system characteristics are closely correlated with one another. An example is the relationship of the sensitivity and power output characteristics of transceivers. The consequences of this are that it is virtually impossible to estimate with any degree of accuracy the separate effects of each; in fact, many of the regression coefficients estimated by least-squares often have implausible (negative) algebraic signs. This difficulty is particularly severe if one of the objectives of the regression is to establish a basis for performing sensitivity analyses. Until recently, analysts have either had to simply live with the problem or else formulate their relationships to conform to the limitations of their data; i.e., deliberately exclude certain system parameters because of the degree to which they were correlated with others in the data available for the analysis. However, a statistical method known as Ridge Analysis, or Ridge Regression, has recently been developed for dealing with this problem. (Hoerl and Kennard, 1970)

Consider a matrix formulation of the general linear regression model:

$$Y = Xb + e$$

where

- Y = $n \times 1$ vector of observation on the dependent variable.
- X = $n \times p$ matrix of observations on the p non-stochastic independent variables.
- b = $p \times 1$ vector of unknown parameters (population regression coefficients) to be estimated.
- e = $n \times 1$ stochastic error term.

On the assumptions that e has mean zero, constant variance and zero covariance, ordinary least squares (OLS) estimators of b,

$$\hat{b} = (X'X)^{-1}X'Y$$

can be shown to have the desirable theoretical properties of (1) unbiasedness and (2) minimum variance among the class of unbiased estimators. However, in practice how "good" an estimation procedure is OLS depends on the conditioning of X'X, the so-called cross-product or moment matrix. If X'X is singular, i.e., if an exact linear relationship exists between same set of X's, the procedure breaks down completely since the moment matrix cannot be inverted. A much more common occurrence is for non-singularity to exist, but for there to be considerable correlation between the independent variables. Manifestations of this problem usually take the form of estimated regression coefficients which are large (too large) in absolute value, and which often have implausible algebraic signs.

For the above reasons, A.E. Hoerl and R.W. Kennard have proposed an estimation procedure based on the criterion of minimizing the sum of squares of the estimated coefficients, subject to an acceptable (to be discussed later) increase over the minimum residual, or error, sum of squares, i.e., the OLS result. They have given the name Ridge Analysis, or Ridge Regression, to this procedure, and the ridge estimators, $\hat{\beta}^*$, are calculated by:

$$\hat{\beta}^* = (X'X + kI)^{-1}X'Y$$

where X'X and X'Y are "coded" in correlation form, k is a number between 0 and 1, and I is the identity matrix of appropriate dimensions. The $\hat{\beta}^*$ are analogous to a vector of "beta" (standardized) coefficients in conventional regression analysis, and must be "decoded" to correspond to the actual units in which the variables are measured. Since the standardized coefficients are useful in themselves (they are directly comparable, one to the other), both these and their decoded (natural) values are printed in the output.

Theoretical justification for the superiority of Ridge Regression over ordinary least squares rests on the general relationship between

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*Note that an absolute minimum would be achieved by setting each parameter estimate equal to zero. However, since such a set of estimates would be totally divorced from reality, Hoerl and Kennard have adopted a constrained minimization criterion.

mean square error of estimation, bias and variance, which is:

$$M.S.E. = \text{var } \hat{\beta} + (\text{bias } \hat{\beta})^2$$

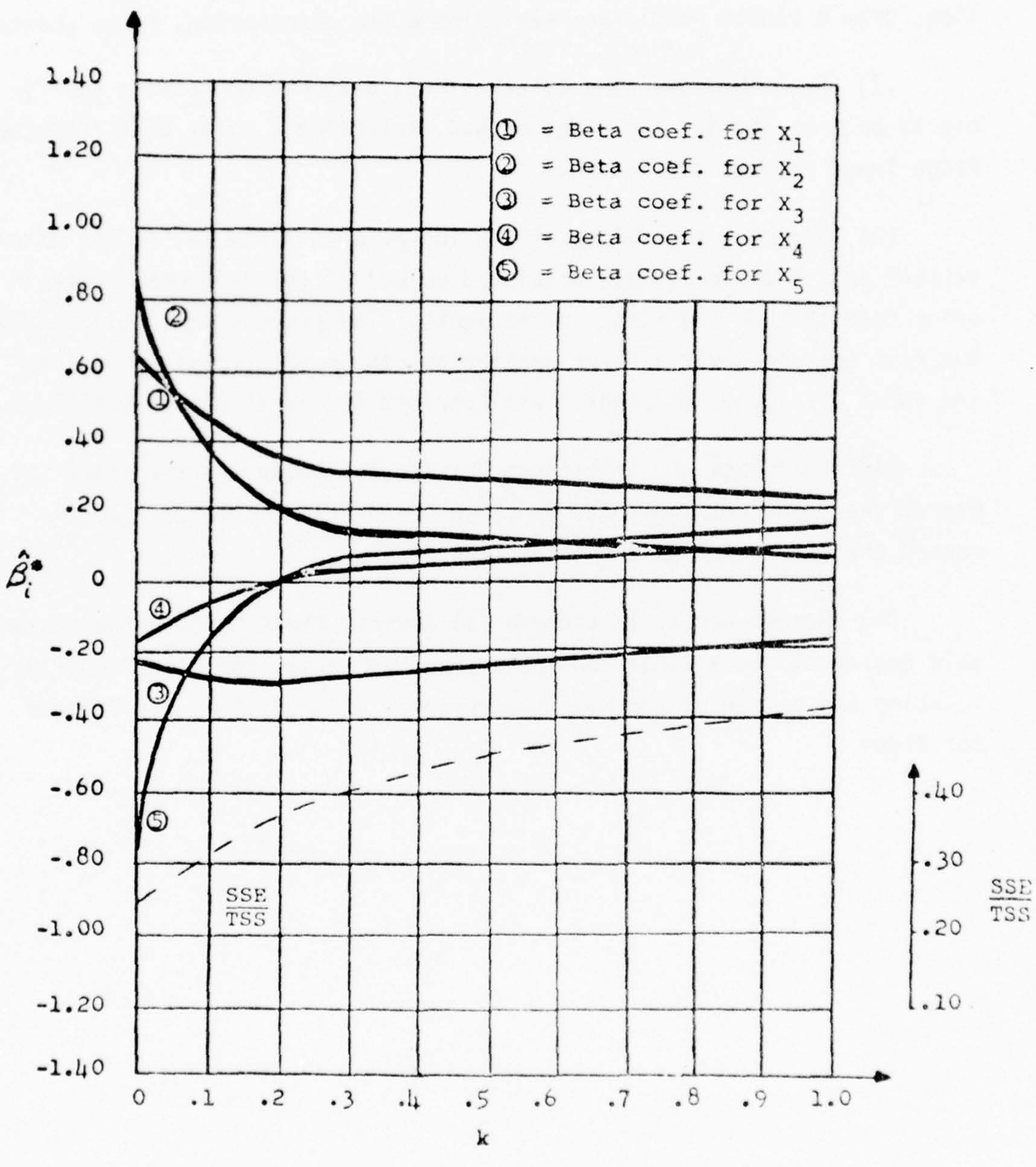
Addition of the quantity k reduces the variance of the estimators but introduces bias into the procedure. However, Hoerl and Kennard have shown that the nature of the bias and variance functions is such that for certain values of k, the increase in bias is more than offset by a reduction in variance, thereby improving the mean square error of estimation. And, it follows that more accurate estimates of regression coefficients will result in greater predictive accuracy. The use of sampling experiments to examine prediction results will be discussed in a later section.

Selection of a "k" Value. In practice, rather than making an *a priori* decision as to what constitutes an acceptable increase over the minimum residual sum of squares (and hence deciding on a value for k), the choice is made by an inspection of the Ridge Trace, a diagram showing values of $\hat{\beta}^*_i$ as a function of $0 < k < 1$. The trace, an example of which is shown in Exhibit A.1, displays the sensitivity of the OLS estimates and often reveals two characteristics of the coefficients mentioned earlier: inflated absolute values and incorrect signs. (Note the change in signs on variables 4 and 5 in the exhibit. In the actual problem from which the exhibit was taken, the changes were in the "right" direction, i.e., they conformed to the signs anticipated for those coefficients). Although no explicit criterion exists for choosing the optimal value of k from the Ridge Trace, it is usually not too difficult to observe where the system stabilizes and to pick the corresponding value of k. In the example shown, stabilization takes place in the neighborhood of k = .3, and the estimates corresponding to that value were the ones selected.

Sampling Experiments to Test Predictive Accuracy. If the ridge estimators are indeed more accurate than their OLS counterparts, one would expect predictions based on these to likewise be more accurate. An approach to verifying this is through what might be called "simulated" prediction. The steps are as follows:

EXHIBIT A.1

EXAMPLE RIDGE TRACE



(1) From a set of data (a sample) available for parameter estimation, draw a random sample and set aside a few observations (data points).

(2) From the remaining observations, estimate regression coefficients both by OLS and the Ridge method, selecting a value of k from the Ridge Trace as discussed above.

(3) Based on the values of the independent variables in the observations set aside, calculate predicted values of the dependent variable using both the OLS and Ridge coefficients. The process is simplified by the fact that observations for prediction can be identified as such in the input data, and the program will perform the necessary calculations.

(4) Calculate the difference between the known and predicted values of the dependent variables in these observations. In each case, record the difference as the prediction error.

(5) Repeat steps (1) through (4) several times to obtain a reasonable degree of statistical confidence, and summarize the results by calculating and comparing the mean square error of prediction for OLS and for Ridge.

Measures of the Goodness of Fit

Coefficient of Determination (R^2). A measure of the goodness of fit of a regression function as compared with the fit attainable by use of a constant alone is the regression function. For a sample in which the estimated values of dependent variable y_i are $\hat{y}_i$ when the regression function is used to predict y_i, R^2 is given by,

$$R^2 = 1 - \frac{\Sigma (y_i - \hat{y}_i)^2}{\Sigma (y_i - \bar{y})^2}$$

$\bar{y}$ is the arithmetic mean of values y_i.

Because $\hat{y}_i$ is most often obtained by use of coefficients estimated from the sample, R^2 tends to overrate the amount of correlation between the time regression function and values y_i. When k coefficients (including a constant term) are so estimated, an adjusted R^2, $\bar{R}^2$, supplies a better indication of the correlation. For a sample of size n,

$$\bar{R}^2 = R^2 - \frac{k - 1}{n - k} (1 - R^2)$$

$\bar{R}^2$ is always less than R^2 and enables a better comparison of regressions in which the number of coefficients used, k, varies.

Standard Error of Estimate (S.E.E.). A measure of the dispersion of predicted values around the regression function.

$$\text{S.E.E.} = \left[\frac{\sum_i (y_i - \hat{y}_i)^2}{n - k} \right]^{\frac{1}{2}}$$

This quantity is the square root of the unbiased estimator of variance of predicted values around the time regression function.

t-Ratio. The ratio between the departure of an estimated $\hat{b}$, the regression coefficient, from a specified number b and the estimator for the standard error of $\hat{b}$, which is proportional to S.E.E. above, but by a factor depending on the values of independent variables in the data. This ratio is distributed as a t-statistic with n-k degrees of freedom in estimating the standard error of $\hat{b}$. It can be used to test whether $\hat{b} = b$ or to set confidence limits on b.

F-Ratio. A generalization of the t-statistic used for similar purposes -- to test whether a set of coefficients has a set of stated values or to obtain a simultaneous confidence region. One application is to test whether all regression coefficients beyond the constant term (presupposed to be present in the regression) are zero. Then,

$$F_{k-1,\, n-k} = \frac{R^2}{1 - R^2} \frac{(n-k)}{(k-1)}$$

is distributed as a ratio of two independent variance estimators with k-1 and n-k degrees of freedom for numerator and denominator respectively.

Durbin-Watson Statistic. A test statistic for serial correlation of errors around the regression indicating, when significant, that consecutive errors are interrelated.

where $e_i = y_i - \hat{y}_i$ is the residual for observation i around the fitted regression,

$$d = \frac{\sum_{i=2}^{n} (e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i{}^2}$$

The Theil-Nagar statistic is interchangeable with the Durbin-Watson statistic as to purpose. It is,

$$r = \frac{n^2 (2 - d) + 2 k^2 - 2}{2n^2 - 2k^2 - 1}$$

and measures the correlation between consecutive residuals directly.

Table of Residuals (Table of Percentage Variations). This makes available values y_i, $\hat{y}_i$, and $e_i = y_i - \hat{y}_i$ for use in residual analysis.

Correlation Matrix. A table of calculated bivariate correlations among values y_i and independent variables. Entry r_{kj} is given by,

$$r_{kj} = \frac{\sum_{i=1}^{n} (x_{ik} - \bar{x}_k)(X_{ij} - \bar{x}_j)}{\left[\sum_{i=1}^{n} (x_{ik} - \bar{x}_k)^2 \sum_{i-1}^{n} (x_{ij} - \bar{x}_j)^2 \right]^{\frac{1}{2}}}$$

where $\bar{X}_k$, $\bar{X}_j$ are sample averages of variables x_{ik}, x_{ij}.

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APPENDIX B

LINEAR PROGRAMMING

LINEAR PROGRAMMING

O.L. Greynolds*

In The Beginning

The Russian mathematician L.V. Kantorovich is generally credited with early pioneering work in formulating linear programming (LP) methodologies (Watson, 1963). W.W. Leontief, the economist, is also given substantial recognition for fostering linear programming techniques in his early work on input-output analyses (Thierauf and Klekamp, 1975). Hitchcock first interpreted the "transportation type" problem as a linear program. He addressed the question of minimizing the costs of shipping commodities from plants to warehouses. Stigler, in a classic work, studied what is called the "Diet" problem, i.e., how to provide a specified level of nutrition at least cost. Later Koopman extended and expanded Hitchcock's work on the Transportation Linear Program. Dantzig developed an innovative computational technique, the Simplex Method, laying the foundation for the ubiquitous linear mathematical program.

Boosted by the advent and growth of the electronic computer, linear programming has so improved in both theoretical foundations and practical applications that it has invaded the daily operations of many industries: oil, chemicals, agriculture, forestry, manufacturing, and transportation, to name but a few. Many day-to-day decisions faced by managers at all levels of private and public organizations are heavily influenced by linear pro-

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gramming and its related methodologies. Furthermore, LP has become the basis for a set of special theories dealing with the economics of the firm (Watson, 1963).

The principal use of LP is to analyze potential resource allocation decisions among competing alternatives in the face of scarcities. In short, it bears on cost-benefit analysis in a very direct way.

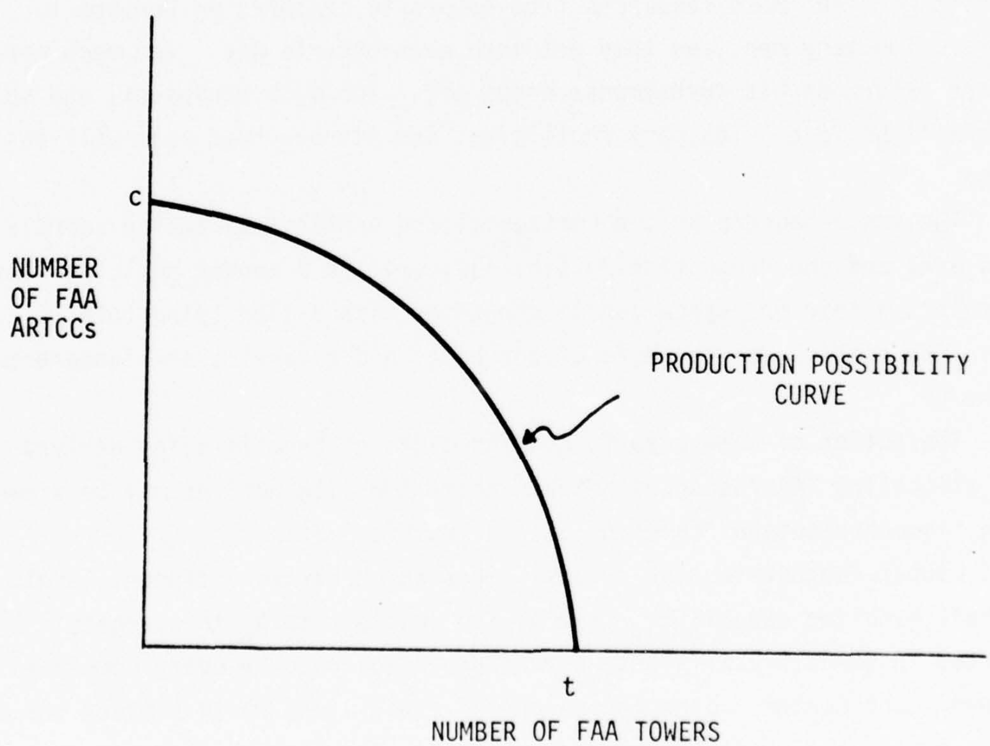
In concept, the LP is simplicity itself. When linked with the computer it can manipulate thousands of alternatives, variables, and constraints, across a wide variety of significant economic, planning, operations, and control problems. The LP attacks the classical economic choice problem faced by every decisionmaker: How can scarce resources be allocated to achieve the greatest output for a given input, or a specified output at least cost?

The Economic Connection

The LP is about the economics of production functions -- Production Possibility Curves, isocosts, and isocontributions. If the decisionmaker must choose between one or two, or proportions of two, products or alternative outputs, given a specified package of inputs, such as labor, material, and money, he may view his decision problem as a Production Possibility Curve, as in Exhibit B.1.

EXHIBIT B.1

PRODUCTION POSSIBILITY CURVE



The "guns and butter" problem of classical economics illustrated in Exhibit B.1 suggests that for a given set of inputs, not specified here, the decisionmaker can develop and operate FAA Towers and/or FAA Air Route Traffic Control Centers. At the extremes, he could create a total of *c* Centers and no Towers, or *t* Towers and no Centers. Each require resources -- men, money, equipment, and facilities. Along the Production Possibility Curve from *t* to *c*, given the existing technologies, the decisionmaker may view the choices he can exercise between these two extremes with regard to his resources. The curvature of the Production Possibility Curve reflects the substitutability that exists, or may be said to exist, when he allocates his resources between Towers and Centers. The curve is convex because the decisionmaker will run into diminishing returns as he attempts to shift more and more resources from Towers to Centers, or Centers to Towers. Too many men, and they get into each other's way. Too much money, and the return on his investments drops off. Too much equipment, and some of it will be idle. Too many facilities, and his overhead rate will increase.

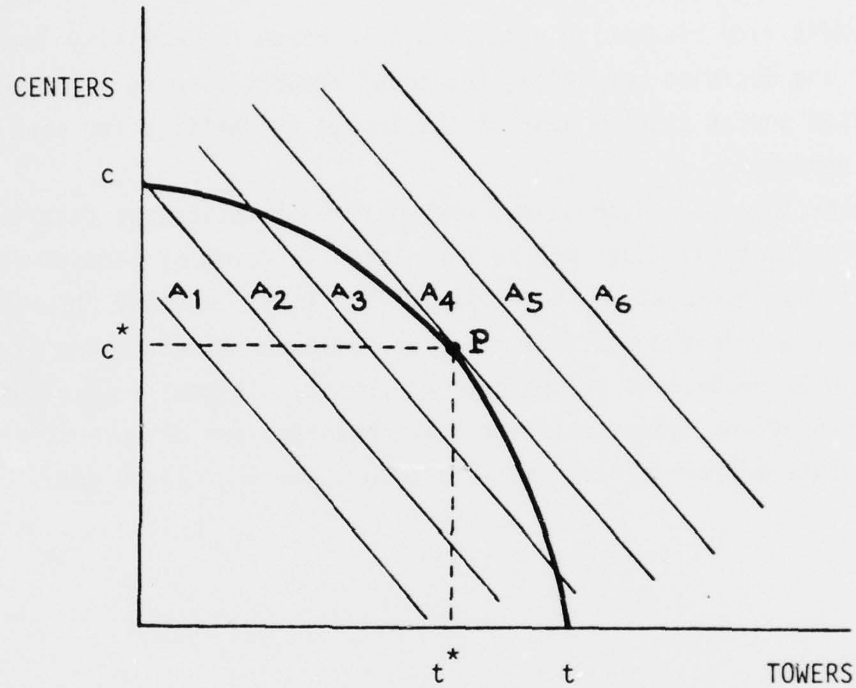
The space bounded by the horizontal and vertical Cartesian coordinates axes and the Production Possibility Curve is a convex hull, for any two points within the space can be connected with a line lying totally within the space. The concepts of convexity and concavity are fundamental to the LP.

The notion of some payoff, contribution, or benefit being derived from allocating FAA resources between these two alternatives may be viewed as an "isocontribution" function, as in Exhibit B.2.

Linear isocontribution curves, lines which depict different levels of aircraft handling capability, from A_1 the smallest to A_6 the largest, are depicted in Exhibit B.2. These curves are "iso" because every combination of Towers and Centers connected on one of these lines would produce the same amount of service, contribution, or benefit. The optimal economic choice would be at point P, the point of tangency between the Production Possibility Curve and isocontribution line A_4. The decisionmaker, given the problem formulated, should operate t^* Towers and c^* Centers to achieve the greatest contri-

EXHIBIT B.2

THE OPTIMUM SOLUTION
PRODUCTION POSSIBILITY AND ISOCONTRIBUTION CURVES



bution -- the number of flights provided service -- within the constraints of resource inputs and technology.

Isocontribution lines A_1, A_2, and A_3 are suboptimal, for they do not fully use the input resources available. On the other hand, isocontribution lines A_5 and A_6 are infeasible production levels, given the quantity of the inputs available and the state of technology in FAA operations and equipment. To reach line A_5 or line A_6, the decisionmaker must allocate more of the same kind of resources, or enhance the productivity of the resources he now has available to him.

These economic concepts enter the LP formulation through two assumptions: divisibility and additivity. Divisibility means that for each production activity the amount of each input and the associated profit are strictly proportional to the level of output, and, furthermore, that each activity can be continuously and proportionally expanded or reduced -- if you double the inputs you double the output and the profits. Divisibility also permits fractional values for a production activity in the optimal solution.

The additivity assumption specifies that given the activity levels for each of the decision variables, the total amounts of each input and the associated profit are the sums of the inputs and profits for each individual process.

Together these two assumptions are equivalent to stating that the underlying mathematical model can be formulated in terms of linear relations. Strictly interpreted, these assumptions imply constant returns to scale and preclude the possibility of economies or diseconomies of scale, both with respect to technology and profit. In real situations these two assumptions seldom hold strictly, but they are often sufficiently accurate approximations that linear programming can be used.

The Method

In much the same fashion as Production Possibility Curves and iso-costs/isocontribution curves seek an optimum allocation of resources, the LP optimizes the value of an objective function according to constraints imposed on the solution. Because of the power of the methodology, the LP has been specialized in many different forms. Aside from all the advantages of any systematic cost-benefit analysis -- specifying objectives, assumptions, cost and benefit variables, and constraints -- the LP also offers great insights into the sensitivity of the optimal solution to changes in the objective function cost coefficients, the technology coefficients of the constraint set, and the constraint quantities.

General Form, Objective Function. Let there be n possible outputs or activities, $x_1, x_2, x_3, \ldots x_n$. Their respective costs of production are $c_1, c_2, c_3, \ldots c_n$. The value of the output Z (value assumed to be equal to cost of production) is $Z = c_1 x_1 + c_2 x_2 + c_3 x_3 + \ldots c_n x_n$. This is the objective function which is optimized. It can be summarized as,

$$Z = \sum_{j=1}^{n} c_j x_j$$

General Form, Constraint Set. To produce the outputs described by the objective function, there are inputs i, where i ranges from 1 to n. These inputs contribute to the outputs x_j at rates determined by the technology coefficients a_{ij}. For example, a_{12} is the contribution of input 1 to a unit of output 2. If the amount of input is specified, this constraint is designated b_i.

The constraint set is of the form,

$$\sum_{j=1}^{n} a_{ij} x_j \geq b \quad \text{for } i = 1, 2, 3, \ldots, m.$$

$$x_j \geq o \qquad j = 1, 2, 3, \ldots, n.$$

This summary equation states that there are available only b_i units of resource i to be used in the production of output x_j. The form of this equation makes necessary an additional constraint, that $x_j \geq o$.

General Form, Possible Results. There are four possible outcomes to the optimization of an objective function according to a set of constraints:

1. There is no feasible solution.
2. There is a unique feasible solution.
3. There is a non-unique feasible optimum solution.
4. The solution is unbounded, that is, the value of the objective can be made arbitrarily large by adding inputs.

These four outcome states are illustrated on the opposite page.

Specific LP Formulation. Various forms of linear programs have evolved, forms which are representative of the more commonly encountered resource allocation problems. These formulations include: maximization, minimization, maximization with blended outputs, transportation, transshipment, assignment, shortest-route, critical path scheduling, theoretic game solutions, and dynamic models.

Formulating Maximization Problems. A form of resource allocation problem often encountered is that a decisionmaker has to decide what quantity of each of n outputs, x_j, to produce. Each x_j yields a profit, p_j. He has m resource inputs, b_i. The amount of input i required for each unit output j is a_{ij}. The objective here is to maximize the profit, given the input resource constraints. The formulation is,

$$\text{Maximize:} \quad Z = \sum_{j=1}^{n} p_j x_j$$

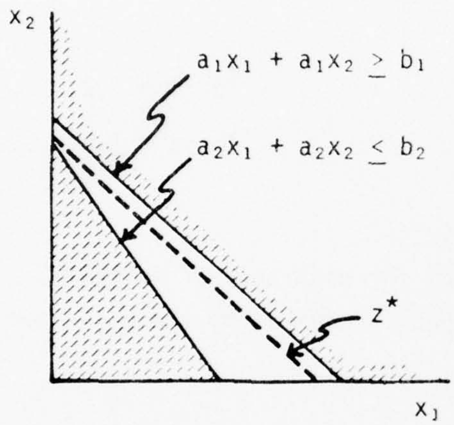
$$\text{Subject To:} \quad \sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad \text{for } i = 1,2, \ldots, m$$

$$x_j \geq o \qquad \text{for } j = 1,2, \ldots, n$$

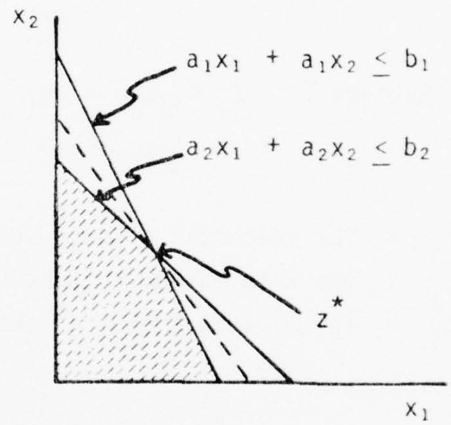
Where: p_j = The unit profit of each activity.
a_{ij} = The i-th input requirement for each unit of activity.
b_i = The maximum amount of input i available to the decisionmaker.

EXHIBIT B.3

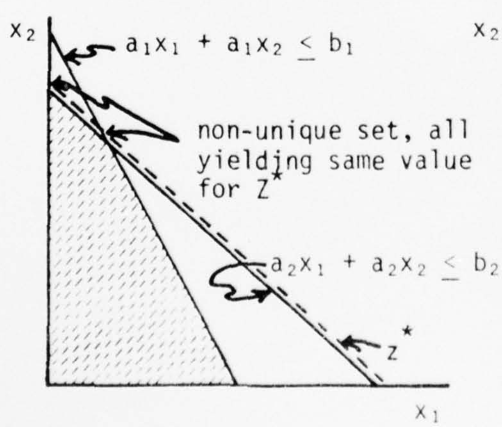
1. NO FEASIBLE SOLUTION



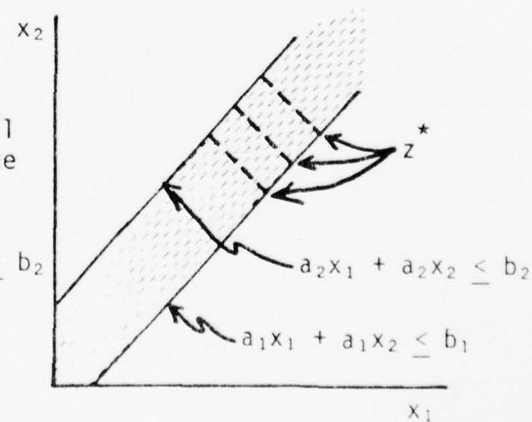
2. UNIQUE FEASIBLE SOLUTION



3. NON-UNIQUE FEASIBLE SOLUTION



4. UNBOUNDED SOLUTION



$z^* = c_1x_1 + c_2x_2$

Formulating Minimization Problems. The general nature of the problem here is that the decisionmaker has a specified output he desires to achieve, and his objective is to minimize the costs of the constituent inputs. The form is,

Minimize: $$Z = \sum_{j=1}^{n} c_j x_j$$

Subject To: $$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \qquad \text{for } i = 1,2, \ldots, m$$

$$x_j \geq 0 \qquad \text{for } j = 1,2, \ldots, n$$

Where: c_j = The unit cost of each activity.

a_{ij} = The i-th output requirement for each unit of activity.

b_i = The minimum amount of output i required by the decisionmaker.

The minimization problem is the complement of the maximization problem. In the minimization problem we desire to produce a specified output at a minimum cost, while in the maximization problem we wanted to produce the maximum output, given the input resources available. Minimization is cost oriented, maximization is benefit oriented.

Formulating the Maximization Problem with Blended Inputs. Often a decisionmaker has a situation in which he desires to maximize his benefit at minimum costs, but with the constraint that outputs must contain specified proportions of the inputs. The objective function is used to maximize the profit of blending input i into output j. The constraint set contains three elements: the supply inputs, the minimum demand levels, and technological properties of each output.

The supply constraints set forth the number of units of input i available during the planning period. Demand constraints would express the minimum levels of the outputs required. The technological properties would establish the relevant mix-proportion of inputs to be achieved in each output. The supply and demand restrictions are similar to those of the maximization and minimization forms. However, the technological restrictions are more complex.

As an example, suppose each input contains a_i of a critical component. A typical constraint might be that output j must contain a_i at some fractional level, r_j. A problem with three inputs, two outputs, and technological constraints on each of the outputs would be formulated as,

$$\text{Maximize:} \qquad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij} x_{ij}$$

$$\text{Subject to:} \quad \sum_{j=1}^{m} x_{ij} \leq S_i \qquad \text{for } i = 1,2, \ldots, m$$

$$\sum_{i=1}^{n} X_{ij} \geq D_j \qquad \text{for } j = 1,2, \ldots, n$$

$$\frac{\sum_{i=1}^{m} a_i x_{ij}}{\sum_{i=1}^{m} x_{ij}} \geq r_j$$

$$\frac{\sum_{i=1}^{m} b_i x_{ij}}{\sum_{i=1}^{m} x_{ij}} \leq r_j$$

$$\frac{x_{ij}}{x_{i+1,\, j}} \geq r_j$$

$$x_{ij} \geq 0$$

Where: p_{ij} = Unit profit of activity x_{ij}.
x_{ij} = Units of input i to be used in output j.
S_i = Maximum units of input i available.
D_j = Minimum units of output j required.
a_i, b_i = Proportion of critical component in input i.
r_j = Specified technological ratios.

In such a blended maximization problem, the coefficients of the outputs would be either 1 or 0, depending on whether the constraint is applicable to x_{ij}. The coefficients of the technological constraints would be numerical values, depending on a_i, b_i, and r_j. A matrix presentation of a two-input, three-output problem formulated above would be:

ITEM	Input 1, Output 1	Input 1, Output 2	Input 1, Output 3	Input 2, Output 1	Input 2, Output 2	Input 2, Output 3	RESTRICTION
Input 1 supply	1	1	1				$\leq S_1$
Input 2 supply				1	1	1	$\leq S_2$
Output 1 demand	1			1			$\geq D_1$
Output 2 demand		1			1		$\geq D_2$
Output 3 demand			1			1	$\geq D_3$
Output 1 technology	a_1-r_1			a_2-r_1			≥ 0
Output 2 technology		b_1-r_2			b_2-r_2		≤ 0
Output 3 technology			1			$-r_3$	≥ 0
Unit Profit	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	MAXIMIZE
Activity Level	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	

Transportation Model. The transportation model is a well-known linear programming formulation. It has been widely used, and it is the basis of many linear programming applications that do not deal directly with "transporting" units between input and output terminals.

The general form of the transportation model is,

Minimize: $$Z = \sum_{i-1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject To: $$\sum_{j=1}^{n} x_{ij} \leq S_i \quad \text{for } i = 1,2, \ldots, m \quad \text{(supply)}$$

$$\sum_{i=1}^{m} x_{ij} \geq D_j \quad \text{for } j = 1,2, \ldots, n \quad \text{(demand)}$$

$$x_{ij} \geq 0 \quad \text{for all } i \text{ and } j$$

Where: c_{ij} = Unit transportation cost from supply i to demand j.

x_{ij} = The number of units shipped from supply i to demand j.

S_i = Maximum units available at supply point i.

D_j = Minimum units required at demand j.

The transportation model has several special aspects which enable the form to be simplified. A key characteristic of network theory is that among all the optimal solutions of the model there is at least one in which each x_{ij} is integer-valued, if S_i and D_j are integers. "Dummy" supply or demand points can be devised to force the equilibrium of $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$. Therefore, with no loss in generality we can rewrite the transportation model as,

$$\text{Minimize:} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{Subject To:} \quad \sum_{j=1}^{n} x_{ij} = S_i \qquad \text{for } i = 1,2, \ldots, m \quad \text{(supply)}$$

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad \text{for } j = 1,2, \ldots, n \quad \text{(demand)}$$

x_{ij} = 0,1,2, ... for all i and j, where S_i and D_j are positive integers satisfying the balancing equation $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$

In this form, the transportation algorithm can be solved using many different computational algorithms for making initial allocations to the network and determining if the iterative allocations can be improved upon. For example, the Vogel Approximation can be used to find a feasible solution, but it may not be optimal. The Northwest Corner Rule, with or without inspection, can be used to make feasible allocations. And the Steppingstone or the Modified Distribution Method (MODI) may be used to evaluate each assignment to test for optimality and iterate if an optimum has not been reached. A discussion of these techniques and others can be found in Thierauf and Klekamp (1975: 213-252).

Of course, the transportation model can be solved, if inefficiently, directly with the linear programming Simplex algorithm. A matrix representation of such a formulation is,

	x_{11}	x_{23}	$\dots x_{1n}$	x_{21}	x_{22}	$\dots x_{2n}$	$\dots x_{m1}$	x_{m2}	x_{mn}	
S_1	1	1	... 1							$= S_1$
S_2				1	1	... 1	...			$= S_2$
.										.
S_m							1	1	..	$= S_m$
D_1	1			1			1			$= D_1$
D_2		1			1			1		$= D_2$
.		.			.	...		.		.
.		.			.			.		.
.			.			.			.	.
.			1			1			1	$= D_n$
D_n	c_{11}	c_{12}	$\dots c_{1n}$	c_{21}	c_{22}	$\dots c_{2n}$	$\dots c_{m1}$	c_{m2}	$\dots c_{mn}$	MINIMIZE

Although the classical form of the transportation model seeks to minimize the costs of an objective function, the same algorithm can be used to maximize the objective function of a network problem if the decisionmaker wants to find this optimum relationship between "supplies" to "demands." This form can be used to allocate work to equipment or functions where the output is profit and the inputs are costs for each work unit on each piece of equipment or in each function.

Transshipment Model. This is an extension of the classical transportation model when, in the network, points can act as transshipment points. that is, they can be both a supply point and a demand point. Such situations are common when divisionalized organizations maintain subunit stocking points from which other subunits are supplied. Many logistics activities have found great applicability for this version of the linear program.

Assignment Problems. This model is a special form of the transportation problem wherein $S_i = D_j = i = j = n$. This form of the linear program is such that the problem confronting the decisionmaker is: How to assign each of n tasks, if each can be performed by any one of n workers or machines that have differential costs associated with them. The costs of accomplishing job i with worker j is c_{ij}. The model assigns one worker to each task so as to minimize the total cost of performing all the tasks.

Shortest-Route Formulation. Situations that suggest this model are those in which a given network has arcs with specified "values," and we wish to find the shortest path to a specified node from any of the other nodes in the network. A broad range of important problems have this structure, including planning equipment replacement schedules and scheduling complex projects.

Critical Path Scheduling. This algorithm is often applied to such projects as facility construction, overhaul maintenance on equipment, research and development programs for new products, and marketing new products. PERT, Program Evaluation and Review Technique, and its many variations are critical path algorithms.

Theoretic Game Solutions. An important model for testing plans, policies, and strategies when there are many uncertainties has been provided by management science -- the theoretic game model. The goal in any gaming model is to determine the strategies available to the players and the game value for a particular game. The limits of gaming algorithms are such that algebraic and graphic techniques are not adequate if the game matrix cannot be reduced to two, in either the row or the column dimension. If this is the case, the linear programming Simplex algorithm is used to solve for the strategies and the game value, after suitable transformations have been made to the game data.

Dynamic Linear Programming. Although the classical linear programming is static, with all elements assigned fixed values for the planning horizon, the linear program has been adapted to optimize objective functions over a time horizon, with special activity in supply, inventorying, logistics, and production smoothing problems. This extension of the basic LP enables a decisionmaker to expand and broaden the analysis in those situations where the value of the objective function varies over time, depending on the values of the variables in the different time periods. A frequent application is smoothing production schedules by deciding when to produce for immediate use and when to produce for inventory, based on the variations in production costs and inventory holding costs over the planning horizon. Wagner offers an extended discussion of dynamic optimization models. (1972)

The Dual Solution. A major value of linear programming cost-benefit analysis is to derive some measures of sensitivity of the value of the output to changes in the inputs. The dual solution of a linear optimization provides us with the marginal incremental value, "shadow values," of each variable in the objective function, the constraints, or the technology coefficients. These are the marginal values cherished by economists everywhere.

LP Summary. These examples of extensions and specializations of the basic LP do not exhaust the many specialized applications one can observe in the use of this cost-benefit methodology. However, they do suggest the major outlines of many that are used straightforwardly. More and more special adaptations are being formulated all the time, especially suited to an organization's needs, especially responsive to a decisionmaker involved in making cost-benefit decisions.

Nonlinear Mathematical Programming. No discussion of linear programming would be complete without a brief synopsis of representative nonlinear formulations. Perhaps the most frequent solution for a nonlinear set is a creative transformation that modifies the nonlinear elements of the problem into equivalent linear forms, which allows the use of the Simplex algorithm. For example, an objective function that has exponential terms can be transformed into linear logarithmic forms, and the optimization is computed on the logarithms. The relevant values for the optimal solution are obtained by taking the appropriate antilogarithms.

Another nonlinear form is the integer linear algorithm. The divisibility assumption of linear optimizations may lead to fractional values for the x_j activities in the optimal solution. Often the decisionmaker cannot implement non-integer values, e.g., three-tenths of a machine or one-half a worker. There is no unambiguous heuristic that allows us to "round off" the optimum non-integer values, for the optimal integer solution will often be quite different. Two principal techniques have been developed to extend the optimal linear solution to provide integer values for x_j: the Gomory Cutting-Plane algorithm and the Branch and Bound technique. The essence of the techniques is to incrementally reduce the size of the feasible constraint set until the "most optimum" integer values are identified. Wagner gives a detailed discussion of these two techniques (Wagner, 1970).

For those problems that are not linear, cannot be transformed into linear equivalents, or cannot be approximated by linear forms, we resort to differential calculus and quadratic programming. Nonlinear forms are of three general types: nonlinear objective function and linear constraints; linear objective function but nonlinear constraints; or both the objective function and the constraints are nonlinear.

A problem having a nonlinear (concave) objective function and linear constraints takes the form illustrated in Exhibit B.4, opposite page. The nonlinear optimum solution is obtained by equating the first derivatives of the objective function and the relevant portion of the constraint set and solving for the pertinent values, x_j^* and Z^*. Polygonal linear approximations are often sufficiently accurate to permit using the Simplex linear programming algorithm directly.

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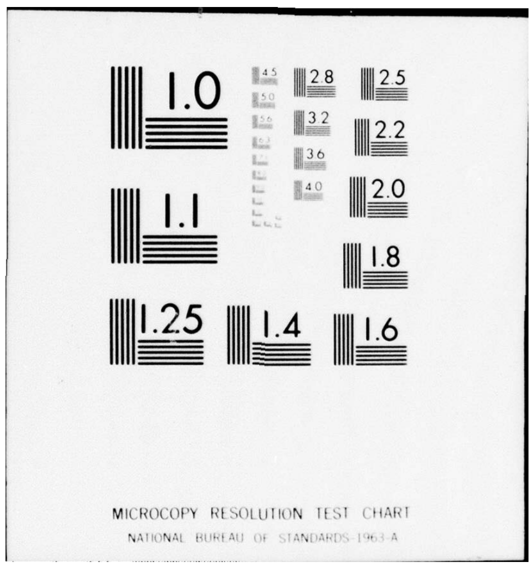
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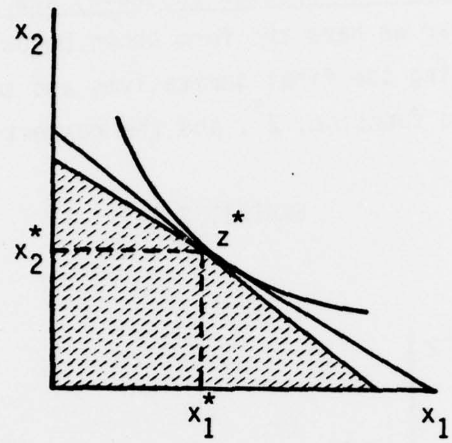
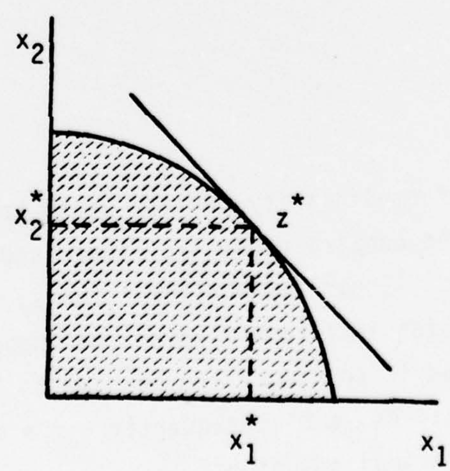
EXHIBIT B.4



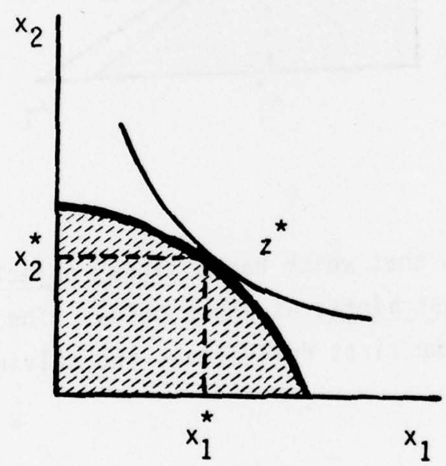
A second form is that which has a <u>linear objective function but nonlinear (convex) constraints</u>, as shown below. The optimum solution is obtained by equating the first derivatives and solving for the relevant values.

EXHIBIT B.5



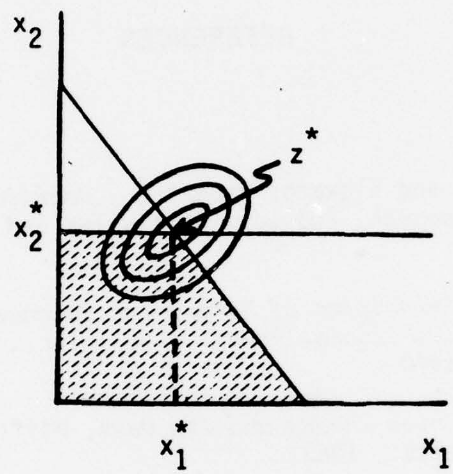
If both the objective function (concave) and the constraint set (convex) are nonlinear we have the form shown below. The optimal solution is obtained by equating the first derivatives and solving for the value of the optimal objective function, Z^*, and the respective x_j^* activities.

EXHIBIT B.6



A special form of nonlinearity permits us to apply a quadratic programming algorithm: the constraint set is linear and the objective function is a combination of linear and nonlinear terms. Such a form for the concave objective function is illustrated on the opposite page. The solution most often used is to reduce the problem to linear equivalents by using partial derivatives, and subsequently use a modified Simplex algorithm to determine the optimal solution.

EXHIBIT B.7



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APPENDIX C

THE INFLUENCE OF LOAD FACTOR ON MARKET EQUILIBRIUM AND THE VALUE OF DELAY TIME

THE INFLUENCE OF LOAD FACTOR ON MARKET EQUILIBRIUM AND THE VALUE OF DELAY TIME

R.A. Groemping

Load Factor as Determinant of Market Equilibrium

Implicit in the cost-demand equilibrium in Section 4 is the concept of load factor. Load factor is a measure of the proportion of paying passengers to total available seats. As the load factor increases, there are more paying passengers among whom the fixed costs of a flight may be distributed. In a hypothetical market suggested by Douglas and Miller (1974) with 800 daily passengers being transported over a distance of 600 miles, it was estimated that the average cost per passenger would be 27 percent higher at an average load factor of 50 percent, typical of the industry in 1970, than at a 75 percent load factor. Exhibit C.1 shows the inverse relationship between average cost, which is the fare which must be charged to the paying passengers to cover the expenses of the flight, and load factor.

The relationship between load factor and average cost implies that there are a series of average cost curves dependent on the load factor. Exhibit C.2 presents average cost curves based on 40, 50, 60, 70, 80, and 90 percent load factors. The demand curve D has been drawn to demonstrate the effects of a constant elasticity of -1.05. Each of the average cost curves implies a distinct market equilibrium dependent on load factor.

Exhibit C.3 shows the relationship between enplanements and load factor. As the load factor increases, the fare decreases, and more trips by air become affordable. Although a lower fare implies more air travelers, a higher load factor allows a reduced number of flights to transport all those wishing to fly. This relationship is shown in Exhibit C.4.

EXHIBIT C.1

DIRECT OPERATING COSTS PER PASSENGER
AS A FUNCTION OF AVERAGE LOAD FACTOR

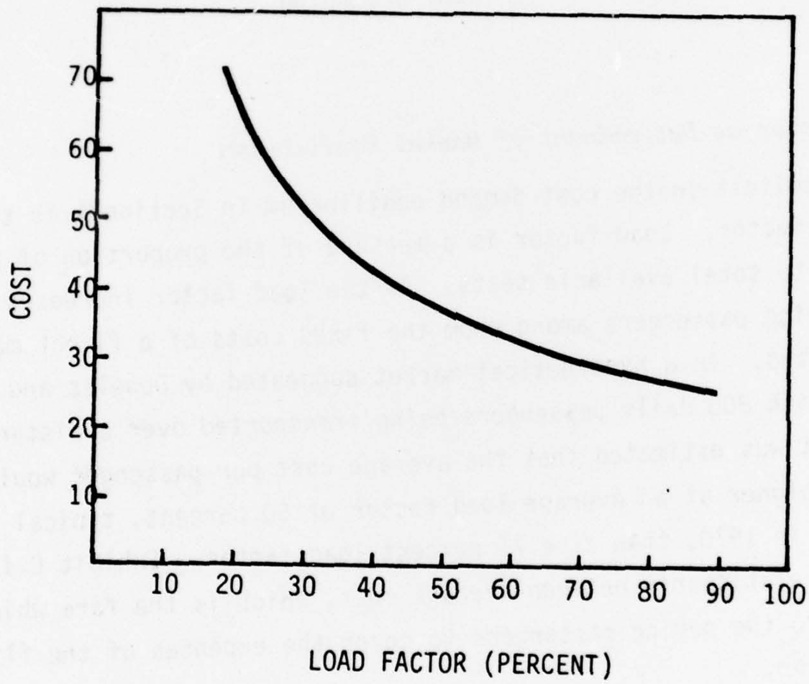


EXHIBIT C.2

LOAD FACTOR AS DETERMINANT OF
EQUILIBRIUM FARE AND ENPLANEMENTS

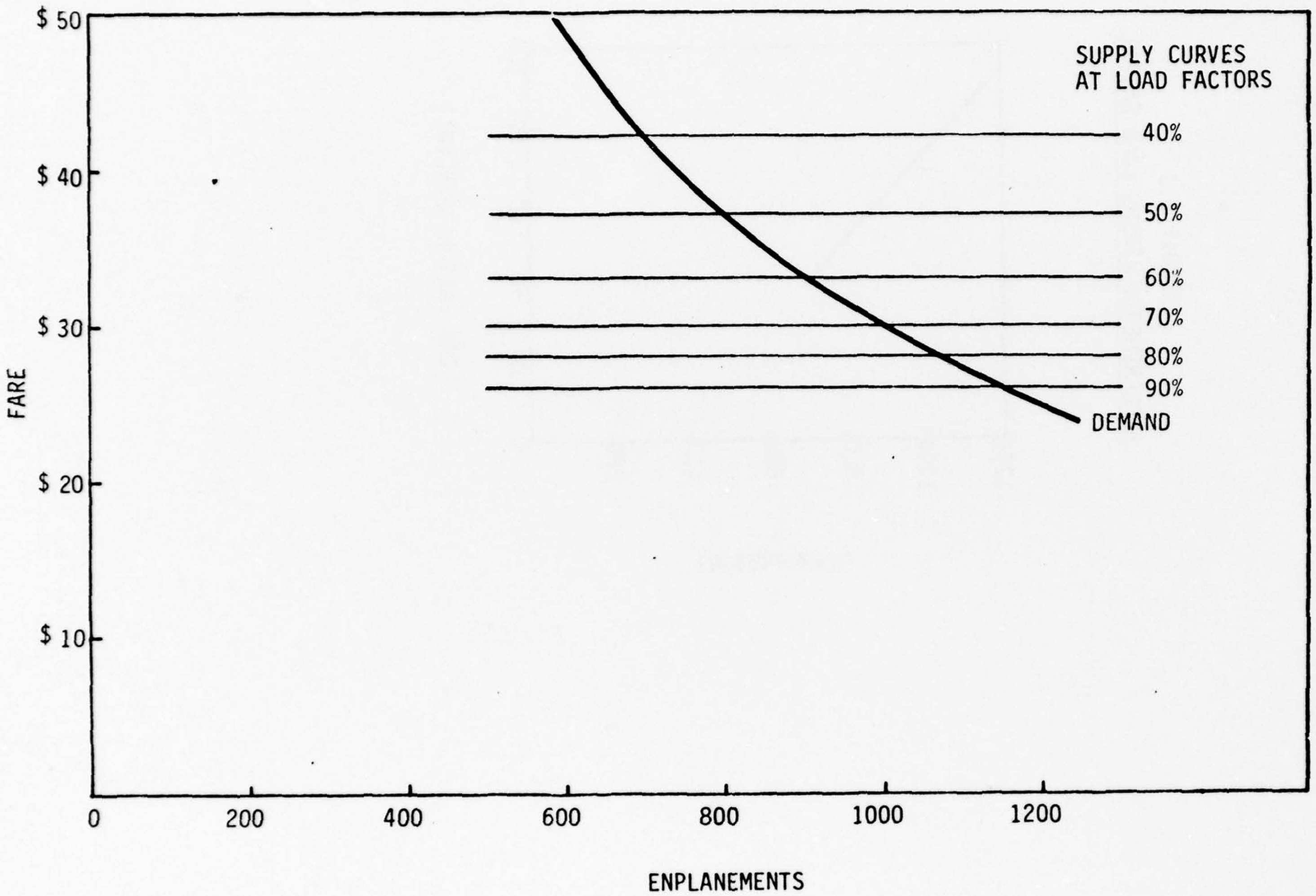


EXHIBIT C.3

ENPLANEMENTS VERSUS LOAD FACTOR

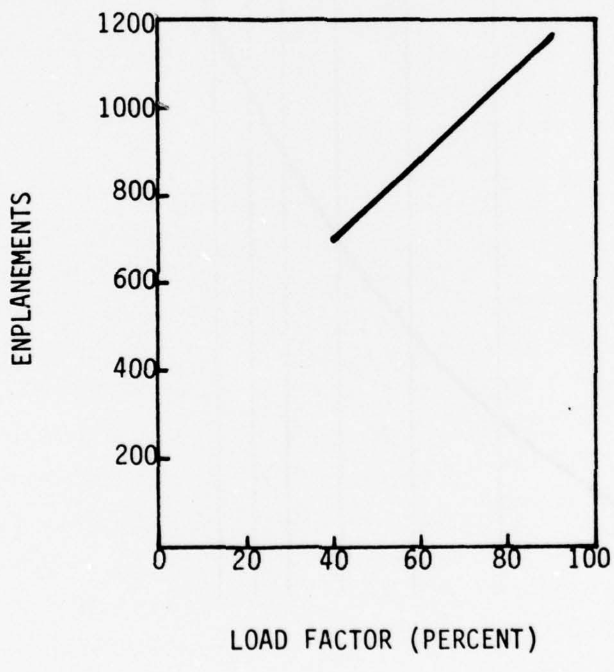


EXHIBIT C.4

NUMBER OF FLIGHTS AS
A FUNCTION OF LOAD FACTOR

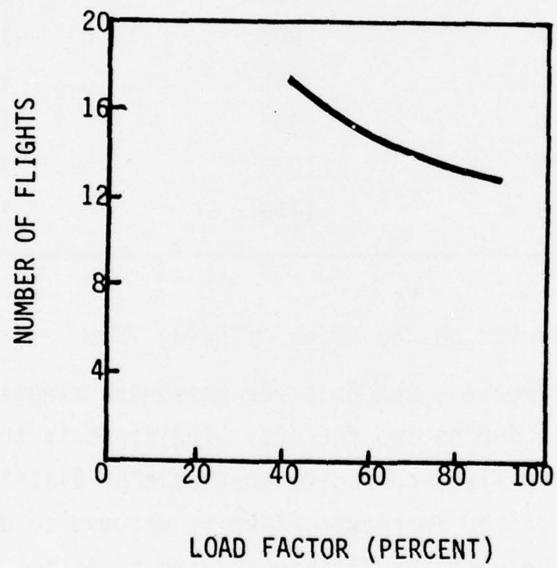


Table C.1 summarizes the data used to construct the example shown in Exhibits C.1 through C.4. The initial equilibrium point was assumed to occur with a load factor of 50 percent, an average cost of $37, and 800 enplanements per day.

TABLE C.1. THE EFFECT OF LOAD FACTOR ON FARE, ENPLANEMENTS, AND NUMBER OF FLIGHTS

Load Factor	Average Cost (Fare)	Enplanements	Number of Flights (100 Seat Aircraft)
40%	$ 42	700	17.5
50	37	800	16.0
60	33	902	15.0
70	30	997	14.2
80	28	1072	13.4
90	26	1159	12.9

The Influence of Load Factor on the Value of Delay Time

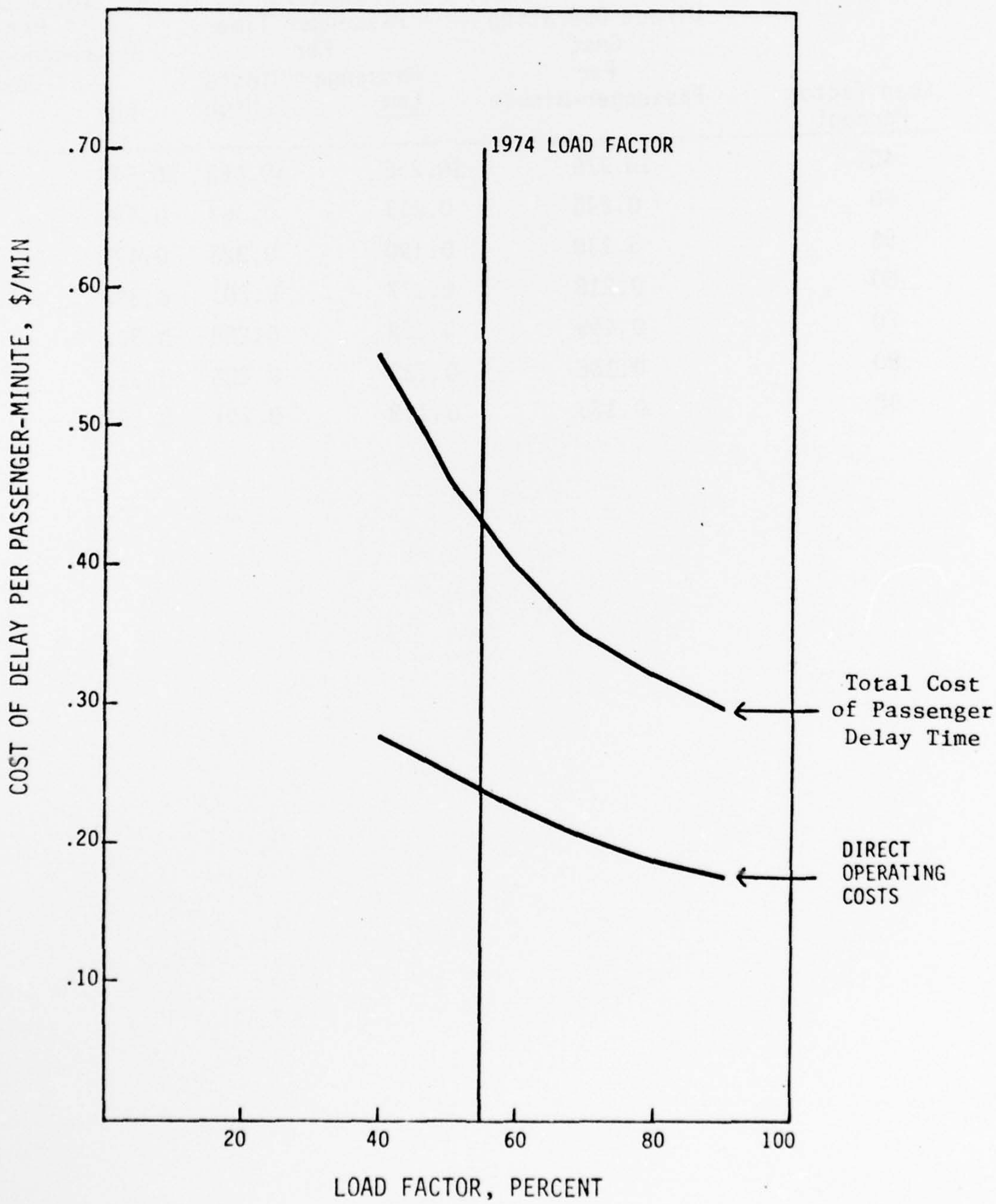
As load factor increases, the cost per passenger minute of delay time decreases. This is due to two factors. The first is that there are more passengers over whom fixed operating costs can be distributed. The second is that as load factor increases, fare is assumed to decrease, and passengers who would otherwise not fly are induced to do so. These passengers are likely to value their time at a rate lower than those who had been willing to pay more to fly at lower load factors. These incremental passengers would then be likely to lower the average value of passenger time. If such a lowering of the value of passenger time follows the cost-quantity relationship expressed by the demand curve, then it can be readily calculated. Table C.2 shows the dollar per passenger-minute figures should load factor change appreciably from the approximately 56 percent load factor of 1974, on which the passenger-minute estimates are based. Exhibit C.5 shows cost per passenger-minute as a function of load factor.

TABLE C.2. COST PER PASSENGER-MINUTE OF DELAY AS A FUNCTION OF LOAD FACTOR

Load Factor Percent	Direct Operating Cost Per Passenger-Minute	Value of Passenger Time Per Passenger-Minute Low	Value of Passenger Time Per Passenger-Minute High	Total Cost Per Passenger-Minute of Delay Low	Total Cost Per Passenger-Minute of Delay High
40%	$0.279	$0.266	$0.452	$0.545	$0.731
50	0.246	0.213	0.362	0.459	0.608
56	0.230	0.190	0.323	0.420	0.553
60	0.219	0.177	0.301	0.396	0.520
70	0.199	0.152	0.258	0.351	0.457
80	0.186	0.133	0.226	0.319	0.412
90	0.173	0.118	0.201	0.291	0.374

EXHIBIT C.5

COST PER PASSENGER-MINUTE OF DELAY TIME
AS A FUNCTION OF LOAD FACTOR



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GLOSSARY

a fortiori analysis

Study based on assumptions contrived to handicap the preferred alternative.

Capital recovery criterion

An investment decision criterion that compares equivalent uniform annual series of outlays/receipts using a stipulated discount rate.

Contingency analysis

Examination of the ranking of alternatives when a relevant change in criteria for evaluating the alternatives is postulated, or when a major change in the general environment is assumed.

Cost-benefit analysis

A formal procedure for comparing costs and benefits of alternative investment actions.

Constant dollars

Dollars adjusted for inflation and specified as being of a particular year (e.g., an historical outlay stream from 1964 through 1974 measured in 1974 dollar prices).

Consumers' Surplus

The monetary value of the difference between the total revenue of a market, and the greater amount of revenue that could be generated if it were possible to charge each buyer the maximum price he is willing to pay.

Continuous compounding

The process by which the present value of a specified sum of money, flowing uniformly through time, is determined. In other words, an infinite or continuous number of compoundings is assumed.

Current dollars

Dollar measures reflecting prevailing-year price levels (e.g., an historical outlay stream from 1964 through 1974 measured in price levels existing in each of those years).

Discount rate

The rate at which future costs and benefits are adjusted to reflect the value of capital over time.

Discrete compounding

The process by which the present value of a specific sum of money, flowing at the end of a specified period of time, is determined.

Force of interest

The nominal rate of interest used in continuous compounding.

Intangible benefits

Those benefits which cannot be valued in monetary terms.

Internal rate of return criterion

An investment decision criterion which seeks the rate of return that equates the present value of alternative net monetary streams to zero.

Learning curve

The relationship between the increasingly more efficient production process and the cumulative quantity of production.

Life cycle cost

The total costs of a system over its lifetime, including research and development, investment, and operating and support costs.

Load factor

For commercial carriers, the proportion of seats paid for to total available revenue-generating seats.

Opportunity cost

The economic cost of alternatives foregone; i.e., the worth of a good or service in some alternative use.

Present value criterion

An investment decision criterion that compares the summation of streams of costs and benefits adjusted by a stipulated discount rate.

Residual value

The usefulness or effectiveness of assets still on hand beyond the selected time horizon of a particular analysis or study.

Risk

A condition ascribed to any outcome which is subject to an uncontrollable random event occurring with a known probability distribution.

Sensitivity analysis

A type of analysis in which key parameters about which the analyst is very uncertain are tested with several values, e.g., high, medium, and low, to determine sensitivity of results.

Tangible benefits

Those benefits which can be valued in monetary terms.

GLOSSARY (CONT'D)

Uncertainty

A condition ascribed to any outcome which is subject to an uncontrollable event occurring with an unknown probability distribution.

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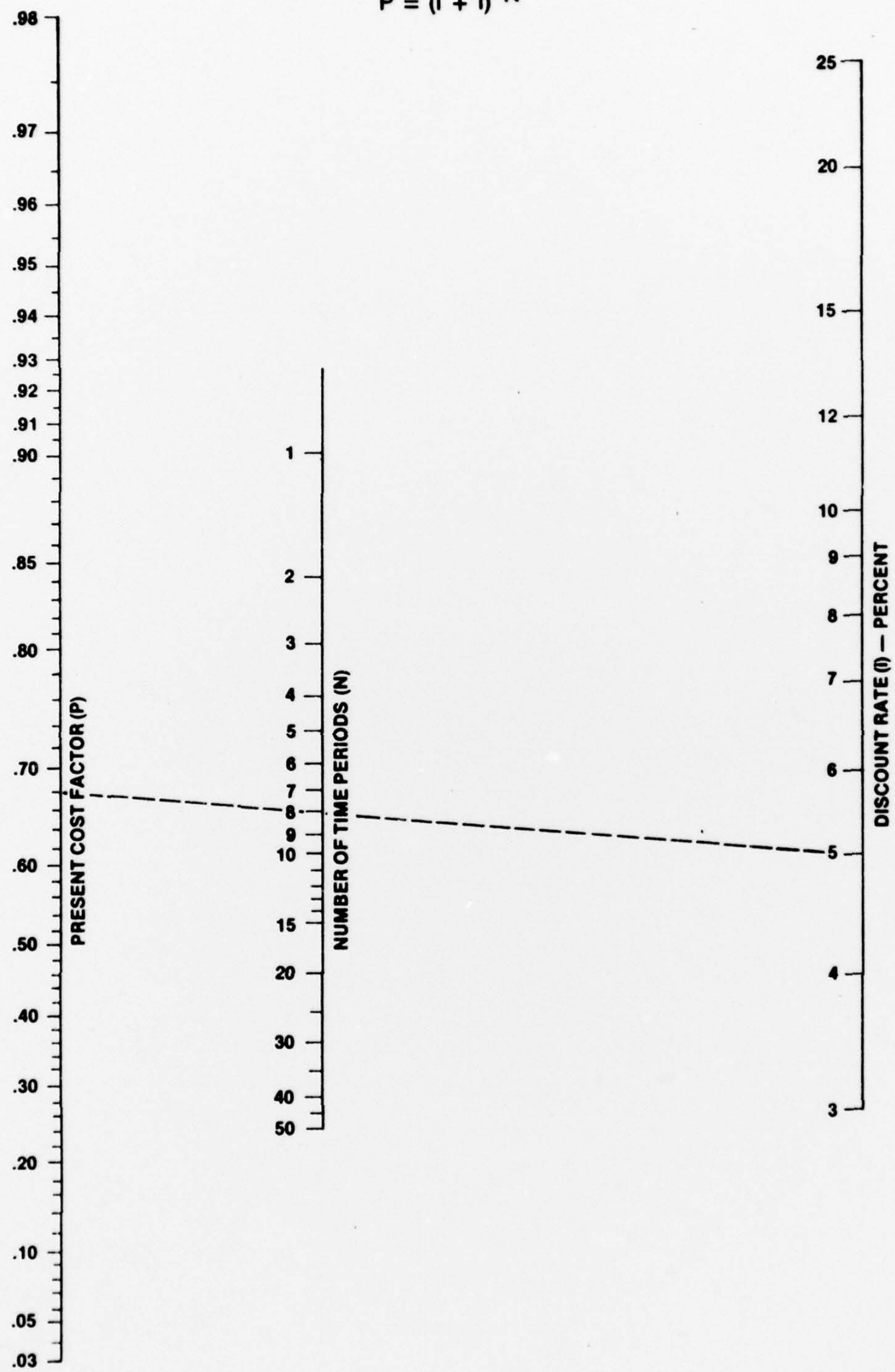
$$P = (1 + I)^{-N}$$



DISCRETE COMPOUNDING