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STATE-SPACE SOLUTION OF TRANSIENT ELECTROMAGNETIC PROBLEMS.(U)
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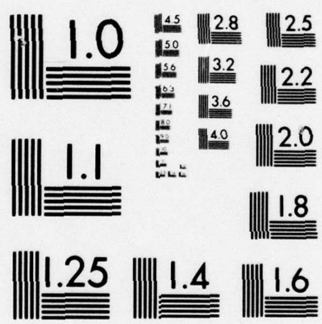
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STATE-SPACE SOLUTION OF TRANSIENT ELECTROMAGNETIC PROBLEMS

Final Technical Report

Grant Nos. DAHC04-75-G-0001

and DAHC04-76-G-0023

United States Army Research Office

Research Triangle Park, NC 27709

by

David K. Cheng
Principal Investigator

(Approved for public release; distribution unlimited)

31 January 1977

Electrical and Computer Engineering Department
Syracuse University
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ABSTRACT

The transient excitation of a cavity through an aperture is used to illustrate the state-space solution of transient electromagnetic problems. Depending on whether the E- or the H-formulation is used, the effect of the aperture can be accounted for by a magnetic current in invoking the induction theorem or by an electric current in invoking the equivalence theorem. The governing second-order differential equation is converted into a set of first-order state equations by defining three new state variables in addition to an appropriate vector potential. These state equations are solved by the method of moments. Two cases are considered: parallel polarization and perpendicular polarization. Some significant singularities for the parallel-polarization case are found and the electric intensities as functions of time at two locations in the cavity are computed for a step excitation.

The Findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

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I. INTRODUCTION

Transient electromagnetic problems arise in the consideration of shielding effectiveness, electromagnetic compatibility, radar target identification, remote environmental sensing, and electromagnetic effects due to X rays and gamma rays generated by a nuclear blast. The time-harmonic behavior of an antenna or a scatterer can be obtained first as a function of frequency by solving the governing integral equations with the given boundary conditions. The transient response can then be determined by performing an inverse Fourier transformation. Numerical methods are used in the frequency-domain solution for many different frequencies. The brute-force numerical Fourier inversion is generally inefficient and convergence problems arise in superposing the steady-state solutions. A few simple situations, such as transient scattering from wire antennas and conducting cylinders [1]-[3], have been analyzed directly in the time domain. Basis-function expansions and inner products over both space and time are required. Space-time integro-differential equations are encountered and the numerical representations of the derivatives lead to very complicated procedures.

A variation of the time-domain approach makes use of a a Hallén-type integral-equation formulation [4]. As a consequence of the absence of space-time derivatives under the integrals, the numerical process is less complicated. However, because of retarded time, a tedious step-by-step iterative procedure has to be used in conjunction with the proper families of trajectories known as characteristic curves in order to determine the homogeneous solutions. All of the methods mentioned above are tedious and do not afford a physical insight in the solution of a transient problem. Moreover, a complete recalculation would be necessary under any change in the wave shape, polarization, or angle of incidence of the source of excitation.

More recently a singularity-expansion method has been used to determine the transient response of scatterers of simple geometries [5]-[11]. Responses to transient excitations are expressed in terms of exterior natural frequencies, modes, and coupling coefficients, and induced currents are represented by a series of damped sinusoidal functions. This method has the advantages of providing a physical sight to the radiation or scattering problem and of allowing the response to be determined for a change in source parameters without a complete recalculation. In the evaluation of the natural-mode and coupling vectors, it is necessary to know the nature of the singularities involved in the inverse Laplace transform. Knowledge in this respect is not yet secure. The transcendental nature of its system impedance matrix results in an infinite number of complex poles whose locations must be numerically searched. It is generally agreed that the singularity-expansion method is not very satisfactory for evaluating early-time responses.

The method of characteristic modes has been used for determining the steady-state response of conducting bodies [12]. A striking similarity appears to exist between this method and the singularity-expansion formulation if the method of moments [13] using a common spatial basis is applied to both cases; but no formal relations have yet been established. The natural modes from the singularity expansion are not orthogonal. The characteristic modes are orthogonal, but they vary with the source frequency and it would be necessary to compute the characteristic modes for all frequencies before a Fourier inversion could be effected to determine the transient behavior.

The problem of transient field behavior inside a conducting cavity due to excitation through an aperture by an incident electromagnetic pulse (EMP) is particularly difficult because of the reflections from the cavity walls and the coupling between the interior and exterior fields at the aperture. Past investigations on EMP excitation of cavity-backed apertures have largely dealt with small openings and have neglected the effect of cavity reflections on the aperture field distribution. For small openings the quasi-static method is used to determine the fictitious magnetic current and charge distributions in the aperture. Equivalent electric and magnetic dipoles are defined, and their radiated fields determined with the aid of scalar and vector potentials [14]-[19]. The fields in the cavity are customarily expanded in terms of unperturbed normal modes. The quasi-static approximation cannot be applied when the aperture is not small and when early-time responses are important. In neglecting the effect of the reflections from cavity walls on the aperture field distribution, one essentially treats the external and internal portions of the problem separately. Since cavity dimensions obviously play an important part in the total problem, this approach may result in significant errors.

In this report we will avoid the quasi-static approximation and solve the internal and external portions of the problem simultaneously. New variables (state variables) will be introduced to convert the governing second-order differential equation into a set of first-order equations which correspond to normalized state equations. The field within the cavity will be expanded in terms of suitably chosen subsectional expansion functions with variable coefficients and the field outside the cavity expressed as a superposition of plane-wave fields. The cavity and the external fields are matched at the aperture where a fictitious equivalent current exists. A combined field expression containing the unknown expansion coefficients is obtained. To determine these coefficients the method of moments [13] is used to convert the first-order equations into matrix equations. It will be shown that the typical coefficient matrix can be expressed in a form for which the singularity-expansion method [5] can be used to advantage.

The general procedure of solution for cavity-backed aperture problems is outlined first. The theoretical formulation for the transient excitation of a rectangular cavity with a slot aperture is then given for both parallel and perpendicular polarizations. Some numerical results are included.

II. SOLUTION PROCEDURE

We consider the problem of a slot aperture in an infinite conducting plane backed by a rectangular conducting box, as shown in Fig. 1. An incident transient electromagnetic wave $(\bar{E}^i, \bar{H}^i)$ impinges normally on the plane and the aperture. The problem is to determine the scattered field in the $y > 0$ region and the field penetrated through the aperture into the conducting cavity.

The induction theorem [20], [21] can be invoked for the solution of this problem. Figure 2(a) represents a simplified 2-dimensional view of the original problem. $(\bar{E}_c, \bar{H}_c)$ and $(\bar{E}_s, \bar{H}_s)$ are, respectively, the cavity field and the external scattered field. In order to determine these unknown fields, we consider the case when the aperture is covered by a conductor. The entire region to the left of the infinite plane will have a null field and, according to the induction theorem, a magnetic current $\bar{M}_o$ on the right surface of the conducting plane will support a different scattered field $(\bar{E}_s^o, \bar{H}_s^o)$, as shown in Fig. 2(b), where

$$\begin{aligned} \bar{M}_o &= \bar{E}_s^o \times \hat{n} \\ &= \hat{n} \times \bar{E}^i \\ &= \hat{y} \times \bar{E}^i \ . \end{aligned} \qquad (1)$$

For a normally incident plane wave $(\bar{E}^i, \bar{H}^i)$, the scattered field $(\bar{E}_s^o, \bar{H}_s^o)$ from an infinite conducting plane without an aperture is easily determined. The null field to the left of the plane will be maintained if the plane is removed and a magnetic current $2\bar{M}_o$ exists in its place which will result in a field $(\bar{E}^i + \bar{E}_s^o, \bar{H}^i + \bar{H}_s^o)$ in the $y > 0$ region, as shown in Fig. 2(c).

Subtracting the fields in Fig. 2(c) from those in Fig. 2(a), we obtain the problem in Fig. 2(d). The magnetic current $\bar{M}$ in the aperture is

$$\bar{M} = -2\bar{M}_o = -2\hat{y} \times \bar{E}^i , \tag{2}$$

which supports the field ($\bar{E}_c$, $\bar{H}_c$) inside the cavity and a field ($\bar{E}_s - \bar{E}^o_s$, $\bar{H}_s - \bar{H}^o_s$) to the right of the infinite plane. We note that the region in which the difference field ($\bar{E}_s - \bar{E}^o_s$, $\bar{H}_s - \bar{H}^o_s$) exists is source-free and that the tangential component of the electric field is required to vanish on conducting walls.

For the problem in Fig. 2(d), we start from the two Maxwell's curl equations

$$\bar{\nabla} \times \bar{E} = -\mu_o \frac{\partial \bar{H}}{\partial t} - \bar{M} \tag{3}$$

$$\bar{\nabla} \times \bar{H} = \varepsilon_o \frac{\partial \bar{E}}{\partial t} . \tag{4}$$

Taking the Laplace transform of Eqs. (3) and (4), we obtain

$$\bar{\nabla} \times \tilde{\bar{E}} = -\mu_o s \tilde{\bar{H}} - \tilde{\bar{M}} \tag{5}$$

$$\bar{\nabla} \times \tilde{\bar{H}} = \varepsilon_o s \tilde{\bar{E}} , \tag{6}$$

where a tilde (~) over a quantity denotes the Laplace transform of that quantity.

Let $\tilde{\bar{F}}$ be the Laplace transform of an electric vector potential $\bar{F}$ such that

$$\tilde{\bar{E}} = -\bar{\nabla} \times \tilde{\bar{F}} . \tag{7}$$

Combining Eqs. (5) to (7) and using the Lorentz gauge, we have an inhomogeneous Helmholtz equation:

$$\bar{\nabla}^2\tilde{\tilde{F}} = \mu_o \varepsilon_o s^2 \tilde{\tilde{F}} = -\tilde{\tilde{M}} . \tag{8}$$

Solution of Eq. (8) for $\tilde{\tilde{F}}$ will give $\tilde{\tilde{E}}$ from Eq. (7) and $\tilde{\tilde{H}}$ from

$$\tilde{\tilde{H}} = \frac{1}{\mu_o s} \bar{\nabla} \times \bar{\nabla} \times \tilde{\tilde{F}} \tag{9}$$

in regions where $\bar{M}$ is zero.

Specialization of these general formulas will depend on the polarization of the incident wave; but as soon as $\bar{E}^i$ is known, Eqs. (7) and (9) can be expanded into component equations and the source term $\tilde{\tilde{M}}$ in Eq. (8) can be found from Eq. (2). A set of new variables can then be defined which will convert the second-order differential equation (8) into a set of first-order equations, and these equations are Laplace-transformed normalized state equations in the new state variables. Solution of the transformed state equations involves four steps. First, the space inside the cavity is divided into subsections and suitable expansion functions are chosen over the subsections. The elements of the unknown state-variable vector are then expressed in terms of the expansion functions within the cavity. Second, the field in the $y > 0$ region is expressed as a superposition of plane waves. Third, the cavity and the half-space fields are matched at the aperture. Fourth, inner products are taken so that the matrix equations for the unknown expansion coefficients are obtained. These steps are outlined separately for the cases of parallel and perpendicular excitations in following sections.

III. RECTANGULAR CAVITY WITH SLOT APERTURE - PARALLEL POLARIZATION

In this section we consider the case of an incident plane wave with the electric field polarized in a direction parallel to the slot. Referring to Fig. 1, we have

$$\bar{E}^i = \hat{z}\, E_z^i \tag{10}$$

and the Laplace transform of Eq. (2) becomes

$$\tilde{\bar{M}} = -\, 2\hat{x}\, \tilde{E}_z^i = \hat{x}\, \tilde{M}_x \tag{11}$$

which has only an x-component. The x-component of Eq. (8) is then

$$\bar{\nabla}^2 \tilde{F}_x - \mu_o \varepsilon_o s^2\, \tilde{F}_x = -\, \tilde{M}_x \delta(y), \tag{12}$$

where $\delta(y)$ is a Dirac delta function. From Eqs. (7) and (9), we have

$$\tilde{E}_x = 0 \tag{13}$$

$$\tilde{E}_y = -\, \frac{\partial}{\partial z}\, \tilde{F}_x \tag{14}$$

$$\tilde{E}_z = \frac{\partial}{\partial y}\, \tilde{F}_x \tag{15}$$

$$\tilde{H}_x = -\, \frac{1}{\mu_o s}\, [\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}]\tilde{F}_x \tag{16}$$

$$\tilde{H}_y = \frac{1}{\mu_o s}\, \frac{\partial^2}{\partial x \partial y}\, \tilde{F}_x \tag{17}$$

$$\tilde{H}_z = \frac{1}{\mu_o s}\, \frac{\partial^2}{\partial x \partial z}\, \tilde{F}_x\ . \tag{18}$$

The second-order differential equation (12) can be represented as a set of first-order equations by defining new quantities u, v, and w such that

$$- \frac{\partial}{\partial x} \tilde{F}_x(r,s) = s\ \tilde{u}(r,s) \tag{19}$$

$$- \frac{\partial}{\partial y} \tilde{F}_x(r,s) = s\ \tilde{v}(r,s) \tag{20}$$

and

$$- \frac{\partial}{\partial z} \tilde{F}_x(r,s) = s\ \tilde{w}(r,s)\ , \tag{21}$$

where r is the space variable. We have, from Eq. (12),

$$\frac{\partial}{\partial x} \tilde{u}(r,s) + \frac{\partial}{\partial y} \tilde{v}(r,s) + \frac{\partial}{\partial z} \tilde{w}(r,s) = - \mu_o \varepsilon_o s\ \tilde{F}_x(r,s) + \frac{1}{s} \tilde{M}_x\ . \tag{22}$$

Comparing Eqs. (21) and (20) with Eqs. (14) and (13) respectively, we see that

$$\tilde{E}_y = s\ \tilde{w} \tag{23}$$

and

$$\tilde{E}_z = - s\ \tilde{v}\ . \tag{24}$$

The introduction of $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ and the use of the first-order equations will result in significantly faster convergence in the numerical solution.

The first-order equations (19) to (22) can be written in a succinct form by defining the following operators and column matrices:

$$L = \begin{bmatrix} 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{\partial}{\partial x} & 0 & 0 & 0 \\ -\frac{\partial}{\partial y} & 0 & 0 & 0 \\ -\frac{\partial}{\partial z} & 0 & 0 & 0 \end{bmatrix} \tag{25}$$

$$P = \begin{bmatrix} -\mu_o\varepsilon_o & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{26}$$

$$\tilde{f}(r,s) = \begin{bmatrix} \tilde{F}_x \\ \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} \tag{27}$$

and

$$\tilde{e}_g(s) = \begin{bmatrix} \frac{1}{s}\tilde{M}_x \\ 0 \\ 0 \\ 0 \end{bmatrix} . \tag{28}$$

Equations (19) to (22) become

$$L\,\tilde{f}(r,s) = s\,P\,\tilde{f}(r,s) + \tilde{e}_g(r,s) . \tag{29}$$

Note that the inverse Laplace transform of Eq. (29) is a set of normalized state equations in the four state variables F_x, u, v, and w.

In order to solve Eq. (29) by the method of moments we subdivide the space within the cavity in the x,y, and z directions and choose expansion functions $F_{x(i,j,k)}(r)$, $u_{(i,j,k)}(r)$, $v_{(i,j,k)}(r)$, and $w_{(i,j,k)}(r)$ over the subsections. The expansion functions must satisfy the required boundary conditions. For convenience, we define the following column vectors:

$$f^F_{(i,j,k)}(r) = \begin{bmatrix} F_{x(i,j,k)}(r) \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad f^u_{(i,j,k)}(r) = \begin{bmatrix} 0 \\ u_{(i,j,k)}(r) \\ 0 \\ 0 \end{bmatrix} \tag{30}$$

$$f^v_{(i,j,k)}(r) = \begin{bmatrix} 0 \\ 0 \\ v_{(i,j,k)}(r) \\ 0 \end{bmatrix} \quad f^w_{(i,j,k)}(r) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_{(i,j,k)}(r) \end{bmatrix} .$$

In view of Eq. (27), we can then write the expanded form of $\tilde{f}(r,s)$ inside the cavity as

$$\tilde{f}(r,s) = \sum_{i,j,k} \{\tilde{\alpha}_{(i,j,k)}(s)\, f^F_{(i,j,k)}(r) + \tilde{\beta}_{(i,j,k)}(s)\, f^u_{(i,j,k)}(r) + \tilde{\gamma}_{(i,j,k)}(s)\, f^v_{(i,j,k)}(r) + \tilde{\delta}_{(i,j,k)}(s)\, f^w_{(i,j,k)}(r)\}. \tag{31}$$

Note that the expansion functions F_x, u, v, and w are functions of position only and that the inverse transformation of $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\delta}$ will yield the time-varying expansion coefficients.

The field in the half-space $y \geq 0$ is expressed as a superposition of plane waves and the internal and external fields are matched at the aperture $y = 0$. A combined field expression for $\tilde{f}(r,s)$ can be obtained which holds inside the cavity, in the $y > 0$ half-space, as well as in the slot. This is then substituted in Eq. (31) which, after inner products with the expansion functions in Eq. (30) have been taken, leads to the following matrix equation:

$$\begin{bmatrix} 0 & \ell^{Fu}_{mn} & \ell^{Fv}_{mn} & \ell^{Fw}_{mn} \\ \ell^{uF}_{mn} & 0 & \ell^{uv}_{mn} & 0 \\ \ell^{vF}_{mn} & \ell^{vu}_{mn} & \ell^{vv}_{mn} & \ell^{vw}_{mn} \\ \ell^{wf}_{mn} & 0 & \ell^{wv}_{mn} & 0 \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\beta} \\ \tilde{\gamma} \\ \tilde{\delta} \end{bmatrix} = s \begin{bmatrix} p^{FF}_{mn} & 0 & p^{Fv}_{mn} & 0 \\ 0 & p^{uu}_{mn} & p^{uv}_{mn} & 0 \\ p^{vF}_{mn} & p^{vu}_{mn} & p^{vv}_{mn} & p^{vw}_{mn} \\ 0 & 0 & p^{wv}_{mn} & p^{ww}_{mn} \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\beta} \\ \tilde{\gamma} \\ \tilde{\delta} \end{bmatrix}$$

$$+ \begin{bmatrix} q^{F} \\ q^{u} \\ q^{v} \\ q^{w} \end{bmatrix} \{\frac{1}{s}\, \tilde{m}_{x(i',n_y,k')}(s)\} + \begin{bmatrix} 0 \\ 0 \\ \tilde{C}(s) \\ 0 \end{bmatrix} \qquad (32)$$

where $\tilde{m}_{x(i',n_y,k')}$ are expansion coefficients for the magnetic current $\tilde{M}_x$, $\tilde{C}(s)$ is a column matrix and ℓ_{mn}'s, p_{mn}'s and q's are themselves matrices arising from the inner products. m and n are indices locating the position of a particular subsection over which an inner product is taken. The expressions for $\tilde{C}(s)$, ℓ_{mn}'s, p_{mn}'s and q's are very complicated. They have been given in a previous report [22], and will not be repeated here.

The unknown coefficient matrices $\{\tilde{\alpha}(s)\}$, $\{\tilde{\beta}(s)\}$, $\{\tilde{\gamma}(s)\}$ and $\{\tilde{\delta}(s)\}$ can be solved from Eq. (32). An outline of the method of solution will be given in Section V. Determination of these coefficient matrices enables the calculation of $\tilde{f}(r,s)$ in Eq. (27) from Eq. (31), which, in turn, leads to the field inside the cavity.

IV. RECTANGULAR CAVITY WITH SLOT APERTURE – PERPENDICULAR POLARIZATION

When the electric field of the incident plane wave is polarized in a direction normal to the slot shown in Fig. 1; that is, if

$$\bar{H}^i = \hat{z}\, H_z^i \,, \tag{33}$$

it is more convenient to use an H-formulation. Instead of invoking the induction theorem, we apply the equivalence theorem [20], [21]. The H-field discontinuity at the aperture in Fig. 2(d) is supported by an electric current

$$\begin{aligned} \bar{J} &= -\,2\hat{n} \times \bar{H}^i \\ &= -\,2\hat{x}\, H_z^i = \hat{x}\, J_x \,. \end{aligned} \tag{34}$$

To solve this problem, we start with the following Maxwell's equations

$$\bar{\nabla} \times \bar{E} = -\,\mu_o \frac{\partial \bar{H}}{\partial t} \tag{35}$$

$$\bar{\nabla} \times \bar{H} = \varepsilon_o \frac{\partial \bar{E}}{\partial t} + \bar{J} \,, \tag{36}$$

which transform to

$$\bar{\nabla} \times \tilde{\bar{E}} = -\,\mu_o s\, \tilde{\bar{H}} \tag{37}$$

$$\bar{\nabla} \times \tilde{\bar{H}} = \varepsilon_o s\, \tilde{\bar{E}} + \tilde{\bar{J}} \,. \tag{38}$$

Let $\tilde{\bar{A}}$ be the Laplace transform of a magnetic vector potential $\bar{A}$ such that

$$\tilde{\bar{H}} = \bar{\nabla} \times \tilde{\bar{A}} \,. \tag{39}$$

We have, from Eqs. (37) to (39),

$$\bar{\nabla}^2\tilde{A}_x - \mu_o\varepsilon_o s^2\tilde{A}_x = -\tilde{J}_x\,\delta(y) \tag{40}$$

and

$$\tilde{H}_x = 0 \tag{41}$$

$$\tilde{H}_y = \frac{\partial}{\partial z}\tilde{A}_x \tag{42}$$

$$\tilde{H}_z = -\frac{\partial}{\partial y}\tilde{A}_x \tag{43}$$

$$\tilde{E}_x = -\frac{1}{\varepsilon_o s}\left[\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\tilde{A}_x \tag{44}$$

$$\tilde{E}_y = \frac{1}{\varepsilon_o s}\frac{\partial^2}{\partial x\partial y}\tilde{A}_x \tag{45}$$

$$\tilde{E}_z = \frac{1}{\varepsilon_o s}\frac{\partial^2}{\partial x\partial z}\tilde{A}_x\ . \tag{46}$$

The second-order differential equation (40) can be converted into a set of first-order equations by defining new quantities $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ similar to those in Eqs. (19) to (21) with $\tilde{F}_x$ replaced by $\tilde{A}_x$. Instead of Eqs. (22) to (24), we now have

$$\frac{\partial}{\partial x}\tilde{u}(r,s) + \frac{\partial}{\partial y}\tilde{v}(r,s) + \frac{\partial}{\partial z}\tilde{w}(r,s) = -\mu_o\varepsilon_o s\tilde{A}_x(r,s) + \frac{1}{s}\tilde{J}_x \tag{47}$$

$$\tilde{H}_y = -s\,\tilde{w} \tag{48}$$

$$\tilde{H}_z = s\,\tilde{v}\ . \tag{49}$$

An operator equation (29) which represents Laplace transformed normalized state equations in the four state variables A_x, u, v, and w is obtained when operators L and P and column matrices $\tilde{f}(r,s)$ and $\tilde{e}_g(s)$ are defined as in Eqs. (25) to (28) respectively with $\tilde{F}_x$ replaced by $\tilde{A}_x$ and $\tilde{M}_x$ replaced by $\tilde{J}_x$. This operator equation can be solved by the method of moments as before by choosing suitable expansion functions $A_{x(i,j,k)}(r)$, $u_{(i,j,k)}(r)$, $v_{(i,j,k)}(r)$, and $w_{(i,j,k)}(r)$ over cavity subsections. We can write the expanded form of $\tilde{f}(r,s)$ inside the cavity as [22]

$$\tilde{f}(r,s) = \sum_{i,j,k} \{\tilde{\alpha}_{(i,j,k)}(s)\, f^{A}_{(i,j,k)}(r) + \tilde{\beta}_{(i,j,k)}(s)\, f^{u}_{(i,j,k)}(r)$$

$$+ \tilde{\gamma}_{(i,j,k)}(s)\, f^{v}_{(i,j,k)}(r) + \tilde{\delta}_{(i,j,k)}(s)\, f^{w}_{(i,j,k)}(r)\}, \qquad (50)$$

where the column matrices $f^{u}_{(i,j,k)}(r)$, $f^{v}_{(i,j,k)}(r)$, and $f^{w}_{(i,j,k)}(r)$ are the same as those defined in Eq. (30),

$$f^{A}_{(i,j,k)}(r) = \begin{bmatrix} A_{x(i,j,k)}(r) \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (51)$$

and $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\delta}$ are the Laplace transforms of the expansion coefficients.

Analogously to the parallel-polarization case treated in Section III, we can expand $\tilde{H}_z$ in the half-space $y \geq 0$ as a superposition of plane waves:

$$\tilde{H}_z = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{g}^{H}(k_x,k_z) e^{-j(k_x x + k_y y + k_z z)}\, dk_x\, dk_z\ , \qquad (52)$$

where

$$jk_y = \sqrt{(s/c)^2 + k_x^2 + k_z^2} \qquad (53)$$

and the new quantity $g^{H}(,_x,k_z)$ can be determined from the boundary condition at the slot [22]. There is a discontinuity in H_z at the slot on account of the equivalent electric current J_x which can be expanded as

$$\tilde{J}_x = \sum_{i,k} \tilde{j}_{x(i,n_y,k)}(s)\, v_{(i,n_y,k)}(x,\ 0,\ z) \qquad (54)$$

A system of three equations are obtained by matching $\tilde{A}_x$, $\tilde{u}$, and $\tilde{v}$ at the slot. A combined field expression which holds inside the cavity, in the $y > 0$ half-space, as well as in the slot can be written. This expression is extremely complicated. Suffice it to say that when it is substituted in Eq. (50) and inner products with the expansion functions are taken, we obtain a matrix equation similar to Eq. (32), from which the unknown coefficient matrices $\{\tilde{\alpha}(s)\}$, $\{\tilde{\beta}(s)\}$, $\{\tilde{\gamma}(s)\}$, and $\{\tilde{\delta}(s)\}$ can be solved.

V. SOME NUMERICAL RESULTS

In this section we outline the procedure for determining the unknown coefficient matrices $\{\tilde{\alpha}(s)\}$, $\{\tilde{\beta}(s)\}$, $\{\tilde{\gamma}(s)\}$, and $\{\tilde{\delta}(s)\}$ from Eq. (32). Although the component matrices (ℓ_{mn}^{uv}, ℓ_{mn}^{vu}, ℓ_{mn}^{vv}, ℓ_{mn}^{vw}, p_{mn}^{Fv}, p_{mn}^{uv}, p_{mn}^{vF}, p_{mn}^{vu}, p_{mn}^{vw}, and p_{mn}^{wv}) representing the coupling between cavity and external fields appear highly complex, they are relatively sparse. Typically an equation of the following form is obtained from Eq. (32):

$$[\tilde{Z}(s)]\ \{\tilde{\alpha}(s)\} = [H(s)]\ \{\frac{1}{s}\,\tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\}\ , \qquad (55)$$

where $[\tilde{Z}(s)]$ and $[\tilde{H}(s)]$ are square matrices containing ℓ_{mn}'s, p_{mn}'s, and q_{mn}'s in Eq. (32), and $\{\tilde{K}(s)\}$ is in general not the same as $\{\tilde{C}(s)\}$. From Eq. (55) we have

$$\{\tilde{\alpha}(s)\} = [\tilde{Z}(s)]^{-1}\ ([\tilde{H}(s)]\ \{\frac{1}{s}\,\tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\})\ . \qquad (56)$$

Similar expressions are obtained for $\{\tilde{\beta}(s)\}$, $\{\tilde{\gamma}(s)\}$, and $\{\tilde{\delta}(s)\}$.
Let s_α be the zeros of $|\tilde{Z}(s)|$ or the roots of the equation

$$\det[\tilde{Z}(s)] = 0\ . \qquad (57)$$

In circuit-theory terminology, $[\tilde{Z}(s)]$ corresponds to the system impedance matrix and s_α are the natural frequencies. $[\tilde{Z}(s)]^{-1}$ can be expanded in a partial-fraction form as follows:

$$[\tilde{Z}(s)]^{-1} = \sum_\alpha \frac{[R_\alpha]}{s - s_\alpha}\ , \qquad (58)$$

where the constant square matrix $[R_\alpha]$ is the system residue matrix at the pole s_α. $[R_\alpha]$ can be written as the product of a natural mode vector $\{R_\alpha^m\}$ and the transpose of a coupling vector $\{R_\alpha^c\}$ [5], [9]:

$$[R_\alpha] = \{R_\alpha^m\} \{R_\alpha^c\}^T , \tag{59}$$

where $\{R_\alpha^m\}$ is a solution of the equation

$$[Z(s_\alpha)] \{R_\alpha^m\} = 0 \tag{60}$$

and $\{R_\alpha^c\}$ is a solution of

$$[Z(s_\alpha)]^T \{R_\alpha^c\} = 0 \quad . \tag{61}$$

A close examination of the composition of the matrices $[\tilde{Z}(s)]$ and $[\tilde{H}(s)]$ reveals that their poles coincide and therefore cancel. We have, from Eqs. (56), (58) and (59),

$$\{\tilde{\alpha}(s)\} = \sum_\alpha \frac{\{R_\alpha^m\}\{R_\alpha^c\}^T}{s - s_\alpha} ([\tilde{H}(s)] \{\frac{1}{s} \tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\}) \quad . \tag{62}$$

Now define

$$[\tilde{H}(s)] \{\frac{1}{s} \tilde{m}_{x(i,n_y,k)}(s)\} + \{\tilde{K}(s)\} = \tilde{N}(s) \{\tilde{V}_o(s)\} , \tag{63}$$

where $\{\tilde{V}_o(s)\}$ is the excitation vector when the incident wave is a pulse. We can then write Eq. (62) as

$$\{\tilde{\alpha}(s)\} = \sum_\alpha \frac{\{R_\alpha^m\}\{R_\alpha^c\}^T}{s - s_\alpha} \tilde{N}(s) \{\tilde{V}_o(s)\}$$

$$= \sum_\alpha \frac{\{R_\alpha^m\}}{s - s_\alpha} \tilde{\eta}_\alpha(s) \tilde{N}(s) , \tag{64}$$

where

$$\tilde{\eta}_\alpha(s) = \{R_\alpha^c\}^T\{\tilde{V}_o(s)\} \tag{65}$$

is called the coupling coefficient [5]. We note that $\tilde{N}(s)$ itself may contain poles in the finite plane, but this fact does not result in any serious difficulty.

We are now in a position to write the expressions for the field distributions within the cavity. From Eq. (31),

$$\tilde{F}_x(x,y,z,s) = \sum_{i,j,k} \tilde{\alpha}_{(i,j,k)}(s)\; F_{x(i,j,k)}(x,y,z)$$

$$= \{\tilde{\alpha}(s)\}^T \{F_{x(i,j,k)}(x,y,z)\} \tag{66}$$

which, in view of Eq. (64), becomes

$$\tilde{F}_x(x,y,z,s) = \sum_{\alpha} \tilde{\eta}_\alpha(s)\; \{R_\alpha^m\}^T \{F_{x(i,j,k)}(x,y,z)\}\; (s - s_\alpha)^{-1}\, \tilde{N}(s)$$

$$= \sum_{\alpha} \tilde{\eta}_\alpha(s)\; \nu_\alpha^F(x,y,z)\; (s - s_\alpha)^{-1}\, \tilde{N}(s)\ . \tag{67}$$

In Eq. (67),

$$\nu_\alpha^F(x,y,z) = \{R_\alpha^m\}^T \{F_{x(i,j,k)}(x,y,z)\} \tag{68}$$

is a natural mode for F_x. In a similar manner, we will get

$$\tilde{E}_y(x,y,z,s) = \sum_{\alpha} \tilde{\eta}_\alpha(s)\; \nu_\alpha^{E_y}(x,y,z)\; (s - s_\alpha)^{-1}\, \tilde{N}(s) \tag{69}$$

and

$$\tilde{E}_z(x,y,z,s) = \sum_{\alpha} \tilde{\eta}_\alpha(s)\; \nu_\alpha^{E_z}(x,y,z)\; (s - s_\alpha)^{-1}\, \tilde{N}(s), \tag{70}$$

where $\nu_\alpha^{E_y}$ and $\nu_\alpha^{E_z}$ are the natural modes for $\tilde{E}_y$ and $\tilde{E}_z$ respectively.

For numerical computation, a rectangular cavity of dimensions 4×6×2 was chosen and the slot width was a/10 or 0.4. Considerable difficulties were experienced in the determination of the singularities (natural frequencies) s_α. After having compared several root-finding procedures and checked meticulously our computer programs, we obtained the locations of the singularities in the upper part of the left halfplane for the parallel-polarization case with E-formulation as shown in Fig. 3. Two layers of singularities were found: the first layer was very close to the $j\omega$-axis and the second layer had a positive slope. The singularities in the first layer fall on a smooth locus, and they appear to be irregularly spaced. No method exists which could ascertain the effect of numerical noise, subdivision scheme, etc. It is known that numerical results are sensitive to round-off-errors and may diverge with an increasing sampling density [23]. Problems of numerical anomaly appear in even the simplest cases [23], and precluded the determination of the natural frequencies of some well-defined geometric structures such as oblate spheroids [24].

The natural modes of the first four singularities of the first layer were determined and combined to give the total $|E_z|$ at two locations within the cavity. Assuming a step excitation, the normalized cavity field $|E_z|/E_z^i$ is plotted in Fig. 4 as a function of time at the point (2, -5, 1), which is (5/6)th of the way in from the slot aperture. Figure 5 shows a similar plot at the point (2,-3,1). The figures indicate that the maximum value of $|E_z|$ inside the cavity is in the order of one-thousandth of the incident field intensity, E_z^i, for parallel polarization (good shielding effectiveness). Maxima and nulls exist as t varies. However, a physical interpretation of their spacings is difficult because of the complicated nature that the penetrated field is reflected from the cavity walls.

Previous studies [25], [26] on the penetration of transient electromagnetic excitation through apertures in an infinite ground screen (without a cavity) have not yielded numerical results. Various difficulties were encountered in attempts to obtain self-consistent pole locations because of numerical instability [26]. The problem is vastly more complicated when there is a cavity behind the aperture. It has been shown [27] that the structure of a conducting cylinder within a parallel-plate region gives rise to two types of singularities; namely, poles and branch points, in the complex-frequency plane. Whether a cavity-backed slot has branch points or multiple poles is an unanswered question.

VI. CONCLUSION

In this report the transient excitation of a cavity through an aperture is used to illustrate the state-space solution of transient electromagnetic problems. Depending on whether the E- or the H-formulation is used, the effect of the aperture is accounted for by a magnetic current in invoking the induction theorem or by an electric current in invoking the equivalence theorem. The governing second-order differential equation is converted into a set of first-order normalized state equations by defining three new state variables in addition to the appropriate vector potential. These state equations subject to the associated boundary conditions are solved by the method of moments. The cavity region is first divided into subsections and the fields within the cavity expressed in terms of appropriate expansion functions with time-dependent coefficients. The fields in the half-space outside the cavity are represented as superpositions of plane waves. At the aperture, the cavity and external fields are properly matched. Inner products are taken with testing functions and the first-order equations are converted into matrix equations containing the expansion coefficients as unknown column vectors. Evaluation of these expansion-coefficient vectors leads to the determination of field distributions. The procedure for evaluating a typical coefficient vector by the singularity-expansion method has been outlined.

This solution procedure was applied to the problem of electromagnetic excitation of a rectangular cavity through a slot aperture. Two cases were considered: an E-formulation for parallel polarization and an H-formulation for perpendicular polarization. For the case of parallel polarization, some significant pole singularities (natural frequencies) in the upper left complex plane were formed for a sample cavity. The corresponding natural modes were determined and combined to give the electric intensities as functions of time at two locations in the cavity for a step-excitation. Both the shape and the magnitude of the penetrated electric field appear reasonable. However, it would not be wise to claim absolute accuracy because of the inherent noise and instability in the numerical procedure and because of a lack of a theoretical proof that there would be no multiple poles or branch points.

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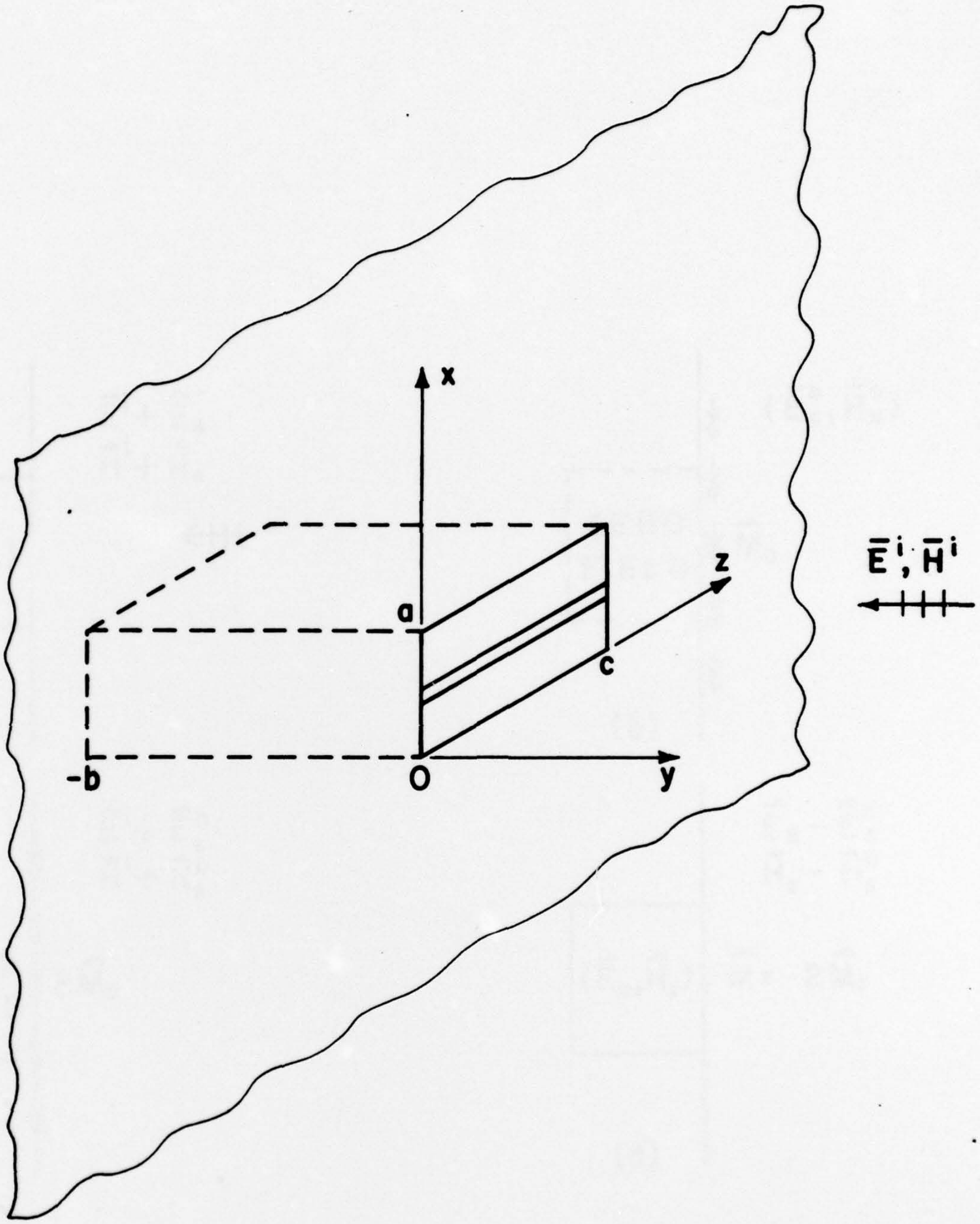

Fig. 1. A Cavity-Backed Slot Aperture Problem

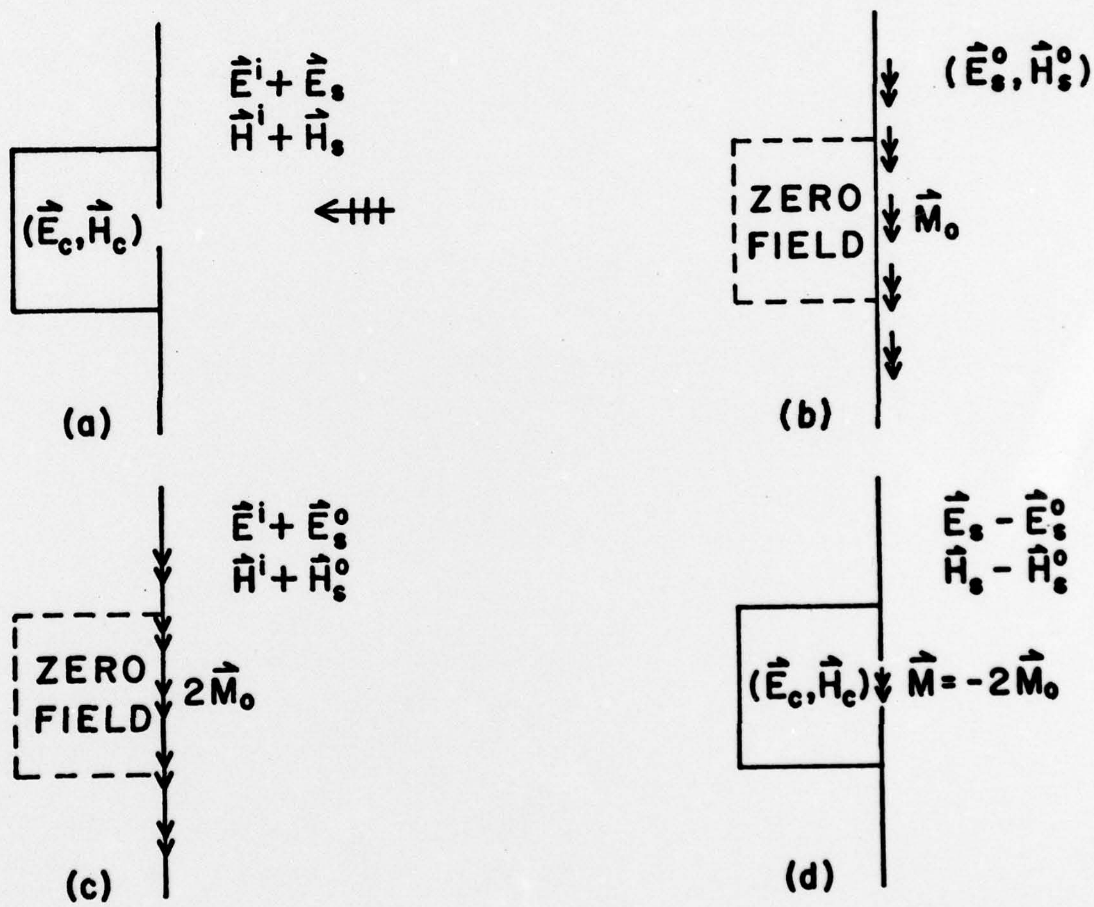

Fig. 2. Application of induction theorem

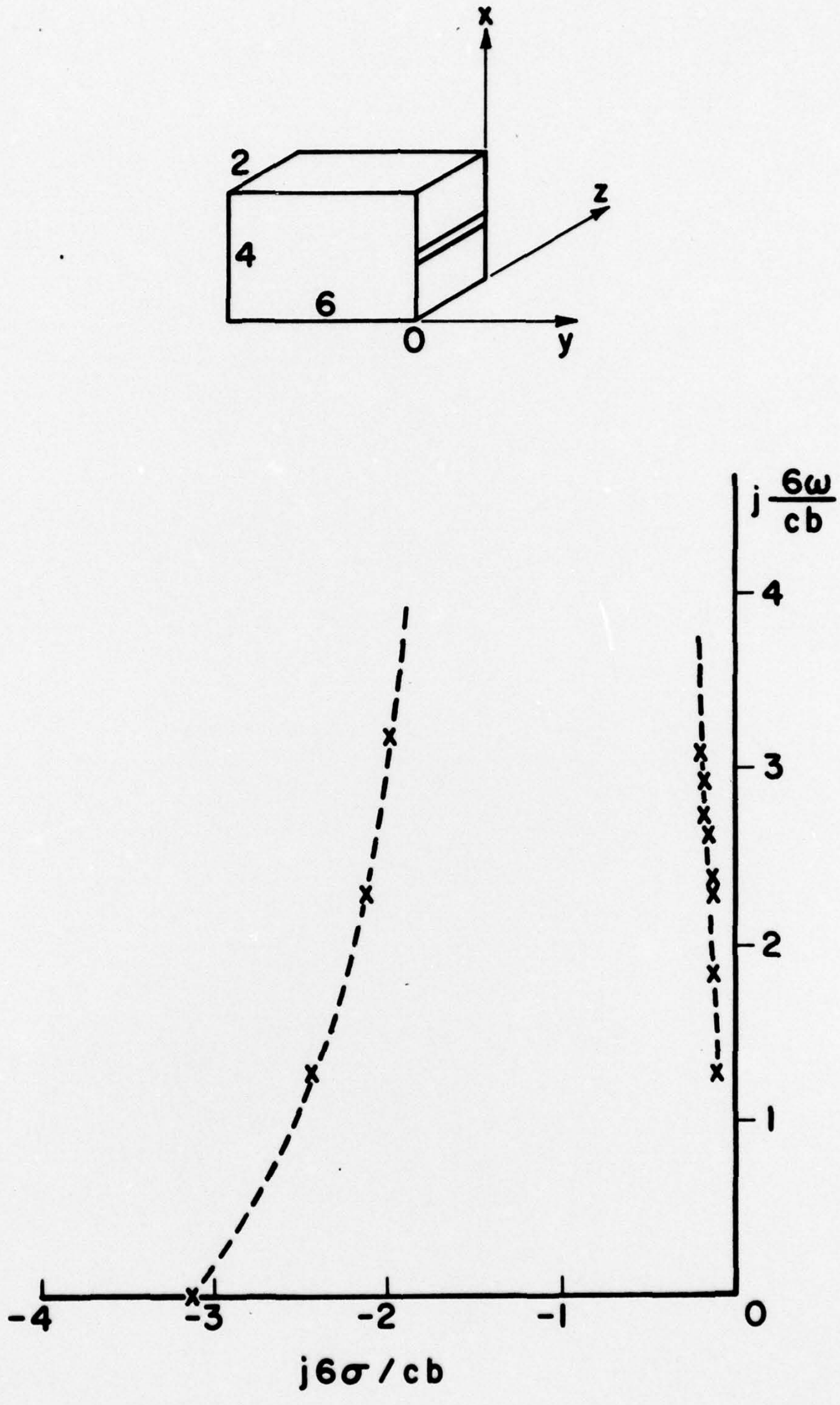

Fig. 3. Singularities of Cavity-Backed Slot (Parallel Polarization)

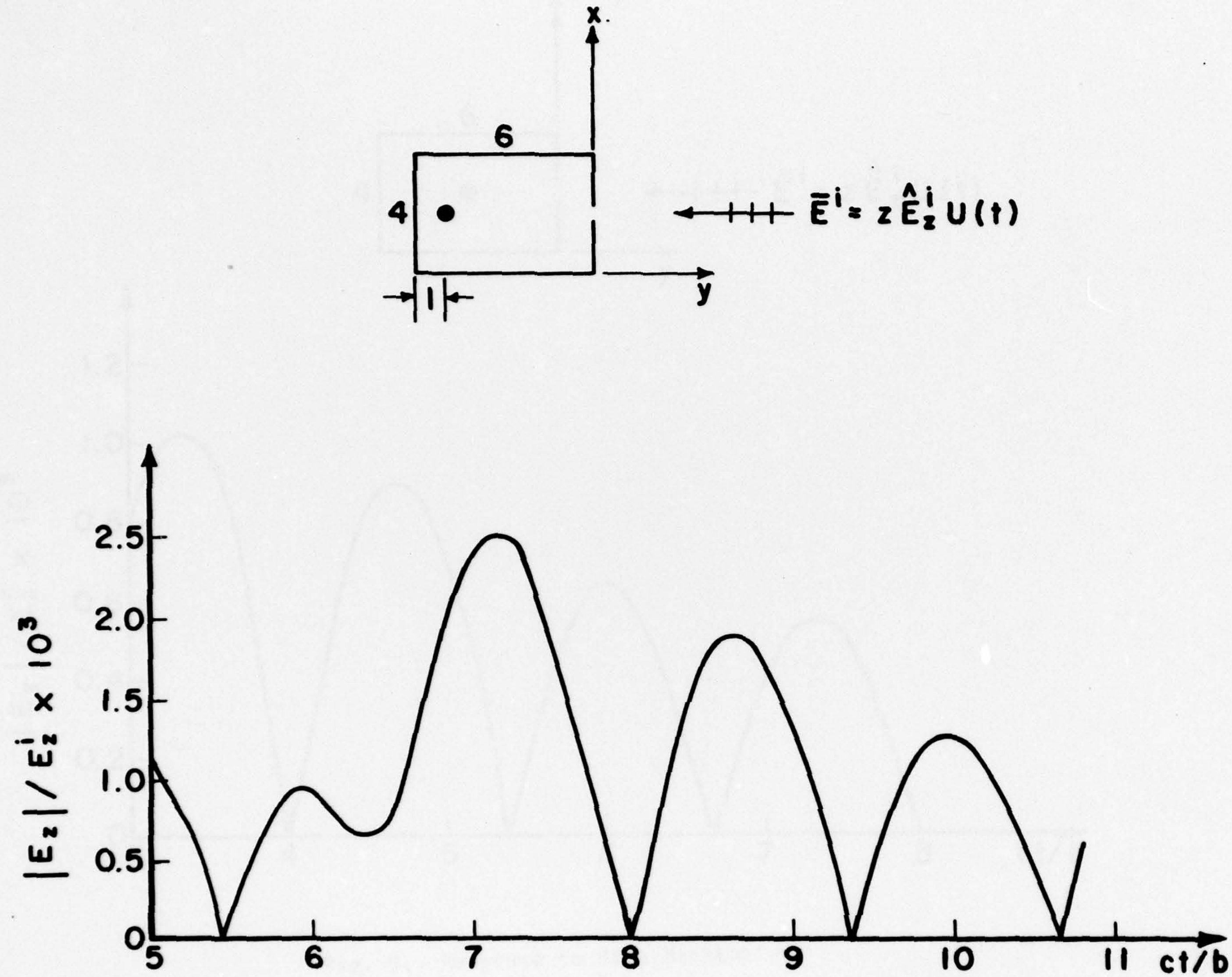

Fig. 4. Response to Step Excitation

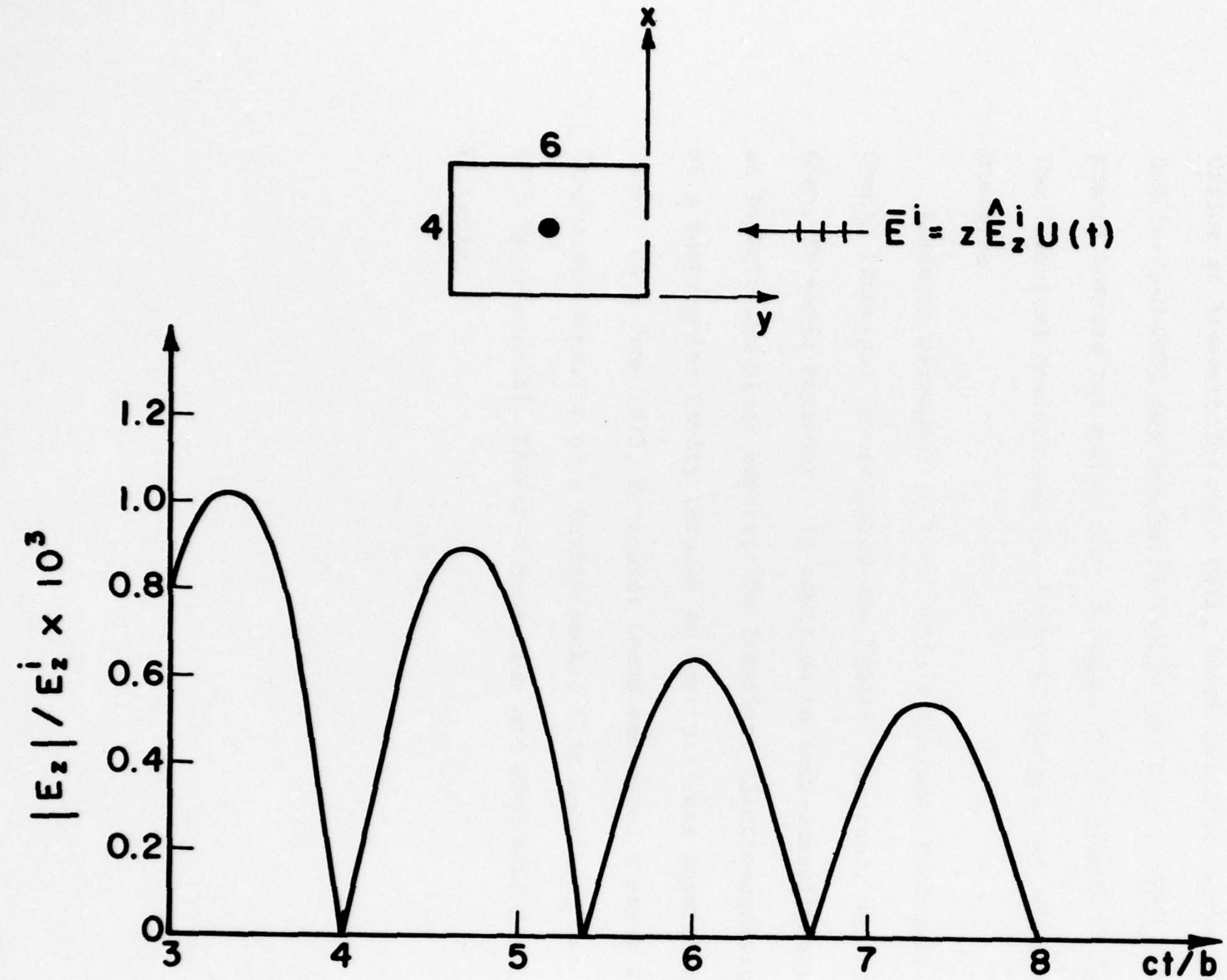

Fig. 5. Response to Step Recitation

APPENDIX

(Grant Brief and Research Personnel)

The work reported herein was sponsored by the U. S. Army Research Office at Research Triangle Park, North Carolina, under Grant Nos. DAHC04-75-G-0001 and DAHC04-76-G-0023 (Project P-11991-EL). These grants covered the period from 1 August 1974 through 31 December 1976. The Technical Monitor was Dr. Horst R. Wittmann of ARO's Electronics Division.

Research personnel on this project included Professor David K. Cheng, Principal Investigator and Project Director, and Dr. Chien-An Chen, Research Engineer. In addition to semi-annual progress reports, an interim technical report, "On Transient Electromagnetic Excitation of a Rectangular Cavity through an Aperture" was submitted in February 1975. On 5 June 1975, Professor Cheng presented a paper entitled, "Transient Behavior of a Cavity-Backed Slot Aperture," at the Joint 1975 International IEEE/AP-S Symposium and USNC/URSI Meeting in Urbana, Illinois.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

9

4. TITLE (and Subtitle)

(6) State-Space Solution of Transient Electromagnetic Problems.

5. TYPE OF REPORT & PERIOD COVERED

Final Technical Report. 1 Aug. [redacted]974 [redacted]-31 Dec. [redacted]976.

6. PERFORMING ORG. REPORT NUMBER

(14) TR-77-3

7. AUTHOR(s)

(10) David K. Cheng

8. CONTRACT OR GRANT NUMBER(s)

(15) DAHC04-75-G-0001 and DAHC04-76-G-0023

9. PERFORMING ORGANIZATION NAME AND ADDRESS

Electrical & Computer Engineering Department
Syracuse University
Syracuse, NY 13210

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

P-11991-EL

11. CONTROLLING OFFICE NAME AND ADDRESS

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709

12. REPORT DATE

(11) 31 Jan[redacted]977

13. NUMBER OF PAGES

34 (12) 38p.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

(18) ARO
(19) 11991.1-EL

15. SECURITY CLASS. (of this report)

UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

NA

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Transient Electromagnetic Problems
State-Space Solution
Cavity-Backed Aperture

406 737

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The transient excitation of a cavity through an aperture is used to illustrate the state-space solution of transient electromagnetic problems. Depending on whether the E- or the H-formulation is used, the effect of the aperture can be accounted for by a magnetic current in invoking the induction theorem or by an electric current in invoking the equivalence theorem. The governing second-order differential equation is converted into a set of first-order state equations by defining three new state variables in addition to an

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