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PLANNING AND CONTROL UNDER RISK

by

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ABSTRACT

This report describes work in the modelling of stochastic phenomena and the development of decision-making techniques under risk and uncertainty. Research areas which received major emphasis were

(1) Basic risk decision models, with emphasis on determining the structure of optimal policies and examining the implications of different risk objectives;

(2) Problems of data collection, estimation, and updating for realistic decision models.

I. INTRODUCTION

Almost all decision problems in military and industrial operations contain elements of risk or uncertainty. These unknowns not only limit the information we can gather about the current state of system, but make future states unknowable, except in a statistical sense, and strongly influence attitudes towards the decision to be made. In other words, the utility of any given decision can only be forecast in a probabilistic manner, and sometimes cannot be evaluated retrospectively without residual uncertainty. Thus, our planning and control functions must always reflect these risks and uncertainties.

There are a variety of models and methods which appear under the heading of risk theory; generally they share the following characteristics:

(1) There is usually a probabilistic law of motion governing the system. In simple cases, this may be a binary gamble, increasing or decreasing the wealth available for future risk taking; in more complicated models, a Markov-renewal or diffusion law may be appropriate.

(2) In addition to the uncertain dynamics, there is usually a concern about boundary conditions, which can lead to ruin (financial, insurance, gambling models), to catastrophes (design of dams and nuclear reactors, responses to fires and epidemics, etc.), or simply to the termination of the game (reliability and human- or corporate-lifetime models). Sometimes the ability to deliberately terminate a (losing) game is the feature of primary concern, as in optimal stopping rules. Conversely, in some stochastic optimization problems, there may be some mathematical embarrassment associated with continuing "forever," and turnpike theorems,

discounting, or absorbing states may have to be invoked to avoid analytic difficulties.

(3) Expected total return may not be a satisfactory system objective; for instance, the decision-maker may be just as concerned about the fluctuations of reward under a certain policy as about the average reward. This may lead to the use of utility theory as an axiomatic way of specifying the decision-maker's risk-aversion, to more explicit multiple-objective formulations or to nature-as-the-opponent minimax functionals. Many interesting risk-sharing problems require multiple-player pareto-optimality approaches. Problems with long horizons usually require specific recognition of the utility of time.

(4) Finally, there are always important data measurement and parameter estimation problems associated with decision under risk. Not only may the observations themselves be subject to error, but there are usually many more parameters to estimate for stochastic problems than for deterministic ones. An important part of many dynamic decision models is the provision for continuing updating of the estimators. The mechanics of this are often laborious in a real problem and the costs of screening and updating information are too often neglected in setting up a model.

During the past eleven years under this contract, "Planning and Control Under Risk," (DA-31-124-ARO-D-331 and predecessor contracts), we have tackled a variety of planning and control problems with increasing emphasis on decision under risk; a list of publications since 1965 is in the Appendix.

We feel strongly that this will continue to be a fruitful research area in the years to come. In fact, a heightened sensitivity to the problems of risk within our industries and government agencies has been demonstrated in recent years, as new problems of the environment, concerns over worker and population safety, resource and energy shortages, etc. have been encountered. Economic planning must now take into account possible sudden shifts in available resources, the uncertainty of long-term plans, and the financial interdependence of world markets. We no longer have the luxury of a certain world, but must develop decision options for a variety of alternate scenarios.

II. AREAS OF RESEARCH

A. Models of Decision Under Risk

1. Optimal Allocations in the Construction of Systems

In [1] a model of optimal allocation under risk was developed; the problem is to build an n component system which is said to function if at least k (out of n) components function. To add a component we must first decide how much money to allocate to that task; when x dollars are invested in a component then the component will function with probability $P(x)$. Given a total budget of A dollars the problem of interest is to determine how much money should be invested in each component so as to maximize the probability of attaining a functioning system. The problem is considered both in the sequential and nonsequential case, and conditions under which it is optimal to invest A/n dollars for each of n components are presented. The special case where $P(x) = \min(x,1)$ is considered in some detail. For this case the optimal strategy is determined in the sequential case when $k = 2$ and a conjecture is made in the case of arbitrary k.

A model similar to the above but with a different cost structure is also considered in [2] where once again the problem is to build an n component system which is said to function if at least k components function. However it is now supposed that the components are constructed sequentially (with full information as to previous attempts being known) and if, after constructing n components, there are only $k - i$ working ones, then a penalty cost $C(i)$ is imposed. It is shown in (2) that if $C(i)$ is convex and if $y_r(i)$ denotes the optimal amount to invest in a component when we have had a total of $k - i$ successes in $n - r$ trials (the objective being to minimize the expected construction plus penalty costs) then

$$y_r(i) \text{ is nondecreasing in } i$$

and

$$y_r(i) \text{ is nonincreasing in } r$$

It was then shown how the above qualitative results could be used to substantially reduce the computation necessary to obtain the optimal policy.

2. Stochastic Sequential Allocation Models

In [3] a model in which D units of resources is available for investment (i.e., allocation) within a fixed time frame was considered. During this time frame opportunities to invest were assumed to occur in accordance with a Poisson process, and, as soon as an opportunity occurs, a decision as to how much of our available resources to invest must be immediately made. If y is invested then an expected return $P(y)$ is gained and the amount y then becomes unavailable for future investment. The problem is to decide how much to invest at each opportunity so as to maximize the expected total return.

It was shown in [3] that if $P(y)$ is a concave function then if $y(A,t)$ denotes the optimal amount to allocate when our present resources is A, a time t remains, and an opportunity is present then

(i) $y(A,t)$ is a nondecreasing function of A, and

(ii) $y(A,t)$ is a nonincreasing function of t.

These structural results can be used to simplify the necessary computation. In the special cases $P(y) = \log y$ and $P(y) = y^{\alpha}$, $0 < \alpha < 1$, more explicit solutions for $y(A,t)$ were presented. When $P(y)$ is convex it was easily shown that $y(A,t) = A$.

3. Gambling Models

In order to obtain some insight into the structure of optimal policies in risk models, a class of gambling models, useful as simple prototypes for risk models, was considered in [4]. For a variety of objectives, it was shown that if the game is favorable to the player, then he should play as timidly as possible; that is, always make the smallest bet. A model in which the gambler is also given the option of working is considered, and it is shown that if the available gambles are unfavorable then the strategy which minimizes the gambler's expected time to reach some preassigned goal is the strategy that always calls for working. For the same model it is also shown that if the work option is only available at certain times (namely, when the gambler is broke) then the optimal gambling strategy is to play boldly. These results were obtained by developing some new general results in dynamic programming, also given in [4].

Currently under investigation is the problem where a decision-maker is allowed to gamble with his objective being to maximize the (expected) utility of his final fortune. It is supposed that if he bets an amount y then his return from this gamble will be yR, where R is a random variable whose distribution is known to the gambler, the classical case being when

$$R = \begin{cases} 2 & \text{with probability } p \\ 0 & \text{with probability } 1 - p \,. \end{cases}$$

The objective of this research is to determine properties of $y_n(x)$, the optimal bet when the gambler's fortune is x and there are n gambles to go, under the assumption of an (aribtrary) concave utility function. Some of the initial results obtained are that, in the classical case, the optimal amount to save and to strive for are both increasing in the gambler's fortune. That is, in the classical case, both

$$x - y_n(x)$$

and

$$x + y_n(x)$$

are increasing functions of x .

4. Investment Models

Many simple decision problems can be paraphrased in the following way: An entrepreneur, with limited capital, is faced with a random stream of various investment opportunities. Each opportunity is a simple gamble, that is, it requires a given investment for a fixed time period, has a known probability of losing the wager, and a known "multiple" of gain. Each gamble is usually superfair. Although some steady-state optimal, policies are known for this model, the risk problem of interest is usually defined over some uncertain lifetime, such as the life of an individual or corporation, or the time until a boundary (going broke) is reached. How does the desirability of the investment vary with size of capital, the average remaining horizon, and investment stream fluctuations?

Another investment model of interest is a consumption-savings scheme in which, at the beginning of each time period, a decision is made as to how much of one's available capital is to be consumed (the remainder to be invested). The interest rate for savings changes stochastically from period to period. While previous models (see [6] for relevant references) seem to suppose that the decision-maker does not know the interest rate at which his savings will grow when the decision is made (the interest rate is assumed to be a random variable whose value is found out *after* the decision) we propose to study the case where the value of the interest rate is known before the decision is made. Thus, for example our state space now consists of a three-dimensional vector (n,x,r) where n is the number of time periods remaining in the problem, x is the present fortune, and r is the present interest rate. Another model, with possibilities for buying insurance against risk, is described in the next sector. Of course, models in the investment area are closely related to the gambling models described in the last section.

5. Risk-Sharing Contracts

In many situations, it is possible to reduce a risk by paying a premium to a third party, such as an insurance company, or by trading similar, but more nearly uncorrelated risks with other risk-takers. If such an arrangement continues over many years, there are a variety of "fair" arrangements, taking into account the differing utility functions of the parties involved, the differing risks, the need for capital reserves (to avoid ruin), etc. Thus a variety of secondary criteria must be invoked to decide between these different "fair" arrangements. Most of the current literature is in the actuarial journals, with particular emphasis on the problems of reinsurance corporations. However, the principles involved are applicable to any business arrangement where liability for financial risks may be shared.

A recent thesis [6] explores in detail the trade-off between payout (in the form of dividends) and hedging (in the form of insurance) which is available to a corporation (here described as an insurance company). In one part a dynamic programming model is formulated for the dynamic dividend --(re)insurance decision problem, and closed-form optimal policies are found for a particularly interesting class of utility functions; this model is similar to those described in the last two sections. The other part of the thesis considers what happens when several such corporations meet to arrange a mutual pooling of risk. The existence of an equilibrium price function in the resulting (re)insurance market is then demonstrated.

Of current interest are problems in which risk contracts can be broken at any time, and neither party has complete knowledge; information updating occurs via linear Bayesian methods (see next section on credibility theory). The definition of what constitutes a "fair" dynamic risk premium is of central importance here.

B. Estimation Problems

Bayesian Estimation

Most decision problems under uncertainty require processing of large amounts of data, either for initial estimates of the parameters, or to provide continuing updating as new information is received from prior decisions. Thus, efficient methods of estimation are important.

Suppose we can observe a random variable $\tilde{x}$ which depends upon a parameter θ in a known way, via the *likelyhood density* $p(x|\theta)$; we assume a *prior* distribution, $u(\theta)$, is available. Prior predictions about an average $\tilde{x}$ can then be made through the mixed density $p(x) = E_\theta p(x|\theta)$. Now suppose that, as a result of some decision about experimentation and sampling, we can observe n independent samples of the random variable, $\underline{x} = \{\tilde{x}_t = x_t \; ; \; t = 1,2, \ldots, n\}$. By using Bayes' theorem, the density of θ is updated as follows:

$$(1) \qquad u(\theta|\underline{x}) = \frac{\Pi p(x_t|\theta)u(\theta)}{\int \Pi p(x_t|\psi)u(\psi)d\psi}$$

In decision problems, we are not usually so interested in forecasting the parameter θ , as we are in the *forecast density* for $\tilde{x}_{n+1} = y$, the next observation. Thus, practical control problems require the calculation of:

(2) $$p(y|\underline{x}) = E_{\theta|\underline{x}}\, p(y|\theta) = \int p(y|\theta)u(\theta|\underline{x})d\theta$$

or its moments. For instance, in decision analysis, as decisions to perform experiments are made, the information from the experiments is used to recompute expected utilities of one or more future actions.

Practically speaking, computations via (1) and (2) are laborious, and require either large computer capability, or the use of a few well-known *natural conjugate prior families* of likelyhood and prior. This requires knowing (or being able to assume) a great deal of structure information about $u(\theta)$ and $p(x|\theta)$. But, as indicated above, most of this information is "wasted," especially if we only want moments or an expected utility.

Credibility Estimation

In the 1920's American actuaries developed a class of estimators called "credibility formulae" to predict the *mean* of the next risk outcome, $\tilde{x}_{n+1}$, given the "experience data," $\underline{x}$. Using simple heuristic arguments, they derived a linear forecast formula $f(\underline{x})$:

(3)
$$E\{\tilde{x}_{n+1}|\underline{x}\} \approx f(\underline{x}) = (1 - z)\cdot m + z\left(\frac{1}{n}\sum_{t=1}^{n} x_t\right).$$

$$z = \frac{n}{n + N}$$

where m is the prior mean of $p(y)$ (no data), $\frac{1}{n}\sum x_t$ is the sample mean of data, and N is a time constant, chosen originally in an ad hoc manner. The most interesting features of this formula are: (1) It uses the data in a simple linear fashion, mixing it with the prior estimate; (2) it worked extremely well for 40 years in casualty insurance experience rating.

In the 1950's it was discovered that (3) was in fact the *exact* Bayesian result $E\{\tilde{x}_{n+1}|\underline{x}\}$ (gotten from (1) and (2)) if the prior and likelyhood were the natural conjugate priors: Beta-Binomial, Gamma-Poisson, and Normal-Normal. Then in 1967, it was shown that the credibility forecast (3) is the best *least-squares approximation* for arbitrary distributions, if N is chosen as the ratio of the two components of prior variance.

Recent Research

In a research effort supported by ARO since 1973, we have been able to find a variety of interesting and useful extensions to credibility estimations. In [10] [12], the class of likelyhoods for which (3) is exact was extended to all distributions where the range of $\tilde{x}$ does not depend upon θ $\forall$ and for which the sample mean is (one of) the sufficient statistics. It is well known that this implies $p(x|\theta)$ belongs to the Koopman-Pitman-Darmois *exponential family*, which includes a very large number of statistically important distributions, and for which the natural conjugate prior families is easy to obtain. The forecast (3) *only* requires estimation of the prior mean and two components of variance in $\tilde{x}$-space; the prior itself does not enter explicitly. Thus, credibility forecasts have very valuable properties for forecasting average values:

(1) They are almost "distribution-free," requiring the estimation of very few prior moments, always in the space of observables;

(2) The data are used in a simple way, as a sample mean or other sufficient statistic.

Furthermore, the credibility approach can be generalized. If other sufficient statistics are known (such as the 2nd moment) to be appropriate, it is pointed out in [9] that a formula similar to (3) also applies to these generalized moments.

The basic model has also been extended to the case where $\tilde{x}$ is a *vector-valued random variable,* whose prior distribution is to be updated using a matrix X of observed data. Reference [7] gives the least-squares linearized approximation for the Bayesian forecast; reference [10] shows that the forecast is exact for multi-dimensional exponential families, such as the multinomial and multinormal distributions. The resulting forecast formula is a vector version of (3), requiring only prior estimates of the (vector) mean and (matrix) covariances.

In another direction [8], it has been shown that a credibility formula can be obtained for an estimate of the *probability* $P(y|\underline{x}) = \Pr\{\tilde{x}_{n+1} \leq y|\underline{x}\}$ for a fixed value of y. Updating involves mixing one's prior estimate of this probability with the sample frequency of success in a form similar to (3), instead of direct computation via (1) and (2). This procedure is exact in only one trivial case (Beta-Bernoulli distributions), but computer simulations have shown that it is a good approximation to the exact Bayesian result, being unbiased and having a variance less than twice the theoretical minimum. And its simplicity recommends itself for updating models with a large data base.

Paper [13] explores a large class of model extensions which are of practical use, such as special data structures, correlated observations, evolutionary models, and integration of cohort data. When this paper appeared, the Principal Investigator was host to an Actuarial Research Conference on Credibility Theory at Berkeley, sponsored by the American Society of Actuaries and Casualty Actuarial Society. Interest in credibility theory thus continues high within actuarial circles, in addition to the many different applications possible in industrial and military estimation problems.

The next five papers [14] [15] [16] [17] [18] appeared while one of the Investigators was on sabbatical leave at the International Institute of Applied Systems Analysis at Laxenburg, Austria, from September 1974 to June 1975. [14] analyzes the two classes of covariance matrices for which one is able to get closed-form solutions to least-squares problems. [15] continues the work of [13] on collateral data, and develops a hierarchical model which explains how related data structures may be integrated in a single forecast. [16] develops Bayesian credibility estimators for simple inverse regression problems which arise, for example, in the calibration of instruments and other simple linear laws. [17] provides a unified overview of credibility as applied to general Bayesian regression problems, based upon initial work by Hachmeister and Taylor which was given at the Actuarial Research Conference on Credibility; this approach has important ties with the Kalman filtering methods used in statistical control theory. [18] explores the application of credibility to a problem in material verification, such as might arise in a Safeguards program for radioactive material accountability. Because of the relative inaccessibility of these papers to Army personnel, it is planned to reissue them in O.R. Center format in the near future, as soon as proper permission can be obtained.

An important problem in any forecast is the estimation of trend components. Although this can be easily formalized in any Bayesian regression problem by expanding the number of unknown coefficients. [19] indicates how credibility trend analysis can be carried out in a compact and efficient manner by using the structure explicitly to reduce the problem dimensions.

Paper [20] explores the application of credibility theory to the Bayesian estimation of network flows, which arise in various logistic and material accountability problems. This problem is notoriously "unidentifiable" within the framework of classical estimation theory, and a full Bayesian analysis would require very large-dimensional multi-dimensional priors and likelyhoods to be specified.

Also part of this research effort is [21], which develops an exact Bayesian model for life-testing models, in which many of the observations are of the $\{\tilde{X} > x\}$ type. This report was supported by ARO under Contract DAA629-76-6-0042 with Richard Barlow.

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DEGREES AWARDED

June 1, 1975 - May 31, 1976

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December 1975

Donald Smith, Ph.D.
December 1975