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MARGINAL STABILITY ANALYSIS: A SIMPLER APPROACH TO ANOMALOUS TR--ETC(U)

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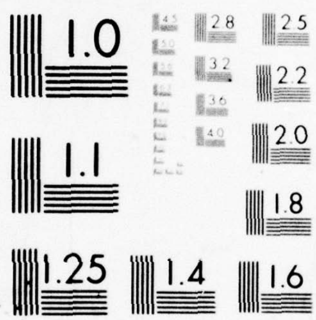

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Marginal Stability Analysis: A Simpler Approach to Anomalous Transport in Plasmas

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It is shown that an unstable plasma evolves toward a marginally stable state. From the knowledge that the plasma is marginally stable, one can calculate profiles, transport coefficients and turbulence levels quite accurately.

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MARGINAL STABILITY ANALYSIS:
A SIMPLER APPROACH TO ANOMALOUS TRANSPORT IN PLASMAS

Understanding anomalous transport quantitatively is one of the most fundamental and difficult problems in plasma physics. Anomalous transport is generally harmful to plasma confinement. This is the case, for instance, with anomalous electron thermal conduction in tokamaks. Occasionally it is beneficial, as is sometimes the case with anomalous absorption of laser light in laser fusion schemes. In all cases, anomalous transport complicates the prediction of plasma behavior. This paper discusses a marginal stability approach to anomalous transport which has broughtexcellent results and has greatly simplified analyses.

We will illustrate our discussion of anomalous transport in plasmas by looking at two examples. The first is the problem of low-Mach-number cross-field shocks. The shock width L_s here is determined by the resistivity (i.e., electron-ion momentum exchange) and is given by $L_s \sim \nu_{ei} c^2/\omega_{pe}^2 V_A$ where ν_{ei} is the electron ion collision frequency, c is the speed of light, ω_{pe} is the electron plasma frequency and V_A is the Alfven speed. Experimentally, the shock width is about 10 c/ω_{pe}. The problem, of course, is that using the classical collision frequency gives a much smaller shock width, implying that the effective collision frequency must be strongly enhanced over the classical value. The generally accepted explanation is that the currents required to support the dB/dx gradients drive ion acoustic instabilities[1] which then govern the

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anomalous resistivity. These microinstability fluctuations have been observed in the shock front.[2]

The other example we shall use is the anomalous transport determining the tokamak temperature profile. The energy deposited in the electrons is ηJ^2 while the energy flux-out is $\chi_e \nabla T_e$. Experimentally, one finds that the resistivity η is approximately classical but the electron thermal conduction χ_e is greatly enhanced. Again, the explanation is that the temperature gradient drives trapped-electron instabilities[3] which in turn govern the heat transport. These electrostatic fluctuations, at $k\rho_i \leq 1$ have been observed in both TFR[4] and ATC.[5]

In each of these cases conventional wisdom dictates the development of a nonlinear theory of the instability to derive anomalous transport coefficients. For ion acoustic instabilities, there has been a great deal of both theory[6-8] and numerical simulation.[9-11] Generally, these simulations start a plasma off in a decidedly unstable state and show that the waves grow exponentially. There is a generally strong electron heating and the wave amplitude is limited by ion trapping or some other nonlinear mechanism. After perhaps 10 or 15 growth times, the situation settles down again, often with the fluctuation amplitude at much less than its maximum value. Trapped particle instabilities are more difficult to simulate, but some preliminary work[12] has been done in this area. These simulations appear to come to the same general conclusions as simulations of ion acoustic instabilities, namely that the plasma exists in an unstable state for a time of about $10/\gamma$.

How well do the simulations describe the magnetosonic acoustic shock and tokamak anomalous transport? The typical growth time of an ion acoustic wave is about $10/\omega_{pi}$ so ten growth times, the length of time of just about the longest simulation, is about $4 \times 10^3/\omega_{pe}$. On the other hand, the time for the plasma to traverse the shock is roughly the shock width $L_s \sim 10c/\omega_{pe}$ divided by the Alfven speed $V_A = c\,\omega_{ci}/\omega_{pi}$, or about $4 \times 10^4/\omega_{pe}$. Thus even a very long microscopic simulation can only simulate 10% of the structure.

For the tokamak, the situation is much worse. At typical tokamak densities, the basic time scale of the simulation is ω_{pe}^{-1}, about 3×10^{-12} sec. The growth time of a trapped particle instability is about 3×10^{-6} sec, and the energy confinement time is about 10^{-2} sec. Thus to get any result at all from the microscopic simulation, the time scales have to be artificially compressed by orders upon orders of magnitude.

The crucial dilemma is this: Why do real plasmas seem to exist in an unstable state for long times when detailed simulations of the basic plasma mechanisms show they should last in unstable states for only about $10/\gamma$? If, on the other hand, the plasma is stable, why are fluctuations and enhanced transport observed?

A partial answer is that instability can only be important for long periods in a steady-state plasma if some external (to the instability) mechanism continually drives the system toward instability. Let us look at what these external mechanisms providing free energy are in the case of the shock and tokamak.

For the shock, it is the $u \frac{du}{dx}$ term in the fluid equation which causes natural steepening of the velocity and density profile. The source of the energy is the strong flow behind the shock. Since the magnetic field is essentially frozen into the flow, B also steepens and therefore $\frac{dB}{dx}$ increases. Once this current in the shock front exceeds an instability threshold, unstable waves grow until the greatly increased nonlinear resistivity successfully fights the tendency of the fluid to steepen by dissipating the current at the same rate it is generated by steepening.

For the tokamak, the mechanism driving the plasma toward instability is the fact that the resistivity is proportional to $T_e^{-3/2}$. Thus current channels into the hotter regions, heating them further. This channeling tends to increase the temperature gradient, and thereby drives trapped-electron instabilities. An induced trapped-electron instability causes anomalous electron thermal conduction, broadens the electron temperature profile, and combats the channeling.

In steady state, these two conflicting tendencies fight each other. The natural meeting ground is at a configuration of marginal stability for the relevant instability. The marginal stability hypothesis then is simply an assumption that the system is at marginal stability.

The system may either just sit at marginal stability, as the solid line in Figure 1, or it may evolve as a relaxation oscillation about marginal stability as in the dashed curve in Figure 1. In this marginal stability approach to anomalous transport, we want to stress, the transport coefficient is not the fundamental quantity to be sought first; rather it is the plasma profile. Once the marginally stable profile is known, one can calculate the value of the anomalous transport coefficient needed to produce that profile. Then, knowing the transport coefficient, one can use quasi-linear theory to calculate the turbulent fluctuation amplitudes needed to give that required value of anomalous transport. Thus, a marginal stability anomalous transport calculation proceeds in a direction just opposite to what conventional theory would dictate, as shown in Fig. 2.

We will now discuss the relation between the marginal stability theory and conventional nonlinear and turbulence theory, and also discuss how to incorporate marginal stability theory into a fluid or transport computer code. Usually a nonlinear theory is worked out assuming that the background plasma profile is somehow held fixed. Then a nonlinear theory would predict some value of fluctuation amplitude which we will denote φ_{NL}. From this φ_{NL} one calculates a transport coefficient which then governs the spatial and temporal evolution of the profile.

As we have just seen, however, the marginal stability theory also predicts a fluctuation amplitude necessary to maintain the profile at marginal stability. We denote this level by φ_{MS}. Whenever the inequality

$$\varphi_{MS} < \varphi_{NL} \tag{1}$$

is satisfied, marginal stability theory will be viable. Indeed, the nonlinear saturation mechanism would never become operative. If, on the other hand, Inequality (1) is violated, then the transport coefficient is limited to the smaller nonlinear value and marginal stability conditions could not be maintained. This would mean, for instance, a shorter shock width or a hotter tokamak than predicted by the marginal stability anomalous transport theory.

At this point, it is worthwhile to ask just how accurately φ_{NL} can be predicted anyway. Even where the basic nonlinear physical mechanism is well understood, the predictions of transport coefficients can be notoriously inaccurate. For instance, numerous theories[6-8] and simulations[9-11] of ion acoustic instability have shown stabilization by ion trapping or a simple variation thereof. This nonlinear theory predicts a value of $\varphi_{NL}^{1/2}$. Given the rigor of most nonlinear theories, this value of $\varphi_{NL}^{1/2}$ can easily be off by a factor of two. Since the transport coefficient (resistivity in this case) goes as φ_{NL}^{2}, this turns into an uncertainty of a factor 16 in resistivity! Thus, if Eq. (1) is violated, one really cannot predict shock width to better than an order of magnitude. For the problem of electron thermal conduction in a tokamak, the situation is much worse if $\varphi_{NL} < \varphi_{MS}$. Here it seems to us, the basic nonlinear mechanisms are not even understood.

Now consider how accurately profiles can be predicted when Eq. (1) is satisfied. To determine the profile one needs only the marginal stability condition. This is determined by linear theory, which is generally well understood. Even in cases where the linear theory is not well understood, it is certainly much better understood than the corresponding nonlinear theory. Therefore, one can reasonably expect to predict profiles to at least within a factor of two when Eq. (1) is satisfied.

Since calculations of plasma profiles are generally made using fluid or transport codes, it is useful to discuss just how the marginal stability approach fits into a transport code. Fig. 3 depicts the dependence of the effective transport coefficient as a function of a relevant physical parameter such as the current in the shock, or the temperature gradient in the tokamak. There is a sharp jump in transport coefficient at the point of marginal stability. To the left of this jump, the transport coefficient is determined by its classical value, to the right by φ_{NL}.

Now envision what happens as the strength of the external mechanism forcing the plasma toward instability is gradually increased. Starting with a very weak source, the profile is determined classically. The dot on Figure 3 represents the actual system transport coefficient. As the strength of the external mechanism increases, the dot moves to the right along the curve in Fig. 3 until it comes to the marginal stability point. Then, as the mechanism continues to get stronger, the dot climbs the vertical part of the curve (i.e., keeping the same profile). It is only when the source strength has greatly increased that the dot reaches the top of the curve, where the profile is determined by φ_{NL}. At this point, the profile can onee again become sensitive to the strength of the driving mechanism.

This situation is rather like a phase change. Below zero degrees centigrade, one knows that for every half-calorie added to a gram of ice, the temperature increases by one degree. At zero degrees, however, one can add up to eighty calories without changing the temperature. The temperature is no longer determined by the heat added, but is fixed at a particular value. To find the energy content, one measures the fraction ice and fraction water. Finally, after one has added eighty calories, there is only water. At this point, the temperature is again determined by the heat content. For every calorie added the temperature increases one degree.

To summarize, if one wishes to determine profiles using a computer code, a transport coefficient having the functional dependence shown in Fig. 3 will automatically give profiles determined by marginal stability as long as the external mechanism forcing the system toward instability is sufficiently weak.

We conclude this article by very briefly showing how the marginal stability hypothesis works for the shock and tokamak and also discuss other works which use this basic concept. Much more detailed discussions can be found elsewhere[13,14]. For the case of the shock, the condition that ion acoustic waves be at marginal stability reduced to

$$\frac{c}{4\pi ne}\frac{dB}{dx} = V_S\left(1 + \left(\frac{M_i}{m_e}\right)^{3/2}\left(\frac{T_e}{T_i}\right)^{3/2}\exp - \frac{MV_s^{\ 2}}{2T_i}\right) \qquad (2)$$

where V_s is the wave phase speed and all other notation is standard. Since the jump in B across the shock is known from Rankine-Hugniot conditions, Eq. (2) above is an expression for L_s in terms of a single parameter, the electron to ion temperature ratio. This was computed in Reference 13 by setting the temperature ratio equal to the ratio of heating rates of electrons and ions. Doing so, the shock width was found to be of order $10c/\omega_{pe}$, in reasonable agreement with transverse shock experiments. Once one knows the shock width, one can determine the resistivity. From the quasi-linear expression for resistivity, one can determine fluctuation amplitude. The result is $e\varphi/T_e \sim 0.05$, also in reasonable agreement with laser scattering experiments.

For the case of the tokamak, the temperature profile is determined by the condition that the growth rate induced by trapped electrons and temperature gradient is just balanced by the damping rate due to shear. (A self-contained discussion of these aspects of trapped particle instabilities is found in Reference 15.) The marginal stability condition relating these two quantities is

$$\sqrt{\frac{r}{R}}\,\frac{1}{q^2}\frac{dq}{dr} = -\,0.25\,T_e^{-1}\frac{dT_e}{dr} \qquad (3)$$

where q is the inverse rotational transform $q = rB_o/RB_p$, B_o is the toroidal field, B_p is the poloidal field, R is the major radius of the torus and r is a variable denoting radial position in the discharge. Another relation between T_e and q is found from Ampere's law which says that the current density at position r is inversely proportional to the resistivity at that point. Assuming classical resistivity, this relation is

$$\frac{dq(r)}{dr} = 2q(r)\left[1 - \left(\frac{T_e(r)}{T_e(o)}\right)^{3/2} \frac{q(r)}{q(o)}\right] . \qquad (4)$$

Equations (3) and (4) are then two simultaneous equations for T_e and q. Solving them, we find reasonable solutions for the relative temperature and current profile which depend only on a single parameter, q(a), that is q evaluated at the tokamak's limiter. It is important to stress that recent experiments in TFR[16] also find this same basic "universal" dependence. Even though the main magnetic field and current in the tokamak are independently varied (and the central temperature therefore also varies), the temperature half width depends only on the single parameter q(a).

Once one has the temperature profile, one can then calculate the self-consistent thermal conductivity needed to maintain this profile. From quasi-linear theory, one then can find fluctuation amplitudes. In Reference 14, we have found $e\varphi/T_e \sim 0.015$ which is comparable to what is measured in ATC.

Another example of a system which comes to a dynamic equilibrium at a marginally stable state is an initially cold plasma accelerated by an electric field.[17] Numerical simulations of this system[17] show that the plasma electrons accelerate freely for very early times. Then, a strong Buneman two-stream instability is excited which slows down the electrons and heats them up. When the electron thermal velocity V_e exceeds the drift velocity V_D, the instability is turned off. This corresponds roughly to the point of linear stability for the Buneman modes. The D. C. electric field then accelerates the electrons further, turning the instability back on. As is clear from the graphs in Reference 17, there are oscillations about the marginal stability condition, $V_D = V_e$ which exist for long times. It is particularly interesting that the marginal stability condition is apparently valid even though the system appears to be in a very strongly turbulent state.

By applying quasi-linear theory along with the marginal stability criterion, one can estimate the unstable wave amplitude as a function of time. Since the unstable waves have phase velocity $\omega/k \ll V_e$ (in the reference frame in which the ions are at rest), one can show by quasi-linear theory for a drifted Maxwellian distribution that the average force acting on each electron is

$$F \approx \sqrt{\frac{\pi}{2}}\ \frac{m V_D}{k V_e^3} \left(\frac{q\delta E}{m}\right)^2 \exp - \frac{V_D^2}{2V_e^2} \qquad (5)$$

The average energy loss for each electron is given by

$$W = \omega/k\ F \approx 0. \qquad (6)$$

The equations for the drift velocity and total energy of the electrons are

$$\frac{dV_D}{dt} = \frac{eE}{m} - F/m \qquad \text{(a)}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (7)$$

$$\frac{d}{dt}\tfrac{1}{2}m\overline{V}^2 = q\,E\,V_D, \qquad \text{(b)}$$

where $V_e^2 + V_D^2 = \overline{V}^2$. Now applying the marginal stability condition, $V_E = V_E$ to Eq. (7b) gives $\frac{dV_D}{dt} = qE/2m$, just half the free acceleration as found in Reference 17. Then Eq. (7a) gives $F = eE/2$ so that one can find the fluctuating field strength from Eq. (5). The result is

$$\left(\frac{e\varphi}{T_e}\right)^2 \sim \sqrt{\frac{2e}{\pi}}\;\frac{q\,E}{2m\,V_e\,\omega_{pe}} \qquad (8)$$

where we have assumed $k \sim k_D$.

As a final example, Christiansen and Roberts[19,20] have developed a fluid simulation code to model the time development of reversed field pinches. As soon as the local Suydam condition is violated, they switch on an anomalously large thermal conduction. They found that the plasma profile was nearly unchanged almost independent of the size of this thermal conduction anomaly. In fact, the system hovered near the pressure profile necessary for Suydam stability.

We feel that the marginal stability approach to instability-dominated transport is extremely promising. It provides a simple and reliable way of estimating plasma profiles. The field amplitudes it has predicted for the shock and tokamak are small enough that quasi-linear theory for determining fluctuations is believable. The profile and transport coefficient can be determined accurately even when the details of the fluctuation spectrum are "blurred" as long as the marginal stability conditions of linear theory are correct. The method also fits in very well with conventional approaches using fluid codes as the work of Christiansen and Roberts shows.

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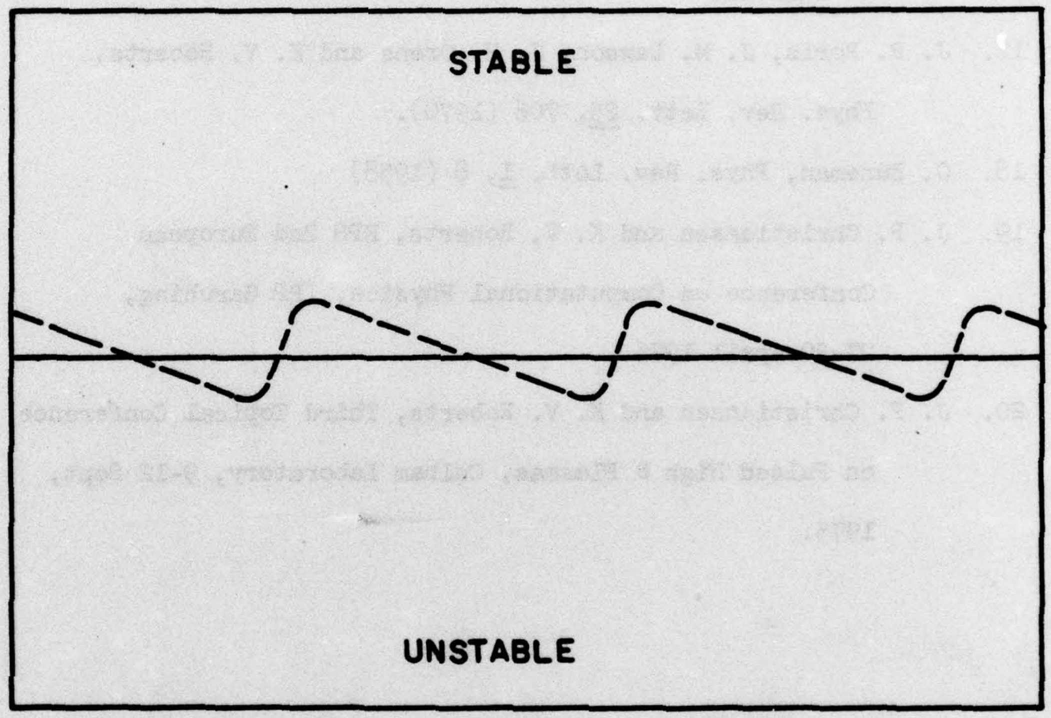

Fig. 1 — Possible loci of the system

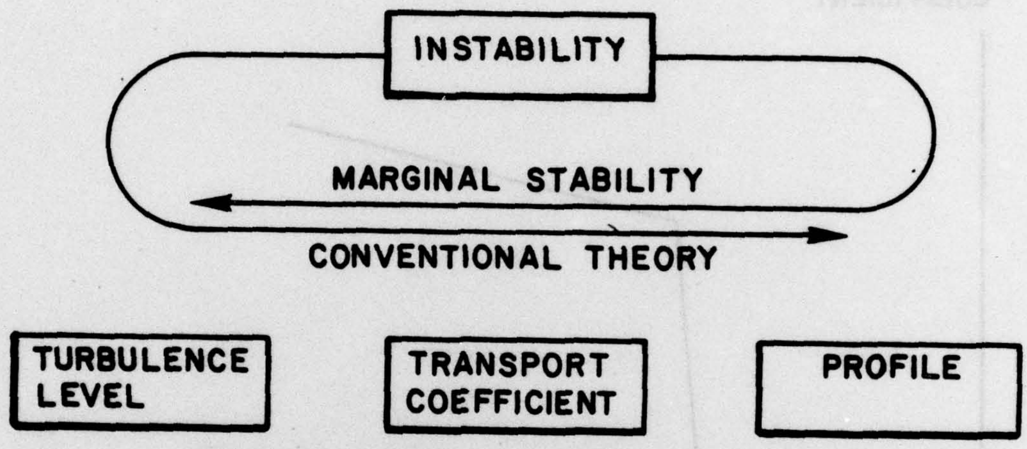

Fig. 2 — The logical paths for marginal stability and conventional nonlinear theory

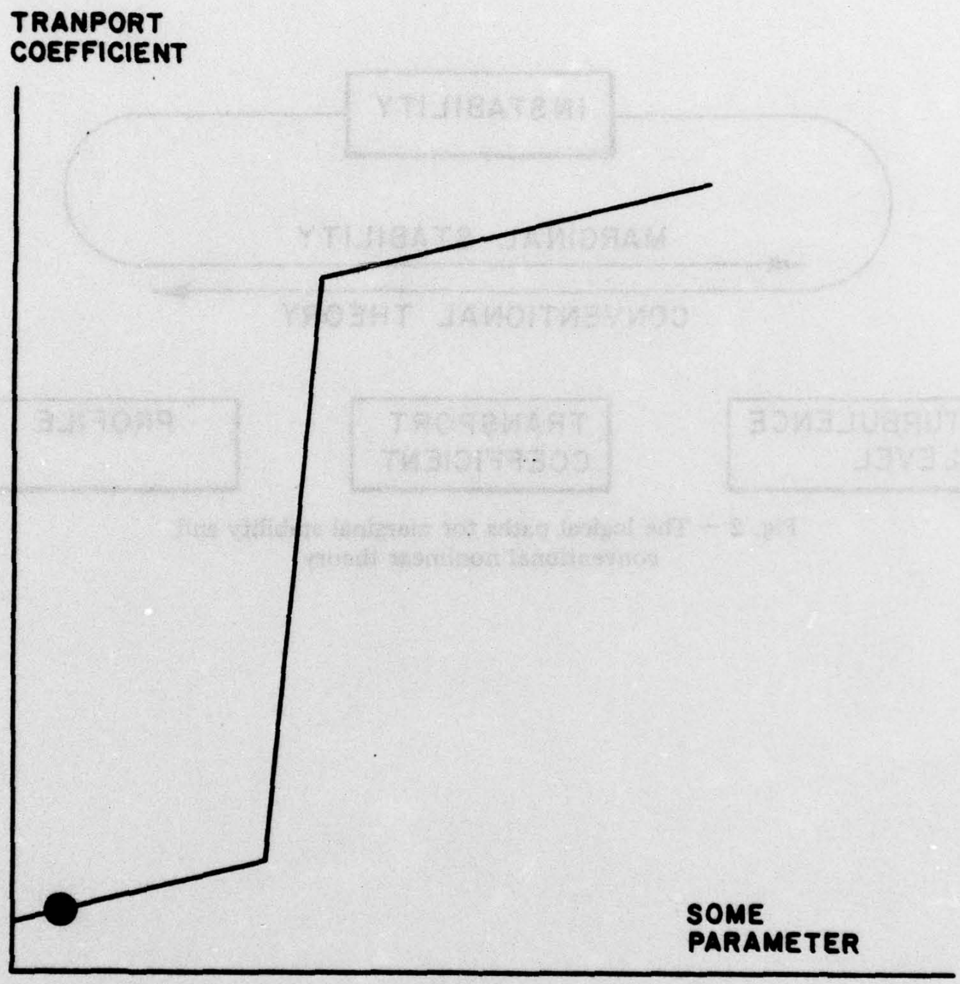

Fig. 3 — Functional form of the relevant transport coefficient

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