

DEPARTMENT OF THE NAVY NAVAL INTELLIGENCE SUPPORT CENTER TRANSLATION DIVISION 4301 SUITLAND ROAD WASHINGTON, D.C. 20390 Fuks 6 G. Savchenka V.R. œ CLASSIFICATION: UNCLASSIFIED 36 APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED Concerning the Effect of Sea Currents on Free Internal Gravitational Faves K yoprosu o vliyanii morskikh techeniy na svobodnyye ynutrenniye gravitatsionnyye volny) AUTHOR(S): Savchenko, V.G., Fuks, V.R. (USSR) Trans. PAGES: Arkticheskiy i Antarkticheskiy Institut, Trudy V301 SOURCE: 0114-123 1972. 17eh R 15 1977 ORIGINAL LANGUAGE: Russian M C TRANSLATOR: NISC TRANSLATION NO. 3899 APPROVED P. T.K. 1 February 1977 NISC-THANS-3899 DATE COPY AVAILABLE TO DDG DOES NOT PERMIT FULLY LEGIBLE PRODUCTION 407682

CONCERNING THE EFFECT OF SEA CURRENTS ON FREE INTERNAL GRAVITATIONAL WAVES

[Savchenko, V. G. and V. R. Fuks, K voprosu o vliyanii morskikh techeniy na svobodnyye vnutrenniye gravitatsionnyye volny, Arkticheskiy i Antarkticheskiy Institut, Trudy, Vol. 301, 1972, pp. 114-123; Russian]

Free internal gravitational waves arise as a reaction of a stably stratified heterogeneous liquid to an energetic external action. They carry energy from the region of an initial disturbance to the ambient space, tending to return the liquid to the initial state of equilibrium.

In the majority of theoretical studies of free internal gravitational waves in the ocean, the basic assumption is that they are small harmonic oscillations of an ideal incompressible liquid about a certain state of equilibrium. The state of rest is usually taken as this state of equilibrium. In this case, the system of fluid mechanics equations describing the unperturbed state of equilibrium is reduced to a single equation of statics. Such a choice of the principal state is equivalent to neglecting the convection terms in linearized equations for perturbations, in comparison with local changes. V. Krauss⁵ notes that this operation is inadmissible in the study of tidal internal waves. A similar situation arises in the study of other types of internal gravitational waves, whose phase velocities are comparable to the velocities of sea currents existing in the initial state of equilibrium.

Since the spectrum of phase velocities of internal gravitational waves in the ocean is fairly wide, and a certain average transport of waters is the rule rather than the exception for all regions of the World Ocean, it appears that sea currents (the main flow) must be considered in order to explain many aspects of the dynamics of internal gravitational waves. Sea currents may cause significant changes in the frequencies of internal waves, including the dynamic instability of oscillations.

The article presents certain considerations concerning the dependence of the parameters of free internal gravitational waves on the characteristics of steady sea currents, derives the necessary condition for the existence of internal gravitational waves, and explains the phenomenon, frequently observed during actual observations, of an abrupt change of phase of internal oscillations with depth. Also discussed is the question of errors in the determination of the true periods of internal waves based on observations of hydrological elements used as indicators of internal oscillations. These errors are due to the existence of the main flow and are generated by the Doppler effect.

/11!

1. In the case of an incompressible heterogeneous liquid, the initial state of which is the state of rest, the exact lower boundary of the periods of free internal gravitational waves was first indicated by Groen.¹⁶ Subsequently, on the assumption of this principal state of equilibrium, estimates of frequencies of progressive harmonic free internal gravitational waves were made in Refs. 2, 6, 9, .10, 15, and 17.

Thus, for example, Monin and Obukhov 9,10 showed for an incompressible liquid that the frequencies of free gravitational waves lie within the interval

$$l^2 \leq \sigma^2 < g\gamma a^{-2},$$

Numbers in the right margin indicate pagination in the original text.

where *l* is the Coriolis parameter;

- σ is the oscillation frequency;
- g is the gravitational acceleration;
- a is the velocity of sound;
- $\gamma = (\kappa 1)g + \frac{da^2}{dz}$ is the thermal stability parameter;
 - κ is the expansion exponent;
 - z is the vertical coordinate, directed upward.

Since in adiabatic processes $\gamma = a^2 \Gamma$ (where $\Gamma = -\left(\frac{1}{\rho_0} \cdot \frac{d\rho_0}{dz} + \frac{g}{a^2}\right)$ is the Sverdrup-Hesselberg stability and ρ_0 is the density in the unperturbed state), by introducing the Väisälä-Brunt frequency $N = \sqrt{g\Gamma}$ and passing to an incompressible liquid $(a + \infty)$, we find that in the ocean, the frequencies of the waves in question satisfy the inequality $l^2 \leq \sigma^2 < N^2$.

Reference 2 discusses the case of an isothermal atmosphere in the absence of the Coriolis force and shows that all the frequencies of free gravitational oscillations are bounded by the frequency

$$\sigma_0 = \sqrt{(\kappa - 1)g\kappa^{-1}h^{-1}},$$

where h is the height of the atmosphere.

In the general case, the height of the equivalent homogeneous atmosphere is related to the sound velocity a by the relation $h(z) = a^2 \kappa^{-1} g^{-1}$, and for an isothermal atmosphere $\gamma = (\kappa - 1)g$.

Obviously, in an incompressible liquid, this case corresponds to a constant gradient of the density logarithm, independently of the height, and the set of gra-vitational wave frequencies has the following frequency as the upper bound:

$$N = \sqrt{-g} \frac{d}{dz} \ln \rho_0 = \text{const.}$$

It will be shown that the characteristics of internal gravitational waves, /116 treated as small oscillations, should depend on the properties of the initial state of equilibrium.

Let the sea be an ideal, incompressible, stably stratified, horizontally infinite liquid of thickness H. We will assume that the density of this layer ρ_0 as well as the velocity u_0 and direction of horizontal mass transfer therein are functions of only the vertical coordinate z. We direct the z axis of the rectangular coordinate system vertically upward, and the x and y axes, horizontally. We place the origin at a flat horizontal seabed. We denote by $\beta(z)$ the angle between the positive direction of the x axis and the direction of mass transfer at any fixed level in the layer. Let the perturbation in the liquid be caused by a progressive free internal gravitational wave traveling at angle α to the positive direction of the x axis. We represent the internal oscillations as small harmonic perturbations of the three velocity components, pressure and density in relation to the principal state of equilibrium. Let

 $r' = r(z)exp[i(\sigma t - kx - sy)],$

where r' is any of five perturbed hydrodynamic elements;

r is a complex amplitude factor;

k and s are horizontal wavenumber components for which α = arctan s/k; t is the time;

i is an imaginary unit.

We then have the following second-order ordinary homogeneous linear differential equation in the amplitude of the vertical velocity of internal waves w:*

$$\frac{d^{2}w}{dz^{2}} - \left\{ \Gamma - \frac{l}{l^{2} - m^{2}(c - |\overline{u}|)^{2}} \left[lm \frac{d\overline{v}}{dz} + \frac{l}{(c - \overline{u})} \cdot \frac{d\overline{u}}{dz} \right] \right\} \frac{dw}{dz} - \frac{m^{2}}{l^{2} - m^{2}(c - \overline{u})^{2}} \left\{ N^{2} - (c - \overline{u}) \left[m^{2}(c - \overline{u}) - \frac{1}{\gamma_{0}} \frac{d}{dz} \left(p_{0} \frac{d\overline{u}}{dz} \right) \right] - \frac{ll}{m} \left[\frac{1}{p_{v}} \frac{d}{dz} \left(p_{0} \frac{d\overline{v}}{dz} \right) + \frac{1}{(c - \overline{u})} \cdot \frac{d\overline{u}}{dz} \cdot \frac{d\overline{v}}{dz} \right] \right\} w = 0, \quad (1)$$

where

$$\overline{u} = u_0 \cos(\alpha - \beta);$$

$$\overline{v} = u_0 \sin(\alpha - \beta);$$

$$c = \sigma m^{-1}.$$

The boundary conditions for Eq. (1) will be written as

$$w(0) = w(H) = 0$$
(2)

/117

With l = 0, we obtain from expression (1)

$$\frac{d}{dz}\left(\rho_0 \frac{dw}{dz}\right) - \left[\gamma_0 m^2 + \frac{R}{(c-\bar{u})^2} \frac{d\gamma_0}{dz} - \frac{1}{(c-\bar{u})} \frac{d}{dz}\left(\rho_0 \frac{d\bar{u}}{dz}\right)\right] w = 0.$$
(3)

We will hereinafter assume that the function u has continuous derivatives up to second order inclusive, and the function ρ_0 has a continuous first derivative inside the layer [0, H].

The conclusion that there is a lower period of internal waves in the presence of a steady main flow is obtained most simply from the equation for the deviation ξ of the liquid particles from the equilibrium position, related to the amplitude of the vertical velocity w as follows:

$$w(z) = \xi(z)im(c - u).$$
 (4)

From expressions (3) and (4), we obtain an equation in ξ :

$$\left[\Gamma + \frac{2}{(c-\overline{u})} \cdot \frac{d\overline{u}}{dz}\right] \frac{d\overline{z}}{dz} + \left[\frac{N^2}{(c-\overline{u})^2} - m^2\right] \overline{z} = 0.$$
 (5)

Equation (5) should be examined for homogeneous boundary conditions $\xi(0) = \xi(H) = 0$.

*This equation can be obtained by simple transformations of the equation given in the monograph of V. Krauss.⁵

3

To estimate the lower period of free internal gravitational waves, we will use Groen's* method,¹⁶ applying it to Eq. (5). Since the function ξ inside the interval is assumed not identically equal to zero and twice continuously differentiable, for the stated boundary conditions, according to the Rolle theorem, it should reach an extreme value in this interval. In this case, there is at least one level z_j for which

$$\frac{d\xi}{dz}\Big|_{z \to z_j} = 0; \quad \frac{1}{\xi} \frac{d^2\xi}{dz^2}\Big|_{z = z_j} < 0.$$

Hence, according to Eq. (5), the following inequality holds for such levels:

$$N^{2}(c - \bar{u})^{-2} - m^{2} > 0.$$
 (6)

In deriving inequality (6), c was assumed to be a real quantity having the meaning of phase velocity of the internal wave.

Considering that the internal wave period $\tau = 2\pi (mc)^{-1}$, we obtain the following estimate from inequality (6):

$$\tau > \frac{2\pi}{N} \left| 1 - \frac{\overline{u}}{c} \right|.$$

Hence, the following inequality is all the more valid:

$$r > r_0 \left| 1 - \frac{\overline{u}}{c} \right|,$$

where $\tau_0 = 2\pi N_0^{-1}$ is the Väisälä period, where $N_0 = \max_{0 \le t \le H} N(z)$. Therefore, in the

presence of the main flow, there exists no accurate lower period of free internal gravitational waves. The actual period of these waves may be larger or smaller than the Väisälä period, depending on the sign of the projection of the velocity of steady flow on the direction of propagation of the wave, with the period of internal waves $\tau \rightarrow +0$ in the coincidence layer ($c = \overline{u}$). The meaning of the latter statement is consistent with Krauss' conclusion⁵ that internal waves disappear at the critical depth, i.e., the level corresponding to the coincidence layer.

2. Applying to Eq. (3) arguments similar to those given in the derivation of inequality (6), we arrive at the inequality

$$m^{2} < \frac{N^{2}}{(c-\bar{u})^{2}} + \frac{1}{\varepsilon_{0}(c-\bar{u})} \frac{d}{dz} \left(\rho_{0} \frac{d\bar{u}}{dz} \right).$$
(7)

Obviously, inequality (7) can be fulfilled only when

$$N^2 > c_0^{-1} \left(\overline{u} - c \right) \frac{d}{dz} \left(c_0 \frac{d\overline{u}}{dz} \right). \tag{8}$$

Hence, internal waves with a given phase velocity in the presence of steady flows in the liquid can exist only when there exists at least one layer for which inequality (8) is fulfilled. Let us note that when $\overline{u} = \text{const}$ (in the special case $\overline{u} = 0$), a known condition of the existence of internal waves follows from inequality (8)

^{*}This method was used by M. Yasui²⁰ in a study of the stability conditions of internal waves traveling in the frontal zone of a horizontally inhomogeneous ocean.

/118

namely: $N^2 > 0$, i.e., if the state of uniform motion (or the state of rest) is the principal state of equilibrium, internal waves can exist only when the inhomogeneous liquid under study includes layers in which the density decreases with height.

3. We will examine the question of limiting frequencies of free internal gravitational waves from the standpoint of the eigenvalues of Eq. (1). We will assume for simplicity that $\alpha = \beta = 0$ (plane motion). Then Eq. (1) will be written in the form

$$\frac{l^{2}w}{z^{2}} - \left\{ \Gamma - \frac{l^{2}}{(c-u_{0})\left[l^{2} - m^{2}\left(c-u_{0}\right)^{2}\right]} \frac{du_{0}}{dz} \right\} \frac{dw}{dz} - \frac{m^{2}}{l^{2} - m^{2}\left(c-u_{0}\right)^{2}} \times \\ \times \left\{ N^{2} - \left(c-u_{0}\right) \left[m^{2}\left(c-u_{0}\right) - \frac{1}{f_{0}} \frac{d}{dz} \left(f_{0} \frac{du_{0}}{dz}\right) \right] \right\} w = 0.$$
 (9)

To obtain the dispersion equation, we set

 $w = w_n \sin \theta z$, where $w_n = \text{const.}$ (10)

/119

Then, boundary conditions (2) will be met when

$$\theta = \frac{n\pi}{H} ,$$

where

n = 1, 2... is the mode of the internal wave; $\theta = \theta(n)$ is the vertical wavenumber.

Substituting equality (10) into Eq. (9), we obtain

$$b^{2} \sin \theta z + \left\{ 1^{*} - \frac{l^{2}}{(c - u_{0}) \left[l^{2} - m^{2} \left(c - u_{0} \right)^{2} \right]} \frac{du_{0}}{dz} \right\} \theta \cos \theta z + \frac{m^{2}}{l^{2} - m^{2} \left(c - u_{0} \right)^{2}} \times \left[N^{2} - m^{2} \left(c - u_{0} \right)^{2} + \varphi_{0}^{-1} \left(c - u_{0} \right) \frac{d}{dz} \left(\varphi_{0} \frac{du_{0}}{dz} \right) \right] \sin \theta z = 0.$$
 (11)

We will consider the levels $z = z*_j$ for which $\cos \theta z = 0$, i.e., $z*_j = Hn^{-1}(j + \frac{1}{2})$. Since we are interested only in $z*_j \in (0, 4)$, for each fixed n we have j = 0, 1, 2..., n-1.

For levels z*, Eq. (11) will be written

$$m^{2}c^{2}\left(1-\frac{u_{0}}{c}\right)^{*}\left(\theta^{2}+m^{2}\right)=\theta^{2}l^{2}+m^{2}\left[N^{2}+\varrho_{0}^{-1}\left(c-u_{0}\right)\times\frac{d}{dz}\left(\varrho_{0}\frac{du_{0}}{dz}\right)\right].$$

Hence

$$\sigma^{2} = \frac{l^{2} + \frac{1}{n^{2}} \left(\frac{2H}{\lambda}\right)^{2} \left[N^{2} + \rho_{0}^{-1} (c - u_{0}) \frac{d}{dz} \left(\rho_{0} \frac{du_{0}}{dz}\right)\right]}{\left[1 + \frac{1}{n^{2}} \left(\frac{2H}{\lambda}\right)^{2}\right] \left(1 - \frac{u_{0}}{c}\right)^{2}}, \quad (12)$$

where $\lambda = 2\pi m^{-1}$ is the length of the internal wave.

The right-hand side of equality (12) is positive in accordance with inequality (8).

If $u_0 \equiv 0$, we find with a high degree of accuracy from expression (12) that the frequency of fairly long waves $\sigma \neq \ell$, i.e., approaches the frequency of inertial oscillations, and the frequency of very short waves $\sigma \neq N$, i.e., approaches the Väisälä-Brunt frequency.

In the absence of currents in a horizontally homogeneous ocean, the frequency of internal gravitational waves is invariant relative to their direction of propagation. It is easy to see that in the presence of a horizontal current, it is substantially dependent on the direction of the current, its velocity, and the direction of propagation of the wave. Indeed, in the presence of the main flow, for /12(fairly long waves, we find from equality (12)

(13)

 $\sigma^2 \rightarrow l^2 \left(1 - \frac{u_0}{c}\right)^{-2},$

 $\tau \rightarrow \frac{2\pi}{l} \left| 1 - \frac{u_0}{c} \right|,$

and for very short waves

a? ->

$$\left[N^2 + \varphi_0^{-1}(c - u_0)\frac{d}{dz}\left(\varphi_0 \frac{du_0}{dz}\right)\right] \left(1 - \frac{u_0}{c}\right)^{-2}$$

$$\rightarrow 2\pi \left| 1 - \frac{u_0}{c} \right| \left[N^2 + c_0^{-1} (c - u_0) \frac{d}{dz} \left(p_0 \frac{du_0}{dz} \right) \right]^{-1/2}.$$
 (14)

It is significant that in general, in the presence of the main flow, there may exist free internal gravitational waves with periods exceeding the period of inertial oscillations. As follows from formula (13), this takes place when a very long wave travels in the direction opposite to that of the main flow, this excess being directly proportional to the ratio of the velocity of the main flow to the velocity of the internal wave.

All of the above considerations pertain to mutually stable internal waves (c being a real quantity). Consideration of the possible dynamic instability of internal oscillations apparently will not lead to any fundamentally different conclusions concerning the effect of currents on the frequency of internal oscillations, but may significantly limit the actual frequency region of existence of internal waves.

Let us note that the direct application of Groen's method to Eq. (9) will lead to the double inequality

 $\frac{2\pi \left|1-\frac{u_0}{c}\right|}{\sqrt{N^2+\varepsilon_0^{-1}(c-u_0)\frac{d}{dz}\left(\cos\frac{du_0}{dz}\right)}} < \tau < \frac{2\pi}{l} \left|1-\frac{u_0}{c}\right|,$

which confirms the validity of the approximations used in the derivations of formulas . (13) and (14).

4. The literature frequently metions the phenomenon of oscillation phase discontinuity in actual observations of internal waves.^{3,4,11,13} The phase discontinuity of internal oscillations is easy to explain if it is observed in layers of density discontinuity, since the conclusions of interface wave theory can be applied to this case.⁷

or

the second state of the state of the second s

In a continuously stratified liquid, the presence of a phase discontinuity of /121 internal oscillations can, generally speaking, be formally explained on the basis of a determination of the normal oscillations (modes) of the system. In this case, it can easily be shown that the depth at which the phase discontinuity is located in disturbed motion does not necessarily coincide with the level of the density discontinuity layer. Moreover, the level at which the phase discontinuity takes place can be accurately determined in this case as a function of the mode of the internal wave.

It is our view that the phenomenon of phase discontinuity may also be explained by using the theory of hydrodynamic stability of the motion of a heterogeneous liquid.

Tollmin⁸ showed theoretically in the case of the motion of a viscous homogeneous liquid that a phase discontinuity in oscillations determining the perturbing motion may arise during the passage through the critical layer. This conclusion was confirmed by the experiments of Schubauer and Scramsted.

For a heterogeneous ideal liquid, a shift in the horizontal velocity of the main flow frequently causes a phase discontinuity during the passage through the critical layer. This assertion results from an analysis of the work of Miles.18,19

It may be postulated that the phase discontinuity for a given mode of an internal wave will occur at the level where the phase velocity of this mode is equal to the projection of the velocity of the main flow on the direction of propagation of the wave. This assumption admits in principle an experimental check involving special observations that may be carried out by means of standard oceanographic instruments.

Let us note that a similar phase discontinuity phenomenon during the passage through the coincidence layer takes place in the process of generation of windgenerated waves.14

5. In oceanographic observations, it is sometimes useful to differentiate the transformation of the wave process by currents, i.e., to consider the Doppler effect. In a study of internal tidal waves, such an estimate was made by Bukhteyev¹ on the assumption that the waves travel along the phase boundary of two homogeneous liquids with a uniform principal motion in the entire system. These results can be easily extended to the case of internal gravitational waves of any period, traveling in an arbitrary direction in a continuously stratified inhomogeneous liquid, in which the horizontal velocity vector of the main flow is an arbitrary function of the vertical coordinate.

Then the distortion ΔT of the period of internal waves (difference between the observed T₁ and true T₂ periods) may be obtained from the equality

$$\Delta T = T_2 \left(\left| \frac{c}{c - \overline{u}} \right| - 1 \right). \tag{15}$$

Since in this case u = u(z), the magnitude of the distortion of the internal wave /12: period substantially depends on the relation between the phase velocity of the wave and the value of the projection of the velocity of the main flow on the direction of propagation of the wave. Hence, in the general case, the values of ΔT will be different at different observation levels.

7

The Doppler effect is usually considered when processing observations made from a moving ship. The corresponding procedure was given by Krauss⁵ and Sabinin.¹² However, in processing observations of internal waves, made from an anchored ship or by means of instruments mounted on autonomous buoy stations, the Doppler effect is usually neglected. It should be kept in mind, however, that the internal wave spectra obtained may be markedly distorted because the velocity of the sea currents at the level of the observations is not zero.

If the actual observations of internal waves are organized so that one can calculate the phase velocity of a wave, direction of its propagation, as well as the speed and direction of nonperiodic currents, then according to equality (15), the true wave period can be determined from the formula

$$T_2 = T_1 \left| 1 - \frac{\overline{u}}{c} \right|.$$

We will note in conclusion that the "observational" Doppler effect and frequency change effect (discussed in sections 1 and 3), physically caused by sea currents, are not, generally speaking, identical. In particular, it appears that resonance phenomena in the ocean occur at frequencies modified by the presence of the main flow.

REFERENCES

- 1. Bukhteyev, V. G., On the necessity of considering the Doppler effect in analyzing observations of internal waves. "Okeanologiya," Vol. 4, No. 6, 1964.
- 2. Dikiy, L. A., Theory of Oscillations of the Earth's Atmosphere (Teoriya kolebaniy zemnoy atmosfery), Gidrometeoizdat Publishing House, Leningrad, 1969.
- 3. Studies of Tidal Phenomena in an Inhomogeneous Sea (Issledovaniya prilivnykh yavleniy v neodorodnom more), V. V. Timonov, Ed. Gidrometeoizdat Publishing House, Leningrad, 1965.
- 4. Koks, K. S., Internal waves. In: The Sea (translated from the English), Gidrometeoizdat Publishing House, Leningrad, 1965.
- 5. Krauss, V., Internal Waves (Vnutrenniye volny), Gidrometeoizdat Publishing House, Leningrad, 1968.
- 6. Krupin, V. D., One property of internal waves. Akustich. Zh. No. 1, Vol. 15, 1969.
- 7. Lamb, G. (H?), Hydrodynamics. Gostekhizdat, Moscow, 1947.
- 8. Lin Chia-chiao, Theory of Hydrodynamic Stability (Teoriya gidrodinamicheskoy ustoychivosti), Foreign Literature Publishing House, Moscow, 1958.
- 9. Monin, A. S. and A. M. Obukhov, Small oscillations of the atmosphere and adaptation of meteorological fields. Izv. AN SSSR, ser. geofiz., No. 11, 1958.
- 10. Monin, A. S. and A. M. Obukhov, Basic types of motion of baroclinic atmosphere in the field of the Coriolis force. Dokl. AN SSSR, Vol. 122, No. 1, 1958.

- Sabinin, K. D., Relationship between short-period internal waves and the verti- /123 cal density gradient in the sea. Izv. AN SSSR, fizika atmosfery i okeana, Vol. 2, No. 8, 1966.
- Sabinin, K. D., Contribution to the determination of internal wave parameters from data of a towed chain of thermistors. Izv. AN SSSR, fizika atmosfery i okeana, Vol. 5, No. 2, 1969.
- Sabinin, K. D. and V. A. Shulepov, Short-period internal waves of the Norwegian Sea. "Okeanologiya," Vol. 5, No. 2, 1965.
- 14. Phillips, O. M., Dynamics of the Upper Layer of the Ocean (Translated from the English). "Mir" Publishing House, Moscow, 1969.
- 15. Eckart, C., Hydrodynamics of Oceans and Atmospheres (Translated from the English), Foreign Literature Publishing House, 1963.
- 16. Groen, P., Contribution to the theory of internal waves. Koninkl. Ned. Met. Inst. de Bilt., Med. en Vern., ser. B, Vol. 2, No. 11, 1948.
- Heyna, B. and P. Groen, On short-periodic internal gravity waves. Physica, Vol. 24, No. 5, 1958.
- Miles, J. W., On the stability of heterogeneous shear flows. J. Fluid Mech., Vol. 10, No. 4, 1961.
- 19. Miles, J. W., On the stability of heterogeneous shear flows. Part 2. J. Fluid Mech., Vol. 16, No. 2, 1963.
- 20. Yasui, M., Internal waves in the open ocean (an example of internal waves progressing along the oceanic frontal zone). Oceanog. Mag., Vol. 12, No. 2, 1961.



9