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TURBULENT WAKE IN A STRATIFIED MEDIUM (TRUBULENTNYY SLED V STRA--ETC(U)
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DEPARTMENT OF THE NAVY
NAVAL INTELLIGENCE SUPPORT CENTER
TRANSLATION DIVISION
4301 SUITLAND ROAD
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10 AUTHOR(S): A.T. Onufriyev, A.T.

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TURBULENT WAKE IN A STRATIFIED MEDIUM

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The article discusses the problem of determining the shape of a turbulent wake /68*
formed behind a self-propelled body in a medium with a density variable in the direction of action of gravity. A schematic picture of the development of the wake behind the moving object is as follows: at the outset, the diffusion is the same in all directions, and the wake expands symmetrically; with increasing distance from the object, the diffusion becomes strongly anisotropic, it decreases in the vertical direction under the action of gravity, and the wake assumes a flattened shape; in the volume accupied by the wake, turbulent mixing within the wake gives rise to a more homogeneous density distribution than in the ambient medium; this volume of liquid is no longer in a state of equilibrium and, acted upon by gravity, tends to return to this state; the wake collapses, and this is accompanied by its further expansion in the horizontal direction and excitation of internal waves.

The article deals with the problem of the first stage of wake development (up to collapse), i.e., the problem of turbulent diffusion in a stratified medium. The medium itself is at rest. The description of the diffusion process is based on the equations obtained in Ref. 1, which make it possible to describe anisotropic diffusion. Simplifications are introduced which are usually employed in discussions of the problem of turbulent wake propagation; they are related to the experimental fact that the free turbulent current zones are relatively narrow. Molecular diffusion is neglected. The density fluctuations are assumed to be small, and therefore they should be considered only in the terms containing gravitational acceleration.[2]

The z axis points upward, parallel to the action of gravity, and the x axis is parallel to the motion. In the body axis coordinate system, the flow pattern is stationary. The velocity U_0 of the incoming flow is assumed to be much greater than the velocity components in the wake. The scales along the ℓ_x, ℓ_y and ℓ_z axes and the characteristic values of fluctuation velocities are different. The above-mentioned facts make it possible to introduce simplifications which are used in deriving the boundary layer equations. In addition, the following simplifications are introduced: (a) third-order moments corresponding to diffusion processes are neglected in all the equations, this being the usual assumption in a wake problem; otherwise, it is necessary to introduce assumptions on third moments with average flow characteristics; (b) it is assumed that the scale magnitudes of turbulence L and energy E of fluctuational motion are constant over the wake cross section and vary only as a function of the longitudinal coordinate.

The variation of E across the wake can be considered additionally.

The system of equations is as follows:

$$U_0 \frac{\partial U}{\partial x} = -\frac{\partial}{\partial y}\langle u_x' u_y'\rangle - \frac{\partial}{\partial z}\langle u_x' u_z'\rangle$$

$$\langle u_x' u_y'\rangle = -\frac{\tau}{2}\left[\langle u_y'^2\rangle \frac{\partial U}{\partial y} + \langle u_y' u_z'\rangle \frac{\partial U}{\partial z} + U_0 \frac{\partial}{\partial x}\langle u_x' u_y'\rangle\right]$$

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*Numbers in the right margin indicate pagination in the original text.

$$\langle u_x'u_z'\rangle = -\frac{\tau}{2}\Big[g\frac{\langle\rho' U_x'\rangle}{\rho} + \langle u_z'u_y'\rangle\frac{\partial U}{\partial y} + \langle u_z'^2\rangle\frac{\partial U}{\partial z} + U_0\frac{\partial}{\partial z}\langle u_x'u_z'\rangle\Big]$$ /69

$$\langle u_y'u_z'\rangle = -\frac{\tau}{2}\Big[g\frac{\langle\rho' u_y'\rangle}{\rho} + U_0\frac{\partial}{\partial z}\langle u_y'y_z'\rangle\Big]$$

$$\langle\rho' u_x'\rangle = -\frac{2}{3}\tau\Big[\langle u_x'u_z'\rangle\frac{d\rho}{dz} + \langle\rho' u_z'\rangle\frac{\partial U}{\partial z} + U_0\frac{\partial}{\partial z}\langle\rho' u_x'\rangle\Big]$$

$$\langle\rho' u_y'\rangle = -\frac{2}{3}\tau\Big[\langle u_y'u_z'\rangle\frac{d\rho}{dz} + U_0\frac{\partial}{\partial z}\langle\rho' u_y'\rangle\Big]$$

$$\langle\rho' u_z'\rangle = -\frac{2}{3}\tau\Big[\langle u_z'^2\rangle\frac{d\rho}{dz} + U_0\frac{\partial}{\partial z}\langle\rho' u_z'\rangle\Big]$$

$$\langle u_y'^2\rangle = \frac{E}{3} - \frac{\tau}{2}U_0\frac{\partial}{\partial z}\langle u_y'^2\rangle$$

$$\langle u_z'^2\rangle = \frac{E}{3} - \tau g\frac{\langle\rho' u_z'\rangle}{\rho} - \frac{\tau}{2}U_0\frac{\partial}{\partial z}\langle u_z'^2\rangle$$

$$U_0\frac{\partial E}{\partial x} + \Big\{\langle u_x'u_y'\rangle\frac{\partial U}{\partial y} + \langle u_y'u_z'\rangle\frac{\partial U}{\partial z}\Big\} = -\frac{E}{\tau} - g\frac{\langle\rho' u_z'\rangle}{\rho}$$

$$\xi_0 L\Big\{\langle u_x'u_y'\rangle\frac{\partial U}{\partial y} + \langle u_x'u_z'\rangle\frac{\partial U}{\partial z}\Big\} + U_0\frac{\partial}{\partial x}LE = -\alpha_s\frac{EL}{\tau} - 2\alpha g\frac{\langle\rho' u_z'\rangle}{\rho}L$$

$$\tau = ALE^{-1/2}$$

The quantities { } are averaged over the wake cross section, A = 3.86 and $\alpha_s = 0.8$[1] and α are empirical constants.

As in the case of wake diffusion in a homogeneous medium,[3,4] the solution is sought in the form of exponential dependences on the longitudinal coordinate.

On the basis of the system of equations, the following estimates can be obtained (terms containing derivatives with respect to x with multiplier U_0 are not small quantities, since $\tau \sim x$, but their consideration leads to small corrections):

$$\langle\rho' u_y'\rangle \cong -{}^2/_3\,\tau\langle u_y'u_z'\rangle\frac{d\rho}{dz}, \quad \langle\rho' u_z'\rangle \cong -{}^2/_3\,\tau\langle u_z'^2\rangle\frac{d\rho}{dz}$$

$$\langle\rho' u_x'\rangle \cong -{}^2/_3\,\tau\langle u_x'u_z'\rangle\frac{d\rho}{dz} + {}^4/_9\,\tau^2\langle u_z'^2\rangle\frac{d\rho}{dz}\frac{\partial U}{\partial z}$$

$$\langle u_z'^2\rangle \cong \frac{E}{3[1+{}^2/_3\tau^2N^2]}, \quad \langle u_y'^2\rangle \cong \frac{E}{3}, \quad N^2 = g\left|\frac{1}{\rho}\frac{d\rho}{dz}\right|$$

The quantity $\langle u_y'u_z'\rangle \cong -{}^1/_2\tau^2\langle u_y'u_z'\rangle N^2$, i.e., in the approximation considered,

$$\langle u_y'u_z'\rangle = 0, \quad \langle u_x'u_y'\rangle = -\varepsilon\frac{\partial U}{\partial y}, \quad \langle u_x'u_z'\rangle = -\varepsilon\varphi(\tau N)\frac{\partial U}{\partial z}$$

$$\varepsilon = \frac{\tau E}{6}, \quad \varphi(\tau N) = \frac{1-{}^4/_9\tau^2N^2}{(1+{}^1/_3\tau^2N^2)(1+{}^2/_3\tau^2N^2)}$$

The following equations are obtained for U, E, and L:

$$U_0 \frac{\partial U}{\partial x} = \frac{\partial}{\partial y}\left[\varepsilon \frac{\partial U}{\partial y}\right] + \frac{\partial}{\partial z}\left[\varepsilon\varphi \frac{\partial U}{\partial z}\right]$$

$$U_0 \frac{\partial E}{\partial x} - \varepsilon \left\{\left(\frac{\partial U}{\partial y}\right)^2 + \varphi \left(\frac{\partial U}{\partial z}\right)^2\right\} = -\frac{E}{\tau} \frac{1 + {}^8/_9 \tau^2 N^2}{1 + {}^2/_3 \tau^2 N^2}$$

$$U_0 \frac{\partial LE}{\partial x} - \xi_0 L\varepsilon \left\{\left(\frac{\partial U}{\partial y}\right)^2 + \varphi \left(\frac{\partial U}{\partial z}\right)^2\right\} = -\alpha_s \frac{EL}{\tau} \frac{1 + \tau^2 N^2 ({}^2/_3 + {}^4/_9 \alpha/\alpha_s)}{1 + {}^2/_3 \tau^2 N^2}$$

$$\tau = ALE^{-1/2}$$

To these equations should be added the approximate condition[4]

$$\int U y^2 \, dy \, dz = \text{const}, \qquad \int U z^2 \, dy \, dz = \text{const}$$

which in the axisymmetric case gives /70

$$\int U r^3 \, dr = \text{const}$$

for a wake with a zero momentum and

$$\int U \, dy \, dz = \text{const}$$

for a wake with a constant momentum.

We introduce the following relations:

$$U(0, 0, x) / U_0 \sim (x')^{-\alpha}, \quad E' = E / E_0 = (x')^{-\beta}, \quad L / L_0 = (x')^{\gamma}, \quad l_y / l_y^{\circ} = (x')^{\delta}$$

$$l_z / l_z^{\circ} = (x')^{\varepsilon}, \quad x_k' = x_k / d, \quad l_k' = l_k / d$$

where E_0, L_0, ℓ_y°, etc. are the characteristic values of the quantities and d is the diameter of the body. Then

$$\tau' = \tau/\tau_0 = (x')^{\gamma + 1/2 \beta}, \quad \tau' E' = (x')^{\gamma - 1/2 \beta}$$

Comparing the terms in the equations, we have

$$\beta = 2 - 2\delta, \; \alpha = 1 - \delta, \; \gamma = \delta$$

At large distances, the wake behaves like a two-dimensional one, and this follows from the fact that the scale ℓ_z' tends to a constant value due to the rapid decrease of $\langle u_z'^2 \rangle$ as x' increases. This may be ascertained by integrating the equation of motion

$$\frac{\partial U}{\partial x'} = \frac{\partial}{\partial y'}\left[\varepsilon' \frac{\partial U}{\partial y'}\right] + \frac{\partial}{\partial z'}\left[\varepsilon' \varphi \frac{\partial U}{\partial z'}\right], \; \varepsilon' = \varepsilon_0 \tau' E', \quad \varepsilon_0 = \frac{\tau_0 E_0}{6 U_0 d}$$

To find the solution of this equation for specified initial data U (0, y', z'), use may be made of the Fourier transformation method[5]

$$U^{\circ}(x', k_y, k_z) = \frac{1}{(2\pi)^2} \int U(x', y', z') \exp[-ik_y y' - ik_z z'] \, dy' \, dz'$$

The equation becomes

$$\frac{dU^\circ}{dx'} = -U^\circ \varepsilon' (k_y^2 + k_z^2\varphi)$$

Its solution will be

$$U^\circ(x', k_y, k_z) = U^\circ(0, k_y, k_z)\exp\left[-k_y^2\int_0^{x'}\varepsilon' dx' - k_z^2\int_0^{x'}\varepsilon'\varphi\, dx'\right]$$

Then, using the method described in the paper cited, we obtain the solution in the form

$$U(x', y', z') = \frac{1}{\pi}\int U(0, y'', z'')\exp\left[-\left(\frac{y''-y'}{l_y'}\right)^2 - \left(\frac{z''-z'}{l_z'}\right)^2\right]\frac{dy''\,dz''}{l_y' l_z'}$$

where

$$l_y' = \left[4\int_0^{x'}\varepsilon' dx'\right]^{1/2}, \quad l_z' = \left[4\int_0^{x}\varepsilon'\varphi\, dx'\right]^{1/2}$$

The vertical scale ℓ'_z tends to a constant value, passing through a maximum (by virtue of the form of the function $\phi(x')$). The maximum corresponds to the value of the coordinate

$$m^* = \tau_0 N x^*/d$$

and equals /71

$$\max l_z' = \left[\frac{4}{\tau_0 N}\int_0^{m^*}\frac{\varepsilon'(1 - {}^4/_9\zeta^2)\,d\zeta}{(1 + {}^2/_3\zeta^2)(1 + {}^1/_3\zeta^2)}\right]^{1/2}$$

For a wake with a constant value of the momentum, the initial velocity distribution may be given in the form of the δ function

$$\int U(0, y'', z'')\,dy''\,dz'' = U_0 D$$

and the velocity distribution is

$$U(x', y', z') = \frac{U_0 D}{\pi l_y' l_z'}\exp\left[-\left(\frac{y'}{l_y'}\right)^2 - \left(\frac{z'}{l_z'}\right)^2\right]$$

For large values of the x' coordinate, we obtain

$$U(x, 0, 0) \sim x^{-1/2}, \quad E \sim x^{-1}, \quad L \sim l_y \sim x^{1/2}$$

In the case of a wake with a zero momentum, one can obtain an estimate of the velocity on the wake axis for large x'. Assuming also that $(\ell'_z)^2 \gg 1$

$$U(x', 0, 0) \simeq \frac{-1}{\pi l_y'(l_z')^3}\int U(0, y'', z'')(z'')^2\,dy''\,dz''$$

we obtain

$$U(x, 0, 0) \sim x^{-3/2}, \quad E \sim x^{-1}, \quad L \sim l_y \sim x^{1/2}$$

To get an idea of the pattern of wake development, we will use in the expressions for ℓ_y' and ℓ_z' the dependences obtained for large x. Then

$$l_y' = \left[\frac{4\varepsilon_0}{2-\beta}\right]^{1/3}(x')^{\beta} = (4\varepsilon_0)^{1/3}(N\tau_0)^{-2/3}(N\tau_0 x')^{2/3}$$

$$l_z' = (4\varepsilon_0)^{1/3}(N\tau_0)^{-1/3}\left[\int_0^m \frac{(1-4/9\,\zeta^2)\,d\zeta}{(1+2/3\,\zeta^2)(1+1/3\,\zeta^2)}\right]^{1/3}$$

The distance to m* covers the initial stage of wake development, in which the action of turbulent diffusion may be considered the predominant effect. The decrease in vertical wake dimensions obtained and the subsequent pattern of its development require further examination. The solution obtained makes it possible to establish the similarity criterion characterizing the length of the initial stage of wake development up to collapse

$$N\tau_0 x^*/d = \text{const} \quad \text{or} \quad A\,\frac{L_0}{d}\,\frac{U_0}{E_0^{1/2}}\,\frac{x^* N}{U_0} = \text{const}$$

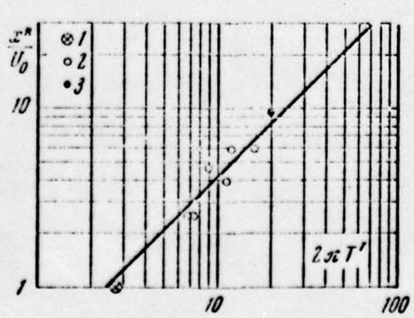
Concerning the factors L_0/d and $U_0/D_0^{1/2}$, they may be assumed to have a weak dependence on the Reynolds number and to be constant with an adequate approximation. The constancy of the ratio L_0/d in jets and wakes is usually postulated in the treatment of these problems. The constancy of $U_0/E_0^{1/2}$ over a certain range of Reynolds numbers is indicated by the experimental data of Ref. 6. Thus, the similarity criterion is the proportionality of x^*/U_0, i.e., of the time after passage of the object to the instant of wake collapse, to the period of natural oscillations of the medium

$$T' = N^{-1} = \left[g\left|\frac{1}{\rho}\frac{d\rho}{dz}\right|\right]^{-1/2}$$

This conclusion corresponds to the experimental data obtained with models (Fig. 1). Here the data are denoted as follows: (1) ⊕ - from Ref. 7, (2) O - from Ref. 9, and (3) ● - from Ref. 8. /72

Comparison with experimental data gives

$$\frac{x^*}{U_0 T'} = 2.6$$



The following relation is obtained for the maximum value of the vertical scale of the wake:

$$\max l_z' \sim \left[\frac{T'}{\tau_0}\right]^{1/2} \left[\frac{\tau_0 E_0}{U_0 d}\right]^{1/2} \sim \left[\frac{T' U_0}{d}\right]^{1/2}$$

The wake flattening coefficient increases with distance and may reach a considerable value

$$n = \frac{l_y'}{l_z'} = m^{1/2}\left[\int_0^m \varphi(\zeta)\, d\zeta\right]^{-1/2} \sim (x')^{1/2} (N^2 \tau_0^2)^{1/4}$$

The motion of models in a stratified medium was studied in Ref. 7 and 8. The experimental conditions described below were different, and the ratios of the vertical dimension of the wake to the diameter of the model were found to be almost the same in these experiments. It is obvious from the solution obtained that this agreement corresponds to the prevailing conditions.

The experimental data according to Ref. 7 and 8 were:

U_0 cm/sec = 45, 60; d cm = 2.2, 15; $|d\rho/dz|$ g cm^{-4} = 0.0052, 0.001;

T' sec = 0.438, 3.16; $[T'U_0/d]^{1/2}$ = 3.0, 3.6

i.e., the values of max ℓ_z' should be fairly similar.

Comparison with existing data suggests that the equations used make it possible to obtain a solution giving a satisfactory pattern of wake development in the initial stage.

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