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A SURVEY OF NETWORK APPROACHES TO COMPLEX SYSTEM RELIABILITY.

JAN 77 NANDA, P.; OKUMOTU, K. F30602-71-C-0312

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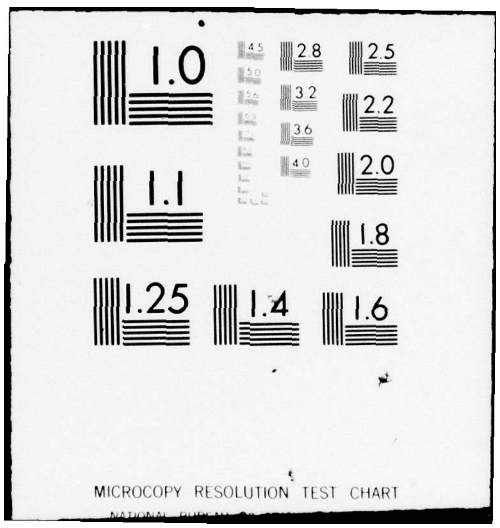
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January 1977

A SURVEY OF NETWORK APPROACHES TO COMPLEX SYSTEM RELIABILITY

Syracuse University

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ROME AIR DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
GRIFFISS AIR FORCE BASE, NEW YORK 13441

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The determination of complex systems reliability has become a subject of great concern to reliability engineers. As systems become increasingly complex, determination of system reliability becomes increasingly difficult.

The purpose of this report is to survey some of the most recent developments in this area with a view to expose reliability engineers to these techniques. An algorithm that seems to be the most efficient to date is delineated. The

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) concepts surveyed are then extended to calculation of time dependent complex systems reliability and the availability of repairable systems.

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EVALUATION

This study was one of many responsive to Technical Planning Objective #13 (Reliability) performed by Syracuse University under Contract F30602-71-C-0312. Its objective was to evaluate recent developments in algorithms for determining the reliability of complex systems. This objective was achieved. No follow-on development is expected, but the results of this study will be disseminated as widely as possible to potential users.

Anthony Coppola

ANTHONY COPPOLA
Project Engineer

A SURVEY OF

NETWORK APPROACHES TO COMPLEX SYSTEM RELIABILITY

1. INTRODUCTION:

The determination of complex systems reliability has become a subject of great concern to reliability engineers. As systems become increasingly complex, determination of system reliability becomes increasingly difficult.

The purpose of this report is to survey some of the most recent developments in this area with a view to expose reliability engineers to these techniques. An algorithm that seems to be the most efficient to date is delineated. The concepts surveyed are then extended to calculation of time dependant complex systems reliability and the availability of repairable systems.

1. Assumptions and Notation

1.1. Assumptions

1) A complex system can be expressed as a network composed of many nodes and components.

2) A set of nodes which are all reliable includes two distinct nodes, i.e. the input node s and the output node t .

3) Each component may be represented as a two-terminal device and has its own reliability.

4) The state of each component or of the network is either good (operating) or bad (failed).

5) The states of all components are statistically independent.

1.2. Notation

X_i = random variable which denotes the state of i-th component

0 = bad state

1 = good state

$P_i = P(X_i=0)$, $p = p_i$ if all components are the same

$q_i = 1 - p_i = P(X_i=1)$, $q = q_i$ if all components are the same

R_s = system reliability

R_f = system unreliability

$= 1 - R_s$

2. Classification of Complex Systems

2.1. Series - parallel system

For m components in series

$$R_s = \prod_{i=1}^{m} q_i$$

For n components in parallel

$$R_f = \prod_{j=1}^{n} p_j$$

For m components in series and n components in parallel

$$R_f = \prod_{j=1}^{n} (1 - \prod_{i=1}^{m} q_i).$$

2.2. k - out of - n system

$$R_s = \sum_{r=k}^{n} \binom{n}{r} p^{n-r} q^r$$

where the components are identical.

2.3. Non series-parallel systems

A complex system may be composed of non series-parallel sub-systems e.g. a bridge circuit. However, it is not easy to obtain the reliability of such systems. We next introduce the concept of a tie set and a cut set.

Definition: A tie set is a set of components which forms a path connected from s to t .

Definition: A cut set is a set of components which separates all connections from s to t if it is deleted from the system.

If b represents the total number of components, we have

$$R_s = \sum_{i=0}^{b} B(i)\, p^{b-i} q^{i}$$

$$R_f = \sum_{i=0}^{b} C(i)\, p^{i} q^{b-i}$$

where

$B(i)$ = total number of tie sets of size i.

$C(i)$ = total number of cut sets of size i.

If we have the information about the smallest number of cut gets say c, then the following upper and lower bounds on system unreliability can be obtained

$$R_f = \sum_{i=0}^{b} C(i) p^{i} q^{b-i} \leq \sum_{i=c}^{b} \binom{b}{i} p^{i} q^{b-i} \qquad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$

$$R_s = \sum_{i=0}^{b} B(i) p^{b-i} q^{i} \leq \sum_{i=0}^{b-(n-1)} \binom{b}{i} p^{i} q^{b-i} \qquad \text{\_\_\_\_\_\_\_\_\_\_(2)}$$

The second inequality comes from the fact that it takes at least (n-1) components to connect a network with n nodes.

From (2) we have

$$R_f = 1 - R_s \geq 1 - \sum_{i=0}^{b-(n-1)} \binom{b}{i} p^{i} q^{b-i}$$

$$= \sum_{i=b-(n-2)}^{b} \binom{b}{i} p^{i} q^{b-i}$$

Combining this with (1) we get

$$\sum_{i=b-(n-2)}^{b} \binom{b}{i} p^{i} q^{b-i} \leq R_f \leq \sum_{i=c}^{b} \binom{b}{i} p^{i} q^{b-i} \qquad \text{\_\_\_\_\_\_\_\_\_\_(3)}$$

which implies the upper and lower bounds on system unreliability, [10].

Alternatively, Esary and Proshan, [1], have obtained the following tie and cut bounds on system reliability

$$\prod_{j=1}^{k} (1 - \prod_{i \varepsilon C_j} p_i) \leq R_s \leq 1 - \prod_{j=1}^{r} (1 - \prod_{i \varepsilon B_j} q_i)$$

where C_j = jth minimal cut set.

B_j = jth minimal tie set.

In the calculation of system reliability it has been pointed out that computation time would be reduced by calculating network reliability using cut sets instead of tie sets.

Now

$\overline{C}_j$ = event that all components fail in C_j.

Since $\overline{C}_j$ might not be mutually exclusive, we have, [9],

$$R_f = P(\bigcup_{j=1}^{k} \overline{C}_j)$$

$$= \sum_{j=1}^{k} P(\overline{C}_j) - \sum_{1 \leq j_1 < j_2 \leq k} P(\overline{C}_{j_1} \quad \overline{C}_{j_2})$$

$$+ \sum_{1 \leq j_1 < j_2 < j_3 \leq n} P(\overline{C}_{j_1} \cap \overline{C}_{j_2} \cap \overline{C}_{j_3})$$

$$- \, . \, . \, . \, . \, . \, . \, . + (-1)^{k+1} P(\bigcap_{j=1}^{k} \overline{C}_j) \qquad (4)$$

which is the exact expression for system unreliability. However, it is not good for a large network to calculate exact unreliability according to the above formula.

Although minimal cut sets C_j are not mutually exclusive events, it has been suggested, [3], that equation (4) be approximated by

$$R_f = P(\bigcup_{j=1}^{k} \overline{C}_j) \approx \sum_{j=1}^{k} P(\overline{C}_j)$$

which would be a good approximation when p_i is small.

3. Several Approaches to Calculate System Reliability

3.1. Inspection methods [8]

In this method reliability is obtained in a series or parallel manner by inspection.

If the system is composed of a small number of units, it is easy to write down the probability of success of the combination.

3.2. Event-space methods [8]

A list of all possible logical occurrences,i.e. success or failure, in the system is made. Since all these events are mutually exclusive, the sum of the probability of each event yields the reliability.

3.3. Path-tracing methods [8]

In this method, only successful paths (which are generally not mutually exclusively) form favorable events. Since they are not mutually exclusive, the reliability is given by expansion and cancellation of terms.

Misra and Rao, [7], gave the path tracing algorithm taking account of the loop in the network. This is summarized as follows:

Let n = number of nodes, b = number of components.

Step 1: Find out all possible paths from s to t and sum them up.

Step 2: Find all paths with only one loop and assign a negative sign to their sum.

Step 3: Find all paths with two loops and assign a positive sign to their sum.

Step 4: Repeat step 2 and step 3 for all loops until the maximum number of loops is b - (n-1).

Assign a negative sign to that sum whose number of loops is odd.

Assign a positive sign to that sum whose number of loops is even.

Summing up the probability for all loops yields the reliability.

Another approach to find a path from s to t was derived by Kim, Case and Ghare, [5]. They used the n-step path matrix P^n_{ij} to enumerate all possible paths from s to t.

The n-step path matrix P^n can be written as

$$P^n = \begin{bmatrix} 0 & P^n_{s2} & \cdots & P^n_{st} \\ P^n_{2s} & 0 & \cdots & P^n_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ P^n_{ts} & P^n_{t2} & \cdots & 0 \end{bmatrix}$$

where component (i,j) of P^1 is defined by

$$P_{ij}^{1} = \begin{cases} (i,j) & \text{if there exists a component from i to j} \\ 0 & \text{otherwise} \end{cases}$$

Then P^n is found by the recursive relationship:

$$P^n = P^{n-1} + P^1$$

where the operation symbol + is defined as follows:

$$P^n_{ijr} = \begin{cases} P^{n-1}_{ikl},j & \text{for } k = s,2,\ldots,t \text{ if all three conditions below are met} \\ 0 & \text{otherwise} \end{cases}$$

$$P_{ik\ell}^{n-1} \neq 0$$
$$P_{kj}^{1} \neq 0$$
$$j \notin P_{ik\ell}^{n-1}$$

where P_{ijr}^{n} represents ℓ-th n-step path from i to j.
This implies that if three conditions aboe are met, then we add node j to a sequence of nodes for the (n-1) step path ℓ from i to k to form the n-step path r from i to j. In other words, the operation + implies Boolean OR summation. The solution of this operation + can be obtained by finding P_{st}^{n} for all n for which $P_{st}^{n} \neq 0$.

Introducing the following operation

$$[\prod_j p_j^{i_j}]^* = \prod_j p_j$$

$$[\sum_k (\prod_j p_j^{i_j})]^* = \sum_k [\prod_j p_j^{i_j}]^*$$

$$[\prod_k (\prod_j p_j^{i_j})]^* = [\prod_k (\prod_j p_j^{i_j})^*]^*$$

yields the approximate reliability, where p_j is any nonnegative real number and i_j is any nonnegative integer.

3.4. Decomposition methods [8]

We can decompose a complex system into suitable equivalents whose reliabilities are readily calculated using successive application of a conditional probability theorem. The technique must begin to select a suitable equivalent. Suppose that one can find a suitable equivalent, say X.

Then the system reliability is given by

$$R_s = \sum_{i=0}^{1} P\,(\text{system good} \mid X=i)P(X=i).$$

Krishnamurthy and Komissar, [6], gave the algorithm to find suitable equivalents by using the input matrix, the reliability matrix, and the reachability matrix.

We shall introduce this algorithm below.

Denote the reachability matrix by U whose element U_{ij} is such that

$$U_{ij} = \begin{cases} 1 & \text{if node j can be reached from noe i} \\ 0 & \text{otherwise} \end{cases}$$

Step 1: Pick (i,j) such that (i) $i \neq j$ (ii) $U_{ij} = 1$

(iii) $i \neq s$, $j \neq t$.

Step 2: Put k such that $U_{ik} = 1, U_{kj} = 1$, $k \neq i \neq j$ in the set L which is the set of nodes of the equivalent.

Step 3: Add i and j to L as end points.

Step 4: If each component with a node internal to L has its other node in L, then label the set of components as M. Add to M the component whose nodes are the endpoints of L.

Step 5: Treat the equivalent, i.e. M and L, as a new network and search it for smaller equivalents.

Repeating until all equivalents are suitable equivalents yields an easy calculation of system reliability.

3.5. Cut set and tie set methods [8]

As mentioned before, (sec. 2.3), this might be the most efficient technique to calculate the reliability of a complex system.

Jensen and Bellmore, [4], have presented an algorithm to determine the minimal cut set which provides for the construction of a tree.

Step 1: Create three vertices for the tree indexed 0, 1 and 2, and edges (0,1) and (1,2) labeled sT and tF, respectively.

Step 2: Choose the unscanned vertex with the greatest index and mark it scanned. If there are no scanned vertices, the algorithm terminates.

Find the unique simple path ℓ_i.

Let $Y_{1i} = \{x \mid \text{an edge in } \ell_i \text{ is labeled } xT\}$

$Y_{2i} = \{x \mid \text{an edge in } \ell_i \text{ is labeled } xF\}$

$Y_{3i} = \{x \mid x \text{ is in N but not in } Y_{1i} \text{ or } Y_{2i}\}$.

$W_i = \{x \mid x \text{ is in } Y_{3i} \text{ and it is a terminal of a component whose other terminal is } Y_{1i}\}$

If $W_i = \phi$, then go to step 7.

Otherwise, choose $y \in W_i$.

$Y_{4i} = Y_{2i} \cup Y_{3i} - y$

If the subnetwork Y_{4i} is not connected, then go to step 4. Otherwise go to step 3.

Step 3: Create two new vertices indexed k and k+1 where k is 1 greater than the number of vertices currently in the tree.

Create two new edges (i,k) and (i, k+1), labeled yT and yF, respectively.

Go to step 2.

Step 4: Find the set of nodes Y_5 which defines the connected subnetwork including t .

If $Y_{2i} \subset Y_5$, go to step 5.

Otherwise go to step 6.

Step 5: Create vertex k and edge (i,k) labeled yT where k is one greater than the number of vertices currently in the tree. Determine the set $Y_6 = Y_4 - Y_5$.

For each number $z \in Y_6$, create a vertex of the tree and an edge labeled zT. If $|Y_6|$ is the number of members in Y_6, vertices k+1, k+2, . . ., $k+|Y_6|$ will be created. Edges (k,k+1), (k+1,k+2),. . ., $(k+|Y_6|-1, k+|Y_6|)$ will also be created.

Finally, create vertex $k+|Y_6| + 1$ and edge $(i, k+ |Y_6| +1)$ labeled yF.

Go to step 2.

Step 6: Create one new vertex indexed k and an edge (i,k) labeled yF.

Go to step 2.

Step 7: A minimal cut has been generated at this step.

$X_i = Y_{1i}$

$\overline{X}_i = N - Y_{1i}$.

This heuristic algorithm cannot, however, yield system reliability easily since the components in the minimal cut set are not mutually exclusive. We shall give a more heuristic algorithm to find the modified cut at next section.

4. An algorithm for Network Reliability

We shall introduce the procedure by Hansler et.al., [3], which produces modified cut sets and seems to be the best algorithm to calculate system reliability to date. The minimal set of mutually exclusive events produced by this algorithm are called modified cut sets. This algorithm, generally speaking, performs a depth first search of the given network starting at node s and traversing several components at the same time.

Step 1: (Initialization)

a) N = set of all nodes except s

b) C = set of all components not incident to s

F_1 = set of all components incident ot s and t.

S_1 = set of all components incident to s and not to t.

c) Construct a binary number B_1 consisting of $|S_1|$ digits b_{1k}, $b_{1k}=1$ for $1 \leq k \leq |S_1|$

d) $i = 1$

Step 2:

a) T_i = subset of $S_i = \{s_{ik}\}$ such that $s_{ik} \in T_i$ if $b_{ik} = 1$

$s_{ik} \notin T_i$ if $b_{ik} = 0$

b) M_i = set of all nodes contained in N and incident to components contained in T_i

c) $N = N - M_i$

d) F_{i+1} = set of all components contained in C and incident to t and adjacent to any components contained in T_i

e) S_{i+1} = set of all components contained in C and incident to nodes in N other than t and adjacent to any component contained in T_i

f) $C = C - (S_{i+1} \cup F_{i+1})$

Step 3: If $S_{i+1} \neq \phi$

then a) Construct a binary number B_{i+1} consisting of $|S_{i+1}|$ digits $b_{i+1,k}$

$b_{i+1,k} = 1$ for $1 \leq k \leq |S_{i+1}|$

b) $i = i + 1$

c) Go to step 2

Otherwise

d) $T_{i+1} = \phi$

e) For a modified cut set

$$CS = \overset{i+1}{\underset{l=1}{\quad}} [F_\ell \quad \overline{T}_\ell \quad (S_\ell - T_\ell)]$$

where $\overline{T}_\ell$ = all components in T_ℓ are assumed to be operative.

Step 4:

a) $C = C \quad F_{i+1} \quad S_{i+1}$

b) $N = N \quad M_i$

c) $B_i = B_{i-1}$ (module 2)

If $B_i \geq 0$

then d) Go to step 2

Otherwise

e) Go to step 5.

Step 5:

a) $i = i - 1$

if $i > 0$

then b) Go to step 4

Otherwise

c) Stop.

<u>Theorem</u>: The modified cut set generated above is

(i) collective exhaustive

(ii) mutually exclusive

(iii) contains minimal cuts

This theorem can be recognized from the following example.

4.2. Example

As an example of this algorithm, we consider a bridge circuit as shown in Figure 1.

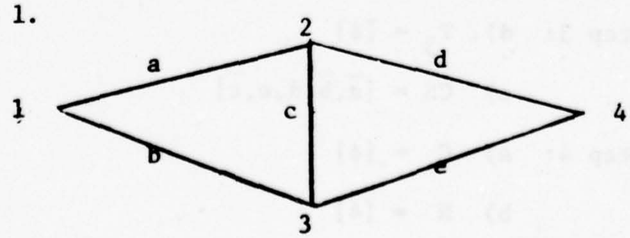

Figure 1

Step 1: a) $N = [2,3,4]$

b) $C = [c,d,e]$

$F_1 = [\phi]$

$S_1 = [a,b]$

c) $B_1 = 2$

d) $i = 1$

Step 2: a) $T_1 = [a,b]$

b) $M_1 = [2,3]$

c) $N = [4]$

d) $F_2 = [d,e]$

e) $S_2 = [c]$

f) $C = [\phi]$

Step 3: a) $B_2 = 1$

b) $i = 2$

Step 2: a) $T_2 = [c]$

b) $M_2 = [\phi]$

c) $N = [4]$

d) $F_3 = [\phi]$

e) $S_3 = [\phi]$

f) $C = [\phi]$

Step 3: d) $T_3 = [\phi]$

e) $CS = [\bar{a},\bar{b},d,e,\bar{c}]$

Step 4: a) $C = [\phi]$

b) $N = [4]$

c) $B_2 = 0$

Step 2: a) $T_2 = [c]$

b) $M_2 = [3]$

c) $N = [4]$

d) $F_3 = [c]$

e) $S_3 = [\phi]$

f) $C = [\phi]$

Step 3: d) $T_3 = [\phi]$

e) $CS = [\bar{a},b,d,e,\bar{c}]$

Step 4: a) $C = [e]$

b) $N = [3,4]$

c) $B_2 = 0$

d) Go to step 2

Step 2: a) $T_2 = [\phi]$

b) $M_2 = [\phi]$

c) $N = [3,4]$

d) $F_3 = [\phi]$

e) $S_3 = [\phi]$

f) $C = [e]$

Step 3: d) $T_3 = [\phi]$

e) $CS = [\bar{a},b,d,c]$

Step 4: a) $C = [e]$

b) $N = [3,4]$

c) $B_2 = -1$

Step 5: a) $i = 1$

Step 2: a) $T_2 = [\phi]$

b) $M_2 = [\phi]$

c) $N = [4]$

d) $F_3 = [\phi]$

e) $S_3 = [\phi]$

f) $C = [\phi]$

Step 3: d) $T_3 = [\phi]$

e) $CS = [\bar{a},\bar{b},d,e,c]$

Step 4: a) $C = [\phi]$

b) $N = [4]$

c) $B_2 = -1$

Step 5: a) $i = 1$

Step 4: a) $C = [c,d,e]$

b) $N = [2,3,4]$

c) $B_1 = 1$

Step 2: a) $T_1 = [a]$

b) $M_1 = [2]$

c) $N = [3,4]$

d) $F_2 = [d]$

e) $S_2 = [c]$

f) $C = [e]$

Step 3: a) $B_2 = 1$

b) $i = 2$

Step 4: a) $C = [c,d,e]$

b) $N = [3,4]$

c) $B_1 = 1$

Step 2: a) $T_1 = [b]$

b) $M_1 = [3]$

c) $N = [4]$

d) $F_2 = [e]$

e) $S_2 = [c]$

f) $C = [d]$

Step 3: a) $B_i = 1$

b) $i = 2$

Step 2: a) $T_2 = [c]$

b) $M_2 = [\phi]$

c) $N = [4]$

d) $F_3 = [d]$

e) $S_3 = [\phi]$

f) $C = [\phi]$

Step 3: d) $T_3 = [\phi]$

e) $CS = [\overline{b},a,\overline{c},d,e]$

Step 4: a) $C = [d]$

b) $N = [4]$

c) $B_2 = 0$

Step 2: a) $T_2 = [\phi]$

b) $M_2 = [\phi]$

c) $N = [4]$

d) $F_3 = [\phi]$

e) $S_3 = [\phi]$

f) $C = [d]$

Step 3: d) $T_3 = [\phi]$

e) $CS = [\overline{b},a,e,c,]$

Step 4: a) $C = [d]$

b) $N = [4]$

c) $B_2 = -1$

Step 5: a) $i = 1$

Step 4: a) $C = [c,d,e]$

b) $N = [3,4]$

c) $B_1 = 0$

Step 2: a) $T_1 = [\phi]$

b) $M_1 = [\phi]$

c) $N = [3,4]$

d) $F_2 = [\phi]$

e) $S_2 = [\phi]$

f) $C = [c,d,e]$

Step 3: d) $T_2 = [\phi]$

e) $CS = [a,b]$

Step 4: a) $C = [c,d,e]$

b) $N = [3,4]$

c) $B_1 = -1$

Step 5: a) $i = 0$

c) Stop.

The example above produces seven modified cut sets;

$$CS_1 = [\bar{a},\bar{b},\bar{c},d,e]$$

$$CS_2 = [\bar{a},\bar{b},c,d,e]$$

$$CS_3 = [\bar{a},b,\bar{c},d,e]$$

$$CS_4 = [\bar{a},b,c,d]$$

$$CS_5 = [a,\bar{b},\bar{c},d,e]$$

$$CS_6 = [a,\bar{b},c,e]$$

$$CS_7 = [a,b]$$

which satisfy the statements (i) - (iii) of theorem.

5. Application to Time Dependent Reliability

Using the modified cut sets generated by the previous algorithm, we attempt to develop some expressions for time dependent complex systems reliability.

Notation:

$X(t)$ = the state of a component at time t.

$\overline{F}(t) = 1 - F(t)$

$= P(X(t)= 1)$

= probability that the component performs adequately over $[0,t]$.

CS_j = jth modified cut set

$r_j = |\overline{CS}_j|$

= number of operative components in jth modified cut set

$s_j = |CS_j|$

= number of failed components in jth modified cut set

Assumption:

1. Each component fails in accordance with the same distribution, i.e. $F(t)$.
2. More than two components don't fail at the same time.

$R(t)$ = time dependent system reliability

Theorem: The time dependent system unreliability is given by

$$\overline{R}(t) = 1 - R(t)$$

$$= \sum_{j=1}^{c} s_j! [\overline{F}(t)]^{r_j} \int_0^t [F(x)]^{s_j-1} dF(x)$$

where c is number of modified cut sets.

Proof: Note that

$[\overline{F}(t)]^{r_j}$ = probability that r_j components don't fail at time t.

$s_j! \int_0^t [F(x)]^{s_j-1} dF(x)$

= probability that the last one of s_j components fail at time t.

The results follows from the fact that each modified cut set is a mutually exclusive event.

From this theorem it can be easily shown

(i) The hazard rate function z(t) of complex system is given

$$z(t) = d\overline{R}(t)/R(t)$$

$$= \frac{\sum_{j=1}^{c} s_j! \left[r_j \int_0^t [F(x)]^{s_j-1} dF(x) + \overline{F}(t)[F(t)]^{s_j-1} \right] [\overline{F}(t)]^{r_j-1} f(t)}{1 - \sum_{j=1}^{c} s_j! [\overline{F}(t)]^{r_j} \int_0^t [F(x)]^{s_j-1} dF(x)}$$

where $f(t) = \frac{dF(t)}{dt}$

(ii) The mean time to system failure (MTTF) is given by

$$MTTF = \int_0^{\infty} R(t)dt$$

$$= \int_0^{\infty} td\bar{R}(t)$$

$$= \sum_{j=1}^{c} S_j! \int_0^{\infty} \{-\gamma_j \int_0^{t} [F(x)]^{S_{j-1}} dF(X) + \bar{F}(t) [F(t)]^{S_{j-1}}\} [F(t)]^{\gamma_{j-1}} tdF(t)$$

(iii) The upper and lower bound on time dependent system unreliability is given by

$$\sum_{i=b-(n-2)}^{b} \binom{b}{i} [F(t)]^{i} [\bar{F}(t)]^{b-i} \leq \bar{R}(t)$$

$$\sum_{i=c}^{b} \binom{b}{i} [F(t)]^{i} [\bar{F}(t)]^{b-i} \geq \bar{R}(t)$$

where b = number of components

n = number of nodes

c = smallest number of cut sets

6. Application to a Repairable Complex Systems

For convenience, we make an assumption that each component has exponential failure and repair distribution with rate λ and μ, respectively.

Let $X(t)$ = random variable of the each component state at time t.

0 = failure state

1 = operating state

$P_{ij}(t)$ = probability that a component will be in state j at time t, given that it was in state i at time 0.

Since each component generates two-state continuous time Markov chain, we have pointwise availability, $P_{11}(t)$, and unavailability,

$P_{10}(t)$, of each component:

$$P_{11}(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}$$

$$P_{10}(t) = \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}$$

where $P_{11}(\infty) = \mu/(\lambda+\mu)$ represents limiting availability.

$P_{10}(\infty) = \lambda/(\lambda+\mu)$ represents limiting unavailability.

Now our interest is to determine pointwise (or limiting) availability of a complex system.

Buzacott [2] suggested network approaches which are based on successive reduction of the network into series-parallel system and on minimal paths or cuts concepts.

Singh and Billinton [9] extended Buzacott's methods into the explicit formulae for system availability mean cycle time and failure frequency in the steady - state.

As pointed out before, application of Hänsler's algorithm to a complex repairable system yields more explicit formulae for pointwise or limiting availability.

Let $A(t)$ = pointwise availability of complex system

$A(\infty)$ = limiting availability of complex system.

Using the same notation as Chap. 5, we have Theorem: If Hänsler's algorith is available, then the pointwise unavailability of the complex system is given by

$$\bar{A}(t) = \sum_{j=1}^{c} \left[P_{10}(t)\right]^{S_j} \left[P_{11}(t)\right]^{\gamma_j}$$

and limiting unavailability will be given by

$$\bar{A}(\infty) = \sum_{j=1}^{c} \left[P_{10}(\infty)\right]^{S_j} \left[P_{11}(\infty)\right]^{\gamma_j}$$

$$= \sum_{j=1}^{c} \left(\frac{\lambda}{\lambda+\mu}\right)^{S_j} \left(\frac{\mu}{\lambda+\mu}\right)^{\gamma_j}$$

where $\bar{A}(t) = 1 - A(t)$ and $\bar{A}(\infty) = 1 - A(\infty)$ yield the pointwise and limiting availability. Since this comes directly from Hansler's theorem, the proof can be omitted.

Also the upper and lower bound on pointwise or limiting unavailability is given by

$$\sum_{i=b-(n-2)}^{b} \binom{b}{i} \left[P_{10}(t)\right]^{i} \left[P_{11}(t)\right]^{b-i} \le \bar{A}(t)$$

$$\sum_{i=c}^{b} \binom{b}{i} \left[P_{10}(t)\right]^{i} \left[P_{11}(t)\right]^{b-i} \ge \bar{A}(t)$$

$$\sum_{i=b-(n-2)}^{b} \binom{b}{i} \left(\frac{\lambda}{\lambda+\mu}\right)^{i} \left(\frac{\mu}{\lambda+\mu}\right)^{b-i} \le \bar{A}(\infty)$$

$$\sum_{i=c}^{b} \binom{b}{i} \left(\frac{\lambda}{\lambda+\mu}\right)^{i} \left(\frac{\mu}{\lambda+\mu}\right)^{b-i} \ge \bar{A}(\infty) \quad .$$

However, this approach might not derive the explicit formula for the failure frequency.

CONCLUSIONS

We have reviewed some of the major new algorithms for calculating the reliability of a complex system. An application of two of the algorithms to the calculation of time dependent reliability and to the calculation of reliability for a rapairable complex system is also discussed.

REFERENCES

[1] R. E. Barlow and F. Proshan, Mathematical Theory of Reliability, Wiley, 1965.

[2] J. A. Buzacott, "Network approaches to finding the reliability of repairable systems," IEEE Trans. Rel., vol. R-19, pp. 140-146, 1970.

[3] E. Hansler, G. K. McAuliffe, and R. S. Wilkov, "Exact calculation of computer network reliability," Networks, vol. 4, pp. 95-112, 1974.

[4] P. A. Jensen and M. Bellmore, "An algorithm to determine the reliability of a complex system," IEEE Trans. Rel., vol. R-18, pp. 160-174, 1969.

[5] Y. M. Kim, K. E. Case, and P. M. Share, "A method for computing complex system reliability," IEEE Trans. Rel., vol. R-21, pp. 215-219, 1972.

[6] E. V. Kirshnamurthy and G. Komissar, "Computeraided reliability analysis of complicated networks," IEEE Trans. Rel., vol. R-21, pp. 86-89, 1972.

[7] K. B. Misra and T.S.M. Rao, "Reliability analysis of redundant networks using flow graphs," IEEE Trans. Rel., vol. R-19, pp. 19-24, 1970.

[8] M. L. Shooman, Probabilistic Reliability: An Engineering Approach, McGraw-Hill, 1969.

[9] C. Singh and R. Billinton, "A new method to determine the failure frequency of a complex system," IEEE Trans. Rel., vol. R-23, pp. 231-234, 1974.

[10] R. VanSlyke and H. Frank, "Network reliability analysis: Part I," Networks, vol. 1, pp. 279-290, 1972.

METRIC SYSTEM

BASE UNITS:

Quantity	Unit	SI Symbol	Formula
length	metre	m	...
mass	kilogram	kg	...
time	second	s	...
electric current	ampere	A	...
thermodynamic temperature	kelvin	K	...
amount of substance	mole	mol	...
luminous intensity	candela	cd	...

SUPPLEMENTARY UNITS:

plane angle	radian	rad	...
solid angle	steradian	sr	...

DERIVED UNITS:

Acceleration	metre per second squared	...	m/s
activity (of a radioactive source)	disintegration per second	...	(disintegration)/s
angular acceleration	radian per second squared	...	rad/s
angular velocity	radian per second	...	rad/s
area	square metre	...	m
density	kilogram per cubic metre	...	kg/m
electric capacitance	farad	F	A·s/V
electrical conductance	siemens	S	A/V
electric field strength	volt per metre	...	V/m
electric inductance	henry	H	V·s/A
electric potential difference	volt	V	W/A
electric resistance	ohm		V/A
electromotive force	volt	V	W/A
energy	joule	J	N·m
entropy	joule per kelvin	...	J/K
force	newton	N	kg·m/s
frequency	hertz	Hz	(cycle)/s
illuminance	lux	lx	lm/m
luminance	candela per square metre	...	cd/m
luminous flux	lumen	lm	cd·sr
magnetic field strength	ampere per metre	...	A/m
magnetic flux	weber	Wb	V·s
magnetic flux density	tesla	T	Wb/m
magnetomotive force	ampere	A	...
power	watt	W	J/s
pressure	pascal	Pa	N/m
quantity of electricity	coulomb	C	A·s
quantity of heat	joule	J	N·m
radiant intensity	watt per steradian	...	W/sr
specific heat	joule per kilogram-kelvin	...	J/kg·K
stress	pascal	Pa	N/m
thermal conductivity	watt per metre-kelvin	...	W/m·K
velocity	metre per second	...	m/s
viscosity, dynamic	pascal-second	...	Pa·s
viscosity, kinematic	square metre per second	...	m/s
voltage	volt	V	W/A
volume	cubic metre	...	m
wavenumber	reciprocal metre	...	(wave)/m
work	joule	J	N·m

SI PREFIXES:

Multiplication Factors	Prefix	SI Symbol
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto*	h
10 = 10^{1}	deka*	da
0.1 = 10^{-1}	deci*	d
0.01 = 10^{-2}	centi*	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a

* To be avoided where possible.

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