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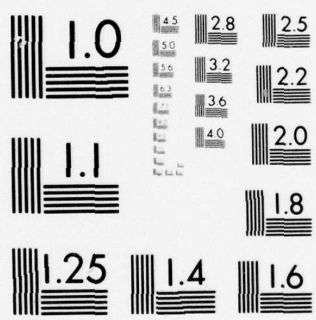

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SPATIAL SAMPLING CRITERIA FOR NEARFIELD MEASUREMENTS MADE ON CYLINDRICAL SURFACES

BY

MARTIN J EARWICKER

A.U.W.E. Publication 42341

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BY

(10) MARTIN J. EARWICKER

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SPATIAL SAMPLING CRITERIA FOR NEARFIELD MEASUREMENTS MADE ON CYLINDRICAL SURFACES

by

M. J. Earwicker

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SPATIAL SAMPLING CRITERIA FOR NEARFIELD MEASUREMENTS MADE ON CYLINDRICAL SURFACES

PRÉCIS

1. This report establishes the spatial sampling criteria for nearfield measurements, made on a cylindrical surface enclosing the sources, that enable the farfield of those sources to be calculated faithfully from the nearfield data. Examples are given of the criterial applied to both omnidirectional and directional sources: their nearfield and calculated farfield.

CONCLUSIONS

2. The sampling criteria for nearfield measurements made on a cylindrical surface have been established in this report for both axial and circumferential samples.

3. In general the common practice of using half wave length samples circumferentially is shown to be adequate for omnidirectional fields (narrow spatial bandwidths) but not adequate for the broad circumferential spatial bandwidth fields encountered with directional sources. The appropriate sampling criteria may be deduced from equations (8) and (9).

NOTATION

$\underline{X}_0 = \underline{X}_0\ (a,z,\phi)$	Nearfield co-ordinate (see Fig. 2).
$\underline{X}_1 = \underline{X}_1\ (\rho,y,\phi^1)$	Farfield co-ordinate (see Fig. 2).
$T(\underline{X}_1)$	The acoustic pressure (amplitude and phase) at the point $\underline{X}_1$.
$\hat{T}(....)$	The Fourier transform of the acoustic pressure w.r.t. ϕ or z.
$\hat{\hat{T}}(....)$	" " " " " " " w.r.t. ϕ and z.
$\frac{\partial G}{\partial n}(....)$	The Fourier transform of the normal derivative of the exact Green's function.
$\phi \xleftrightarrow{F.T.} \omega_\phi;\ \omega_\phi$	The circumferential spatial wave number.
$z \xleftrightarrow{F.T.} k_z;\ k_z$	The axial spatial wave number.
$H_\nu\ (....)$	Hankel functions of the first kind of order ν.
$\tau\ (....)$	sampled field.
$T_E\ (....)$	sampling error field.
$k = 2\pi f/_c$	the temporal wave number.
ν	an integer $-\infty$, ... $-1,0,+1$, ... $+\infty$
$\delta\ (....)$	The Dirac delta function.
$\bar{z}$	sampling spacing in z.
$\bar{\phi}$	sample spacing in ϕ.

$$\alpha_\nu = \frac{e^{i(\pi(\nu+1)/2)}}{H_\nu(ka \sin y)}$$

INTRODUCTION

4. It has been shown by several authors that the farfield of an acoustic source can be calculated from knowledge of the acoustic field on a specific surface enclosing the source (see for example ref. 1, 2, 3). This calculation involves the evaluation of a surface integral of the pressure field of the source on the enclosing surface.

5. In the case of a planar surface of integration this surface integral reduces to the form of a Fourier transform. Now the acoustic field on the surface of integration can only be determined practically at discrete points, and the required spacing of these samples can be determined by applying the standard sampling theory to the field and its Fourier transform. The result is familiar that the samples must be made at least at twice the highest spatial frequency (assuming a spatially band limited field function with its highest spatial frequency component less than the limit of propagating waves).

6. In this report the equivalent sampling process is applied to a cylindrical surface of integration and the corresponding sampling criteria established. This is achieved by making use of the fact that the surface integral relating the near and farfields can be considered as a convolution integral in terms of the cylindrical polar co-ordinate, and hence use may be made of the convolution integral for Fourier transforms which enables the sampling problem to be readily analysed in terms of the Fourier transform of the cylindrical field. The mathematical analysis describes both axial and circumferential sampling, but as the axial problem is essentially the same as for a plane surface of integration, the discussion is restricted for brevity to the circumferential sampling criteria.

7. The theory applies equally to nearfield measurements of both sources and receivers.

8. Examples are given of the sampling criteria applied to both directional and omnidirectional fields.

THEORY (see Appendix)

9. The farfield of an acoustic source may be written in terms of its nearfield pressure (amplitude and phase) measured on a closed surface surrounding the source as, (reference 1,3)

$$T(\underline{X}_1) = \iint_S T(\underline{X}_0) \frac{\partial G(\underline{r})}{\partial n} \, dS \qquad (1)$$

where $T(\underline{X})$ is the complex pressure at the point $\underline{X}$, $\underline{X}_0$ is a point on the surface S, $\underline{X}_1$ is a point exterior to the surface S, $\underline{r} = \underline{X}_1 - \underline{X}_0$ and $\frac{\partial G(\underline{r})}{\partial n}$ is the normal derivative of the exact Green's function for Dirichlet boundary conditions on the particular surface S, with respect to the outward normal.

10. Now in the co-ordinates of figure 2 it can be shown (Appendix) that for a cylindrical surface S with Dirichlet boundary conditions the farfield pressure is given in the form of a convolution in ϕ of the nearfield pressure with the normal derivative of the exact Green's function, i.e.,

$$T(\rho, y, \phi^1) = \iint_S T(a, z, \phi) \frac{\partial G(a, z, \rho, y, \phi - \phi^1)}{\partial n} \, d\phi \, dz \qquad (2)$$

Relation 2 may be rewritten, making use of the convolution theorem for Fourier transforms (reference 4), in terms of the Fourier transforms of $T(\underline{X}_0)$ and $\frac{\partial G(\underline{r})}{\partial n}$ with respect to ϕ as,

$$T(\rho,\gamma,\phi^1) = \int_z \frac{1}{2\pi} \int_{\omega_\phi} \hat{T}(z,\omega_\phi) \frac{\partial \hat{G}(z,\omega_\phi)}{\partial n} e^{(i\,\omega_\phi\,\phi)} \, d\omega_\phi \, dz \tag{3}$$

where ω_ϕ is the Fourier mate of ϕ and represents the circumferential spatial wave number. Now by using the particular form of the Green's function for a cylindrical surface with Dirichlet boundary conditions this integral may be rewritten as (see Appendix).

$$T(\rho,\gamma,\phi^1) = \frac{e^{(i\,k\,\rho)}}{2\pi^2\, a\, \rho} \sum_{\nu=\infty}^{+\infty} \hat{\hat{T}}(k_z,\omega_\phi)\, \alpha_\nu(\gamma)\, e^{(i\,\gamma\,\phi^1)} \tag{4}$$

where $\hat{\hat{T}}(k_z,\omega_\phi)$ is the Fourier transform of the nearfield pressure with respect to both ϕ and z, k_z is the Fourier mate of z and represents the spatial wave number in the z axis.

11. The errors introduced by sampling the nearfield pressure can now be found by substituting for $\hat{\hat{T}}(k_z,\omega_\phi)$ by the wave number spectrum of the sampled nearfield which then allows relation 4 to be expanded into a term which gives the correct field and an error term, (see Appendix)

$$\tau(\rho,\gamma,\phi^1) = T(\rho,\gamma,\phi^1) + T_E(\rho,\gamma,\phi^1) \tag{5}$$

where $T(\rho,\gamma,\phi^1)$ is the correct field given by relation 4, and $T_E(\rho,\gamma,\phi^1)$ is the error term given by,

$$T_E(\rho,\gamma,\phi^1) = \frac{e^{(i\,k\,\rho)}}{2\pi^2\, a\rho\bar{\phi}\bar{z}} \sum_{\nu=-\infty}^{+\infty} \left\{ \sum_{\substack{p=-\infty \\ p\neq o}}^{+\infty} \sum_{q=-\infty}^{+\infty} \hat{\hat{T}}(k_z - 2\pi p/\bar{z},\ \omega_\phi - 2\pi q/\bar{\phi}) + \sum_{\substack{q=-\infty \\ (q\neq o)}}^{+\infty} \hat{\hat{T}}(k_z,\omega_\phi - 2\pi q/\bar{\phi}) \right\} \alpha_\nu(\gamma)\, e^{(i\,\nu\,\phi^1)}$$

Thus the farfield calculated from the sampled nearfield has been separated into the correct field, and an error term which contains the true wave number spectrum of the nearfield repeated about points $2\pi p/\bar{z}$ and $2\pi q/\bar{\phi}$ in k_z and ω_ϕ respectively. Now by a suitable choice of the sample spacings $\bar{z}$ and $\bar{\phi}$ these repeated spectra may be removed from the ranges of ω_ϕ and k_z that contribute to the farfield thus allowing the correct field to be calculated from the sampled nearfield. This is discussed further in the next section.

DISCUSSION

12. It is shown above that the farfield of a source can be described by the inverse Fourier transform of, the product of the transform of the nearfield pressure on the cylindrical surface with the transform of the normal derivative of the exact Green's function for the surface. The normal derivative of the exact Green's function for a cylindrical surface with Dirichlet boundary conditions is shown in Fig. 3a,b together with its Fourier transform with respect to ϕ calculated from relation (A4). The modulus of the Fourier transformed derivative of the Green's function decays rapidly beyond some value of ω_ϕ as can be seen in Fig. 3b. The region of ω_ϕ for which the modulus of the transform is effectively non zero will depend on the accuracy required for the particular problem and will also depend on the value of ka sin γ. This is shown in Fig. 3c. For most practical purposes values of the normalised modulus,

$$\left| \frac{\partial \hat{G}(\omega_\phi)}{\partial n} \right| \Big/ \left| \frac{\partial \hat{G}(o)}{\partial n} \right| > 10^{-10}$$

are considered non zero and the corresponding region of ω_ϕ can be seen from Fig 3c to be well approximated by the relation

$$|\omega_\phi| \leqslant M \text{ ka } \sin\gamma + C$$

where $M = 1.4$ and $C = 16$ in the range, $10 < \text{ka} \sin\gamma < 50$. In the limit of large ka sin γ the gradient of the line tends to 1. Should greater accuracy be required then the approximate value of M and C can be determined from Fig 3c. Now the Fourier transform of the nearfield pressure on the cylindrical surface has been shown due to the sampling, to be repeated about points in ω_ϕ spaced $2\pi/\bar{\phi}$ apart (see Reference 5, Fig 4a and Appendix). The region of this transformed field that contributes to the farfield is that part which is enclosed by the effectively non zero region of the modulus of the Fourier transformed derivative of the Green's function,

$$\left| \frac{\partial \hat{G}(\omega_\phi)}{\partial n} \right|$$

Thus the sampling interval ϕ must be small enough to ensure that the repeated spectra do not overlap into this region. This may be summarised as (see Fig 4a)

$$\frac{2\pi}{\bar{\phi}} > \omega_{\phi\max} + M \text{ ka } \sin\gamma + C \tag{7a}$$

where $\omega_{\phi\max}$ is the highest circumferential spatial wave number of the nearfield pressure. Rearranging equation 7a gives the required angular sample spacing ϕ as,

$$\bar{\phi} < 2\pi \Big/ (M \text{ ka } \sin\gamma + C + \omega_{\phi\max}) \tag{7b}$$

and the circumferential sample spacing as,

$$\bar{x} < \lambda \Big/ \{\sin\gamma\, [M + (C + \omega_{\phi\max})/\text{ka} \sin\gamma]\} \tag{8}$$

Consider now two special cases:-

Broad Circumferential Spatial bandwidth

13. The nearfield pressure in this example is taken as,

$$T(z,\phi) = \cos^4 (\phi/2) \exp (i\ 2\pi a(1 - \cos \phi)/\lambda)$$

which is assumed for the purpose of analysis to be typical of the field of a directional source. This function and its Fourier transform are shown in Fig 5a,b for ka sin γ = 25.8.

14. The maximum spatial frequency in this example can be taken approximately as 50. Hence from relation 7 the angular sample spacing is given as $\bar{\phi} < 0.06$ radians or in terms of the circumferential samples $\bar{x} < \lambda/(4 \sin \gamma)$.

15. The effect of various sampling intervals on the predicted farfield is shown in Fig 6a,b together with the spectra of the nearfield for ka sin γ = 25.8. It can be seen that $\lambda/4$ samples will indeed allow the farfield to be calculated with an error term less than about 95 dB below the pattern maximum; somewhat more accurate than normally required! However with nearfield samples every $\lambda/2$ spacing the error term in this example is only 25 dB below the pattern maximum. So although a sampling criterion of $\lambda/2$ spacing is in this case shown to be inadequate a criterion of $\lambda/4$ spacing is certainly sufficient.

Narrow circumferential spatial bandwidth

16. The nearfield pressure in this example is taken as

$$T(z,\phi) = 1$$

which is an ideal non-directional circumferential field. The maximum circumferential spatial wave number is obvious, ω_ϕ max = 0. Hence from relation 7b and taking ka sin γ = 25.8, M = 1.4 and C = 16 we have that the circumferential sampling criterion is $\bar{x} < \lambda/(2 \sin \gamma)$.

17. The effect of various sampling intervals on the calculated farfield is shown in Fig 7a,b together with the nearfield spectra. It can be seen that the nearfield is faithfully calculated with $\lambda/2$ samples; with 0.8λ samples the error is about 0.5 dB and with 1.6λ samples about 8 dB.

Circumferential and Axial spatial sampling criteria

18. It can be seen above that the error term, relation 6, will be negligible if the following inequalities are satisfied, (see Fig 4). For axial samples,

$$\frac{2\pi}{\bar{z}} > k + k_{zmax} \qquad (9)$$

i.e. $\bar{z} < 2\pi/(k + k_{zmax})$

where $k = \frac{2\pi f}{c}$, and is the limit of real angles for γ, and k_{zmax} is the highest axial spatial wave number present. This leads to the familiar $\lambda/2$ sampling criterion when k_z max = k.

For circumferential sampling,

$$\bar{\phi} < 2\pi / (M\ ka \sin y + C + \omega_{\phi max}) \tag{7b}$$

or

$$\bar{x} < \lambda / \{\sin y\ [M + (C + \omega_{\phi max})/ka \sin y]\} \tag{8}$$

where $\omega_{\phi max}$ is the highest circumferential spatial wave number and M = 1.4 and C = 16 for most practical purposes. Equations (8) and (9) are shown evaluated in figs 8a,b.

CONCLUSIONS

19. The sampling criteria for nearfield measurements made on a cylindrical surface have been established in this report for both axial and circumferential samples.

20. In general the common practice of using half wave length samples circumferentially is shown to be adequate for omnidirectional fields (narrow spatial bandwidths) but not adequate for the broad circumferential spatial bandwidth fields encountered with directional sources. The appropriate sampling criteria may be deduced from equations (8) and (9).

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21. My thanks are due to Mrs Dilys Grant for writing the computer programs and to Mr C. J. S. Grant for producing the computer graphics.

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APPENDIX 1: MATHEMATICAL DETAILS

1. The field at a point $\underline{X}_1$ exterior to a closed surface S containing the sources may be written for Dirichlet boundary conditions on the surface S.

$$T(\underline{X}_1) = \iint_S T(\underline{X}_0) \frac{\partial G(\underline{r})}{\partial n} ds \qquad \text{A1}$$

where $T(\underline{X}_0)$ is the field at a point $\underline{X}_0$ on S and $\frac{\partial G(\underline{r})}{\partial n}$ is the normal derivative of the Green's function for Dirichlet boundary conditions on S.

2. For an infinite cylindrical surface S we may write (see Fig 2)

$$\frac{\partial G(\underline{r})}{\partial n} = \frac{e^{ik(\rho - z\cos\gamma)}}{2\pi^2 a\rho} \sum_{\nu=-\infty}^{+\infty} \frac{\cos(\nu(\phi - \phi^1))\, e^{i(\pi(\nu+1)/2)}}{H_\nu(ka\sin\gamma)} \qquad \text{A2}$$

or for simplicity

$$\frac{\partial G(\underline{r})}{\partial n} = \beta(z,\gamma) \sum_{\nu=-\infty}^{+\infty} \alpha_\nu(\gamma)\cos(\nu(\phi - \phi^1))$$

where $\beta(z,\gamma) = \frac{e^{ik(\rho - z\cos\gamma)}}{2\pi^2 a\rho}$ and $\alpha_\nu(\gamma) = \frac{e^{i(\pi(\nu+1)/2)}}{H_\nu(ka\sin\gamma)}$

Equation A1 may now be rewritten,

$$T(\rho,\gamma,\phi^1) = \iint_S T(a,z,\phi) \frac{\partial G}{\partial n}(a,z,\rho,\gamma,\phi - \phi^1)\, d\phi\, dz \qquad \text{A2}$$

which has the form of a convolution in ϕ and hence we may Fourier transform $T(a,z,\phi)$ and $\frac{\partial G}{\partial n}(a,z,\rho,\gamma,\phi - \phi^1)$ with respect to ϕ and use the convolution theorem for Fourier transforms to give,

$$T(\rho,\gamma,\phi^1) = \int_z \frac{1}{2\pi}\int_{\omega_\phi} \hat{T}(z,\omega_\phi) \frac{\partial \hat{G}}{\partial n}(z,\omega_\phi)\, e^{i\omega_\phi\phi^1}\, d\omega_\phi\, dz \qquad \text{A3}$$

Now we may write the Fourier transform of $\frac{\partial G}{\partial n}(z,\phi)$ with respect to ϕ as,

$$\int_{-\infty}^{+\infty} \frac{\partial G(z,\phi)}{\partial n} e^{-i\omega_\phi\phi}\, d\phi = \int_{-\infty}^{+\infty} \left\{\beta(z,\gamma) \sum_{\nu=-\infty}^{\infty} \alpha_\nu(\gamma)\cos(\nu\phi)\right\} e^{-i\omega_\phi\phi}\, d\phi$$

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which as the series is uniformly convergent can be written,

$$\int_{-\infty}^{+\infty} \frac{\partial G(z,\phi)}{\partial n} e^{-i\omega_\phi \phi} d\phi = \beta(z,\gamma) \sum_{\nu=-\infty}^{\infty} \alpha_\nu(\gamma) \int_{-\infty}^{+\infty} \cos(\nu\phi)\, e^{-i\omega_\phi \phi} d\phi$$

Now

$$\int_{-\infty}^{+\infty} \cos(\nu\phi)\, e^{-i\omega_\phi \phi} d\phi = \pi\,\{\delta(\omega_\phi - \nu) + \delta(\omega_\phi + \nu)\}$$

hence

$$\frac{\partial \hat{G}(z,\omega_\phi)}{\partial n} = \int_{-\infty}^{+\infty} \frac{\partial G(z,\phi)}{\partial n} e^{-i\omega_\phi \phi} d\phi = \beta(z,\gamma)\pi \sum_{\nu=-\infty}^{+\infty} \alpha_\nu(\gamma)\,\{\delta(\omega_\phi - \nu) + \delta(\omega_\phi + \nu)\} \qquad \text{A4}$$

This then is the Fourier transform with respect to ϕ of $\frac{\partial G(z,\phi)}{\partial n}$ which we may now use in the evaluation of equation A3.

Hence,

$$T(\rho,\gamma,\phi^1) = \int_{-\infty}^{+\infty} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{T}(z,\omega_\phi)\, \beta(z,\gamma)\pi \sum_{\nu=-\infty}^{+\infty} \alpha_\nu(\gamma)\,\{\delta(\omega_\phi - \nu) + \delta(\omega_\phi + \nu)\}\, e^{i\omega_\phi \phi^1}\, d\omega_\phi\, dz$$

or

$$T(\rho,\gamma,\phi^1) = \sum_{\nu=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \beta(z,\gamma)\, \hat{T}(z,\nu)\, dz\, \alpha_\nu(\gamma)\, e^{i\nu\phi^1} \qquad \text{A5}$$

This then gives the farfield pressure $T(\rho,\gamma,\phi^1)$ in terms of the Fourier transformed, in ϕ, nearfield pressure $\hat{T}(z,\nu)$.

Sampling of the Neafield Pressure

3. Suppose that the nearfield pressure $T(z,\phi)$ is not known as a continuous function of z and ϕ but is sampled at discrete points $\bar{z}$, $\bar{\phi}$ apart then we may write the sampled nearfield as

$$\tau(z,\phi) = \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} T(p\bar{z}, q\bar{\phi})\, \delta(z - pz)\, \delta(\phi - q\bar{\phi})$$

and hence its Fourier transform as, (ref 5)

$$\hat{\tau}(z,\omega_\phi) = \frac{1}{\bar{\phi}} \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} \hat{T}(pz, \omega - \frac{2\pi q}{\bar{\phi}})\, \delta(z - pz)$$

Thus we may rewite equation A5 as

$$T(\rho,\gamma,\phi^1) = \sum_{\nu=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \hat{\tau}(z,\omega_\phi)\ \beta(z,\gamma)\ dz\ \alpha_\nu(\gamma)\ e^{i\nu\phi^1}$$

which by expanding $\beta(z,\gamma)$ and letting $k_z = k \cos \gamma$ may be written,

$$T(\rho,\gamma,\phi^1) = \frac{e^{ik\rho}}{2\pi^2\ a\rho} \sum_{\nu=-\infty}^{+\infty} \hat{\hat{T}}\ (k_z,\omega_\phi)\ \alpha_\nu(\gamma)\ e^{i\nu\phi^1} \qquad \text{A6}$$

and as $\hat{\hat{\tau}}(k_z,\omega_\phi)$ may be written

$$\hat{\hat{\tau}}(k_z,\omega_\phi) = \frac{1}{\bar{\phi}\ \bar{z}} \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} \hat{\hat{T}}\ (k_z - \frac{p2\pi}{\bar{z}},\ \omega_\phi - \frac{2\pi q}{\bar{\phi}}) \qquad \text{A7}$$

we have,

$$\tau(\rho,\gamma,\phi^1) = \frac{e^{ik\rho}}{2\pi^2 a\rho\bar{\phi}\bar{z}} \sum_{\nu=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} \hat{\hat{T}}(k_z - \frac{2\pi\rho}{\bar{z}},\ \omega_\phi - \frac{2\pi q}{\bar{\phi}})\ \alpha_\nu(\gamma) e^{i\nu\phi^1} \qquad \text{A8}$$

or substituting for $\alpha_\nu(\gamma)$ and letting ka sin (γ) be rewritten as $(k^2 - k_z^{\ 2})^{1/2}$ we have,

$$\tau(\rho,\gamma,\phi^1) = \frac{e^{ik\rho}}{2\pi^2\ a\rho\bar{z}\ \bar{\phi}} \sum_{\nu=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} \hat{\hat{T}}(k_z - \frac{2\pi p}{\bar{z}},\ \omega_\phi - \frac{2\pi q}{\bar{\phi}})\ \frac{e^{(i(\pi(\nu+1)/2)\nu\phi^1)}}{H_\nu((k^2-k_z^{\ 2})^{1/2}a)}$$

Now this may be separated into the correct field $T(\rho,\gamma,\phi^1)$ and the error term $T_E(\rho,\gamma,\phi^1)$ as,

$$\tau(\rho,\gamma,\phi^1) = T(\rho,\gamma,\phi^1) + T_E(\rho,\gamma,\phi^1) \qquad \text{A9}$$

where, the correct field is given by,

$$T(\rho,\gamma,\phi^1) = \frac{e^{(ik\rho)}}{2\pi^2\ a\rho} \sum_{\nu=-\infty}^{+\infty} \hat{\hat{T}}(k_z,\omega_\phi)\ \alpha_\nu(\gamma)\ e^{(i\nu\phi^1)}$$

and the error term is given by,

$$T_E(\rho,\gamma,\phi^1) = \frac{e^{(ik\rho)}}{2\pi^2 a\rho\,\bar{\phi}\bar{z}} \sum_{\nu=-\infty}^{+\infty} \left\{ \sum_{\substack{p=-\infty \\ (p \neq o)}}^{+\infty} \sum_{q=-\infty}^{+\infty} \hat{\hat{T}}(k_z - \frac{2\pi p}{\bar{z}}, \omega_\phi - \frac{2\pi q}{\bar{\phi}}) \right.$$

$$\left. + \sum_{\substack{q=-\infty \\ (q \neq o)}}^{\infty} \hat{\hat{T}}(k_z, \omega_\phi - \frac{2\pi q}{\bar{\phi}}) \right\} \alpha_\nu(\gamma)\; e^{(i\nu\phi^1)}$$

This error term will be negligible as long as the following inequalities are obeyed,

$$\frac{2\pi}{\bar{z}} > k_o + k_{zmax} \qquad \text{A10}$$

where $k = \frac{2\pi f}{c}$ and k_{zmax} is the highest axial spatial wave number present, and

$$\frac{2\pi}{\bar{\phi}} > M\ ka\ \sin\gamma + C + \omega_{\phi max} \qquad \text{A11}$$

where $\omega_{\phi max}$ is the highest circumferential spatial wave number and M and C are constants depending on the accuracy required, which for most practical purposes take the values $M = 1.4$ and $C = 16$.

Asymptotic limits of the circumferential sampling Criteria

4. It is interesting to consider the circumferential sampling criteria for the limit of large radius a. Let equations A10 and A11 be rewritten

$$\bar{z} < \frac{2\pi}{k(1 + \frac{k_{zmax}}{k})} \qquad \text{A11}$$

$$\bar{x} < \frac{2\pi a}{(M\ ka\ \sin\gamma + C)\left(1 + \frac{\omega_{\phi max}}{(ka\ \sin\gamma + C)}\right)} \qquad \text{A12}$$

Now let $\frac{k_{zmax}}{k} = \frac{\omega_{\phi max}}{(M\ ka\ \sin\gamma + C)} = \Delta$ a constant, then

we have $\bar{z} < \frac{\lambda}{(1 + \Delta)}$

$$\bar{x} < \frac{\lambda}{(1 + \Delta)\ \sin\gamma\ (M + \frac{c}{ka\ \sin\gamma})}$$

Now for large a, $\frac{c}{ka \sin \gamma}$ will tend to zero and M tends to 1 (see reference 6 equation 9.2.3, and equation A2) and the limit becomes

$$\bar{x} < \frac{\lambda}{(1 + \Delta) \sin \gamma}$$

which for the equitorial plan $\gamma = 90^\circ$ is the same criterion as for the axial field.

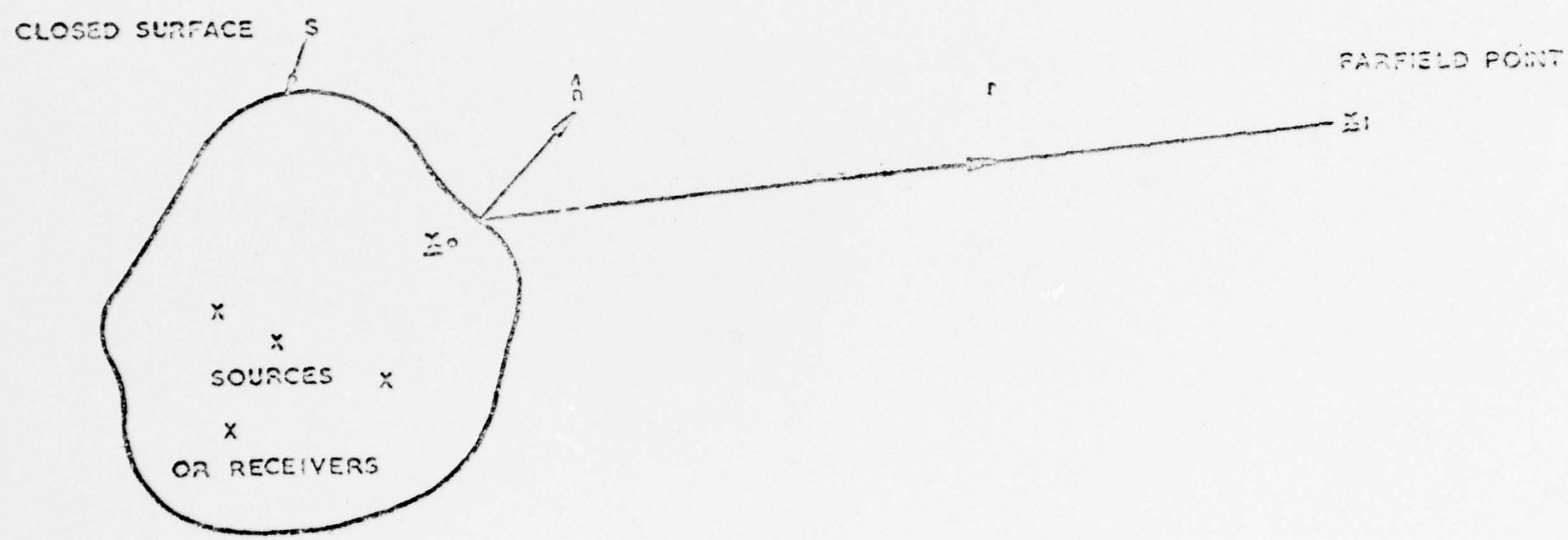
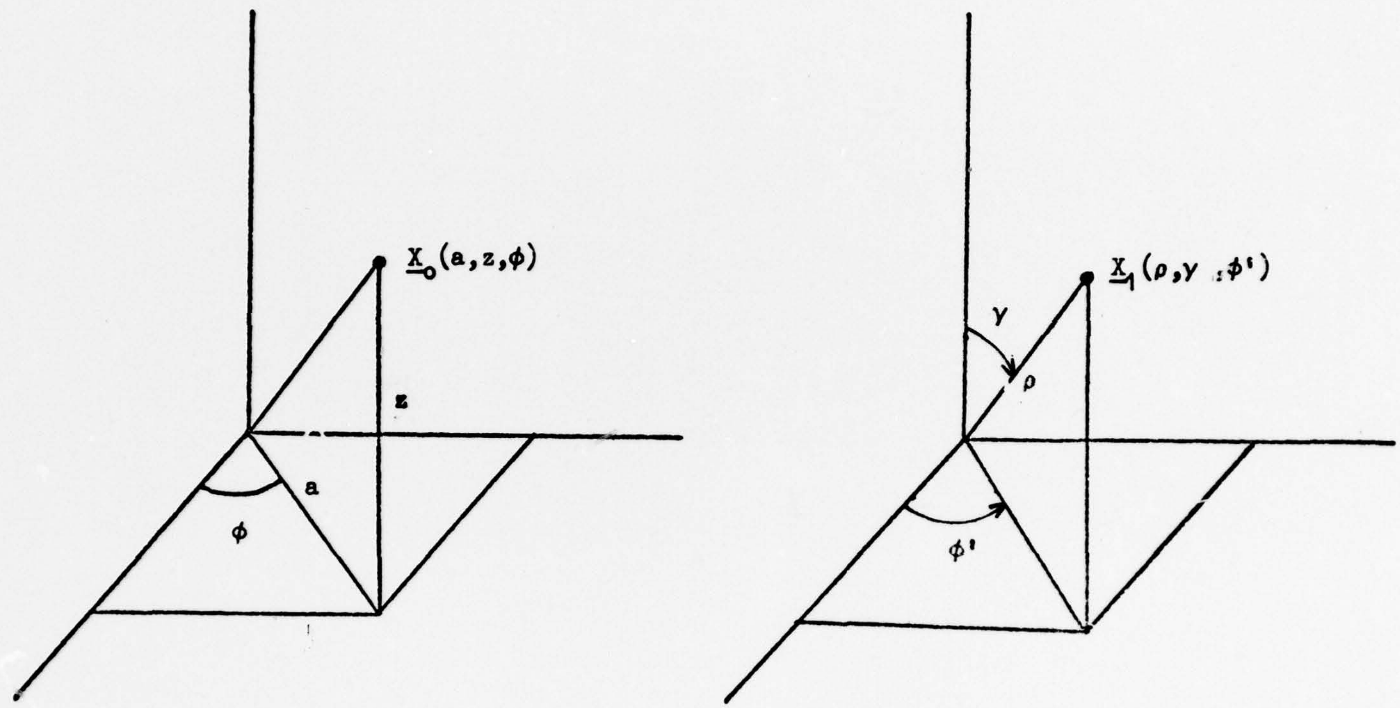


FIG. 1. GENERAL CO-ORDINATES



NEARFIELD COORDINATE

FARFIELD COORDINATE

FIG. 2 NEAR AND FARFIELD COORDINATES

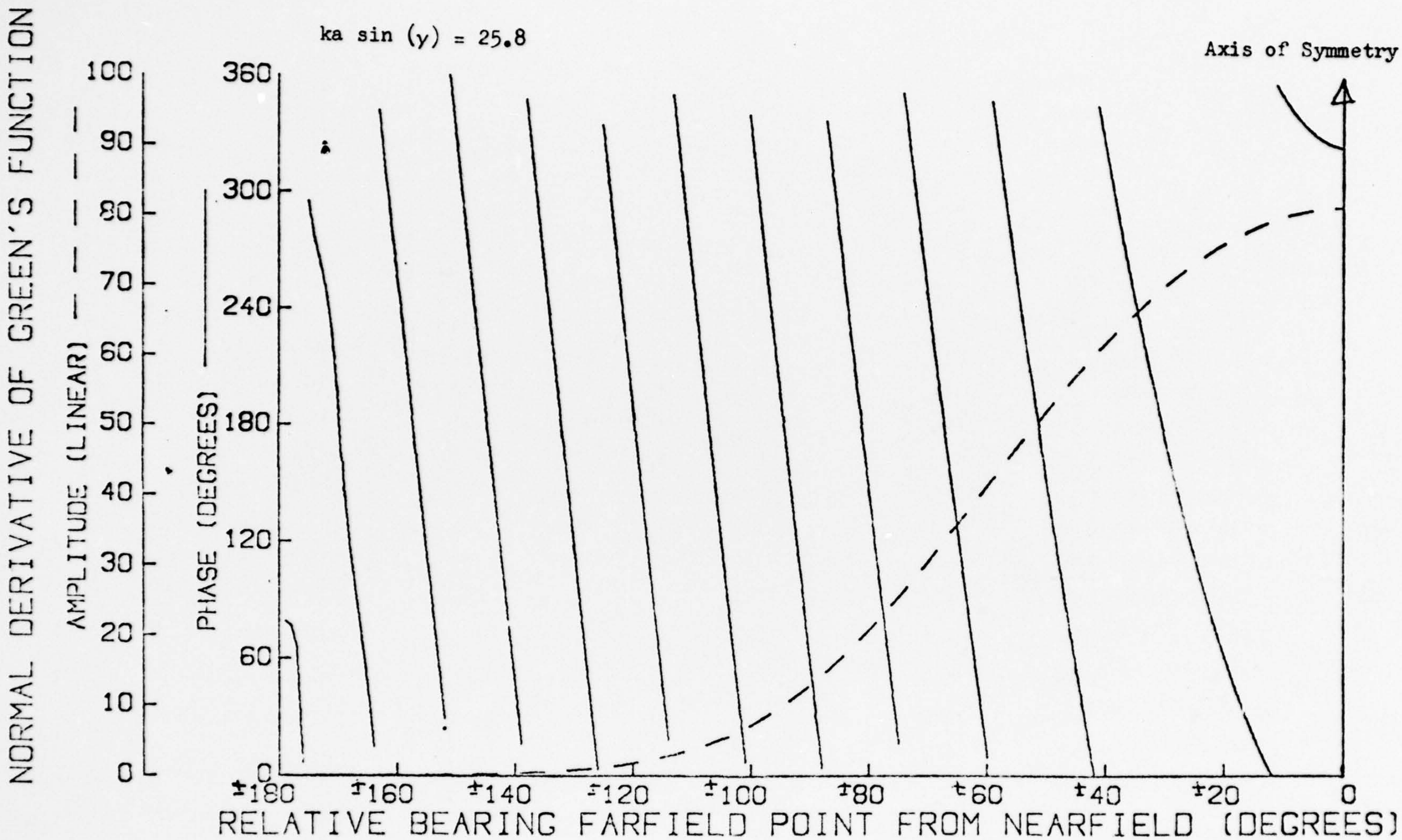


FIG. 3a. NORMAL DERIVATIVE OF THE GREEN'S FUNCTION FOR A CYLINDER

FIG. 3b

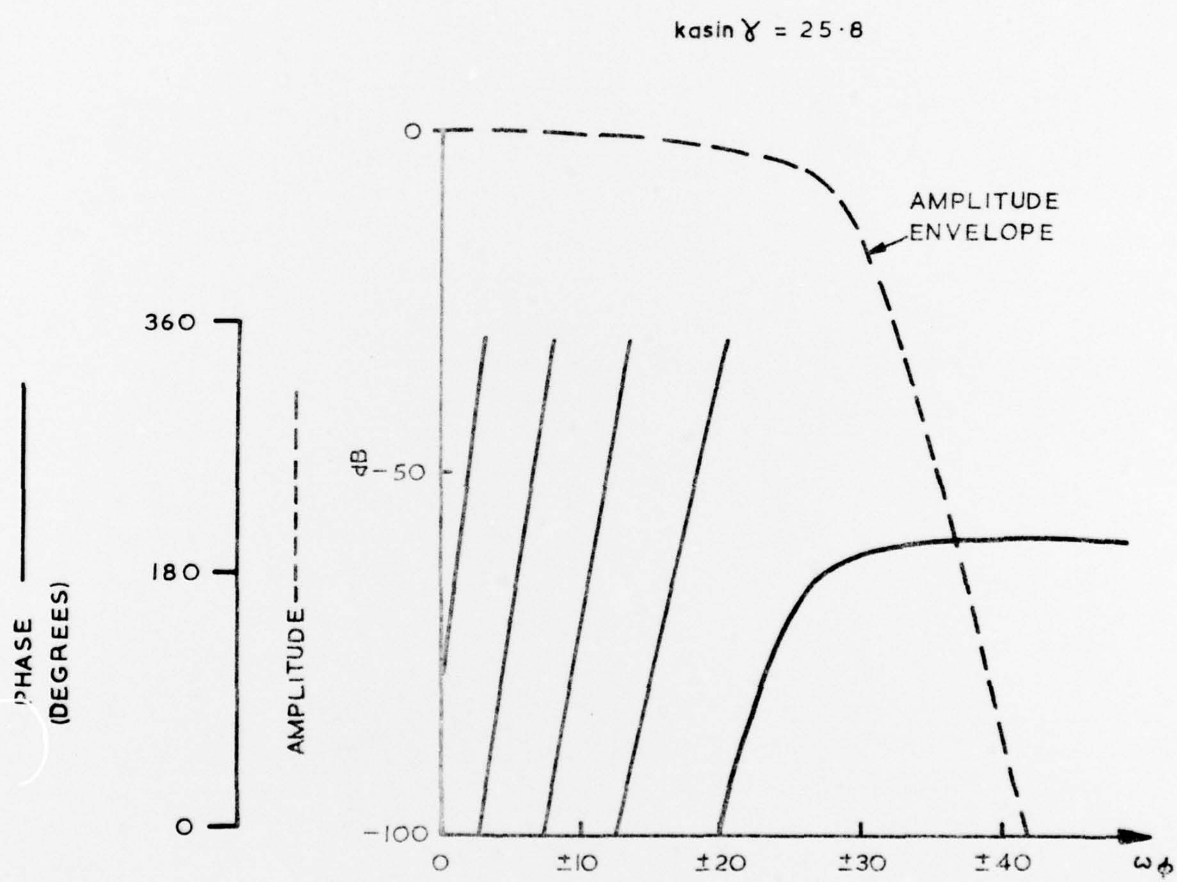


FIG. 3b FOURIER TRANSFORM OF THE NORMAL DERIVATIVE OF THE GREEN'S FUNCTION FOR A CYLINDER

FIG. 3C

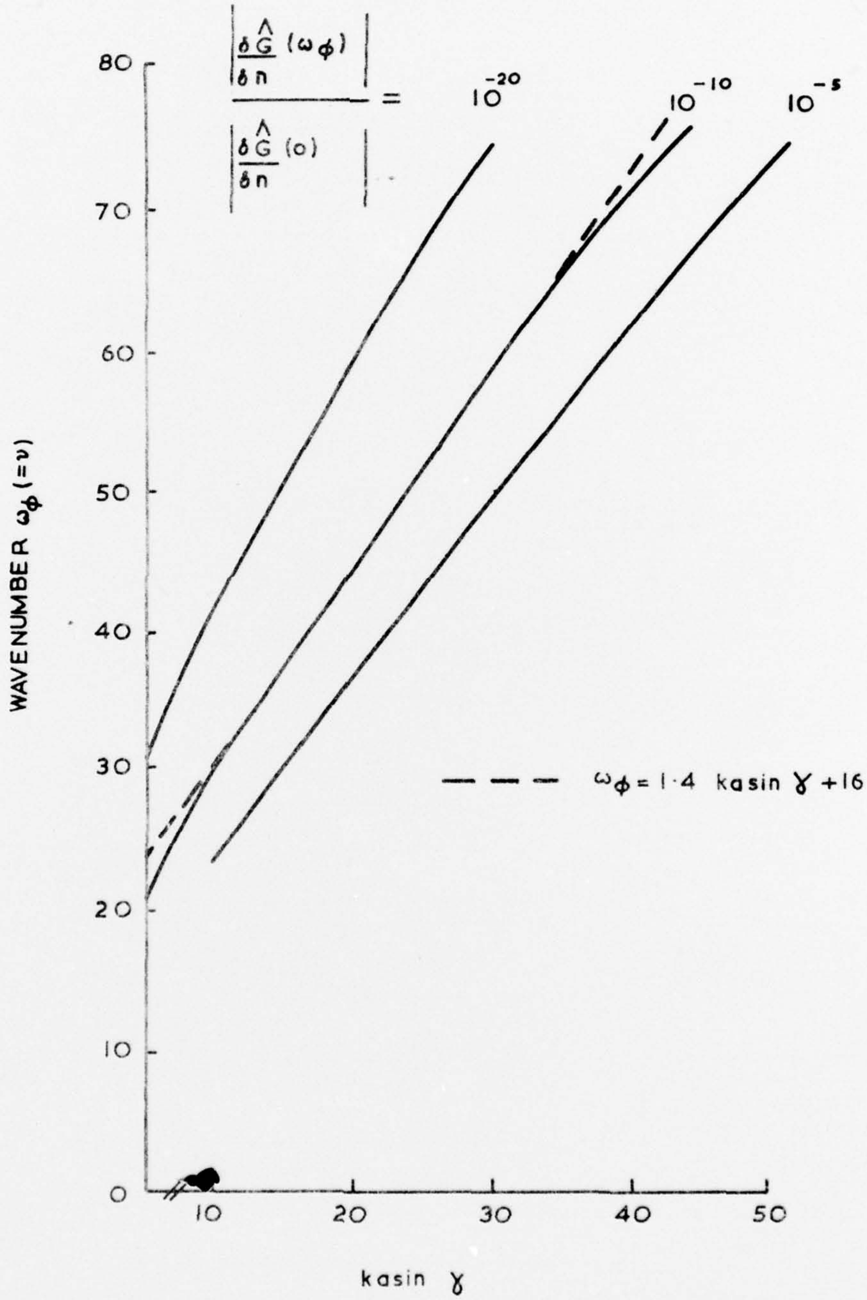


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FIG. 3C FOURIER TRANSFORM OF NORMAL DERIVATIVE OF THE GREEN'S FUNCTION FOR A CYLINDER: MAGNITUDE

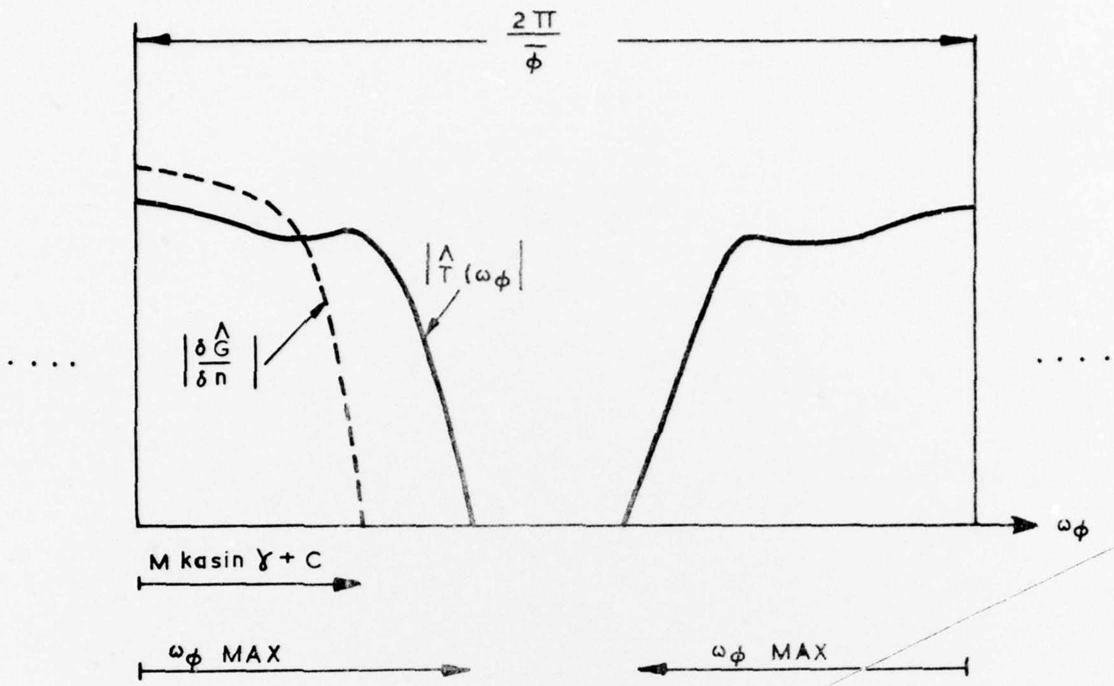

FIG. 4a CIRCUMFERENTIAL SPECTRUM

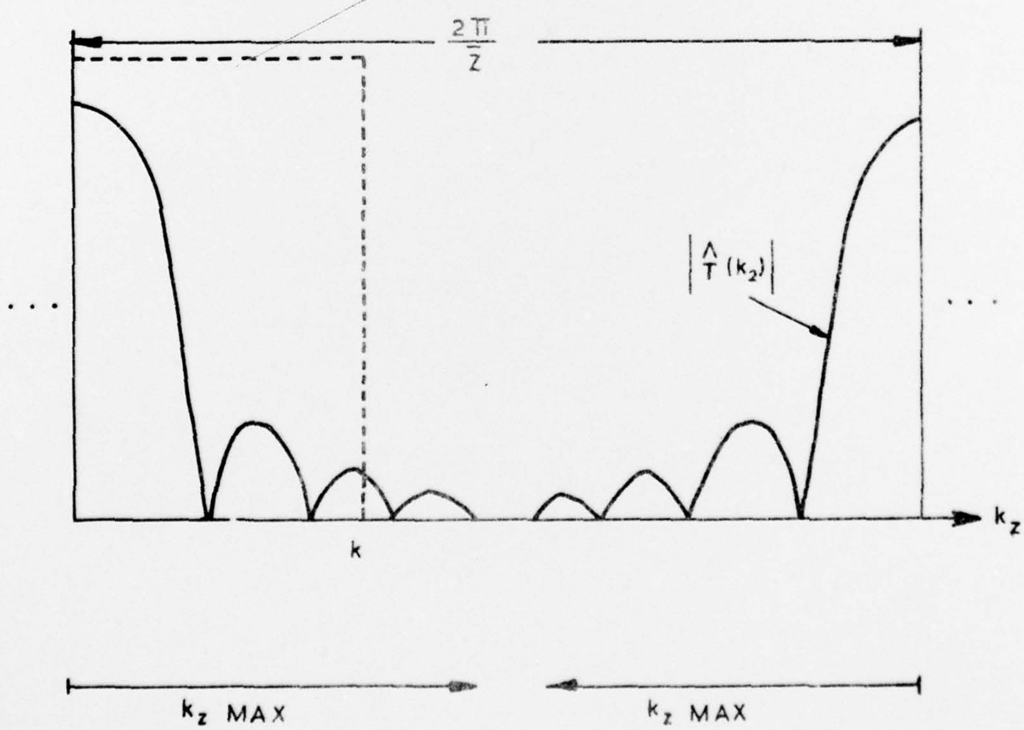

FIG. 4b AXIAL SPECTRUM

FIG. 4a, 4b SAMPLED SPECTRA

FIG. 5a, 5b

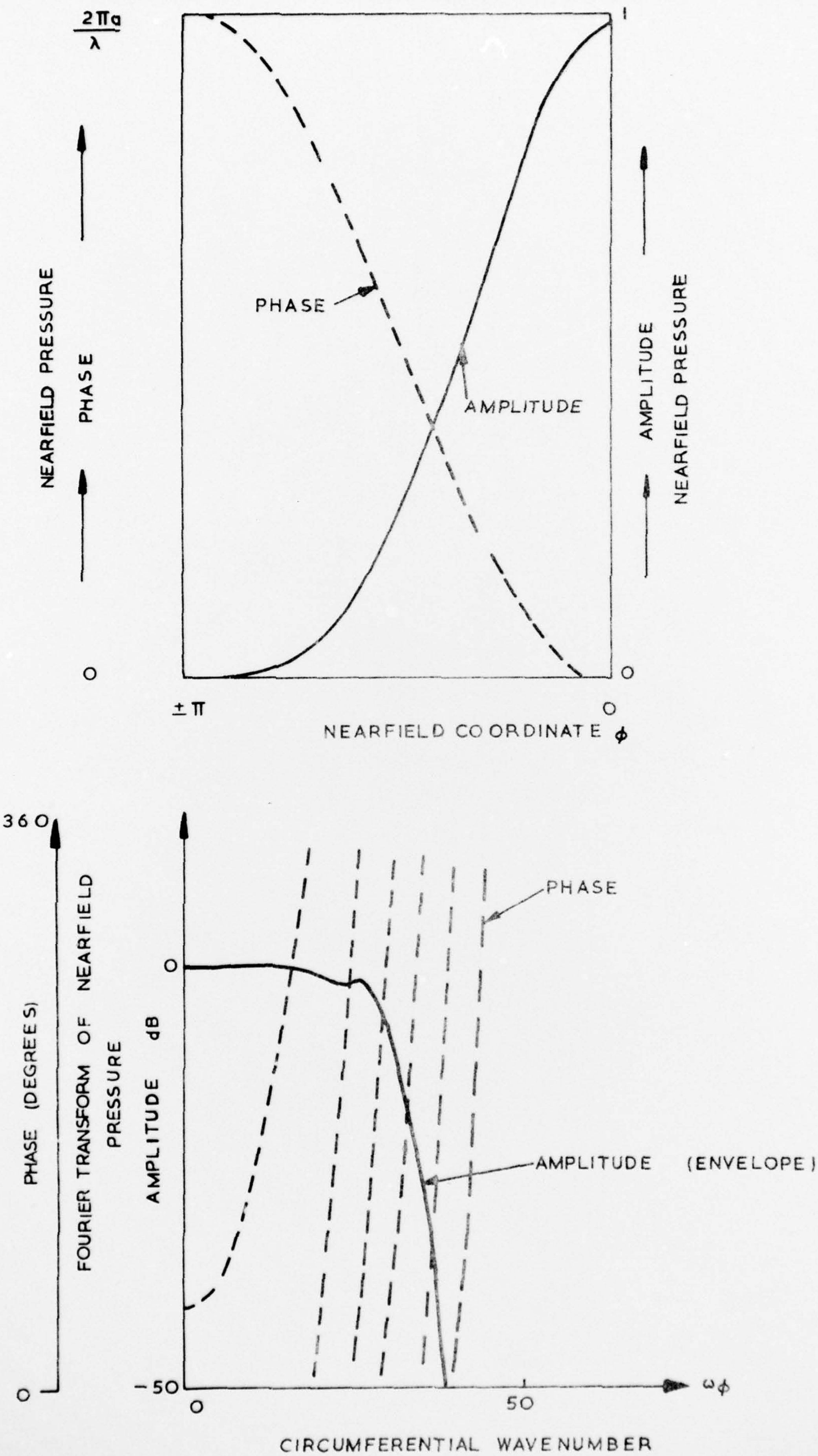

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FIG. 5a, 5b NEARFIELD PRESSURE AND IT'S FOURIER TRANSFORM

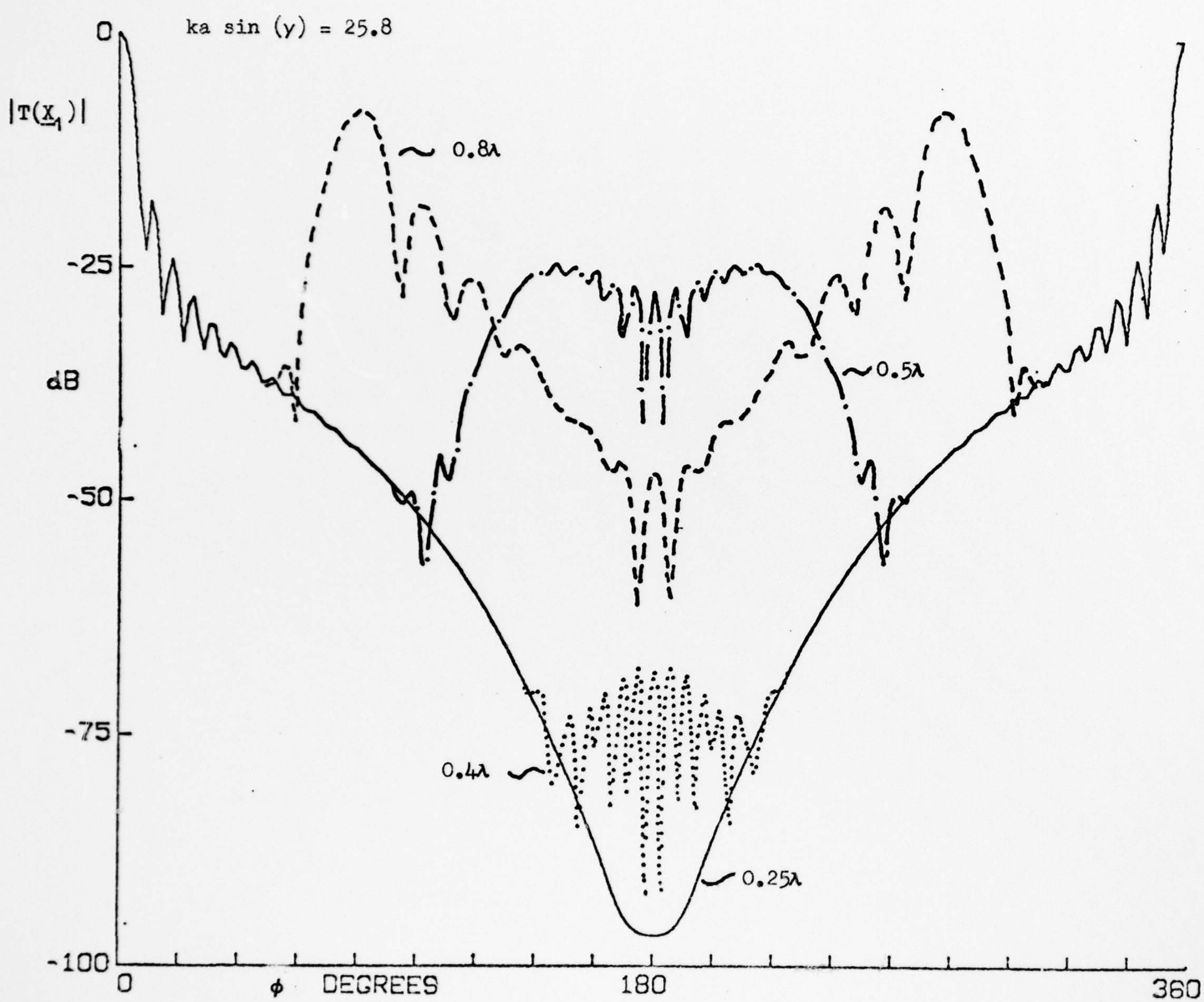

FIG. 6a. PREDICTED FARFIELD FOR VARIOUS SAMPLING CRITERIA

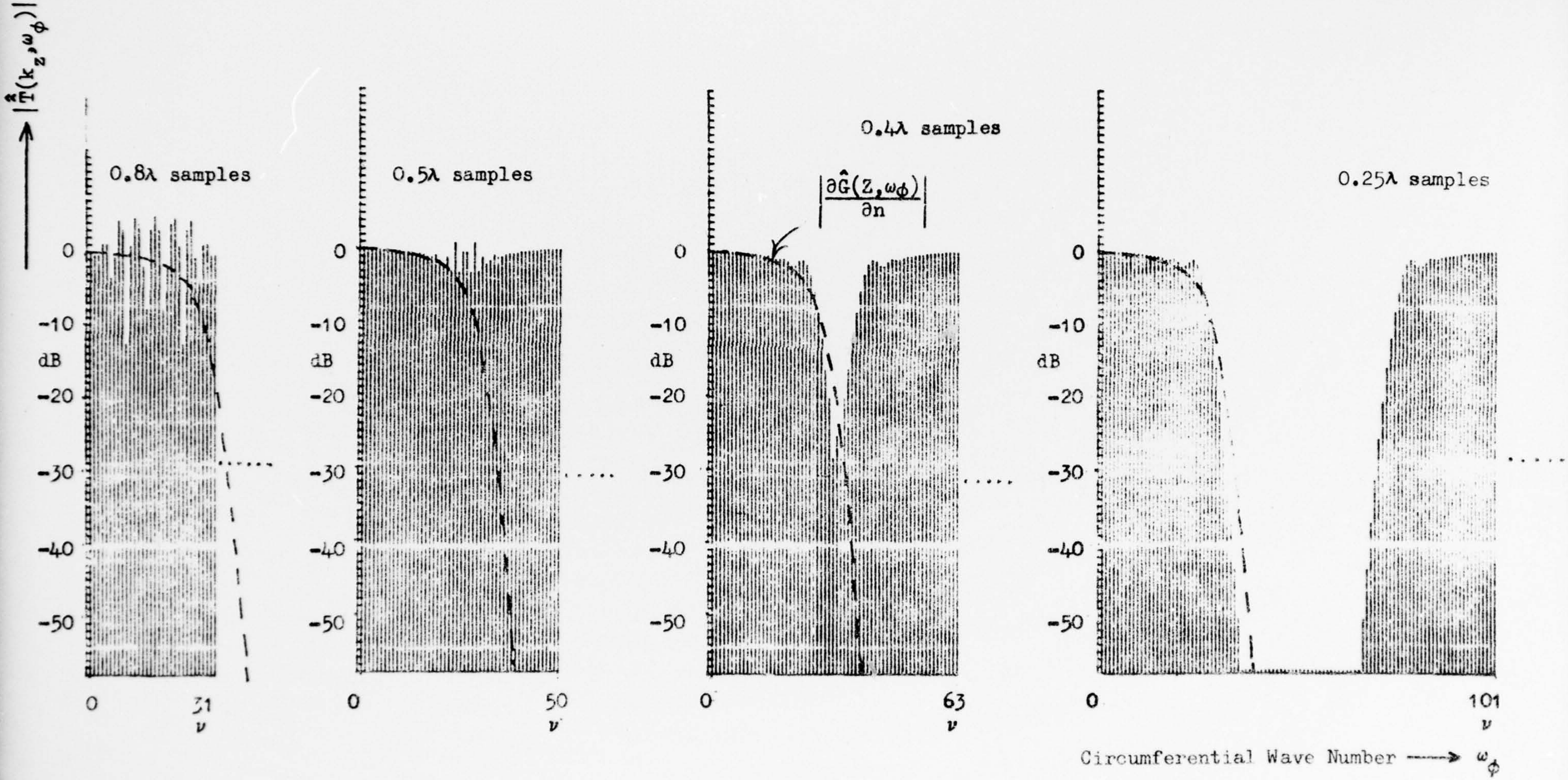

FIG. 6b. FOURIER TRANSFORM (w.r.t. ϕ) OF NEARFIELD CIRCUMFERENTIAL PRESSURE FOR VARIOUS SAMPLE SPACINGS

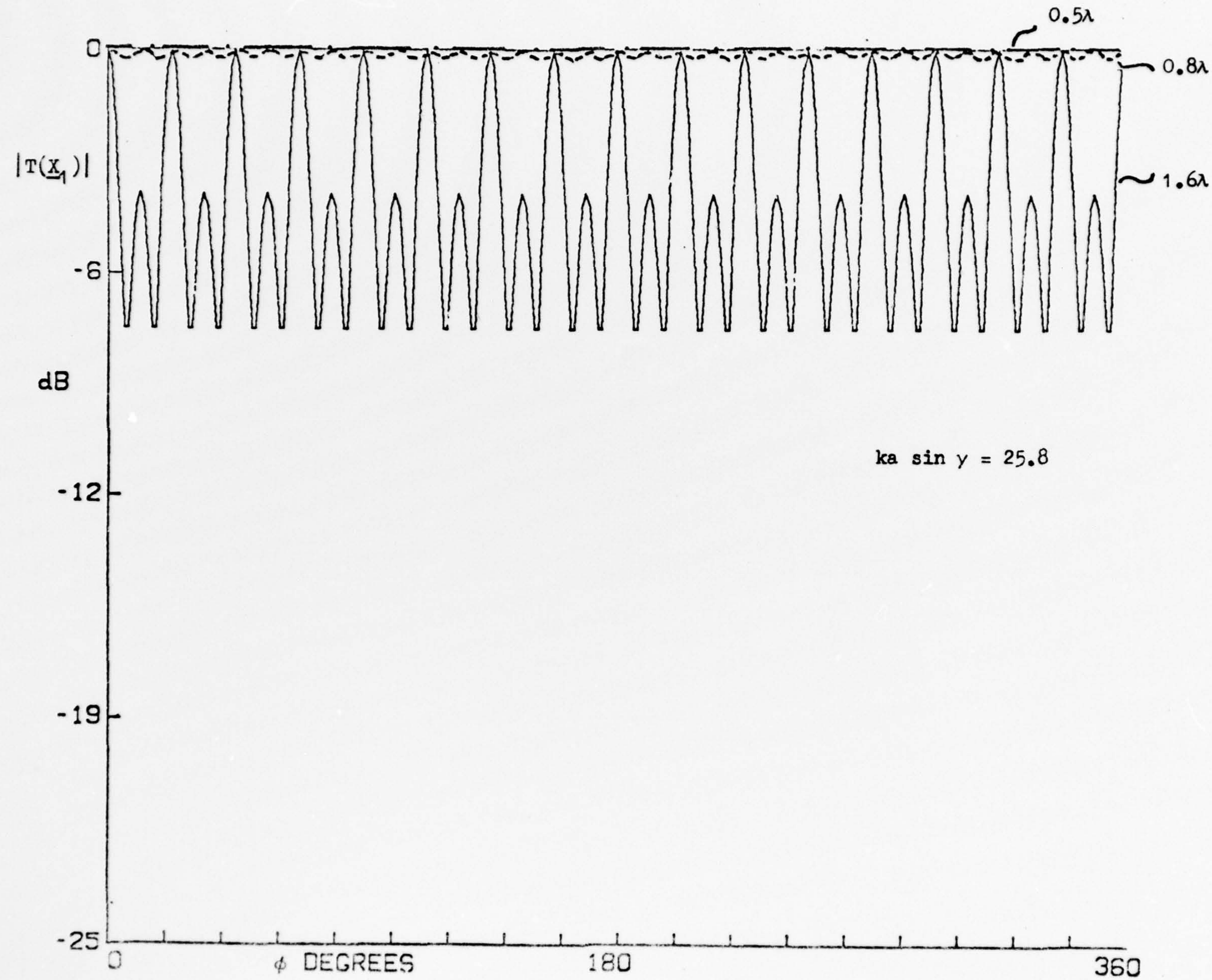

FIG. 7a. PREDICTED FARFIELD FOR VARIOUS SAMPLING CRITERIA

FIG. 7b

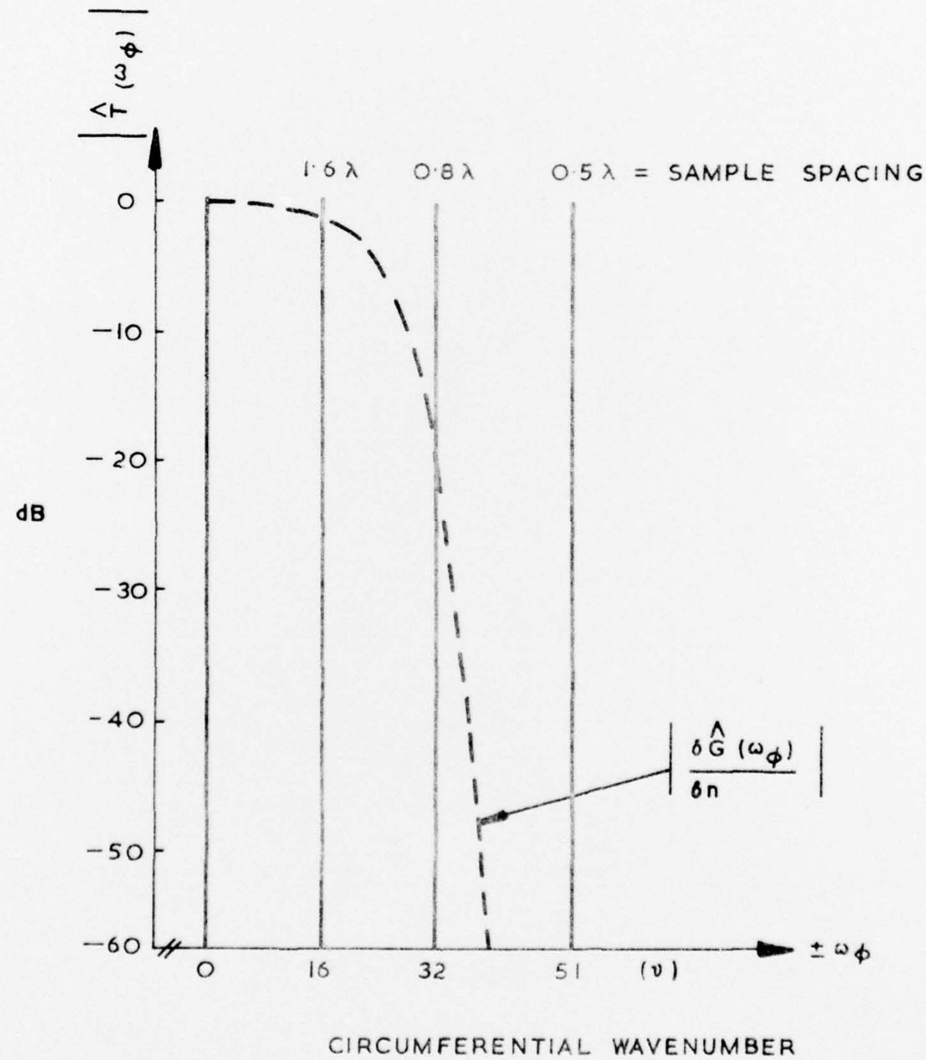

FIG. 7b FOURIER TRANSFORM OF SAMPLED NEARFIELD (OMNIDIRECTIONAL)

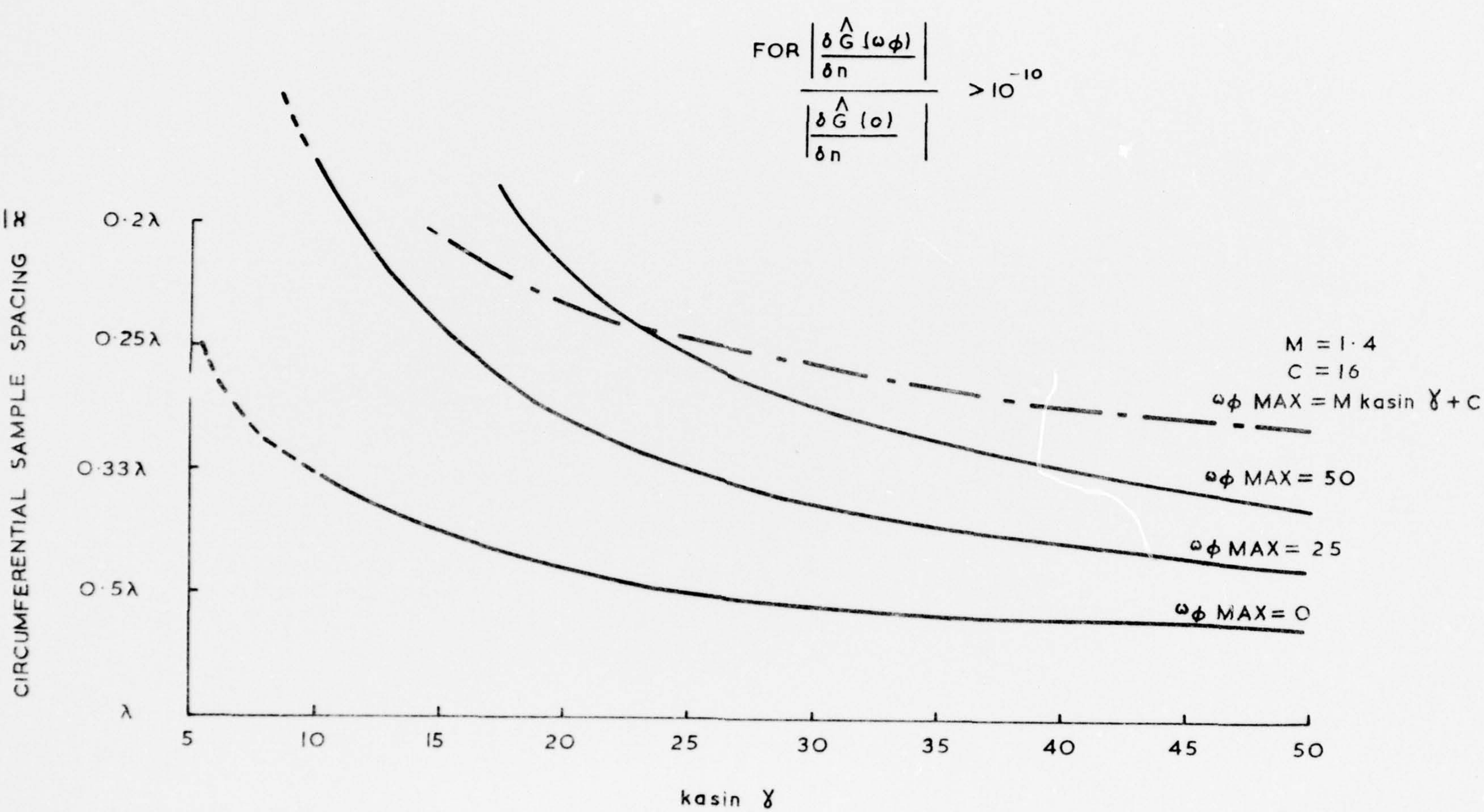

FIG. 8a CIRUMFERENTIAL SAMPLING CRITERIA

FIG. 8a

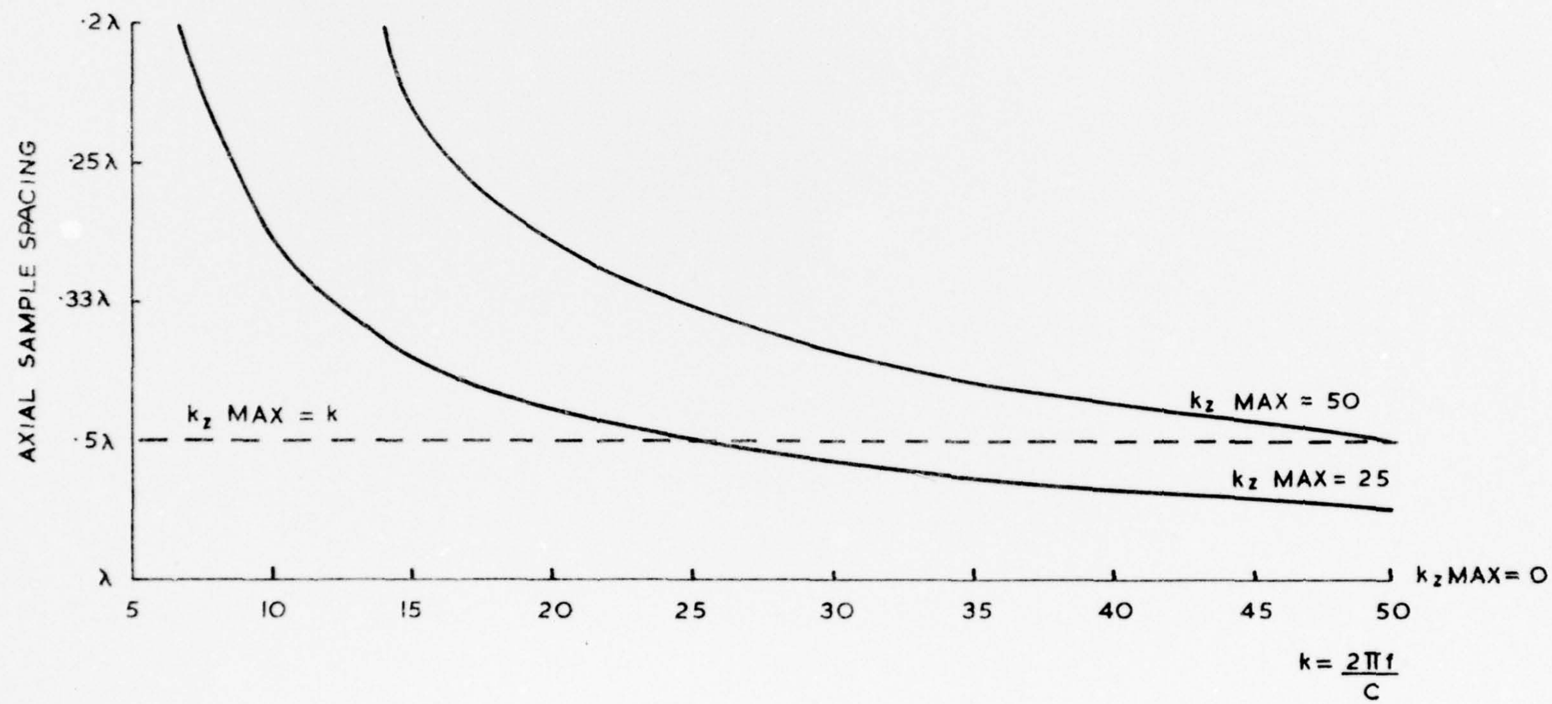

FIG. 8b AXIAL SAMPLING CRITERIA

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