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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

AN APPLICATION AND COMPARISON OF
STATIC MARGINAL ANALYSIS AND
GENERALIZED LAGRANGE MULTIPLIERS
IN GENERATING A U.S. NAVY
REPAIR MATERIALS REQUIREMENT LIST

by

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December 1976

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(20. ABSTRACT Continued)

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Static Marginal Analysis and
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by

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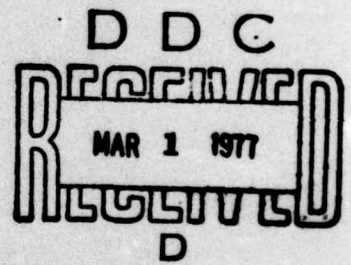
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ABSTRACT

An analysis is made of current Single Supply Support Control Point procedures for developing a Repair Material Requirements List. The objective is to minimize the expected cost of stockouts over all line items subject to a budget constraint. Static Marginal Analysis and Generalized Lagrange Multipliers are utilized in the generation of a revised Repair Material Requirements List. The revised and the present generation techniques are compared by the use of a simulation of a R3350 aircraft engine overhaul production facility. Both the Static Marginal Analysis and the Generalized Lagrange Multipliers techniques drastically reduced the number of stockouts and the number of subsequent orders. Given a choice between these techniques the Generalized Lagrange Multiplier approach appears preferable because it requires substantially less computer time to generate the list than did the Static Marginal Analysis.

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I. BACKGROUND

The Naval Air Systems Command Instruction 4700.5B of April 30, 1975 is the most recent in a series of instructions defining policy and prescribing procedures for supply support in commercial rework of aeronautical weapon systems and aircraft engines. The implementation of this series of instructions is through the Single Supply Support Control Point (SSSCP) concept. This concept involves and organization, referred to as the SSSCP, which is charged with two objectives of interest to this thesis: first, to achieve dollar savings by providing available government furnished material (GFM) to the commercial contractor for the support of a rework program and secondly, to minimize the rework turnaround time by reducing the overall supply response time through dedicated single point management.

Upon award of a commercial rework contract, an initial supply of available GFM is provided the contractor. The quantity of material provided is determined using a Repair Material Requirements List (RMRL). The RMRL is used by the SSSCP and the contractor as a guide for positioning and requisitioning GFM, respectively, to support an initial 90 day rework production schedule of end items. Timely receipt of this material insures support for the end items first inducted for rework and allows for an orderly implementation of follow-on material support procedures.

Before the development of the RMRL in the early sixties, a contractor was provided 100% of requirements of each line item for each end item to be reworked in the first 90 days of the contract. As an example, if the end item contained ten units of line item Y and 36 end items were to be reworked in the first 90 days, then 36 x 10 or 360 units of issue of item Y would be provided. During the contract performance phase, the contractor was charged to maintain a moving average of the usage rate of each line item and to use this information to order the expected demand for the next increment of end items to be reworked under the contract. The information gathered was subsequently formalized into the current Usage and Assets Report which gives the number of end items reworked and quantities of each line item used since the time of the last report and the quantity of each line item on hand at the time of the report.

By accumulating these records over several contracts the SSSCP was able to devise a replacement factor for each line item, according to the following formula:

$$ R_i = \frac{U_i}{Q_i \cdot N_c} \quad ; \qquad i = 1, 2, \ldots, n \quad , $$

where R_i = the replacement factor for the ith line item.

U_i = the total number of line item i used over the several contracts.

Q_i = the quantity of line item i required for each end item.

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N_c = the total number of end items requiring item i
completed over the several contracts.

n = the total number of different line items
applicable to the particular end item.

The resultant R_i is expressed as a percentage and rounded
to the nearest integer value. Items with historical usages
too low to produce a R_i of 1% or greater after rounding are
not included in the RMRL. The combination of the quantity
required per end item and the historical demand resulting
in such a low R_i, apparently does not warrant the inclusion
of these items in an initial inventory.

The replacement factors that are 1% or greater after
rounding become the key elements in the generation of the
RMRL. As presently structured, the RMRL is a computer-
based listing giving National Item Identification Number
(NIIN)/Manufacture Part Number, nomenclature, unit of issue,
number of units of issue required per end item (Q_i),
replacement factor (R_i), gross requirement (explained below),
unit of issue cost, cost of the gross requirement and total
cost for the RMRL. The gross requirement (G_i) is the
quantity to initially be shipped to the contractor. It is
determined from the quantity required (Q_i) per end item and
the replacement factor (R_i), as follows:

$$G_i = \frac{R_i}{100} \times Q_i \times N \; ; \quad i = 1,2,\ldots,n$$

where R_i is expressed as a percentage

N = the estimated number of end items to be reworked
during the initial 90 days.

n = the number of different line items on the particular RMRL.

It should be noted that G_i is rounded to the next higher integer value and that G_i is never less than one.

The SSSCP, through the RMRL, will provide a contractor with the quantities calculated according to the above formulae as material for initial support. These quantities are the nearest integer value above the mean historical usage as long as the replacement factor, after rounding, is at least 1%. The occasional demand for an item not provided via the RMRL is satisfied by the follow-on material support procedures instituted at the time of contract award.

In an earlier time when there was much less concern over the allocation of limited budgets, the RMRL would not have been required. By providing 100% of engineering requirements, the disruption and cost associated with a stockout and with an order placement could be kept to a minimum during the first 90 days. Of course the amount of funds required to provide inventory storage, protection and control would be high and excessive funds would be spent shipping the very low usage material to one contractor after another until they are finally incorporated in the project or discarded due to wear and tear.

Today, however, with the multitude of military programs vying for a limited budget, a continuing search for cost-saving efficiencies is being carried out at all levels. The RMRL is an example of just such an efficiency, for it provides a much more realistic level of inventory (the

expected demand for 90 days) than was provided prior to the implementation. It should be noted, however, that the present generation technique does not consider any budget constraint as such. The budget consumed is simply the cost of an item times the expected demand for 90 days summed over all items included in the RMRL.

However, because further improvements appeared possible for the RMRL generation technique, an analysis was recently conducted and reported in [1]. The problem addressed in that paper can be stated as follows:

"Given a probability distribution of demand, develop a RMRL generation technique that minimizes the total expected cost of stockouts over all items during the initial contract period, subject to a budget constraint."

If s_i represents the number of units of item i to be stocked initially, then the problem can be stated mathematically as:

Find the value of $s_i \geq 0$, i = 1,2,...,n, which

$$\text{minimizes} \quad \sum_{i=1}^{n} \pi_i \sum_{x=s_i}^{\infty} (x - s_i)\, p_i(x) \qquad (1)$$

$$\text{subject to} \quad \sum_{i=1}^{n} c_i s_i \leq C$$

where n = the number of different line items

c_i = the unit cost for the ith item

x = the demand for a line item

$p_i(x)$ = the probability that x units of line item i will be demanded

π_i = the weight (penalty cost or essentiality) of a stockout for item i.

As was noted in [1], one of the problems associated with providing an initial inventory is the lack of knowledge concerning the underlying demand generation probability distribution function. This lack of knowledge usually leads to the use of an assumed distribution or to an inventory based on expected values (the present RMRL approach). To be more specific relative to demand generation a record of demand data for a recently completed contract for the overhaul of 167 R3350 engines was obtained from SSSCP. Demand data for a sample of 200 items out of a total of 2106 items was analyzed. Under the assumption that all items follow the same type of distribution, the Poisson distribution was found to provide the best description of the actual demand data (see Table VII of [1]).[1]

Reference [1] proposed static marginal analysis as a solution procedure for (1). The notion of marginal analysis is that the efficient mix of productive inputs is the mix for which the "marginal product equals marginal costs". In [1] that meant that the composition of the RMRL should be

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[1] Historical mean demands for the individual items were used as the Poisson parameters and simulated demands were compared with the actual usage on the completed contract.

such that the inclusion of an additional unit of an item
is solely dependent on the decrease in expected stockout
cost per budget dollar consumed. This was mathematically
expressed as

$$\frac{\pi_i \, P_i(s_i)}{c_i} \quad ,$$

where $P_i(s_i)$ is the probability that s_i units are used.

The marginal analysis procedure progressively assigns
a unit to the inventory of that item which yields the
greatest reduction in expected stockout cost per unit
increase in budget usage. The first step is to set all
$s_i = 0$ and compute

$$\max_i \left\{ \frac{\pi_i}{c_i} \, P_i(s_i + 1) \right\} = \max_i \left\{ \frac{\pi_i}{c_i} \, P_i(1) \right\} . \qquad (2)$$

If the maximum is taken on for item j, set $s_j = 1$ and deduct
the unit price c_j from the budget C. This process will
continue, using the generalization of (2), as follows:

$$\max \left\{ \max_{i \neq j} \left\{ \frac{\pi_i}{c_i} \, P_i(s_i) \right\} , \frac{\pi_i}{c_j} \, P_j(s_j + 1) \right\} , \qquad (3)$$

until adding an additional unit of item i would exceed the
budget constraint.

This marginal analysis technique was applied in [1] to
the random sample of 200 line items (with $\pi_i = 1$ for all
items) in order to generate an RMRL. This RMRL was then

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compared with the RMRL generated for the new contract by SSSCP using the current procedures through a simulation of the repair of 167 R3350 engines. It was found that marginal analysis provided reductions of 40% in total number of stockouts and 26% in total number of orders during the rework of the engines over the current procedure for the 200 items. Although improvements were observed, the algorithm used did not produce optimal solutions since, as mentioned in [1],

1. Static marginal analysis is a heuristic process that, by itself, does not guarantee optimality. In particular, the algorithm might stop too soon. If the item i selected from the marginal analysis has a c_i value greater than the remaining budget, the procedure terminates even though some other item j may have a c_j value less than the remaining budget.

2. Due to computer limitations in calculating "powers of e" outside the range -180.218 to +174.673, twenty-three line items were excluded from the marginal analysis and included in the RMRL with the number of items calculated according to current procedures.

3. Severe computer rounding errors occurred when the incremental protection obtained by adding one more item was very small for all line items even though double precision was used.

II. SOLUTION PROCEDURES

A. MARGINAL ANALYSIS, POISSON DEMAND WITH NORMAL APPROXIMATION FOR $\lambda > 15$

In order to circumvent some of the problems and limitations described in Chapter I, several actions have to be taken.

To overcome the problem of the inability to calculate "powers of e" outisde the range -180.218 to +174.673 (further limited in [1] to the range ±150.0) the application of a Normal approximation to the Poisson distribution for high mean demands was used in (3). Under Poisson demands (3) takes the following explicit form

$$\max\left\{\max_{i \neq j}\left\{\frac{\pi_i}{c_i} \cdot \frac{e^{-\lambda_i}\lambda_i^{s_i}}{s_i!}\right\}, \frac{\pi_j}{c_j} \cdot \frac{e^{-\lambda_j}\lambda_j^{s_j+1}}{(s_j+1)!}\right\}. \quad (4)$$

The theory justifying the use of the Normal approximation to the Poisson distribution for high mean values is well known [2,3,4]. The approximation equation is:

$$P_i(s_i) = e^{-\lambda}\frac{\lambda^{s_i}}{s_i!} \doteq \phi\left(\frac{s_i+\frac{1}{2}-\lambda}{\sqrt{\lambda}}\right) - \phi\left(\frac{s_i-\frac{1}{2}-\lambda}{\sqrt{\lambda}}\right) \quad (5)$$

Equation (5) can then be used in Equation (4) for $\lambda > 15$. selection of the value 15 as appropriate mean demand for (5) was a result of several test runs in which different values of λ was used. These runs showed that for $\lambda > 15$

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the resulting inventory vector did not change significantly, while for lower values of λ significant changes occured. With a cutoff value of 15, 36% or 72 items would use Equation (5).

The problem of not exhausting the budget C completely can easily be corrected by including a "clean up" algorithm. If the optimal item j selected by the marginal analysis has a c_i > remaining budget then use marginal analysis to select the optimal item i from the set of all items having $c_j \le$ remaining budget. If selection of items continues in this manner, total budget exhaustion is guaranteed in this case. The approach of static marginal analysis with the "clean up" algorithm, Poisson demands for $\lambda \le 15$, and Normal approximation to demand for items having $\lambda > 15$ is called the "P,N" procedure in the remainder of this thesis.

The last problem, the problem of rounding errors cannot, at present, be circumvented if the technique of static marginal analysis is used. However, the technique of Generalized Lagrange Multipliers (GLM) might circumvent this problem and will be discussed next.

B. GENERALIZED LAGRANGE MULTIPLIER APPROACH ASSUMING POISSON DEMAND RATE FOR $\lambda \le 15$; OTHERWISE ASSUMING NORMAL

Lagrange multipliers are usually introduced in the context of differentiable functions, and are used to produce constrained stationary points. The validity normally appears to be connected with differentiation of the function to be

optimized. However, most real-world problems (e.g. the
present multi-item inventory problem) involve discontinuous
functions which are to be optimized subject to constraints.

It has been shown [5] that with another viewpoint the
use of Lagrange multipliers constitutes a technique whose
goal is maximization – rather than location of a stationary
point – of a function with constraints, and that in this
light there are no restrictions such as continuity or
differentiability on the function itself.

Let us suppose there is a set S that is interpreted
as the set of possible combinations of items in an inven-
tory. Defined on this strategy set is a real valued pay-
off function H where $H(s)$ is the payoff obtained by employing
the strategy vector $s \in S$. In addition there are n real
valued functions c_i, $i = 1,2,...,n$ defined on S, which are
called resource functions. The interpretation of c_i is that
the employment of strategy vector $s \in S$ will require $c_i(s)$
of the ith resource. The objective is then to maximize
the payoff (or minimize a penalty function) subject to a
resource constraint on each resource.

Now recall the inventory problem at hand. We want to
minimize the penalty π_i resulting from a stockout of the
i^{th} item in a situation where the total resource expenditure
over all items is subject to a constraint C. Let s_i be
the inventory position after the initial RMRL is generated
and let x_i be the demand for item i in the initial 90 days

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period (the RMRL is intended to cover demand in an initial 90 days period).

The expected number of fulfilled demands for item i is then

$$\sum_{x_i=0}^{s_i} x_i \, P_i(X_i = x_i) + s_i \, P_i(X_i > s_i), \qquad (6)$$

which is equivalent to

$$E(X_i) - \sum_{x_i=s_i+1}^{\infty} (x_i - s_i) \, P_i(X_i = x_i). \qquad (7)$$

Then, when we try to minimize the expected penalty incurred, or, equivalently to maximize the expected penalty avoided, we get the objective function:

maximize $Z(s) =$

$$\sum_{i=1}^{n} \pi_i [E(X_i) - \sum_{x_i=s_i+1}^{\infty} (x_i - s_i) P_i(X_i = x_i)]$$

$$\qquad (8)$$

Therefore (1) can be rewritten as:

maximize $Z(s)$

subject to $\sum_{i=1}^{n} c_i s_i \leq C \qquad (9)$

In a GLM context the problem on hand can be formulated as:

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$$\text{maximize } L(s,\theta) = Z(s) - \theta[(\sum_{i=1}^{n} c_i s_i) - C] \tag{10}$$

where the vector $s \in S$, and $\theta \geq 0$.

Problem (10) is obviously the Lagrangian problem associated with (9). From Everett [5], we know that if a vector S solves (10), then it also solves (9). Guidance on how to adjust θ in the event that $\sum_i c_i s_i - C \neq 0$ can be obtained from Everett's second theorem. This theorem states that, given two solutions produced by the Lagrange multipliers technique for which only one resource expenditure differs, the ratio of the change in optimum payoff to the change in that resource expenditure is bounded between the two multiplier values that correspond to the changed resource. Let θ^1 and θ^2 be two values of θ that produce solutions $S_1^*(\theta^1)$ and $S_2^*(\theta^2)$. If we assume that the resource expenditures of the two solutions differ only in the jth resource, i.e.

$$c_i(s_1^*) = c_i(s_2^*) \quad \text{for } i \neq j$$

and that

$$c_j(s_1^*) > c_j(s_2^*)$$

then

$$\theta^2 \geq \frac{Z(s_1^*) - Z(s_2^*)}{c_j(s_1^*) - c_j(s_2^*)} \geq \theta^1 .$$

This indicates very simply in which direction to make changes when employing a trial and error method for adjusting θ in order to achieve some given constraint on the resource. Decreasing the non-negative multiplier value tends to increase the use of the resource, increasing it use less.

An alternative way to look at the above problem is as a separable or cell problem, in which there is a number of independent areas or cells into which the resources may be committed, and for which the overall payoff is the sum of the payoffs from each independent cell. The advantage of having N single variable problems instead of one N variable problem lies especially in the temporary conversion of the constrained problem to a series of unconstrained maximization problems. In the cell problem with constraints on total resource expenditures, the conversion to unconstrained maximization of the Lagrangian function uncouples what was essentially a combinatorial problem into a vastly simpler problem involving independent strategy selection in each cell.

In the context of the inventory problem on hand, it can be restated such that the objective is to find a strategy set, one element for each cell, which maximizes the total payoff, subject to constraint C on the total resource expenditure. The separated Lagrangian function, one for

each line item, then takes the following form [5]:

$$L_i(s_i, \theta) = Z_i(s_i) - \theta c_i(s_i)$$

The separated Lagrangian expression is maximized by utilizing the following theorem [6].

Theorem: Let S_i be the set $0, 1, 2, \ldots, 3$. Then $L_i(s_i, \theta)$ is maximized over $s_i \in S_i$ at the smallest value of $s_i \in S_i$ which satisfies the inequality

$$\pi_i P_i(x_i > s_i) \leq \theta c_i. \tag{11}$$

If the Lagrangian in each cell has been correctly maximized, then Everett's Theorem 1 [5] guarantees that the result is a global maximum to the overall problem. It should be noted however that, due to the integer nature of the problem, exact equality between resources used and available may be impossible because of so-called "duality-gaps." In the present case duality-gaps can be explained as abrupt discontinuities in the consumed resource levels generated when θ is continuously varied. Everett [5] suggested that these "gaps" could be filled by comparing inventory vectors $s^*(\theta_1)$ and $s^*(\theta_2)$, one feasible and near-optimal and the other slightly infeasible. By identifying those items whose unit levels change in going from $s^*(\theta_1)$ to $s^*(\theta_2)$, it may be possible to get closer to optimality

by incrementing those items in the near-optimal solution until there is only a very small slack remaining in the constraint. It should be pointed out that such a procedure will be very time consuming in a real-life situation having thousands of items in the inventory vector.

C. COMPARATIVE EXAMPLES

Let's assume an initial inventory consisting of three items with the following unit costs and mean historical demands (λ) as shown in Table I, and a budget constraint of $143.37, determined as the cost of the inventory that would be shipped under the present system. Finally we assume $\pi_1 = \pi_2 = \pi_3 = 1.0$

TABLE I

	ITEM 1	ITEM 2	ITEM 3
λ	7.2	10.8	2.52
Cost	$16.75	$0.05	$2.94

(1) Standard RMRL calculation

If the inventory is provided according to the present procedure as described in Chapter I, the starting inventory would consist of the units shown in Table II. The probability of a stockout is given as a comparative measure for the three methods.

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TABLE II

	ITEM 1	ITEM 2	ITEM 3	COST
INVENTORY	8	11	3	$143.37
P(STOCKOUT)	.4075	.4207	.3528	

(2) Revised RMRL calculation.

Using the approach of marginal analysis described in Chapter II A, the inventory would be calculated according to marginal protection per $ unit cost. The process begins with Table III.

TABLE III

MARGINAL PROTECTION PER $ UNIT COST

X	ITEM 1	INCLUDED AS #	ITEM 2	INCLUDED AS #	ITEM 3	INCLUDED AS #
1	.05968	27	20.000	1	.32319	20
2	.05952	28	19.996	2	.27241	21
3	.05887	29	19.976	3	.19619	22
4	.05717	30	19.902	4	.12000	24
5	.05375	31	19.698	5	.06282	26
6	.04828	32	19.250	6	.02853	36
7	.04099	34	18.428	7	.01134	40
8	.03266	35	17.136	8	.00404	44
9	.02432	37	15.360	9	.00129	49
10	.01691	39	13.190	10	.00037	52
11	.01099	41	10.802	11	.00010	54
12	.00660	43	8.414	12	.00003	57
13	.00380	46	6.226	13	-	
14	.00204	47	4.374	14	-	
15	.00103	50	2.920	15	-	
16	.00048	51	1.852	16	-	
17	.00022	53	1.118	17	-	
18	.00009	55	.644	18	-	
19	.00003	56	.354	19	-	
20	.00001	58	.186	23	-	
21	-		.094	25	-	
22	-		.046	33	-	
23	-		.020	38	-	
24	-		.010	42	-	
25	-		.004	45	-	
26	-		.002	48	-	
27	-		-			

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Since the budget constraint is $143.37 all 20 units of
item one, 26 units of item 2 and 12 units of item 3 cannot
be included, so we must decide on an initial inventory
composition that yields the best marginal protection. This
is simply done by including units of each of the three
items according to the column "Included as (step) #". But
when we reach step number 35 and attempt to include the 8th
unit of item one, we find this is not possible since we
have already used $133.05 of the budget. The inclusion of
the 8th unit of item one would increase expenditures to
$148.80. We therefore use up the rest of the budget with
units of item three. Table IV gives the final inventory.

TABLE IV

	ITEM 1	ITEM 2	ITEM 3	COST
INVENTORY	7	26	8	$142.07
P(STOCKOUT)	.5470	0	.0038	

As can be seen, the budget is not quite exhausted.
However, the decrease in P(Stockout) between Tables II and
IV is very significant for items two and three. It has
increased by approximately 25% for item one.

3. GLM Inventory Composition

Applying the GLM procedure as described in Chapter
II B gives the following cell equations for the data
provided:

$$P(x_1 > s_1) \leq 16.75 \, \theta$$

$$P(x_2 > s_2) \leq 0.05 \, \theta$$

$$P(x_3 > s_3) \leq 2.94 \, \theta$$

Table V presents the results of the GLM procedure. The resulting solution is $s_1 = 7$, $s_2 = 23$, and $s_3 = 6$. The total budget consumed is $136.04.

TABLE V

θ	COST	CELL 1 (ITEM 1)	CELL 2 (ITEM 2)	CELL 3 (ITEM 3)
.01	$242.47	13	24	8
.04	$152.79	8	23	6
.05	$119.09	6	19	6
.041	$136.04	7	23	6
.042 + .046	$136.04	7	23	6
.048	$135.99	7	22	6

As can be seen from Table V, the results of this example are very insensitive for a wide range of θ values. It should be noted however, that with an increasing number of items, solutions closer to optimality (i.e., a more complete budget consumption) are to be expected. This expectation is not based on actual knowledge of closing of duality gaps when the number of items increases, but

rather on empirical observation. Further, in the present
context with unit prices ranging from a low of $0.01 to a
high of $1,770.00, the inclusions of more items, many of
which have a very low price, tends to close any gap.

The probability for a stockout in the above
example is 0.547 for item 1, 0.036 for item 2 and 0.0254
for item 3.

NOTE: In Tables IV and V the mean demand for the three
items was rounded to the next high integer value to simplify
computation. This results in a slight disadvantage for the
last two examples when compared to example one.

III. RESULTS OF THE ANALYSIS

The first step of the analysis involved implementing
the Normal approximation for items with mean greater than
15 and a "clean up" routine in the static marginal analysis
inventory generation program.

The results of the simulation with inventories generated
by the current procedure system (STANDARD), the approach
used in [1] (REVISED), and the approaches described in
Chapter II (the (P,N) and the GLM procedures) are given
in Table VI.

TABLE VI

SIMULATION OF THE REPAIR OF 167 ENGINES

METHOD	INITIAL BUDGET USED	# OF ORDERS	# OF STOCKOUTS	RESIDUAL VALUE
STANDARD	$138,062.63	2510	165	$20,406.85
REVISED	$138,061.48	1841	99	$24,795.09
P,N	$138,062.62	1124	50	$33,789.35
GLM	$138,044.61	1125	49	$33,763.40

In Table VI above, column 1 (initial budget used)
gives the total value of the initial inventory generated
by each method. In all cases the budget constraint was
$138,062.63, which was the budget used under the present
system. Column 2 (# of orders) gives the total number of

orders submitted by the contractor for additional units of the 200 line items during the simulation of the repair of 167 engines. Column 3 gives the total number of stock-outs observed during the simulation, and the last column gives the total value of the residual line items in the contractor's inventory after completion of the repair of all 167 engines.

As can be seen from Table VI substantial reductions in both the number of orders and the number of stockouts were obtained by using a Normal demand distribution for high demand items instead of merely their historic mean demands in the development of the initial inventories.

The behavior of the residual value of the inventory remaining at the contractor's facility at the end of the contract is not unexpected. As better protection against stockouts are provided, the probabilities of larger final inventories increases. Unfortunately these inventories must be retrieved by the SSSCP at contract termination time. A reduction in this residual inventory might be obtained by changing the contractor's ordering policy in the late part of an overhaul.

As was shown in the Chapter II example, a trial and error approach is required for the GLM method. This in-volves assuming a value for θ, determining the associated s_i values from Equation (11) and then the total amount of the budget consumed. Table VII summarizes the analysis.

TABLE VII

LAGRANGE MULTIPLIERS AND THE
ASSOCIATED RMRL COST

θ VALUE	RMRL COST	BUDGET CONSTRAINT
.00100	$160,714.22	
.00200	$144,330.26	
.00300	$142,659.47	
.00400	$140,663.48	
.00410	$140,552.29	
.00411-13	$140,204.61	$138,062.61
.00414	$137,684.61	

As can be seen from Table VII, the optimal solution must be associated with a θ value between .00413 and .00414; the θ's between these values were then investigated and the change in the inventory vector was observed. This investigation revealed that there existed one of the previously described duality gaps resulting in the number of units of only one item changing between these two values. The item mean was 440.64 units, its cost was $120.00, and the number of units jumped from 440 units at θ = .00414 to 461 units at any θ value greater than .00414. A solution, as suggested by Everett, to this gap problem is to apply θ = .00414 and then to include 3 more units of that specific item, leaving a gap of $18.02.

Table VI suggests that for large numbers of items the P,N and GLM procedures gives quite comparable results. Perhaps this should not be too surprising since both methods use the ratio of the form $R_i(s_i)/c_i$ in determining optimal s_i values.

A disadvantage of the P,N procedure is that the inventory vector must be generated in steps, each step requiring comparison of the marginal protection of each of the N items in the vector. In contrast, the GLM procedure can generate the inventory vector in one step if θ is known. The generation of the inventory vector by the P,N approach required close to two minutes of CPU time on the Naval Postgraduate School IBM 360/67 computer. The GLM procedure solved the problem in a little less than 10 seconds for a single θ value. It should be noted however that in the GLM approach several runs might be required before the θ value that gives budget exhaustion or close to budget exhaustion is found. On the other hand, with continued use of the GLM procedure in the context of generating inventory vectors, a priori knowledge of the approximate value of θ could reduce the number of trial runs considerably.

The potential reduction in computer time by using the GLM method should be even more significant in a real-life situation with tenfold as many items in the inventory vector. This is because the comparison of the marginal protection for each of the items in each step in the P,N approach would result in more than linear growth in CPU time usage, whereas

the GLM procedure would have a close to linear growth. A closer inspection and comparison of the two inventory vectors generated by the P,N and the GLM methods uncovered minor differences in a few line items (under one method the initial inventory position for a given line item was 10 whereas using the second method was 11).

After the above simulations with the original budget constraint were conducted, the sensitivity to changes in the budget constraint C was examined. The results of this sensitivity analysis are given in Table VIII.

TABLE VIII

SENSITIVITY ANALYSIS WITH
REGARD TO INITIAL BUDGET CONSTRAINT

INITIAL BUDGET	REVISED		P,N		GLM	
	#STOCK-OUTS	# OF ORDERS	#STOCK-OUTS	# OF ORDERS	#STOCK-OUTS	# OF ORDERS
$138,062.61	99	1841	50	1124	49	1125
$125,000.00	102	1921	51	1151	50	1150
$115,000.00	132	2031	61	1231	61	1230
$105,000.00	152	2125	81	1297	81	1298
$ 95,000.00	174	2322	82	1382	83	1382
$ 85,000.00	223	2604	95	1416	95	1416

Table VIII shows that the P,N and the GLM procedures continued to give comparable results and that both performed far better than the REVISED approach as developed in [1] and the current approach (STANDARD) which had 165 stockouts and 2510 orders and required a budget of $138,062.61.

As was noted earlier a budget reduction is not possible in the STANDARD since the generation technique is to have an inventory equal to the mean historic demand regardless of the associated costs. Any reduction would mean a deviation from this technique.

Table VIII shows that even for a budget reduction of approximately 39% the P,N and the GLM procedures will still perform better than REVISED and STANDARD, i.e. will result in less total stockouts and orders. At the 39% budget reduction point REVISED performed comparable to STANDARD at full budget.

It is important to emphasize any savings in the initial budget are only temporary since the total usage over the entire contract will be the same regardless of when the material is delivered. The new generation technique merely allows for a temporary reallocation of the initial fund savings to other programs.

IV. SUMMARY AND RECOMMENDATIONS

Two improved methods for the generation of RMRL's were developed, employing in the first case static marginal analysis with a "clean up" procedure and, in the second case, the use of a generalized Lagrange multiplier (GLM) approach to a cell problem. The use of both static marginal analysis and GLM requires that the underlying probability distribution for demand be known.

A RMRL giving the initial inventory vector was generated using both techniques and the subsequent demands and ordering during overhaul of 167 R3350 engines were determined using simulation. Historical data [1] suggested that item demand followed the Poisson distribution. For items with a mean historical demand greater than 15, the Normal approximation to the Poisson distribution was used.

The numbers of stockouts and orders were then compared with the numbers resulting from a similar simulation using the present RMRL generation technique and the technique employed in [1]. Both methods reduced the numbers of stockouts and orders. Reductions obtained were 70% in stockouts and 55% in orders when compared with the present technique (based on mean historic demand), and 50% and 39%, respectively, when compared with the approach employed in [1].

The GLM procedure appears to be the more economical of the two methods in terms of computer time usage.

Even though the number of stockouts and the number of orders were drastically reduced, no general claim of optimality can be made. In the case of static marginal analysis the budget was nearly exhausted (1 cent left), but the heuristic nature of the process does not guarantee optimality. The existence of optimal solutions that can be found by the GLM procedure depends upon an approximate concavity requirement (see Everett [5]) in the region of the solution. As was previously discussed, a duality gap was found in the area of interest in the present case, and hence only a feasible, sub-optimal solution was reached. An inspection and comparison of a feasible, sub-optimal inventory vector and a slightly infeasible inventory vector was performed as suggested in references [5] and [6]. This inspection resulted in the inclusion of three more units of a given line item but the final solution was still believed to be sub-optimal. A duality gap remained although it had been reduced.

As mentioned above, previous work had shown that the Poisson distribution was applicable for the R3350 engine overhaul. In any new attempt to apply either the GLM or the static marginal analysis techniques in the generation of an RMRL, the necessary first step is to determine the specific demand distribution.

The large size (value) of the residual inventory after the simulated repair of all 167 engines suggests that a study should be made of the ordering policy after the

determination and delivery of the initial inventory. An ordering policy which reduces these residuals without creating excessive stockouts seems appropriate. The ordering policy used in this context in order to be able to make comparisons of the influence of the generation technique has been that of the SSSCP. According to this policy, the initial RMRL is the basis for the contractor's future orders, i.e. as soon as the inventory drops below a certain level he is allowed to order the difference between the RMRL quantity and what he has on hand minus backorders. When the number of end items left to rework is less than the number used for generating the RMRL (in this case 36 engines) the reorder policy changes in a way such that the reorder quantity limit becomes the expected usage per line item times the number of end items left to rework.

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APPENDIX A

DATA BASE

NIIN	QTY PER ENG	RPL FCT	UNIT PRICE	CONTRACT USAGE
7047523	0	0	20.00	0
1749497	0	0	2.04	35
916593	0	0	69.00	0
184096	1	1	334.00	2
242896	2	10	16.75	32
316599	4	1	40.00	6
379260	9	1	0.08	10
379363	1	7	5.21	11
379423	2	13	0.10	45
379691	2	15	0.05	50
458865	2	5	0.69	17
1416693	1	6	0.05	10
1711509	1	7	2.94	12
2062981	1	3	0.30	5
2095394	1	7	0.01	12
2131789	1	4	54.05	6
2131813	3	1	107.80	7
2173185	1	32	12.18	53
2440514	5	2	0.10	15
2537554	1	16	0.36	27
2750475	7	8	1.05	97
2762769	18	1	0.05	32
2906984	1	17	0.57	28
2913285	1	16	0.06	26
2913303	2	10	0.04	34
2923120	2	4	0.20	12
2986868	2	1	0.14	4
3036123	1	2	37.43	3
3049019	1	37	35.00	61
3075570	2	4	3.33	15
3102870	1	1	88.75	2
3109004	1	13	86.19	21
3128836	1	3	129.00	5
3133636	7	5	0.47	61
3133653	1	6	36.84	10
3144651	7	3	0.12	30
3236729	1	7	12.00	12
3236730	1	7	5.20	11
3260802	1	14	136.21	24
3266635	1	28	29.00	46

NIIN	QTY PER ENG	RPL FCT	UNIT PRICE	CONTRACT USAGE
3266649	1	1	6.05	1
3266652	1	12	9.10	20
3266657	1	5	100.00	8
3320476	1	7	3.00	12
3320477	2	6	21.50	20
3320485	1	5	77.04	8
3354807	4	3	40.00	17
3357073	22	1	0.27	20
3421180	1	38	0.72	63
3441409	3	6	36.50	30
3459562	1	5	6.13	8
3596844	1	1	0.74	2
4423415	2	7	4.58	23
4451522	1	3	129.96	5
4460530	1	4	66.00	6
4788907	1	34	0.49	56
4789077	2	8	0.96	26
4848265	2	1	0.08	2
5063334	18	1	138.20	24
5085494	6	5	19.00	55
5126425	1	20	29.61	33
5129631	1	7	1.23	11
5129635	1	3	37.21	5
5129707	1	2	48.50	3
5129739	1	2	225.00	3
5129777	1	15	111.00	25
5129790	1	13	7.39	22
5150800	1	8	2.88	14
5163785	2	10	4.90	35
5164844	1	29	54.63	48
5255110	6	6	0.06	63
5285683	1	10	0.98	17
5516876	4	7	2.21	49
5555751	1	25	14.00	42
5668943	1	4	592.00	6
5727165	1	33	6.76	55
5739655	3	5	218.00	25
5849563	4	4	0.02	30
5918215	1	8	849.00	14
5941171	1	5	42.43	8
6023691	2	13	144.00	42
6048493	8	5	0.25	68
6048494	1	6	0.25	10
6058293	1	2	78.09	3
6066965	2	7	170.10	22
6182527	1	14	0.21	24
6233794	1	11	10.61	19
6250754	1	4	399.91	7
6322052	1	5	0.02	9
6384070	6	1	1.10	9

NIIN	QTY PER ENG	RPL FCT	UNIT PRICE	CONTRACT USAGE
6501192	1	23	50.56	39
6501194	1	2	15.50	4
6514692	1	1	1770.00	1
6547284	4	4	42.37	24
6547287	4	7	55.00	46
6598523	1	2	563.00	3
6622281	8	1	12.00	15
6622476	1	35	0.50	58
6736677	1	2	950.00	4
6969469	1	8	60.00	13
6969477	1	1	8.08	2
6974802	4	2	58.06	12
7047531	1	3	176.00	5
7161469	1	23	29.50	39
7162944	1	2	292.52	3
7162955	9	2	530.00	24
7172218	1	2	146.26	4
7172404	4	2	40.19	11
7204894	1	15	18.98	25
7303275	1	8	99.00	14
7575069	1	6	0.24	10
7974052	27	1	0.08	50
8117017	1	23	10.50	38
8301942	1	3	35.00	5
8303008	1	2	26.80	4
8303010	1	2	48.50	3
8303012	1	4	61.41	7
8303040	1	7	57.83	11
8846264	2	10	775.22	32
8991790	1	1	0.50	1
9038282	1	10	0.03	16
9152018	2	1	3.50	3
9317218	2	3	36.10	10
9631387	3	13	161.95	65
9631388	3	4	152.00	22
9670092	1	17	13.00	29
9773423	6	3	17.50	32
9782993	9	1	8.45	17
330368	20	12	35.30	407
489131	45	3	62.40	190
1006170	2	102	0.86	340
2076434	18	5	0.42	157
2105349	9	41	0.06	617
2250470	2	51	74.00	172
2750632	18	9	0.05	263
2913291	4	18	0.06	123
2978384	6	11	1.32	112
3036014	1	43	2.70	71
3037779	1	44	0.09	73

NIIN	QTY PER ENG	RPL FCT	UNIT PRICE	CONTRACT USAGE
3065839	4	69	18.18	461
3108941	9	14	9.60	208
3108946	8	48	2.21	637
3109005	6	12	85.12	120
3133661	8	7	0.16	100
3133672	84	2	15.44	243
3144629	3	51	0.22	256
3144661	9	16	0.34	245
3148139	18	10	1.40	302
3306478	16	14	10.07	376
3306479	16	9	21.97	237
3963912	8	48	0.11	646
4335422	2	65	3.60	218
5058611	150	1	2.99	140
5058634	3	22	854.00	108
5129694	2	60	0.01	199
5129730	2	89	0.03	298
5155558	2	56	0.14	187
5224835	2	34	9.60	113
5277488	2	78	0.15	260
5285651	32	10	0.12	510
5676397	6	7	0.97	70
5796324	2	23	1.48	76
5804444	2	73	1.53	245
5901802	150	1	9.20	150
5995989	5	44	0.06	368
5996406	2	71	0.14	238
6138001	168	2	8.40	588
6563168	3	60	0.01	303
6621800	6	25	10.00	249
6807297	5	13	0.03	110
6807628	24	2	0.59	98
7172223	4	35	0.10	235
7194426	2	30	72.57	99
7197729	8	30	1.25	401
8032651	4	61	0.22	410
9500039	118	1	3.68	102
1187490	9	75	0.10	1132
1476306	24	21	0.10	833
1984735	218	31	0.23	11329
2105221	36	82	0.08	4944
2686041	16	45	0.07	1200
3036651	36	12	0.69	727
3133641	9	48	0.15	726
3439277	72	92	0.05	11074
4790482	6	111	0.28	1111
5159073	6	85	0.16	855
5309323	72	22	0.09	2589
5513093	35	17	0.37	977

NIIN	QTY PER ENG	RPL FCT	UNIT PRICE	CONTRACT USAGE
5804634	102	12	120.00	1961
5961865	54	17	0.17	1510
5966095	26	48	0.23	2078
6061829	205	4	1.12	1495
6118234	300	4	12.16	1922
6527000	52	28	0.02	2431
6621790	36	54	0.53	3258
6724938	18	60	0.04	1804
7220101	18	50	0.04	1491
8641347	150	9	6.52	2285
9086292	15	33	0.17	820
2973756	0	0	0.69	8001

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