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BEATS IN COCHLEAR MODELS

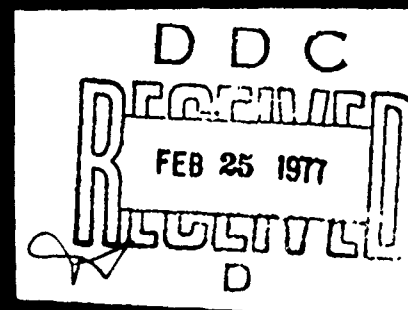
SCHOOL OF AVIATION MEDICINE  
RANDOLPH AIR FORCE BASE, TEXAS

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## BEATS IN COCHLEAR MODELS

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## BEATS IN COCHLEAR MODELS

Beats in cochlear models were studied from two aspects:

1. Their effect upon cochlear fluid motion which, occurring along elliptic orbits, essentially represent Lissajous figures. The orbits expanded and contracted synchronously with the beat rate. Thereby, a partial rectification (asymmetric expansion) became apparent which, occurring in opposite directions in the two scalae across the partition, constituted the beat. This latter phenomenon was caused by the stiffness gradient of the cochlear partition.

2. Their effect on the traveling wave pattern along the partition was originally described by Bekesy. Stroboscopic examination revealed that the beat frequency modulates the two primaries thus producing two additional side bands. The displacement pattern of the membrane when viewed by high-speed motion picture photography, however, did not give any evidence of these five frequencies. Only the intertone (and the beat) were present. It then became apparent that the model is really a pattern analyzer of complex sound. In terms of cochlear fluid motion, it is able to resolve complex Lissajous figures by a spatial filter action along the partition into simple Lissajous figures. Any deviation from steady Lissajous figures, which ordinarily result from primaries in relation of integral numbers, produces beats. Thus beats of imperfect unison as well as of mistuned consonances have essentially the same cause: a cyclic phase change between the two primaries.

## INTRODUCTION

The phenomenon of beats has continued to attract the attention of investigators in the auditory field for many years because of its significance to Auditory Theory. G. S. Ohm (1), in 1843, first formulated what later became known as "Ohm's Law of Acoustics." In the words of Helmholtz (2): "The human ear perceives pendular vibrations alone as simple tones, and resolves all other periodic motions of the air into a series of pendular vibrations, hearing the series of simple tones which correspond with these simple vibrations." In other words, the ear was thought to perform a Fourier analysis in resolving a complex sound signal. That this ability of the ear is limited became obvious from the observation of beats. When two tones of nearly the same frequency are sounded together the ear, instead of perceiving them separately, hears a single tone. The pitch of the latter lies between that of the two primary tones and is commonly referred to

as the *intertone*. Helmholtz (2), in line with his Resonance Theory, maintained that in such a case "the same elastic apperages of the auditory nerve [are set] in sympathetic vibrations at the same time." Or as later writers have put it, two such tones stimulate overlapping areas along the basilar membrane causing a sensory *fusion*. It had to be admitted, therefore, that the ear is not quite a perfect analyzer of sound. That such a negative definition is not very satisfactory hardly needs to be pointed out.

The latter difficulty seems rather trifling, however, compared with those arising from the phenomenon of beats of mistuned consonances. In contrast to beats between nearly identical frequencies which are known as *imperfect unisons*, the term *mistuned consonance* refers to beats between a given frequency and another one which is not quite in harmonic relationship with the former (e.g., 500 cps and 1002 cps). Helmholtz (2) indicated a simple

way in which the beat rate can be calculated for a given set of primary frequencies. The following formulation is Wever's (3): "If  $m/n$  is the simplest perfect ratio of the interval and  $M$  and  $N$  the numerical values of the primaries, the beat rate is given by  $nM - mN$ ." In the present example (500 cps and 1002 cps)  $n/m = 1/2$ ; hence  $1 \times 1002 - 2 \times 500 = 2$  beats per second. For 502 cps and 1000 cps, however, the result is:  $2 \times 502 - 1 \times 1000 = 4$  beats per second. There is a difference in whether the lower or the higher primary is mistuned.

Helmholtz (2) had thought that the beats which are perceived in the case of mistuned consonances were due to an interaction between overtones of the fundamentals in question. Thus his concept of sympathetic vibrations in overlapping areas was still valid in his opinion. In opposition to this, R. Koenig (4) maintained that "beats of mistuned consonances as well as of imperfect unison . . . arise from the periodically altering coincidences of similar maxima of the generating tones, and of maxima with the opposite sign." In Koenig's opinion, then, this was definitely not a case of beating between upper partials, a fact which can be easily corroborated by means of modern electronic sound-generating equipment. Chocholle and Legoux (5), probably not being aware of Koenig's earlier communication, recently voiced a similar opinion concerning the origin of mistuned consonances.

This author became interested in the problem of aural beats in the course of his studies on cochlear models of the Bekesy-type (6). In an earlier paper (7) the appearance of the beat frequency ( $f_1 - f_2$ ) in such models has been described.

The experiments to be reported in the present paper were designed in an attempt to find answers to the following questions: Are there any phenomena in the hydrodynamic events associated with cochlear responses to beat signals which might account for (a) the origin of the beat frequency as a separate entity; (b) the similarity between beats of imperfect unison and of mistuned consonances; and (c) the appearance of the intertone?

## PROCEDURE

For reasons of their easy adaptability, cochlear models after Bekesy (6) were chosen for the purpose of the present study. These models as used by the present author have been described at several earlier occasions (7, 8, 9). The following description is a brief summary.

The models consisted of transparent Lucite. The "basilar membranes" were held in metal frames and were made of rubber cement in which fine aluminum particles had been embedded to facilitate better observation of membrane motion. The "perilymphatic cochlear" fluids were glycerin-water solutions in which aluminum "dust" was suspended for visualization of particle motion within the fluid. In the present study, "basilar membranes" were 50 mm. long (i.e., not quite twice the size found in human ears). Such an increase lowers the frequency range to which a model may respond — a fact which makes investigation easier. Response of the present model was to frequencies from as low as approximately 8 cps to 1000 cps.

Routinely, the "helicotrema" is represented by a small circular opening, 2 mm. in diameter, which is cut into the frame of the basilar membrane, 1 mm distal to the "apical" end of the membrane. However, this is not quite accurate according to Hilding's (10) anatomic findings. In reality, a portion of the border of the helicotrema is formed by a free edge of the basilar membrane close to its apical end. In some respects, models built to this latter specification give a more "correct" response, especially with regard to low frequencies as has been shown in an earlier study (7). However, such membranes have a tendency to tear from their free edge under the effect of low-frequency signals. Since in comparative studies it is imperative to perform all measurements with the aid of the same membrane, a separate "helicotrema" was considered safer and its shortcomings, as indicated, had to be taken into account.

Vibratory stimulation was provided by a loud-speaker driver unit (Jensen DD 100) which was connected to the "oval window" by

a small air-filled, closed coupler. Illumination was either by stroboscopic light (for observation of instantaneous particle or membrane positions) or by continuous light (for observation of particle orbits or of "eddy" movement). The observation of fluid motion and its photographic recording was greatly facilitated by "dark field" illumination: Light entered the model through a narrow slit at  $90^\circ$  to the plane of observation (the latter being parallel to the cochlear partition) so that particles appeared brightly illuminated against a dark background. A dissecting microscope was used to assure uniform conditions of observation. A micrometer disk built into one eyepiece of the microscope served for measurement of particle amplitude, etc. Magnification was usually 16X, but was sometimes stepped up to 40X.

## RESULTS

### Particle motion

The first experiments concerned the particle motion within the cochlear fluids in response to *beats of imperfect unison*. It has been reported in previous papers that particle motion within both perilymphatic scalae occurs along elliptic orbits (7). These orbits are essentially Lissajous figures composed of two vectors,  $90^\circ$  apart in phase — viz, the (forced) motion of the window membrane and the (induced) motion of the cochlear partition. Such Lissajous figures can easily be constructed on the screen of an oscilloscope. In the case of two closely related primaries (e.g., 1000 cps and 1003 cps), one sees an elliptic Lissajous figure which *expands* and *contracts* alternately at the rate of three times a second (fig. 1A). The degree of this *amplitude variation* depends on the amplitude ratio between the two primaries, contraction to a point occurring only when both are equal in amplitude. It is noted that the amplitude variation takes place about the true centerpoint of the ellipses.

When one observes particle motion within the model, one sees a similar amplitude variation of the particle orbits in response to two beating primaries. (Actually, since particles revolve along their orbits, this variation occurs

in the form of expanding and contracting spirals.) On closer examination, however, it is found that the expansion of the orbits does not take place about their true centerpoints everywhere within the model. For low-frequency primaries (e.g., 25 cps and 27 cps), the orbital expansion became distinctly asymmetric within the distal regions of the model (i.e., beyond the locus of maximum membrane displacement in response to these frequencies). This asymmetric expansion, which is shown in figure 1B, concerns *both* orbital vectors.

In both perilymphatic scalae, the wider expansion occurs always in a given direction. As shown in figure 1C, longitudinal expansion in scala vestibuli is always wider in the apical direction than in the fenestral one, and transversal expansion is always wider toward the partition than away from it. In scala tympani, these directions are reversed. Consequently, the resulting asymmetric motion of the particle orbits in the two scalae is in *phase opposition* across the partition. Thus it has been found that two primaries of imperfect unison when applied to the cochlear model produce by partial rectification a *low-frequency* component. This new component is equal to the *beat frequency*.

When the average frequency of the primaries was increased, the point at which asymmetric beat motion set in moved closer toward the windows. At approximately 60 cps the asymmetric beat motion became universal throughout the model; that is, symmetrical amplitude variation of orbits ceased to exist within the model.

If the beat rate was as low as in the present example (e.g., 2 cps), the energy of the newly created component was seen to exchange through the helicotrema, where it was best observed and measured. The amplitude of the primaries, which otherwise might interfere with such measurements, reduces to zero before the region of the helicotrema is reached, since their frequency is considerably higher than the beat frequency. If higher beat frequencies were used — e.g., 10 cps between primaries of 400 cps and 410 cps — the new component frequency formed a traveling wave of its own.

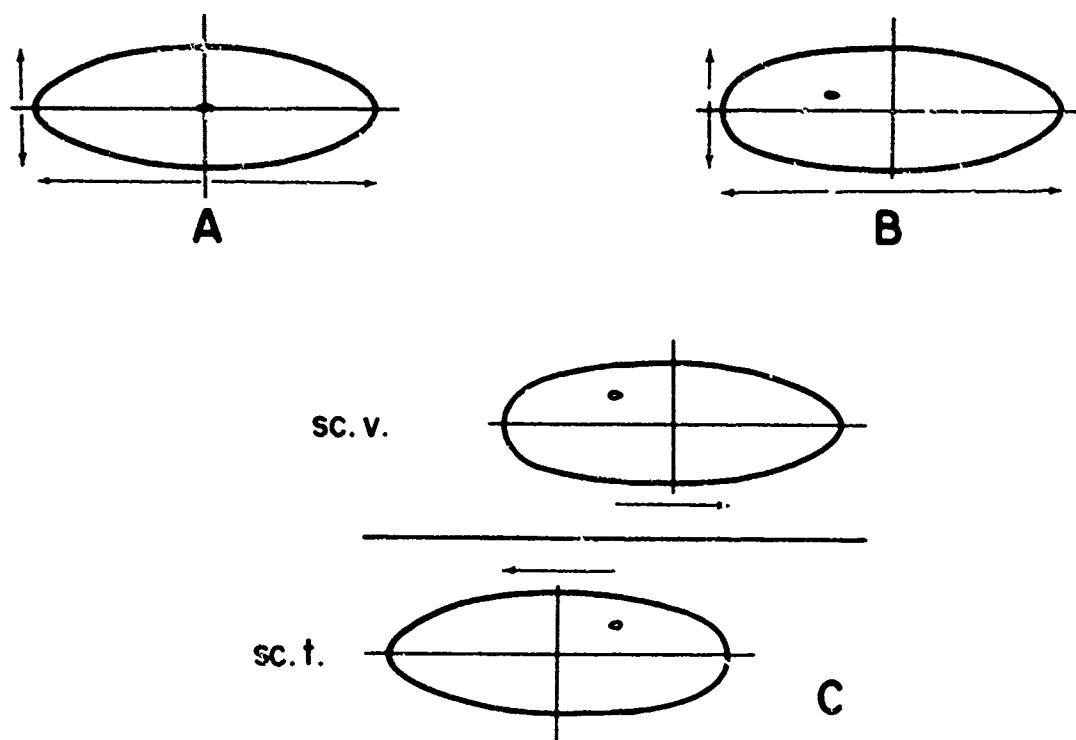


FIGURE 1

*Beats of imperfect unison (schematic). A: Beating Lissajous figure as seen on the oscilloscope. Shown are the instances of extreme contraction and expansion. The latter occurs symmetrically along both vectors (cf. the two pair of horizontal and vertical arrows). B: Beating particle orbit. Note asymmetry along both axes. C: Phase opposition of orbital expansion in scala vestibuli (sc. v.) and scala tympani (sc. t.).*

However, when this happened, intensity was sufficiently high so as to introduce distortion. On the basis of the present results, it cannot be decided whether the low-frequency component in this case is really the difference tone or the beat.

Before discussing the possible cause of the phenomenon of beat production, I shall describe the pattern of fluid motion in response to *mistuned consonances*. A running pattern of such a combination of primaries as it appears on the oscilloscope is shown in figure 2. It is the result of a cyclic shift in phase between the two mistuned primary frequencies. The pattern of Lissajous figures (and/or particle orbits) in response to complex stimuli, with the components in true harmonic relationship,

has been described elsewhere (7). Such Lissajous figures are characteristically multiple-looped. For example, in the case of a 1:2 interval (e.g., 50 cps and 100 cps) the Lissajous is double-looped. (Some other Lissajous figures will be described later in relation to figure 4.) Its pattern may be any of the four shown in figure 3A to D or any other pattern between these, depending upon the phase relationship between the two primaries. However, if one of the two primaries is slightly mistuned (e.g., 50 cps and 102 cps) the Lissajous figure revolves constantly through patterns A to D of figure 3 in a cyclic manner. This revolving is caused by the cyclic phase shift indicated in figure 2. There is one complete revolution per *audible* beat as is easy to ascertain by monitoring the beating signal.

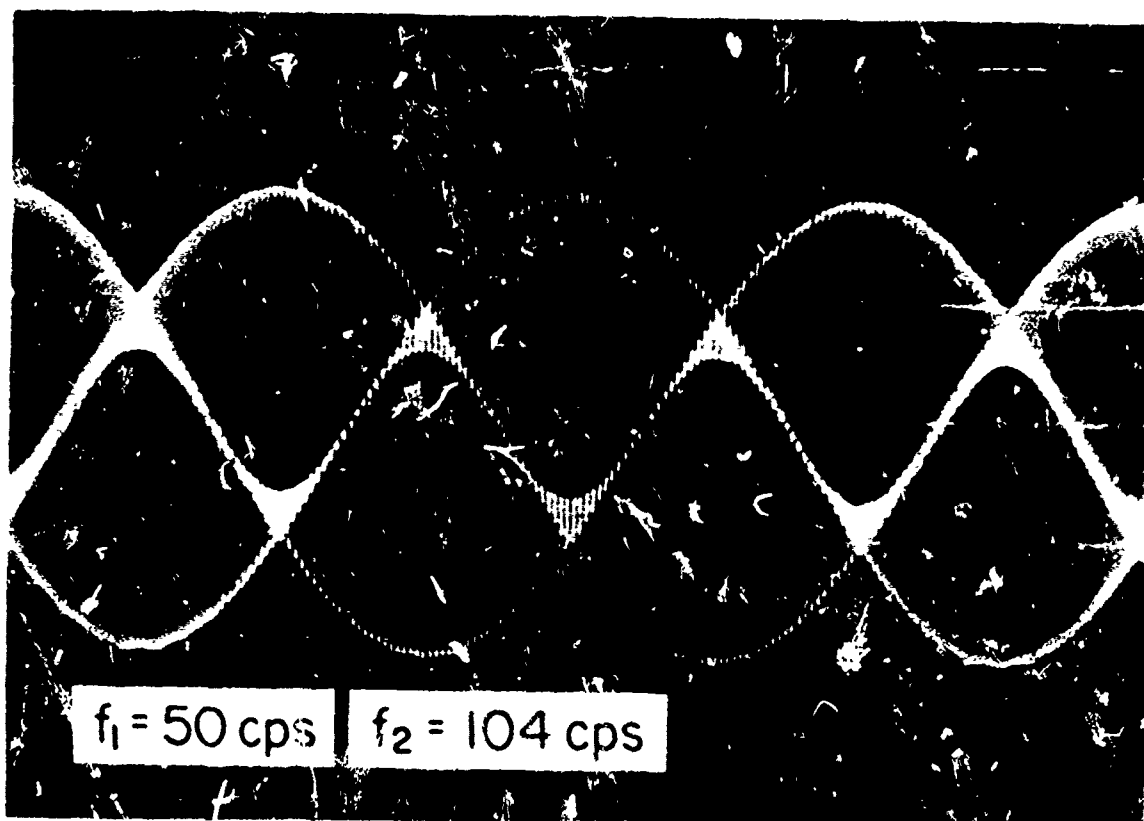


FIGURE 2

*Waveform of a mistuned (1:2) resonance, oscilloscopic recording; frequencies as indicated. Note the cyclic phase shift within the wave form.*

During their resolution, these Lissajous figures display an asymmetry which is shown in figure 3 and is also caused by the cyclic phase shift. Within the model, the phase relationship between particle motion in the two scalae across the partition is  $180^\circ$ . The *phase shift asymmetry* of figure 3, therefore, appears in opposite phases of the two scalae. For example, while orbits in scala vestibuli might be in the position of figure 3A, the corresponding orbits in scala tympani should be in the position of figure 3C. Theoretically, therefore, the phase shift asymmetry by itself could be sufficient to introduce a low-frequency component — viz, the beat frequency. This assertion was examined by observation within the model.

As is apparent from figure 3, the asymmetric motion of Lissajous figures in response

to mistuned 1:2 intervals when measured in the plane of the longitudinal axis is exactly one-third of that axis, provided the two primaries are of equal amplitude. It is recalled that in case of beats of imperfect unison the beat motion was not visible in the window region when low-frequency primaries were employed. Similarly, particle orbits, in the case of mistuned 1:2 consonances and low-frequency primaries (e.g., 24 cps and 50 cps), when observed in the window region showed only the degree of asymmetric motion found in phase-shifting Lissajous figures. However, when one approached the region of maximal membrane displacement, the amplitude of asymmetric motion increased gradually. Finally, within the region of maximal displacement, the asymmetric motion, when measured in the longitudinal plane, was found to be equal to the longitudinal orbital diameter. Thus, it had increased by a factor of 3.



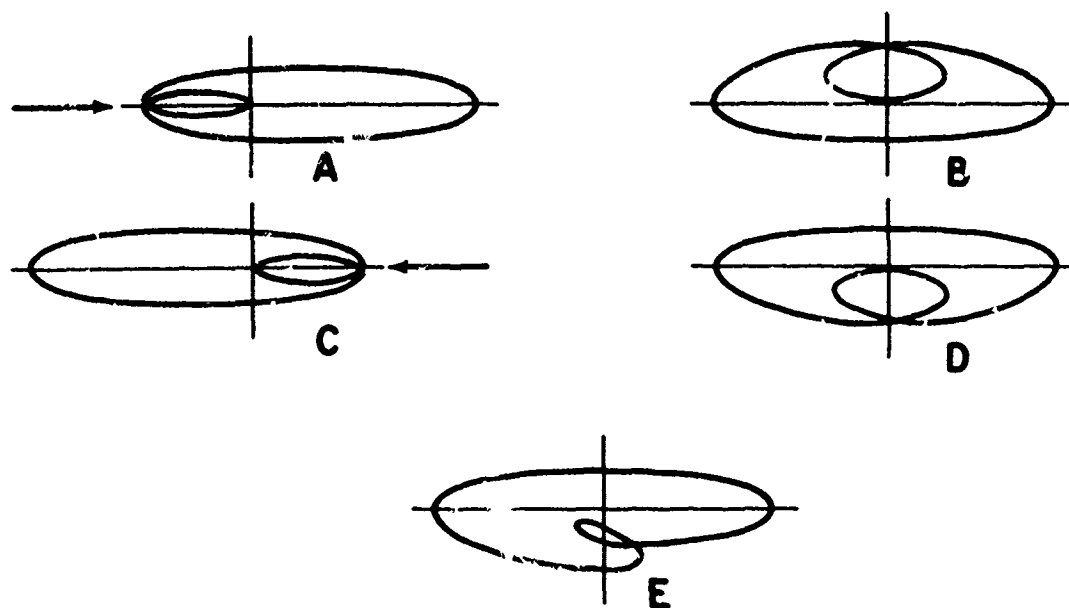


FIGURE 3

*Beating Lissajous figures of a mistuned (1:2) resonance (drawn after oscilloscopic recordings). Both components are equal in amplitude. The Lissajous revolves through patterns A to D once per beat. Note the structural asymmetry by comparing patterns A and C (position of arrows). Such figures are produced only when the  $90^\circ$  phase difference between signals on both pair of deflection plates is obtained independently for each frequency component. If only one phase shifter is used, patterns like that of E are produced.*

This difference in asymmetry between the Lissajous figures and the resulting particle orbits was even larger when primaries of higher frequencies were employed. For two primaries of 248 cps and 500 cps, the asymmetric motion of particle orbits when measured in the longitudinal plane was found to be approximately 10 times as large as the longitudinal particle amplitude. Thus it had increased by a factor of 30. Therefore, it appears that the phase-shift asymmetry of the Lissajous figures in the case of mistuned consonances does not simply represent the beat itself but it plays essentially the same role in its origin as did the amplitude variation of the Lissajous figures in the case of beats of imperfect unison. The latter amplitude variation also is caused by a cyclic phase shift between the primary frequencies. It has been found, therefore, that in both cases (imperfect unisons and mistuned consonances) the beat which appears in the cochlear model as due to

partial rectification is caused by the same property of the signal, the cyclic phase shift between the two primaries.

The examples of figures 2 and 3 concerned the mistuned 1:2 interval only. It is well known that audible beats arise whenever any integral ratio between two primaries is mistuned, although the strength of the resulting beats is not uniform. For example, the beat of imperfect unison is always stronger than that of a mistuned 1:2 consonance. This difference is paralleled by the amplitude (or asymmetric) variation of the appropriate Lissajous figures: 100 percent in the case of imperfect unisons but only 33 percent in the case of mistuned 1:2 intervals (provided primaries are of equal amplitude in both cases). It was of interest, therefore, to attempt to find a general rule concerning the strength of audible beats of various mistuned intervals of integral numbers and the degree of asymmetric amplitude variation of

the appropriate Lissajous figures. Figure 4 shows a series of complex Lissajous figures for various such intervals. Some of these Lissajous figures have been shown in an earlier publication (7). These Lissajous figures are steady when the interval is perfect, but begin to revolve when it becomes mistuned. First the rate of asymmetric amplitude variation for one revolution of the Lissajous figure was determined for one complete period of the underlying phase shift. For example, one such variation takes place in case of the mistuned 1:2 interval through the sequence of figures 3A to D. The values of this rate for all Lissajous figures are indicated in figure 4. As a rule of thumb: if  $n_1$  and  $n_2$  are the two numbers making up the interval, the rate is given as  $n_1 + n_2 - 2$ . (This rate is not identical to the number of loops of which each of the Lissajous figures is composed. The latter values are also indicated in figure 4). Next, the Lissajous figures were produced on the screen of an oscilloscope and the magnitude of the asymmetric amplitude variation determined in each case. The ratio between the size of the

Lissajous and the degree of variation along one of its axes ( $I/V$ ) expresses the inverse of the relative magnitude. In figure 5 (left-hand figure) the rate of asymmetric amplitude variation is plotted vs. the ratio  $L/V$ . The result is a straight-line function indicating that the magnitude of the amplitude variation of Lissajous figures decreases with what might be considered their *dynamic complexity* (not with their structural complexity — i.e., the number of their loops).

Helmholtz (2) has given the relative strength of audible beats, at least for some of the intervals of figure 5 (left). These are plotted in graph at right (figure 5) again vs. the rate of amplitude variation of the Lissajous figures. Given in the same graph are the inverse values of curve at left ( $V/L$ )—i.e., the relative magnitude of the amplitude variation. (For the sake of continuity, the imperfect unison has been assigned a rate of zero and a relative amplitude variation of 1.0.) The two curves in right-hand graph of figure 5 are members of the same family. Thus Helmholtz'

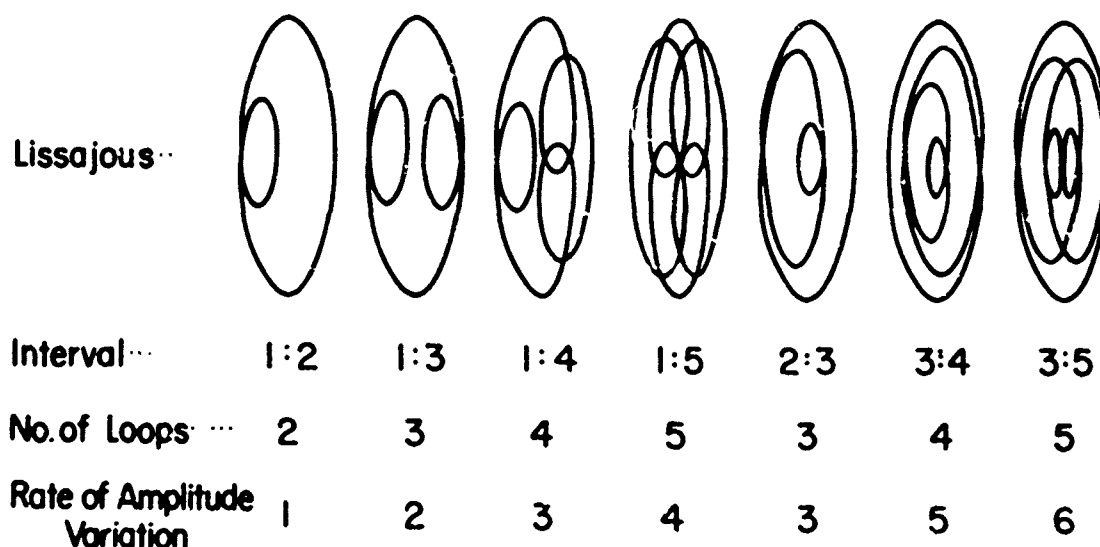


FIGURE 4

Complex Lissajous figures (drawn after oscilloscopic recordings). In each case, amplitudes of both components are equal. Such Lissajous figures remain steady when the interval is perfect, but revolve when the interval is mistuned. For further explanation see text.

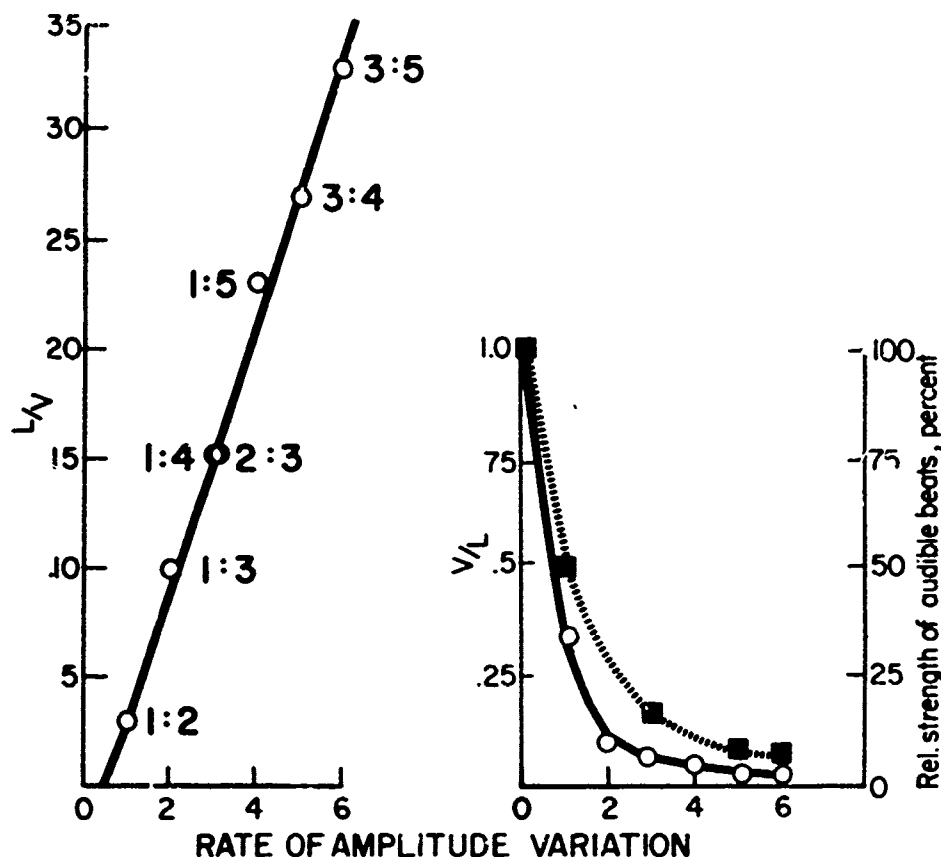


FIGURE 5

Left: Rate of amplitude variation of mistuned complex Lissajous figures vs. the ratio of the size of the Lissajous figures to the degree of their amplitude variation ( $L/V$ ). Ratios of (mistuned) intervals as indicated. Right: Inverse plot of figure at left, expressing the rate of amplitude variation of mistuned complex Lissajous figures vs. their relative magnitudes ( $V/L$ ). Plotted in the same graph are the relative strengths of audible beats after Helmholtz (2) for some of the intervals (black squares).

measurement of the relative strength of audible beats is provided with a rationale in terms of the phase-shifting Lissajous.

#### Cause of asymmetry

The next task was to find out what might be the cause of the asymmetric motion of particle orbits observed in the model. Within linear systems, beats represent a variation of amplitude synchronous with the beat rate; but there is never any energy present in the latter frequency. All the energy there is, is contained in the two primaries, a fact which can be confirmed either by Fourier analysis or by

wave-filtering. This statement is also true with respect to the case of mistuned consonances. It is not correct to call the pattern of figure 2 an "amplitude modulation of the higher frequency by the lower one," an opinion recently expressed by Chocholle and Legouix (5). (However, as it turns out now they were more correct in saying "that the ear is able to follow the cyclic variations of the wave form.") Figure 2 represents the result of a mere mixing of two frequencies. No new frequencies are generated in such a process. In nonlinear systems, however, the frequency ( $f_1 - f_2$ ) appears as a product of distortion. Therefore, it is possible that the appearance of the beat

frequency within the cochlear model, as reported above, results from some *nonlinear* property of the model.

First, it was ascertained that the postulated nonlinearity was not due to overloading. If that were the case the degree of asymmetry displayed by the beating particle orbits should depend on stimulus amplitude. Repeated checks failed to verify this proposition although it is admitted that the sound pressure range covered was rather small — viz, slightly less than 20 db. In an effort to elucidate this point further, the following measurements were made. Best beats of imperfect unison of a constant beat rate (2 cps) for various pairs of frequencies were applied to the model. Determined under the microscope were the voltage levels at the driver needed to produce: (a) the just-noticeable onset of particle motion; (b) the just-noticeable onset of beat motion; and (c) the just-noticeable onset of eddy motion. In a previous paper (9), the onset of motion of Bekey's eddies had been associated with amplitude distortion. The measurements were carried out for three pairs of frequencies in octave intervals. Since the onset of particle and of eddy motion was measured for one of the primaries only, a 6 db correction was applied to the onset of the beat. Figure 6 presents the results concerning the just-noticeable onset of particle motion. Whereas at 25 cps beat motion did not become apparent until particle motion had almost doubled in amplitude, at higher frequencies the situation was reversed; that is, beat motion was detected at lower amplitudes than particle motion. Invariably, eddy motion is observed at amplitudes considerably higher than that at which beat motion was first noticed, the ratio being 16 db at 25 cps and 19 db at 100 cps. Because of the findings of figure 6, it is not likely that the beat motion is caused by Bekey's eddies, although the slope of the two curves is similar. This conclusion is supported by the fact that the onset of eddy motion, which is a DC motion, can be assessed with more certainty than the onset of an AC motion such as the beat. The latter measurement depends more than the former on the magnification used and on particle size. Yet, beat motion was always

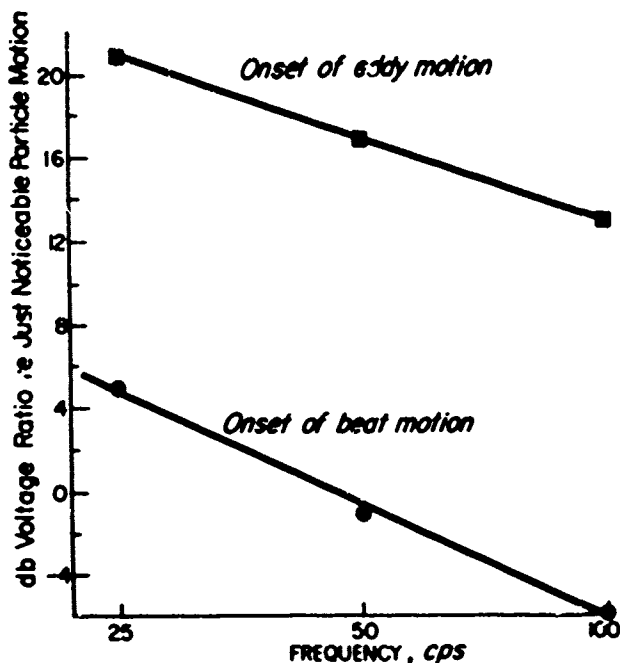


FIGURE 6

*The onset of beat motion and of eddy motion for three frequencies in octave intervals in regard to the onset of particle motion. Values are given as db-voltage ratios.*

detected at much smaller amplitudes than eddy motion.

Next, the cochlear partition was examined as a possible source of the postulated nonlinearity. A membrane was made with uniform stiffness; that is, along this membrane stiffness did not lessen with distance as it does along the membranes which are ordinarily used and which are made according to Bekey's measurements of the stiffness gradient (11). A perfect membrane of uniform stiffness is the boundary layer between two nonmixing fluids, provided the width of this boundary is uniform from end to end. A rectangular slot, 50 mm. long and 2 mm. wide, was cut into a frame of the same type as that used normally to hold the membranous partition. The model was filled with equal parts of ether and of the usual glycerin-water solution. Then the two scalae were arranged, one above the other, so that the boundary was located exactly within the rectangular slot. When this model was

driven by two primaries of imperfect unison (e.g., by 50 cps and 52 cps), the *asymmetric beat motion did not appear*. The particle orbits were exactly asymmetric like the beating Lissajous figures produced upon the oscilloscope (cf. fig. 1A). It is apparent from this observation that the *asymmetric beat motion found in the Bekesy-type models results from the stiffness gradient of the partition*.

In order to find the dependence of the degree of asymmetric beat motion upon the steepness of the stiffness gradient, a series of four interchangeable partitions was made into which slots of varying taper were cut. All slots were 50 mm. long and 1 mm. wide at their proximal end. At their distal end, they varied from 1 to .5 mm. Covering these slots with a uniform amount of rubber cement produced membranes with a linear gradient of stiffness (in contrast to the membranes normally used in which stiffness varies exponentially with distance). The steepness of these linear gradients varied in proportion to the taper of the slots.

All experiments were carried out for low-frequency primaries because of the inverse relationship between frequency and particle amplitude. Both primaries were set so as to produce best beats. In the first experiment, the ratio was determined between the amplitude of the beats originating within the model and the maximal orbital amplitude ( $a_b/a_{o \max}$ ). This ratio was obtained for identical sets of primaries along each of the four different membranous partitions. The beat amplitude ( $a_b$ ) was conveniently measured in the region of the helicotrema (where there are no primaries in evidence) and the maximal orbital amplitude ( $a_{o \max}$ ) in the proximal regions of the model (where there is no beat motion in response to low-frequency primaries). Figure 7 gives the results for primaries of 30.2 cps and 31.0 cps (beat rate: 0.8 cps). The alteration of the stiffness gradient is expressed as the percentage increment in width of the membranous partition with distance ( $\Delta W/L$ ). A straight-line function, with the limited set of data at hand, was obtained when the amplitude ratio was expressed as  $e^{.5a_b/a_{o \max}}$ . Figure 7

then indicates that the beat amplitude (for equal amplitudes of the primaries) increases with the steepness of the stiffness gradient in a logarithmic fashion.

Next, a membrane was employed with the usual exponential gradient of stiffness, the ratio of stiffness at the two ends being approximately 10:1. The amplitude ratio  $a_b/a_{o \max}$  was determined for various sets of low-frequency primaries, but for a uniform beat rate ( $b = 0.5$  cps). In figure 8, the results are plotted as the db ratio of  $a_b/a_{o \max}$  vs. the average frequency of the primaries. The values follow a straight-line function with a positive slope of approximately 6 db/octave. In other words, the beat amplitude (for equal amplitudes of the primaries) increases as an exponential function of frequency along such membranes. Because of the dependence of beat amplitude upon the value of the stiffness gradient (cf. fig. 7) (which in turn alters with distance along membranes of exponential stiffness gradients), it is reasonable to assume that the beat for each set of primaries is mainly produced in the region of maximal membrane displacement in response to those primaries. This explains the finding reported earlier that in the case of low-frequency primaries the beat was not in evidence in the fenestral region. However, the locus of origin of the beat cannot be too sharply defined in distance along the membrane as will become apparent from other results to be reported presently.

Last in this series, a membrane of a linear gradient of stiffness was used ( $\Delta W/L = 3$  percent). Again, the amplitude ratio  $a_b/a_{o \max}$  was determined for various sets of low-frequency primaries and for a uniform beat rate ( $b = 0.8$  cps). In figure 9, the results are again plotted as the db ratio  $a_b/a_{o \max}$  vs. the average frequency of the primaries. The beat amplitude (for equal amplitudes of the primaries) was fairly uniform for primaries down to approximately 40 cps. A sharp drop of the curve occurs below this limit. The reason for this drop is explained by the findings of figure 8. In the vicinity of the helicotrema a nonlinear gradient of stiffness is formed because of the sudden transition from the membrane to the frame and to the opening. This

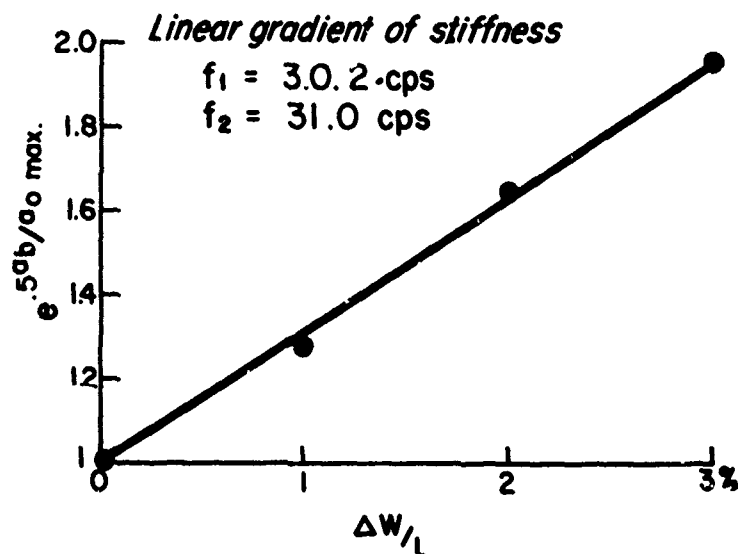


FIGURE 7

The relative beat amplitude ( $a_b/a_{o \text{ max.}}$ ) vs. the steepness of (linear) gradients of stiffness of four different partitions for a given set of primaries. The relative beat amplitude is given as an e-function and the stiffness gradient is expressed as the percentage increase in width ( $\Delta W/L$ ) of the slots into which the partitions were fitted.

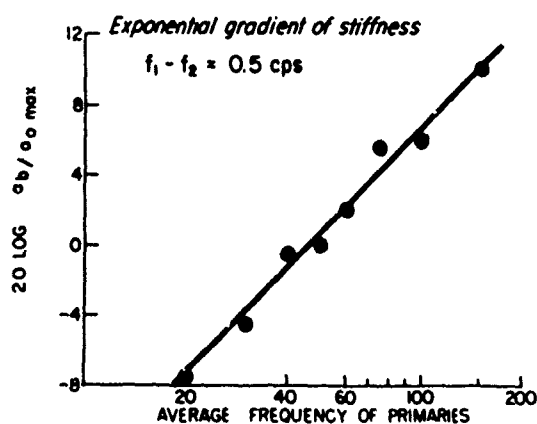


FIGURE 8

The average frequency of the primaries for a given beat rate vs. the relative beat amplitude ( $a_b/a_{o \text{ max.}}$ ); the latter expressed as a db ratio. Exponential gradient of stiffness.

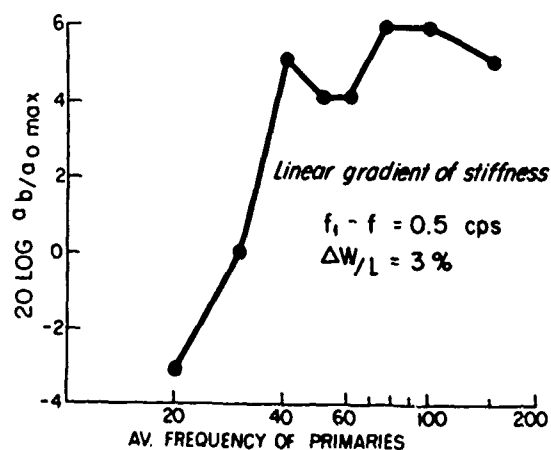


FIGURE 9

Average frequencies of the primaries for a given beat rate vs. the relative beat amplitude ( $a_b/a_{o \text{ max.}}$ ); the latter expressed as a db ratio. Linear gradient of stiffness.

nonlinear gradient affects only frequencies which form their locus of maximal membrane displacement close to the helicotrema and is an artifact due to the present model as was explained earlier. Otherwise, *beat production along membranes with linear gradients of stiffness is independent of frequency.*

### Membrane motion

The motion of the membranous partition in response to beats will be reported next. The motion was studied under stroboscopic illumination and with the aid of high-speed motion pictures. The stroboscopic findings will be reported first.

**Stroboscopic illumination.** A stroboscope can be considered an optical wave analyzer. Therefore, it was not surprising that a running beat pattern as one sees it upon the oscilloscope was never observed under the stroboscope. When a beat pattern of imperfect unison was set up along a membrane of *uniform* stiffness and the stroboscope was set to the frequency of the *lower* primary, an undistorted traveling wave pattern was observed which corresponded to the *higher* primary. Switching the lower primary off and on had hardly any effect upon the observed pattern.

When the stroboscope was set to the intermediate frequency

$$\frac{(f_1 - f_2)}{2}$$

a peculiar "flip-flop" motion was observed. Alternately, bulges traveled forward and backward which seemed to originate from slightly overlapping areas. This phenomenon indicates that the stroboscope brings, as it were, alternately the lower and the higher primaries into focus. From both observations it is obvious that only the two primary frequencies are present in the traveling waves along membranes with uniform stiffness. It is recalled that no beating had occurred in the fluid motion along such membranes either.

When along the same membrane, beats of a mistuned 1:2 interval were set up, the low-frequency component ( $f_1 - f_2$ ) was clearly in

evidence. For instance, with the stroboscope set at the lower primary, the higher primary was seen to travel along a membrane which simultaneously swung with the low-frequency component. However, when compared to findings to be reported later, this seemed to be a case of a mere *mixing* of frequencies. Therefore, the low-frequency component in this case must be considered the "rate of amplitude variation of the particle orbits," instead of the "beat frequency."

Next, membranes with a linear gradient of stiffness were examined. When along such a membrane beats of imperfect unison were produced and the stroboscope set to the lower of the two primaries, a whiplike motion of the higher primary was observed which was very different from the undistorted motion along a membrane with uniform stiffness. This whiplike motion started suddenly and decayed more slowly and was synchronized with the beat frequency. When the stroboscope was set to the intermediate frequency the "flip-flop" motion was still seen, but by comparison with the membrane with uniform stiffness it was jerky and very limited in extent. These results seemed to indicate that in addition to the two primaries other frequencies were present; presumably the primaries were being *modulated* by the beat frequency.

If there really were an amplitude modulation of the two primaries by the beat frequency the following four side bands should be produced:  $f_1 \pm f_b$  and  $f_2 \pm f_b$ . Two of these ( $f_1 + f_b$  and  $f_2 - f_b$ ) simply represent the alternate primary. The other two side bands ( $f_1 - f_b$  and  $f_2 + f_b$ ), however, are *new* frequencies which should be observable if they existed. With stroboscope settings slightly lower than  $f_1$  or slightly higher than  $f_2$  the phenomena one observes are highly complex. However, if one slowly alters the strobe-frequency further (i.e.,  $< f_1$  or  $> f_2$ ) a distinct setting is reached, though not too striking, at which the complexity is somewhat reduced. This setting which is quite reproducible corresponds to the two sidebands ( $f_1 - f_b$  and  $f_2 + f_b$ ). This is the only direct proof that

is offered for the presence of the sidebands in the motion of the cochlear partition. (Another indirect proof will be given later.)

With mistuned consonances, comparable complex phenomena were seen upon stroboscopic illumination. Now the three frequencies ( $f_1$ ;  $f_2$ ; and  $f_b$ ) were not merely "mixed" together as along the membrane of uniform stiffness. When the stroboscope was set to either primary, the whiplike motion of the other one, synchronous with the beat, was very pronounced, making the difference between mixing and modulation quite obvious.

As the next step, membranes with *exponential* stiffness gradients were examined. In response to imperfect unisons, the results were similar to those observed along the membranes with a *linear* stiffness gradient. However, both the whiplike motion (when the stroboscope was set to the lower primary) and the jerky "flip-flop" motion (when the stroboscope was set to the intermediate frequency) appeared to be restricted to the region proximal to the locus of maximal membrane displacement. In response to mistuned consonances, results were very complex, almost defying description. Some similarity to the events along a membrane with a linear stiffness gradient was recognized.

Next, along all three membranes (uniform stiffness, linear gradient, and exponential gradient of stiffness) beats were introduced after the pattern had been distorted within the electrical driving circuit. As seen in figure 10 (upper trace) the positive and negative amplitudes had been made unequal. When such an asymmetrically distorted pattern was applied to the model with the membrane of uniform stiffness, its response became very similar to that of the model with a membrane of a linear stiffness gradient. In other words, the same stroboscopic pattern of modulation was produced when the source of nonlinearity was outside or was within the model.

Conversely, the beat within the fluid motion of the model with a membrane of a linear gradient of stiffness was canceled by proper

choice of the degree and the polarity of asymmetry of the applied signal. The pattern of figure 10, incidentally, achieved exactly this result for primaries of 50 cps and 51 cps along a membrane with a stiffness gradient corresponding to  $\Delta W/L = 3$  percent. The asymmetry is 1:1.3. With this particular stimulus pattern, the events along the membrane when viewed stroboscopically were practically like those along a membrane of uniform stiffness to which an undistorted signal had been applied. The results of these two experiments indicate clearly that the *stiffness gradient*, which is responsible for the generation of the beat frequency, is a *nonlinear property* of the model. This nonlinearity in turn leads to a true *amplitude modulation* of the primaries by the beat frequency as was shown earlier.

Such cancellations were never completely achieved along a membrane with an *exponential* gradient of stiffness. One had the impression that by adjusting the degree and the polarity of asymmetry of the signal the membrane motion became more regular over a limited area. However, the results never came near those achieved along a membrane with a *linear* stiffness gradient. The reason for this failure is obvious from figure 9. Since the amplitude of the beat produced within the model depends upon the value of the stiffness gradient (which in this case alters with distance), it is not possible by application of an extraneously distorted signal to cancel the beat uniformly throughout its region or origin. Simultaneously, this indicates that the locus of origin of the beats cannot be too sharply defined along the partition.

Stroboscopic examination of the model has thus shown the membrane motion to be highly complex. Along a membrane with an exponential gradient of stiffness, five frequencies are in evidence: the original pair of primaries, the beat, and two sidebands produced by amplitude modulation of the primaries by the beat.

*High-speed motion pictures.* High-speed motion picture photography was used for observation of the actual displacement pattern of the cochlear partition under the effect of beats.



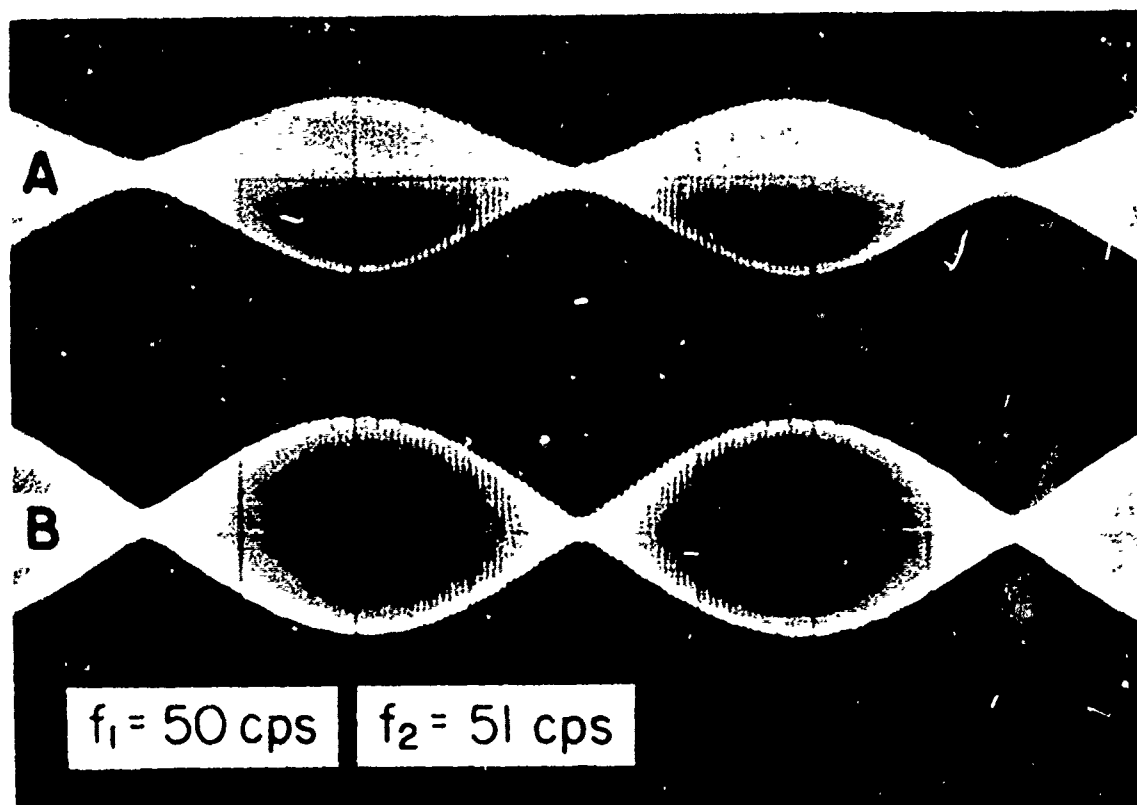


FIGURE 10

Oscilloscopic tracings of a beat pattern. A: Asymmetric amplitude distortion (ratio of 1:1.3 between positive and negative amplitudes). B: Symmetric, undistorted wave form.

A camera speed of approximately 1,500 frames per second and primaries of 50 and 55 cps were used in most instances. Traveling waves were seen of a frequency of 52.5 cps which increased and decreased in amplitude five times a second. No other frequencies were in evidence. It was not even possible to say with certainty whether or not the displacement of the membrane was asymmetric as it should be, judging from the results of stroboscopic examination. However, such judgments are difficult to make as had been learned in earlier studies (9). The pattern of motion appeared to be essentially the same as that seen on an oscilloscope in the case of beats.

During the course of these studies an accidental observation was made which is worthy of description. When the first trial shots were made with the high-speed camera for the purpose of determining optimal illumination and

exposure, beats were applied to the model without adjusting the amplitude of the primaries very carefully for best beats. When these first rolls of film were reviewed, a peculiar phenomenon was immediately noted: the frequency did not remain uniform but varied between the periods of maximal and minimal amplitude. It was recalled that Helmholtz (2) had stated that one "can hear a slight variation in pitch with the beat," provided the two primaries are not equal in amplitude. Helmholtz, in giving credit to G. Geroult (the French translator of his book) for having brought this matter to his attention, treated the ... mathematically and derived the following two terms for the frequencies during the periods of maximal and of minimal amplitude:

$$f_{a \max} = \frac{f_1 a_1 + f_2 a_2}{a_1 + a_2}$$

and

$$f_{\text{result}} = \frac{f_1 a_1 - f_2 a_2}{a_1 - a_2}$$

$f_1, f_2$  — frequencies of the primaries

$a_1, a_2$  — amplitudes of the primaries,  $a_1 > a_2$

"Hence [in the words of Helmholtz] during maximum strength, the pitch of the variable tone lies between the pitches of the two separate tones. But during minimum strength, if the stronger tone is also the sharper, the pitch of the variable tone is sharper than that of either of the single tones; and if the stronger tone is the flatter the pitch of the variable tone is flatter than either of the single tones."

S. Taylor (12), whom Helmholtz mentions in the fourth edition of his book, has treated the problem graphically. The lower portion of figure 11 is one of Taylor's original drawings:

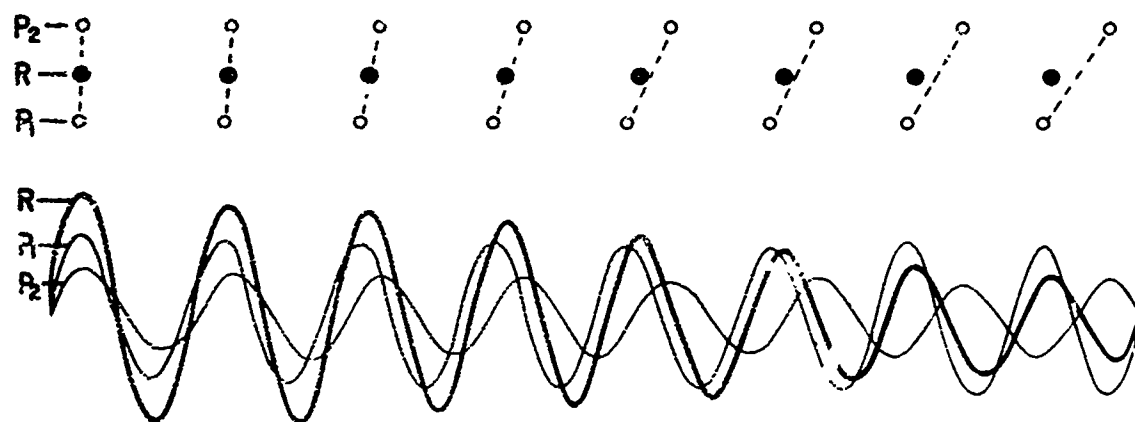


FIGURE 11

Section of a beat pattern ( $R$ ) resulting from two primaries ( $P_1$  and  $P_2$ ) of unequal amplitude (adapted from S. Taylor (12)). The lower frequency has the larger amplitude. The three rows of dots on top correspond to the amplitude maxima of the wave pattern and indicate the gradual decrease of frequency of the resultant ( $R$ ) as it approaches its amplitude minimum.

the composition of a small section of a beat pattern from primaries of unequal amplitude. The rows of dots on top have been added only to show the pitch variation more clearly. Taylor, incidentally, gave credit to deMorgan for being the first to report upon this pitch variation in 1857. Taylor also showed mathematically that in the case of equal amplitude

of both primaries, when Helmholtz' second term leads to a discontinuity, the pitch is really constant — viz, that of the intertone.

High-speed motion pictures of the model when driven by two beating primaries of unequal amplitude show this frequency variation very clearly. Figure 12 gives a comparison between frequencies calculated with the aid of Helmholtz' equations and those observed in the high-speed motion pictures. The latter values were obtained in the following manner. While the film was projected, a bright but small spot produced by a stroboscope was flashed upon the same screen close to the projected image. In this way, it is possible after some practice to synchronize the stroboscope with the observed wave motion and thus to obtain its relative frequency. This method was found less difficult than frame-by-frame analysis,

since the resolution of the wave pattern on single frames is none too good, whereas its observation as a dynamic phenomenon is much easier. The small discrepancies between calculated and observed values in figure 12 probably are due to slight variations in camera speed between films, since the deviations are systematic.

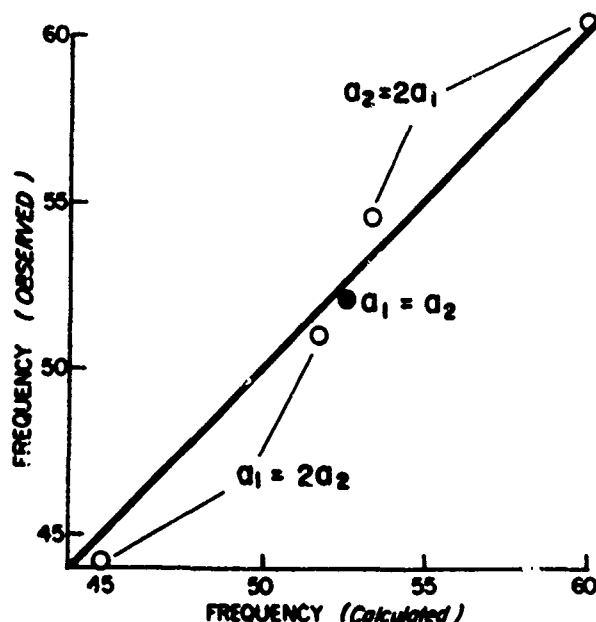


FIGURE 12

Frequency variation in beats ( $f_1 = 50$  cps;  $f_2 = 55$  cps). Values calculated after Helmholtz (2) compared to those observed in high-speed motion pictures. Amplitude relation between primaries as indicated.

According to Helmholtz and Taylor, the variation in apparent frequency does not arise on account of the nonlinear stiffness gradient of the cochlear partition. Therefore, the variation was expected to be found along partitions of uniform stiffness as well. This was, indeed, confirmed by observation in high-speed motion pictures.

After reading Helmholtz' statement and Taylor's rather detailed description on the audibility of the pitch variation in the case of unequal amplitudes of the primaries, this author tried repeatedly but did not quite succeed in convincing himself that he was able to hear the pitch variations in all cases. In the case of mistuned 1:2 consonances it seemed relatively easy. However, in the case of imperfect unisons it was not so convincing. It is apparently very difficult to separate the variation in loudness, which is very distinct, from that in pitch, since both of them occur at the same time. However, it was easy to demonstrate the following phenomenon: the variation of the average pitch with the amplitude ratio of

the primaries. One of the primaries (e.g., the higher one) was set at a constant amplitude. When the other primary was gradually introduced from below audibility by means of a continuously variable, motor-driven attenuator, one heard the pitch becoming gradually flatter as soon as beating was noticed. There was a continuous, gradual variation in pitch. Finally, when the second primary was much louder than the first one and beats began to disappear, the pitch remained that of the variable tone.

#### Frequency resolution of the model

The fact that the cochlear partition vibrates with the frequency of the so-called intertone

$$\frac{(f_1 + f_2)}{2}$$

and even participates in its variation, while the products of distortion do not become visible, needs further discussion. In beat patterns within electronic circuits, the frequency of the intertone is only *apparent* (e.g., upon the screen of an oscilloscope) without having any energy content. Nonlinear distortion introduces the difference tone,  $f_1 - f_2$ , and the summation tone,  $f_1 + f_2$ , but again the intertone is not in evidence. And yet, the intertone is what one *hears*.

The answer to this problem may be provided by consideration of the mode of *frequency resolution* in cochlear models which had been described elsewhere (7). When a complex signal is applied to the model (e.g., one consisting of two frequencies in a true 1:2 harmonic relationship), one sees that the particle orbits (which originally look like those in figure 3) become gradually simpler with distance along the partition. The loop due to the higher harmonic gradually disappears. By adjusting the relative amplitude of both components, it is indeed possible to obtain completely simple particle orbits in the region where the lower frequency forms its maximal membrane displacement. This process of gradual elimination of the higher harmonic has been described as the *spatial filter action* of the model (7).

In the case of beats of imperfect unison, however, particle orbits already display the form of simple Lissajous figures when viewed over the duration of one complete cycle. No further resolution takes place as was shown in the present paper. Instead, the variation in orbital amplitude results in beats. It appears then that *the model is capable of resolving only such particle orbits which are complex over one complete cycle.* (Some of the most common examples of such orbits were given in figure 4.) Consequently, one may consider the cochlear model a *pattern analyzer* (13), instead of an imperfect Fourier analyzer which produces fusion between two adjacent frequencies as Helmholtz and others had postulated. The ability to act as a pattern analyzer is a *hydrodynamic* property of the model.

Recognition of the model as a pattern analyzer has solved a puzzle this writer was confronted with when first observing the resolution of complex sound signals along the cochlear partition (7). From Beke's earlier investigations (14) it is well known that in response to a pure-tone signal a very characteristic cumulative phase shift develops along the partition. Figure 13 which shows this phenomenon is one of Beke's own graphs. The phase shift, according to figure 13, is in some relation to the distribution of displacement maxima of each single frequency, although the slopes of the resulting curves are definitely not identical to each other. Much to his surprise, the present writer when studying particle orbits in response to complex sound signals always observed that the original phase relationship between the component frequencies was maintained along the cochlear partition as far as the complex orbits were observable. Therefore, this writer stated (7) "that [the slope of the phase-shift curves] must be more uniform than indicated in [Beke's] figure." However, this statement solved the dilemma only partially. For one had to expect from Beke's measurements (figure 13) that the most striking phase alteration between component frequencies should occur as soon as the displacement maximum of the higher frequency component was passed. That definitely was not the case. It appears now that the above statement was a

little premature. The phase shift in the traveling waves in response to a single-frequency signal, and that occurring in response to complex sound signals, apparently are two different entities. Beke's measurements, which in essence have been confirmed by Diestel (15), are very probably correct for single frequencies. One would expect that a system which works like a Fourier analyzer retains all essential features displayed by single frequencies when a complex sound signal is applied to the system. The failure to do so as in the present instance appears to be a characteristic property of pattern analyzers. It is paralleled by the phenomenon reported earlier that the traveling waves along the cochlear partition do not give evidence of the frequency components present in a pattern of beats. For if such were the case, the varying phase relationship between the components would be the prime indication of their presence in the displacement pattern.

It is recalled that under stroboscopic observation a Fourier analysis was realized of the events along the cochlear partition. However, considering the displacement pattern of the same events simply as a space-time sequence as revealed by high-speed motion picture photography indicates the performance of a pattern analysis. Therefore, it seems that Fourier analysis and pattern analysis are merely two aspects of the same phenomenon. If one is performed, execution of the other one should be feasible. What matters, of course, is which of the two is *utilized* in the analysis actually performed. Speaking in terms of the ear now, it appears from all evidence at hand that the sensory-neural processes elicited in the cochlea are the product of a sequence of events of which the displacement pattern of the basilar membrane is an essential part. Consequently, it is logical to assume that pattern analysis is utilized in the auditory process.

## DISCUSSION

The present concept of beats is supported by several known psychophysical facts. The correlation between the strength of *audible* beats (after Helmholtz) and the amplitude

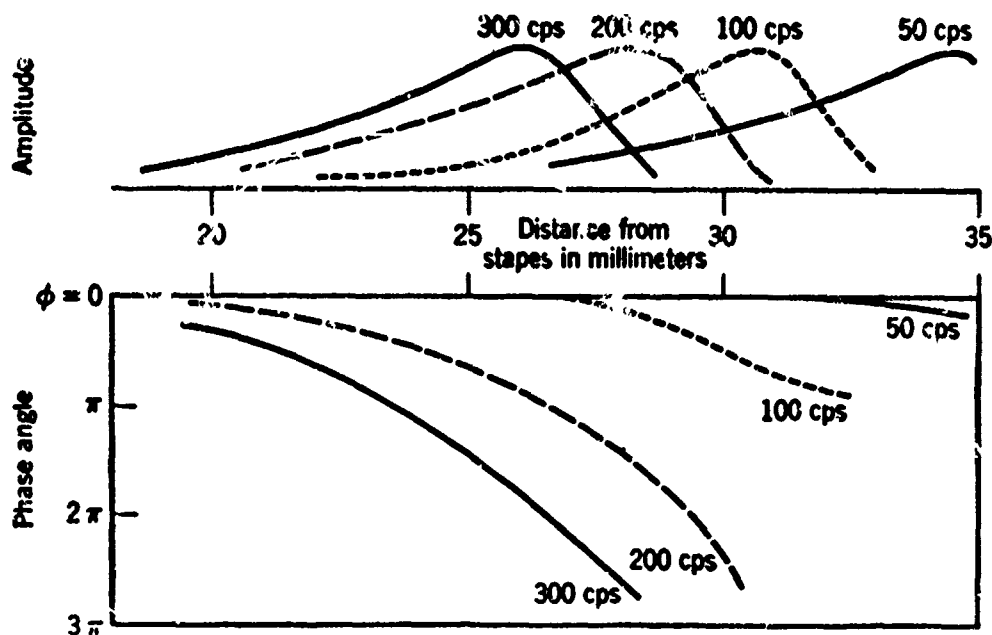


FIGURE 13

*Amplitude and phase angle of the cochlear partition at various distances from the stapes (from Bekesy (14)).*

variation of the Lissajous figures, which cause the modulation of particle orbits in the model, has been mentioned earlier. In addition, Helmholtz (2) has stated that the ear differentiates dissonances and consonances by the presence or absence of beats. It appears that this behavior is paralleled by the mode of pattern analysis found in cochlear models — viz, that there are steady orbital patterns in the case of consonances and revolving orbital patterns in the case of dissonances.

Further support for the present concept that beats are produced hydrodynamically (i.e., extraneurally) is given by the common clinical experience concerning tinnitus and beats. Most commonly, tinnitus originates at the sensory-neural level, although it may also be produced peripherally (e.g., muscular noises within the middle ear muscles). There are only two cases reported in the literature — by Wegel (16) and by Fowler and Wegel (17) — as far as this writer knows in which "phenomena suggesting something akin to beats" (17) were produced between tinnitus and an applied pure-tone signal. Fowler states categorically that the

occurrence of beats must be taken as an indication of the fact that the tinnitus is of extraneural origin. Davis et al. (18), in their study of noise-induced temporary threshold shifts, reported that they had failed to produce beats in all of their cases in which tinnitus was associated with the hearing loss. There can be no doubt that such tinnitus is of sensory-neural origin. This writer had the opportunity of verifying this observation on two well-trained listeners who, as members of the departmental staff, have served as listening subjects at numerous occasions. Both have a profound high-tone loss and tinnitus in the 3000 to 4000 cps region, the frequency of which they can match to pure tones. However, in repeated trials it was impossible for either subject to detect beats between the tinnitus and applied pure-tone signals of nearly the same frequency.

## CONCLUSION

If it can be assumed that the human ear behaves under acoustic stimulation as do

cochlear models (for which there is considerable evidence in Bekesy's earlier work), there now exists a physical basis for the understanding of beats, of consonances and dissonances, of the generation of harmonic distortion, and of difference and summation tones. (Work on the latter two phenomena is now in progress.) For explanation of the intertone the concept of fusion is no longer needed. The differences between consonances and dissonances which some writers have thought to be an acquired faculty because of the contrasting preference in occidental and oriental (Chinese) music may well have its roots in the hydrodynamic phenomena of the ear. The postulate that the ear acts like a pattern analyzer, instead of an imperfect Fourier analyzer, simplifies a number of psycho-acoustic problems. This concept, however, does not preclude the occurrence of other events at the sensory-neural level which may be responsible for additional limitations as to the tonal analysis performed by the auditory system.

## SUMMARY

The present paper describes the hydrodynamic phenomena observed in cochlear models when responding to beat signals. In addition, these observations give evidence as to the mode of frequency analysis performed by the model in the case of complex stimulation.

I. 1. *Particle motion within the cochlear fluids* occurs along elliptic orbits which are essentially Lissajous figures, as described in previous papers.

2. In response to beats of imperfect unison, these orbits execute an amplitude variation; that is, they expand and contract synchronously with the beat rate.

3. The beat rate becomes a physical reality by virtue of a partial *rectification* effect: the orbital expansion is *asymmetric* along both orbital vectors and this asymmetric expansion is in phase opposition in the two perilymphatic scalae.

4. The rectification effect is caused by the fact, known from Bekesy's studies, that the *stiffness* of the cochlear partition varies with distance from high to low values in an exponential manner. The resulting relative beat amplitude for given sets of primaries varies with the degree of the stiffness gradient displayed by different partitions. Furthermore, for a given beat rate and various sets of primary frequencies, the relative beat amplitude increases in an exponential manner with frequency along membranes with exponential gradients of stiffness. It is independent of frequency along membranes with linear gradients of stiffness.

5. In response to signals consisting of two components in harmonic relationship, *complex multiple-looped particle orbits* are formed. If such relationships are mistuned, the orbits revolve around themselves, thus introducing an amplitude variation. The latter, again by a partial rectification effect, produces the beat (*beats of mistuned consonances*).

6. In both forms of beats (imperfect unison and mistuned consonances), the variation of orbital amplitude, which leads to formation of the beat, has the same cause: a *cyclic phase shift between the two primaries*.

7. In the case of various mistuned musical intervals (dissonances), the amplitude variation of the complex Lissajous figures as observed on the oscilloscope (i.e., without rectification) is closely paralleled by the relative strength of *audible* beats after Helmholtz.

II. 1. The *displacement pattern of the cochlear partition* in response to beats when observed *stroboscopically* reveals the presence of five frequencies: the two primaries, the beat, and two side bands thus indicating that the primaries are *modulated* by the beat frequency. Actual traveling waves are not seen, since a stroboscope acts as an optical wave analyzer.

2. Along membranes with linear gradients of stiffness, the beat and the subsequent modulation can be *canceled* by application of a

driving signal which has been asymmetrically distorted to the same degree (but of opposite polarity) as that occurring within the model. Along membranes of exponential gradients of stiffness, such cancellations are only partially successful.

3. Comparison of the displacement patterns along membranes with and without gradients of stiffness make the difference between amplitude modulation and mere mixing of frequencies very obvious.

III. 1. When the displacement pattern of the cochlear partition in response to beats is viewed by high-speed motion picture photography, traveling waves are displayed with the frequency of the intertone (the average of the two primaries). Their amplitude increases and decreases with the beat rate. It is not possible to assess with certainty whether or not the displacement is asymmetrical as it should be, judging from the observations on fluid particle motion (cf. I.3).

2. When the two primaries are not adjusted for best beats (i.e., are unequal in amplitude), the frequency displayed varies between the periods of maximal and minimal amplitude. This frequency variation is in accordance with laws established by Helmholtz and by Taylor describing the variation of audible pitch in the same cases.

IV. 1. The displacement pattern of the cochlear partition in response to complex sounds displays a spatial filter action. In terms of fluid motion, particle orbits with distance along the partition approach gradually the form of simple Lissajous figures by

filtering out their higher-frequency components.

2. In response to beats of imperfect unison, particle orbits are already simple Lissajous figures. No further frequency analysis takes place. Instead, the amplitude variation displayed by these orbits results in beats.

3. The frequency analysis thus performed by the model is not a somewhat "imperfect Fourier analysis" (as assumed by classical theory), but a different form for which the term *pattern analysis* is suggested.

4. Pattern analysis is characterized by two properties: (a) by the resolution of particle orbits to the form of simple Lissajous figures, described above (IV. 1); and (b) by the fact that the cumulative phase shift of the traveling waves along the basilar membrane, as first described by Bekesy, occurs in a common pattern for all components instead of each component following its own.

5. Pattern analysis and Fourier analysis are merely two aspects of the same phenomenon. The results of stroboscopic examination of the membrane motion (II. 1) represent a Fourier analysis. However, in the detection process of the cochlea the displacement pattern of the basilar membrane plays a decisive role. Therefore, it appears that with reference to this problem a pattern analysis is performed.

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## REFERENCES

1. Ohm, G. S. Pogg. Annal. 135:513 (1843); 138:1 (1844).
2. Helmholtz, H. v. On the sensations of tone as a physiological basis for the theory of music. New York: Dover Publications, 1954; 2d English edition of 1884.
3. Wever, E. G. Theory of hearing. New York: Wiley & Sons, 1949.
4. Koenig, F. Pogg. Annal. 157:177 (1876).
5. Chochoille, R., and P. J. Legoux. J. Acoust. Soc. Am. 29:750 (1957).

6. Bekesy, G. v. *Physik. Zachr.* 29:733 (1928).
7. Tonndorf, J. *J. Acoust. Soc. Am.* 29:558 (1957).
8. Tonndorf, J. *Ann. Otolaryng.* 66:766 (1957).
9. Tonndorf, J. *J. Acoust. Soc. Am.* 39:929 (1958).
10. Hilding, A. C. *Ann. Otolaryng.* 64:278 (1955).
11. Bekesy, G. v. *J. Acoust. Soc. Am.* 20:227 (1948).
12. Taylor, S. *Phil. Mag.* 2:56 (1876).
13. This term has been chosen after some consideration. It is meant much in the same sense in which H. Fletcher [*Speech and Hearing in Communication*, Van Nostrand & Co., Inc., 1953] has referred to his theory of cochlear function as a space-time pattern theory. It is definitely not meant in the sense of Ewald's sound pattern theory.
14. Bekesy, G. v. *J. Acoust. Soc. Am.* 19:452 (1947).
15. Diestel, H. G. *Acustica* 4: 421 (1954).
16. Wegel, B. L. *Arch. Otolaryng.* 14:158 (1931).
17. Fowler, E. P. Sen., *Ann. Otolaryng.* 50:139 (1941).
18. Davis, H., C. T. Morgan, J. E. Hawkins, Jr., R. Galambos, and F. W. Smith. *Acta Otolaryng.* (Stockh.), suppl. 88 (1950).