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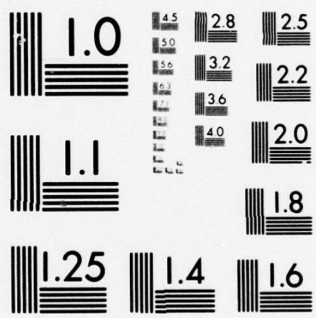
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METHOD OF CALCULATING THE FREEZING RATE OF SINGLE-COLUMN WATER-PERMEABLE SOILS

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METHOD OF CALCULATING THE FREEZING RATE OF SINGLE-COLUMN WATER-PERMEABLE SOILS

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I. Introduction

As is well known, the artificial freezing of soils is accomplished by means of implanting freezing columns in which brine is circulated in the ground at certain intervals. The brine is cooled by cooling devices. Initially individual frozen-soil cylinders are formed around each column, and these columns then fuse, thus forming a solid body. Depending on the nature of the construction operations, the columns are placed in very different ways: in a straight line or in a circle, in a single row or in several rows, etc.

Predicting soil freezing consists primarily of selecting the type and power of the cooling devices and the dimensions, number and arrangement of the freezing columns. In design it is also necessary to develop a production graph of the soil freezing operations which must indicate the operating mode of the freezing devices. It is quite obvious that to solve all of these problems and to select the most economical version the designer must be able to determine the soil freezing rate.

However, the existing methods of calculating the freezing rate. of water-permeable soils are insufficiently precise for practice, which is primarily due to the excessively rough allowance for the influx of heat from the unfrozen ground. Usually the thermal influx value adopted in the calculations is far below the actual value. This is confirmed by experimental materials on the freezing of water-permeable soils obtained in recent years by the Ice-Heat Laboratory of the B. Ye. Vedeneyev All-Union Scientific Research Institute of Hydraulic Engineering and data from field observations of a frozen-soil cofferdam in the construction of a hydroelectric power plant, as well as the works of B. V. Proskuryakov [1].

The precision with which freezing rate is calculated is also reduced because of shortcomings in the methods used to determine the heat absorption factor of the columns.[1]

[1]See the author's next article in this collection.

This article analyzes the method for calculating the freezing rate of homogeneous water-permeable soils by a single column; the method has been developed in works of B. V. Proskuryakov [1] and I. A. Charnyy [2].

Our task can be formulated as follows.

A single freezing column of radius r_0 is implanted perpendicularly in a homogeneous, infinite water-permeable layer of ground (Figure 1). The base and top of the layer are two parallel planes which are permeable to water and heat. The length of the column L is equal to the thickness of the layer h.

At a considerable distance from the column, the filtration flow has velocity v and temperature ϑ. The ground freezes at temperature ϑ_0. The physical constants of frozen and unfrozen ground are given. The column's heat absorption is a known function of the frozen ground's thermal resistance. It is necessary to determine how the dimensions of the frozen ground depend on time.

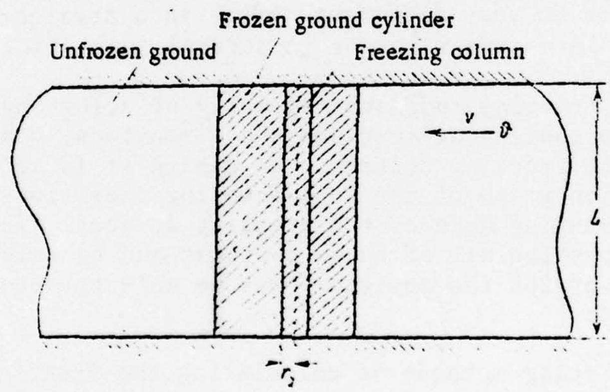

Figure 1. Freezing of Water-Permeable Soil by a Single Column.

In solving this problem, we will introduce the following basic assumptions, the influence of which will be examined later:

1) the thermal flux which changes the temperature of the frozen ground may be taken into account by increasing the computed value of the latent ice formation heat;

2) the temperature distribution in frozen ground at any moment of time corresponds to the established thermal state;

3) the frozen ground has the shape of a cylinder;

4) the axis of the frozen ground cylinder coincides with the axis of the freezing column.

Of the other less significant assumptions which we will introduce but whose influence we will not examine, let us note the following:

1) the thermal-physical constants of the frozen and unfrozen ground are not a function of temperature;

2) all of the ground water freezes at the same temperature;

3) the coefficient of heat transfer from the surface of frozen ground to the freezing water layer is infinitely great; i.e., we will ignore the transfer heat resistance from the water to the frozen ground cylinder.

2. Thermal Balance Equation and Its Components

The thermal balance equation of frozen ground has the following form:

$$Q_f = Q_c - Q_{fi} - Q_g, \tag{1}$$

where

Q_i is the thermal flux from the latent ice formation heat which is given off at the freezing boundary;

Q_c is the thermal flux which passes from the frozen ground to the freezing column (the thermal absorption of the column);

Q_f is the thermal flux which passes from the filtration flow to the frozen ground cylinder;

Q_g is the thermal flux due to the reduced heat content in the frozen ground.

Because of the above-mentioned assumption that the thermal flux Q_g can be taken into account by simply increasing the calculated value of the latent ice formation heat, thermal balance equation (1) is written in the following form:

$$Q_f = Q_c - Q_{fi}. \tag{2}$$

The thermal flux from the latent ice formation heat is equal to

$$Q_f = 2\pi L\sigma r \frac{dr}{d\tau}, \tag{3}$$

where

σ is the amount of heat given off when a unit of ground volume freezes;

L is the length of the freezing column;

r is the radius of the frozen ground cylinder;

τ is time.

B. V. Proskuryakov [1] has demonstrated that the thermal flux which passes from the unfrozen water-permeable ground can be represented by an equation with the following form:

$$Q_{fl} = 8\sqrt{\frac{\lambda_1 c_w \gamma_w v r}{\pi}}\, L(\vartheta - \vartheta_0); \tag{4}$$

here

λ_1 is the thermal conductivity factor of unfrozen ground;

c_w is the thermal capacity of water;

γ_w is the specific weight of water;

v is the velocity of the filtration flow;

ϑ is the temperature of the filtration flow;

ϑ_0 is the freezing temperature of the ground.

The last term in equation (2), the column's heat absorption, is in each particular case a completely determined function of the radius of the frozen ground cylinder:

$$Q_c = f(r). \tag{5}$$

3. Solving the Thermal Balance Equation

By substituting expressions (3), (4) and (5) into heat balance equation (2) we will have

$$2\pi L \sigma r \frac{dr}{d\tau} = f(r) - A\sqrt{r}, \tag{6}$$

where

$$A = 8\sqrt{\frac{\lambda_1 c_w \gamma_w v}{\pi}}\, L(\vartheta - \vartheta_0). \tag{7}$$

By separating variables r and τ and by integrating within the limits of $\tau = \tau_1$ to $\tau = \tau_2$ and $r = r_1$ to $r = r_2$, we find

$$\tau_2 - \tau_1 = 2\pi L \sigma \int_{r_1}^{r_2} \frac{r\,dr}{f(r) - A\sqrt{r}}. \tag{8}$$

The integral of the right-hand portion of equation (8) can be most simply determined by graphic or tabular integration.

The maximum size of the frozen ground cylinder $r = r_{max}$ is determined from the condition that the thermal absorption of the column is equal to the thermal influx from the filtration flow:

$$f(r) = A\sqrt{r}. \tag{9}$$

Computation equation (8) which is used to determine the velocity of freezing is correct with any form of the dependency $Q_c = f(r)$[1]. The uncertain form of the latter [dependency] made it possible to present computation equation (8) in integral form, which is very inconvenient for practical use. This fact causes us to search for short-cuts.

The course of freezing consists of two periods. During the first period the temperature of the brine in the column drops; during the second period the temperature of the brine is constant.

The first period changes to the second when a frozen ground cylinder cylinder is formed which has a radius determined from the equation:

$$f(r) = \frac{2\pi\lambda_2 L(\vartheta_0 - \vartheta_m)}{\ln\frac{r}{r_0}}. \tag{10}$$

where

r_0 is the radius of the freezing column;

λ_2 is the thermal conductivity factor of the frozen ground;

θ_m is the maximum low temperature of the brine.

Equation (10) is obtained by jointly solving equation (5) with the expression for the heat flow in a cylinder which, as is known from heat transfer courses, has the following form:

$$Q_c = \frac{2\pi\lambda_2 L(\vartheta_0 - \vartheta_m)}{\ln\frac{r}{r_0}}. \tag{11}$$

We will determine the freezing time separately for the first and second periods of freezing.

During the first freezing period we will assume that the heat absorption of the column is constant (averaged); if desired, it is

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[1]The method for determining this dependency is given in another article by the author later in this collection.

possible to refine the calculation, for which we should determine the freezing time in sections while assuming that each section has its own constant column heat absorption value.

During the second freezing period the brine temperature is constant, and therefore the heat absorption of the column will be expressed by equation (11) during this time.

It must be noted that in a number of cases the entire process of freezing takes place during the first period, and in some other cases the first period comprises an insignificant portion of the total freezing time. In these cases it is obvious that the freezing time is determined by calculating one period.

Thus, let us determine the calculated dependency for establishing freezing time when the column's heat absorption is constant.

Heat balance equation (6) acquires the following form:

$$2\pi L\sigma r \frac{dr}{d\tau} = Q_c - A\sqrt{r}, \tag{12}$$

from which we have

$$d\tau = \frac{r}{\frac{Q_c}{2\pi L\sigma} - \frac{A}{2\pi L\sigma}\sqrt{r}}\, dr.$$

In order to integrate the latter differential equation, we will introduce a new variable

$$z = \sqrt{r},$$

and stipulating that

$$B_1 = \frac{2\pi L\sigma}{A} \tag{13}$$

and

$$N_1 = \frac{Q_c}{A}, \tag{14}$$

we find

$$d\tau = B_1 \frac{z^3}{N_1 - z}\, dz,$$

from which we have

$$d\tau = B_1\left(\frac{2N_1^3}{N_1 - z} - 2z^2 - 2N_1 z - 2N_1^2\right) dz.$$

By integrating this differential equation within the limits of $\tau = \tau_1$ to $\tau = \tau_2$ and $z = z_1$ to $z = z_2$ and by returning to the initial variable r, we finally find the calculated dependency which is used to determine the freezing and thawing rates of the frozen ground cylinder when the heat absorption of the column is constant:

$$\tau_2-\tau_1=-B_1\left[\frac{2}{3}\left(r_2^{\frac{3}{2}}-r_1^{\frac{3}{2}}\right)+N_1(r_2-r_1)+2N_1^2\left(r_2^{\frac{1}{2}}-r_1^{\frac{1}{2}}\right)+2N_1^3\ln\frac{r_2^{\frac{1}{2}}-N_1}{r_1^{\frac{1}{2}}-N_1}\right], \tag{15}$$

where r_1 is the initial radius of the frozen ground cylinder at time τ_1.

From formula (15) it is evident that the freezing rate noted when the column's heat absorption is constant is a function of only two parameters: B_1 and N_1. This makes it possible to plot graphs which are convenient for making the calculations (Figures 2 and 3).

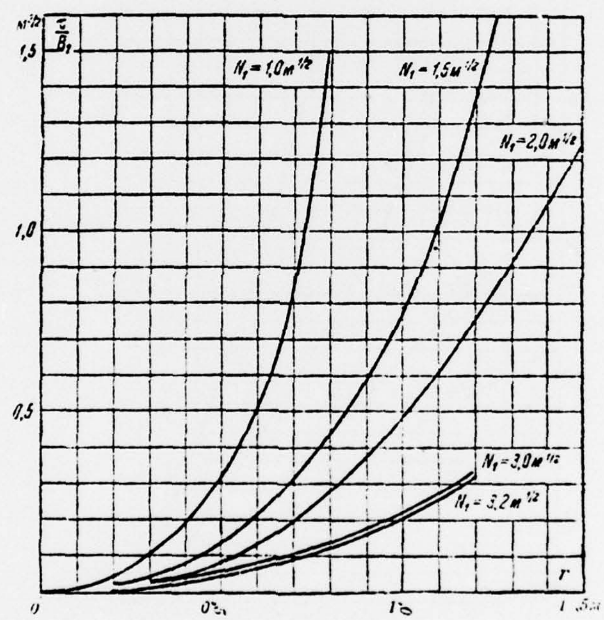

Figure 2. Graph for Determining the Rate with Which a Frozen Ground Cylinder is Formed when the Heat Absorption of the Column is Constant.

Assuming that the left-hand portion of equation (12) is equal to zero, we find the maximum radius of the frozen ground cylinder:

$$r_{max} = N_1^2 . \tag{16}$$

Let us now attempt to determine the calculated dependency of the speed with which the frozen ground cylinder is formed when the brine temperature is constant, which is what occurs during the second freezing period. In this case the heat absorption of the column is expressed by formula (11) and heat balance equation (2) acquires the following form:

$$2\pi L\sigma r \frac{dr}{d\tau} = \frac{2\pi\lambda_2(\theta_0 - \theta_m)L}{\ln\left(\frac{r}{r_0}\right)} - A\sqrt{r}. \tag{17}$$

Designating

$$a = 2\pi\lambda_2(\theta_0 - \theta_m)L, \tag{18}$$

we find

$$2\pi L\sigma r \frac{dr}{d\tau} = \frac{a}{\ln\left(\frac{r}{r_0}\right)} - A\sqrt{r}. \tag{19}$$

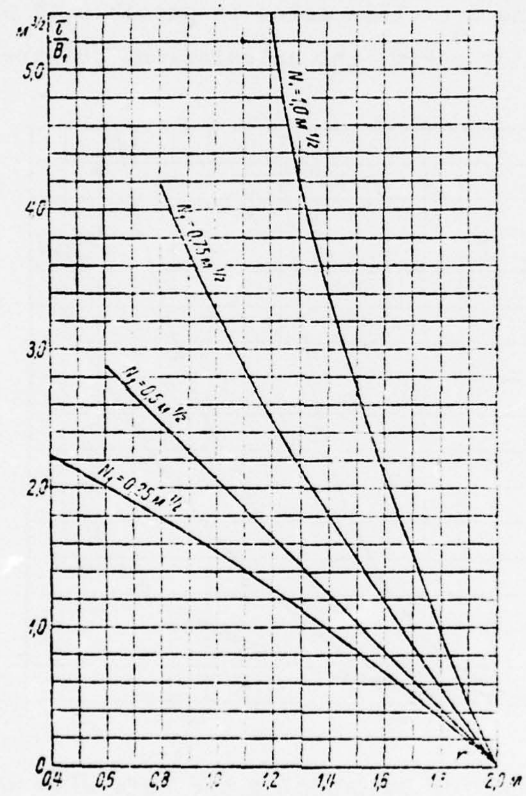

Figure 3. Graph for Determining the Thawing Rate of the Frozen Ground Cylinder When the Column's Heat Absorption is Constant.

In order to make it easier to integrate equation (19) we replace the function $\sqrt{r}\,\ln\left(\frac{r}{r_0}\right)$ with the simpler dependency:

$$\sqrt{r}\,\ln\left(\frac{r}{r_0}\right) \approx cr + f, \tag{20}$$

where c and f are constant coefficients.

By substituting (20) into (19), we find:

$$d\tau = \frac{2\pi L\gamma}{a}\,\frac{cr^{\frac{3}{2}} - fr^{\frac{1}{2}}}{1 - \frac{A}{a}(cr - f)}\,dr. \tag{21}$$

By integrating this latter equation, we find:

$$\tau = \frac{2\pi L\gamma}{a}\int\frac{cr^{\frac{3}{2}} - fr^{\frac{1}{2}}}{1 - \frac{A}{a}(cr - f)} = \frac{2\pi L\gamma}{a}J + C_0, \tag{22}$$

where C_0 is the integration constant.

We find:

$$J = c\int\frac{r^{\frac{3}{2}} - hr^{\frac{1}{2}}}{k - gr}, \tag{23}$$

where

$$h = \frac{f}{c}, \tag{24}$$

$$k = 1 - \frac{Af}{a}, \tag{25}$$

$$g = \frac{Ac}{a}, \tag{26}$$

from which we find

$$J = \frac{c}{g}\int r^{\frac{1}{2}}\,dr - \frac{ck}{g}\int\frac{r^{\frac{1}{2}}}{k + gr}\,dr + hc\int\frac{r^{\frac{1}{2}}}{k + gr}\,dr.$$

With an increase in the frozen ground cylinder of $\frac{k}{g} > 0$, then

$$J = -\frac{2c}{3g}r^{\frac{3}{2}} + \left(hc - \frac{ck}{g}\right)\frac{2}{k\left(\frac{g}{k}\right)^{\frac{3}{2}}}\left[\operatorname{Arth}\sqrt{\frac{gr}{k}} - \sqrt{\frac{gr}{k}}\right]. \tag{27}$$

After very elementary transformations of (27) and allowing for (24), (25) and (26), we will have:

$$J = -\frac{2}{3}\frac{a}{A}r^{\frac{3}{2}} - \frac{a}{A}\left(\frac{2\frac{a}{A}}{c}\right)r^{\frac{1}{2}} - \left(\frac{\frac{a}{A} - f}{c}\right)\frac{N}{N - f}\operatorname{Arth}\sqrt{\frac{c}{N - f}r}. \tag{28}$$

By substituting (28) into (22), we find

$$\tau = B_2\left[-\frac{2}{3}r^{\frac{3}{2}} - \frac{2N_2}{c}r^{\frac{1}{2}} + 2\left(\frac{N_2-f}{c}\right)^{\frac{3}{2}}\frac{N_2}{N_2-f}\operatorname{Ar\,th}\sqrt{\frac{c}{N_2-f}r}\right] + C_0;$$

here

$$B_2 = \frac{2\pi L\gamma}{A}, \tag{29}$$

$$N_2 = \frac{a}{A}. \tag{30}$$

By determining the integration constant from the initial condition that when $\tau = \tau_1$ we have $r = r_1$, we obtain the final computation equation for calculating the growth of the cylinder when the brine temperature is constant:

$$\tau_2 - \tau_1 = B_2\left[-\frac{2}{3}\left(r_2^{\frac{3}{2}} - r_1^{\frac{3}{2}}\right) - \frac{2N_2}{c}\left(r_2^{\frac{1}{2}} - r_1^{\frac{1}{2}}\right) + 2\left(\frac{N_2-f}{c}\right)^{\frac{3}{2}}\frac{N_2}{N_2-f}\left(\operatorname{Ar\,th}\sqrt{\frac{c}{N_2-f}r_2} - \operatorname{Ar\,th}\sqrt{\frac{c}{N_2-f}r_1}\right)\right] \tag{31}$$

If the influence of the changing freezing conditions, for instance an increase in the temperature of the brine or the filtration flow, reduces (melts) the frozen ground cylinder, then inequality $\frac{k}{g} < 0$ occurs, and then instead of (27) we obtain:

$$J = \frac{2c}{3g}r^{\frac{3}{2}} + \left(hc - \frac{ck}{g}\right)\frac{2}{k\left(\frac{g}{k}\right)^{\frac{3}{2}}}\left[\operatorname{Ar\,cth}\sqrt{\frac{gr}{k}} - \sqrt{\frac{gr}{k}}\right]. \tag{32}$$

Therefore when the cylinder melts, it is necessary to substitute expression (32) into equation (22) instead of equation (27); then after making the appropriate transformations and determining the integration constant, we find the following computation equation for the speed with which the cylinder melts when the brine temperature is constant:

$$\tau_2 - \tau_1 = B_2\left[-\frac{2}{3}\left(r_2^{\frac{3}{2}} - r_1^{\frac{3}{2}}\right) - \frac{2N_2}{c}\left(r_2^{\frac{1}{2}} - r_1^{\frac{1}{2}}\right) + 2\left(\frac{N_2-f}{c}\right)^{\frac{3}{2}}\frac{N_2}{N_2-f}\left(\operatorname{Ar\,cth}\sqrt{\frac{c}{N_2-f}r_2} - \operatorname{Ar\,cth}\sqrt{\frac{c}{N_2-f}r_1}\right)\right]. \tag{33}$$

From formulas (31) and (33) it follows that the freezing rate is the function of two parameters: B_2 and N_2. The coefficients f and c are constant and are determined from expression (20); thus, for instance, if the radius of the column is $r_0 = 0.05$ meters, then we can assume (up to the radius of the frozen ground cylinder $r \leqslant 0.5$ m) that $c = 3.8\ m^{-1/2}$ and the coefficient is $f = cr_0 = -3.8 \cdot 0.05 =$ $= 0.19\ m^{1/2}$. Dependencies (31) and (33) are used to make computations for the case where $r_0 = 0.05$ m. The results of these calculations are shown in Figures 4 and 5, and they make it possible to find a solution for them fairly quickly instead of finding them from the formulas.

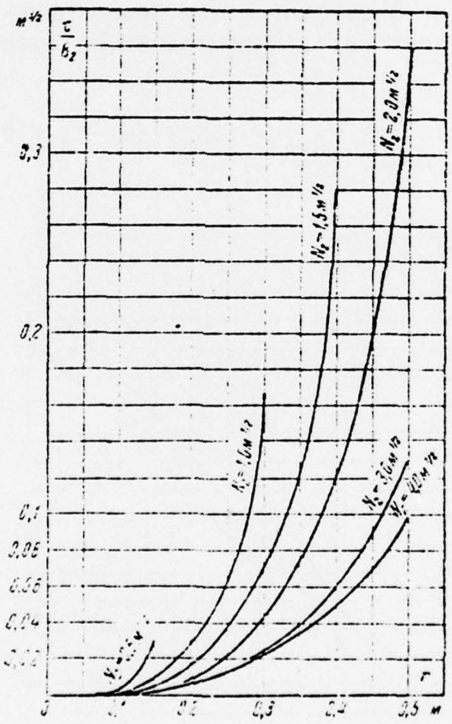

Figure 4. Graph to Determine the Speed with which a Frozen Ground Cylinder is Formed at Constant Brine Temperature.

The maximum value of the frozen ground cylinder radius can be determined if the left-hand portion of equation (17) is made equal to zero:

$$r_{max} \ln \frac{r_{max}}{r_0} = N_2. \tag{34}$$

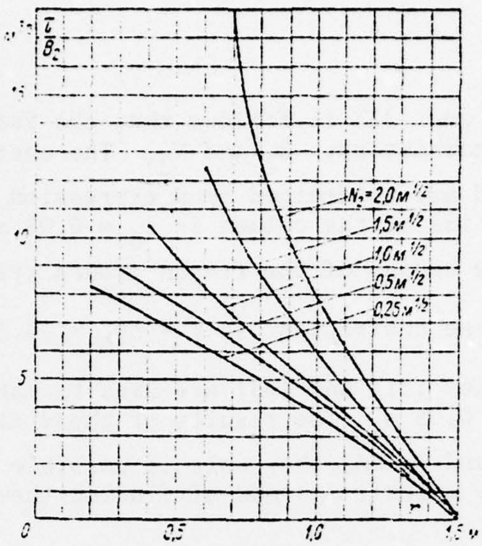

Figure 5. Graph for Determining the Speed with Which the Frozen Ground Cylinder Melts When the Brine Temperature is Constant.

The results of calculating $r_{max} = f(N_2)$ for $r_0 = 0.05\text{-}0.075\text{-}0.1$ m are shown in Figure 6.

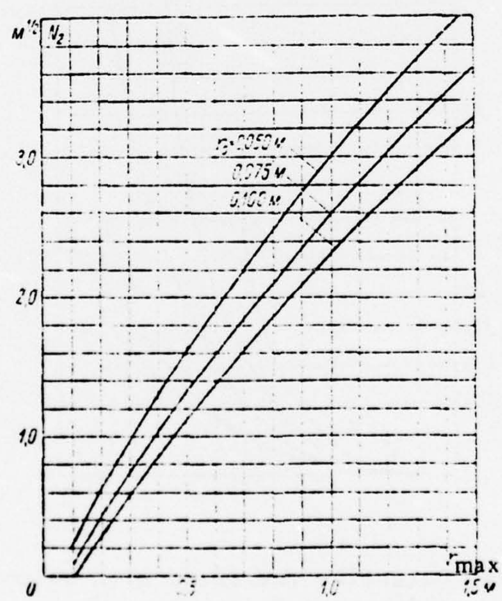

Figure 6. Graph for Determining the Maximum Radius of the Frozen Ground Cylinder.

Thus it is found that the maximum radius of the frozen ground cylinder depends only on parameter N_2 when the radius of the column is given. In order to analyze the influence of the individual factors on the maximum cylinder radius, we will determine the value of N_2.

For this purpose we insert expressions (7) and (18) into (30), and then we find

$$N_2 = 1{,}39 \frac{\lambda_2}{\sqrt{\lambda_1 c_w \gamma_w v}} \frac{(\vartheta_0 - \vartheta_1)}{(\vartheta - \vartheta_0)} . \tag{35}$$

An examination of this equation and Figure 6 makes it possible to draw the following conclusions:

1) changes in the brine temperature and the filtration flow temperature influence the course of freezing more significantly than changes in the flow rate of the filtration flow;

2) when water-permeable ground freezes, the maximum cylinder radius can only be increased by reducing the brine temperature or by increasing the column radius. The other values are usually given and cannot be changed;

3) the maximum radius of the frozen ground cylinder is not a function of the amount of latent ice formation heat.

4. Evaluation of Previous Assumptions

The assumptions adopted in this article have been previously introduced by many authors to one extent or another [1, 2, 3, 4, 5], but the influence of these assumptions, as far as we know, have not been evaluated in detail. Therefore the evaluation made below of these assumptions is one of the first attempts undertaken in this direction.

First assumption: the thermal flux which changes the temperature of the frozen ground can be taken into account by increasing the calculated value of the latent ice formation heat.

The thermal flux Q_g is equal to:

$$Q_g = \frac{dw}{d\tau} , \tag{36}$$

where w is the heat content of the frozen ground.

The heat content of the elementary ring of the frozen ground cylinder is equal to

$$dw = 2\pi c_2 \gamma_2 L (\vartheta_0 - t_\rho) \rho \, d\rho , \tag{37}$$

where

c_2 is the thermal capacity of the frozen ground;

t_ρ is the temperature of the elementary ring of the frozen ground;

γ_2 is the ~~percentage~~ of frozen ground;

ρ is the radius of the elementary ring.

As is known from heat transfer courses, the temperature distribution established in the cylinder is determined by the following equation [6]:

$$t_\rho = \theta - \frac{\theta - \theta_0}{\ln\frac{r}{r_0}} \ln\frac{\rho}{r_0}. \tag{38}$$

The temperature of the brine is equal to:

$$\theta = \theta_0 - \frac{Q_K}{2\pi\lambda_2 L} \ln\frac{r}{r_0}. \tag{39}$$

By substituting (39) into (38) we find:

$$t_\rho = \theta_0 - \frac{Q_K}{2\pi\lambda_2 L} \ln\frac{r}{\rho}; \tag{40}$$

then for the heat content of the elementary ring of the cylinder we find

$$dw = \frac{c_2\gamma_2 Q_c}{\lambda_2} \ln\frac{r}{\rho}\, d\rho.$$

By integrating the latter expression within the limits of $\rho = r_0$ to $\rho = r$, we find the heat content of the frozen ground cylinder:

$$w = \frac{c_2\gamma_2 Q_c}{\lambda_2}\left(\frac{r^2 - r_0^2}{4} - \frac{r_0^2}{2}\ln\frac{r}{r_0}\right), \tag{41}$$

from which we find

$$Q_g = \frac{dw}{d\tau} = \frac{c_2\gamma_2 Q_c}{2\lambda_2}\left(r - \frac{r_0^2}{r}\right)\frac{dr}{d\tau}. \tag{42}$$

If $r \geq 4r_0$, then it is obvious that

$$\frac{\frac{r_0^2}{r}}{r} 100 \leq \frac{\frac{r_0^2}{4r_0}}{4r_0} 100 \approx 6\%;$$

and therefore by making a minor error on the side of increasing the thermal flux Q_g, we assume $\frac{r_0^2}{r} \approx 0$, and therefore we will have

$$Q_g = \frac{c_2\gamma_2 Q_c}{2\lambda_2} r \frac{dr}{d\tau}. \tag{43}$$

This expression makes it possible to represent the sum of the thermal fluxes Q_i [expression (3)] and Q_g [equation (43)] in the following form:

$$Q_f + Q_g = (\sigma + \sigma_g) 2\pi L r \frac{dr}{d\tau},$$

where

$$\sigma_g = \frac{c_2 \gamma_2 Q_c}{4\pi \lambda_2 L}. \tag{44}$$

Designating

$$\sigma' = \sigma + \sigma_g, \tag{45}$$

we finally obtain:

$$Q_f + Q_g = \sigma' \cdot 2\pi L r \frac{dr}{d\tau}. \tag{46}$$

Expression (46) makes it possible to allow for the thermal flux which reduces the temperature of the frozen ground by increasing the calculated value of the latent ice formation heat. We should emphasize the fact that the freezing time of the ground is directly proportional to the value of the latent ice formation heat, but the latter exerts no influence on the maximum size of the frozen ground cylinder.

An examination of expressions (44) and (45) makes it clear that for practical purposes the increase in the calculated value of the latent ice formation heat does not exceed 10-20%. Consequently, the value σ_g in (44) can be determined approximately, and this has little effect on the final results of calculating freezing time.

Second assumption: at every moment in time the temperature distribution in the frozen ground corresponds to the established thermal state.

This assumption is reflected in the calculated values of the column's heat absorption. The heat absorption of the column Q_c is equal to the product of three values, i.e.: the coefficient of the frozen ground's thermal conductivity, the area of the column's lateral surface and the temperature gradient in the frozen ground near the column surface.

The temperature gradient in the ground near the column wall is always less than when the thermal state is established than when it is not (Figure 7). Therefore the above-mentioned assumption about the established temperature distribution in the frozen ground should mean that the calculated heat absorption values of the column will be below the actual values. Consequently, the heat balance equation (1) $Q_f = Q_c - Q_f - Q_g$ provides understated values of the thermal flux from the latent ice formation heat Q_f, and therefore understated calculated values for the freezing rate.

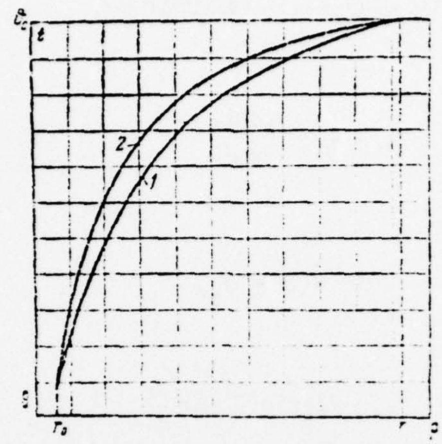

Figure 7. Temperature Distribution in the Cylinder. 1, Established temperature distribution; 2, unestablished temperature distribution.

However, it is obvious that the difference between the actual value of the column's heat absorption and the calculated value is less than the value of the thermal flux Q_g which changes the temperature of the frozen ground. This allows us to state that if we use heat balance equation (2) $Q_f = Q_e - Q_{fi}$, then, quite the contrary, understated calculated values will be obtained for the thermal flux from the latent ice formation heat, and consequently understated calculated values for the freezing rate. The true value lies between the two calculated values.

The relative difference between the two solutions is equal to:

$$\xi = \frac{Q_g}{Q + Q_g} 100\% \qquad (47)$$

and because of what is stated above concerning the first assumption, this difference is found to be within 20% for practical purposes.

Third assumption: frozen ground has the shape of a cylinder.

The reason for the circular shape to be violated may be non-uniform distribution of the thermal influx from the unfrozen ground over the cylinder surface.

The non-uniform distribution of the thermal influx is characterized by the relationship:

$$\varepsilon_1 = \frac{q_{fi}}{q_{fi\ av}}, \qquad (48)$$

where

q_{fi} is the intensity of the thermal influx from the unfrozen ground at a given point on the cylinder surface;

$q_{fi\ av}$ is the average intensity of the thermal influx from the unfrozen ground.

We will determine the intensity of the thermal influx Q_{fi} (the position of the coordinate axes relative to the direction of the filtration flow is shown in Figure 8).

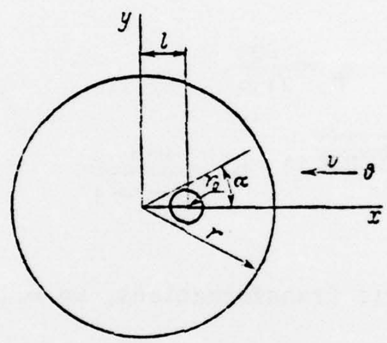

Figure 8. Frozen Ground Cylinder in the Filtration Flow.

By using the solution to the task of the cooling of the laminar flow when the flow around the cylinder is transverse [7], with regard to our case, we can write

$$q_{fi} = \sqrt{\frac{\lambda_1 c_w \gamma_w}{\pi}}\, v \frac{(\vartheta - \vartheta_0)}{\sqrt{\eta}},$$

where η_1 is the current function of the filtration flow.

The current function η is expressed by the equation

$$\eta = v\left(2r - x - \frac{xr^2}{x^2 + y^2}\right). \qquad (49)$$

On the cylinder surface where $\sqrt{x^2 + y^2} = r$, we have:

$$\eta = 2vr\left(1 - \frac{x}{r}\right);$$

or, noting that $x/r = \cos \alpha$ where α is the radius-vector argument of the point on the cylinder surface (Figure 8), we find

$$\eta = 2vr\ (1 - \cos\alpha). \tag{50}$$

By substituting expression (50) into equation (49) and integrating in terms of η, we obtain:

$$Q = \int q_{fi}\, d\eta = 2\sqrt{\frac{2\lambda_1 wiw v}{\pi}}\ (\vartheta - \vartheta_0)\, L\, \sqrt{1 - \cos\alpha} + C_0,$$

where C_0 is the integration constant.

By differentiating the latter expression in terms of α and by substituting the result into equation

$$q_{fi} = \frac{dQ}{Lr\, d\alpha},$$

we will have:

$$q_{fi} = \sqrt{\frac{2\lambda_1 c wiw v}{\pi r}}\ (\vartheta - \vartheta_0)\, \frac{\sin\alpha}{\sqrt{1 - \cos\alpha}},$$

or, after certain trigonometric transformations, we will finally obtain

$$q_{fi} = 2\sqrt{\frac{\lambda_1 c \gamma v}{\pi r}}\,(\vartheta - \vartheta_0) \cos\frac{\alpha}{2}. \tag{51}$$

The average intensity of the thermal influx from the unfrozen ground is equal to:

$$q_{fi\ av} = \frac{Q_f}{2\pi r L}.$$

Substituting expression (4) into this expression, we obtain:

$$q_{fi\ av} = \frac{4}{\pi}\sqrt{\frac{\lambda_1 c wiw v}{\pi r}}\,(\vartheta - \vartheta_0). \tag{52}$$

By substituting (51) and (52) into (48), we find:

$$\varepsilon_1 = \frac{\pi}{2} \cos\frac{\alpha}{2}. \tag{53}$$

The dependency $\varepsilon_1 = f(\alpha)$ is plotted in Figure 9. An examination of it shows that the thermal influx from the unfrozen ground is distributed along the surface of the cylinder in an extremely uneven fashion. This fact, however, does not violate the circular form of the cylinder since the half-sum of the thermal influx intensities at diametrically opposed points on the cylinder surface is equal to the average intensity of the thermal influx.

This condition is written in the following fashion:

$$\varepsilon_2 = \frac{(q_{fl})_\alpha + (q_{fl})_{\alpha+\pi}}{2q_{fl\ av}}. \tag{54}$$

By substituting expressions (51) and (52) into (54), it is easy to see that:

$$\varepsilon_2 = \frac{\pi}{4}\left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right). \tag{55}$$

The dependency $\varepsilon_2 = f(\alpha)$ is shown in Figure 9. An examination of it shows that the values of ε_2 deviate from unity by no more than 22%. From this it follows that, despite the extremely uneven distribution of the thermal influx from the water-permeable ground, the frozen ground has a shape which is similar to that of a cylinder, but its axis is shifted in the direction of the flow: an eccentricity arises between the cylinder axis and the axis of the freezing column.

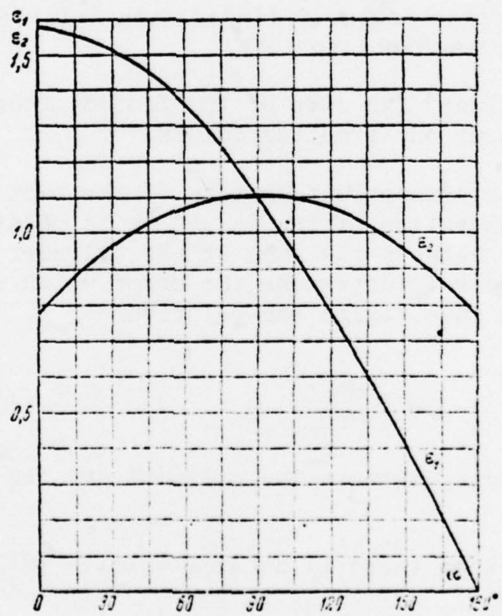

Figure 9. Distribution of Heat Influx from Unfrozen Ground Over Circumference of Cylinder.

The correctness of this conclusion has also been confirmed by the carried out by the Ice Heat Laboratory of the B. Ye. Vedeneyev All-Union Scientific Research Institute of Hydraulic Engineering.

Figure 10 shows a photograph of an frozen ground cylinder formed in sandy ground at a flow speed of v = 0.06 m/hr and a water temperature of ϑ = 10.6°C.

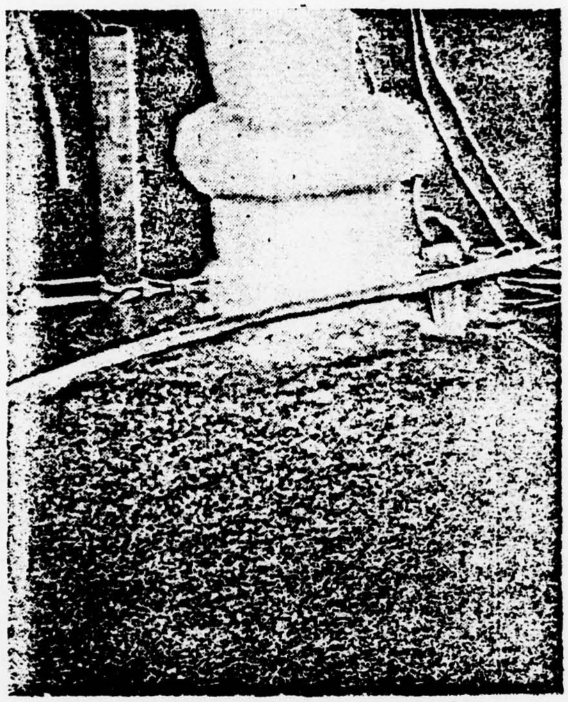

Figure 10. Frozen Ground Cylinder (the Filtration Flow was Directed From Right to Left).

The fourth assumption: the axis of the frozen ground cylinder coincides with the axis of the freezing column.

The essence of this assumption consists of the fact that in calculating the heat absorption of the column it is possible to ignore the eccentricity between the axes of the cylinder and the freezing column. In order to determine the error which is committed in this process, let us investigate the relationship

$$\eta = \frac{(Q_c)_{e\neq 0}}{(Q_c)_{e=0}}, \tag{56}$$

where e is the eccentricity between the cylinder and the freezing column.

As is well known, when there is an eccentricity [8],

$$(Q_c)_{e\neq 0} = \frac{(\vartheta_0 - \vartheta)\, 2\pi\lambda_2 L}{\ln\left[\frac{r}{r_0}\left(1 - \frac{e^2}{r^2}\right)\right]}, \tag{57}$$

-from this we find

$$\eta = \frac{\ln \frac{r}{r_0}}{\ln\left[\frac{r}{r_0}\left(1 - \frac{e^2}{r^2}\right)\right]}. \tag{58}$$

From Figure 11, which is plotted with the aid of dependency (58), it follows that since $e \leqslant 0.5\ r$, then the error which is committed in calculating the thermal absorption of the column without allowance for the eccentricity is small (less than 10%).

Thus, we may consider that it has been proven that the introduction of these four assumptions introduces no large errors into the calculations of the freezing of water-permeable ground.

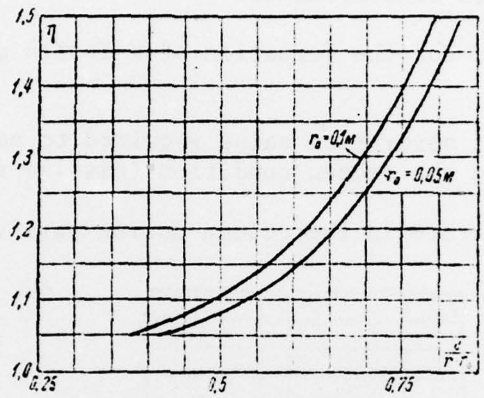

Figure 11. Function $\eta = f\left(\frac{e}{r - r_0}\right)$

5. Sample Calculation

Let us show by way of an example how we use the calculated dependencies obtained here.

The task may be formulated as the following: it is necessary to form a frozen ground cylinder of radius $r = 0.7$ meters in a filtering layer of ground 10 m thick. The cylinder is set perpendicular to the direction of the flow.

The following data are known:

speed of filtration flow..........	$v = 0.1$ m/hr
temperature of filtration flow.....	$\vartheta = 3°C$
freezing temperature of ground.....	$\vartheta_0 = -1°C$
thermal conductivity factor of unfrozen ground..................	$\lambda_1 = 1$ kcal/m·deg·hr
thermal conductivity factor of frozen ground....................	$\lambda_2 = 2$ kcal/m·deg·hr
latent heat of ice formation per unit of ground volume............	$\sigma = 24{,}000$ kcal/m^3
thermal capacity of water..........	$c_w = 1$ kcal/kg·deg
specific weight of water...........	$\gamma_w = 1{,}000$ kg/m^3
radius of freezing column..........	$r_0 = 0.10$ m
length of freezing column..........	$L = 10$ m.

The cooling device and its operating mode are selected in such a way that the heat absorption of the column changes as the size of the frozen ground cylinder increases, as shown in Figure 12; in this process the temperature of the brine does not reach a constant maximum low temperature.

The following have to be determined:

1) the time required for the formation of a frozen ground cylinder of the given dimensions;

2) the column's heat absorption value required to maintain the frozen ground cylinder in the frozen condition (passive freezing);

3) the brine temperature in the column during passive freezing.

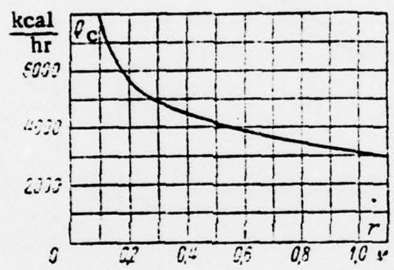

Figure 12. Change in Heat Absorption of Freezing Column.

Solution. We will determine the freezing time. First according to expression (7) we find the value of parameter A:

$$A = 8\sqrt{\frac{\lambda_1 q w_i w}{\pi}}\,L\,(\vartheta - \vartheta_0) = 8\sqrt{\frac{1\cdot 1\cdot 1000\cdot 0{,}1}{\pi}}\,10\,(3+1) =$$

$$= 1810 \text{ kcal/hr}\cdot\text{m}^{1/2}.$$

By substituting the initial data into formula (8), we find that the freezing time is determined by the following expression:

$$\tau = \tau_2 - \tau_1 = 2\pi L \sigma \int_{r_1}^{r_2} \frac{r\,dr}{f(r) - A\sqrt{r}} = 2\pi\cdot 10\cdot 24\,000 \int_{r_1=0{,}10}^{r_2=0{,}7} \frac{r}{f(r) - 1810\sqrt{r}}\,dr =$$

$$= 1{,}51\cdot 10^6 \int_{r_1=0{,}10}^{r_2=0{,}7} \frac{r}{f(r) - 1810\sqrt{r}}\,dr.$$

We will carry out the calculation by the tabular integration method, for which purpose we divide the integration limits into n sections: each section will be designated by a serial number i. Then we will have

$$\tau_1 = 1{,}51 \cdot 10^{6} \sum_{i=1}^{i=n} \frac{r_i}{f(r_i) - 1810\sqrt{r_i}} (r_i - r_{i-1}).$$

The value of the column's heat absorption will be selected according to Figure 12, allowing for the fact that

$$f(r) = \frac{f(r_i) + f(r_{i-1})}{2}.$$

The rest of the calculations require no explanation and are summarized in the following table:

i	r_i, м	$f(r_i)$, kcal/hr	$1810\sqrt{r_i}$, kcal/hr	$f(r_i) - 1810\sqrt{r_i}$, kcal/hr	$\frac{r_i}{f(r_i) - 1810\sqrt{r_i}}$, м · hr/kcal	Δr_i, м	$\frac{r_i \Delta r_i}{f(r_i) - 1810\sqrt{r_i}}$, м² · hr/kcal	$\Delta\tau$, hr	τ, hr
1	0,15	7200	700	6500	$23{,}08 \cdot 10^{-6}$	0,05	$1{,}15 \cdot 10^{-6}$	2	2
2	0,25	5900	905	4995	$50 \cdot 10^{-6}$	0,10	$5 \cdot 10^{-6}$	8	10
3	0,35	4950	1070	3880	$90{,}3 \cdot 10^{-6}$	0,10	$9{,}03 \cdot 10^{-6}$	14	24
4	0,45	4400	1210	3190	$141 \cdot 10^{-6}$	0,10	$14{,}1 \cdot 10^{-6}$	21	45
5	0,55	4100	1340	2760	$198 \cdot 10^{-6}$	0,10	$19{,}8 \cdot 10^{-6}$	30	75
6	0,70	3750	1510	2240	$312 \cdot 10^{-6}$	0,15	$46{,}8 \cdot 10^{-6}$	71	146

Commas indicate decimal points.

The calculation results show that the formation of an frozen ground cylinder will take 146 hours, i.e., approximately six days (Figure 13).

Let us compare this result with that which is obtained if we carry out the calculation according to formula (15).

The column's heat absorption drops from an additional value of 8,000 kcal/hr to 3,600 kcal/hr at the beginning of the freezing process (see Figure 12). We will assume that the column's heat absorption is a constant value, equal to

$$Q_c = \frac{8000 + 3600}{2} = 5800 \text{ kcal/hr}.$$

From expressions (13) and (14) we find:

$$B_1 = \frac{2\pi L \tau}{A} = \frac{2\pi \cdot 10 \cdot 24\,000}{1810} = 834 \text{ hr } \text{м}^{\frac{3}{2}}.$$

$$N_1 = \frac{Q_c}{A} = \frac{5800}{1810} = 3{,}2 \text{ м}^{\frac{1}{2}}.$$

Then, according to formula (15), we obtain

$$\tau = -834\left[\frac{2}{3}\left(r_2^{\frac{3}{2}} - 0{,}1^{\frac{3}{2}}\right) + 3{,}2\,(r_2 - 0{,}1) + 2\cdot 3{,}2^2\left(r_2^{\frac{1}{2}} - 0{,}1^{\frac{1}{2}}\right) + 2\cdot 3{,}2^3 \ln \frac{3{,}2 - r_2^{\frac{1}{2}}}{3{,}2 - 0{,}1^{\frac{1}{2}}}\right];$$

for r_2 = 0.7 m we find τ = 84 hours.

The results obtained previously (by the tabular integration method) τ = 146 hours is more precise. Thus, we see that averaging the column's heat flow within such large limits and for the entire freezing period provides only a very approximate result (τ = 84 hours). It is much more precise to determine the freezing time in sections, assuming that each section has its own averaged, constant value for the column's heat absorption. We will demonstrate this by means of the same example.

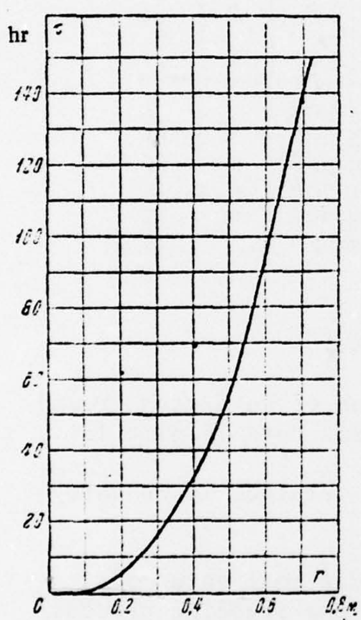

Figure 13. Graph of Freezing Speed.

In carrying out the calculations we will utilize the graph shown in Figure 2.

We will divide the desired freezing time into four parts:

$$\tau = \tau_1 + \tau_2 + \tau_3 + \tau_4.$$

As before, A = 1810 kcal/hr·$m^{1/2}$ and B_1 = 834 hr/$m^{3/2}$.

We will first determine the formation time (τ_1) for a frozen ground cylinder 0.3 m in radius.

The average heat absorption value is equal to (Figure 12):

$$Q_c = \frac{8000 + 4800}{2} = 6400 \text{ kcal/hr.}$$

and consequently

$$N_1 = \frac{Q_c}{A} = \frac{6400}{1810} = 3{,}5\ m^{\frac{1}{2}}.$$

According to Figure 2 we find that when r = 0.3 m and N_1 = 3.5 $m^{1/2}$, we have

$$\frac{\tau}{B_1} = 0{,}01\ m^{\frac{3}{2}}.$$

from which we obtain

$$\tau_1 = 0{,}01 B_1 = 834\cdot 0{,}01 \approx 8 \text{ hrs.}$$

Now we will determine the freezing time from r_1 = 0.3 m to r_2 = 0.5 m.

The average heat absorption of the column is equal to:

$$Q_c = \frac{4800 + 4000}{2} = 4400 \text{ kcal/hr.}$$

and then

$$N_1 = \frac{4400}{1810} = 2.43 \, m^{\frac{1}{2}}.$$

From Figure 2 we find that when r = 0.5 m and N_1 = 2.43 $m^{1/2}$, we have

$$\frac{\tau}{B_1} = 0.07 \, m^{\frac{3}{2}}.$$

From the same Figure 2, we find that when r = 0.3 m and N_1 = 2.43 $m^{1/2}$ we have

$$\frac{\tau}{B_1} = 0.02 \, m^{\frac{3}{2}}.$$

Then we find

$$\tau_2 = 834 (0.07 - 0.02) = 42 \text{ hrs.}$$

For the third freezing section from r_1 = 0.5 m to r_2 = 0.6 m, we find:

$$Q_c = \frac{4000 + 3800}{2} = 3900 \text{ kcal/hr.}$$

and

$$N_1 = \frac{3900}{1810} = 2.15 \, m^{\frac{1}{2}}.$$

From Figure 2 we determine:

for r_2

$$\frac{\tau}{B_1} = 0.12 \, m^{\frac{3}{2}},$$

and for r_1

$$\frac{\tau}{B_1} = 0.08 \, m^{\frac{3}{2}}.$$

Consequently,

$$\tau_3 = 834\,(0{,}12 - 0{,}08) = 33 \text{ hrs.}$$

Let us determine the freezing time from $r_1 = 0.6$ m to $r_2 = 0.7$ m. We find:

$$Q_c = \frac{3800 + 3600}{2} = 3700 \text{ kcal/hr}$$

and

$$N_1 = \frac{3700}{1810} = 2{,}05\ m^{\frac{1}{2}}.$$

According to Figure 2 we have:

for r_2

$$\frac{\tau}{B_1} = 0{,}19\ m^{\frac{3}{2}},$$

for r_1

$$\frac{\tau}{B_1} = 0{,}13\ m^{\frac{3}{2}}.$$

Consequently

$$\tau_4 = 834\,(0{,}19 - 0{,}13) = 50 \text{ hrs.}$$

Finally we find that the freezing time is equal to

$$\tau = 8 + 42 + 33 + 50 = 133 \text{ hrs.}$$

This result is considerably more precise than the previous one (τ = 84 hours) since it makes sufficient allowance for the change in the column's thermal absorption during freezing.

Let us solve the second portion of the task: determining the value of the column's heat absorption during passive freezing.

The following equality should be valid

$$Q_c = Q_{fi}.$$

According to expressions (4) and (7),

$$Q_{fi} = A\sqrt{r}.$$

In our case A = 1810 kcal/hr$\cdot$m$^{1/2}$ and r = 0.7 m. We find that in order to keep the frozen ground cylinder in a frozen state, the heat absorption of the column must be:

$$Q_c = A\sqrt{\tau} = 1810\sqrt{0.7} = 1510 \text{ kcal/hr.}$$

In order to answer the third question of what the temperature of the brine during passive freezing should be equal to, we will utilize formula (11), according to which

$$\vartheta_m = \vartheta_0 - \frac{Q_c}{2\pi\lambda_2 L} \ln \frac{r_r}{r_0}.$$

By substituting our known data into this expression, we find

$$\vartheta_m = -1 - \frac{1510}{2\pi \cdot 2 \cdot 10} \ln \frac{0{,}7}{0{,}1} = -24{,}1^\circ \text{C}.$$

BIBLIOGRAPHY

1. В. В. Проскуряков. Тепловой расчет замораживающей скважины в фильтрующем грунте, Изв. ВНИИГ, 1950, т. 45.
2. И. А. Чарный, Расчет скорости замораживания грунтов, „Советский метрополитен", 1940, № 1.
3. Л. С. Лейбензон, Руководство по нефтепромысловой механике, ч. I, ОНТИ, 1931.
4. Г. М. Марипольский. Расчет искусственного замораживания грунта, Горный журнал, 1940, № 5 и 6.
5. Х. Р. Хакимов, Искусственное замораживание грунтов для строительных целей, Машстройиздат, 1949.
6. М. А. Михеев, Основы теплопередачи, Госэнергоиздат, 1947.
7. Ф. Франк и Мизес, Дифференциальные и интегральные уравнения математической физики, т. II, ОНТИ, 1937.
8. И. А. Чарный, Подземная гидромеханика, Гостехиздат, 1948.