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FRACTURE STATISTICS OF BRITTLE MATERIALS WITH SURFACE CRACKS.(U)

JAN 77 S B BATDORF, H L HEINISCH, W J KNAPP

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- Tables of material properties and experimental parameters.
- Textual descriptions of the fracture statistics and experimental methods.

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**FRACTURE STATISTICS OF BRITTLE MATERIALS  
WITH SURFACE CRACKS**

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**FRACTURE STATISTICS OF BRITTLE MATERIALS**

**WITH SURFACE CRACKS**

**S.B. Batdorf and H.L. Heinisch, Jr.**

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#### ABSTRACT

Several different statistical fracture theories are developed for materials with cracks confined to the surface. All assume that crack planes are normal to the surface, but are otherwise randomly oriented. The simplest theory assumes that only the component of stress normal to the crack plane contributes to fracture. This theory is in fair agreement with biaxial fracture data on pyrex glass obtained by Oh. When the contribution of shear is included in the analysis, the crack shape has to be considered. Several shapes are examined, and the corresponding fracture statistics are derived. The failure criterion employed is that fracture occurs when the maximum tensile stress on some part of the crack surface reaches the intrinsic strength of the material. The assumption of shear-sensitive cracks leads to improved agreement with experiment, but really good agreement appears to require the assumption that the cracks have a preferred orientation.

TABLE OF CONTENTS

	<u>Page</u>
Figures. . . . .	vii
I. Introduction . . . . .	1
II. Theory: Shear on Crack Plane Neglected. . . . .	2
III. Oh's Theory. . . . .	7
IV. Present Theory . . . . .	9
V. Concluding Discussion. . . . .	20
References . . . . .	21

v

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LIST OF FIGURES

	<u>Page</u>	
Fig. 1	Probability of failure for pyrex tubes under biaxial tensile stress states 1:1, 1:0.5, and 1:0 assuming only normal components of stress on the cracks contribute to failure. The curves for 1:0.5, and 1:0 are generated from the curve for 1:1, which is fitted to the experimental results of Oh (1970). Data points for the three stress states are plotted. . . . .	6
Fig. 2	Failure probability curves fitted to 1:1 data of Oh (1970). Curve (O) is determined from parameters reported by Oh for the three-parameter Weibull function $P_f = 1 - \exp\{-((\sigma - \sigma_u)/\sigma_0)^m\}$ . Curve (A) is a Weibull function, and (B) is a piecewise function consisting of two Weibull curves joined by a straight line such that $P_f$ and its slope remain continuous. The values of the parameters are as follows:	
	O: $\sigma_u = 1.50$ $\sigma_0 = 3.32$ $m = 2.20$	
	A: $\sigma_u = 0.25$ $\sigma_0 = 4.64$ $m = 3.60$	
	B: $\sigma_u = 0.50$ $\sigma_0 = 4.45$ $m = 2.73$ $\sigma_u \leq \sigma < 3.5$	
	$P_f = 0.221(\sigma - 0.5) - 0.373$ $3.5 \leq \sigma < 6.0$	
	$\sigma_u = 0.50$ $\sigma_0 = 4.69$ $m = 4.02$ $6.0 \leq \sigma < \infty$	8
Fig. 3	Probability of failure for uniaxial tension by Oh's method. The curves labeled A(1:0), B(1:0) and O(1:0) are Weibull functions whose parameters are determined graphically from computed values of $P_f \ll 0.01$ starting from the A(1:1), B(1:1), and O(1:1) fits to the equibiaxial data, respectively. The unlabeled curve was plotted using the Weibull parameters for 1:0 reported by Oh. . . . .	10
Fig. 4	Surface crack models: (a) Griffith through-crack, (b) Griffith notch, (c) half ellipsoid. $P_1, P_2, P_3$ indicate the locations of the maximum stress in each model. $P_3$ lies in the $X_1 - X_3$ plane, but not on the $X_1$ axis when shear is present. Its exact position depends on the stress state . . . . .	11
Fig. 5	Failure probability curves for uniaxial tension (1:0) determined from the B(1:1) fit to the equibiaxial data assuming three different crack models: the shear insensitive (SI), the Griffith notch (GN), and the half-ellipsoid (HE). The points are the 1:0 data of Oh (1970). . . . .	15

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LIST OF FIGURES (Cont'd)

	<u>Page</u>
Fig. 6. Failure probability curves for uniaxial tension calculated using the B(1:1) fit to the equibiaxial data for Griffith notch cracks assuming a simple anisotropy of crack orientation in which the cracks are uniformly distributed over only a certain fraction R of the possible angular orientations. Curves for R = .6 and R = .75 are shown in comparison to the data points. . . . .	18
Fig. 7 Failure probability curves for stress state 1:0.5 for isotropically and anisotropically (R = .6) distributed cracks calculated using the B(1:1) fit to the equibiaxial data. The points are the 1:0.5 data of Oh (1970). . . . .	19

## I. INTRODUCTION

There are many cases in which structures must be fabricated using brittle materials. The strongest and most refractory materials tend to be brittle. Also, materials transparent to microwave, infrared, or visible radiation are generally brittle. Brittle structures characteristically exhibit a large variation in fracture stress which must be taken into account in design.

The most widely used statistical theory of fracture is due to Weibull (1939). He attributed the variation in fracture stress of nominally identical specimens to the presence of unidentified, invisible flaws. The flaws were assumed to have a distribution in strength, and the specimen or structure was assumed to fail when the strength of the weakest flaw or link was exceeded.

Batdorf and Crose (1974) revised weakest link theory by assuming the flaws to be cracks, and therefore to have strengths which depend on the orientation of the cracks with respect to the applied stresses. All orientations were considered equally likely, i.e., the material was assumed to be macroscopically isotropic. It was further assumed that only the component of stress normal to the crack plane contributed to fracture. The latter assumption was a convenient approximation which permitted development of a general theory without having to specify crack shapes. The shear parallel to the crack plane also contributes to the fracture, but by an amount that depends on the crack shape, which is something one usually does not know. In some cases, however, we may be able to derive information about crack shapes by examining the fracture statistics for a number of different stress states.

Only volume distributed cracks were treated by Batdorf and Crose (1974). In the case of some materials, e.g., glass, it is generally accepted that all cracks are located at the surface, and also that the crack planes are normal to the glass surface (McClintock and Argon, 1966). For such materials

the crack orientation is given by a single parameter rather than two, as in the case of volume distributed cracks.

In the present paper the theory of Batdorf and Crose is modified for surface distributed cracks and is applied to fracture data for glass obtained by Oh (Oh, 1970; Oh et al., 1973). The agreement with experiment leaves something to be desired. It is evident that including the effects of shear on the crack plane would decrease the discrepancy.

A more refined theory including the influence of shear and based on the assumption that the cracks are Griffith cracks and that failure occurs when the local tensile stress on the crack surface exceeds the intrinsic strength of the material was developed by Oh (1970). Some improvement in agreement with test data resulted. The present authors believe that the crack model employed by Oh is not appropriate for surface cracks. Alternative models are therefore proposed, but the improvement is marginal. Good agreement with experiment is obtained by assuming that in addition to being shear-sensitive, the cracks have a preferred orientation.

## II. THEORY: SHEAR ON CRACK PLANE NEGLECTED

Consider first a single crack of arbitrary orientation. Since it is assumed to be small and located at the surface, it is subjected at most to plane stress. In the principal axis system, the tensile component of stress normal to the crack line and in the plane of the surface is

$$\sigma_n = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta \quad (1)$$

where  $\theta$  is the angle between the x-axis and the crack normal. In accordance with the preceding assumptions, the material will rupture when  $\sigma_n > \sigma_{cr}$

where  $\sigma_{cr}$  is the macroscopic normal stress required to rupture the crack.

If the crack is randomly oriented, the probability of failure is given by

$$P_f = \frac{\omega}{\pi} \quad (2)$$

where  $\omega$  is the radian measure of the angular range in the positive  $\sigma_x$  half-plane within which  $\sigma_n > \sigma_{cr}$ .

In a real material, there will be a number of cracks of varying orientation and critical stress. If we assume that the cracks are uniformly distributed over the surface, the material can be characterized by a density function  $N(\Sigma, \sigma_{cr})$  where  $\Sigma$  is the applied stress state. This function represents the number of cracks per unit area having a critical stress less than or equal to  $\sigma_{cr}$ . The number of cracks per unit area having critical stresses between  $\sigma_{cr}$  and  $\sigma_{cr} + d\sigma_{cr}$  is, then,

$$dN = \frac{dN}{d\sigma_{cr}} d\sigma_{cr} \quad (3)$$

The probability that failure will occur in a uniformly stressed surface of area  $A$  due to a crack having a critical stress in the range  $\sigma_{cr}$  to  $\sigma_{cr} + d\sigma_{cr}$  is the product of the probability that a crack is present and the probability that the crack, if present, will fail; i.e.,

$$P_f(\Sigma, d\sigma_{cr}) = \left( A \frac{dN}{d\sigma_{cr}} d\sigma_{cr} \right) \left( \frac{\omega}{\pi} \right) \quad (4)$$

The probability that such cracks will survive is

$$P_s(\Sigma, d\sigma_{cr}) = 1 - P_f = 1 - A \frac{\omega}{\pi} \frac{dN}{d\sigma_{cr}} d\sigma_{cr} \\ \approx \exp \left[ - A \frac{\omega}{\pi} \frac{dN}{d\sigma_{cr}} d\sigma_{cr} \right] \quad (5)$$

The probability that cracks in every stress range  $d\sigma_{cr}$  will survive is the product of the probabilities of survival of cracks in the individual ranges, i.e.,

$$P_s = \exp \left[ - A \int \frac{\omega}{\pi} \frac{dN}{d\sigma_{cr}} d\sigma_{cr} \right] = 1 - P_f \quad (6)$$

The fracture probability for a surface of area A subjected to stress state  $\Sigma$  can be evaluated with the use of Eq. (6) when  $\omega$  and N are known. For any stress state  $\Sigma$ , we can determine  $\omega$  by using Eq. (1), while N must be obtained by experiment.

In the uniaxial case, Eq. (1) reduces to

$$\sigma_n = \sigma_x \cos^2 \theta$$

Thus for this case,  $\pm\theta_{cr}$ , the angle within which  $\sigma_n > \sigma_{cr}$ , is given by

$$\sigma_{cr} = \sigma_x \cos^2 \theta_{cr}$$

or

$$\theta_{cr} = \cos^{-1} \sqrt{\frac{\sigma_{cr}}{\sigma_x}} \quad (7)$$

Since  $\omega = 2\theta_{cr}$ ,

$$\frac{\omega}{\pi} = \frac{2}{\pi} \cos^{-1} \sqrt{\frac{\sigma_{cr}}{\sigma_x}} \quad (8)$$

In the equibiaxial case,

$$\sigma_x = \sigma_y = \sigma \quad (9)$$

as a result of which,  $\sigma_n = \sigma$  and

$$\frac{\omega}{\pi} = 1 \text{ for } \sigma_{cr} < \sigma \quad (10a)$$

$$= 0 \text{ for } \sigma_{cr} > \sigma \quad (10b)$$

As a result of (10a, b), in the equibiaxial tension case Eq. (6) takes the simple form

$$P_f(\sigma) = 1 - \exp[-AN(\sigma)] \quad (11)$$

From a comparison of Eqs. (6) and (8) with (11), it becomes clear that it is computationally advantageous to determine N from the fracture statistics for equal biaxial tension. From (11), we find AN is given by

$$AN = \ln(1 - P_f)^{-1} \quad (12)$$

thereby avoiding the necessity encountered in the theory of volume-distributed cracks of solving simultaneous linear algebraic equations or an integral equation to obtain N. Actually, in the volume distribution case, a simplification like Eq. (12) occurs for equitriaxial tension, but this state of stress cannot be realized in practice.

Under general biaxial stress states, Eq. (1) can be rewritten as

$$\sigma_n = \sigma_x (\cos^2 \theta + K \sin^2 \theta) = \sigma_x [\cos^2 \theta (1 - K) + K] \quad (13)$$

where

$$K = \frac{\sigma_y}{\sigma_x} \quad (14)$$

Thus,

$$\theta_{cr} = \cos^{-1} \sqrt{\frac{(\sigma_{cr}/\sigma_x) - K}{1 - K}} \quad (15)$$

$$\frac{\omega}{\pi} = \frac{2}{\pi} \cos^{-1} \sqrt{\frac{(\sigma_{cr}/\sigma_x) - K}{1 - K}} \quad (16)$$

Using Eqs. (6), (12), and (16) we can obtain the probability of failure of a surface of area A subject to biaxial tension.

The results of the theory just outlined are compared with the experimental results of Oh in Figure 1. It is evident that although the general trend of the theoretical results are in accord with the data, the theoretical curves for stress ratios 1:1, 1:0.5, and 1:0 are too far apart. Taking account of the contribution of shear on a crack to the failure process would increase the probability of failure for stress ratios 1:0.5 and 1:0, and therefore bring the curves closer together. We therefore turn to theories in which shear effects are taken into account.

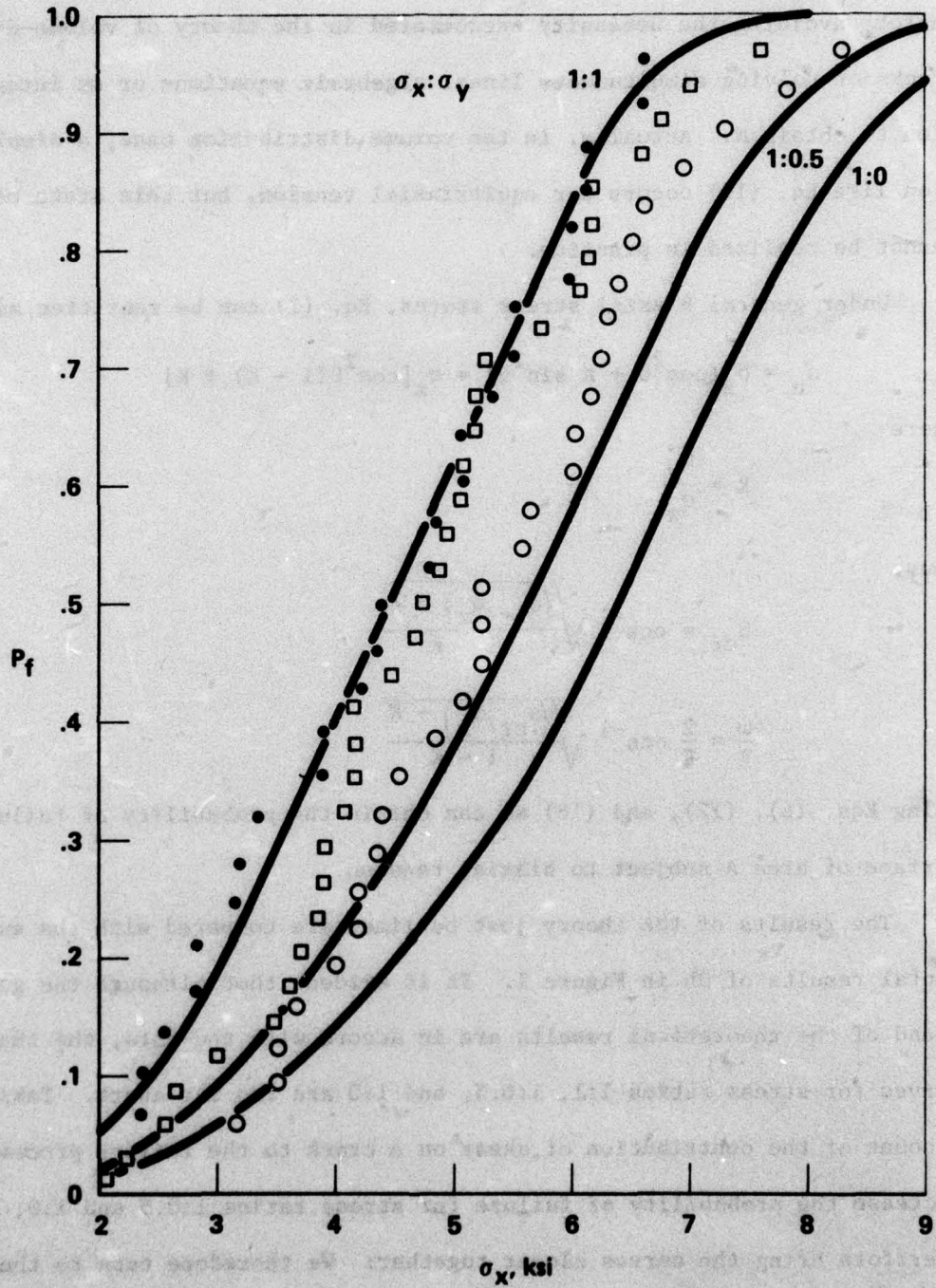


Figure 1. Probability of Failure for Pyrex Tubes Under Biaxial Tensile Stress States 1:1, 1:0.5, and 1:0 Assuming Only Normal Components of Stress on the Cracks Contribute to Failure. The curves for 1:0.5 and 1:0 are generated from the curve for 1:1, which is fitted to the experimental results of Oh (1970). Data points for the three stress states are plotted.



### III. OH'S THEORY

It was shown by Oh (1970) that when a Griffith crack is subjected to principal stresses  $\sigma_x$  and  $\sigma_y$ , the maximum tensile stress at any point on the surface of a crack is (in our notation)

$$\sigma_{\max} = \frac{1}{\xi} \left( T + \sqrt{T^2 + S^2} \right) \quad (17a)$$

where the tensile and shear forces on the crack are given by

$$T = \sigma_x \left( \frac{1+K}{2} + \frac{1-K}{2} \cos 2\theta \right) \quad (17b)$$

$$S = \sigma_x \left( \frac{1-K}{2} \sin 2\theta \right) \quad (17c)$$

In these equations  $K \equiv \sigma_y/\sigma_x < 1$ ,  $\xi$  is the ratio of the minor to the major axis of the ellipse, while  $\theta$  is the complement of the angle between the larger principal stress and the crack plane. All values of  $\theta$  were assumed equally likely, and the distribution of cracks with respect to  $\xi$  was chosen to fit a three-parameter Weibull representation of the data for the stress ratio 1:1.

To obtain the failure probabilities for stress ratios 1:0.5 and 1:0 it was arbitrarily assumed that these would also be three-parameter Weibull distributions. The corresponding parameters were found by applying the graphical approach for Weibull parameter estimation to synthetic data computed for  $P_f \ll 0.01$ . Such a procedure would be theoretically justified if failure always occurred for  $\sigma_x - \sigma_u \ll \sigma_u$ , a condition not applying to Oh's data. In spite of this, when the procedure is applied using Oh's Weibull parameters for equibiaxial tension, good agreement is obtained with uniaxial test data. However, two other fits to the equibiaxial data were devised by the present authors (Figure 2). One, labeled A, is an alternative Weibull function. The other, labeled B and identified in Figure 2, is a closer fit to the data, but also more complicated than any Weibull function. Although the three fits to

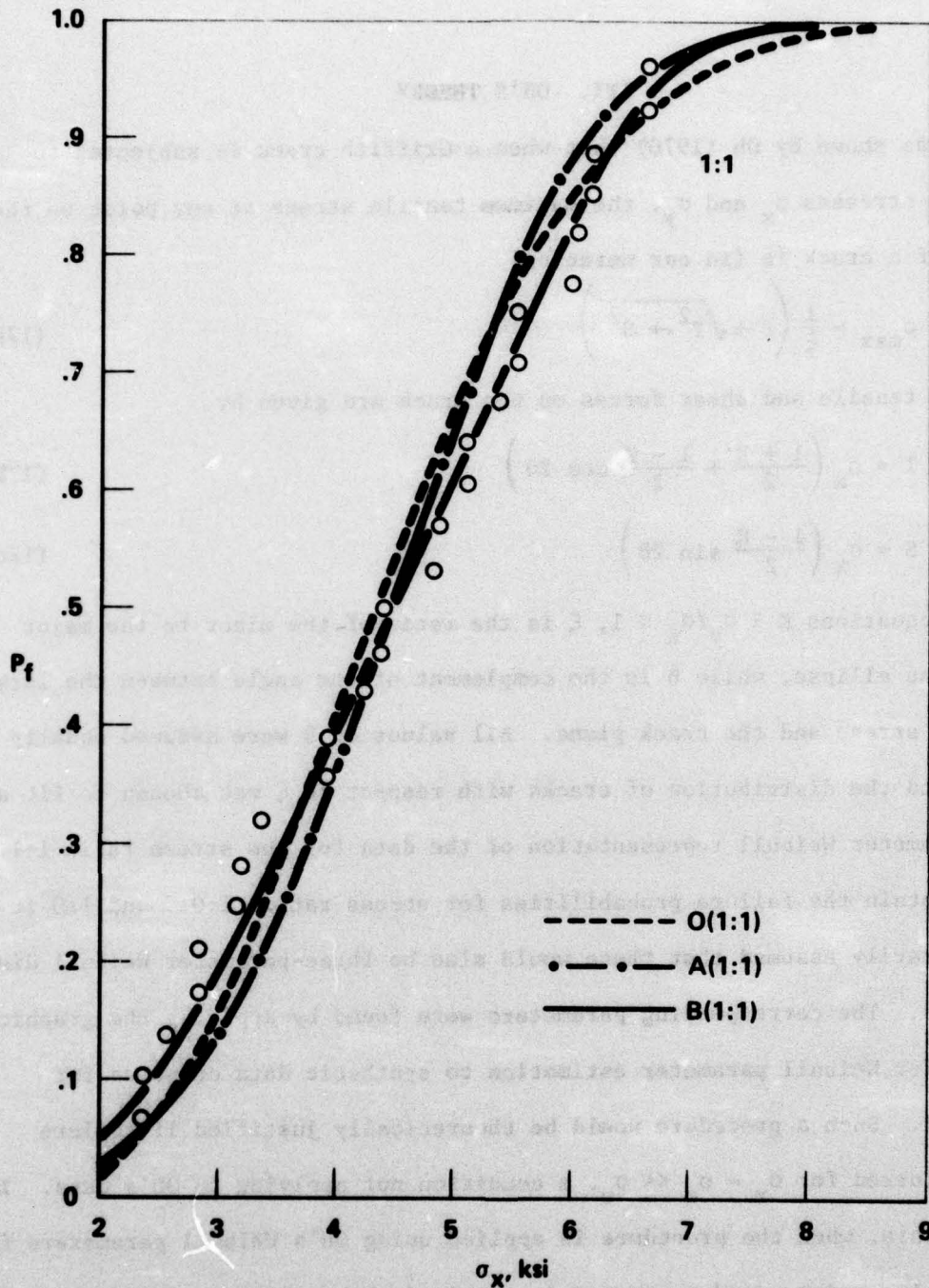


Figure 2. Failure Probability Curves Fitted to 1:1 Data of Oh (1970). Curve (O) is determined from parameters reported by Oh for the three-parameter Weibull function  $P_f = 1 - \exp\{-((\sigma - \sigma_u)/\sigma_0)^m\}$ . Curve (A) is a Weibull function, and (B) is a piecewise function consisting of two Weibull curves joined by a straight line such that  $P_f$  and its slope remain continuous. The values of the parameters are as follows:

O:	$\sigma_u = 1.50$	$\sigma_0 = 3.32$	$m = 2.20$	
A:	$\sigma_u = 0.25$	$\sigma_0 = 4.64$	$m = 3.60$	
B:	$\sigma_u = 0.50$	$\sigma_0 = 4.45$	$m = 2.73$	$\sigma_u \leq \sigma < 3.5$
	$P_f = 0.221(\sigma - 0.5) - 0.373$			$3.5 \leq \sigma < 6.0$
	$\sigma_u = 0.50$	$\sigma_0 = 4.69$	$m = 4.02$	$6.0 \leq \sigma < \infty$

the 1:1 data are quite close, the Oh procedure leads to very significant differences in the predicted uniaxial failure curves for the three cases, as shown in Figure 3. This suggests that the good agreement found using Oh's function may be fortuitous.

Oh's analysis is based on the tacit assumption that the Griffith crack is a through-crack (Figure 4a). This is not considered by the present authors to be an appropriate model for the surface crack. Section IV is concerned with two other types of cracks illustrated in Figures 4b and 4c. The first, which we shall call a Griffith notch, is a half-elliptic cylinder with the principal axis of the cylinder at the surface of the specimen. The second is a half-ellipsoid in which  $a \gg b \gg c$ .

#### IV. PRESENT THEORY

A fracture criterion under combined stress can be formulated on any of several different bases. These include use of an energy criterion, use of critical stress concentration factors, and use of maximum tensile stress occurring at a point on a surface of the crack. The simplest one of these to apply is the last.

Mirandy and Paul (1975; also Paul and Mirandy, 1975) have recently worked out the stress state at any point on the surface of an ellipsoidal cavity having axes  $a$ ,  $b$ , and  $c$ , such that  $c \ll b \leq a$  for arbitrary applied stresses. For present purposes the principal findings are: (1) If the applied stress is simple tension  $T$  normal to the plane of the crack (i.e., parallel to the polar or  $c$ -axis of the crack), the maximum stress occurs at the intersection of the cavity and the  $a$ - $b$  or equatorial plane. The local stress is the same at all points in the equatorial plane and is given by

$$\sigma = \frac{b}{c} \frac{2T}{E} \quad (18)$$

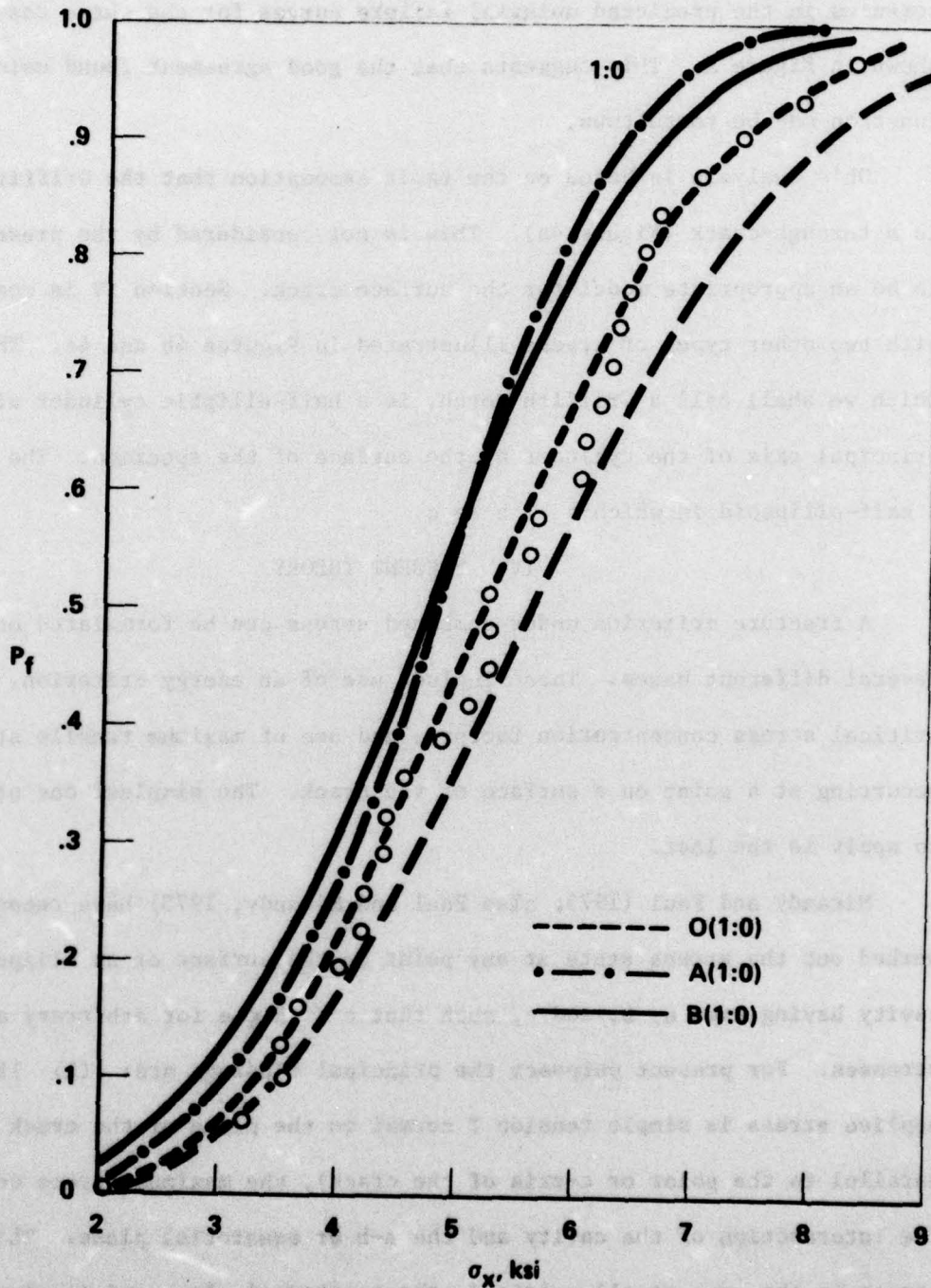
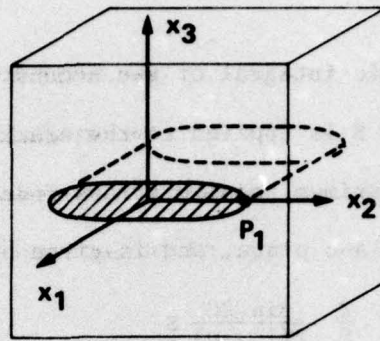
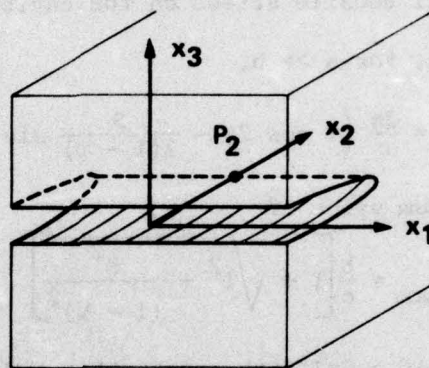


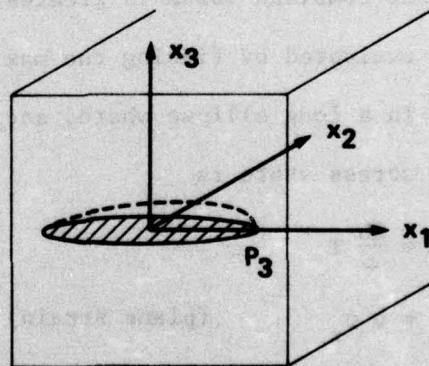
Figure 3. Probability of Failure for Uniaxial Tension by Oh's Method. The curves labeled A(1:0), B(1:0) and O(1:0) are Weibull functions whose parameters are determined graphically from computed values of  $P_f \ll 0.01$  starting from the A(1:1), B(1:1), and O(1:1) fits to the equibiaxial data, respectively. The unlabeled curve was plotted using the Weibull parameters for 1:0 reported by Oh.



(a)



(b)



(c)

**Figure 4. Surface Crack Models: (a) Griffith Through-Crack, (b) Griffith Notch, (c) Half Ellipsoid.  $P_1$ ,  $P_2$ ,  $P_3$  indicate the locations of the maximum stress in each model.  $P_3$  lies in the  $X_1$ - $X_3$  plane, but not on the  $X_1$  axis when shear is present. Its exact position depends on the stress state.**

where  $E$  is an elliptic integral of the second kind. For  $b/a \ll 1$ ,  $E = 1$ .

(2) If shear stress  $S$  is applied to the crack plane in a direction parallel to the  $a$ -axis, the maximum tensile stress induced on the surface of the cavity occurs in the  $a$ - $c$  plane, and is given by

$$\sigma = \frac{b}{c} \frac{2}{E} \frac{\sin 2\beta}{2(1-\nu)} S \quad (19)$$

where  $\beta$  is the local latitude. (3) When both  $S$  and  $T$  as described above are present, the local tensile stress on the cavity surface is largest in the  $a$ - $c$  plane and becomes, for  $a \gg b$ ,

$$\sigma = \frac{2b}{c} \left( T \cos 2\beta - \frac{S}{2(1-\nu)} \sin 2\beta \right) \quad (20)$$

which has a maximum value of

$$\sigma_{\max} = \frac{b}{c} \left[ T + \sqrt{T^2 + \frac{S^2}{(1-\nu)^2}} \right] \quad (21)$$

(4) In the case of a Griffith crack subjected to tension normal to the crack plane and shear on the crack plane applied parallel to the cylinder axis, the tensile stress under combined loads is greatest along the line of maximum curvature. It is evaluated by finding the maximum principal stress at the end of the  $b$ -axis in a long ellipse where, according to Mirandy and Paul (1975), the local stress state is

$$\sigma_z = \frac{2b}{c} T \quad (22a)$$

$$\sigma_x = \nu \sigma_z \quad (\text{plane strain}) \quad (22b)$$

$$\tau = \frac{b}{c} S \quad (22c)$$

Using Mohr's circle, this is readily shown to be

$$\sigma_{\max} = \frac{b}{c} \left[ T(1+\nu) + \sqrt{T^2(1-\nu)^2 + S^2} \right] \quad (23)$$

The above results are for ellipsoids in an isotropic elastic body located far from any free surfaces. In the case of surface cracks such as those shown in Figure 4, the presence of the free surface will modify to some degree the stresses on the surface of the half-ellipsoid and Griffith notch. An estimate of the amount of this modification can be made for the Griffith notch as follows: We note that as  $a \rightarrow \infty$  the ellipsoid approaches an elliptic cylinder, so that (23) should be valid for the cylinder. In the case of the half cylinder (Griffith notch) of Figure 4b, it has been shown (Paris and Sih, 1965) that the free surface causes the stress concentration factor for tension to be increased by a factor of 1.12, while that for shear is unchanged. Thus, we modify (23) to read

$$\sigma_{\max} = 1.12 \frac{b}{c} \left[ T(1 + \nu) + \sqrt{T^2 (1 - \nu)^2 + \left(\frac{S}{1.12}\right)^2} \right] \quad (24)$$

This equation differs appreciably in appearance from that applying to the through-crack (17a). Among other things, it depends on Poisson's ratio. However, for the range of values appropriate to glass,  $\nu = 0.2$  to  $0.3$ , the relative contribution of the applied tensile and shear stresses to the maximum tensile stress on the surface of the cavity is almost the same.

The present method of determining failure probability curves for various stress states is to solve the integral in (6) numerically over the entire range of applied stress. The data for equibiaxial tension are used to determine  $dN/d\sigma_{cr}$  via (12). While it is not necessary to assume a Weibull form for  $P_f(\sigma)$ , several Weibull fits to the data were investigated. The best fit, however, was a piecewise function defined over three regions of the applied stress range (Figure 2) and used exclusively in the computations of the present theory.

The fraction  $\omega/\pi$  is a function of the applied stress state and  $\sigma_{cr}$  and is determined in the following manner. The maximum tensile stress on the surface of the crack (equations 17, 21 or 24) can be written as

$$\sigma_{max} = \frac{2}{\xi} \sigma_x g(\theta, K) \quad (25)$$

where  $g(\theta, K)$  is the function of crack orientation  $\theta$  and stress state  $K$  appropriate to the crack model and  $\sigma_x$  is the applied stress. The failure criterion is  $\xi\sigma_{max} \geq 2\sigma_{cr}$  or  $\sigma_x g(\theta, K) \geq \sigma_{cr}$ . The fraction  $\omega/\pi$  is the portion of the range  $0 \leq \theta \leq \pi/2$  for which  $g(\theta, K) \geq \sigma_{cr}/\sigma_x$ , and in general can be determined from the roots of the nonlinear equation  $g(\theta, K) = \sigma_{cr}/\sigma_x$  at each value of  $\sigma_{cr}/\sigma_x$ .

Figure 5 compares the failure probability curves for stress ratio 1:0 deduced from the experimental data for stress ratio 1:1 using (13), (21) and (24). The results using (21) are somewhat closer to the experimental data, but not much. A correction of unknown magnitude is needed to account for the presence of the free surface. Moreover, it is doubtful whether a fracture criterion should be based on a maximum stress occurring at the end of the major axis. If the material strength is exceeded there, the crack should lengthen along the surface but not penetrate in from the surface in the manner required to partition the specimen. Accordingly, we conclude that (24) is the more appropriate fracture criterion.

Use of the critical stress concentration factor for the fracture criterion leads to the same conclusion. The critical stress concentration factors for a notch subjected to tension and out-of-plane shear are (Paris and Sih, 1965).

$$K_I = 1.12 T \sqrt{\pi b} \quad (26)$$



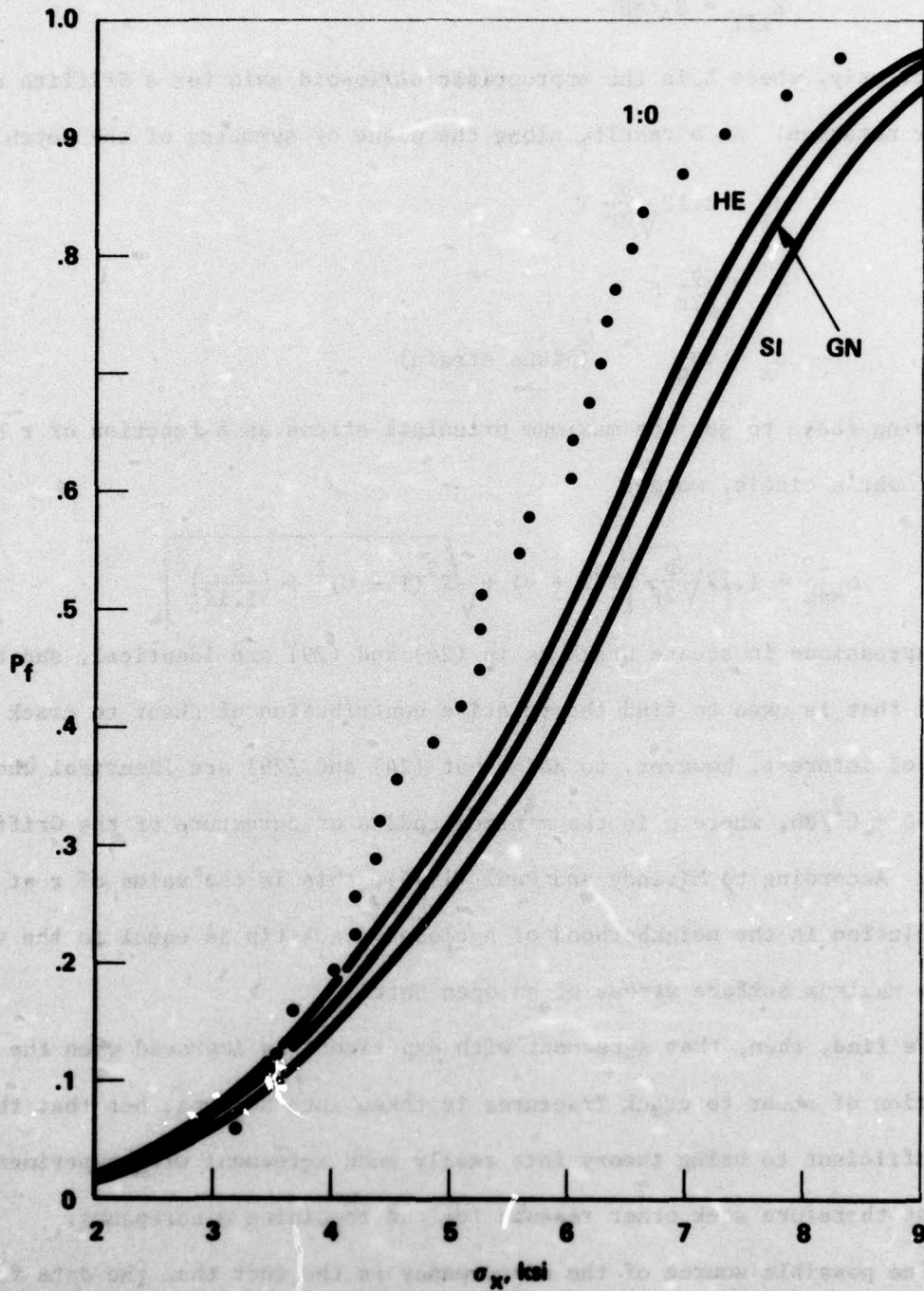


Figure 5. Failure Probability Curves for Uniaxial Tension (1:0) Determined from the B(1:1) Fit to the Equibiaxial Data Assuming Three Different Crack Models: the Shear Insensitive (SI), the Griffith Notch (GN), and the Half-Ellipsoid (HE). The points are the 1:0 data of Oh (1970).

and

$$K_{III} = S\sqrt{\pi b} \quad (27)$$

respectively, where  $b$  is the appropriate ellipsoid axis for a Griffith notch in our notation. As a result, along the plane of symmetry of the notch

$$\sigma_z = 1.12\sqrt{\frac{b}{2r}} T \quad (28a)$$

$$\tau = \sqrt{\frac{b}{2r}} S \quad (28b)$$

$$\sigma_x = \nu\sigma_z \quad (\text{plane strain}) \quad (28c)$$

Combining these to get the maximum principal stress as a function of  $r$  by using Mohr's circle, we get

$$\sigma_{\max} = 1.12\sqrt{\frac{b}{2r}} \left[ T(1 + \nu) + \sqrt{T^2(1 - \nu)^2 + \left(\frac{S}{1.12}\right)^2} \right] \quad (29)$$

The expressions in square brackets in (24) and (29) are identical, and this is all that is used to find the relative contribution of shear to crack failure. It is of interest, however, to note that (24) and (29) are identical when  $r = \frac{1}{8} \rho = C^2/8b$ , where  $\rho$  is the minimum radius of curvature of the Griffith notch. According to Mirandy and Paul (1975), this is the value of  $r$  at which the solution in the neighborhood of a closed crack tip is equal to the value of the maximum surface stress of an open notch.

We find, then, that agreement with experiment is improved when the contribution of shear to crack fractures is taken into account, but that this is not sufficient to bring theory into really good agreement with experiment. We must therefore seek other reasons for the remaining discrepancy.

One possible source of the discrepancy is the fact that the data for failures at a stress ratio 1:1 cover a somewhat lower stress range than failures at stress ratio 1:0. Thus, an analytical expression for  $AN(\sigma_{cr})$  obtained from 1:1 data should not be extrapolated to higher values of  $\sigma_{cr}$ .

However, the 1:1 failure stress range covers most of the 1:0 failure stress range so the agreement between theory and experiment in this large overlap region should be good, but it is not.

Another possible source for the discrepancy is the assumption in the theory that crack planes are always normal to the free surface of the material. This explanation also fails. It has been shown (by methods to be reported elsewhere) that the 1:0 failure curve deduced from 1:1 data is the same using the surface crack theory described here as it is using the volume distributed crack theory of Batdorf and Crose, employing a normal stress failure criterion in both cases.

The final possible explanation to be discussed here is that the assumption of uniform distribution of crack orientation is not valid for the pyrex tubes tested. There is no a priori method of predicting what form an anisotropic distribution should take. A crude but useful check on the hypothesis of anisotropy is to investigate the consequences of assuming that cracks are uniformly distributed through a given range of angles and are absent outside this range. For instance, one might assume that all crack planes are within  $\psi$  radians of the axis of the pyrex tubes, thus occupying a fraction  $R = 2\psi/\pi$  of the total available angular orientation. Figures 6 and 7 show the results of assuming shear-sensitive Griffith notch cracks for  $R = 0.6$  and  $R = 0.75$ . It is evident that the anisotropy assumption greatly improves agreement between theory and experiment. Unfortunately, the tubes were tested in simple tension in only one (the circumferential) direction, so that a direct test of the anisotropy hypothesis is lacking.

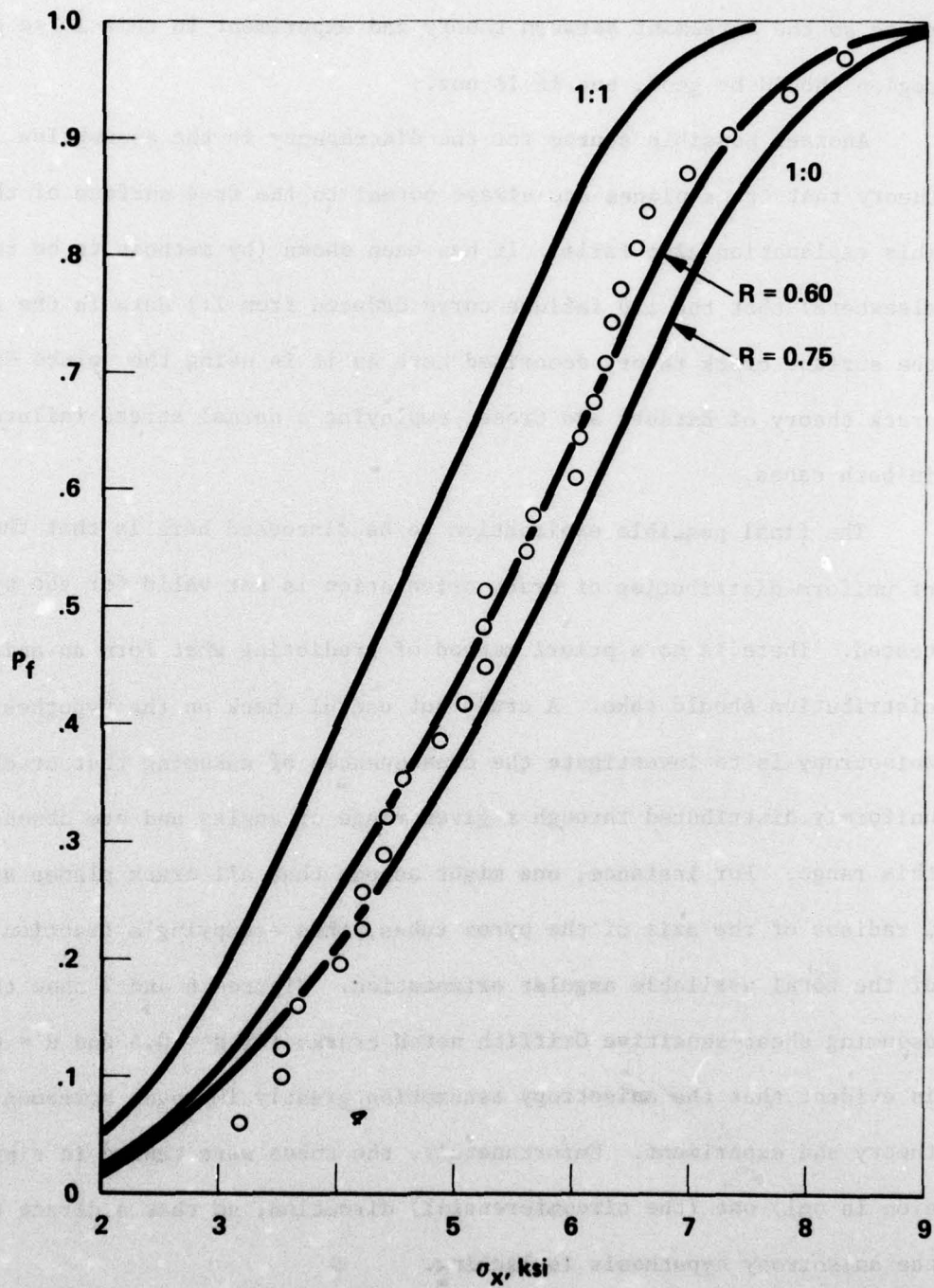


Figure 6. Failure Probability Curves for Uniaxial Tension Calculated Using the B(1:1) Fit to the Equibiaxial Data for Griffith Notch Cracks Assuming a Simple Anisotropy of Crack Orientation in Which the Cracks are Uniformly Distributed Over Only a Certain Fraction  $R$  of the Possible Angular Orientations. Curves for  $R = .6$  and  $R = .75$  are shown in comparison to the data points.

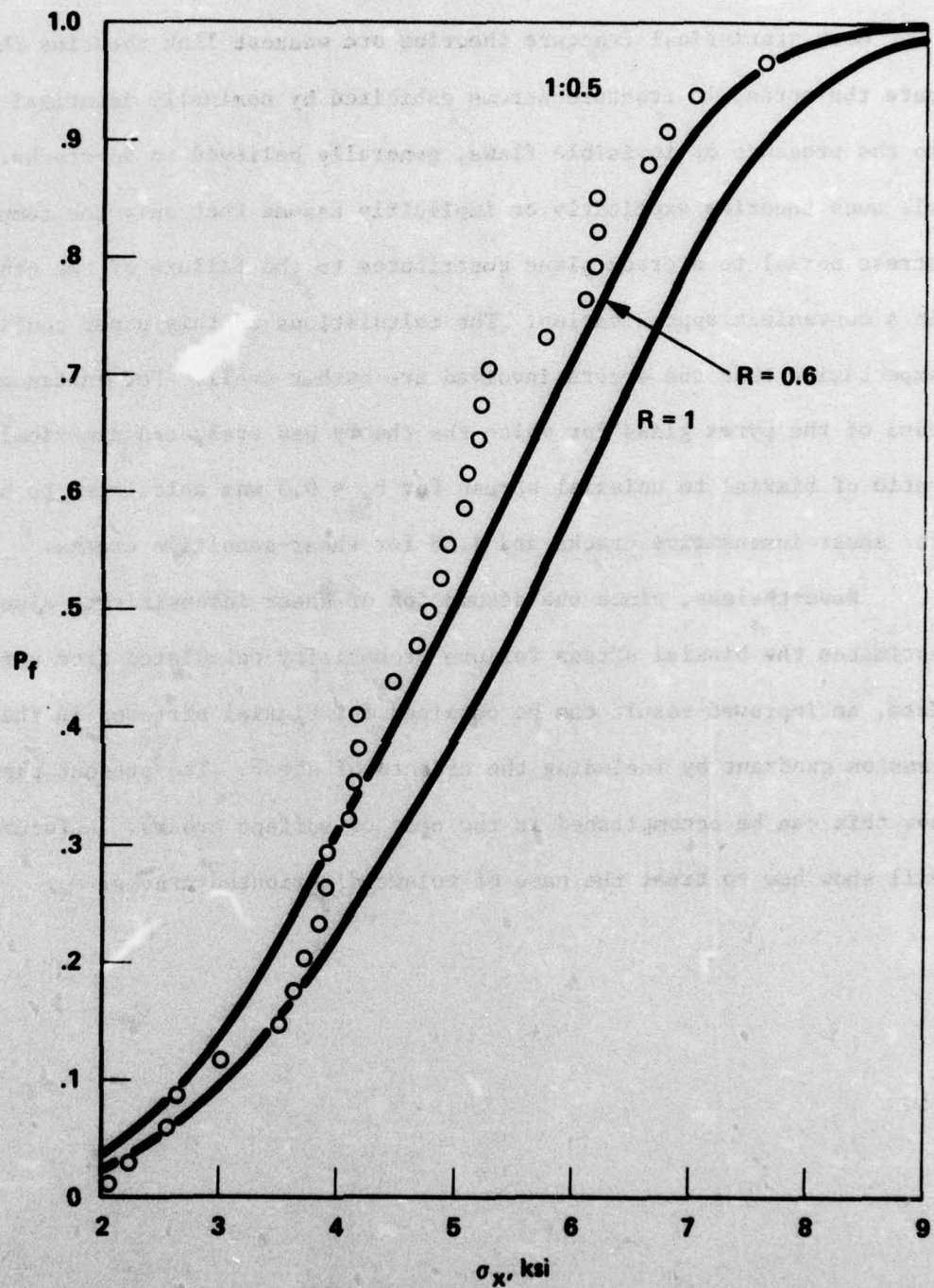


Figure 7. Failure Probability Curves for Stress State 1:0.5 for Isotropically and Anisotropically ( $R = .6$ ) Distributed Cracks Calculated Using the B(1:1) Fit to the Equibiaxial Data. The points are the 1:0.5 data of Oh (1970).

## V. CONCLUDING DISCUSSION

Most statistical fracture theories are weakest link theories that attribute the spread in fracture stress exhibited by nominally identical specimens to the presence of invisible flaws, generally believed to be cracks. Nearly all such theories explicitly or implicitly assume that only the component of stress normal to a crack plane contributes to the failure of the crack. This is a convenient approximation. The calculations in this paper confirm the expectation that the errors involved are rather small. For instance, in the case of the pyrex glass for which the theory was evaluated numerically, the ratio of biaxial to uniaxial stress for  $P_f = 0.5$  was calculated to be 1.44 for shear-insensitive cracks and 1.38 for shear-sensitive cracks.

Nevertheless, since the assumption of shear insensitivity always overestimates the biaxial stress failure probability calculated from uniaxial data, an improved result can be obtained for biaxial stresses in the tension-tension quadrant by including the effects of shear. The present paper shows how this can be accomplished in the case of surface cracks. A future paper will show how to treat the case of volume-distributed cracks.

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Wright-Patterson AFB, OH 45433

AFFDL/FBEdr John Halpin  
Wright-Patterson AFB, OH 45433

AFML (MBM) Dr. S.W. Tsai  
Wright-Patterson AFB, OH 45433

AFML (MXS)  
Capt. C. Budde  
Wright-Patterson AFB, OH 45433

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Aerospace Corporation  
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Los Angeles, CA 90009

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The Pennsylvania State University  
University Park, PA 16802

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University of California  
Berkeley, CA 94720

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Dept. 248, Bldg 13-3  
McDonnell Douglas Aircraft Co.  
5301 Bolsa Ave  
Huntington Beach, CA 92647

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