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SUMMARY

The orbit of Proton 4, 1968-103A, has been redetermined, in greater detail and with better accuracy, in order to clarify previously puzzling features in the variation of orbital inclination. Orbital parameters have been determined at 25 epochs between December 1968 and July 1969, using about 1600 optical and radar observations with the RAE orbit refinement program PROP6.

During January 1969 the orbit passed through 31:2 resonance -- when the ground track over the Earth repeats every two days after 31 revolutions of the satellite. A simultaneous least-squares fitting of theoretical curves to the values of inclination and eccentricity between 14 December 1968 and 6 March 1969 has yielded values for two pairs of lumped 31st-order geopotential coefficients, appropriate to an inclination of 51.5°. This is the first specific evaluation of 31st-order coefficients.

The 15 values of inclination after the resonance, from March to near decay in July 1969, have been used to determine mean, morning and afternoon-evening values for the rotation rate of the atmosphere at a height near 260km; the values of rotation rate, namely 1.1, 0.9 and 1.3rev/day respectively, confirm the trends already established from analysis of other satellite orbits.

Departmental Reference: Space 513

CONTENTS

1 INTRODUCTION

Proton 4, 1968-103A, was launched on 16 November 1968 into an orbit of inclination 51.5°; initially the perigee height was near 250km and apogee near 480km. The satellite was extremely massive, weighing 17000kg, and was in the form of a squat cylinder 3m long and 4m in diameter; it remained in orbit for 250 days and decayed on 24 July 1969.

The orbit seemed promising for use in geophysical studies, and was determined[1] in 1970 at 20 epochs during its life, using US Navy, optical and radar observations. Analysis of the decrease in inclination yielded an acceptable value of the atmospheric rotation rate, but there were some peculiarities in the variation of inclination, particularly a 'dip' between mid January and early March 1969. It was suggested in 1970 that this might be due to neglected effects of tesseral harmonics in the geopotential. The orbit has now been redetermined at 25 epochs, from about 1600 US Navy, optical and radar observations, using the improved version[2,3] of the RAE computer program for orbit refinement, PROP6, and assigning more realistic *a priori* accuracies to the US Navy observations.

The new orbit determination shows that the 'dip' in inclination was the result of a strong resonance effect associated with tesseral harmonics in the geopotential of order 31, at the time when the ground track of the satellite over the Earth was repeating every two days, after 31 revolutions. Since Proton 4 was so massive, it was less affected by drag than most satellites at similar heights, and passed through the resonance slowly enough to allow values of lumped 31st-order geopotential coefficients to be determined. Values of atmospheric rotation rate are also obtained for the last four months of the satellite's life.

2 ORBIT DETERMINATION

2.1 Observations

More than 1600 observations were available, made between December 1968 and July 1969. In the previous orbit determination, all the US Navy observations were used, but in this redetermination of the orbit 165 US Navy observations having elevations less than 20° were omitted, because they are known to be of poorer accuracy than observations at higher elevations. Many of the US Navy observations are expressed relative to the Earth's centre, and the *a priori* angular accuracy assigned to these observations has been changed from 0.02° to 0.01°. Six observations were used from the Hewitt camera at Malvern, seven from the 200mm camera at Meudon, and 24 from the kinetheodolite at the South African Astronomical Observatory. There were about 700 US Navy observations, about 180 visual observations and about 140 observations from the Malvern radar. In the course of the orbit determinations about 400 observations were rejected.

2.2 Observational accuracy

Table 1 summarizes the residuals obtained for stations contributing at least seven observations to the orbit determination and for the Malvern Hewitt camera (with six observations). The residuals were evaluated using the computer program ORES[4], and the results have been sent to the individual observers. The most accurate observations are those from the Malvern camera (rms residual 4 seconds of arc). The next most accurate are the seven from the 200mm camera at Meudon (6 seconds of arc). The 24 observations from the kinetheodolite at the South African Astronomical Observatory are accurate to about 40 seconds of arc. The US Navy observations (totalling about 700) have topocentric accuracies between 3.4 and 4.5 minutes of arc (station 29 is geocentric and the residuals need to be multiplied by a factor of about 5 to obtain an equivalent topocentric accuracy, which is 3.2 minutes of arc). The 199 Malvern radar observations (which include about 60 removed from the orbit determinations to avoid bias on occasions when radar observations were too numerous) have rms residuals of about 3.2 minutes of arc. The 180 or so visual observations, obtained from either the Appleton Laboratory at Slough or the (now disbanded) Moonwatch Division of the Smithsonian Astrophysical Observatory, have residuals varying between 4 and 14 minutes of arc.

These residuals cannot be compared directly with those determined in 1970, since different observations have sometimes been used. The US Navy observations now appear more accurate than before, but this is because of the omission of low-elevation observations and the reduction from 0.02° to 0.01° in the pre-assigned accuracy of observations expressed relative to the Earth's centre (with the result that more were rejected). On average, the visual observations show little change in their residuals. The abc residuals (the arithmetic mean of the best 70 to 80 per cent of the visual observations) are rather higher, but differences are not significant since the choice of observations to be eliminated is not an exact procedure.

Table 1

Residuals for selected observing stations

Station		Number of observations	rms residuals				abc residuals minutes of arc	
			Range km	Minutes of arc				
				RA	Dec	Total	RA	Dec
1	US Navy	71		2.9	2.8	4.1		
2	US Navy	14		1.9	2.8	3.4		
4	US Navy	9		2.5	2.6	3.6		
5	US Navy	26		2.7	3.2	4.2		
6	US Navy	32		3.1	3.3	4.5		
29	US Navy	530	0.7	0.5	0.4			
719	Rodewisch	18		2.8	2.9	4.0	1.9	1.8
2265	Farnham	23		6.4	5.3	8.3	4.8	2.7
2303	Malvern (Hewitt camera)	6		0.06	0.03	0.07		
2304	Malvern (Radar)	199	0.9	2.4	2.1			
2403	Stevenage 3	7		3.7	1.6	4.0	2.2	1.6
2409	Ribbleton*	29		5.1	3.3	6.1	2.2	1.6
2512	Mountcastle**	30		5.9	2.9	6.6	2.4	2.1
2525	Crawley Ridge	8		5.2	5.5	7.6	3.4	2.4
2531	Ditton Park East†	13		7.7	4.5	8.9	5.7	3.0
2539	Dymchurch	7		4.3	1.9	4.7	1.9	1.4
2573	Genoa 1	9		9.2	10.2	13.7	7.9	8.4
2577	Cape kinetheodolite	24		0.45	0.53	0.70		
8030	Meudon camera	7		0.06	0.08	0.10		

* Includes Moonwatch station 667
** Includes Moonwatch station 656
† Includes Moonwatch station 661

2.3 Orbital accuracy

The 25 sets of orbital elements as determined by PROP6 are given in Table 2. The mean anomaly is represented in PROP by a polynomial in time t from epoch,

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + M_4 t^4 + M_5 t^5 \quad . \qquad (1)$$

The drag proved to be rather variable, and 9 of the 25 orbits required all six terms, as shown by values up to M_5 in Table 2. With the other orbits, the observations were better fitted using less coefficients.

Orbit 1 played no part in the final analysis because it proved to be too distant from the 31:2 resonance. Orbits 2-10 were used in the analysis of 31:2 resonance effects. The sd in inclination for these 9 orbits varied from 0.0002° (for orbit 2 with Hewitt camera observations) to 0.0016°, the rms being 0.0011°. The sd in eccentricity varied from 3×10^{-6} (equivalent to about 20m in perigee or apogee) to 21×10^{-6}, with an rms of 14×10^{-6}. The corresponding rms values for the first determination[1] made in 1970, were 0.0014° in inclination and 17×10^{-6} in eccentricity - so the accuracy has been improved by about 20%. The rms value for ε, the measure of fit, is 0.75 here, compared with 0.51 in 1970. This indicates that the pre-assigned errors for the US Navy observations were too large previously and are nearer the mark here.

Orbits 11-25 were used to determine the atmospheric rotation rate: their sd in inclination varied over the range 0.0001° to 0.0018°, the rms value being 0.0011°. The sd in eccentricity varied from 10×10^{-6} to 38×10^{-6}, with an rms of 22×10^{-6}. The corresponding 1970 mean standard deviations in i and e were 0.0011° (no change) and 29×10^{-6} (30% worse).

For all 25 orbits, the sd in Ω varied between 0.001° and 0.004°. M_0 and ω have similar standard deviations, varying between 0.02° and 0.17° for orbits 1-19, but increased, for orbits 20-25, to between 0.18° and 0.50°. The decay rate M_2 had an rms sd of 0.0008deg/day^2 for the first 24 orbits (an accuracy of about 0.2%); on the final (25th) orbit, the sd increased to 0.0044deg/day^2 (but is accurate to 0.1%).

Some difficulty was encountered in determining a few of the orbits. For orbits 12 and 17, for example, magnetic storms occurred near epoch, and observations on the magnetic-storm days had to be removed before satisfactory orbits were obtained. Orbit 20 was particularly difficult to determine, with both Hewitt camera and Meudon camera observations present. Although the Hewitt camera observations appeared to fit satisfactorily, they had to be removed before achieving an orbit which fitted in with the other orbits. Orbit 24 had observations from only two stations, mostly from a US Navy station. Although a good orbit appears to have been obtained, it may suffer from bias and should be treated with caution.

3 ANALYSIS OF THE 31:2 RESONANCE

3.1 Theoretical equations

Near 31:2 resonance, the appropriate resonance angle Φ is given by

$$\Phi = 2(\omega + M) + 31(\Omega - \nu) \quad , \tag{2}$$

where ν is the sidereal angle. Fig.1 shows the variation of Φ for Proton 4 and indicates that exact 31:2 resonance occurred on 18 January 1969 (MJD 40239). The resonance affected the orbit mainly between 14 December 1968 and 25 February 1969, when Φ was within $1\frac{1}{2}$ cycles of its value at resonance (318°).

The Earth's gravitational potential U at an exterior point (r,θ,λ) may be written[5] in normalized form as

$$U = \bar{U} + \frac{\mu}{r} \sum_{\ell=2}^{\infty} \sum_{m=1}^{\ell} \left(\frac{R}{r}\right)^{\ell} P_{\ell}^{m} (\cos \theta) \left\{\bar{C}_{\ell m} \cos m\lambda + \bar{S}_{\ell m} \sin m\lambda\right\} N_{\ell m} \quad , \tag{3}$$

where $\bar{U}$ is the longitude-averaged potential, r the distance from the Earth's centre, θ the co-latitude, λ the longitude (positive to the east), μ the gravitational constant for the Earth ($398601 km^3/s^2$), and R the Earth's equatorial radius (6378.1km). $P_{\ell}^{m} (\cos \theta)$ is the associated Legendre function of order m and degree ℓ, and $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ are the normalized tesseral harmonic coefficients, of which those of order $m = 31$ particularly concern us here. The normalizing factor $N_{\ell m}$ is given by[5]:

$$N_{\ell m}^{2} = \frac{2(2\ell + 1)(\ell - m)!}{(\ell + m)!} \quad . \tag{4}$$

Near the 31:2 resonance the theoretical variation of inclination may be written[6,7,8]

$$\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{31}\Bigg[\frac{R}{a}(31 - 2\cos i)\,\bar{F}_{32,31,15}\left\{\bar{S}_{31}^{0,2}\sin\Phi + \bar{C}_{31}^{0,2}\cos\Phi\right\}$$

$$+ 17e(31 - \cos i)\,\bar{F}_{31,31,15}\left\{\bar{C}_{31}^{1,1}\sin(\Phi - \omega) - \bar{S}_{31}^{1,1}\cos(\Phi - \omega)\right\}$$

$$+ 13e(31 - 3\cos i)\,\bar{F}_{31,31,14}\left\{\bar{C}_{31}^{-1,3}\sin(\Phi + \omega) - \bar{S}_{31}^{-1,3}\cos(\Phi + \omega)\right\}$$

$$+ \text{terms in } e^{|q|}\begin{matrix}\cos\\ \sin\end{matrix}(\gamma\Phi - q\omega)\Bigg] \quad . \qquad (5)$$

In equation (5), γ and q are integers, with γ taking the values 1,2,3,... and q the values 0, ±1, ±2 ... . Only the terms with $\gamma = 1$ and $q = 0$ and ±1 are given explicitly, since it is believed that the others will be small: the terms with $|q| = 2$ have e^2 as a multiplying factor (where $e \simeq 0.015$ for Proton 4), while the terms with $\gamma = 2$ are associated with harmonics of order 62, which should be much smaller than those of order 31.

The quantities $\bar{S}_{31}^{0,2}$, $\bar{C}_{31}^{1,1}$, etc. in equation (5) are what might be called 'natural lumped coefficients' of order 31. In defining them it is helpful to consider the general $\beta{:}\alpha$ resonance[7] and, for given γ and q, to introduce integer suffixes m, k and p such that, for suitable values of ℓ (Ref.6): $m = \gamma\beta$; $k = \gamma\alpha - q$; and $2p = \ell - k$. Then the lumped coefficients may be written[6]

$$\bar{C}_m^{q,k} = \sum_\ell Q_\ell \bar{C}_{\ell m} \,, \qquad \bar{S}_m^{q,k} = \sum_\ell Q_\ell \bar{C}_{\ell m} \quad , \qquad (6)$$

where ℓ increases in steps of two from a minimum value ℓ_0 (normally equal to m or $m + 1$) , and the Q_ℓ are constant factors with $Q_{\ell_0} = 1$.

We are concerned with the 31:2 resonance and are considering only the terms with $\gamma = 1$; so we have $m = \gamma\beta = 31$, and $k = \gamma\alpha - q = 2 - q$. Thus, for the three terms included in equation (5), with q = 0, 1 and −1, the affixes [q,k] are [0,2], [1,1] and [−1,3] respectively, as shown in equation (5). The relevant values of ℓ , for use in equation (6), must be such that $\ell \geqslant m$ and $(\ell - k)$ is even: with $\bar{C}_{31}^{1,1}$ for example, $\ell \geqslant 31$ and $(\ell - 1)$ must be even; hence ℓ = 31, 33, 35, ...

The quantities $\bar{F}$ appearing in equation (5) are the functions of inclination $\bar{F}_{\ell mp}$ defined by Allan[7]. The factors Q_ℓ in equations (6) depend on the $\bar{F}_{\ell mp}$ and on R/a, and for 31:2 resonance the lumped coefficients may be written explicitly in terms of the geopotential harmonic coefficients $\bar{C}_{\ell m}$ as follows[8]:

$$\bar{C}_{31}^{0,2} = \bar{C}_{32,31} - \frac{\bar{F}_{34,31,16}}{\bar{F}_{32,31,15}}\left(\frac{R}{a}\right)^2 \bar{C}_{34,31} + \frac{\bar{F}_{36,31,17}}{\bar{F}_{32,31,15}}\left(\frac{R}{a}\right)^4 \bar{C}_{36,31} - \dots \quad (7)$$

$$\bar{C}_{31}^{1,1} = \bar{C}_{31,31} - \frac{18\bar{F}_{33,31,16}}{17\bar{F}_{31,31,15}}\left(\frac{R}{a}\right)^2 \bar{C}_{33,31} + \frac{19\bar{F}_{35,31,17}}{17\bar{F}_{31,31,15}}\left(\frac{R}{a}\right)^4 \bar{C}_{35,31} - \dots \quad (8)$$

$$\bar{C}_{31}^{-1,3} = \bar{C}_{31,31} - \frac{14\bar{F}_{33,31,15}}{13\bar{F}_{31,31,14}}\left(\frac{R}{a}\right)^2 \bar{C}_{33,31} + \frac{15\bar{F}_{35,31,16}}{13\bar{F}_{31,31,14}}\left(\frac{R}{a}\right)^4 \bar{C}_{35,31} - \dots \quad (9)$$

and similarly for S, on replacing C by S throughout. The numerical values of the three most important $\bar{F}$ functions used here are:

$$\left.\begin{aligned}
\bar{F}_{32,31,15} &= 0.23668 \sin^{29} i\,(16\cos i - 1)(1+\cos i)^2 = 4.6336\times 10^{-3} \text{ for Proton 4 } (i = 51.53^\circ)\\
\bar{F}_{31,31,15} &= 0.49905 \sin^{30} i\,(1+\cos i) = 0.52694\times 10^{-3} \text{ for Proton 4}\\
\bar{F}_{31,31,14} &= 0.44035 \sin^{28} i\,(1+\cos i)^3 = 1.9949\times 10^{-3} \text{ for Proton 4}
\end{aligned}\right\} \quad (10)$$

The theoretical equation for the variation of eccentricity near 31:2 resonance, involves the same lumped coefficients $\bar{C}_m^{q,k}$ and $\bar{S}_m^{q,k}$ as the equation for di/dt, and may be written[6,7]

$$\begin{aligned}
\frac{de}{dt} = n\left(\frac{R}{a}\right)^{31}\Bigg[&-\frac{R}{a}\bar{F}_{32,31,15}\, e\left\{\bar{S}_{31}^{0,2}\sin\Phi + \bar{C}_{31}^{0,2}\cos\Phi\right\}\\
&- 17\bar{F}_{31,31,15}\left\{\bar{C}_{31}^{1,1}\sin(\Phi-\omega) - \bar{S}_{31}^{1,1}\cos(\Phi-\omega)\right\}\\
&+ 13\bar{F}_{31,31,14}\left\{\bar{C}_{31}^{-1,3}\sin(\Phi+\omega) - \bar{S}_{31}^{-1,3}\cos(\Phi+\omega)\right\}\\
&+ \text{terms in } e^{|q|-1}\left\{q - \tfrac{1}{2}(k+q)e^2\right\}\begin{matrix}\cos\\ \sin\end{matrix}(\gamma\Phi - q\omega)\Bigg] \quad . \quad (11)
\end{aligned}$$

Since $e \simeq 0.015$ for Proton 4, the first of the three terms within the square brackets is likely to be negligible.

3.2 Results

At dates between 14 December 1968 and 6 March 1969, when the orbit of Proton 4 was appreciably perturbed by the effects of 31:2 resonance, there are nine sets of PROP orbital elements available (orbits 2-10 of Table 2), and ten sets of US Navy elements. The 19 values of inclination from these orbits are plotted in Fig.2. The values were then cleared of (a) lunisolar and zonal-harmonic perturbations (with a combined maximum value of 0.0023°) and (b) the effects of an atmosphere rotating at 1.0rev/day (maximum value 0.0028°). The PROP values of inclination were also cleared of tesseral harmonic perturbations due to the $J_{2,2}$ term in the geopotential (maximum value 0.0018°). The 19 values of inclination after removal of perturbations are plotted in Fig.3. The corresponding observational values of eccentricity are plotted in Fig.4, and then replotted (on a much larger scale) in Fig.5 after removal of zonal-harmonic and lunisolar perturbations.

The 19 modified values of inclination and eccentricity in Figs.3 and 5 have been fitted with least-squares theoretical curves, first separately and then simultaneously, using the THROE[9,6] and SIMRES[6] computer programs respectively, to obtain values of lumped 31st-order harmonics in the geopotential.

First, the values of i in Fig.3 were fitted with equation (5), in integrated form, using THROE. The fitting was good, with the measure of fit, ε, having a value of 0.96. This confirms that the variation in i is consistent with a 31:2 resonance effect, but the numerical values of the C and S coefficients are indeterminate. They are as follows:

$$\left.\begin{aligned} 10^6\bar{S}_{31}^{0,2} &= 0.1 \pm 1.8 & 10^6\bar{C}_{31}^{0,2} &= 0.03 \pm 0.8 \\ 10^6\bar{C}_{31}^{1,1} &= -87 \pm 46 & 10^6\bar{S}_{31}^{1,1} &= -20 \pm 18 \\ 10^6\bar{C}_{31}^{-1,3} &= +8 \pm 12 & 10^6\bar{S}_{31}^{-1,3} &= 7 \pm 15 \end{aligned}\right\} \quad (12)$$

The first two values here, representing the $(\gamma,q) = (1,0)$ terms in equation (5), are (presumably by chance) very small, and, on substituting them into (5), we find that they comprise less than 5% of the total. These $(\gamma,q) = (1,0)$ terms are also very likely to be negligible in equation (11) for de/dt ; so they were discarded, and the values of i were fitted with $(\gamma,q) = (1,1)$ and $(1,-1)$ only. The resulting value of ε was 0.93, and the values of the coefficients are:

$$\left.\begin{array}{llll} 10^6\bar{C}_{31}^{1,1} = -86 \pm 22 & & 10^6\bar{S}_{31}^{1,1} = -18 \pm 13 \\ 10^6\bar{C}_{31}^{-1,3} = 8 \pm 6 & & 10^6\bar{S}_{31}^{-1,3} = 6 \pm 6 \end{array}\right\} \quad (13)$$

The values are not significantly altered, but the standard deviations are approximately halved.

In fitting i, the sd of the US Navy values was taken as 0.003°; the sd of the PROP values was taken from Table 2, except that the sd for orbit 2 was increased from 0.0002° to 0.0005°, because of the neglect of Earth-tide perturbations; and the density scale height was taken as 50km, appropriate to a height of 280km ($\frac{3}{2}H$ above perigee). The fitted curve, shown as a broken line in Fig.3, satisfactorily follows the variations in the observational values.

Next, the values of e in Fig.5 were fitted with equation (11), in integrated form, using THROE. The $(\gamma,q) = (1,0)$ term was dropped - the values obtained when it was included being absurdly large and indeterminate, as expected. The measure of fit, ε, was 1.48 and the values of the lumped coefficients are:

$$\left.\begin{array}{llll} 10^6\bar{C}_{31}^{1,1} = -20 \pm 15 & & 10^6\bar{S}_{31}^{1,1} = 50 \pm 11 \\ 10^6\bar{C}_{31}^{-1,3} = 12 \pm 4 & & 10^6\bar{S}_{31}^{-1,3} = 11 \pm 4 \end{array}\right\} \quad (14)$$

In fitting the eccentricity, the sd of the US Navy values was taken as 0.0001 (taking the sd as 0.00004 gave unacceptably large values of ε, though the standard deviations of the lumped coefficients were not much altered); the density scale height H was taken as 55km, appropriate to a height of 325km (3H/2 above perigee). In the course of the analysis, the sd of e was doubled on four of the PROP orbits (orbits 2, 4, 5 and 10), since this change improved the fitting. The fitted curve is shown as a broken line in Fig.5, and it is clear that the US Navy values display an oscillation, of unknown origin.

In the US Navy elements as received, the odd-harmonic perturbations in e have been largely removed and have to be restored (using the SDCELS subroutine). In an attempt to reduce the oscillation, the restoration was made using a non-standard value of J_3, -2.40×10^{-6}, which includes an allowance for J_5. This change reduced the amplitude of the oscillation, but it remains quite large.

The second pair of (C,S) coefficients in (14) agrees well with the second pair in (13), but the only agreement between the [1,1] pairs of (C,S) coefficients in (13) and (14) is that both are larger than the [-1,3] coefficients. The discrepancy between the values of the [1,1] coefficients in equations (13) and (14) can be resolved by a simultaneous fitting of the variations in i and e. This has been done, using the SIMRES computer program[6], where there is a choice of weighting. Two alternatives were tried: (a) with i and e having equal weights, and (b) with e degraded by a factor of 1.6 in recognition of its larger value of ε (1.48 as against 0.93 for i). The second alternative was chosen because it is more logical and because it gave values more consistent with (13) and (14). (The equal weighting gave slightly lower standard deviations, however.) The fitting (b), for which $\varepsilon = 1.14$, is shown by the unbroken lines in Figs.3 and 5, and the values of the coefficients given by SIMRES are:

$$\left.\begin{array}{llll} 10^6\bar{C}_{31}^{1,1} = -45 \pm 14 & & 10^6\bar{S}_{31}^{1,1} = 11 \pm 7 \\ 10^6\bar{C}_{31}^{-1,3} = 10 \pm 3 & & 10^6\bar{S}_{31}^{-1,3} = 2 \pm 3 \end{array}\right\} \quad (15)$$

The values in equations (15) are quite close to the means of the values in equations (13) and (14).

3.3 Discussion

The values of the lumped coefficients in equations (15) are fairly consistent with those obtained from analysis of i alone, equations (13), and with those derived from e alone, equations (14) - they are within about twice the sum of the sd, the [-1,3] coefficients being much the closer. Inevitably, the simultaneous fitting is a compromise between i and e, and neither is fitted so well by the combined solution: the values of ε are 1.18 for i and

2.16 for e , as compared with 0.93 for i and 1.48 for e in the separate solutions. Nevertheless, the fitting of the unbroken curves in Figs.3 and 5 is quite satisfactory, so the combined solution is obviously preferable.

The expected order of magnitude of the lumped coefficients can be estimated by assuming that the individual coefficients of degree ℓ have numerical values[5] of order $10^{-5}/\ell^2$, so that those with $31 \leqslant \ell \leqslant 43$ should be of order 10^{-8}. For Proton 4 the numerical versions of equations (8) and (9) are:

$$\bar{C}_{31}^{1,1} = \bar{C}_{31,31} - 15\bar{C}_{33,31} + 80\bar{C}_{35,31} - 244\bar{C}_{37,31} + 466\bar{C}_{39,31} - 534\bar{C}_{41,31} + 256\bar{C}_{43,31} + 190\bar{C}_{45,31} - 328\bar{C}_{47,31} + \dots \qquad (16)$$

$$\bar{C}_{31}^{-1,3} = \bar{C}_{31,31} - 12\bar{C}_{33,31} + 52\bar{C}_{35,31} - 122\bar{C}_{37,31} + 164\bar{C}_{39,31} - 96\bar{C}_{41,31} - 47\bar{C}_{43,31} + 104\bar{C}_{45,31} - 8\bar{C}_{47,31} + \dots \qquad (17)$$

and similarly for S , on replacing C by S throughout.

Since the $(\gamma,q) = (1,0)$ terms have been dropped, both pairs of coefficients give lumped values of geopotential coefficients of order 31 and *odd* degree (ℓ = 31, 33, 35 ...). Equations (16) and (17) show that the largest contributions are most likely to come from the coefficients of degree 37, 39 and 41.

Equations (16) and (17) indicate that the C and S coefficients with affixes [1,1] might be expected to be about 3 or 4 times greater than those with affixes [-1,3]. This is in conformity with the actual results, since $\sqrt{45^2 + 11^2}/\sqrt{10^2 + 2^2} = 4.5$.

Equations (16) and (17) also suggest that if the individual coefficients are of order 10^{-8}, the expected magnitude of the lumped coefficients $\bar{C}_{31}^{1,1}$ and $\bar{C}_{31}^{-1,3}$ would be about 10×10^{-6} and 3×10^{-6} respectively: the actual values in equations (15) are about three times larger. This discrepancy cannot be regarded as significant, since the assumed magnitude of the coefficients $(10^{-5}/\ell^2)$ may be in error by a factor of up to 2, and the terms in equations (16) and (17) might happen to add up to considerably more than the root of the sums of their squares. Nevertheless these numerical values of 31st-order coefficients should be treated with the caution appropriate in any first determination of physical constants.

4 ATMOSPHERIC ROTATION RATE

The rotation rate of the upper atmosphere was estimated using 15 values of inclination, given in Table 2 by orbits 11-25, between March and July 1969. These values were cleared of lunisolar and geopotential perturbations and the modified values are shown plotted, together with their standard deviations, in Fig.6 (from MJD 40293 to 40424). Similarly modified US Navy values, shown by crosses, are included for general comparison. Fig.6 also shows the values during the resonance period and the broken curve of Fig.3, after the restoration of the atmospheric rotation perturbation.

The theoretical change in inclination was calculated for various values of atmospheric rotation rate (expressed as Λ times the Earth's rotation rate), using oblate-atmosphere theory[10], with numerical integration at intervals of about 6 days (corresponding to 22.5° steps in argument of perigee).

A theoretical curve fitted to the 15 values, as shown in Fig.6, gives a mean value of rotation rate,

$$\Lambda = 1.10 \pm 0.05 \text{rev/day} ,$$

equivalent to an average zonal wind of 40 ± 20m/s west to east, at a mean height of 260km, effectively averaged over latitudes up to about 35° (since atmospheric rotation has little effect on an orbit of inclination 51°, at latitudes of $35-51^{\circ}$).

Recent results from analysis of many satellite orbits[11] have indicated a difference between evening and morning winds, so the values were separated into two groups, with the division where the local time at perigee is 12h (MJD 40375), and fitted with two curves as shown in Fig.7 to give morning (4-12h) and afternoon-evening (12-24h) values of Λ ; the local time at perigee is marked at the top. The values of Λ obtained, 1.3 ± 0.1 for afternoon-evening and 0.9 ± 0.1 for the morning, agree with those obtained from other satellites[11].

5 CONCLUSIONS

Our analysis shows that the orbit of Proton 4 was substantially affected by 31:2 resonance with the geopotential between December 1968 and March 1969.

The sets of orbital parameters obtained from observations were accurate enough for the variations in inclination and eccentricity near resonance to be analysed to determine, for the first time, numerical values of lumped geopotential coefficients of 31st order and odd degree.

The simultaneous least-squares fitting of the values of inclination and eccentricity at 19 epochs, using the SIMRES program, leads to the following values of lumped 31st-order harmonic coefficients:

$$10^6 \bar{C}_{31}^{1,1} = -45 \pm 14 \qquad 10^6 \bar{S}_{31}^{1,1} = 11 \pm 7$$

$$10^6 \bar{C}_{31}^{-1,3} = 10 \pm 3 \qquad 10^6 \bar{S}_{31}^{-1,3} = 2 \pm 3 \quad .$$

Equations (16) and (17) give these lumped coefficients explicitly in terms of the individual geopotential coefficients of order 31, $\bar{C}_{\ell,31}$ and $\bar{S}_{\ell,31}$.

Although the numerical values of the lumped coefficients obtained here are not of high accuracy, this work, and the recent analysis of 29:2 resonance[12], show that it should be feasible to evaluate individual coefficients of order 31 and 29 - and possibly also 27 and 25 - from analysis of a number of resonant orbits at different inclinations, if enough accurate observations can be obtained near the time of resonance.

The atmospheric rotation rate Λ has been evaluated from the change in inclination of Proton 4 between March and July 1969. The mean value is found to be 1.10 ± 0.05rev/day at 260km height, corresponding to a zonal wind of 40 ± 20m/s from west-to-east, at a representative latitude near 30°. Dividing the values into two groups gave evening and morning values of Λ , as follows: Λ = 1.3 ± 0.1rev/day for afternoon-evening (12-24h) at 270km height, equivalent to a west-to-east wind of 120 ± 40m/s; and Λ = 0.9 ± 0.1rev/day in the morning (4-12h), for 240km height, corresponding to an east-to-west wind of 40 ± 40m/s. These three values of Λ, which are biased towards latitudes of 0-35°, agree well with the recent findings from other satellites[11].

Acknowledgment

We thank R.H. Gooding for the version of THROE extended to general $\beta:\alpha$ resonance.

Table 2

ORBITAL PARAMETERS FOR PROTON 4, WITH STANDARD DEVIATIONS

	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	M_4	M_5	c	D	N
1	0184.0	1968 Nov 24	6738.9380 10	0.017281 17	51.5430 6	201.133 3	108.53 10	283.64 10	5649.2862 12	0.1797 6	0.0007 2	0.00064 5	-0.00008 1	0.75	8.9	43
2*	204.0	Dec 14	6732.2318 5	0.015570 3	51.5451 2	98.722 1	182.75 2	313.55 2	5657.7291 6	0.2063 1	-0.0028 1			0.60	8.2	61
3	211.0	Dec 21	6730.0263 3	0.014944 6	51.5457 12	62.794 1	210.23 3	326.81 3	5660.5104 4	0.2058 1				0.64	8.3	50
4	226.0	1969 Jan 5	6725.3647 4	0.013949 6	51.5424 8	345.664 1	271.02 4	316.86 5	5666.3958 5	0.1769 3	0.00057 3	0.00022 1		0.39	9.9	46
5	236.0	Jan 15	6722.3747 6	0.013875 16	51.5419 11	294.148 2	311.94 6	117.32 6	5670.1764 8	0.1983 4	0.00119 5	-0.00007 2		0.70	9.6	57
6	246.0	Jan 25	6719.4259 13	0.014120 11	51.5370 10	242.546 1	351.67 4	317.48 4	5673.9084 16	0.1916 6	0.0042 3	-0.00029 3	-0.00008 1	0.77	8.9	57
7	257.0	Feb 5	6715.9820 12	0.014511 15	51.5368 12	185.682 2	33.44 5	114.47 5	5678.2729 16	0.2271 6	-0.0039 2	-0.00017 5	0.00011 1	0.75	9.3	59
8	266.0	Feb 14	6712.8496 10	0.014665 19	51.5335 16	139.086 2	66.76 10	118.27 10	5682.2472 12	0.1976 9	-0.00332 7	0.00069 4		1.09	9.4	36
9	275.0	Feb 23	6709.8501 11	0.014386 12	51.5342 12	92.407 2	99.50 9	157.16 9	5686.0576 14	0.2478 5	0.0025 1	0.00013 3		0.84	9.3	39
10	286.0	Mar 6	6705.0312 11	0.013686 21	51.5365 11	35.247 2	140.48 6	99.25 6	5692.1883 13	0.2907 13	0.0073 3	-0.00035 13	-0.00022 3	0.74	8.0	56
11	293.0	Mar 13	6701.7020 18	0.012999 16	51.5375 13	358.790 2	166.80 6	0.35 6	5696.4303 24	0.3075 10	-0.0061 6	-0.00013 7	0.00031 3	0.78	7.6	38
12	30[illegible].0	Mar 26	6694.4755 16	0.011479 14	51.5350 15	290.909 2	218.74 6	309.27 7	5705.6554 20	0.3804 5	-0.0072 2	-0.00011 2	0.00011 1	1.14	10.4	49
13	317.0	Apr 6	6688.3353 6	0.010580 10	51.5335 9	233.266 2	266.03 10	110.56 11	5713.5135 8	0.3477 4	-0.00154 5	0.00020 2		0.73	9.4	52
14*	328.0	Apr 17	6682.0904 9	0.010385 18	51.5358 5	175.448 2	313.20 11	358.09 11	5721.5245 12	0.4076 6	0.00007 8	-0.00049 4		0.79	8.8	38
15	341.0	Apr 30	6673.5211 13	0.010355 25	51.5302 12	106.840 2	6.55 7	285.45 7	5732.5469 17	0.4638 5	-0.0004 2	-0.00035 4		0.79	8.9	41
16	349.0	May 8	6667.7932 7	0.010323 17	51.5323 12	64.463 1	37.65 8	95.96 8	5739.9348 9	0.4781 4	0.00349 6	0.00025 2		0.66	8.8	32
17	362.0	May 21	6656.3168 20	0.009575 15	51.5309 14	355.293 3	85.42 16	292.49 16	5754.7845 26	0.5926 8	0.0036 5	-0.00026 5	-0.00009 2	0.84	8.5	30
18	375.0	Jun 3	6644.5689 10	0.008326 17	51.5283 14	285.689 2	132.72 17	328.89 16	5770.0519 13	0.5940 7	0.0050 1	0.00100 5		0.68	8.2	22
19	386.0	Jun 14	6632.1288 5	0.006626 27	51.5272 3	226.438 2	173.88 4	166.96 4	5786.2926 7	0.8474 6	0.0051 4			0.80	4.0	25
20*	392.0	Jun 20	6624.5940 14	0.005547 37	51.5251 8	193.947 4	198.81 18	354.43 18	5796.1664 19	0.7886 3	0.0020 2			0.83	5.9	26
21*	395.0	Jun 23	6621.0056 23	0.005247 25	51.5230 1	177.650 2	211.18 18	109.97 18	5[illegible]0.8785 30	0.7818 18	0.0131 16	0.0018 4	-0.0008 2	0.53	5.0	45
22	400.0	Jun 28	6614.8547 13	0.004636 14	51.5223 8	150.415 2	234.33 21	332.08 21	5808.9707 17	0.7682 11	-0.0046 6	0.0015 2		0.42	4.8	29
23	408.0	Jul 6	6603.9381 14	0.003956 15	51.5245 18	106.646 2	272.45 39	52.51 40	5823.3794 19	1.0999 6	0.0253 2	0.00087 4		0.73	9.4	34
24	420.0	Jul 18	6572.0992 12	0.003038 38	51.5213 9	40.326 4	327.60 49	304.56 50	5865.7444 16	2.7417 18	0.0981 3	0.0063 3		1.01	5.0	45
25	424.0	Jul 22	6549.8476 17	0.002189 25	51.5153 6	17.890 1	342.95 26	61.74 26	5895.6578 23	5.7649 44	0.7256 31	0.1367 18	0.0130 10	0.76	3.6	42

KEY:
- MJD Modified Julian Day
- a semi major axis (km)
- e eccentricity
- i inclination (deg)
- Ω right ascension of node (deg)
- ω argument of perigee (deg)
- M_0 mean anomaly at epoch (deg)
- M_1 mean motion, n (deg/day)
- M_2–M_5 additional coefficients in polynomial for M
- c measure of fit
- D time coverage of observations (days)
- N number of observations used

* orbits with Hewitt-camera observations.

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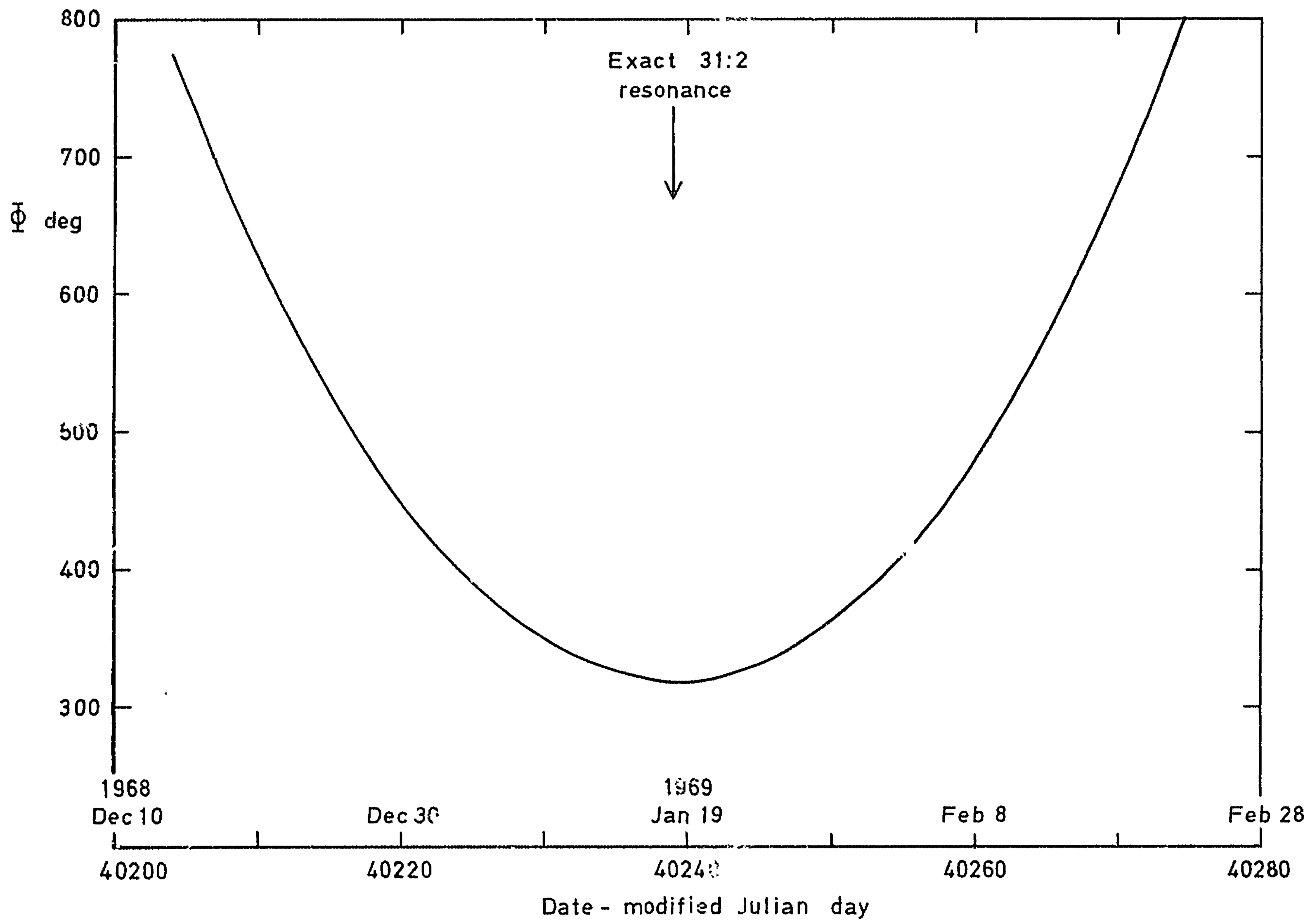

Fig.1 The resonance angle, Φ

Fig.1

Fig. 2

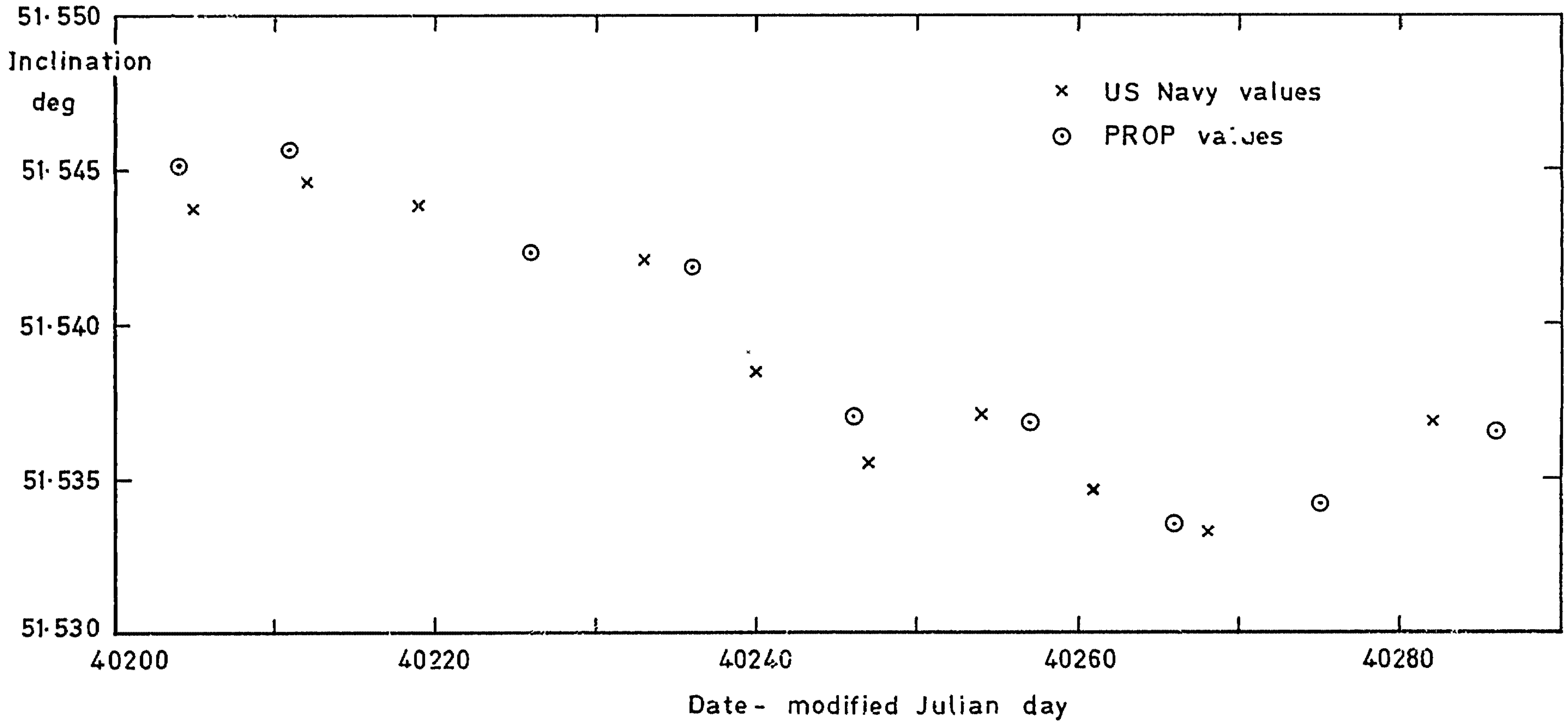

Fig. 2 Observational values of inclination

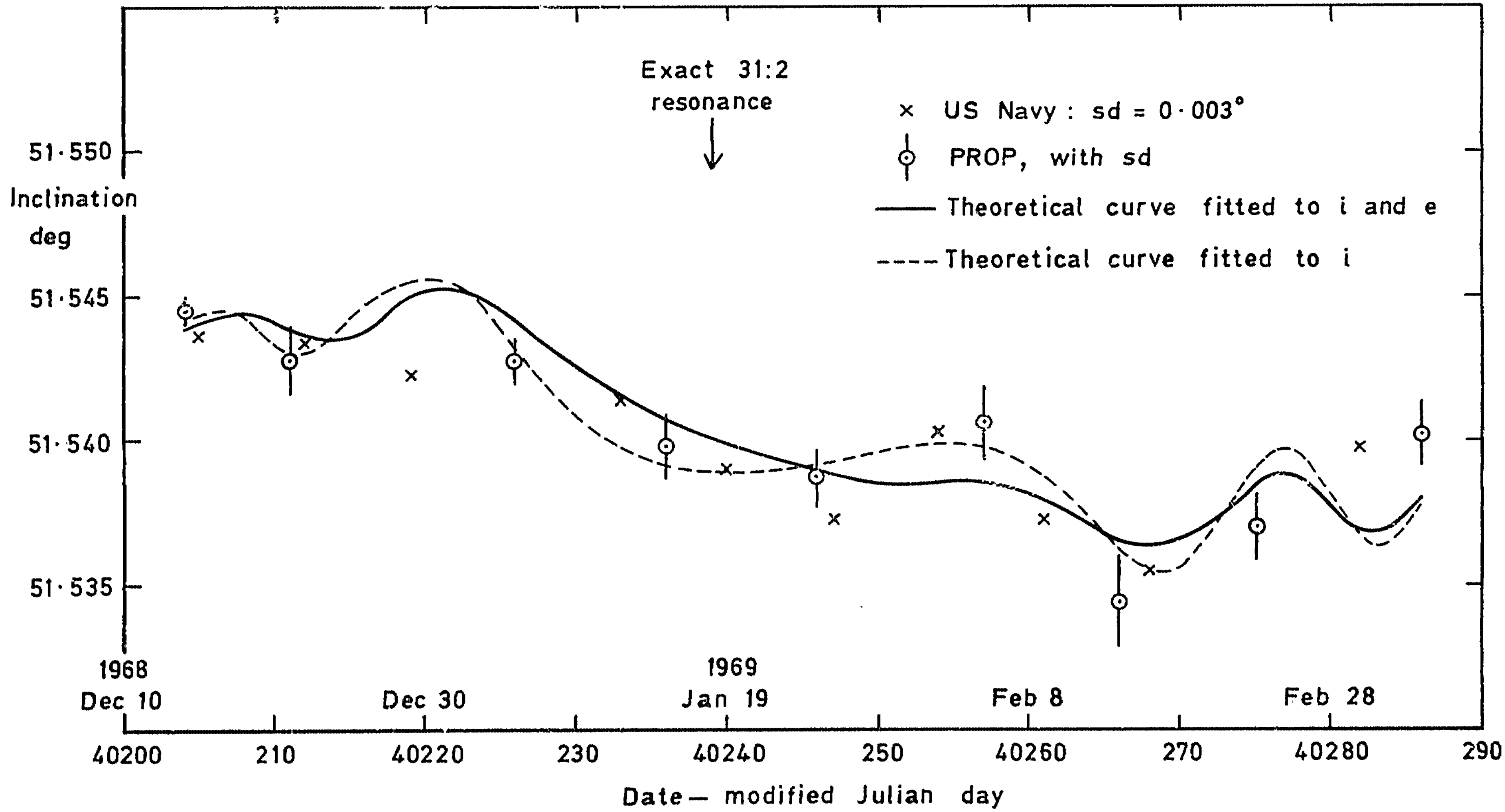

Fig. 3 Variation in inclination during 31:2 resonance, cleared of perturbations

Fig.3

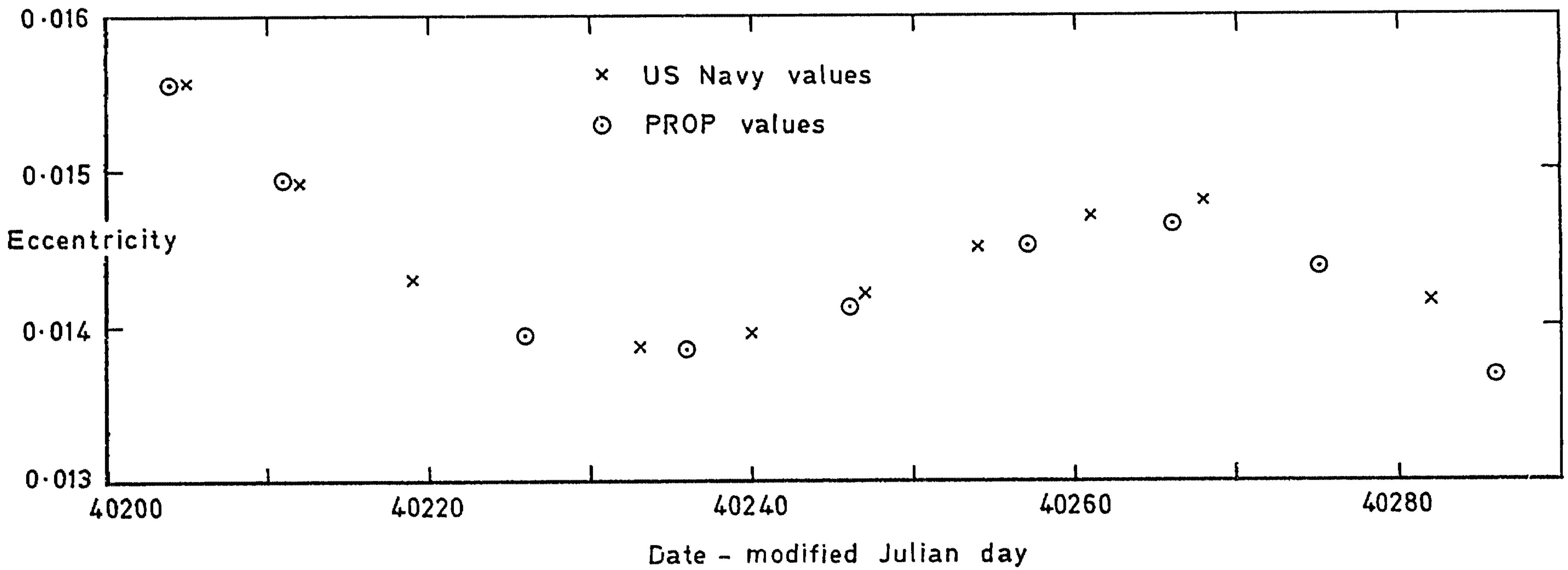

Fig.4 Observational values of eccentricity

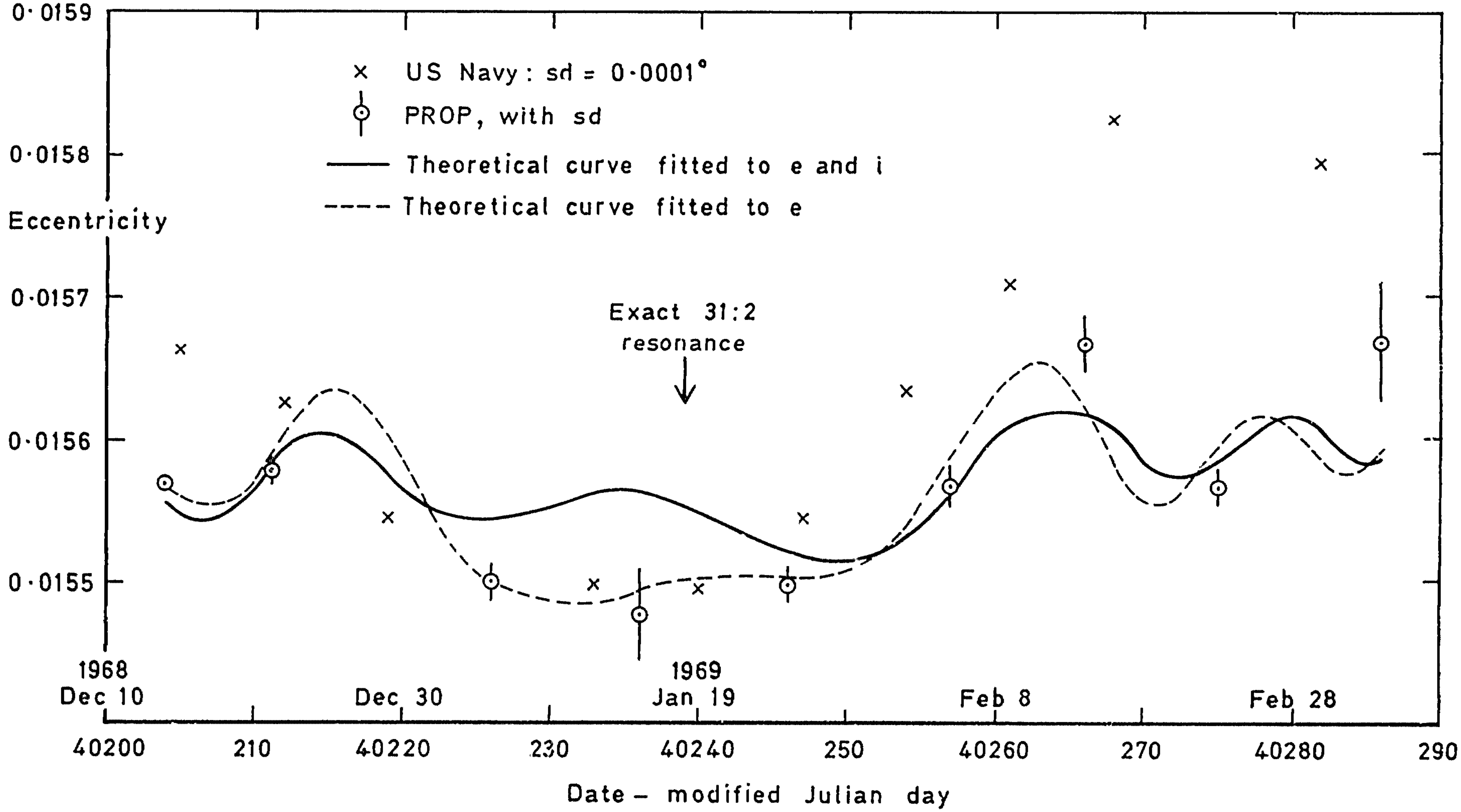

Fig. 5 Variation in eccentricity during 31:2 resonance, cleared of perturbations

Fig.5

Fig.6

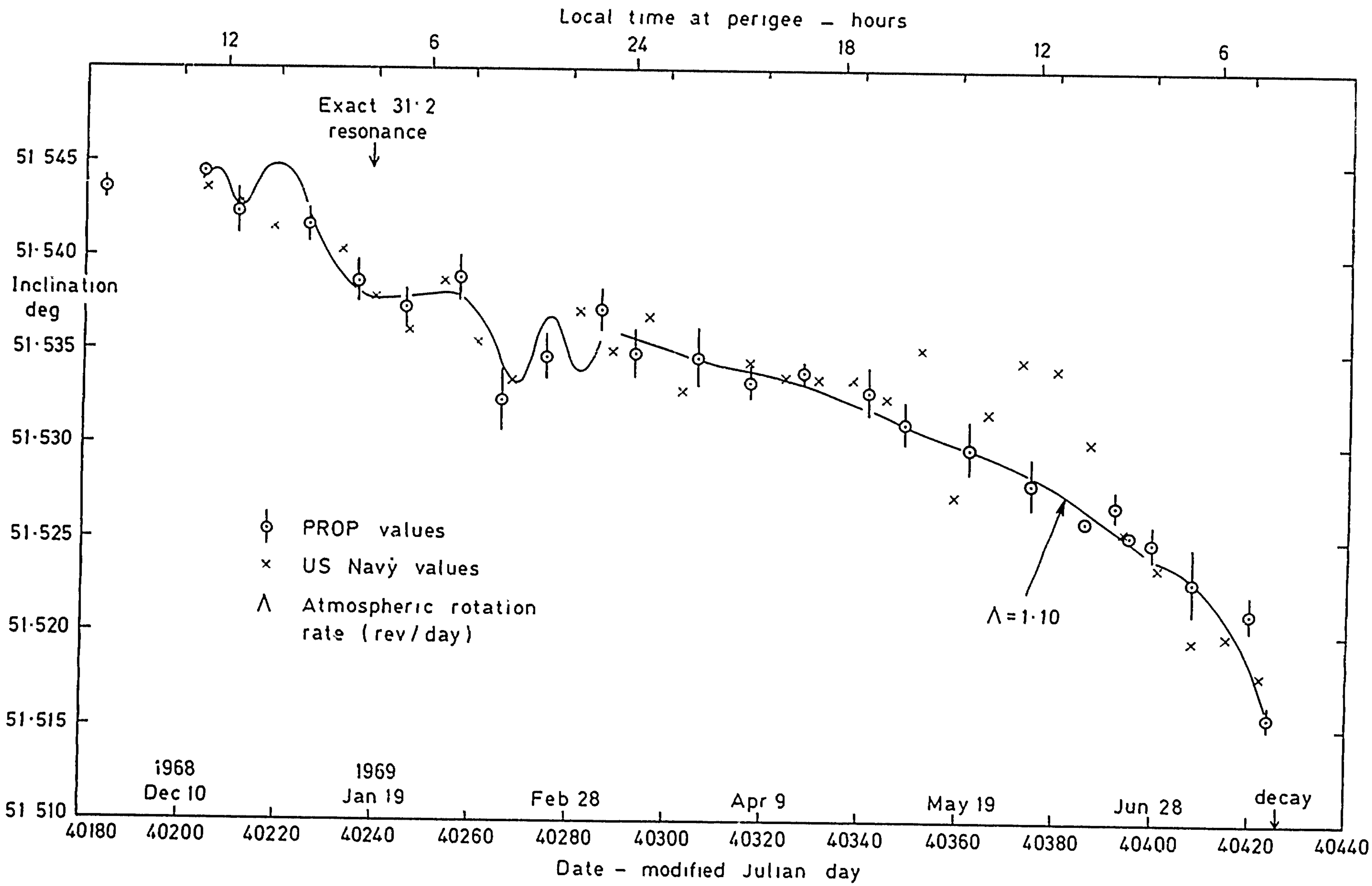

Fig.6 Inclination of Proton 4 orbit

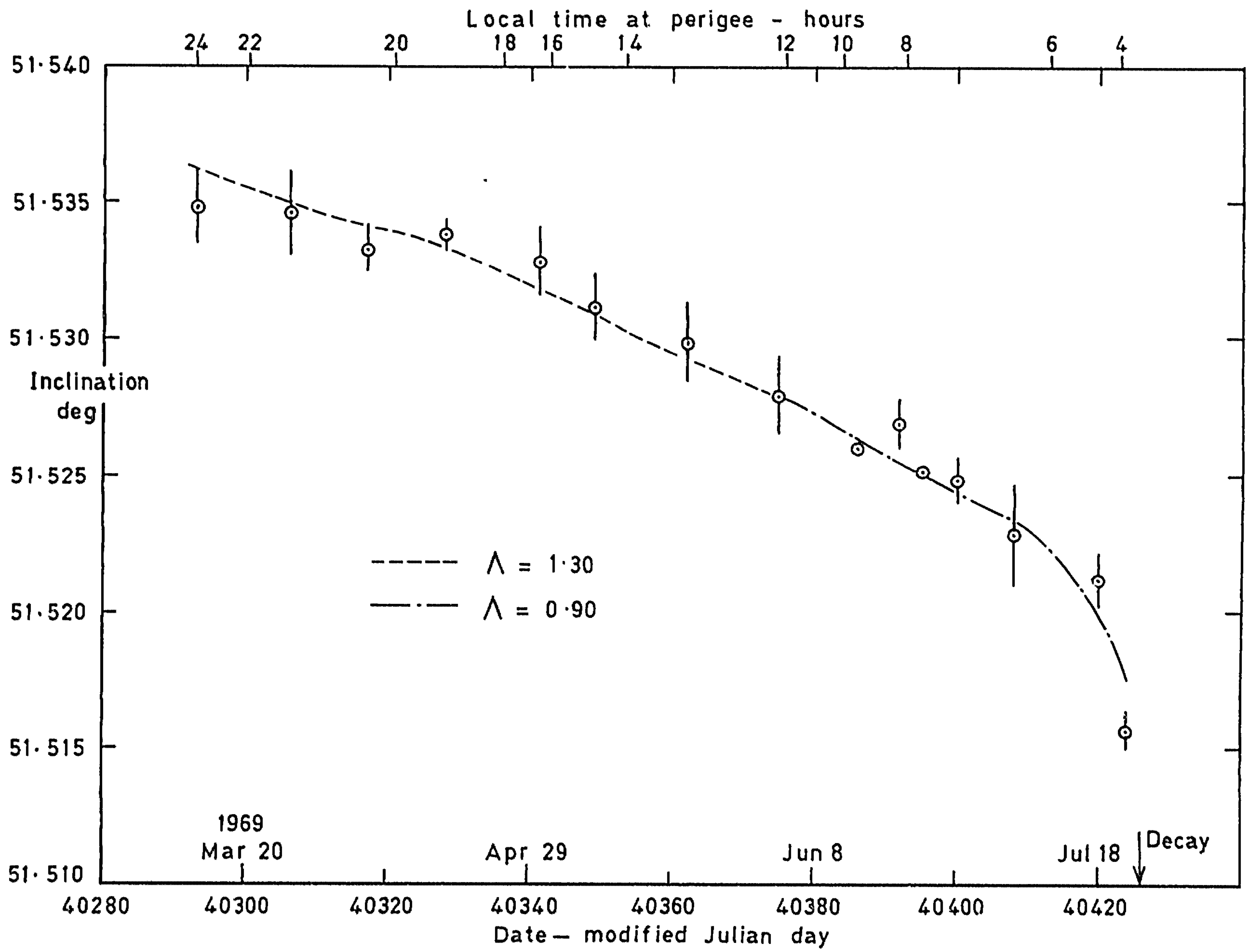

Fig.7 Values of inclination after removal of lunisolar and geopotential perturbations

Fig.7

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17. Abstract

The orbit of Proton 4, 1968-103A, has been redetermined, in greater detail and with better accuracy, in order to clarify previously puzzling features in the variation of orbital inclination. Orbital parameters have been determined at 25 epochs between December 1968 and July 1969, using about 1600 optical and radar observations with the RAE orbit refinement program PROP6.

During January 1969 the orbit passed through 31:2 resonance – when the ground track over the Earth repeats every two days after 31 revolutions of the satellite. A simultaneous least-squares fitting of theoretical curves to the values of inclination and eccentricity between 14 December 1968 and 6 March 1969 has yielded values for two pairs of lumped 31st-order geopotential coefficients, appropriate to an inclination of 51.5°. This is the first specific evaluation of 31st-order coefficients.

The 15 values of inclination after the resonance, from March to near decay in July 1969, have been used to determine mean, morning and afternoon-evening values for the rotation rate of the atmosphere at a height near 260km; the values of rotation rate, namely 1.1, 0.9 and 1.3rev/day respectively, confirm the trends already established from analysis of other satellite orbits.