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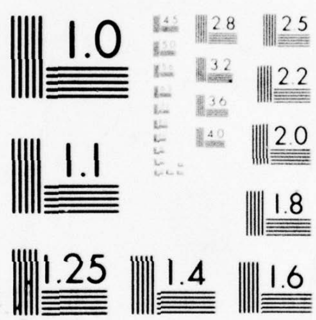
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A MATHEMATICAL STRUCTURE FOR REFINEMENT OF SOUND RANGING ESTIMATES

By

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In this report, a mathematical framework is introduced in which the "sound ranging" problem is defined, together with requisite structure to permit solution of certain problems generally known under this heading. The information presented shows that under hypothesis the sound ranging problem admits to a unique solution, and may be found among the zeroes of a related family of nonlinear equations. A form of solution is offered based on the concept of the Moore-Penrose Pseudoinverse; and a description of preliminary results is offered.

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CONTENTS

INTRODUCTION

This presentation offers a mathematical framework for a series of problems normally encountered under the heading "sound ranging," and developments are directed toward this end. However, the techniques may be subject to a broader range of interpretation. The language is intended to be strongly suggestive of actual application and will be kept of such a nature as to agree with intuitive "feel." As considered here, the fundamental problem of "sound ranging" will be the location of a "sound source" when the approximate position of an expanding wave front is known at a number of times; or more appropriately, if the approximate time of arrival of such a wave front is known at a series of prescribed locations in space.

The approach taken is nonlinear and consists of two distinct steps: a preliminary estimation of sound source and influencing parameters, and an iterative technique for refinement of such estimates.

Of particular interest is the interplay which arises between points chosen as "receiving stations," sound source position, accuracy of arrival times, and integrity of meteorological information.

The guidelines chosen for the mathematical framework were sufficiently general to allow for the consideration of many differing sound wave fronts without destroying an undue amount of intuitive flavor. Several interesting and occasionally surprising relationships emerge which are not generally evident, especially with regard to the role of microphone placement.

The main flow proceeds from the general to the specific, with an ultimate goal the improvement of overall "sound ranging capability." This goal should be approached by the augmentation of current or proposed techniques with a class of "vernier" methods. The authors believe that these techniques may be initiated at a modest computational level and will be consistent with modern concepts of automation.

In this initial presentation, a general mathematical skeleton is developed and then applied in a simple problem. The example presented will consider a spherical sound wave propagating in a medium of uniform temperature and in the presence of a wind of fixed magnitude and direction. This model will be sufficient to demonstrate several pertinent features, yet the lack of complication provides ease of presentation and interpretation. As more complex models are examined, they may be contrasted in light of their differences from the basic model and perhaps in this manner be more easily interpreted themselves.

To this point, random variations within the structure have not been considered. However, the authors have explored this area to the extent that a probabilistic approach is expected to be highly rewarding, both from the point of view of sensitivity analysis and from the standpoint of information recovery versus receiver accuracy. The authors believe stochastic modeling will provide a fruitful avenue of approach, and preliminary results are encouraging.

THE SOUND RANGING PROBLEM, A GENERAL MATHEMATICAL APPROACH

Fundamental in the structure to be developed is the notion of an extended wave generating function. For the highly specialized application encountered here, the following definitions will be employed.

Definition 1

Let w be a real-valued function whose domain is a compact connected region of Euclidean 4-space (E^4) and whose range contains the real number zero. Assume, moreover, that w is of class C^2, that is, possesses continuous partial derivatives of the second order. Then w will be called an <u>extended wave generating function</u>.

Let $\mathcal{D}$ denote the domain of definition of an extended wave generating function w. A typical point in $\mathcal{D}$ will be written as (x,y,z,t); with $D = \{(x,y,z) \mid (x,y,z,t) \in \mathcal{D}\}$ carrying a connotation of "points in space," while $T = \{t \mid (x,y,z,t) \in \mathcal{D}\}$ has a context of "time." The notation for $\mathcal{D}$ and the projections D and T will be consistently utilized throughout the discussions to follow.

Observe that the continuity of projection maps assures that both T and D are compact and connected; hence D may be visualized as a bounded volume in Euclidean 3-space (E^3), and T as a real interval.

Attention will now be focused on sets of the form $W_t = \{(x,y,z) \mid (x,y,z,t) \in \mathcal{D};\ w(x,y,z,t) = 0\}$. Some descriptive terminology concerning these sets is given in the following definition.

Definition 2

Let w be an extended wave generating function defined on the region $\mathcal{D}$ of E^4. For each $t \in T$, the set of points W_t in E^3 defined by $W_t = \{(x,y,z) \mid (x,y,z,t) \in \mathcal{D};\ w(x,y,z,t) = 0\}$ will be called the wave front at t generated by w. A point (x,y,z) in W_t will be said to lie on the appropriate wave front.

Suppose that $\{w_j \mid j \in J\}$ is an indexed family of extended wave generating functions; and let $\{\mathcal{D}_j \mid j \in J\}$, $\{D_j \mid j \in J\}$, and $\{T_j \mid j \in J\}$ have their obvious interpretations. Let (x,y,z) be a point in $\bigcap_{j \in J} D_j$, which will be assumed to be nonempty. If there exists $\{t_j \mid t_j \in T_j;\ j \in J\}$ such that $(x,y,z) \in \bigcap_{j \in J} (W_j)_{t_j}$, where for each $j \in J$, $(W_j)_{t_j}$ denotes the wave front at t_j generated by w_j, the point (x,y,z) will be called a point of contingency for the family $\{w_j \mid j \in J\}$. A special point of this nature will be of interest in the solution of the sound ranging problem as presented in the current context.

At this point, it will be expedient to define a special point which will emerge as a point of contingency for certain families of extended wave generating functions. These will arise in a logical manner from a single wave generating function, and a known family of discrete points in the domain of this latter function. This point will be called a source point, and its description is the content of the following definition.

Definition 3

Let w be an extended wave generating function defined on $\mathcal{D}$. For any $(x,y,z) \varepsilon D$, let $\Gamma(x,y,z)$ designate the set of all linear graphs in E^3 with origin (x,y,z).

Now suppose that (x_o, y_o, z_o, t_o) is a point in $\mathcal{D}$ with the following properties

a. For any $\gamma \varepsilon \Gamma$, $t \varepsilon T$, $\gamma \bigcap W_t \neq \Phi$.

b. For any $\gamma \varepsilon \Gamma$, and sequence $\{t_n \mid t_n \varepsilon T; n\ 1,2,\ldots\}$ such that $\lim_n t_n = t_o$; if (p_n,q_n,r_n) is in $\gamma \bigcap W_{t_n}$, it follows that $\lim_n \|\ (p_n,q_n,r_n) - (x_o,y_o,z_o)\ \| = 0$.

Then (x_o,y_o,z_o,t_o) will be called a <u>source point</u> of w.

<u>Theorem 1</u>. The source point as designated by Definition 3 is unique; that is, an extended wave generating function can have at most one source point.

<u>Proof</u>. Suppose there exist two distinct source points (x_1,y_1,z_1,t_1), (x_2,y_2,z_2,t_2). For times sufficiently near t_1 and t_2, all wave fronts belonging to these times may be separated by open spheres. By choosing a point outside either of these spheres so that a line connecting this point to the center of one of the spheres is not tangent to the other, part a of Definition 3 is violated. This proves the theorem and demonstrates uniqueness of the source point.

A fundamental property of extended wave generating functions with source points which will be of considerable importance in the development to follow, and which will offer one approach to the problem of numerical determination of source points, can now be demonstrated.

<u>Theorem 2</u>. Let w be a wave generating function with source point (x_0,y_0,z_0,t_0). Let J be a finite index set, and for $j \in J$, let (x_j,y_j,z_j,t_j) lie in $\mathcal{D}$. Assume that $\{(x_j,y_j,z_j,t_j) \mid j \in J\}$ forms a distinct set of points. It is now possible to define a family of transformations from E^4 onto E^4 by letting τ_j for each $j \in J$ be

$$\tau_j: \begin{cases} x' = x - (x_j - x_0) \\ y' = y - (y_j - y_0) \\ z' = z - (z_j - z_0) \\ t' = t - (t_j - t_0) \end{cases}$$

For each $j \in J$, τ_j possesses a differential equal to the appropriate identity matrix; therefore, the Jacobian is of constant value 1 through E^4. In particular, τ_j is a pure translation in E^4, and hence carries compact, connected sets into sets of similar nature. Now, for each $j \in J$, let w_j be defined for each $p' \in \tau_j(\mathcal{D})$ by $w_j(p') = w(\tau^{-1}(p'))$. Clearly w_j is well-defined for each $j \in J$. It follows that $\{w_j \mid j \in J\}$ is a family of extended wave generating functions, and (x_0,y_0,z_0) is a point of contingency for this family.

Proof. As has been seen, the nature of the family of translations $\{\tau_j \mid j \in J\}$ assures that each member of $\{w_j \mid j \in J\}$ is a real-valued function defined on a compact connected region of E^4. It also follows that each is of class C^2, since w possesses this nature.

Observe now that for any $j \in J$, $(x_j,y_j,z_j,t_j) \equiv p_j$ is mapped by τ_j onto $(x_0,y_0,z_0,t_0) \equiv p_0$; hence, p_0 is in the domain of w_j for each $j \in J$. In addition, $w_j(p_0) = w(\tau^{-1}(p_0)) = w(p_j) = 0$; therefore, zero is in the range of w_j for each $j \in J$ and each w_j is an extended wave generating function. Examination also yields that (x_0,y_0,z_0) lies in $(W_j)_{t_j}$; that is, the wave front at t_j generated by w_j; and this for each $j \in J$. It therefore follows that (x_0,y_0,z_0) is a point of contingency for $\{w_j \mid j \in J\}$. This proves the theorem.

In reality, an important corollary has essentially been proved.

Corollary 2.1. If w, p_0, and $\{w_j \mid j \in J\}$ are as given in Theorem 1, the source point of w lies among the common zeroes of the family $\{w_j \mid j \in J\}$.

Corollary 2.1 links the problem of determining a source point of an extended wave generating function to the problem of determining the proper zero of $\{w_j \mid j \in J\}$. As will be seen later, a series of computational devices for determining source points may arise from this fact.

At this point, a precise definition of the sound ranging problem may be phrased in current context.

Definition 4

Let w be an extended wave generating function and $\{p_j \mid j \in J\}$ be a finite family of points in D with $\{(x_j,y_j,z_j) \mid j \in J\}$ distinct in E^3. The problem of determining the source point of w, if such exists, will be called a sound ranging problem.

In the study of general sound ranging problems, an especially revealing form of the extended wave generating function is described in the following definition.

Definition 5

Let w be an extended wave generating function having the following properties

a. For any t,t' in T, $W_t \neq W_{t'}$

b. There exists $(x,y,z) \in D$ having the property that if $\gamma \in \Gamma(x,y,z)$, and t, t' are in T with $t \leq t'$; and if in addition, $(p,q,r) \in \gamma \cap W_t$ and $(p',q',r') \in \gamma \cap W_{t'}$, then $\|(x,y,z) - (p,q,r)\| \leq \|(x,y,z) - (p',q',r')\|$.

Then w is said to be an extended wave generating function with wave fronts expanding about (x,y,z). If strict inequalities hold in b the wave fronts in question will be said to be strictly expanding.

The following theorem exhibits some of the more salient features of extended wave generating functions with wave fronts expanding about a source point.

Theorem 3. Let w be an extended wave generating function with source point p_0, and which generates wave fronts expanding about (x_0,y_0,z_0). Then the following conditions hold

a. The set W_{t_0} must contain only the point (x_0,y_0,z_0).

b. The point t_0 is the greatest lower bound of T.

In this case, t_0 is called the blast time; and (x_0,y_0,z_0) the point of origin of the expanding family of wave fronts.

Proof: Let t' be given such that $t' \leq t_0$. As w generates wave fronts expanding about (x_0,y_0,z_0), it must follow tnat $|| (x_0,y_0,z_0) - (p,q,r) || = 0$ for any $(p,q,r) \in W_{t'}$, hence $W_{t'} \equiv \{(x_0,y_0,z_0)\}$. By condition a of Definition 4, it must follow that $t' = t_0$. Conditions a and b follow at once.

It might be recalled from Corollary 2.1 that a source point of an extended wave generating function lies among the common zeroes of a family of functions of class C^2, each defined on a compact, connected region of E^4. If the entire set of zeroes of this family may be determined, there still remains the problem of isolating the source point from the remaining zeroes. If a set of restraints is known which assures this separation, the resulting system will produce what will be called a completely determinate sound ranging problem. As will be seen, however, the problem of selecting a proper zero may be approached from another avenue.

The problem of solving a set of simultaneous, nonlinear equations of the form $\{w_j(x,y,z,t) = 0 \mid j \in J\}$ is well known in the literature of numerical analysis, and an introductory presentation is given for example in [2]. The particular approach which will be examined in context with the previously stated problems will be the familiar Newton-Raphson iterative technique, or a slight variant thereof, utilizing the concept of the Moore-Penrose Pseudoinverse [3].

The notation to be followed hereafter is largely that of [1], and has the advantage of smoothness in manipulation and interpretation. In this notation, if $\{w_i \mid i = 1,2,\ldots.n\}$ is a family of real-valued functions defined on a region of E^4, and of class C^1 or greater, this defines a <u>transformation</u>, call it τ, from E^4 into E^n. For each p in the common domain of the family $\{w_i \mid i = 1,2,\ldots.n\}$, an n x 4 matrix, called the <u>differential</u> of the transformation (in symbols $d\tau(p)$), is defined by letting row k be given by $\frac{\partial w_k}{\partial x} \mid p$, $\frac{\partial w_k}{\partial y} \mid p$, $\frac{\partial w_k}{\partial z} \mid p$, $\frac{\partial w_k}{\partial t} \mid p$, for each k among $1,2,\ldots.n$.

In other words, $d\tau(p)$ is the matrix whose k^{th} row is the gradient of w_k at p.

If τ is as previously defined, but with $n = 4$, and p' is any zero of τ; that is, $\tau(p')$ is the zero vector in E^4, there is a neighborhood of p' such that for any p_1 in this neighborhood, the sequence $\{p_k \mid k = 1,2,\ldots..\}$ defined for $k = 1,2,\ldots$, by

$$p_{k+1} = p_k - [d\tau(p_k)]^{-1}\ \tau(p_k) \qquad (1)$$

will converge to p' if the determinant of $d\tau(p')$ (in symbols $|\ d\tau(p')\ |$) is nonzero.

A more general scheme, and one which will prove of wider application than the above, may be defined by utilizing the concept of the Moore-Penrose Pseudoinverse. This allows n to be any finite number at least four. Advantages of such a scheme are obvious, but caution must be exercised in the numerical inversion of the involved matrix; as a worsening of the condition of the linear system may occur as n becomes large. Such a situation may be accompanied by computational difficulties. This scheme will be discussed more fully as matters progress.

WAVE GENERATING FUNCTIONS WITH ONE OR MORE PARAMETERS

Let $\Omega(\mathcal{D})$ be the set of all wave generating functions on E^4 having a common domain $\mathcal{D}$. Let A be a region of Euclidean m-space which is compact and connected. Interest will be with a function f whose domain is A and whose range is $\Omega(\mathcal{D})$. The set $f\{A\} = \{f(a)\ |\ a \in A\}$; that is, the range of f will be called the family of wave generating functions with parameters in A. If a is in A and $p \in \mathcal{D}$, the value of $f(a)$ at p will be denoted by $[f(a)](p)$.

The set f[A] will be said to define a family of wave generating functions with continuous parameters if for any a, b in A and any real $\varepsilon > 0$, there exists $\delta > 0$ such that $|\ [f(b)]\ (p) - [f(a)]\ (p)\ | < \varepsilon$ when $||\ b - a\ || < \delta$.

and this condition holds for any $p \in D$. In a similar fashion, one may define wave generating functions with absolutely continuous parameters, of class C^1, C^2, etc., the interpretation being straightforward.

A source point of $f(a)$ may not be a source point of $f(b)$ if $a \neq b$. More attention will be given this important concept later in this discussion where the applications to the sound ranging problem will be clarified.

AN APPLICATION OF THE MOORE-PENROSE PSEUDOINVERSE

Let A be a region of E^m and let $f: A \to \Omega(\mathcal{D})$ define a family of wave generating functions with parameters $a = (a_1, a_2, \ldots a_m) \in A$. It will be assumed that the family $f\{A\}$ is of class C^1 or greater. As has been discussed previously, any member of $f\{A\}$ is a wave generating function, and for a, b in A with $a \neq b$, the source point of $f(a)$ will not in general be equal to that of $f(b)$, and it is possible that such a point fails to exist for either. However, the assumption will now be made for each $a \in A$, $f(a)$ has a source point.

Now suppose for some $a \in A$, a family of n points $\{(x_i, y_i, z_i, t_i) \mid i = 1,2,\ldots.n\}$ exists with $n \geq 4$. Suppose it is further known that if W_i is the wave front at t_i of $f(a)$, (x_i, y_i, z_i) lies in W_i, and that each (x_i, y_i, z_i, t_i) is distinct. One may then define a transformation of E^4 into E^n exactly as done previously, obtaining:

$$T = \begin{cases} f_1(a)(x', y', z', t') \\ f_2(a)(x', y', z', t') \\ \ldots\ldots\ldots \\ f_n(a)(x', y', z', t') \end{cases}$$

Note that the set of points p such that $T(p) = 0$ contains the source point of $f(a)$ which will be called $p_0(a)$.

It is not overly difficult to show that if p_1 is chosen within a sufficiently small neighborhood of $p_0(a)$, the iterative scheme defined by

$$p_{k+1} = p_k - \{[dT(p_k)]^t [dT(p_k)]\}^{-1} [dT(p_k)]^t T(p_k) \qquad (2)$$

will converge to $p_0(a)$ if $dT(p_k)$ is always of rank at least 4. This fact will be of interest in several practical applications and will be of particular interest when a random model is to be considered.

At this time, suppose that $n \geq m + 4$, and that $f\{A\}$ is of class C^1 with respect to the parameters $(a_1, a_2, \ldots. a_n) \in A$. Consider the transformation from $(m + 4)$-space to n-space defined by

$$T: \begin{cases} f_1(a)(x,y,z,t) \\ f_2(a)(x,y,z,t) \\ \cdots\cdots\cdots \\ f_n(a)(x,y,z,t) \end{cases} \qquad (3)$$

The iterative procedure defined by choosing $p_1 = (x'_1, y'_1, z'_1, t'_1, a'_1, a'_2, \ldots. a'_m)$ and for $k > 1, p'_k$ by

$$p'_{k+1} = p'_k - \{[dT'(p'_k)]^t [dT'(p'_k)]\}^{-1} [dT'(p'_k)]^t T'(p'_k) \qquad (4)$$

will converge to $p_0' = (x_0,y_0,z_0,t_0,a_1,a_2,\ldots a_m)$ if $\| p_k' - p_0' \|_{m+4}$ is sufficiently small and $d\tau'$ is of rank m + 4 at the source point of $f(a)$. Note what this states. If f{A} is a family of extended wave generating functions with parameters $(a_1,a_2,\ldots a_n) \varepsilon A$, one may determine the source point of $f(a)$ and the parameters $(a_1,a_2,\ldots a_n)$ from knowledge of points and the wave fronts of adjoining or nearby wave generating functions. In a physical situation, it means that if sufficient devices are placed in the field to determine necessary arrival times, these times may contain information about parameter sets describing the family of extended wave generating functions, as well as information concerning the source point.

Applications of the Moore-Penrose Pseudoinverse will prove to be of special interest when one considers various random models for field application, and vulnerability is a factor to be considered. It might be noted that in the case that n = 4, expressions (1) and (2) coincide, and if n = (m + 4), expressions (1) and (4) coincide. More specific information can be derived if a particular example is considered for f{A}, which leads into the following section.

SOUND RANGING: A SIMPLE MODEL AND SOME RAMIFICATIONS

Consider the following highly idealized situation. A sound source is located at some point in a region of space over which is specified a rectangular Cartesian coordinate system (X,Y,Z). It will be assumed that a steady wind of velocity (u,v,q) is blowing throughout the region of interest, and that the temperature is a constant k degrees Kelvin. A candidate function for describing the evolution of sound wave fronts has been suggested by some authors, for example [4], as

$$[(x - x_0) - u(t - t_0)]^2 + [(y - y_0) - v(t - t_0)]^2$$
$$+ [(z - z_0) - q(t - t_0)]^2 - c^2k(t - t_0)^2 \equiv w(x,y,z,t) \quad (5)$$

where c is of constant value.

If the region concerned is a compact (closed and bounded) connected region of E^4; w, as defined by (5), is an extended wave generating function. In fact the following theorem may be proved.

Theorem 3. Let w be as given by condition (5) and be defined on a compact connected region of E^4 containing the point (x_0,y_0,z_0,t_0). Then w is an extended wave generating function with source point (x_0,y_0,z_0,t_0). If $\sqrt{u^2 + v^2 + q^2} \leq c\sqrt{k}$; that is, if the windspeed is less than Mach 1; and in addition, t_0 is the greatest lower bound of T, then w generates a family of wave fronts expanding about (x_0,y_0,z_0). If windspeed is less than Mach 1, the family of wave fronts is strictly expanding.

Proof. Since by hypothesis, w is defined on a compact connected region of E^4, the fact that w is an extended wave generating function is a matter of routine. It is clear from the geometry that any linear trace whose origin is at (x_0,y_0,z_0) must intersect the spherical surface w_t for any $t \in T$ such that $t > t_0$, and that any sequence of times in T have property b of Definition 3. Hence (x_0,y_0,z_0,t_0) is the source point of w. The remaining verifications are equally straightforward and are left to the reader.

This extended wave generating function may be visualized as giving rise to a family of undeformed, concentric spherical wave fronts propagating outward at a rate of $c\sqrt{k}$, and being translated along (u,v,q) with a speed of $\sqrt{u^2 + v^2 + q^2}$.

As has been mentioned, this is not a complex, or even a particularly realistic "real world," model; and many physical phenomena which are normally encountered are not included. However, there remains an adequate amount of information to warrant its careful examination. In addition, in such an uncomplicated model, one is not overly hampered by problems of notation; and some degree of intuitive flavor is retained which could become lost if a more complicated model were chosen.

For the purposes of this presentation, all quantities will be assumed deterministic, that is, not random in nature. The ramifications of probabilistic models of varying complexity are of considerable interest, and sufficient progress has already been made to assure that this is a highly productive area, with certain elegant relationships emerging. However, sufficient results are available from the deterministic model to bring a number of significant facts to the surface.

With w as given by relation (5), it can be seen that for $\{(x_j,y_j,z_j,t_j) \mid j = 1,2,....n\}$, a set of points in $\mathcal{D}$ with

$\{(x_j,y_j,z_j) \mid j = 1,2,\ldots.n\}$ distinct, one may define a family $\{w_j \mid j = 1,2,\ldots.n\}$ as in Theorem 2 by utilizing the transformations

$$\tau_j: \begin{cases} x' = x - (x_j - x_o) \\ y' = y - (y_j - y_o) \\ z' = z - (z_j - z_o) \\ t' = t - (t_j - t_o) \end{cases} \qquad j = 1,2,\ldots.n$$

Substitution in expression (5) yields, for $j = 1,2,\ldots.n$

$$\begin{aligned} w_j(x',y',z',t') &= w[\tau_j^{-1}(x',y',z',t,)] \\ &= [x' + x_j - 2x_o - u(t' + t_j - 2t_o)]^2 \\ &+ [y' + y_j - 2y_o - u(t' + t_j - 2t_o)]^2 \\ &+ [z' + z_j - 2z_o - f(t' + t_j - 2t_o)]^2 \\ &- c^2k[t' + t_j - 2t_o]^2 \end{aligned}$$

Observe that for any j among $1,2,\ldots.n$,

$$\begin{aligned} w_j(x_o,y_o,z_o,t_o) &= [(x_j - x_o) - u(t_j - t_o)]^2 \\ &+ [(y_j - y_o) - v(t_j - t_o)]^2 \\ &+ [(z_j - z_o) - q(t_j - t_o)]^2 \\ &- c^2k[t_j - t_o]^2 \\ &= 0 \end{aligned}$$

This may be derived through algebraic manipulation bearing in mind the fact that (x_j,y_j,z_j) lies on the t_j wave front of w for each j among $1,2,\ldots.n$.

Now suppose that $n \geq 4$, and that the parameters u, v, q, k are known. Consider the transformation τ from E^4 into E^n defined by

$$\tau: \begin{cases} w_1(x,y,z,t) \\ w_2(x,y,z,t) \\ \cdots\cdots \\ w_n(x,y,z,t) \end{cases} \tag{6}$$

The differential of τ at the point $p_o = (x_o, y_o, z_o, t_o)$ is an $n \times 4$ matrix whose j^{th} row is given by

$$(d\tau(p_o))_{1,j} = 2[(x_j - x_o) - u(t_j - t_o)]$$

$$(d\tau(p_o))_{2,j} = 2[(y_j - y_o) - v(t_j - t_o)]$$

$$(d\tau(p_o))_{3,j} = 2[(z_j - z_o) - q(t_j - t_o)]$$

$$(d\tau(p_o))_{4,j} = -2[u(d\tau(p_o))_{1,j} + v(d\tau(p_o))_{2,j}(t_j - t_o) + q(d\tau(p_o))_{3,j} - c^2k(t_j - t_o)]$$

Recall that the sequence $\{p_n \mid n = 1,2,\ldots.\}$ as defined by (2) will converge to p_o if the matrix $d\tau(p_o)$ is of rank 4, and if p_1 is chosen within a sufficiently small neighborhood of p_o. Consider the following chain of reasoning. The matrix $d\tau(p_o)$ is known to be of unaltered rank if $(d\tau(p_o))_{4,j}$ is replaced by

$$(d\tau(p_o))_{4,j} = (d\tau(p_o))_{4,j} + u(d\tau(p_o))_{1,j} + v(d\tau(p_o))_{2,j} + q(d\tau(p_o))_{3,j} = -2c^2k(t_j - t_o)$$

for each j among $1,2,\ldots.n$. Through a similar reasoning process, one makes the following substitutions

$$(d\tau(p_o))_{1,j} = (d\tau(p_o))_{1,j} - \frac{u}{c^2k}(d\tau(p_o))_{4,j}$$

$$(d\tau(p_o))_{2,j} = (d\tau(p_o))_{2,j} - \frac{v}{c^2k}(d\tau(p_o))_{4,j}$$

$$(d\tau(p_o))_{3,j} = (d\tau(p_o))_{3,j} - \frac{q}{c^2k}(d\tau(p_o))_{4,j}$$

Note that the matrices $d\tau(p_o)$ and $d\tau'(p_o)$ have identical rank.

Finally, the observation can be made that if $d\tau''(p_o)$ is defined as $d\tau''(p_o) = -2^n c^2 k(d\tau'(p_o))$, the rank of $d\tau(p_o))$ and $d\tau''(p_o)$ are also identical, and $d\tau''(p_o)$ may be written as

$$\left.\begin{aligned} (d\tau''(p_o))_{1,j} &= (x_j - x_o) \\ (d\tau''(p_o))_{2,j} &= (y_j - y_o) \\ (d\tau''(p_o))_{3,j} &= (z_j - z_o) \\ (d\tau''(p_o))_{4,j} &= (t_j - t_o) \end{aligned}\right\} \quad j = 1,2,\ldots.n$$

Now suppose that the points $\{(x_j,y_j,z_j,t_j) \mid j = 1,2,\ldots.n\}$ are of such a nature that constants a,b,c,d may be found which are not all zero, and for which

$$a(x_j - x_o) + b(y_j - y_o) + c(z_j - z_o) + d(t_j - t_o) = 0. \qquad (7)$$

It follows that the matrix $d\tau(p_o)$ must be of column rank 3 or less, hence no iterative scheme similar to the one given by (2) can converge to p_o.

A special case of the above which appeals to geometric intuition occurs when d of condition (7) is known to be zero. For this case, condition (7) is equivalent to the statement that all points $\{(x_j,y_j,z_j) \mid j = 1,2,\ldots.n\}$ lie on a single straight line through the point (x_o,y_o,z_o). This condition will be quickly recognized by those with sound ranging experience.

Considering alternately the conditions a = 0, b = 0, c = 0, together with d = 0, it can be shown that $d\tau''(p_0)$ and therefore $d\tau(p_0)$ will be of rank less than 4, if a straight line connecting the proper points can be constructed in the appropriate projection. For example, if a line connecting $\{(x_j, y_j) \mid j = 1,2,\ldots.n\}$ and (x_0, y_0) can be constructed in the (X,Y) plane, and so forth.

It would appear that among all of the various possible configurations for $\{(x_j, y_j, z_j) \mid j = 1,2,\ldots.n\}$, that one which permits the above set of points, or projections thereof, to lie on a straight line, is intrinsically of more than usual risk from a standpoint of source point determinations and information recovery through a scheme such as given by iterative scheme 2.

RECOVERY OF PARAMETERS ASSOCIATED WITH A GIVEN WAVE GENERATING FUNCTION

Consider now the following situation. Let A be a given compact connected region of E^4. Let $a = (a_1, a_2, a_3, a_4)$ be a point in A, and define $f(a)$: $A \to E^1$ by choosing $p = (x,y,z,t) \in \mathcal{D}$, and let

$$\begin{aligned}[][f(a)](p) = {} & [(x - x_0) - a_1(t - t_0)]^2 \\ & + [(y - y_0) - a_2(t - t_0)]^2 \\ & + [(z - z_0) - a_3(t - t_0)]^2 \\ & - c^2 a_4 \, [t - t_0]^2 \end{aligned} \tag{8}$$

Let $\mathcal{D}$ be the domain of the wave generating function given by (5). Then for each $a \in A$, $f(a) \in \Omega\ (\mathcal{D})$, so that f is well defined on the set of wave generating functions with domain $\mathcal{D}$. The family f{A}, then, is a set of wave generating functions in $\mathcal{D}$ with parameters in A. In fact, (5) is a special case of (8) with A = {u,v,q,k}. It is seen from (8) that f{A} is also a family of wave generating functions with parameters of class C^2 over A.

Now let $a' \in A$ be given, and $\{p_j \mid j = 1,2,....n\}$ be a set of points in $\mathcal{D}$. One may define a transformation

$$\Psi : \begin{cases} [f(a_0')]_1(p') \\ \ldots\ldots \\ [f(a')]_n(p') \end{cases}$$

by allowing $[f(a')]_j = [f(a')][\tau_j^{-1}(p')]$ for j = 1,2,....n.

Suppose that $n \geq 8$. If (a_1,p_1) is in a sufficiently small neighborhood of (a_0,p_0), the iterative scheme specified by (4) will converge to (a_0,p_0) if the rank of d $\Psi(p_0)$ is 8 or greater. To determine conditions which may present problems with regards to convergence of the iterative scheme mentioned earlier, procedure will be as follows. Ψ may clearly be considered a transformation of E^8 into E^n of class $\geq C^2$. The differential of this transformation at (a_0,p_0) may be calculated from (8) as in the previous section, obtaining columnwise

$$[d\,\Psi(a',p_o)]_{1,j} = 2[(x_j - x_o) - a_1(t_j - t_o)]$$

$$[d\,\Psi(a',p_o)]_{2,j} = 2[(y_j - y_o) - a_2'(t_j - t_o)]$$

$$[d\,\Psi(a',p_o)]_{3,j} = 2[(z_j - z_o) - a_3'(t_j - t_o)]$$

$$[d\,\Psi(a',p_o)]_{4,j} = -\{2\,a_1'[d\,\Psi(a',p_o)]_{1,j} + a_2'[d\,\Psi(a',p_o)]_{2,j} + a_3'[d\,\Psi(a',p_o)]_{3,j} + c^2 a_4'[t_j - t_o]^1\}$$

$$[d\,\Psi(a',p_o)]_{5,j} = -(t_j - t_o)[d\,\Psi(a',p_o)]_{1,j}$$

$$[d\,\Psi(a',p_o)]_{6,j} = -(t_j - t_o)[d\,\Psi(a',p_o)]_{2,j}$$

$$[d\,\Psi(a',p_o)]_{7,j} = -(t_j - t_o)[d\,\Psi(a',p_o)]_{3,j}$$

$$[d\,\Psi(a',p_o)]_{8,j} = -c^2(t_j - t_o)$$

for $j = 1,2,....n$.

Observe that by reasons similar to those employed in the preceding section, the matrix $d\,\Psi(a',p_o)$ will have rank the same as a matrix in which the substitution

$$[d\,\Psi(a',p_o)]_{4,j} = [d\,\Psi(a',p_o)]_{4,j} + a_1'[d\,\Psi(a',p_o)]_{1,j} + a_2'[d\,\Psi(a',p_o)]_{2,j} + a_3'[d\,\Psi(a',p_o)]_{3,j}$$

$$= -c^2 a_4'(t_j - t_o)$$

has occurred.

One may, after a series of substitutions like those of the preceding section, arrive at a matrix d $\Psi''(a',p_o)$ having a rank identical to d $\Psi(a',p_o)$ but with components given columnwise by

$$[d\,\Psi''(a',p_o)]_{1,j} = (x_j - x_o)$$

$$[d\,\Psi''(a',p_o)]_{2,j} = (y_j - y_o)$$

$$[d\,\Psi''(a',p_o)]_{3,j} = (z_j - z_o)$$

$$[d\,\Psi''(a',p_o)]_{4,j} = (t_j - t_o)$$

$$[d\,\Psi''(a',p_o)]_{5,j} = (t_j - t_o)(x_j - x_o)$$

$$[d\,\Psi''(a',p_o)]_{6,j} = (t_j - t_o)(y_j - y_o)$$

$$[d\,\Psi''(a',p_o)]_{7,j} = (t_j - t_o)(z_j - z_o)$$

$$[d\,\Psi''(a',p_o)]_{8,j} = (t_j - t_o)^2$$

Observe that all remarks of the preceding section concerning $d\tau(p_o)$ may also apply to $\Psi(a',p_o)$, a situation of more ramifications than are at first apparent. In addition, it might be observed that for any choice of $\{p_j \mid j = 1,2,....n\}$ such that $t_j - t_o$ is constant for all j among 1,2,....n, the matrix d $\Psi(a',p_o)$ becomes of rank less than 8, and in particular, less than 4, hence neither of the iterative schemes given by (2) or (4) will converge. Such a situation may occur in any of several low probability situations, depending on the geometry as specified by $\{(x_j,y_j,z_j) \mid j = 1,2,....n\}$, and on the nature of (x_o,y_o,z_o). If, for example, $(a_1',a_2',a_3') = (0,0,0)$ and (x_j,y_j,z_j) are points on the circumference of a circle at whose focal point the point (x_0,y_0,z_0) is located, then d $\Psi(a',p_0)$ is of rank 3. In this case, however, there is but a single point of discontinuity near (x_0,y_0,z_0). It is of interest to note that in this case the condition would hold wherein the failure of a scheme to converge could specify the source point.

It may be observed that if an extended wave generating function is of the type specified by (5) or (8), and a suitably restricted estimate of the parameters in question is available, the solution of the sound ranging problem is equivalent to the transformation d $\Psi(a',p_0)$ being of rank 8.

In practice, restraints on the physical system may come from a primary estimation scheme, or may be specified by the user in the form of a query *"Is there, based on knowledge of* $\{p_j \mid j = 1,2,\ldots,n\}$ *and* a', a sound source near p'^2?" If so, the iterative scheme will converge to the proper value; if not, the scheme will diverge, or converge to the sound source itself.

CURRENT INTERPRETIVE TENDENCIES

In the examination of behavior characteristics exhibited by the proposed theory in a real numerical setting, the authors decided to conform with established practice and consider a six sensor system. This would give rise to at most a system of six nonlinear equations. The reduced system was derived from a more general presentation ("Sound Ranging: A Simple Model and Some Ramifications") by ignoring vertical variations in both microphones, sound source, and wind velocity. The variables examined were at a minimum, the x and y components of sound source position and blast time, and, at a maximum, these variables, together with wind component velocities, and temperature.

The variety of possible behavior patterns is large, even in such an uncomplicated sample as the one presented. Differing microphone configurations, sound source locations, meteorological conditions, and departures in preliminary estimates each give rise to differing but overlapping behavior patterns. More than a thousand numerical examples have been examined for the purpose of clarifying such patterns, and various consistencies have begun to emerge. Some are recognizable at once from preliminary analysis; others are not so apparent. These latter results are useful in developing a more realistic slant toward further analysis, while the former serve to verify the model's validity.

In the particular system of numerical examples, arrival times based on sound source position, microphone positions, and wind and temperature information were calculated to the limits of accuracy of the Hewlett-Packard 9830A. Among the information sought were possible degree of information recovery, number of iterations in order to accomplish recovery, and system reaction to a progressive degree of divergence between "guess" vectors of source points and related atmospheric parameters. Three wind velocity situations were considered: no wind conditions, winds directed toward the centroid of the microphone configuration, and a limited number of arbitrary windspeeds and directions. Results tend to agree with behavior patterns which might be suspected, but some interesting phenomena have occurred - the action of which is more obscure, and which will be of considerable interest to further exploratory analysis. A detailed description of these and more results will be presented in a more visual mode in a subsequent report. For the present situations described (i.e., high level timing accuracy), if no wind was encountered, information recovery was in almost all cases complete, even when extreme departures in estimated temperature, wind velocity, and source point estimates were permitted. In general, the number of iterations to recover this information was four. Recovery was considered complete when estimated and known quantities differed in the hundredths of meters. If arbitrary winds were introduced, a mixing seemed to occur between temperature behavior and wind velocity which indicated that error behavior and stability for the wind and no-wind situations differ considerably. This should not be too difficult to verify from a purely analytic viewpoint, and may possibly require an increase in accuracy for arrival time accuracies when wind is considered.

The greatest problems in total recovery occurred in a purely linear array as predicted. The various other arrays which were examined did not appear to offer any really significant differences in behavior in the limited cases considered.

The documentation of the above-mentioned material will probably be followed by at least two reports: a sensitivity analysis on timing and meteorological errors, and an examination of the differing behavior of contemporary sound ranging techniques employing data derived through simulation. In this manner, sufficient statistical control is available for proper data interpretation.

Note that each sensor configuration requires a certain level in accuracy of arrival times to accomplish recovery either of a complete parameter set or of a source point. For a given configuration, then, knowledge of system behavior may be specified by a set of "level curves" of error values. This will be the approach taken in later studies when the effect of configuration on error behavior is examined. In addition, more general extended wave generating functions will be examined, in both a determinative and random setting. When all ground work has been accomplished, techniques will be compared by use of a simulation model and then with actual data. This should place the techniques as described in the proper functional position in the class of sound ranging schemes.

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